

Accounting for non-annuitization ^{*}

Preliminary version

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Abstract

Why don't people buy annuities? Several explanations have been provided by the previous literature: large fraction of preannuitized wealth in retirees' portfolios; adverse selection; bequest motive; medical expense uncertainty. This paper uses a quantitative model to assess the importance of these impediments to annuitization and also studies three newer explanations: government safety net in terms of means-tested transfers; illiquidity of housing wealth; restrictions on minimum amount of investment in annuity. This paper shows that quantitatively the last three explanations play a big role in reducing annuity demand. The minimum consumption floor turns out to be important to explain the lack of annuitization, especially for people in lower income quintiles, who are well insured by this provision. The minimum annuity purchase requirement involves big upfront investment and is binding for many, especially if housing wealth cannot be easily annuitized. Among the traditional explanations, preannuitized wealth has the largest quantitative contribution to the annuity puzzle.

1 Introduction

In the canonical life-cycle model people choose to smooth the marginal utility of consumption throughout their entire lifespan. In presence of lifespan uncertainty, households risk outliving their assets. This risk can be insured by buying life annuities, which are

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financial instruments that allow an individual to exchange a lump-sum of wealth for a stream of payments that continue as long as he is alive.

In practice few people buy annuities. This empirical fact was called "annuity puzzle". Several explanations for this puzzle has been suggested by the literature, however their relative importance is still not quantified. This paper provides quantitative analysis of the different factors contributing to the annuity puzzle.

To do this I develop a quantitative model of saving after retirement in which individuals face lifespan uncertainty and medical expense risk. Retirees choose how much to save and the composition of their portfolios given that they can invest in risk-free bonds and annuities.

The model allows for rich heterogeneity of the individuals. This is motivated by the fact that observed annuity demand varies a lot by quintiles of permanent income distribution. To account for this observation the following dimensions of heterogeneity are included in the model: initial wealth, preexisting annuity income, life expectancy and medical expense risk.

In modeling annuity market this paper considers two information structures. In the first, the insurer and the annuity buyer have the same information about the mortality of the latter. In the second, there is asymmetric information in a way that the insurer can only observe the age of the annuity buyer. The latter scenario creates environment for adverse selection. The adverse selection is intensified by the negative correlation between wealth and mortality. This happens since retirees with low mortality buy more annuities because not only they expect to live longer but also because they are wealthier. I compare the outcomes of the models with two information setups to identify the effect of adverse selection on the different group of retirees.

The main quantitative exercise of this paper consists of comparing annuity market participation rates between the models that incorporate different impediments to annuitization. I start with studying traditional explanations for the annuity puzzle. In the previous literature the lack of interest in annuities has been mainly attributed to the following four factors: a substantial fraction of preannuitized wealth in retirees' portfolios, actuarially unfair prices, bequest motives and uncertain health expenses. Next, I explore the role of another three factors that have been studied much less: government provided social assistance, difficulties with annuitizing housing wealth, and minimum purchase requirement set by insurance companies.

The consumption minimum floor, among other things, provides financial support for people if they outlive their assets and, thus, has a longevity insurance element in it. This public longevity insurance may partially substitute private annuity at least for low-income retirees. In the presence of health uncertainty the consumption floor can also be considered as one hundred percent tax on annuity income in the states when an individual

cannot finance his medical expenses.

Another possible impediment to annuitization is related to the fact that private annuity brings return over long period of time and, as such, involves big upfront investment. So when it comes to buying annuity, liquidity constraint may become an issue because, first, housing wealth is not easily annuitized, and second, insurance firms place restrictions on the minimum amount that can be invested in annuity.

Housing constitutes a significant portion of retirement wealth. In principle housing wealth can be annuitized by taking a reverse mortgage. In reality housing and non-housing wealth differ in terms of costs of annuitizing it. For example, a 70 years old woman having \$100,000 in liquid wealth can get an annuity that will pay her around \$700 every month until she is alive. If on the other hand she has \$100,000 worth of housing wealth and chooses to take a reverse mortgage with an annuity option she will get only around \$300 per month¹.

Another consideration is that from the point of view of an economic model an individual may find it rational to buy \$10 worth of annuity. In reality insurance companies set some restrictions on the minimum amount of investment in annuity. The minimum premium varies from one firm to another but can go up to \$100,000. This minimum amount of investment constitutes a significant barrier to annuitization for many retirees.

I find that the following four factors play major role in reducing annuity market participation rates: preannuitized wealth, consumption minimum floor, minimum annuity purchase requirements and illiquidity of housing wealth. All the impediments to annuitization combined account for a significant decline in annuity demand compared to the simple life-cycle model. However the full version of the model still overpredicts the annuity market participation rates. The predictions of the model can come close to the data under three conditions. First, the model has to be fed with the annuity prices from the data. Second, the consumption minimum floor has to be set to \$8,000. Third, minimum available for purchase annuity income should be equal to \$3,000 per year.

The paper is organized as follows. Section 2 review the literature. Section 3 looks at the data. Section 4 presents the model. In Section 5 I describe calibration. Section 6 presents and discusses the results. Section 7 concludes.

2 Related literature

This paper is related to three strands of literature. First, it is the literature on the annuity puzzle. Since the seminal work of Yaari (1965), the role of annuities in saving decisions of consumers with uncertain life spans has been widely studied in economic

¹Data for reverse mortgages was taken from the website <http://www.reversecalculator.com>, and for annuities - <http://www.immediateannuities.com>

literature. Yaari's famous result is that under certain assumptions an individual should fully annuitize all of his wealth. These assumptions include the absence of a bequest motive, actuarially fair prices and no uncertainty except about the time of death.

These theoretical results have been followed by a number of empirical papers measuring the insurance value of annuitization for representative consumers in a calibrated life cycle models (Mitchell, et al. 1999, Brown 2001, Brown et al. 2005). A general finding of the literature is that there are substantial gains to some annuitization, though full annuitization is not always optimal.

The literature seeking to explain the annuity puzzle identified four factors that may play a major role in reducing the demand for annuities from the part of single retirees. First, individuals already have a substantial fraction of annuities in their portfolio provided by Social Security and Define Benefits (DB) pension plans (Dushi and Webb 2004).

Second, the prices for annuities are actuarially unfair due to the presence of adverse selection. Mitchell, Poterba and Warshawsky (1997) showed that annuity prices in the US are around 20% higher than implied by the value of an actuarially fair annuity if the price of the latter is calculated with population average mortality.

Third, annuitized wealth cannot be bequeathed. Thus individuals with bequest motives should have lower demand for annuities. Lockwood (2008) suggested that bequest motives play a central role in explaining low annuity demand.

Fourth, the attractiveness of annuities can decrease in the presence of a health uncertainty. The possibility of incurring high medical expenses increases preferences for liquid wealth as opposed to illiquid annuity (Turra and Mitchell 2004). Also big medical expenses coincide with health deterioration which increases mortality and decreases the value of an annuity (Sinclair and Smetters 2004).

The contribution of this paper to the literature on the annuity puzzle is two-fold. First, it provides relative quantitative assessment of all these impediments to annuitization. And second, it considers other three possible explanations that are barely studied but have an important quantitative contribution to the annuity puzzle.

The second strand of literature this paper is related to studies retirees' saving decisions in the presence of medical expense risk. Palumbo 1999, De Nardi, French, Jones 2006 analyze decumulation decisions when retirees can only save in bonds. Pang and Warshawsky (2008) introduce portfolio choice problem by allowing retirees to save in bonds, stocks and annuities. Yogo (2008) studies more complicated portfolio problem where retirees can allocate their assets between bonds, stocks, annuities and housing. At the same time he treats medical expenditures as endogenous investments in health. This paper restricts the portfolio choice of retirees to only two assets - bonds and annuities, and treats medical expenses as exogenous shocks to income.

The third strand of literature studies equilibrium in annuity markets in the presence

of adverse selection. Hosseini (2009) evaluates the benefits from the mandatory annuitization feature of the Social Security. He considers equilibrium where agents differ only by their mortality. Walliser (1999) studies the effect of the Social Security on the private annuity markets. He constructs the environment where agents are heterogeneous both by mortality and income and allows for the income-mortality correlation. This paper augments the heterogeneity of individuals by health and medical expenses. This framework allows to get a more detailed picture of the effect of adverse selection on different categories of population.

3 Data

In the US only around 8% of people aged 70 years and older report having income from private annuities in the HRS/AHEAD dataset. The participation in the private annuities market varies a lot by quintiles of permanent income distribution (see Table 1). Almost 16% of people in the highest income quintile report having income from private annuities while among the bottom quintile this fraction is less than 1%.

Table 1: Participation in annuity market for people aged 70 years and older in 2006

Income quintile	Percentage
All	7.8
1	0.8
2	1.5
3	3.7
4	5.7
5	15.9

To get some idea of what causes such a significant variation one can compare life expectancy for people in different income quintiles. Annuities provide longevity insurance and as such should be more valuable for people who live longer. Indeed, Table 2 shows that on average at age 70 people in the fifth income quintile expect to live almost four years longer than people in the first quintile.

On the other hand, besides providing longevity insurance, annuities are also saving instruments. As such they are more valued by people who chose to keep large amount of assets very late in life. Table 3² compares asset decumulation rates for single retired

²In Table 3 the numbers for the bottom income quintile should be taken with caution because people in this quintile have almost zero assets

Table 2: Life expectancy at age 70 (Source: De Nardi et al, 2009)

Income quintile	Life expectancy
1	11.1
2	12.4
3	13.1
4	14.4
5	14.7

individuals in different income quintiles. More specifically, it shows percentage change in median assets between 1995 and 2002 for the retirees who were still alive in 2002. For higher permanent income levels the decumulation rate is slower than for the bottom ones. Since people in high quintiles spend down their wealth slower they are more interested to keep part of their assets in annuity as one of the available saving options.

Table 3: Percentage change in median assets, 1995-2002 (Source: De Nardi et al, 2009)

Income quintile	Ages 72-81	Ages 82-91
1	-83.4	-98.2
2	-33.4	-60.1
3	-23.2	-34.5
4	-27.5	-42.2
5	-7.7	2.7

In general, the heterogeneity in saving behavior and life expectancy for different income quintiles results in a significant variation in observed annuity demand. This heterogeneity should be taken into account when analyzing annuity puzzle because the reasons for low annuitization may differ by income quintile.

Another dimension to consider is how participation in the annuity market changes with age. Table 4 presents participation rates for two groups: people aged from 70 to 80 years and those older than 80 in 2006. Percent of people receiving income from private annuity for the older cohort is 10.6% while for the younger cohort it is only 6.2%. Part of this increase is due to survival bias: people who buy annuity have higher life expectancy. However, it may also indicate that people increase annuity purchase as they age. In general, the pattern of annuity purchase by age is worth exploring because it can convey information about people's preferences for financial instruments in retirement and how well annuities meets these needs.

Table 4: Participation in annuity market for people aged 70 years and older in 2006

Income quintile	Age 70-80	Older than 80
All	6.2	10.6
1	1.2	0.0
2	1.3	1.9
3	2.7	5.4
4	4.5	7.5
5	13.0	21.2

4 Model

Consider a portfolio choice model in which retired people decide how much to save, and how to split their net worth between bonds and annuities. These retirees face uncertain lifespans and out-of-pocket medical expenses.

Agents have an initial endowment of wealth, part of which is exogenously annuitized. Preannuitized wealth is the expected present value of the stream of annuities that an agent is entitled to and consists of Social Security and DB pension wealth.

Agents are heterogeneous by age, health status, and permanent income. Permanent income is an indicator of the agents' lifetime earnings and is important for two reasons. It determines agent's initial endowment of wealth, and affects the survival probability, health evolution and medical expenses. The association between income, health and mortality is a well documented phenomenon (Hurd, McFadden, Merrill 1999) and should be taken into account in modeling annuity demand.

4.1 Households

4.1.1 Preferences

Denote age of the individual by t , $t = 1, \dots, T$, where T is the last period of life. Households are assumed to have CRRA preferences:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

and enjoy leaving a bequest. Utility from a bequest takes the following functional form:

$$v(k_t) = \eta \frac{(\phi + k_t)^{1-\sigma}}{1-\sigma}$$

with $\eta > 0$. Here $\phi > 0$ is a shift parameter making bequests luxury goods thus allowing for zero bequest among low-income individuals.

4.1.2 Health, survival and medical expenses

In specifying medical expenses and survival uncertainty I follow De Nardi, French and Jones (2006) (DFJ). Their framework is well-suited for studying heterogeneity in annuitization decisions because they explicitly model the relationship between several factors affecting demand for annuities: income, life expectancy and medical expenses.

Each period an individual's health status m_t can be good ($m_t = 1$) or bad ($m_t = 0$). The transition between health states is governed by a Markov process with a transition matrix depending on age (t) and permanent income (I).

The individual survives to next period conditional on being alive today with probability s_t , where $s_T = 0$. Survival is a function of age, permanent income and current health status: $s_t = s(m, I, t)$.

Each period, an agent has to pay medical costs, z_t that are assumed to take the following form:

$$\ln z_t = \mu(m, t, I) + \sigma_z \epsilon_t \quad (1)$$

where ϵ_t consists of persistent and transitory components.

$$\epsilon_t = \zeta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2)$$

Persistent component is modeled as AR(1) process:

$$\zeta_t = \rho_{hc} \zeta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (2)$$

I denote the joint distribution of ζ_t, ξ_t by $F(\zeta_t, \xi_t)$. Unconditional mean of medical expenses is: $\exp(\mu(m, t, I) + 0.5\sigma_z^2)$ where

$$\mu(m, t, I) = \beta_{0t}^h + \beta_{1t}^h 1\{m_t = 1\} + \beta_{2t}^h I + \beta_{3t}^h I^2 \quad (3)$$

Here $\beta_{0t}^h, \beta_{1t}^h, \beta_{2t}^h, \beta_{3t}^h$ are some coefficients.

4.1.3 Government transfers

An agent who doesn't have enough assets to pay her medical expenses receives transfer from the government in the amount τ_t . This transfer maintains the agent's consumption at a minimum level guaranteed by the government $c_{\min}$.

4.1.4 Portfolio choice

Individuals have two investment options: a risk-free bond and an annuity, and cannot borrow. As usually assumed in the literature, once the annuity is bought, it cannot

be sold. The annuity is modeled in the following way: by paying the amount $q_t \Delta_{t+1}$ today, an individual buys a stream of payments Δ_{t+1} that she will receive each period, conditional on being alive.

4.1.5 Optimization problem

Each period the individual decides how to distribute her current wealth between consumption (c_t) and investments in bonds (k_{t+1}) and annuities (Δ_{t+1}), given that she has to pay medical expenses. I denote as X_t a set of state variables I, t, n_t, k_t : $X_t = (I, t, n_t, k_t)$. The recursive formulation can be represented in the following form:

$$V(X_t, m_t, \zeta_t, \xi_t) = \max_{c_t, k_{t+1}, \Delta_{t+1}} \left\{ \begin{array}{l} u(c_t) + \beta s_t \Pr(m_{t+1} = 0 | m_t, t, I) \int_{\zeta, \xi} V(X_{t+1}, 0, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1}) + \\ \beta s_t \Pr(m_{t+1} = 1 | m_t, t, I) \int_{\zeta, \xi} V(X_{t+1}, 1, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1}) + \\ \beta(1 - s_t)v(k_{t+1}) \end{array} \right\} \quad (4)$$

s.t. the budget constraint:

$$c_t + z_t + k_{t+1} + q_t \Delta_{t+1} = k_t(1 + r) + n_t + \tau_t,$$

government transfers

$$\tau_t = \min \{0, c_{\min} - k_t(1 + r) - n_t + z_t\},$$

the annuities evolution equation

$$n_{t+1} = \Delta_{t+1} + n_t,$$

borrowing and annuity illiquidity constraints: $k_{t+1}, \Delta_{t+1} \geq 0$ and initial conditions k_0, n_0, m_0 .

Notation is as follows: c_t is consumption, m_t is health status, t is age, I permanent income, k_t is investment in risk-free bond with return r , Δ_{t+1} is investment in annuity, n_t total annuity level, $v(\cdot)$ is bequest function, $\Pr(m_{t+1} = 0 | m_t, t, I)$ probability of being in a bad state tomorrow given current health status, permanent income and age, τ_t is government transfer.

4.2 Insurance sector

A common approach in the literature is to model annuities as non-exclusive insurance contracts (Chiappori, 2000). Individuals are free to buy an arbitrary number of contracts from different insurance companies, which makes it impossible to condition the contract design on the amount purchased. I assume contracts are linear: to purchase Δ units of annuity coverage an individual pays $q\Delta$ in premiums. This assumption rules out market separation through menus of contracts.

I assume insurance firms set a restriction on the minimum amount than can be invested in annuities equal to $\bar{\Delta}$. This minimum purchase requirement can be rationalized as follows. From the point of view of an insurance company what matters is not only how many annuities are sold but also how many accounts are open. Keeping track of too many small accounts is costly so insurance company needs to put some limit on the number of small customers.

Another restriction that annuity buyers face is the maximum issue age $\bar{t}$. Individuals older than $\bar{t}$ cannot buy annuity. This restriction reflects the rule according to which in most states insurance companies are prohibited from selling annuities to individuals beyond certain age (Levy et al. 2005).

In terms of information structure I consider two scenarios. Under the first scenario insurance firms are allowed to observe all state variables of the individuals that are relevant for forecasting her survival probability. I call this setup the "full information scenario".

In the second scenario the insurers know the aggregate distribution of individuals over states, however it cannot observe any characteristics of the annuity purchaser except age. I call this setup the "asymmetric information scenario". In this environment all people of the same age buy annuity at a uniform price. This outcome of such information structure resembles current situation in the market for longevity insurance in the US: annuity prices are conditioned usually only on age and gender.

I assume insurance firms act competitively: they take the price of annuity q_t as given. Expected payout per unit of insurance sold to individual of age t can be expressed as follows:

$$\pi_t(\Omega_t) = q_t(\Omega_t) - \gamma \sum_{i=1}^{T-t} \frac{\hat{s}_{t+i}}{(1+r)^i}$$

where γ is administrative load that assumed to be proportional to the total expected payment for the contract, $\gamma \geq 1$. Ω_t is the information available to insurer about individual of age t , and $\hat{s}_{t+i}$ is expected by insurer future survival probability of the individual buying annuity. It can be expressed as follows:

$$\widehat{s}_{t+i} = E_t(s_{t+i}|\Omega_t)$$

In the full information case insurer and annuity buyer have the same information. Thus Ω_t includes all variables relevant for determining survival probability of a person of a given age:

$$\Omega_t = (m_t, I)$$

Under asymmetric information environment the firm does not know anything about the individual of a given age except the fact that she bought annuity:

$$\Omega_t = (\Delta_{t+1}(k, n, m, I, t, \zeta, \xi) \geq \overline{\Delta})$$

In this case $\widehat{s}_{t+i}$ is the firm's belief about the probability that an individual who buys an annuity will survive till period $t+i$. In equilibrium $\widehat{s}_{t+i}$ has to be consistent with the individual behavior.

Firms chooses the amount of annuity to sell (N_t) by solving the following maximization problem:

$$\max_{N_t} N_t \pi_t \tag{5}$$

4.3 Competitive equilibrium

Before defining the competitive equilibrium, denote the distribution of individuals of age t over states by $\Gamma_t(k, n, m, I, \zeta, \xi)$ where $k \in K = R^+ \cup \{0\}$, $n \in N = R^+ \cup \{0\}$, $m \in M = \{0, 1\}$, $I \in \{I_1, I_2, I_3, I_4, I_5\}$, $\zeta \in R$, $\xi \in R$.

The competitive equilibrium for the asymmetric information case can be defined as follows:.

Definition 1 *A competitive equilibrium is*

- (i) *a set of belief functions $\{\widehat{s}_{t+i}, i = 0, \dots, T-t\}_{t=1}^T$*
- (ii) *a set of annuity prices $\{q_t\}_{t=1}^T$*
- (iii) *a set of decision rules for households*
 $\{c_t^*(k, n, m, I, t, \zeta, \xi), k_{t+1}^*(k, n, m, I, t, \zeta, \xi), \Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi), m \in \mathcal{M}, I \in \mathcal{I}, k \in \mathcal{K}, n \in \mathcal{N}\}_{t=1}^T$
and for insurance firms $\{N_t^\}_{t=1}^T$*

such that

1. Each annuity seller earns zero profit:

$$N_t^* \pi_t = 0$$

2. Firms' belief functions are consistent with households' decision rules:

$$\widehat{s}_{t+i} = \frac{\int_{\mathcal{K}, \mathcal{N}, \mathcal{M}, \mathcal{I}, \zeta, \xi} \Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi) \Gamma_{t+i}^t(k, n, m, I, \zeta, \xi)}{\int_{\mathcal{K}, \mathcal{N}, \mathcal{M}, \mathcal{I}, \zeta, \xi} \Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi) \Gamma_t(k, n, m, I, \zeta, \xi)}$$

where $\Gamma_{t+i}^t(k, n, m, I)$ is measure of people of age $t + i$ who bought annuity in the amount $\Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi)$ at age t . It can be defined recursively in the following way:

$$\Gamma_{t+1}^t(k, n, m, I, \zeta, \xi) = s(m, I, t) \Gamma_t(k, n, m, I, \zeta, \xi)$$

$$\Gamma_{t+i}^t(k, n, m, I, \zeta, \xi) = \int_{\widetilde{m} \in \mathcal{M}} s(\widetilde{m}, I, t + i - 1) \Gamma_{t+i-1}^t(k, n, m, I, \zeta, \xi, \widetilde{m})$$

Here $\Gamma_{t+i-1}^t(k, n, m, I, \widetilde{m})$ is distribution of people aged $t + i - 1$ who bought annuity in the amount $\Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi)$ at age t across their current health status $\widetilde{m}$. It can be recursively expressed as follows:

$$\Gamma_{t+1}^t(k, n, m, I, \zeta, \xi, \widetilde{m}) = \Pr(\widetilde{m} | m, t, I) s(m, I, t) \Gamma_t(k, n, m, I, \zeta, \xi)$$

$$\Gamma_{t+i}^t(k, n, m, I, \zeta, \xi, \widetilde{m}) = \int_{\overline{m} \in \mathcal{M}} \Pr(\widetilde{m} | \overline{m}, t + i - 1, I) s(\overline{m}, I, t + i - 1) \Gamma_{t+i-1}^t(k, n, m, I, \zeta, \xi, \overline{m})$$

3. Given annuity prices $\{q_t\}_{t=1}^T$ households' decision rules solve optimization problem 4 and N_t^* solves equation 5.

4. The market clears

$$N_t^* = \int_{\mathcal{K}, \mathcal{N}, \mathcal{M}, \mathcal{I}, \zeta, \xi} \Delta_{t+1}^*(k, n, m, I, t, \zeta, \xi) \Gamma_t(k, n, m, I, \zeta, \xi)$$

The definition of the competitive equilibrium for the full information scenario is similar with the following modifications: the annuity prices now depend on m_t and I ; and the second condition for the equilibrium takes the form:

$$\widehat{s}_{t+i} = E_t(s_{t+i} | m_t, I_t)$$

5 Data and calibration

5.1 Mortality, health and medical expenditures

The parameters governing the evolution of health, survival and medical expenses come from papers of French and Jones (2004) and De Nardi, French, and Jones (2009) (DFJ) which use the AHEAD dataset. These parameters include coefficients from the relationships (3), σ_z and characteristics of the stochastic component of medical expenses process (2): ρ_{hc} and σ_ε^2 .

In DFJ model there is additional state variable that affects health uncertainty and mortality - gender. The model in this study does not have gender, so when using DFJ estimates the effect of the gender on all the variables was averaged out.

French and Jones (2004) found that AR(1) component of health costs is quite persistent: $\rho_{hc} = 0.922$. They found that innovation variance of persistent component σ_ε^2 is equal to 0.0503 and innovation variance of transitory component is 0.665. Thus 66.5% of the cross sectional variance of medical expenses comes from the transitory component, and 33.5% - from the persistent component. The variance of the log medical expenses σ_z^2 is equal to 2.53.

5.2 Parameters calibration

Parameters of the model that needs to be calibrated include: r , γ , $\bar{t}$, $\bar{\Delta}$, β , σ , η , ϕ , and $c_{\min}$. Annual interest rate r was set to be equal to 2%, which corresponds to the historical mean of twenty-year US government bonds. The administrative load γ was assumed to be equal 10%. This number is based on the results of the study of Mitchell et al (1997) who showed that on average the US insurance companies add 10% to the annuity price because of administrative load. Maximum issue age was set to be equal to 88 years. In general maximum issue age varies by insurance company and can go from 80 years old to mid-90s.

Minimum purchase requirement was set to \$2,500. This means in order to buy an annuity the individual should be willing to initiate a contract that will bring him at least \$2,500 per year or \$208 per month. Given prices produced by the model this is equivalent to minimum initial premium ($q\Delta$) of \$25,000 for 70 year old persons and \$11,000 for 88 year old persons. The minimum premium for life annuity varies by insurance companies and can go up to \$100,000. As an example, two big annuity distributors, Vanguard and Berkshire-Hathaway, put restrictions of \$20,000 and \$40,000 correspondingly on minimum premium for life annuity.

For preferences parameters β , σ , η , and ϕ , and consumption minimum floor $c_{\min}$ I used structural estimates from the DFJ's study. Later on I will report results for several alter-

native values of the coefficient of risk aversion and discount factor. Table 5 summarizes all the parameters values.

Table 5: Parameters of the model

Parameter	Value
Risk aversion σ	4
Discount factor β	0.97
Strength of bequest motive η	2360
Shift parameter ϕ	\$273,000
Interest rate r	2%
Administrative load γ	10%
Consumption floor $c_{\min}$	\$2,663
Maximum issue age $\bar{t}$	88 years
Minimum premium $\bar{\Delta}$	\$2,500

5.3 Initial distribution

Initial wealth (k_0) and preexisting annuity holdings (n_0) that individuals start their retirement with are calibrated from the AHEAD dataset. Since in the model individuals start their retired life at age 70 to calibrate initial wealth I used cohort that was aged 69-76 at 1993. The sample used for calibration includes only retired individuals who were single (divorced, separated or never married) at the time of the survey. The total number of observation is equal to 1114.

Initial financial wealth (k_0) includes the value of housing and real estate, vehicles, value of business, IRA, Keogh, stocks, bonds, checking, saving and money market accounts, less mortgages and other debts. Preexisting annuity holdings (n_0) correspond to annuity-like income that an individual is entitled to receive during his retirement years. To measure annuity-like income I sum the values of Social Security benefits, defined benefit pensions, and annuities that individuals receive each year and then take the average over all years that individuals is observed in the data. This measure of preexisting annuity income also proxies permanent income (I). Since both Social Security benefits and DB payments are closely linked to lifetime earnings this provides a good approximation of permanent income.

The joint distribution of retirees over k_0 and n_0 was estimated using two-dimensional kernel density.

6 Results

This section presents results for different versions of the model above. It starts with the analysis of a model without medical expenses ($z_t = 0$ for $\forall t$), bequest motives ($\eta = 0$), minimum annuity purchase requirements ($\bar{\Delta} = 0$) and with full information annuity pricing. This simplified model is used to study annuitization decisions for people in different income quintiles given heterogeneity in life expectancy and initial wealth holdings.

This simplified version of the model is then augmented with, first, deterministic and then uncertain medical expenditures. The last model with uncertain medical expenditures is the baseline for further comparisons.

The following features of the baseline model are changed one-at-a-time. First, I drop full information assumption and require insurance firms to price annuities according to scenario two (asymmetric information). Second, I allow for a bequest motive. Third, I increase consumption minimum floor. Fourth, I assume housing wealth is completely illiquid. Finally, I consider the effect of the restrictions on minimum annuity purchase.

6.1 Simplified version of the model: no medical expenses

The model considered here only allows for lifespan uncertainty and preannuitized wealth coming from Social Security and DB pension plans. There are no medical expenses, bequest motives, minimum annuity purchase requirements and unfair annuity pricing.

Figures 1 and 2 show general pattern of annuity purchase for individuals who were given initial wealth and annuity holdings that correspond to the median values in the initial distribution for each permanent income quintile. Several things can be noticed in the graph.

First, people buy annuities only once in the first period. It can be shown (see Appendix) that under certain conditions one-time purchase of annuities in the first period is a general result. The conditions under which this result holds are the following:

- There is no uncertainty except the time of death
- Medical expenditures are constant
- $\beta(1 + r) < 1$
- $n_0 - z > c_{\min}$ or $k_0 + n_0 - z > q_1(c_{\min} - n_0 - z)$ where z is value of medical expenditures that doesn't change over time.

The last condition insures that individual is already guaranteed income that exceeds consumption minimum floor or can afford to get equivalent stream of income through buying annuity in the first period.

The intuition behind this theoretical result is as follows. There are two ways to finance annuity purchase: using financial wealth or existing annuity income. The second way

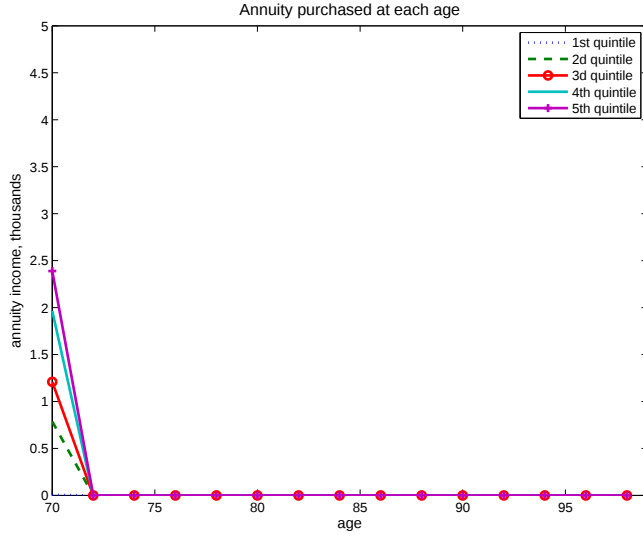


Figure 1: Annuities purchased by people in good initial health

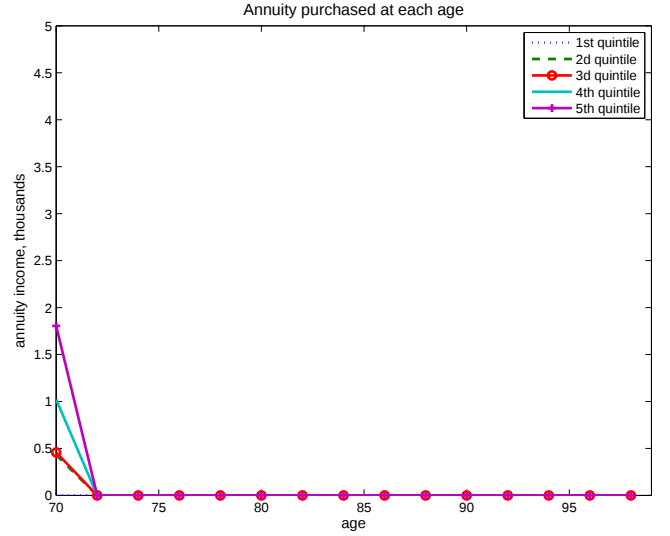


Figure 2: Annuities purchased by people in bad initial health

would imply increasing consumption profile which is not optimal given $\beta(1+r) < 1$. Thus if an individual buys annuity he will use his financial wealth. If the individual waits to buy annuities, he has to save in bonds. But this strategy is dominated by buying annuities from the start because over the long-run an annuity brings higher return.

Second, the amount of annuity bought is increasing with income quintile. People in the highest income quintile who start their retirement in good health buy the stream of annuity income equal to almost \$2,500 per year. People in the second income quintile buy the stream of annuity income equal to only \$700. The median investment of retirees in the lowest quintile is almost zero.

Third, people who start their retirement in bad health invest in annuity less than those retirees whose initial health is good. If healthy retirees in the highest income quintile buy annuity income equal to almost \$2,500, retirees in bad health in the same income quintile invest only \$1,700. This difference is explained by two factors. First, people in bad health have lower life expectancy. Second, people in bad health start their retirement with lower wealth.

To isolate the effect of survival probability on annuity demand I run two counterfactual experiments. In the first experiment individual who starts his retirement in good health was given survival probability of a person whose health is always bad without changing initial wealth. In the second experiment the survival probability was set to be equal to survival probability of someone who is always in bad health and in the lowest income quintile. The results of these experiments are presented in the second column of Table 6 for retirees in the top income quintile (the results for other quintiles are similar)

Everything else equal, the reduction in survival probability decreases demand for

Table 6: Impact of survival probability on annuity demand

Survival probability	Price is fixed	Price is adjusted
Own	2.50	2.50
Always in bad health	1.50	3.50
Always in bad health and lowest income quintile	0.25	5.00

annuities: in the first experiment annuity purchase decreases from \$2,500 to \$1,500 and in the second experiment - to \$250. It is important to note, however, that in these experiments change in the survival probability did not have any effect on price: the price for the annuity was fixed at the level of a price that retiree whose initial health is good faces. In a next set of experiments I change this assumption by allowing the price of annuity to adjust to changes in the survival probability. The results are presented in the third column of Table 6. As can be seen in this case the decrease in survival probability actually *increases* demand for annuities: in the first experiment the amount of annuity bought increases from \$2,500 to \$3,500 and in the second - to \$5,000. This is explained by the fact that decrease in the survival probability triggers two effects: it increases the effective discount factor making people care less about future consumption, and it decreases the price for annuities making future consumption more affordable. The income effect from decreasing price turns out to be more powerful (more on this see Appendix).

Given that in the simplified version of the model considered in this subsection insurance firms have full information about the annuity buyer, prices are based on the individual survival probabilities. Individuals who start their retirement in bad health face lower annuity prices and this should increase their demand for annuity. Lower demand for annuities from the part of this group results from the fact that they have less initial wealth not lower survival probability.

Apart from the amount of annuities bought, another dimension to consider is how many people do invest in annuity. Second column of Table 7 shows the percentage of retirees in each income quintile who have non-zero investment in annuity. First thing to notice is that overall participation rate is very high: 75.3% of retirees invested in annuities despite the fact that they already have part of their wealth annuitized through Social Security and DB pension plans.

For comparison, the third column of Table 7 presents results for the case where individuals do not have any preannuitized wealth. More specifically, initial wealth k_0 was increased by the amount $q_1 n_0$ and annuity holdings n_0 were set equal to zero. In other words retirees were given additional liquid wealth which corresponds to the market

Table 7: Participation in annuity market: model without medical expenses

Income quintile	Percentage	
	With preannuitized wealth	Without preannuitized wealth
All	75.3	91.0
1	51.0	85.3
2	86.7	100.0
3	78.6	100.0
4	81.3	100.0
5	75.5	100.0

value of the annuity income they are entitled to. One can see that in this case the overall participation rate increases to 91%. Except for the first income quintile all retirees buy annuities. The fact that around 15% of people in the first quintile do not invest in annuity is explained by the fact that they rely on minimum consumption floor provided by the government. The comparison between second and third columns of Table 7 shows that preannuitized wealth has significant effect on annuity market participation rate. However even in the presence of Social Security and DB pension plans the interest in private annuities is still high.

Another important observation is that in the model with no medical expenses we do not see a monotone relationship between annuity market participation rate and income quintile that can be observed in the data. On the contrary two lowest participation rates are observed among people in the highest (fifth) and the lowest (first) income quintiles. Around 75% of the retirees in the fifth income quintile choose to invest in annuity and among first income quintile this number is equal to 51%. For comparison, among people in the second income income quintile participation rate is around 87%. The relatively low participation rate of the highest income quintile is not surprising in this environment. People in the highest income quintile have considerable amount of annuity income already and given the absence of precautionary motive they choose not to invest in annuity anymore.

In general what matters for annuity demand in the environment with no uncertainty except the time of death, no bequest motives and actuarially fair prices is the ratio of liquid assets that people have to their annuity-like income (more on this see Appendix). The necessarily condition for retiree to invest in annuity is that the annuity income he is entitled to is relatively small and the amount of liquid wealth is relatively big. Table 8 shows the share of annuity income in total amount of available resources $\frac{n_0}{k_0 + n_0}$ separately for people who did buy annuity and for those who didn't. Those retirees in

Table 8: Share of annuity-like income in available resources

Quintile	Retirees who bought annuity	Retirees who didn't buy annuity
1	20.9	36.3
2	36.6	94.7
3	33.8	90.8
4	32.3	91.4
5	24.6	88.1

top four income quintiles who decide not to invest in annuities have few resources apart from annuity income: the ratio of annuity income to total available funds is around 90%. For people who chose to invest in annuity this ratio is much lower: it does not exceed 40%. The only quintile where this is not true is the bottom one. Even those people who choose not to invest in annuity have this ratio below 40%. It is important to note that for this quintile the actual annuity income that they take into account is the income from the means-tested transfers. The majority of this people (around 68%) cannot afford to buy annuity that will guarantee them income higher than consumption minimum floor.

6.2 The effect of medical expenditures

6.2.1 Deterministic medical expenditures

The next question is how the pattern of annuity holdings changes when medical expenditures are introduced? Figures 3 and 4 show the results for the case when retirees are facing medical expenditures that are deterministically increasing over time. That is to say ϵ_t in expression (1) is set to zero and mean of the medical expenditures is adjusted to match the mean in the stochastic case.

One can notice immediately that the pattern of annuity purchases change compared to the case of no medical expenditures. People still buy annuities in the beginning of retirement but they also increase annuity holdings towards the end of life. The retirees now have to save more because they have to finance not only their consumption but also medical expenditures that are increasing over time. In this case retirees use their annuity income to buy more annuities. Survival probability is decreasing with age and as annuity becomes cheaper it is more actively used to finance medical expenses.

The fraction of retirees involved in annuity market increases in the presence of deterministic medical expenses: Table 9 presents participation rates in the beginning of retirement and in the last period when individuals are allowed to buy annuity (88 years).

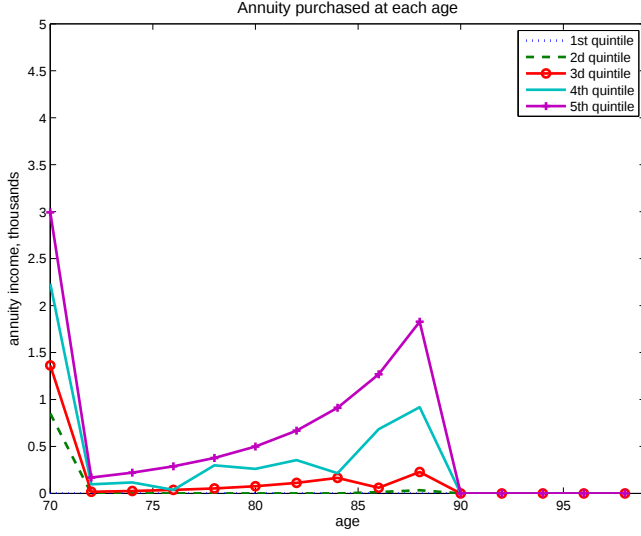


Figure 3: Deterministic medical expenses: annuities purchased by people in good initial health

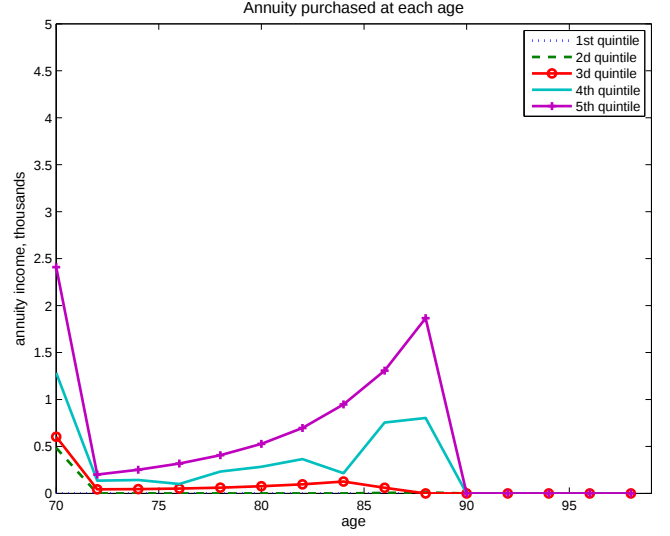


Figure 4: Deterministic medical expenses: annuities purchased by people in bad initial health

The participation rate at age 70 goes up from 75 to 86% and participation rate at age 88 increases from 0 to 77%.

Table 9: Participation in annuity market: deterministic medical expenses

Income quintile	Age 70	Age 88
All	86.1	77.2
1	32.4	0.9
2	90.2	63.8
3	100.0	50.5
4	100.0	99.4
5	99.6	99.2

Thus if anything the presence of deterministic medical expenditures increases demand for annuities in a significant way. This means that medical expenditures by itself cannot be an explanation for the annuity puzzle. The next section will consider how the uncertainty associated with medical expenditures affects annuity demand.

6.2.2 Uncertain medical expenditures

Introduction of uncertain medical expenditures enforces the pattern of annuities purchase observed in the previous experiment: the demand for annuities increases substantially in the advanced ages (see Figures 5 and 6). To understand why retirees now buy even more annuities in advanced ages than in the deterministic case one has to remember that

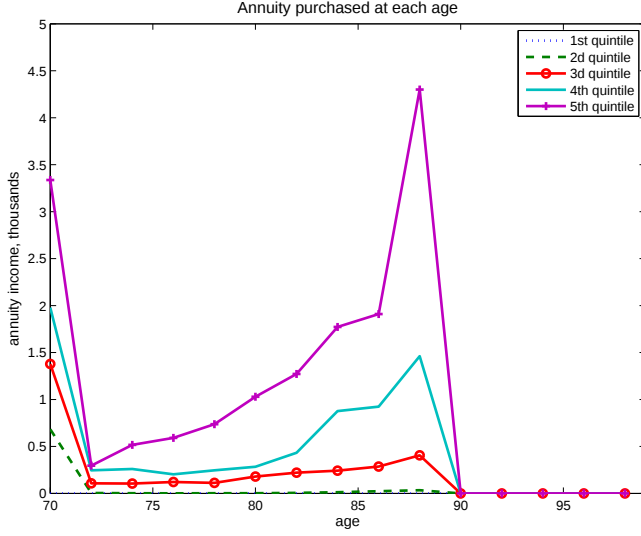


Figure 5: Uncertain medical expenses: annuities purchased by people in good initial health

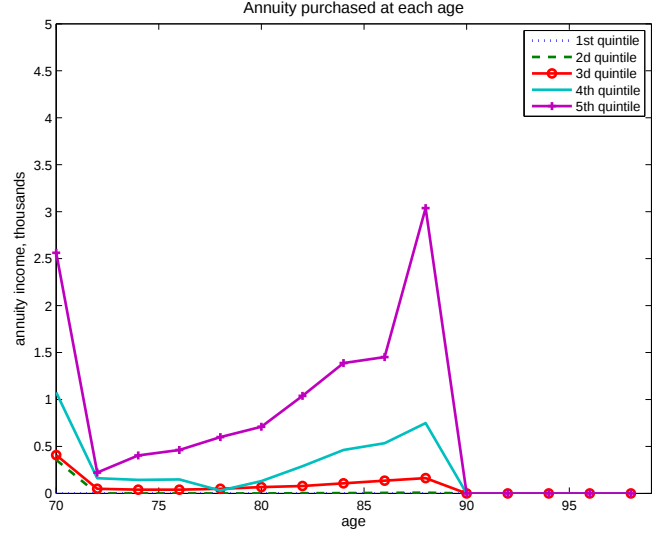


Figure 6: Uncertain medical expenses: annuities purchased by people in bad initial health

the stochastic component of medical expenditures is highly persistent. Thus with some probability retirees will face much higher medical expenses than in the deterministic case.

The participation rate drops comparing to the case of deterministic medical expenditures but still stays higher than in the case of no medical expenditures (see Table 10). Thus uncertainty in medical expenses results in two things. Some retirees, mostly in the lower income quintiles, give up on fully financing their health costs and decrease participation in the annuity markets. At the same time retirees who choose to self-insure against medical shocks increase their annuity holdings.

Table 10: Participation in annuity market: uncertain medical expenses

Income quintile	Age 70	Age 88
All	76.3	72.1
1	40.7	11.1
2	80.7	58.9
3	83.8	76.3
4	85.9	81.9
5	84.8	85.8

One interesting result is that introduction of *deterministic* medical expenses increases participation in the annuity market in each quintile except the bottom one where participation actually decreases. Introduction of *uncertainty* associated with medical expenses decreases participation rate in each quintile except the bottom one where the participation increases. This behavior of people in the bottom quintile is explained by their

reliance on government transfers. Introduction of deterministic medical expenses makes it easier to qualify for means-tested transfers. If medical expenses are uncertain some group of people may not meet requirement for receiving government transfers in case their medical shocks are small. Thus they have to finance their old age consumption themselves so they increase annuity market participation.

Another dimension worth considering here is what is the share of annuity investment in retiree's portfolio. Table 11 shows this percentage for individuals with median wealth holdings in the beginning of retirement for three cases: no medical expenses, deterministic and uncertain medical expenses.

Table 11: Percentage of annuity investment in retiree's portfolio at age 70

Income quintile	Medical expenses		
	None	Deterministic	Uncertain
1	0.0	0.0	0.0
2	100.0	100.0	73.0
3	100.0	100.0	92.5
4	100.0	100.0	81.8
5	61.4	100.0	86.9

The median retiree in the bottom income quintile does not invest in annuity in all three cases. Among other quintiles only retirees in the fifth quintile did not save entirely in annuities in case of no medical expenses. This means that since people in this quintile are entitled to big annuity income already they prefer to decumulate part of their financial wealth in several years as opposed to lifetime investment in annuity. In the case of deterministic medical expenses annuity clearly dominates bond for each income quintile. Finally, in the case of uncertain medical expenses individuals prefer to hold part of their wealth in liquid form in order to be able to finance bad medical shocks.

6.2.3 Summary of the effect of medical expenses

Table 12 presents a summary of the effect of medical expenses on the participation rates. In general if anything uncertain medical expenditures make annuity puzzle harder to explain. Uncertain medical expenditures do decrease the annuity market participation rate in the lower income quintiles but in aggregate it is more than compensated by increase in demand for annuities from the high income quintiles. For people who can afford to finance their medical expenses they will do it at least partially through buying annuities. Annuity pays out in a state when the individual is old and alive, and this state

coincides with the state when he is likely to have high medical expenses. Thus insuring medical expense risk and longevity risk becomes complementary.

Table 12: Participation in annuity market at age 70: summary of the effect of medical expenses

Income quintile	Medical expenses		
	None	Deterministic	Uncertain
All	75.3	86.1	76.3
1	51.0	32.4	40.7
2	86.7	90.2	80.7
3	78.6	100.0	83.8
4	81.3	100.0	85.9
5	75.5	99.6	84.8

The version of the model with uncertain medical expenses, fair prices, and no bequest motives is the baseline for comparison for all further experiments. The pattern of annuity purchased observed in Figures 5 and 6 is robust to all subsequent changes in the baseline model so I omit graphs of annuity purchase in retirement in the further analysis.

6.3 Effect of adverse selection

In this section I consider a version of the model when insurance firms price annuity according to scenario two. This means they do not observe any characteristics of an annuity buyer except age. In this case there exists one pooling price that is above actuarially fair for people with high mortality and below actuarially fair for people with low mortality. Since mortality is negatively correlated with permanent income this means in the pooling equilibrium higher quintiles will face better prices and thus get an implicit subsidy from low income quintiles. Table 13 provides a quantitative assessment of this subsidy. For people in the lowest income quintile and bad health the pooling equilibrium price is around 73% higher than the price they face if the insurance firms observe their mortality. On the other extreme people in the highest quintile and in good health pay for annuity 13% less than in the full information equilibrium.

As a result the participation of high income quintiles in the annuity market increases and the participation of low quintiles decreases. Table 14 compares the participation rates with the baseline model. In the bottom income quintile the fraction of people buying annuities at age 70 drops from 40.7% to 30.8%. In the highest quintile this number

Table 13: Percentage change in price in pooling equilibrium compared to full information equilibrium

Income quintile	Bad health	Good health
1	73.2	25.7
2	53.5	14.0
3	35.7	3.7
4	20.1	-5.3
5	6.8	-13.1

increases from 84.8% to 93.5%. The decline in the participation rates among low income quintiles is partially offset by increase in the participation rates among low quintiles. Thus the overall effect of the adverse selection is quite small: in the pooling equilibrium the percentage of people involved in the annuity market at age 70 decrease by less than 4% comparing to the baseline model.

Table 14: Participation in annuity market at age 70: baseline model vs. model with adverse selection

Income quintile	Baseline	Adverse selection
All	76.3	72.5
1	40.7	30.8
2	80.7	63.1
3	83.8	74.7
4	85.9	89.9
5	84.8	93.5

6.4 Effect of bequest

Annuity is a financial instrument that pays out only in a state when individual is alive. Bequest motive makes individual care about a state when he is not alive and thus decreases attractiveness of annuity. In theory very strong bequest motive can drive demand for annuity to zero. However the empirical evidence suggests that bequest is a luxury good, moreover, only saving decisions of people in the highest income quintile get affected by bequest in a significant way (De Nardi, French, Jones, 2009).

To get some idea of the effect of bequest motives this section uses recent estimates of De Nardi, French, Jones, 2009. Using AHEAD dataset DFJ found that the strength of the bequest motive (η) is equal to 2360 and the shift parameter (ϕ) is equal to \$273,000. The

easiest way to understand the intuition behind these parameters' values is to consider a two period deterministic model where an agent lives for one period and dies with certainty in the second period. In this case using the estimated values for η and ϕ one can compute that only individuals with wealth above \$38,000 will leave a bequest. Once bequest motive become operative its effect is very strong: 88 cents of every additional dollar of wealth will be bequeathed (more on this see De Nardi, French, Jones, 2009).

Table 15 illustrates the effect of bequest motives on participation rates: overall the percent of retirees involved in the annuity market decreases by around 4%. This decrease mostly comes from the highest income quintile. While participation rates for individuals in the first four quintiles are almost not affected the fraction of people buying annuity in the fifth income quintile decreases by almost 16%: from 85.8 to 69%. This is not surprising given high value of the shift parameter ϕ : only people in the highest income quintile get significantly affected by a bequest motive.

Table 15: Participation in annuity market at age 70: baseline model vs. model with bequest motives

Income quintile	Baseline	Bequest motive
All	76.3	71.9
1	40.7	40.6
2	80.7	79.7
3	83.8	83.2
4	85.9	85.7
5	84.8	69.0

6.5 Effect of minimum consumption floor

Consumption minimum floor provides alternative annuity income and the question is how well does it substitute private annuity? In the baseline model of this study government guaranteed consumption minimum floor was set to be equal to \$2,700. This number is on the low side of what is commonly used in the literature (see Kitao, Jeske 2009, Kopecky, Koreschkova 2009). Table 16 demonstrates how participation in the annuity market changes when the consumption floor is raised to \$6,000.

The increase in the consumption minimum floor has a significant effect on overall participation rate: it drops from 76.3 to 53.1%. This decrease mainly comes from the three bottom income quintiles: top two quintiles are affected much less. If in the first quintile increase in consumption minimum floor leads to a decline in participation rate

Table 16: Participation in annuity market at age 70: baseline model vs. model with $c_{\min} = \$6000$

Income quintile	Baseline	$c_{\min} = \$6000$
All	76.3	53.1
1	40.7	8.6
2	80.7	28.8
3	83.8	61.5
4	85.9	73.5
5	84.8	82.9

from 40.7 to 8.6%; in the highest quintile the participation rate goes down from 84.8 to 82.9%. This asymmetric effect is not surprising. People in the highest quintile start their retirement with annuity income that is significantly higher than consumption minimum floor. For them the probability to qualify for means-tested government transfers is low: it happens only if they get hit by a bad sequence of medical shocks. Hence the size of the consumption minimum floor has small effect on their decisions to annuitize. People in the lower quintiles have more chances to rely on government transfers and the increase in public longevity insurance has a big effect on their demand for private annuity.

6.6 Effect of illiquid housing

In the next version of the model I assume that housing wealth is completely illiquid. In other words all the housing wealth individual retirees with stays unchanged throughout the retirement. The assumption that retirees cannot use their housing wealth even to finance medical expenses is the other extreme from the assumption that households can consume out of their housing wealth maintained before. It was shown (Walker, 2004) that housing wealth in retirement is not used to finance consumption, however, it may be used to finance catastrophic medical expenditures. Given computational difficulties with treating housing wealth as a separate state variable, this paper considers two alternative assumptions of perfect liquidity and absolute illiquidity of housing wealth. This may give some idea about the magnitude of the bias of imposing any assumption of this sort. What is important for this study, however, is that housing wealth is not equal to non-housing wealth in terms of the costs of converting it into annuity.

Table 17 shows the percentage of retirees involved in the annuity market if housing wealth is assumed to be illiquid. The ban on using housing wealth decreases participation rate though the effect is not big: on average only around 3% of retirees leave the market. Illiquidity of housing wealth uniformly affect all quintiles though the biggest effect it has on the retirees in the bottom quintile: their participation rate drops by 10%. This big

effect is explained by the fact that for this group of people housing constitutes a major part of their retirement wealth. In the next section I will show that the illiquidity of housing wealth has a much bigger effect on the participation rate once minimum annuity purchase requirement is introduced.

Table 17: Participation in annuity market at age 70: baseline model vs. model with illiquid housing

Income quintile	Baseline	Illiquid housing
All	76.3	73.2
1	40.7	30.5
2	80.7	74.9
3	83.8	78.6
4	85.9	80.3
5	84.8	77.5

6.7 Effect of minimum purchase requirements

In this section I assume insurance firms still price annuity according to scenario one but this time they set minimum purchase requirement equal to \$2,500 (in 2000 dollars). The results of introducing minimum purchase requirement in the baseline model are presented in Table 18. The decline in participation rates is very noticable: percentage of people involved in the annuity markets drops from 76.3 to 39.9%. The effect of minimum purchase requirement on the participation rate becomes even bigger once housing wealth is assumed totally illiquid (third column of Table 18). The participation in annuity market at age 70 drops to 24.2%.

The large effect the restriction on minimum investment in annuity on participation rates results from the fact that a lot of retirees involved in annuity market end up investing only small amounts. One can recall from Figures 5 and 6 that the median annuity purchase is below minimum purchase requirement for all income quintiles except the top one. But even in the highest income quintile for considerable group of retirees the optimal amount of investment in annuity is below the minimum one: the participation in this quintile decreases from 84.8 to 57.4% in case with liquid housing. This reduction is accounted for by people whose financial wealth is relatively small compared to their preexisting annuity income.

Given that on average retirees are willing to participate in the annuity market but are not willing to invest a lot, one question to ask is: how much the individuals value

Table 18: Participation in annuity market at age 70: baseline model vs. model with minimum purchase requirement

Income quintile	Baseline	Minimum purchase	
		Liquid housing	Illiquid housing
All	76.3	39.9	24.2
1	40.7	23.9	11.2
2	80.7	32.5	16.1
3	83.8	35.4	18.6
4	85.9	43.1	23.7
5	84.8	57.4	36.4

access to the annuity market? Table 19 shows how much a median retiree has to be compensated in terms of the percentage of consumption if he loses access to the annuity market.

Table 19: Value of annuity market for median retiree in each income quintile (in percent of consumption)

Income quintile	Good health	Bad health
1	0.0	0.0
2	2.4	0.7
3	2.3	0.3
4	6.3	2.8
5	8.9	7.3

Not surprisingly the value of the annuity market for retiree increases with income quintile and health status. If a median retiree in the lowest income quintile is indifferent between having access to the annuity market or not, for retiree in good health and highest income quintile the loss of opportunity to invest in annuities is equivalent to 8.9% of consumption. In general for people in good health and high income quintiles the opportunity to have access to the annuity market is very valuable. Annuities bring higher return over the long run and can help to finance late in life medical expenses. However, the amount of money that retirees can actually spare to invest in annuity is not big. The issue of liquidity constraint is accentuated by the need to keep part of the wealth as a buffer stock against medical shocks and impossibility to use housing wealth.

6.8 Summary

The quantitative effect of different explanations for non-annuitization is summarized in Table 20. In terms of *overall* participation rate three factors have the largest effect: pre-annuitized wealth, minimum purchase requirement and consumption minimum floor. In terms of participation rates *by quintile* consumption minimum floor affects mostly people in low quintiles while the impact of minimum purchase requirement and preannuitized wealth is approximately uniform. Bequest motives have a significant effect only on people in the highest income quintile. The impact of illiquidity of housing wealth is large only in combination with minimum purchase requirement. Adverse selection and medical expenses have significant effect on participation rates in each quintile but this effect has opposite sign for low- and high-income group. For people in low income quintile both medical expenses and adverse selection reduce participation in the annuity markets while for people in high income quintiles the effect goes the other way.

Table 20: Quantitative importance of different factors behind non-annuitization

Factors	Impact on low quintiles	Impact on high quintiles	Overall effect
Preannuitized wealth	Big (negative)	Big (negative)	Big (negative)
Medical expenses	Big (negative)	Big (positive)	Small (positive)
Adverse selection	Big (negative)	Big (positive)	Small (negative)
Bequest	None	Big (negative)	Small (negative)
Consumption floor	Big (negative)	Small (negative)	Big (negative)
Illiquid housing	Small (negative)	Small (negative)	Small (negative)
Minimum premium	Big (negative)	Big (negative)	Big (negative)

6.9 Combined effect

Now that the separate effects of different factors behind non-annuitization is quantified the next step is to evaluate how much all these factors combined can contribute to the annuity puzzle. To do this I consider the full version of the model: in this version insurance firms are assumed to price according to scenario two (asymmetric information) and restrict minimum amount of annuity purchase. Agents face medical expense uncertainty, have bequest motives, and cannot use their housing wealth. Consumption floor guaranteed by the government is set to be equal to \$6,000 per year. Table 21 compares annuity market participation rates at age 70 produced by the full model with the baseline model and with participation rates observed in the data for people aged from 70 to 80 years old.

Table 21: Participation in annuity market at age 70: baseline model vs. model with all impediments to annuitization

Income quintile	Baseline	Full version	Data
All	76.3	19.6	6.2
1	40.7	1.9	1.2
2	80.7	6.5	1.3
3	83.8	12.8	2.7
4	85.9	20.8	4.5
5	84.8	35.3	13.0

As can be seen the model combining different factors behind non-annuitization goes a long way in explaining low participation rates in the annuity market comparing to the baseline model. However the predicted participation rates are still higher than those observed in the data.

Another way to look at the performance of the model is to compare prices it produces with the ones observed in the data. Figure 7 shows that prices in the model line up well with what we see in the data, though in general the model tends to underpredict the real prices.

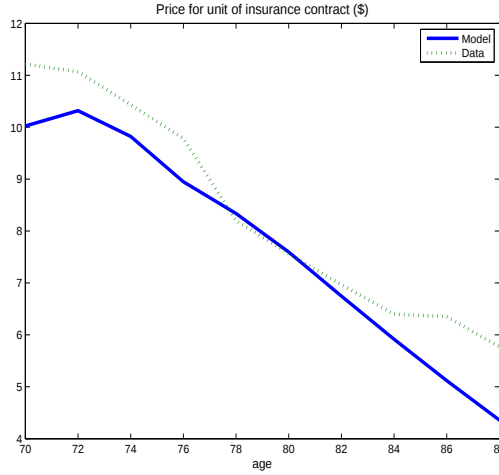


Figure 7: Prices in the model and in the data

There are at least three explanations for this downward bias of the prices in the model. First, the model considers only single individuals while real prices are based on the aggregate statistics. Second, the AHEAD dataset underrepresents wealthy individuals. Given that wealth is negatively correlated with mortality and that rich individuals tend to buy

more annuities the lack of wealthy individuals in the model biases the prices downwards. Third, this paper makes assumption that insurance firms are perfectly competitive and set administrative load to 10%. It may be that the markup insurance firms actually have is more than 10% due to higher administrative expenses or to the violation of perfect competition assumption.

An interesting question to consider is what participation rate would be if the individuals in the full version of the model face the prices based on the real data. Table 22 presents the results of this experiment. Though the participation rates noticeably decrease they are still not as low as the ones in the data.

Table 22: Participation in annuity market at age 70: data vs. model with real prices

Income quintile	Model	Data
All	13.1	6.2
1	1.7	1.2
2	6.6	1.3
3	11.6	2.7
4	19.3	4.5
5	21.2	13.0

Since apart from the preannuitized wealth the most powerful factors affecting the annuity market participation rate in the model are consumption minimum floor and minimum purchase requirement the question is how big the parameters $c_{\min}$ and $\bar{\Delta}$ should be in order to match the data? As a final experiment I consider the model that combines all impediments to annuitization, uses the real prices, sets consumption minimum floor to \$8,000 and minimum purchase requirement to \$3,000. The results are shown in Table 23. In this case the model performs much better in terms of matching the empirical participation rates: overall participation rate predicted by the model (7.3%) is only 1% higher than the ones observed in the data.

The close proximity of the participation rates from the data and from the model observed in Table 23 should be taken with caution. The model of this study underrepresents wealthy individuals who have higher demand for annuities and thus biases the annuity market participation rates downwards.

Table 23: Participation in annuity market at age 70: data vs. model

Income quintile	Model	Data
All	7.3	6.2
1	0.9	1.2
2	2.6	1.3
3	5.9	2.7
4	7.9	4.5
5	15.5	13.0

7 Conclusion

This study considers different explanations for the annuity puzzle in a quantitative heterogeneous agents model. It shows that in the absence of any impediments to annuitization all but the poorest retirees buy annuities. The Social Security and DB pension plans significantly decrease the annuity market participation rate but it still remains high.

Medical expenses uncertainty increases demand for annuities because health expenditures are increasing with age: the state of being old and alive coincides with the state of being old and having to pay high medical costs. Thus insurance against longevity risk and insurance against medical costs uncertainty compliment each other.

Adverse selection decreases demand for annuities for people in low income quintiles because their mortality is high and in equilibrium they face unfair prices. However the equilibrium price is not big enough to crowd out high mortality types out of the market. Thus high-income quintiles face lower prices which increase their demand for annuities.

The quantitative effect of bequest motives is not big because it affects the demand for annuities only from the part of the highest income quintile. Public longevity insurance provided through various means-tested transfers play a big role in reducing demand for private annuity, especially from the part of low income quintiles.

In general people place high value on opportunity to invest in annuity. However, the amount of annuity retirees are actually buying is small. This is the result of the fact that annuity brings return for many years and upfront investment even when buying a stream of small annuity income is not a trivial sum of money. If insurance firms restrict the minimum amount individual can invest in annuity, it substantially decreases the number of retirees on the market, especially if housing wealth cannot be annuitized.

8 Appendix

8.1 Robustness check

This section checks the sensitivity of results to the change in two parameters: discount factor (β) and risk aversion (σ). Table 24 compares participation rates in annuity market with the baseline model when β is first raised to 0.98 and then decreased to 0.8. Note, that in the first experiment β was increased to the point when $\beta(1+r) = 1$.

Table 24: The sensitivity of participation rates to change in discount factor

Income quintile	Baseline	$\beta = 0.98$	$\beta = 0.80$
All	76.3	81.2	43.0
1	40.7	43.9	26.4
2	80.7	85.1	44.1
3	83.8	90.4	45.8
4	85.9	90.1	47.2
5	84.8	89.9	48.3

Not surprisingly, increase in discount factor raises annuity market participation rates and decrease in discount factor has the opposite effect. Patient individuals save more and so they are more interested in annuity as one of the available saving options. The overall increase in participation rate due to higher discount factor is equal to 5%. When discount factor goes down to 0.8 the participation rate drops almost twice. In general, increase in discount factor makes annuity puzzle harder to explain, though the effect is not big for ranges where $\beta(1+r) \leq 1$.

Table 25: The sensitivity of participation rates to change in risk aversion

Income quintile	Baseline	$\sigma = 2$	$\sigma = 6$
All	76.3	69.3	77.4
1	40.7	32.8	44.8
2	80.7	69.2	83.3
3	83.8	74.1	82.6
4	85.9	81.5	88.6
5	84.8	81.6	82.6

In the next experiment risk aversion was, first, set to 2, and then raised to 4. Table 25

compares the results of these experiments with the baseline model. When risk aversion decreases agents want to save less, and thus, buy less annuity. Increase in risk aversion has the opposite effect. Higher risk aversion makes annuity puzzle harder to explain, but again, the effect is not big.

8.2 Computation

I solve the model using backward induction. In the last period ($t = T$) the value function (and policy functions) can be solved for analytically. For every age t prior to T and for each point in the state space I optimize with respect to bonds and annuities. Two-dimensional optimization problem is solved applying, first, coarse grid search, and then Brent method along both dimensions. The numerical integration of tomorrow's value function was performed using Gauss-Legendre quadrature. This numerical integration was complicated by the fact that value function is not concave. The non-concavity arises because each period agent can run out of assets due to the medical expense shocks. The probability to get into this state is endogenous. To address this problem, in numerical integration "kink point" were identified for each grid of tomorrow's assets. "Kink point" is such a value of tomorrow's medical shock that agent can finance her medical expenses and be left exactly with amount equal to consumption minimum floor. Then value function was integrated separately to the left and to the right of the "kink point". I evaluate the value function for points outside the state space grid using a linear interpolation.

8.3 Theory

TO BE COMPLETED

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