

Asset Pricing and Housing Supply in a Production Economy

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Abstract

The equity premium and the housing risk premium are studied in a production economy with habit formation and building restrictions. When combined with a specification of habit formation in the mix of consumption and leisure, building restrictions provide an explanation for the high volatility of house prices and contribute to the resolution of asset pricing puzzles. A high equity premium, a low risk-free rate, and a significant difference between the equity and the housing risk premium can be generated in a two-sector model with endogenous labor supply, investment, and one single source of exogenous shocks.

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1 Introduction

A considerable amount of empirical evidence suggests that housing supply is constrained and that these building restrictions have a substantial impact on the dynamics of house prices (e.g. Glaeser and Gyourko 2003, Green, Malpezzi and Mayo 2005, Quigley and Raphael 2005). Glaeser, Gyourko and Saiz (2008) study the impact of the elasticity of housing supply on the dynamics of house prices and show that, in general, areas with more inelastic supply experience larger increases in prices and much smaller increases in new construction.

Glaeser, Gyourko and Saks (2005a, 2005b) for instance present evidence that, in the United States, rising house prices have been accompanied by reductions in residential developments, and that regulation is constraining the supply of housing¹. Moreover, according to them: "Changes in housing-supply regulations may be the most important transformation that has happened in the American housing market since the development of the automobile, but this change is both under-studied and underdebated."

Given that housing represents seventeen percent of total consumption expenditures, the first main objective of this paper is to study the asset pricing and the business cycle implications of building restrictions. First, by impairing households' ability to smooth their consumption of housing services over time, building restrictions could potentially affect agents' discount factors and may therefore contribute to the resolution of asset pricing anomalies. Second, building restrictions reduce the elasticity of housing supply and are therefore likely to affect the volatility of housing market variables. Finally, restrictions on housing-supply could have feedback effects on the rest of the economy by distorting the allocation of capital and labor across sectors.

The second main objective of this paper is to assess whether a macro-housing model with habit formation and building restrictions could explain the significant difference between the equity and the housing risk premium observed in the data. While the puzzling equity premium has attracted a lot of attention, very few studies have attempted to jointly explain these facts in a model where residential rents and dividends are endogenously determined.

This paper contributes to fill these gaps by studying the questions: What are the asset pricing and the business cycle implications of building restric-

¹Ortalo-Magne and Prat (2007) study the determinants of housing supply regulation. See Di Pasquale (1999) for a review of the housing supply literature and Leung (2004) for a review of the macro-finance/housing literature.

tions? Can a macro-housing model explain asset market facts, and in particular the equity and the housing risk premiums? And is it possible to explain these asset market facts in a model also able to capture the main business cycle regularities?

To study these questions, a two-sector general equilibrium model where new homes are produced by a housing sector is developed. As in Davis and Heathcote (2005) (e.g. Iacoviello and Neri 2009), the housing sector uses labor and residential capital to produce new homes and housing services are determined by the stock of housing which has been accumulated over the years. The non-housing sector is standard and produces a final output good which can be divided between non-housing consumption and investment.

Compared to Davis and Heathcote (2005), the first important difference is that building restrictions are introduced into the analysis. As far as the nature of these restrictions are concerned, we base our approach on the findings reported by Glaeser, Gyourko and Saks (2005b) which emphasize the importance of regulation in constraining large-scale development. To keep the analysis tractable, this aspect is captured by introducing an adjustment cost on housing supply which makes large projects more expensive to implement than smaller ones.

As in a standard model, house prices are determined by an asset pricing formula equalizing the cost of buying a house today with the expected discounted pay-off, where future pay-offs depend on rents and on price appreciation. In the presence of building restrictions, the difference is that the textbook formula has to be adjusted to take into account the effects of supply constraints on the valuation.

The second key difference is that the model is augmented with habit formation (e.g. Abel 1990, Constantinides 1990, Campbell and Cochrane 1999). A crucial assumption is that habits are formed over a mix of total consumption and leisure where total consumption is an aggregate of housing and non-housing consumption. This assumption aims at capturing the idea that agents get hooked to a certain mix of consumption, housing and leisure reflecting their standards of living. This particular specification which only introduces one additional parameter to calibrate considerably improves the asset pricing predictions of standard macro-housing models (e.g. Jaccard 2009b).

Compared to Piazzesi, Schneider and Tuzel (2007), in our study, the main difference is that the impact of housing crucially depends on the extent to which housing supply is constrained. Building restrictions increase the cost

of adjusting the housing stock and make consumption of housing services smoothing more difficult to achieve. With habit formation, these greater fluctuations in the consumption of housing services lead to an increase in the volatility of the stochastic discount factor which makes future pay-offs more uncertain. This increase in uncertainty, which affects asset pricing decisions, has to be compensated by higher risk premiums and enhances the model's ability to resolve asset pricing puzzles.

As far as the dynamics of house prices is concerned, in a model calibrated to match the volatility of residential investment, we find that building restrictions lead to a substantial increase in the volatility of house prices. This result seems in line with the empirical facts reported by Glaeser, Gyourko and Saiz (2008) which emphasize the importance housing supply elasticities in explaining house price dynamics. Our analysis also suggests that, while housing-supply regulation is essential, demand factors are likely to play an important role². Even with high building restrictions, it would be considerably more difficult to explain the high volatility of house prices in a model without habit formation.

Finally, these quantitative implications essentially rely on the model's ability to generate costly business cycle fluctuations (e.g. Alvarez and Jermann 2004, Barro 2006, Gourio 2009) and empirically plausible risk premiums. In such an environment, the role played by housing-supply restrictions very much depends on the amount of risk generated by the model. In highly uncertain environments, housing provides a valuable insurance because of its gradual response to unexpected shocks. This favorable cyclical property creates a precautionary demand for housing services whose intensity depends on uncertainty, and therefore on the model's ability to explain high risk premiums. In such an environment, we find that housing-supply restrictions have important quantitative implications because they reduce agents' ability to use housing as a hedge against business cycle fluctuations.

Section 2 presents a set of stylized facts for the United States. The theoretical environment is developed in section 3 and the asset pricing implications of the model are discussed in section 4. The calibration is presented in section 5 and the results are discussed in section 6. Section 7 concludes.

²Van Nieuwerburgh and Weill (2010) also attribute the increase in house price dispersion to a combination of supply and demand factors. According to their findings, while housing-supply regulation is important, the increase in wage dispersion is an essential part of the explanation.

2 Data description

Empirical facts describing the volatility and the cyclical of the business cycle and the asset pricing variables under study are reported in Table 1 and have been computed using quarterly data. All the variables have been expressed in logs and the cyclical component has been extracted using a HP-filter.

Following the literature on the equity premium puzzle, Table 2 reports the mean and standard deviation of equity and housing returns. The financial statistics presented in Table 2 have been expressed in annualized percent.

Table 1: Volatility and correlation

Total output (HP-filter 1947-2009)			
	σ_{y_T}	$\rho(y_{Tt}, y_{Tt-1})$	$\rho(y_{Tt}, y_{Tt-4})$
y_T	1.68	0.81	0.19
Business cycle statistics (HP-filter 1947-2009)			
	σ_{x_i/y_T}	$\rho(x_{it}, y_{Tt})$	$\rho(x_{it}, x_{it-1})$
c	0.49	0.79	0.83
i_T	3.37	0.82	0.82
y_H	5.93	0.53	0.88
n_T	0.98	0.83	0.90
n_H	2.23	0.76	0.90
w_B	0.57	0.19	0.66
w_H	0.85	-0.16	0.64
Asset prices (HP-filter 1987-2009)			
	σ_{x_i/y_T}	$\rho(x_{it}, y_{Tt})$	$\rho(x_{it}, x_{it-1})$
d	25.3	0.80	0.78
p_E	10.6	0.65	0.83
p_H	4.01	0.57	0.91
z_H	0.43	-0.03	0.78

In Table 1, the volatility of total output is denoted σ_{y_T} , and the relative standard deviation of variable x_i with respect to output is denoted $\sigma_{x_i}/\sigma_{y_T}$. The correlation of variable x_i with respect to output is denoted $\rho(x_{it}, y_{Tt})$

while $\rho(x_{it}, x_{it-1})$ denotes the first-order autocorrelation of variable x_i . Market consumption, which in the data corresponds to real consumption of services and of non-durables goods, is denoted c . Business investment, which corresponds to investment in equipment and software, is denoted i_T while residential investment is denoted y_H . Total hours worked are denoted n_T and correspond to a measure of total employment. Wages in the business sector are denoted w_B and correspond to a measure of real compensation per hour in the nonfarm business sector. Finally, employment and real wages in the housing sector are respectively denoted n_H and w_H , and correspond to a measure of total employment and earnings in the construction sector. Real wages in the construction sector have been deflated using CPI inflation. All eight series are taken from the online database of the St-Louis Fed.

House prices, equity prices and total earnings are denoted p_H, p_E and d . The Case-Shiller index is used as a proxy for house prices, and equity prices and earnings are taken from the online database of Robert Shiller. Compared to other house price indices, the advantage of the Case-Shiller index is that it includes transaction prices which are based on a wider range of mortgage contracts³. Residential rents are denoted, z_H , and are proxied using the housing component of the CPI index.

In Table 2, the equity premium and the housing risk premium are respectively denoted $E(r_M - r_f)$ and $E(r_H - r_f)$, where r_f is the real risk-free rate. The volatility of equity returns, of housing returns and of the risk-free rate are respectively denoted $\sigma(r_M)$, $\sigma(r_H)$ and $\sigma(r_f)$. Finally, the first order autocorrelation of equity returns, housing returns and of the risk-free rate is denoted $\rho(r_{Mt}, r_{Mt-1})$, $\rho(r_{Ht}, r_{Ht-1})$ and $\rho(r_{ft}, r_{ft-1})$. These financial statistics are taken from the study of Piazzesi, Schneider and Tuzel (2007).

Table 2: Financial returns (Piazzesi, Schneider and Tuzel 2007)

Mean		
$E(r_M - r_f)$	$E(r_H - r_f)$	$E(r_f)$
6.19	1.77	0.75

³To be able to compare the volatility and standard deviation of house prices and equity prices, the sample is restricted to the period 1987-2009, since the Case-Shiller index is not available before 1987. Over the restricted sample 1987-2009, the standard deviation of equity price, σ_{p_E} , is 11.7 and the volatility of output, σ_{y_T} , is 1.1. Over the whole sample 1871-2009, the standard deviation of equity price, σ_{p_E} , is 12.1.

Standard deviation		
$\sigma(r_M)$	$\sigma(r_H)$	$\sigma(r_f)$
16.56	2.73	3.68
Autocorrelation		
$\rho(r_{Mt}, r_{Mt-1})$	$\rho(r_{Ht}, r_{Ht-1})$	$\rho(r_{ft}, r_{ft-1})$
-0.06	0.48	0.73

3 The model

The representative firm is composed of two sectors which use capital and labor as factors of production. The business sector produces a standard final output good, which can be divided between consumption and investment, while new homes are produced by a housing sector. The housing sector generates a revenue from renting the stock of new homes that is produced to the representative household. The economy is subject to a single source of exogenous disturbances which takes the form of random shocks to total factor productivity. The specification of preferences and technology is compatible with balanced growth. The deterministic growth rate at which the economy is growing along the balanced growth path is denoted γ .

3.1 The firm

In each period, the representative firm has to decide how much labor to hire in each sector, how much to invest, and how to allocate capital across the two sectors. Managers maximize the value of the firm which is equal to the present discounted value of all current and future expected cash flows:

$$E_t \sum_{k=0}^{\infty} \beta^{*k} \frac{\lambda_{t+k}}{\lambda_t} d_{t+k} \quad (1)$$

with $\beta^{*k} / \frac{\lambda_{t+k}}{\lambda_t}$ being the discount factor of the representative agents who is the owner of the firm, and where dividends are given by:

$$d_t = A_t k_{Bt}^{\alpha} n_{Bt}^{1-\alpha} + z_{Ht} h_t + p_{CBt} b_{t+1} - w_{Bt} n_{Bt} - w_{Ht} n_{Ht} - i_{Tt} - b_t - G_t \quad (2)$$

As far as the business sector is concerned, the capital stock used to produce the final output good is k_{Bt} , n_{Bt} , is the quantity of labor input, w_{Bt} , is

the wage rate, and the stochastic total factor productivity level is denoted A_t . The real estate activity of the representative firm generates a revenue from renting the existing housing stock to the household. The housing stock and the rental rate are denoted h_t and z_{Ht} . The quantity of labor input needed to produce new homes is denoted n_{Ht} , and the wage rate is w_{Ht} . Capital accumulation is financed via retained earnings and total investment is denoted i_{Tt} . Production, rental income, labor costs and investment determine the component of dividends related to operating profits.

Leverage

In practice, borrowing is an important source of financing and the majority of firms finance part of their activity through debt. To capture the impact of leverage on the dynamics of dividends, borrowing is introduced by assuming that firms can issue a short-term corporate bond, b_t . The price of the corporate bond is denoted p_{CBt} .

The capital structure of the firm is chosen by managers who can use debt to reduce the tax bill of the firm. To keep the analysis simple, we assume that the government levy a lump sum tax, which is denoted G_t , and that the total amount of tax is composed of two components:

$$G_t = f_t - \tau(b_{t+1}) \quad (3)$$

The first component, f_t , is independent of the firm's capital structure and is set by the government while the tax advantage provided by debt is captured by assuming that the rebate, $\tau(b_{t+1})$, is an increasing function of b_{t+1} , with $\tau'(b_{t+1}) > 0$ and $\tau''(b_{t+1}) < 0$.

Production of new homes

The evolution of the stock of housing which can be rented to the household depends on the amount of new homes produced each period. The production function of new homes has the standard Cobb-Douglas characterization:

$$y_{Ht} = k_{Ht}^\varphi n_{Ht}^{1-\varphi} \quad (4)$$

where k_{Ht} is the capital stock. Compared to Davis and Heathcote (2005), a production structure which abstracts from land is considered in order to simplify the analysis. New homes and residential investment are therefore equivalent in our economy.

Housing-supply regulation is captured by introducing an adjustment cost affecting the housing stock and the law of motion characterizing its evolution is given by:

$$(1 - \delta_H)h_t + \phi\left(\frac{y_{Ht}}{h_t}\right)h_t = \gamma h_{t+1} \quad (5)$$

where δ_H is the depreciation rate of the housing stock, and where $\phi\left(\frac{y_{Ht}}{h_t}\right)$ is the adjustment cost function. To keep the analysis as general as possible, the type of adjustment costs used by Jermann (1998) in the context of asset pricing models is adopted⁴. The adjustment cost which is a function of the new homes to housing stock ratio, $\frac{y_{Ht}}{h_t}$, is denoted $\phi()$. Concavity of the function $\phi()$ captures the idea that changing the housing stock rapidly is more costly than changing it slowly⁵. To reduce the number of free parameters, $\phi()$ is parametrized to ensure that the model with and without adjustment costs are similar. Housing-supply regulation can therefore be summarized by one single elasticity parameter, ϵ_H , capturing the curvature of the adjustment cost function⁶:

$$\epsilon_H = \frac{\phi''\left(\frac{y_H}{h}\right)\frac{y_H}{h}}{\phi'\left(\frac{y_H}{h}\right)}$$

Our specification which abstracts from land would be consistent with the facts reported by Glaeser, Gyourko and Saks (2005a, 2005b) suggesting that these constraints are not caused by a declining availability of land but are rather the result of housing-supply regulation. The above specification which increases the cost of large projects, as measured by changes in y_{Ht}/h_t , aims at capturing that housing-supply regulation makes large-scale developments more costly to implement.

⁴Lucas and Prescott (1971), Hayashi (1982) and Baxter and Crucini (1993) also study models with similar types of adjustment costs.

⁵Near the steady state, we have that:

$$\phi\left(\frac{y_H}{h}\right) > 0, \quad \phi'\left(\frac{y_H}{h}\right) > 0 \quad \text{and} \quad \phi''\left(\frac{y_H}{h}\right) < 0$$

⁶The case $1/\epsilon_H = \infty$ corresponds to a model without adjustment costs while the case $1/\epsilon_H = 0$ corresponds to a specification with infinite adjustment costs.

Capital accumulation and its allocation across sectors

Capital accumulation which is determined by the firm's investment policy and the intraperiod allocation of total capital across sectors are two distinct decisions. In addition to the amount devoted to capital accumulation, managers have to decide how to allocate the total stock of capital, k_{Tt} , between the housing and the business sectors, where:

$$k_{Tt} = k_{Bt} + k_{Ht} \quad (6)$$

As shown by Jermann (1998), production economy models with investment and habit formation cannot generate plausible asset pricing predictions without capital adjustment costs. While capital adjustment costs also play a key role in our study, as we will show in the next section, a model with adjustment costs but without housing-supply restrictions would fail on several key dimensions.

To keep the analysis as simple as possible, we assume that the accumulation of capital is subject to the same type of adjustment costs as housing. The firm's capital stock obeys an intertemporal accumulation equation which is given by:

$$\gamma k_{Tt+1} = (1 - \delta_K)k_{Tt} + \phi\left(\frac{i_{Tt}}{k_{Tt}}\right)k_{Tt} \quad (7)$$

where the cost of adjusting the capital stock depends on the elasticity parameter, ϵ_I :

$$\epsilon_I = \frac{\phi''\left(\frac{i_T}{k_T}\right)\frac{i_T}{k_T}}{\phi'\left(\frac{i_T}{k_T}\right)}$$

and where δ_K is the depreciation rate of capital.

Our mechanism should be robust to a more realistic specification of capital adjustment costs. As shown by Tuzel (2009) for instance, introducing asymmetric adjustment costs generally contributes to increase risk premiums. This result suggests that assuming costly reversibility could help to further reduce the potential for intertemporal smoothing. The mechanism under study should therefore be robust to a more realistic specification of adjustment costs.

Each period, managers choose $k_{T+1}, h_{t+1}, b_{t+1}, n_{Ht}, n_{Bt}, i_{Tt}, k_{Bt}$ to maximize their objective, (1), subject to the constraints (2) to (7).

$$\begin{aligned}
L = E_t \bigg\{ & \sum_{k=0}^{\infty} \beta^{*k} \frac{\lambda_{t+k}}{\lambda_t} \left[A_{t+k} k_{Bt+k}^{\alpha} n_{Bt+k}^{1-\alpha} + z_{Ht+k} h_{t+k} + p_{CBt+k} b_{t+k+1} \right. \\
& - w_{Bt+k} n_{Bt+k} - w_{Ht+k} n_{Ht+k} - i_{Tt+k} - b_{t+k} - [f_{t+k} - \tau(b_{t+k+1})] \\
& + p_{Ht+k} \left((1 - \delta_H) h_{t+k} + \phi \left(\frac{(k_{Tt+k} - k_{Bt+k})^{\varphi} n_{Ht+k}^{1-\varphi}}{h_{t+k}} \right) h_{t+k} - \gamma h_{t+k+1} \right) \\
& \left. + q_{Tt+k} \left((1 - \delta_K) k_{Tt+k} + \phi \left(\frac{i_{Tt+k}}{k_{Tt+k}} \right) k_{Tt+k} - \gamma k_{Tt+k+1} \right) \right] \bigg\}
\end{aligned}$$

where house prices and Tobin's Q are respectively denoted p_{Ht} and q_{Tt} .

3.2 Households

In this economy, utility is derived from consuming a market consumption good, c_t , from enjoying leisure, l_t , and from the housing stock which has been accumulated over time, h_t . As far as preferences are concerned, the key assumption is that habits are formed over the mix of total consumption and leisure (e.g. Jaccard 2009). The reference level or habit stock is denoted, x_t , and lifetime utility is given by:

$$U = E_t \left\{ \sum_{k=0}^{\infty} \beta^{*k} \frac{1}{1-\sigma} \left[c_{t+k}^{\kappa} h_{t+k}^{1-\kappa} v(l_{t+k}) - x_{t+k} \right]^{1-\sigma} \right\} \quad (8)$$

Net utility is given by the difference between the composite good, $c_t^{\kappa} h_t^{1-\kappa} v(l_t)$, and the reference level, x_t . The modified discount factor and the coefficient of relative risk aversion are respectively denoted⁷ β^* and σ . As in Constantinides (1990), the evolution of the habit stock is governed by a law of motion which allows for memory effects:

$$\gamma x_{t+1} = m x_t + (1 - m) c_t^{\kappa} h_t^{1-\kappa} v(l_t) \quad (9)$$

⁷where $\beta^* = \tilde{\beta} \gamma^{1-\sigma}$

where m captures the rate at which the habit stock depreciates. To restrict the number of degrees of freedom, we assume that the parameter measuring the impact of $c_t^\kappa h_t^{1-\kappa} v(l_t)$ on the habit stock is given by $1 - m$. Compared to a macro-housing model (e.g. Davis and Heathcote 2005), this specification of habit formation therefore only adds one free parameter.

The representative household faces the following sequential budget constraint:

$$w_{Bt}n_{Bt} + w_{Ht}n_{Ht} + s_t d_t + b_t + T_t = z_{Ht}h_t + c_t + p_{Et}(s_{t+1} - s_t) + p_{CBt}b_{t+1} \quad (10)$$

where equity prices are denoted p_{Et} , s_t is equity holding and, T_t , is a lump sum transfer received from the government. As far as the allocation of time is concerned, households decide how to divide their time endowment between leisure activities, hours worked in the business sector and hours worked in the housing sector. Normalizing the total time endowment to 1, we have that:

$$n_{Tt} + l_t = 1$$

where:

$$n_{Tt} = n_{Bt} + n_{Ht} \quad (11)$$

and where n_{Tt} , n_{Bt} and n_{Ht} respectively denote the total number of hours worked, hours worked in the market sector, and hours worked in the housing sector.

Households decide how many corporate bonds to purchase from the representative firm. New bond purchases are denoted $p_{CBt}b_{t+1}$ and the coupon paid by the corporate bond is normalized to one. To pin down the capital structure of the firm, we assume that the tax advantage provided by debt creates an agency problem between lenders and borrowers. Household are not able to directly assess the solvency of the firm but the level of assets own by the firm is observable. Because of imperfect monitoring, households are never willing to hold more than a certain level of corporate bonds which depends on the firm's total asset base, $a_{t+1} = k_{t+1} + h_{t+1}$, and which is taken as given. This agency problem is captured by introducing the following solvency constraint into the optimization problem of the household:

$$b_{t+1} \leq \xi a_{t+1} \quad (12)$$

where ξ is the leverage ratio⁸.

Each period household choose $c_t, h_t, n_{Bt}, n_{Ht}, s_{t+1}, b_{t+1}$ and x_{t+1} so as to maximize lifetime utility, subject to the constraints (9) to (12). The Lagrangian for this problem can be written as:

$$\begin{aligned}
L = & E_t \left\{ \sum_{k=0}^{\infty} \beta^{*k} \frac{1}{1-\sigma} \left[c_{t+k}^{\kappa} h_{t+k}^{1-\kappa} v(l_{t+k}) - x_{t+k} \right]^{1-\sigma} \right. \\
& + \sum_{t=0}^{\infty} \beta^{*t} \lambda_{t+k} \left[w_{Bt+k} n_{Bt+k} + w_{Ht+k} n_{Ht+k} + s_{t+k} d_{t+k} + b_{t+k} + T_{t+k} \right. \\
& \quad \left. - z_{Ht+k} h_{t+k} - c_{t+k} - p_{Et+k} (s_{t+k+1} - s_{t+k}) - p_{CBt+k} b_{t+k+1} \right] \\
& + \sum_{t=0}^{\infty} \beta^{*t} \psi_{t+k} \left[m x_{t+k} + (1-m) \left[c_{t+k}^{\kappa} h_{t+k}^{1-\kappa} v(l_{t+k}) \right] - \gamma x_{t+k+1} \right] \\
& \quad \left. \left. + \sum_{t=0}^{\infty} \beta^{*t} \varphi_{t+k} \left[\xi a_{t+k+1} - b_{t+k+1} \right] \right\}
\end{aligned}$$

where marginal utility and the Lagrange multiplier attached to the constraints (9) and (12) are denoted λ_{t+k} , ψ_{t+k} and φ_{t+k} .

3.3 Government

To avoid introducing any additional source of shocks, we assume that governments follow a simple rule and that taxes are set according to:

$$G_t = \Omega d_t$$

So that the total tax bill paid by the firm is always equal to a fraction Ω of dividends. To keep the fiscal aspect as simple as possible, we further assume that the government is able to levy lump sum taxes and that it keeps a

⁸In the growing economy the solvency constraint is $\Gamma_t \tilde{b}_{t+1} \leq \tilde{\xi} [\tilde{k}_{T+1} + \tilde{h}_{t+1}]$ where the deterministic growth rate of the economy is given by $\gamma = \Gamma_{t+1}/\Gamma_t$. In the detrended economy, we have $b_{t+1} \leq \xi [k_{T+1} + h_{t+1}]$ where the leverage ratio $\xi = \tilde{\xi}/\gamma$ has been adjusted for growth.

balanced budget. Under this set of simplifying assumptions, the government budget constraint is given by:

$$G_t = T_t$$

3.4 Market equilibrium

An equilibrium is a set of prices $\{\lambda_t, \psi_t, \varphi_t, z_{Ht}, p_{Et}, q_T, p_{Ht}, w_{Bt}, w_{Ht}, p_{CBt}\}$ for all possible states and for all $t \geq 0$ such that when households and firms maximize utility and profit taking these prices as given, all markets clear, and the government budget constraint is satisfied. Market clearing for the consumption/investment good market implies that all produced good are either consumed or invested:

$$y_{Bt} = c_t + i_{Tt}$$

Labor supply equals labor demand, the quantity of housing stock produced equals the quantity rented by the households, and the quantity of corporate debt issued by the firm is equal to the amount demanded by the household. Finally, financial market equilibrium requires that the investors hold all outstanding equity shares.

4 Asset pricing implications

The dynamics of equity prices can be characterized by deriving the first-order conditions of the household problem. The standard intertemporal arbitrage equation where the cost of buying the asset today and tomorrow's expected future gains have to be equalized can be derived from the maximization problem⁹:

$$p_{Et} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} [d_{t+1} + p_{Et+1}] \quad (13)$$

Equity returns are given by the standard definition:

$$r_{Mt,t+1} = \frac{p_{Et+1} + d_{t+1}}{p_{Et}}$$

⁹See equation (30) in the appendix

House prices which can be derived from the first-order conditions of the firms can be characterized by a similar intertemporal arbitrage equation linking prices to fundamentals¹⁰:

$$p_{Ht} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [(1 - \delta_H + \omega_{t+1}) p_{Ht+1} + z_{Ht+1}]$$

where:

$$\omega_{t+1} = \phi \left(\frac{y_{Ht+1}}{h_{t+1}} \right) - \phi' \left(\frac{y_{Ht+1}}{h_{t+1}} \right) \frac{y_{Ht+1}}{h_{t+1}}$$

and where $\beta = \tilde{\beta} \gamma^{-\sigma}$. Compared to equity prices, the fact that the housing stock depreciates and that the accumulation of housing is subject to building restrictions creates a wedge which shows up in the asset pricing formula. Building restrictions and physical depreciation affects the dynamics of house prices through the capital gain component of the valuation. The term ω_{t+1} captures the impact of regulation on the accumulation of the housing stock. Compared to a financial asset, the capital gain component also has to be adjusted for the fact that the housing stock depreciates at rate δ_H .

The payoff from increasing the stock of housing is determined by the rental rate, z_{Ht} which is given by the following ratio of marginal utilities:

$$z_{Ht} = \frac{U_h(c_t, h_t, L_t)}{U_c(c_t, h_t, L_t)}$$

where U_c and U_h respectively denote the marginal utility of market consumption and of the housing stock. With the specification of preferences that has been adopted, this expression reduces to:

$$z_{Ht} = \frac{(1 - \kappa)}{\kappa} \frac{c_t}{h_t}$$

Most of the empirical literature studying the determinants of housing returns usually abstracts from the direct impact of building restrictions the return of housing investments (e.g. Piazzesi, Schneider and Tuzel 2007, Flavin and Yamashita 2002). To ensure consistency between the model implications and the empirical facts, we adopt a standard definition and define housing returns, $r_{H,t+1}$, as:

¹⁰See equation (19) in the appendix

$$r_{Ht,t+1} = \frac{(1 - \delta_H)p_{Ht+1} + z_{Ht+1}}{p_{Ht}}$$

The equilibrium level of corporate debt is pinned down by the following equilibrium condition which can be derived using equations (20) and (29) in the appendix:

$$\frac{\varphi_t}{\lambda_t} = \tau'(b_{t+1})$$

where the price of corporate bonds is given by:

$$p_{CBt} + \frac{\varphi_t}{\lambda_t} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t}$$

The assumption that $\tau'(b_{t+1}) > 0$ and $\tau''(b_{t+1}) < 0$ ensures that firms are always willing to borrow as much as possible and therefore that the inequality constraint $b_{t+1} \leq \xi a_{t+1}$ is always strictly binding.

By contrast, the price of a zero net supply risk-free bond would be given by:

$$p_{RFt} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t}$$

where $1 + r_{ft} = 1/p_{RFt}$ is the risk-free rate. The agency problem between the firm and the household therefore introduces a premium between the price of corporate bonds and the price of risk-free bonds.

5 Parameter selection

The parameter selection is carried out in two steps. A first set of parameters is chosen based on National Income Account data, following the standard in the business cycle literature. A second set of parameters, for which a priori knowledge is weak, is chosen to maximize the model's ability to replicate a set of asset pricing moments of interest, namely the equity premium and the risk-free rate.

Preference parameters

With internal habit formation, the steady state coefficient of relative risk aversion is independent of the habit parameter, m , and is exactly equal to the

curvature parameter σ (e.g. Jaccard 2009). To ensure that the conclusion of this study do not rely on an implausible curvature coefficient, we pick a conservative value and set σ to 3, as suggested by Kocherlakota (1996).

To maximize the model's ability to explain the low mean risk-free rate, the subjective discount factor β^* is set to 0.99, which is a standard value used in the literature (e.g. Jermann 1998). Setting β^* to 0.99 implies an annualized deterministic risk-free rate of about 4.8%.

Market sector and growth rate

The quarterly trend growth rate γ is set to 1.005 and the constant capital share in the Cobb-Douglas production function, α , is 0.36. These are standard values used in the literature. Following Davis and Heathcote (2005), the depreciation rate of business capital, δ_K , is set to 0.0136.

Housing sector

Following Davis and Heathcote (2005), the depreciation rate of the housing stock, δ_H , is set to 0.0035. Given that our production function of new homes depends on labor and residential capital and that land is excluded from the analysis, the estimated values reported by Jin and Zeng (2004) are used, and the residential capital share φ is set to 0.13.

Housing and non-housing consumption share

With Cobb-Douglas, the expenditure share of non-housing consumption is given by the utility weight, κ :

$$\kappa = \frac{c_t}{z_{Ht}h_t + c_t}$$

The empirical evidence presented in Piazzesi, Schneider and Tuzel (2007) can therefore be used to calibrate κ using data on the expenditure share of housing and non-housing consumption. Following their empirical findings, we set κ to 0.826, which implies that housing consumption represents about seventeen percent of total consumption.

Hours worked

Evidence from the 2008 time of use survey is firstly used to calibrate the steady state value of n_T . In 2008, households spent on average 3.75 hours on work related activities. Assuming that the available time for leisure and working activities is 16 hours per day, this implies a steady state value for n_T of about 0.23.

Next, a steady state restriction derived from the structure of the model is exploited to calibrate n_H and n_B . In the steady state, the first-order conditions with respect to n_{Ht} , n_{Bt} and c_t can be used to derive the following condition:

$$\frac{n_B}{n_H} = \frac{y_B}{c} \frac{(1 - \alpha)}{(1 - \varphi)} \frac{[1 - \beta(1 - \delta_H)]}{\beta} \frac{\kappa}{(1 - \kappa)} \frac{h}{y_H}$$

Given the above value for n_T , the fact that the ratio n_B/n_H is endogenously determined allows us to pin down the steady state time allocation. Given values for c/y_B and h/y_H which are endogenously determined in the deterministic steady state, we find that: $n_H = 0.0225$ and $n_B = 0.2094$. The model therefore predicts a steady state share of hours worked in the housing sector, n_H/n_T , of about 10%.

To assess the relevance of this model implication, we construct an empirical counterpart using data on total hours worked and on hours worked in the construction sector. Using this empirical measure of time allocation as a proxy for n_H/n_T , we find that hours worked in the construction sector represents on average 6.6% of total employment.

Labor supply

The introduction of endogenous labor supply involves the calibration of two additional parameters controlling the curvature of $v(l)$. The above labor market restrictions which implies that in the steady state $n_T = 0.23$, pins down $v'(l)/v(l)$. The second elasticity parameter, $v''(l)/v'(l)$, determines the elasticity of labor supply. To our knowledge empirical evidence regarding the elasticity of labor supply in the construction sector are not available. Given this lack of a priori knowledge, we choose a value for $v''(l)/v'(l)$ which implies an elasticity of labor supply in the total number of hours worked, n_T , of about 1. Values for the Frisch elasticity of labor supply that are used in the literature usually range from 1 to 4 (e.g. King and Rebelo 1999, Uhlig 2007).

Productivity shock

Following the real business cycle literature, we assume that technology shocks are the only source of business cycle fluctuations. Total factor productivity, A_t , has the usual autoregressive characterization:

$$A_t = \rho A_{t-1} + \varepsilon_t$$

Compared to Davis and Heathcote (2005) and Greenwood and Hercowitz (1991), the number of degrees of freedom is therefore reduced by considering an economy where business fluctuations are entirely driven by one single exogenous shock. Following the business cycle literature, the persistence parameter, ρ , is set to 0.99 and the innovation standard deviation, σ_ε , is set to 0.0078.

As shown by Chari, Kehoe and McGrattan (2007) fluctuations caused by technology shocks can be reinterpreted as variations in the efficiency wedge. The equivalence results presented in their study allow us to interpret technology shocks in a broader sense. Variations in the efficiency wedge can reflect underlying frictions affecting the allocation of factor inputs (e.g. Restuccia and Rogerson 2008). Variations in the efficiency wedge can also capture financial frictions leading to an inefficient allocation of input across firms.

Leverage

The leverage ratio, ξ , is calibrated using available empirical evidence on the debt to equity ratio for nonfarm nonfinancial corporate firms in the United States¹¹. Over the period 1947-2009, the average debt to equity ratio is 0.564. Given that the government balances its budget, leverage only affects the dynamics of dividends and has no impact on the determination of the remaining variables. Calibrating ξ using these empirical facts allows us to pin down b using the solvency constraint:

$$\xi = \frac{b}{a}$$

To keep the analysis as simple as possible, we choose the following specification for $\tau(b_{t+1})$:

$$\tau(b_{t+1}) = \varrho \ln b_{t+1}$$

This assumption implies that, in the steady state, the price of corporate bonds is given by:

$$p_{CBt} + \frac{1}{a_{t+1}} \frac{\varrho}{\xi} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t}$$

In the steady state, the spread between the price of a risk-free and a corporate bond is therefore given by:

¹¹See flow of funds accounts, B.102 Balance Sheet of Nonfarm Nonfinancial Corporate Business, line 105.

$$p_{RF} - p_{CB} = \frac{1}{a} \frac{\varrho}{\xi}$$

The tax advantage of debt generates an agency problem between the firm and the household which vanishes in the special case $\varrho = 0$ or when the leverage ratio or the asset of the firm tend to infinity. Compared to a risk-free bond, the adjustment, $\frac{1}{a} \frac{\varrho}{\xi}$, therefore captures the risk premium that the firm has to pay in order to induce the household to hold its debt. We set ϱ to 0.011 in order to generate a steady state annual risk premium between the return of a corporate bond and of a risk-free bond of about 0.66%.

Government

The parameter governing expenditures in the fiscal rule, Ω , is set to 0.08. This parameter has no impact on the dynamics of dividends and given that the government keeps a balanced budget, it does not affect the equilibrium.

Adjustment costs, building restrictions, and habit formation

The 3 remaining parameters to select are the housing-supply regulation parameter, ϵ_H , the capital adjustment cost parameter, ϵ_I , and the habit parameter, m . This second set of parameters is picked to maximize the model's ability to match the equity premium, the mean risk-free rate, and the volatility of residential investment. The parameter search has been restricted to the following range of values:

$$m = [0 : 1], \quad \epsilon_H = [0.0 : 6.25], \quad \epsilon_I = [0.0 : 6.25]$$

As regards the adjustment cost and the housing-supply regulation parameters, the model without frictions corresponds to the case $\epsilon_I = \epsilon_H = 0$ and the model reduces to the case without habit formation when m is set to 1.

6 Results and discussion

The model's ability to match the equity premium and the mean risk-free rate is maximized at the following values for ϵ_H , ϵ_I and m :

$$m = 0.81, \quad \epsilon_H = 1.045, \quad \epsilon_I = 4.05$$

The results reported in Table 3, where the moments that are targeted are emphasized in bold, confirm that business cycle fluctuations cannot be

entirely explained by a model with only one shock and a low Frish elasticity of labor supply. Despite the low volatility of output, the model is still able to generate a 6.5% equity premium and a 1% mean risk-free rate. This illustrates that the mechanism under study considerably amplifies the impact of business cycle fluctuations on risk premiums.

Table 3: Output and business cycle statistics

Total output (HP-filter)						
	σ_{y_T}		$\rho(y_{Tt}, y_{Tt-1})$		$\rho(y_{Tt}, y_{Tt-4})$	
	Data	Model	Data	Model	Data	Model
y_T	1.68	1.08	0.81	0.72	0.19	0.13

Business cycle statistics (HP-filter)						
	σ_{x_i/y_T}		$\rho(x_{it}, y_{Tt})$		$\rho(x_{it}, x_{it-1})$	
	Data	Model	Data	Model	Data	Model
c	0.49	0.71	0.79	0.99	0.83	0.72
i_T	3.37	2.0	0.82	0.99	0.82	0.72
y_H	5.93	5.92	0.53	0.99	0.88	0.78
n_T	0.98	0.68	0.83	0.99	0.90	0.72
n_H	2.23	6.05	0.76	0.99	0.89	0.72
w_B	0.57	0.81	0.19	0.99	0.66	0.72
w_H	0.85	0.81	-0.16	0.99	0.65	0.72

Asset prices (HP-filter)						
	σ_{x_i/y_T}		$\rho(x_{it}, y_{Tt})$		$\rho(x_{it}, x_{it-1})$	
	Data	Model	Data	Model	Data	Model
d	25.3	12.8	0.80	0.99	0.78	0.76
p_E	10.6	14.0	0.65	0.99	0.83	0.72
p_H	4.01	7.14	0.57	0.99	0.91	0.72
z_H	0.43	0.74	-0.03	0.99	0.78	0.73

Table 3: The theoretical moments of the variable reported in Table 3 have been expressed in logs and the cyclical component has been extracted using a HP-filter.

σ_{x_i/y_T} and $\rho(x_{it}, y_{Tt})$ respectively denote the relative standard deviation of variable x_i with respect to output and the correlation of variable x_i with output. $\rho(x_{it}, x_{it-k})$ is the k order autocorrelation of variable x_i . y_T is total output, c is consumption, i_T is business investment, y_H is residential investment, n_T is total

hours worked, n_H is hours worked in the housing sector. w_B and w_H are the wage rate in the business and in the housing sector. Dividends, equity prices, house prices and residential rents are respectively denoted d, p_E, p_H and z_H .

Table 4: Financial returns

Mean in annualized (% p.a.)					
$E(r_M - r_f)$		$E(r_f)$		$E(r_H - r_f)$	
Data	Model	Data	Model	Data	Model
6.50	6.50	1.0	1.0	1.77	3.13

Standard deviation (% p.a.)					
$\sigma(r_M)$		$\sigma(r_H)$		$\sigma(r_f)$	
Data	Model	Data	Model	Data	Model
16.56	23.7	2.73	12.13	3.68	4.35

Autocorrelation					
$\rho(r_{Mt}, r_{Mt-1})$		$\rho(r_{Ht}, r_{Ht-1})$		$\rho(r_{ft}, r_{ft-1})$	
Data	Model	Data	Model	Data	Model
-0.06	-0.01	0.48	0.01	0.73	0.99

Table 4: The financial moments reported in Table 4 have been computed using theoretical moments. The results are expressed in annualized percent. r_M denote equity returns, r_f is the risk-free rate and housing returns are denoted r_H . $E()$, $\sigma()$ and $\rho()$ respectively denote the unconditional mean, standard deviation, and first-order autocorrelation of the variable under study.

Term premium

As shown by Jermann (1998), risk premiums can be decomposed into a common and an asset specific components. The term premium is the component that is common to every asset and which depends on the yield curve. The payout uncertainty premium is asset specific and is determined by the covariance between the dividend paid by the asset and marginal utility. Given that the stochastic discount factor is known, the term premium can be computed by pricing the following console bond:

$$p_{Ct} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} [1 + p_{Ct+1}]$$

The return on the asset can be defined as:

$$r_{Ct,t+1} = \frac{p_{Ct+1} + 1}{p_{Ct}}$$

where, for simplicity, the constant coupon paid by the asset has been normalized to one. The risk premium on this console bond, $E(r_C - r_f)$, corresponds to the term premium and is entirely determined by the term structure of the interest rates.

The term premium generated by the benchmark model being 3.84%, the cyclical behavior of dividends therefore accounts for $6.50 - 3.84 = 2.66\%$ of the total equity premium. Compared to the benchmark calibration, removing leverage would make dividends considerably smoother and countercyclical. This cyclical behavior of dividends would make equity less risky than a console bond; an implication which would be difficult to reconcile with the empirical facts (e.g. Jermann 1998, Abel 1999).

Figure 1: Impulse response of rents and the housing stock

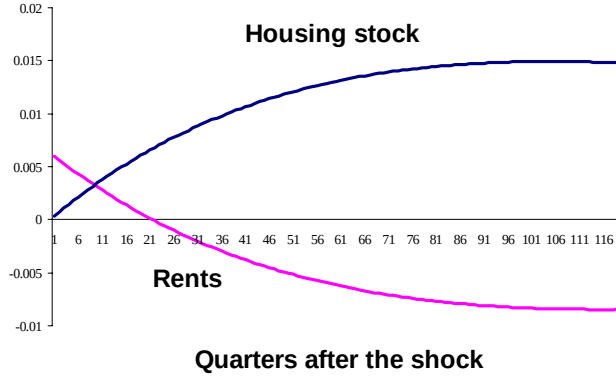


Figure 1: Impulse response of rents and the housing stock in percentage deviation from steady state to a one standard deviation technology shock. The impulse responses are simulated using the benchmark calibration.

Equity and housing risk premium

While the housing risk premium predicted by the model is too large, it is still possible to capture that housing is significantly less risky than equity. The spread between the equity and the housing risk premium, which is larger than 3%, is essentially due to the difference in the volatility of dividends and rents generated by the model. Residential rents, whose dynamics is

determined by the consumption to housing services ratio, react slowly to shocks whereas leverage makes dividends considerably more volatile. The spread reflects that the unfavorable cyclical property of dividends, which are very volatile and pro-cyclical, has to be compensated by a higher risk premium.

The fact that the supply of housing is endogenously determined contributes to decrease the housing risk premium. Figure 1, which shows the impulse response of rents to a positive technology shock, illustrates this point. The increase in residential rents is short-lived because the supply of new homes increases gradually in response to a positive shock. This supply effect, which puts downward pressure on residential rents, reduces the cyclical property of the payoff, and therefore lowers the payout uncertainty component of the housing risk premium.

As shown in Table 4, while the model is able to explain the very low autocorrelation of equity returns, it is not possible to simultaneously explain why the autocorrelation of housing returns observed in the data is so high. The fact that the model also over states the volatility of housing returns is another indication of potential model misspecification.

Wages

Not surprisingly, the rather extreme assumption of perfect mobility of labor across the two sectors implying that $w_B = w_H$ is rejected by the data. Wages in the construction sector are more volatile and less correlated with output than wages in the business sector.

The model also fails to capture the low correlation between wages in the two sectors and output. As far as the cyclical property of wages is concerned, as shown by Christiano and Eichenbaum (1992), introducing preference shocks usually helps to overcome this problem, typical of real business cycle models.

6.1 Asset pricing and business cycle implications of building restrictions

Table 5 below shows the sensitivity of the results to a change in the housing supply coefficient, ϵ_H , which captures the tightness of housing-supply regulation. Compared to the results reported in Table 2 and 3, all other parameters are kept constant.

Table 5: Sensitivity analysisModel with habit formation, $m = 0.81$

Asset Pricing	Data	ϵ_H				
		0	0.5	1.045	1.5	2
$\sigma_{p_H}/\sigma_{y_T}$	4.01	1.91	5.0	7.14	8.1	8.35
$\sigma_{p_E}/\sigma_{y_T}$	10.6	9.52	11.6	14.0	15.6	15.75
$E(r_M - r_f)$	6.50	4.06	4.82	6.50	8.16	9.43
$E(r_H - r_f)$	1.77	0.68	2.08	3.13	4.16	4.82
Business Cycle	Data	ϵ_H				
		0	0.5	1.045	1.5	2
σ_{y_T}	1.68	0.94	1.0	1.08	1.14	1.17
$\sigma_{i_T}/\sigma_{y_T}$	3.37	1.46	1.75	2.0	2.16	2.25
$\sigma_{y_H}/\sigma_{y_T}$	5.93	8.75	7.57	5.92	4.88	3.57
$\sigma_{n_H}/\sigma_{y_T}$	2.23	9.44	7.84	6.05	4.87	4.10

Model without habit formation, $m = 1$

Asset Pricing	Data	ϵ_H				
		0	0.5	1.045	1.5	2
$\sigma_{p_H}/\sigma_{y_T}$	4.01	1.2	1.57	1.70	1.76	1.8
$\sigma_{p_E}/\sigma_{y_T}$	10.6	3.1	3.55	3.68	3.74	3.78
$E(r_M - r_f)$	6.5	0.21	0.23	0.24	0.25	0.25
$E(r_H - r_f)$	1.77	0.08	0.1	0.11	0.11	0.12
Business Cycle	Data	ϵ_H				
		0	0.5	1.045	1.5	2
σ_{y_T}	1.68	0.9	0.92	0.93	0.93	0.93
$\sigma_{i_T}/\sigma_{y_T}$	3.37	0.48	0.51	0.52	0.52	0.52
$\sigma_{y_H}/\sigma_{y_T}$	5.93	2.38	0.92	0.58	0.45	0.36
$\sigma_{n_H}/\sigma_{y_T}$	2.23	2.35	0.89	0.54	0.41	0.32

Table 5: The benchmark calibration corresponds to the column $\epsilon_H = 1.045$ in the case $m = 0.81$. When $\epsilon_H = 0$, the model reduces to a case without building restrictions while $\epsilon_H = 2$ corresponds to a case with very high building restrictions. The lower part of Table 5 shows the sensitivity of the results to changes in ϵ_H when, compared to the benchmark case, the habit formation channel is completely switched off by setting $m = 1$.

House prices

Increasing ϵ_H reduces the elasticity of housing supply and generates a dramatic increase in the relative standard deviation of house prices, $\sigma_{p_H}/\sigma_{y_T}$. The adjustment in quantities occurs in the labor market and leads to an equally dramatic reduction in the relative standard deviation of hours worked, $\sigma_{n_H}/\sigma_{y_T}$. Housing-supply regulation amplifies the response of prices and reduces the response of quantities by acting as an adjustment costs on hours worked.

The large quantitative impact of housing-supply restrictions on house prices very much depends on the closed economy assumption. As shown by van Nieuwerburgh and Weill (2010), the impact of regulation is smaller in a model with perfect mobility across cities because agents can choose to move out in response to a tightening in regulation. While building restrictions generate a reallocation of resources across sectors, agents cannot completely escape regulation in our economy.

While our results suggest that housing-supply regulation could have a major impact on the volatility of house prices, it is however important to stress that this effect is considerably amplified by the introduction of habit formation. Table 5 which also reports the sensitivity of the results to a change in ϵ_H in the case $m = 1$ illustrates this point. In a model without habit, while reducing the elasticity of housing supply still contributes to increase the volatility of house prices, the quantitative impact is however substantially smaller.

Even with implausibly large housing-supply adjustment costs, the model without habit formation could only explain about half of the observed house price volatility. This result suggests that it would be difficult to find a plausible explanation for the high volatility of house prices without combining both supply and demand factors.

Equity and housing risk premiums

The effect of a change in ϵ_H on the equity premium, $E(r_M - r_f)$, is quite striking. With housing services accounting for 17% of total consumption, this large quantitative impact illustrates that introducing housing into the utility function has key asset pricing implications. Housing increases the potential for consumption risk diversification and could in principle generate a significant reduction in the equity premium. The extent to which housing contributes to the resolution of asset pricing puzzles therefore very much depends on the degree of housing supply restrictions.

The impact of building restrictions on the equity premium works through its effect on the stochastic discount factor, $\beta^* E_t \frac{\lambda_{t+1}}{\lambda_t}$. Tighter regulation increases the cost of adjusting the housing stock and generates a decline in the volatility of residential investment, $\sigma_{y_H}/\sigma_{y_T}$. The key is that this reduction in the volatility of residential investment makes consumption of housing services smoothing more difficult to achieve. The reduction in household's tolerance to these variations, which is induced by habit formation, makes marginal utility more volatile and increases the uncertainty of future pay-offs. With future pay-offs being more uncertain, investors need to be compensated by a higher risk premium in order to accept to hold equity.

The housing risk premium, $E(r_H - r_f)$, is more sensitive to changes in ϵ_H because building restrictions not only affect the stochastic discount factor but also have a direct impact on the dynamics of house prices. A reduction in the elasticity of housing supply makes house prices more volatile which increases the capital gain component of housing returns. Combined with the indirect effect on the stochastic discount factor which is common to every asset, this direct effect on house prices makes the housing risk premium very sensitive to changes in the regulatory environment.

As illustrated by the sensitivity analysis reported in Table 5, the impact of building restrictions on risk premiums very much relies on the presence of habit formation. This result illustrates that this is the combination of habit formation and building restrictions which matters for the determination of risk premiums.

Output

As shown by the impact of ϵ_H on σ_{y_T} , building restrictions increase the volatility of output and amplify business cycle fluctuations. This effect works through the impact of building restrictions on the allocation of labor across sectors. Both sectors being equally penalized by the adjustment cost on capital, the higher labor intensity of the construction sector provides a comparative advantage which distorts the allocation of labor. In good times, this difference in labor intensity which makes the construction sector very attractive is the key mechanism generating the boom in residential investment.

Building restrictions reduce the comparative advantage provided by high labor intensity and increase the competitiveness of the final output good sector. By distorting the optimal allocation of labor over the business cycle, this effect leads to an increase in the volatility of output, and at the same time, reduces the volatility of residential investment.

The mechanism is similar to the case studied by van Nieuwerburgh and Weill (2010), where a tightening in housing-supply regulation induces workers to move out. In our case, this is the perfect mobility between sectors which enables workers to escape housing-supply regulation by increasing hours worked in the business sectors.

6.2 Co-movement and lead-lag correlation

As discussed by Boldrin, Christiano and Fisher (2001), explaining the strong positive co-movement between hours worked in the business sector and output is a challenge for models with adjustment costs. While in the data the correlation, $\rho(n_B, y_T)$, is higher than 0.8, standard models with high capital adjustment costs usually generate a negative correlation which is at odd with the facts.

Table 6: Hours worked

		Benchmark	No Habit
	Data	$m = 0.81$	$m = 1.0$
$\rho(n_B, y_T)$	0.86	0.99	-0.99

Table 6: $\rho(n_B, y_T)$ is the correlation between hours worked in the business sector and output. The case $m = 0.81$ corresponds to the benchmark calibration while the case $m = 1.0$ reduces to the case without habit formation.

The model’s ability to explain the positive co-movement observed in the data essentially relies on the introduction of habit formation in the mix of consumption and leisure. In standard models, the root cause of the problem is that the presence of adjustment costs induce a strong negative wealth effect which reduces the incentive to supply labor in good times. In boom periods, this effect dominates the positive substitution effect induced by the change in real wages and generates leftward shifts in labor supply. This labor supply effect which is responsible for the countercyclical variations in hours worked obtained in these models is therefore the source of the problem.

The key is that our specification of habit formation enables to offset the effect of capital adjustment costs on labor supply by reducing the wealth elasticity of labor supply (e.g. Jaccard 2009a). As illustrated by the simulation reported in Table 6, compared to the benchmark calibration with high adjustment costs, removing habit formation would compromise the model’s

ability to generate these procyclical variations. The effect on labor supply is therefore similar to the effect obtained using the type of preferences proposed by Greenwood, Hercowitz and Huffman (1988), and Jaimovich and Rebelo (2009). When it comes to the resolution of asset pricing puzzles in models with housing, adopting a specification based on habit formation is however of the essence (e.g. Jaccard 2009b).

As documented by Fisher (2007), and as shown by Figure 1 in the annex, the fact that residential investment leads business investment over the cycle, *i.e.* $\text{corr}(i_B, y_{Ht-k}) > \text{corr}(i_{Bt-k}, y_H)$ is an important empirical regularity typical of the housing market. As illustrated by the right panel of Figure 1, the fact that the model fails to account for this robust empirical regularity seems to falsify the specification of adjustment costs that has been adopted. When it comes to alternative specifications which could potentially help to fix this problem, the findings presented by Gomme, Kydland and Rupert (2001) suggest that introducing time-to-build into the analysis would be a natural direction for future research.

As shown by the right panel of figure 2, the fact that house prices lead the cycle, *i.e.* $\text{corr}(y_{Tt}, p_{Ht-k}) > \text{corr}(y_{Tt-k}, p_{Ht})$, is another well-documented empirical regularity. House prices and many housing market variables have leading indicator properties which are often used in forecasting. While the exact magnitude cannot be reproduced, as illustrated by the left panel of figure 2, it is encouraging to see that on this dimension the qualitative predictions generated by the model go in the right direction.

6.3 Welfare cost of uncertainty and precautionary demand for housing

The model's ability to explain the equity premium and the mean risk-free rate in an environment with endogenous labor supply essentially relies on the assumption that habits are formed over a mix of consumption and leisure (e.g. Jaccard 2009). Introducing this particular type of habit formation decreases the volatility of $c_t^\kappa h_t^{1-\kappa} v(l_t)$, and at the same time, increases the volatility of marginal utility. This increase in the volatility of marginal utility allows the model to generate the large fluctuations in the stochastic discount factor which are necessary to resolve asset pricing anomalies.

The key is that this increase in the volatility of marginal utility is achieved by inducing a strong willingness to smooth fluctuations in $c_t^\kappa h_t^{1-\kappa} v(l_t)$, as

opposed to fluctuations in c_t . This assumption aims at capturing the idea that agents get hooked to a certain mix of consumption, housing and leisure reflecting their standards of living. With habit formation, abrupt changes in lifestyles, as measured by changes in the composite good, are very costly and lead agents to choose total consumption and leisure so as to maintain the smoothest possible path for $c_t^\kappa h_t^{1-\kappa} v(l_t)$.

This point is illustrated in Figure 2 which shows the impulse response of consumption and of the composite good to a positive technology shock. While this mechanism makes fluctuations in the composite good very costly, it does not lead to excess consumption smoothing. On impact, agents compensate the increase in consumption and in housing services by reducing leisure in order to prevent their habit stock from rising too quickly. This specification of habit in the composite good therefore enables to generate the volatility in marginal utility which is needed to resolve asset pricing puzzles, without compromising the model's ability to capture the dynamics of consumption.

Figure 2: Impulse response, consumption and composite good

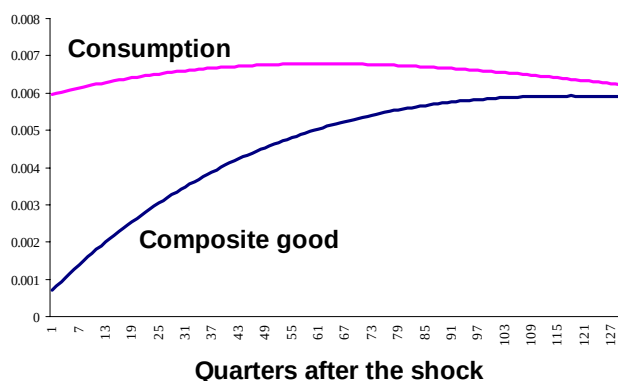


Figure 2: Impulse response of consumption, c_t , and of the composite good, $c_t^\kappa h_t^{1-\kappa} v(l_t)$, in percentage deviation from steady state to a one standard deviation technology shock. The impulse responses are simulated using the benchmark calibration.

This mechanism which makes business cycle fluctuations very costly also increases the welfare cost of uncertainty. To illustrate this point, following Lucas (2003), the cost of uncertainty is evaluated by comparing the stochastic

and the deterministic economy. The outcome of the deterministic economy is obtained by setting the shock standard deviation to zero and corresponds to an economy that has reached its steady state and which is growing at a constant rate along the balanced growth path. The welfare cost of uncertainty is measured by comparing the mean level of consumption in the stochastic case, $E(c)$, with consumption evaluated at the deterministic steady state, $\bar{c}$. The difference $E(c_t) - \bar{c}$, can be interpreted as the risk compensation which is required to make agents indifferent between a deterministic economy and an economy subject to business cycle fluctuations.

As illustrated by Table 7 below, this risk compensation, which is measured in annual percentage of agents' consumption $(E(c_t) - \bar{c})/E(c_t)$, is considerably larger in a model able to generate a 6.5% equity premium. This result confirms that the welfare cost of uncertainty is likely to be substantially higher when measured with models able to resolve asset pricing puzzles (e.g. Tallarini 2000).

Table 7: Habit and welfare cost of uncertainty

	Benchmark	Intermediate cases		No Habit
	$m = 0.81$	$m = 0.92$	$m = 0.99$	$m = 1$
$(E(c) - \bar{c})/E(c)$	17.7	14.5	2.8	0.67
$E(r_M - r_f)$	6.5	5.37	1.25	0.24

Table 7: The case $m = 0.81$ corresponds to the benchmark calibration discussed above. The model without habit formation is obtained by setting $m = 1$. The consumption risk compensation and the equity premium are expressed in annualized percent and are computed using theoretical moments

Finally, in a model producing costly business cycle fluctuations, uncertainty generates a precautionary demand for housing services, which affects the allocation of production and labor across sectors. To illustrate this point, Table 8 compares the allocation in the benchmark case, with the case where uncertainty has been turned off. The residential investment to output ratio is denoted, y_H/y_B , while n_H/n_B is the hours worked in the construction sector to hours worked in the business sector ratio. The large positive impact of uncertainty on both ratios illustrates that the share of resources which is allocated to the housing sector increases as the economy becomes more uncertain.

This confirms that, in models generating a large cost of business cycle fluctuations, housing plays a crucial role. By providing a durable source of consumption which depreciates and adjusts slowly over the business cycle, housing contributes to diversify consumption risk and provides a valuable insurance against unexpected shocks. Compared to an economy without uncertainty, this favorable cyclical property creates a precautionary demand for housing services. As illustrated by Table 8, this precautionary demand, relative to the final output good sector, translates into higher employment and production in the housing sector.

Table 8: Uncertainty and allocation of output and time

	Data	Deterministic	Stochastic
y_H/y_B	6.14	3.19	3.83
n_H/n_B	6.6	9.17	11.49

Table 8: The last column reports the outcome generated by the benchmark model which generates a 6.50% equity premium. The deterministic case corresponds to the allocation obtained when the standard deviation of the shock is set to zero.

7 Conclusion

The recent episode of financial distress has revived the debate over whether monetary policy should react to fluctuations in house prices. While this debate has been ongoing for many years, the answer to this question still very much depends on central banks' ability to distinguish between "excessive" and "fundamentally driven" movements in house prices.

Our analysis suggests that ignoring factors which affect the elasticity of housing supply, such as housing-supply regulation, may lead central banks to overstate the potential for house price misalignment. As we have shown in section 4, in a model with housing supply constraints, the value of a house can deviate from the standard infinite discounted sum formula. Moreover, a model with building restrictions and habit formation can generate "fundamentally driven" fluctuations in house prices which can be very large.

Finally, the equilibrium value of a house is closely linked to the model's financial market implications. Versions which failed to generate a plausible equity premium also generated smaller house price fluctuations. This result illustrates the importance of using well-specified stochastic discount factors. Checking the robustness of the discount factor that is assumed by confronting

additional model implications to the data may contribute to reduce the risk of misspecification.

8 References

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9 Data Appendix

Variable	Source
y_T	St-Louis Fed, GDPC96 (1947-2009)
c	St-Louis Fed, PCNDGC96+PCESVC96 (1947-2009)
i_B	St-Louis Fed, NRIPDC96 (1947-2009)
i_H	St-Louis Fed, PRFIC96 (1947-2009)
n_T	St-Louis Fed, USPRIV (1947-2009)
n_H	BLS, Aggregate hours in construction (1964-2009)
n_B	BLS, Aggregate hours in private sector (1947-2009)
w_B	St-Louis Fed, RCPHBS (1947-2009)
w_H	St-Louis Fed, AHECONS (1947-2009)
p_{Et}	R. Shiller, Real equity prices (1871-2009)
d_t	R. Shiller, Real earnings (1871-2009)
p_{Ht}	R. Shiller, case-Shiller 15 index (1987-2009)
z_H	Housing CPI, BLS (1967-2009)
$E(r_M - r_f), \sigma(r_M)$	Piazzesi, Schneider and Tuzel (2007)
$E(r_H - r_f), \sigma(r_H)$	Piazzesi, Schneider and Tuzel (2007)
$E(r_f), \sigma(r_f)$	Piazzesi, Schneider and Tuzel (2007)

10 Solution Method

The model is solved using perturbation methods. The equity premium and the welfare cost of uncertainty is computed using a second-order approximation around the steady state. The theoretical moments and the impulse responses are computed using the software dynare [see Adjemian, Juillard, Mihoubi, Perendia and Villemot (2009)].

11 Annex

Figure 2: Residential and business investment lead-lag correlation

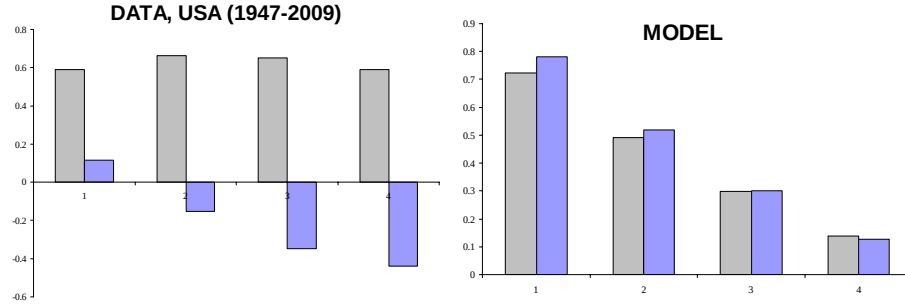


Figure 2: The left panel reports the observed cross correlogram between residential and business investment for the period 1947-2009. The left panel shows the corresponding model implications. The series have been expressed in logs and HP-filtered. For each lag considered ($k=1$ to 4), the left (gray) bar shows the lead correlation between residential investment in $t-k$ and business investment while the right (blue) bar shows the lag correlation between residential investment in t and business investment in $t-k$.

Figure 3: House prices and output lead-lag correlation

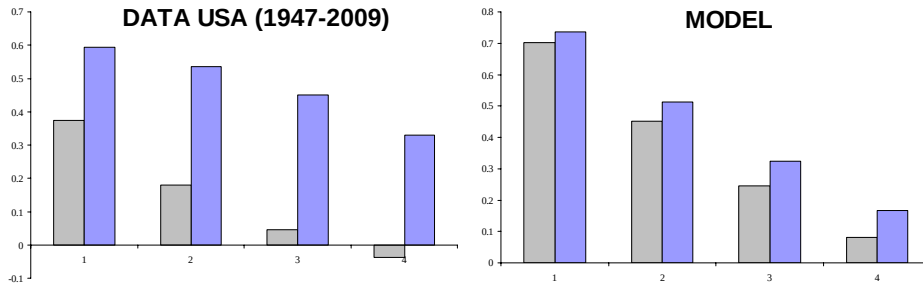


Figure 3: The left panel reports the observed cross correlogram between house prices and output for the period 1947-2009. The left panel shows the corresponding model implications. The series have been expressed in logs and HP-filtered. For each lag considered ($k=1$ to 4), the left (gray) bar shows the lead correlation between house prices in $t-k$ and output in t while the right (blue) bar shows the lag correlation between house prices in t and output in $t-k$.

12 The Competitive Equilibrium

12.1 The Firm

Managers maximize the value of the firm by solving the following dynamic optimization program:

$$\begin{aligned}
 L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{\lambda_t}{\lambda_0} \left[A_t k_{Bt}^{\alpha} n_{Bt}^{1-\alpha} + z_{Ht} h_t + p_{CBt} b_{t+1} \right. \right. \\
 \left. \left. - w_{Bt} n_{Bt} - w_{Ht} n_{Ht} - i_{Tt} - b_t - [f_t - \tau(b_{t+1})] \right. \right. \\
 \left. \left. + p_{Ht} \left((1 - \delta_H) h_t + \phi \left(\frac{(k_{Tt} - k_{Bt})^{\varphi} n_{Ht}^{1-\varphi}}{h_t} \right) h_t - \gamma h_{t+1} \right) \right. \right. \\
 \left. \left. + q_{Tt} \left((1 - \delta_K) k_{Tt} + \phi \left(\frac{i_{Tt}}{k_{Tt}} \right) k_{Tt} - \gamma k_{Tt+1} \right) \right] \right\}
 \end{aligned}$$

First-order conditions:

n_{Bt} :

$$w_{Bt} = (1 - \alpha) \frac{y_{Bt}}{n_{Bt}} \quad (14)$$

n_{Ht} :

$$w_{Ht} = p_{Ht} \phi' \left(\frac{y_{Ht}}{h_t} \right) (1 - \varphi) \frac{y_{Ht}}{n_{Ht}} \quad (15)$$

k_{Bt} :

$$\alpha \frac{y_{Bt}}{k_{Bt}} = p_{Ht} \phi' \left(\frac{y_{Ht+1}}{h_{t+1}} \right) \varphi \frac{y_{Ht}}{k_{Tt} - k_{Bt}} \quad (16)$$

i_{Tt} :

$$1 = q_{Tt} \phi' \left(\frac{i_{Tt}}{k_{Tt}} \right) \quad (17)$$

k_{Tt+1} :

$$q_{Tt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{Tt+1} \left[(1 - \delta_K) + \phi \left(\frac{i_{Tt+1}}{k_{Tt+1}} \right) \right]$$

$$-\phi' \left(\frac{i_{Tt+1}}{k_{Tt+1}} \right) \frac{i_{Tt+1}}{k_{Tt+1}} \Big] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{Ht+1} \phi' \left(\frac{y_{Ht+1}}{h_{t+1}} \right) \varphi \frac{y_{Ht+1}}{k_{Tt+1} - k_{Bt+1}} \quad (18)$$

$h_{t+1} :$

$$\lambda_t p_{Ht} = \beta E_t \lambda_{t+1} \left[\left((1 - \delta_H) + \phi \left(\frac{y_{Ht+1}}{h_{t+1}} \right) - \phi' \left(\frac{y_{Ht+1}}{h_{t+1}} \right) \frac{y_{Ht+1}}{h_{t+1}} \right) p_{Ht+1} + z_{Ht+1} \right] \quad (19)$$

$b_{t+1} :$

$$p_{CBt} + \tau'(b_{t+1}) = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} \quad (20)$$

$\lambda_t :$

$$d_t = A_t k_{Bt}^\alpha n_{Bt}^{1-\alpha} + z_{Ht} h_t + p_{CBt} b_{t+1} - w_{Bt} n_{Bt} - w_{Ht} n_{Ht} - i_{Tt} - b_t - [f_t - \tau(b_{t+1})] \quad (21)$$

$q_{Tt} :$

$$(1 - \delta_T) k_{Tt} + \phi \left(\frac{i_{Tt}}{k_{Tt}} \right) k_{Tt} - \gamma k_{Tt+1} = 0 \quad (22)$$

$p_{Ht} :$

$$(1 - \delta_H) h_t + \phi \left(\frac{y_{Ht}}{h_t} \right) h_t - \gamma h_{t+1} = 0 \quad (23)$$

12.2 Households

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \frac{1}{1-\sigma} \beta^{*t} [c_t^\kappa h_t^{1-\kappa} v(l_t) - x_t]^{1-\sigma} + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [w_{Bt} n_{Bt} + w_{Ht} n_{Ht} + s_t d_t + b_t + T_t \right.$$

$$\left. - z_{Ht} h_t - c_t - p_{Et} [s_{t+1} - s_t] - p_{CBt} b_{t+1} \right] + \sum_{t=0}^{\infty} \beta^{*t} \varphi_t [\xi a_{t+1} - b_{t+1}]$$

$$+ \sum_{t=0}^{\infty} \beta^{*t} \psi_t \left[m x_t + (1-m) \left[c_t^\kappa h_t^{1-\kappa} v(L_t) \right] - \gamma x_{t+1} \right] \Bigg\}$$

To simplify notation, we define:

$$u_t = c_t^\kappa h_t^{1-\kappa} v(L_t) - x_t$$

First-order conditions:

c_t :

$$\left[\kappa c_t^{\kappa-1} h_t^{1-\kappa} v(L_t) \right] u_t^{-\sigma} + \left[\kappa c_t^{\kappa-1} h_t^{1-\kappa} v(L_t) \right] (1-m) \psi_t = \lambda_t \quad (24)$$

n_{Bt} :

$$\left[c_t^\kappa h_t^{1-\kappa} v'(L_t) \right] u_t^{-\sigma} + \left[c_t^\kappa h_t^{1-\kappa} v'(L_t) \right] (1-m) \psi_t = \lambda_t w_{Bt} \quad (25)$$

n_{Ht} :

$$\left[c_t^\kappa h_t^{1-\kappa} v'(L_t) \right] u_t^{-\sigma} + \left[c_t^\kappa h_t^{1-\kappa} v'(L_t) \right] (1-m) \psi_t = \lambda_t w_{Ht} \quad (26)$$

h_t :

$$z_{Ht} \lambda_t = \left[(1-\kappa) c_t^\kappa h_t^{-\kappa} v(L_t) \right] u_t^{-\sigma} + \left[(1-\kappa) c_t^\kappa h_t^{-\kappa} v(L_t) \right] (1-m) \psi_t \quad (27)$$

x_{t+1} :

$$\psi_t = m \beta E_t \psi_{t+1} - \beta E_t u_{t+1}^{-\sigma} \quad (28)$$

b_{t+1} :

$$p_{CBt} + \frac{\varphi_t}{\lambda_t} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} \quad (29)$$

s_{t+1} :

$$p_{Et} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} [d_{t+1} + p_{Et+1}] \quad (30)$$

λ_t :

$$w_{Bt} n_{Bt} + w_{Ht} n_{Ht} + s_t d_t + b_t + T_t = z_{Ht} h_t + c_t + p_{Et} [s_{t+1} - s_t] + p_{CBt} b_{t+1} \quad (31)$$

$\psi_t :$

$$mx_t + (1 - m) [c_t^\kappa h_t^{1-\kappa} v(L_t)] - \gamma x_{t+1} = 0 \quad (32)$$

$\varphi_t :$

$$\xi a_{t+1} - b_{t+1} = 0 \quad (33)$$

12.3 The government

$$G_t = T_t \quad (34)$$

12.4 Aggregate resource constraint

$$A_t k_{Bt}^\alpha n_{Bt}^{1-\alpha} - i_{Tt} - c_t = 0 \quad (35)$$