Trade Dynamics in the Market for Federal Funds

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February 2010

Abstract

We develop a search model of the federal funds market and show that, at each point along the trading session, rates are increasing in the penalty for reserve defficiencies, decreasing in the borrower's bargaining power, and when there are more (less) lenders than borrowers, also decreasing (increasing) in the frequency of meetings. We also study the conditions that shape the time path of the fed funds rate throughout a trading session, and identify the factors that can cause rates to rise or to fall with the time remaining until the end of the trading day.

Keywords: Fed funds market, search, bargaining, over-the-counter
JEL Classification: G1, C78, D83, E44

1 Introduction

Interbank markets facilitate the transfer of funds across the banking system, channeling bank liquidity from institutions with excess balances to those in need of funds. Disruptions to these markets can impede the efficient allocation of liquidity and have the potential to impair the functioning of the financial system. Interbank markets also play a key role in the execution of monetary policy and thus having a well-functioning interbank market is of utmost importance as has been highlighted during the recent financial crisis.[1]

The interbank market, known as the federal funds market in the United States, is an over-the-counter market for unsecured loans (fed funds) traded mostly overnight. To understand how these loans are purchased and sold as well as the characteristics of these transactions we develop a search-theoretic model in which banks are required by the central bank to hold a certain level of end-of-day balances and participate in the fed funds market to achieve this target. In this over-the-counter market, banks randomly contact trading partners and once they meet they bargain over the terms of the loans.

We find that as a borrower gets a stronger position in the bargaining process, the rate the borrower pays for the loan decreases. Also, as banks become more patient (e.g., as measured by their rate of time preference) they become willing to trade at lower rates. We also find that a reform that makes the penalty for holding end-of-day balances below required balances more severe leads to an overall increase in fed funds rates. Similarly, a rise in the overnight overdraft rate charged by the central bank when an institution reaches the end of the operating day with negative balances, causes an increase in the rates banks are willing to pay for fed funds. We also show that rates are decreasing (increasing) in the frequency of meetings between banks when there are more (fewer) lenders than borrowers. Finally, we study the conditions that shape the time path of the fed funds rate throughout a trading session, and identify the factors that can cause rates to rise or to fall with the time remaining until the end of the trading day.

This paper is related to the literature that studies search and bargaining frictions in financial markets. Search-theoretic models such as the frameworks introduced in labor markets by Diamond (1982a), Diamond (1982b), Mortensen (1982) and Pissarides (1985) have been broadly used in different areas of economics and were pioneered in asset pricing by Duffie, Gârleanu, and Pedersen (2005) to model trading frictions characteristic of over-the-counter markets. Their

[1] See Acharya and Merrouche (2009) and Afonso, Kovner and Schoar (2010) among others.

work has been generalized by Lagos and Rocheteau (2007, 2009) to allow for general preferences and unrestricted long positions and by Weill (2008) and Vayanos and Wang (2007) to allow investors to trade multiple assets. Duffie, Gârleanu, and Pedersen (2007) extends their setting to incorporate risk aversion and risk limits and Afonso (2009) endogeneizes investors' entry decision to the market.

Our work is also related to the early theoretical research on the federal funds market which includes the micro-model of Ho and Saunders (1985) and the stochastic general equilibrium model of Coleman, Christian and Labadie (1996). To our knowledge our paper is the first to present a search-theoretic model that captures the over-the-counter features of the federal funds market.

2 The model

There is a large population of agents that we refer to as *banks*, each represented by a point in the interval $[0,1]$. Banks hold integer amounts of an asset that we interpret as *reserve balances*, and can negotiate these balances during a trading session set in continuous time that starts at time 0 and ends at time T. We use τ to denote the time remaining until the end of the trading session, so $\tau = T - t$ if the current time is $t \in [0, T]$. The reserve balance that a bank holds (e.g., at its Federal Reserve account) is denoted by $k(\tau) \in \mathbb{K}$, with $\mathbb{K} = \{-M, ..., -1, 0, 1, ..., N\}$, where N and M are natural numbers. Each bank starts the period with a given balance $k(T) \in \mathbb{K}$, and a target balance $\bar{k} \in \mathbb{K}$ to be held at time T. We let $n_{k\bar{k}}(\tau)$ denote the measure of banks who have target $\bar{k}$ but hold balance k at time $T - \tau$. The initial distribution of balances, $\{n_{k\bar{k}}(T)\}_{k\in\mathbb{K}}$, is given. Let $v_k \in \mathbb{R}$ denote the flow payoff to a bank from holding k balances during the trading session, and let $U(k) \in \mathbb{R}$ be the payoff from holding k balances at the end of the trading session. All banks discount payoffs at rate r.

Banks can trade with each other in an over-the-counter market where meetings are bilateral and random, and represented by a Poisson process with arrival rate $\alpha > 0$. We model these bilateral transactions as loans of reserve balances. Once two banks have contacted each other, they bargain over the size of the loan of reserve balances made by the lender and the quantity of reserve balances to be repaid by the borrower. We assume that every loan gets repayed at a fixed time $T + \Delta$ in the following trading day, with $\Delta \in \mathbb{R}_+$. After the transaction has been completed, the banks part ways. Let $\boldsymbol{x} = \{x_m\}_{m=0}^{\infty}$ be the gross credit position of a bank, where $x_m \in \mathbb{R}$ denotes the size of a loan (received if negative, made if positive) with m remaining time until maturity, i.e., if the current time is t, x_m is a liability or a credit that the bank will settle at time $t+m$. Given a history of trades that left the bank with a gross credit position $\boldsymbol{x}$, the implied present discounted net credit position is $\sum_{m=0}^{\infty} e^{-rm} x_m \equiv x$. Hence, if a bank with a net credit position x makes a new loan at time $T-\tau$ which entitles the bank to collect R at time $T+\Delta$, then the post-transaction discounted net credit position is $x + e^{-r(\tau+\Delta)} R$.

3 Institutional features of the market for federal funds

Depository institutions keep reserve balances at Federal Reserve Banks to clear financial transactions and to meet requirements. The market for federal funds is a market for unsecured loans of reserve balances at the Federal Reserve Banks that allows participants with excess reserve balances to lend balances (or *sell funds*) to those with reserve balance shortages. These unsecured loans, or *fed(eral) funds*, are traded mostly overnight at a rate known as the *fed funds rate*. The fed funds market is an over-the-counter market: in order to trade, a financial institution must find a willing counterparty, and then negotiate the size and rate of the loan bilaterally. We use a search-based model to capture the over-the-counter nature of this market.[2]

Fed funds loans are settled through Fedwire Funds Services (Fedwire), a large-value real-time gross settlement system operated by the Federal Reserve Banks. More than 7,000 Fedwire participants can lend and borrow in the fed funds market including commercial banks, thrift institutions, agencies and branches of foreign banks in the United States, federal agencies, and government securities dealers. In 2008, the average daily number of borrowers and lenders was 164 and 255, respectively.[3]

In practice, the terms of a loan can be negotiated directly between participants or indirectly through a fed funds broker. According to Ashcraft and Duffie (2007), non-brokered transactions represented 73% of the volume of federal funds traded in 2005, and tend to be more common among the largest banks. Consequently, in our baseline model we abstract from brokers and focus on direct lending between banks.

The Federal Reserve imposes a minimum level of reserves on banks and other depository

[2]There is a growing search-theoretic literature on financial markets which includes Afonso (2009), Duffie, Gârleanu, and Pedersen (2005, 2007), Gârleanu (2009), Lagos and Rocheteau (2007, 2009), Lagos, Rocheteau, and Weill (2010), Miao (2006), Rust and Hall (2003), Spulber (1996), Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008), just to name a few. See Ashcraft and Duffie (2007) for a discussion of the over-the-counter nature of the fed funds market.

[3]See Afonso, Kovner and Schoar (2010).

institutions, all of whom we refer to as *banks*, for brevity. This reserve balance requirement applies to the average level of a bank's end-of-day balances during a two-week maintenance period.[4] Banks typically target an average daily level of end-of-day balances, which in our model is represented by $\bar{k}$. End-of-day balances within a period may vary but remain in general positive as overnight overdrafts are considered unauthorized extensions of credit, and are penalized. The rate charged on overnight overdrafts is generally 400 basis points over the effective federal funds rate.[5] On daily basis, banks target an average end-of-day balances and avoid overnight overdrafts. On October 9, 2008, the Federal Reserve began remunerating banks' positive end-of-day balances. Since December 18, 2008, the interest rate paid on both, required reserve balances, and excess balances, is 25 basis points (Federal Reserve, 2008). In our model, these policy considerations can be captured by the end-of-day payoffs $\{U(k)\}_{k\in\mathbb{K}}$. For example, in our quantitative work we will specify

$$U(k) = \begin{cases} i^r \bar{k} + i^e \left(k - \bar{k}\right) & \text{if } \bar{k} \leq k \\ -P^r & \text{if } \bar{k}_0 \leq k < \bar{k} \\ -\left[P^r + P^o + \left(\bar{k}_0 - k\right) i^o\right] & \text{if } k < \bar{k}_0, \end{cases}$$

where $\bar{k}_0$ is the overdraft threshold (typically equal to 0), $i^r > 0$ is the interest paid on required reserves, $i^e > 0$ is the interest paid on excess reserves ($i^r = i^e$ is the case currently in the United States), $i^o > 0$ is the overnight overdraft penalty rate, $P^r > 0$ is the pecuniary value of penalties for failing to meet reserve requirements, and $P^o > 0$ represents additional penalties resulting from the use of unauthorized credit.

Throughout the day, when an institution has insufficient funds in its Federal Reserve account to cover its settlement obligations, it can incur in a daylight overdraft up to an individual maximum amount known as net debit cap. To control the use of intraday credit, the Federal Reserve charges an overdraft fee currently equal to 36 basis points.[6] In our model, the flow payoff to a bank from holding non-negative balances during a trading session is captured by the vector $\{v_k\}_{k=0}^{N}$, e.g., a formulation in which banks are not remunerated for positive balances held during the day would have $v_k = 0$ for $k = 0, ..., N$. The flow payoff from incurring in daylight overdrafts is captured by $\{v_k\}_{k=-M}^{-1}$, e.g., a formulation in which banks are charged a daylight overdraft rate $i^d > 0$ would have $v_k = i^d k$ for $k = -M, ..., -1$.

[4] See Bennett and Hilton (1997) for an explanation of how these required operating balances are calculated.

[5] The penalty fee is increased by 100 basis points if there have been more than three overnight overdraft occurrences in a year.

[6] The fee will be increased to 50 basis points (annual rate) when the revised Payment System Risk policy is implemented in the fourth quarter of 2010 or the first quarter of 2011 (Federal Reserve, 2008a).

Fedwire operates 21.5 hours each business day, from 9.00 pm Eastern Time (ET) to 6.30 pm ET on the following calendar day. However, fed funds activity is concentrated in the last two hours of the operating day. On a typical day, institutions receive the returns corresponding to the fed funds loans sold the previous day before they send out the new loans. In 2006, the average value-weighted time of return was 3.09 pm $\pm$ 9 minutes, while the average time of delivery was 4.30 pm $\pm$ 7 minutes. The average duration of a loan was 22 hours and 39 minutes (Bech and Atalay, 2008). For simplicity, in our theory we take as given that every loan gets repaid at a fixed time $T + \Delta$ in the following trading day.

Until mid afternoon, a typical bank's transactions reflect its primary business activities. Later in the day, the trading and payment activity is orchestrated by the federal funds trading desk and aimed at achieving the target balance. In 2008, more than 75% of the value of fed funds traded among banks was traded after 4:00 pm. By this time, each bank has a balance of reserves resulting from previous activities which is taken as given by the bank's fed funds trading desk. We think of $t = 0$ as standing in for 4:00 pm, and model the distribution of reserve balances given to the bank's fed funds trading desk at this time with the initial condition $\{n_{k\bar{k}}(T)\}_{k\in\mathbb{K}}$.

Fed funds transactions are usually made in round lots of over a \$1 million (Furfine, 1999). The distribution of loan sizes is skewed to the right. In 2008, the average loan size was \$148.5 million while the median loan was \$50 million. The most common loan sizes were \$50 million, \$100 million and \$25 million. To keep the analytics tractable, we assume integer loan sizes in our model.

4 Equilibrium

Let $J_k(x,\tau)$ be the value function of a bank who holds k units of reserve balances, and whose discounted net credit position is x when the time until the end of the trading session is τ. Let $s = (k, x)$ denote the bank's individual state, then

$$J_k(x,\tau) = \mathbb{E}\left\{\int_0^{\min(\tau_\alpha,\tau)} e^{-rz} v_k dz + \mathbb{I}_{\{\tau_\alpha>\tau\}} e^{-r\tau} J_k(x,0)\right. \tag{1}$$

$$\left. + \mathbb{I}_{\{\tau_\alpha\leq\tau\}} e^{-r\tau_\alpha} \int J_{k-b_{ss'}(\tau-\tau_\alpha)}\left(x + e^{-r(\tau-\tau_\alpha+\Delta)} R_{s's}(\tau-\tau_\alpha), \tau-\tau_\alpha\right) dF\left(s', \tau-\tau_\alpha\right)\right\}$$

where $\mathbb{E}$ is an expectation operator over the exponentially distributed random time until the next trading opportunity τ_α, v_k is the flow payoff from holding k balances during a trading session, r is the payoffs discount rate, $s' = (k', x')$ is the individual state of the randomly drawn counterpart in the bilateral trade, and $\left(b_{ss'}\left(\tau-\tau_{\alpha}\right), R_{s's}\left(\tau-\tau_{\alpha}\right)\right)$ denotes the bilateral terms of trade, i.e., $b_{ss'}\left(\tau-\tau_{\alpha}\right)$ is the amount of balances that the bank with state s loans to the bank with state s' when the remaining time is $\tau-\tau_{\alpha}$, and $R_{s's}\left(\tau-\tau_{\alpha}\right)$ is the amount of funds that s' commits to deliver to s at the future date $T+\Delta$. The distribution $F\left(s', \tau\right)$ captures the heterogeneity of potential trading partners over individual states s' with τ time on the clock. The terminal value $J_k\left(x, 0\right)$ is given by

$$J_k\left(x, 0\right)=U\left(k\right)+x. \tag{2}$$

where $U(k)$ is the payoff from holding k balances at the end of the trading session.

In the appendix (Lemma 4) we show that

$$J_k\left(x, \tau\right)=V_k\left(\tau\right)+x, \tag{3}$$

where

$$\begin{aligned} V_k\left(\tau\right)=\mathbb{E}\Bigg\{ & \int_0^{\min\left(\tau_{\alpha}, \tau\right)} e^{-rz} v_k dz+\mathbb{I}_{\{\tau_{\alpha}>\tau\}} e^{-r\tau} V_k\left(0\right) \qquad (4) \\ & +\mathbb{I}_{\{\tau_{\alpha}\leq\tau\}} e^{-r\tau_{\alpha}} \int\left[V_{k-b_{ss'}\left(\tau-\tau_{\alpha}\right)}\left(\tau-\tau_{\alpha}\right)+e^{-r\left(\tau-\tau_{\alpha}+\Delta\right)} R_{s's}\left(\tau-\tau_{\alpha}\right)\right] dF\left(s', \tau-\tau_{\alpha}\right)\Bigg\}. \end{aligned}$$

Note that (2) and (3) imply $V_k\left(0\right)=U\left(k\right)$.

Next, consider a bank with individual state (k, x) who meets a bank with individual state (k', x'). Let $(b_{ss'}, R_{s's})$ denote the Nash bargaining outcome, where $b_{ss'}$ is the amount of balances that the bank with state s lends to the bank with state s', and $R_{s's}$ is the amount that the latter will repay the former at time $T+\Delta$ (so the discounted value of the repayment is $e^{-r(\tau+\Delta)}R_{s's}$). Then the gains from trade are

$$J_{k-b_{ss'}}(x+e^{-r(\tau+\Delta)}R_{s's}, \tau)-J_k\left(x, \tau\right)=V_{k-b_{ss'}}\left(\tau\right)-V_k\left(\tau\right)+e^{-r(\tau+\Delta)}R_{s's} \tag{5}$$

and

$$J_{k'+b_{ss'}}(x'-e^{-r(\tau+\Delta)}R_{s's}, \tau)-J_{k'}\left(x', \tau\right)=V_{k'+b_{ss'}}\left(\tau\right)-V_{k'}\left(\tau\right)-e^{-r(\tau+\Delta)}R_{s's}. \tag{6}$$

The bargaining outcome solves

$$\begin{aligned} & \max_{b,R}\left[V_{k'+b}\left(\tau\right)-V_{k'}\left(\tau\right)-e^{-r(\tau+\Delta)}R\right]^{\theta}\left[V_{k-b}\left(\tau\right)-V_k\left(\tau\right)+e^{-r(\tau+\Delta)}R\right]^{1-\theta} \\ \text{s.t. } & -k'\leq b\leq k \text{ and } b\in\mathbb{K}, \end{aligned}$$

where θ is the bargaining power of the bank with the smaller balance, in this case, $k' \leq k$ (and $\theta = 1/2$ if $k = k'$).

Hereafter, we assume $M = 0$, $N = 2$, and $\bar{k} = 1$, and simplify the notation by letting $n_k(\tau) \equiv n_{k1}(\tau)$. In this case it is convenient to refer to a bank with $k = 0$ and a bank with $k = 2$ as a *borrower*, and *lender*, respectively. The loan agreed upon in the bilateral meeting with time τ left on the clock is

$$b_{ss'}(\tau) = \begin{cases} 1 & \text{if } k' = 0 \text{ and } k = 2 \\ -1 & \text{if } k' = 2 \text{ and } k = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

So there is only trade when a bank with balances above target meets a bank with balances below target, and in this case, using (7), the Nash problem simplifies to

$$\max_R \left[V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)}R\right]^\theta \left[V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)}R\right]^{1-\theta}.$$

Given the value functions $V_k(\tau)$, the repayment amount R is given by the first-order condition

$$e^{-r(\tau+\Delta)}R(\tau) = \theta\left[V_2(\tau) - V_1(\tau)\right] + (1-\theta)\left[V_1(\tau) - V_0(\tau)\right]. \tag{8}$$

We summarize the key properties of the bargaining outcome in the following lemma.

Lemma 1 *The size of the loan, $b_{ss'}(\tau)$, and the repayment, $R_{s's}(\tau)$, are independent of x and x', so we can denote them by $b_{kk'}(\tau)$ and $R_{k'k}(\tau)$, respectively.*

The gain from trade to be divided between a borrower and a lender is

$$S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau). \tag{9}$$

With (8), the gains from trade of the borrower and the lender implied by the bargaining outcome are

$$V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)}R(\tau) = \theta S(\tau) \tag{10}$$

$$V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)}R(\tau) = (1-\theta)S(\tau). \tag{11}$$

Note that $S(0) = 2V_1(0) - V_2(0) - V_0(0) = 2U(1) - U(2) - U(0)$. Also, we can rewrite (8) as

$$e^{-r(\tau+\Delta)}R(\tau) = V_{10}(\tau) - \theta S(\tau), \tag{12}$$

where $V_{10}(\tau) \equiv V_1(\tau) - V_0(\tau)$. Notice that the interest rate implicit in the typical loan that promises to repay $R(\tau)$ at time $\tau + \Delta$ for one unit borrowed at time $T - \tau$ is

$$\rho(\tau) = \frac{\ln R(\tau)}{\tau + \Delta}. \tag{13}$$

The measures of banks with positive and negative balances satisfy

$$\dot{n}_0(\tau) = \alpha n_2(\tau) n_0(\tau)$$
$$\dot{n}_2(\tau) = \alpha n_2(\tau) n_0(\tau)$$

where the initial conditions $(n_0(T), n_2(T))$ are given. This system of differential equations implies

$$n_0(\tau) = \frac{1}{Ke^{-\alpha c\tau} - \frac{1}{c}} \tag{14}$$

$$n_2(\tau) = n_0(\tau) + c \tag{15}$$

$$n_1(\tau) = 1 - n_0(\tau) - n_2(\tau) \tag{16}$$

where $c \equiv n_2(T) - n_0(T)$ and $K \equiv \left[\frac{1}{c} + \frac{1}{n_0(T)}\right] e^{\alpha c T}$.

With Lemma 1 we can write (4) as

$$\begin{aligned} V_k(\tau) &= \Delta_k(\tau) \\ &+ \mathbb{E}\left\{\mathbb{I}_{\{\tau_\alpha \le \tau\}} e^{-r\tau_\alpha} \sum_{k'=0}^{2} n_{k'}(\tau - \tau_\alpha)\left[V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau-\tau_\alpha+\Delta)} R_{k'k}(\tau - \tau_\alpha)\right]\right\} \end{aligned}$$

where

$$\Delta_k(\tau) \equiv \mathbb{E}\left\{\int_0^{\min(\tau_\alpha,\tau)} e^{-rz} v_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} V_k(0)\right\}.$$

With (10) and (11) we can write the value functions as

$$\begin{aligned} V_2(\tau) = \Delta_2(\tau) + \mathbb{E}\{\mathbb{I}_{\{\tau_\alpha \le \tau\}} e^{-r\tau_\alpha} & [n_0(\tau - \tau_\alpha)[V_2(\tau - \tau_\alpha) + (1-\theta) S(\tau - \tau_\alpha)] \\ & + [n_1(\tau - \tau_\alpha) + n_2(\tau - \tau_\alpha)] V_2(\tau - \tau_\alpha)]\} \end{aligned} \tag{17}$$

$$\begin{aligned} V_0(\tau) = \Delta_0(\tau) + \mathbb{E}\{\mathbb{I}_{\{\tau_\alpha \le \tau\}} e^{-r\tau_\alpha} & [n_2(\tau - \tau_\alpha)[V_0(\tau - \tau_\alpha) + \theta S(\tau - \tau_\alpha)] \\ & + [n_1(\tau - \tau_\alpha) + n_0(\tau - \tau_\alpha)] V_0(\tau - \tau_\alpha)]\} \end{aligned} \tag{18}$$

$$V_1(\tau) = \Delta_1(\tau) + \mathbb{E}\left\{\mathbb{I}_{\{\tau_\alpha \le \tau\}} e^{-r\tau_\alpha} V_1(\tau - \tau_\alpha)\right\}. \tag{19}$$

Definition 1 *Given an initial condition* $\{n_k(T)\}_{k=0}^2$*, an equilibrium is a path for the repayment* $R(\tau)$*, a distribution of balances* $\{n_k(\tau)\}_{k=0}^2$ *and value functions* $\{V_k(\tau)\}_{k=0}^2$ *such that* $R(\tau)$ *and* $\{V_k(\tau)\}_{k=0}^2$ *satisfy (12) and (17)–(19), and* $\{n_k(\tau)\}_{k=0}^2$ *is given by (14)–(16).*

Next, we show how to characterize an equilibrium. In order to find the repayment $R(\tau)$, all we need is $S(\tau)$ and $V_{10}(\tau)$. And given $S(\tau)$, we can also obtain the equilibrium value functions $\{V_k(\tau)\}_{k=0}^2$. In the following lemma we derive an explicit expression for the equilibrium trade surplus $S(\tau)$.

Lemma 2 *The surplus of a match between a bank with* $k=2$ *and a bank with* $k'=0$ *is*

$$S(\tau) = \begin{cases} \left(K - \frac{1}{c}e^{\alpha c\tau}\right)e^{-(r+\theta\alpha c)\tau}\left[\frac{S(0)}{K-\frac{1}{c}} + q\xi(\tau)\right] & \text{if } c \neq 0 \\ (a-\tau)e^{-r\tau}\left[\frac{S(0)}{a} + \alpha q\xi(\tau)\right] & \text{if } c = 0 \end{cases} \tag{20}$$

where $q \equiv 2v_1 - v_2 - v_0$*,* $a \equiv \frac{1}{\alpha n_0(T)} + T$*, and*

$$\xi(\tau) \equiv \begin{cases} \frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{(Kc)^{n-1}}{\frac{r}{\alpha c}+\theta-n}\left[1-e^{[r+(\theta-n)\alpha c]\tau}\right] & \text{if } c<0 \\ \frac{1}{\alpha}e^{ar}\left[\ln\left(\frac{a}{a-\tau}\right) + \sum_{n=1}^{\infty}\frac{(-1)^n r^n}{n!}\frac{1}{n}\left[a^n - (a-\tau)^n\right]\right] & \text{if } c=0 \\ \frac{1}{\alpha}\sum_{n=0}^{\infty}\frac{(Kc)^{-n-1}}{\frac{r}{\alpha c}+\theta+n}\left[e^{[r+(\theta+n)\alpha c]\tau}-1\right] & \text{if} c>0. \end{cases} \tag{21}$$

Proposition 1 *There exists a unique equilibrium.*

Given $S(\tau)$, from (12) we see that $V_{10}(\tau)$ is all we need to recover the equilibrium repayment $R(\tau)$. In the following lemma we derive an explicit expression for $R(\tau)$.

Lemma 3 *The equilibrium repayment is*

$$R(\tau) = e^{r(\tau+\Delta)}\left[V_{10}(\tau) - \theta S(\tau)\right] \tag{22}$$

where

$$V_{10}(\tau) = [U(1) - U(0)]e^{-r\tau} + \frac{v_1 - v_0}{r}\left(1 - e^{-r\tau}\right) - \theta\alpha K c e^{-r\tau}\zeta(\tau) \tag{23}$$

where

$$\zeta(\tau) \equiv \begin{cases} \frac{1}{\alpha\theta}\frac{S(0)}{Kc-1}\left(1-e^{-\theta\alpha c\tau}\right)+\frac{q}{\alpha}\sum_{n=1}^{\infty}\frac{(Kc)^{n-1}}{\frac{r}{\alpha c}+\theta-n}\left[\frac{1-e^{-\theta\alpha c\tau}}{\theta\alpha c}+\frac{1-e^{(r-\alpha cn)\tau}}{r-\alpha cn}\right] & \text{if } c<0 \\ \frac{S(0)}{\alpha a}\tau+\frac{q}{\alpha}e^{ar}\left\{(\tau-a)\ln\left(\frac{a}{a-\tau}\right)+\tau+\sum_{n=1}^{\infty}\frac{(-1)^n r^n}{n!}\frac{1}{n}\left[a^n\tau-\frac{a^{n+1}-(a-\tau)^{n+1}}{n+1}\right]\right\} & \text{if } c=0 \\ \frac{1}{\alpha\theta}\frac{S(0)}{Kc-1}\left(1-e^{-\theta\alpha c\tau}\right)+\frac{q}{\alpha}\sum_{n=0}^{\infty}\frac{(Kc)^{-n-1}}{\frac{r}{\alpha c}+\theta+n}\left[\frac{e^{-\theta\alpha c\tau}-1}{\theta\alpha c}+\frac{e^{(r+\alpha cn)\tau}-1}{r+\alpha cn}\right] & \text{if } c>0. \end{cases}$$

5 Free intraday credit

In this section we consider the case with $q=0$. For example, this would be case when banks are not remunerated for holding balances at or above target during the trading session, i.e., $v_k = 0$ for $k = 1, 2$, and banks are not penalized for holding intraday balances below target, i.e., banks have access to intraday credit from the central bank at no cost: $v_0 = 0$.[7]

Proposition 2 *Assume $q = 0$ and $S(0) > 0$.*

(i). *The fed funds rate at each point in time is increasing in the discount rate, i.e., for all τ, $\frac{\partial\rho(\tau)}{\partial r} > 0$.*

(ii). *The fed funds rate at each point in time during the trading session is decreasing in the borrower's bargaining power, i.e., for all $\tau > 0$, $\frac{\partial\rho(\tau)}{\partial\theta} < 0$.*

(iii). *The fed funds rate at each point in time is increasing in the penalty for below-target end-of-day balances, i.e., for all τ, $\frac{\partial\rho(\tau)}{\partial U(0)} < 0$.*

Proposition 2 describes the behavior of the fed funds rate at each point in time along the trading session. Parts *(i)–(iii)* follow from (13) and the fact that the size of the loan repayment $R(\tau)$ agreed in a trade increases with r and the penalty $U(0)$, and decreases with the borrower's bargaining power, θ.

Proposition 3 *Assume $q = 0$ and $S(0) > 0$.*

(i). *The surplus at each point in time during the trading session is decreasing in the discount rate, i.e., for all $\tau > 0$, $\frac{\partial S(\tau)}{\partial r} < 0$.*

[7] Currently, the Fed does not pay interest on intraday balances, and the interest it charges for intraday is relatively small (see Section 3).

(ii). *The surplus is decreasing in the time remaining until the end of the trading session, i.e., for all* $\tau > 0$, $\frac{\partial S(\tau)}{\partial \tau} < 0$. *Equivalently, extending operating hours,* T, *reduces total surplus at each point in time.*

(iii). *If the initial population of lenders is larger (smaller) than that of borrowers, then the surplus at each point in time during the trading session is decreasing (increasing) in the borrower's bargaining power. If the initial populations of lenders and borrowers are equal, then changes in the bargaining power have no effect on the surplus, i.e., for all* $\tau > 0$, $\frac{\partial S(\tau)}{\partial \theta}$ *is equal in sign to* $-c$.

(iv). *The surplus at each point in time is increasing in the penalty for below-target end-of-day balances, i.e., for all* τ, $\frac{\partial S(\tau)}{\partial U(0)} < 0$.

Proposition 3 describes the behavior of $S(\tau)$, namely the value of executing a trade (or the "value of a trade") between a borrower and a lender when the remaining time is τ. With $q = 0$, the value functions $\{V_k(\tau)\}_{k=0}^{2}$ and (9) imply $S(\tau) = e^{-\delta(\tau)} S(0)$, where

$$\delta(\tau) \equiv \int_0^\tau \{r + \alpha [\theta n_2(s) + (1-\theta) n_0(s)]\} \, ds.$$

The value of a trade at the end of the trading session, $S(0)$, is exogenously given by the policy parameters. The previous expression makes clear that the value of a trade at τ, $S(\tau)$, is a discounted version of $S(0)$, with effective discount rate given by $\delta(\tau)$. Intuitively, there are two reasons why $S(\tau)$ is smaller than $S(0)$. First, the actual gain from trade accrues at the end of the trading session, so it is discounted by the pure rate of time preference r. Second, consider a meeting between a borrower and a lender when the remaining time is $\tau > 0$. The value $S(\tau)$ is smaller than $S(0)$ because both agents might meet alternative trading partners before the end of the session, and this increases their outside options. The borrower's outside option, $V_0(\tau)$, is increasing in the average rate at which he is able to contact a lender and reaps gains from trade between time $T - \tau$ and T, i.e., $\alpha\theta \int_0^\tau n_2(s) \, ds$. Similarly, the lender's outside option, $V_2(\tau)$, is increasing in the average rate at which he is able to contact a borrower and reaps gains from trade between time $T - \tau$ and T, i.e., $\alpha(1-\theta) \int_0^\tau n_0(s) \, ds$. The effective discount rate is increasing in r and the time until the end of the trading session, τ, which explains parts *(i)* and *(ii)* in Proposition 3. The effect of θ on $S(\tau) = 2V_1(\tau) - V_0(\tau) - V_2(\tau)$ is more subtle because a higher θ tends to increase $V_0(\tau)$ (benefits borrowers) and at the same time it tends to decrease $V_2(\tau)$ (hurts lenders). In part *(iii)* we show that the former effect dominates if and only if

$n_2(\tau) > n_0(\tau)$, and in this case, the effective discount rate decreases with θ, which implies $S(\tau)$ decreases with θ for all $\tau > 0$. Finally, making the penalty for below-target end-of-day balances more severe (lowering $U(0)$) increases the terminal surplus $S(0)$, and consequently every surplus along the trading session, which explains part *(iv)*.

6 Trade dynamics

In this section we parametrize the model with $M = 0$, $N = 2$, and $\bar{k} = 1$ and use it to illustrate and complement our analytical results.

6.1 Bargaining power

Figure 1 shows the behavior of the trade surplus and the fed funds rate for various values of the borrower's bargaining power, θ (with actual time, $t = T - \tau$, on the horizontal axis). Panels (a) and (c) are for the case in which the initial number of lenders is larger than the initial number of borrowers, i.e., $c \equiv n_2(T) - n_0(T) > 0$. First consider panel (a). Naturally, the trade surplus at the terminal date is the same for all values of θ, since $S(0) = 2U(1) - U(0) - U(2)$. The time-path for the trade surplus, however, becomes steeper for larger values of θ. This is because $n_2(\tau) > n_0(\tau)$ for all τ causes an increase in the borrower's outside option, $V_0(\tau)$, that is larger than the decrease in the lender's outside option, $V_2(\tau)$, at each point along the trading session. Panel (b) illustrates that the opposite is true for parametrizations with $c < 0$. Naturally, from panels (c) and (d), we also see that the rate on a loan traded at any point in the trading session will be smaller for larger values of θ. Note that in this parametrization the interest rate is decreasing in the time remaining until the end of the session.

6.2 Deficiency charges

Figure 2 shows the behavior of the trade surplus and the fed funds rate for various values of $U(0)$, which can be interpreted as the penalty for having balances below the end-of-day target. Panels (a) and (b) show that making the penalty more severe shifts up the path of the surplus, an effect driven by the fact that the first-order implication of a smaller $U(0)$ is to reduce the borrower's outside option, $V_0(\tau)$, making it more valuable for them to be able to trade and avoid paying the penalty at the end of the trading session. This effect is also causing the paths for the interest rate in panels (c) and (d) to shift up in response to a reduction in $U(0)$.

Interestingly, panel (d) shows that the interest rate need to be decreasing in the time remaining until the end of the session.

6.3 Trading delays

Figure 3 shows the behavior of the trade surplus and the fed funds rate for various values of the contact rate, α. Since α does not enter the expression for the terminal surplus, $S(0)$, the trade surplus at the terminal date is the same for all values of α in all four panels. The effect of changes in α on equilibrium payoffs is subtle. On the one hand, all agents benefit from contacting partners faster. But on the other hand, for the agents who are on the long side of the market, e.g., borrowers in the parametrizations with $c < 0$, increases in α have the undesirable effect of taking scarce potential trading partners off the market, and hence adversely affecting their trading rate. Our numerical simulations show that indeed, if $c < 0$, then $V_0(\tau)$ is typically nonmonotonic in α: increasing in α for small values of α, but decreasing in α for large values. In this case with $c < 0$, however, $V_2(\tau)$ is typically increasing in α. We find the converse to be the case for $c > 0$, i.e., $V_0(\tau)$ is increasing in α, while increases in α from relatively small values tend to shift $V_2(\tau)$ up, while increases in α at large values tend to shift $V_2(\tau)$ down. These forces account for the pattern of interest rates displayed in panels (c) and (d).

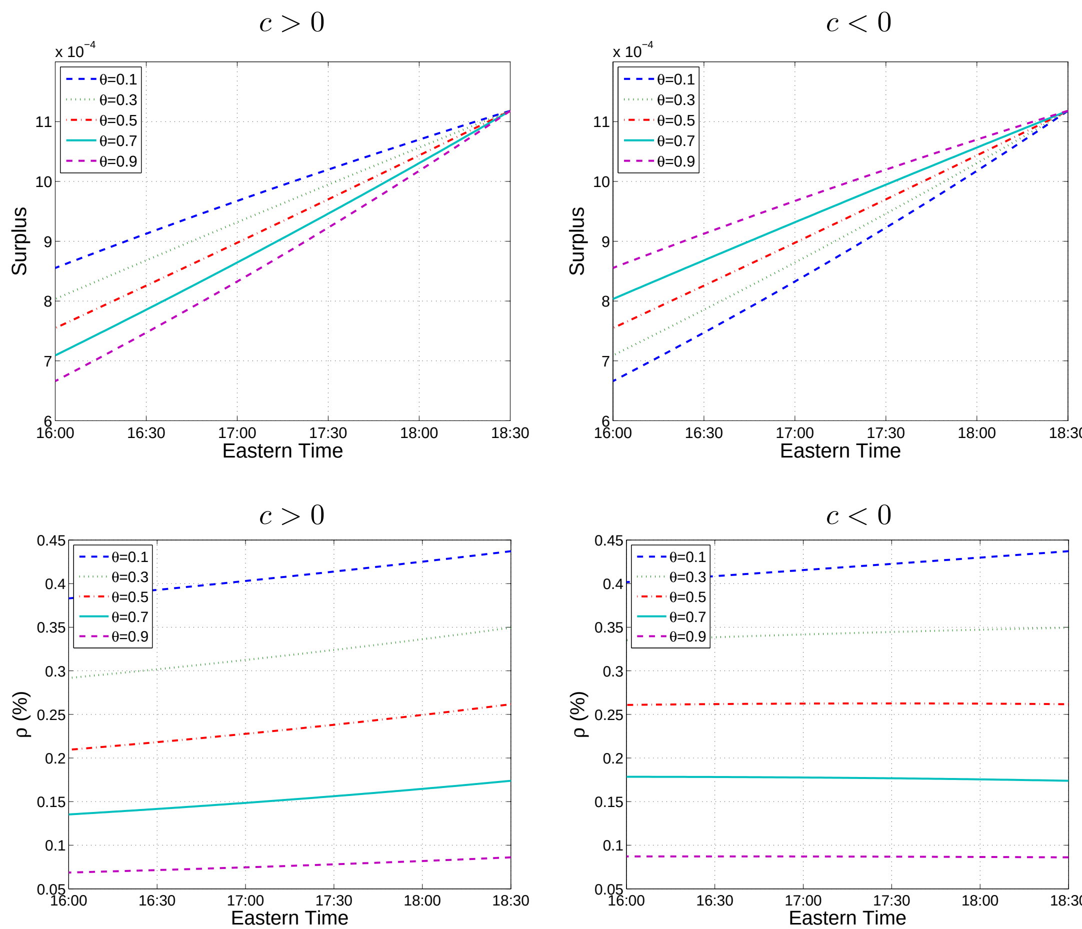

Figure 1: Surplus (top row) and fed funds rate (bottom row) for different values of the bargaining power θ when $c > 0$ (left column) and when $c < 0$ (right column). Parameter values: $\alpha = 10$, $T = 2.5/24$, $\Delta = 22/24$, $n_2(T) = 0.6$ (0.3), $n_0(T) = 0.3$ (0.6), $r = 0.04/365$, $i^d = 0.0036/360$, $i^o = 0.0425/360$, $i^r = i^e = 0.0025/360$ and $P^r = 0.001$.

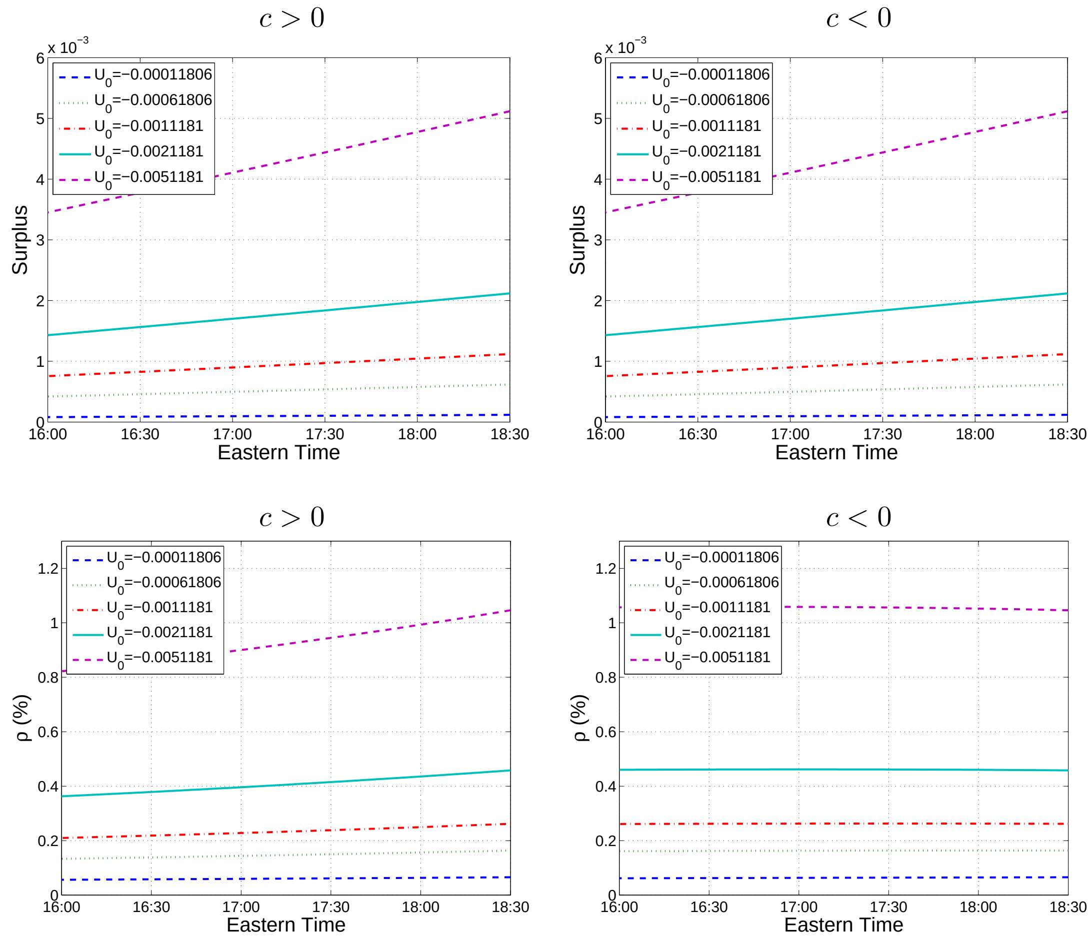

Figure 2: Surplus (top row) and fed funds rate (bottom row) for different values of the payoff from holding an end-of-day balances below target U_0 when $c > 0$ (left column) and when $c < 0$ (right column). Parameter values: $\alpha = 10$, $T = 2.5/24$, $\Delta = 22/24$, $n_2(T) = 0.6$ (0.3), $n_0(T) = 0.3$ (0.6), $r = 0.04/365$, $i^d = 0.0036/360$, $i^o = 0.0425/360$, $i^r = i^e = 0.0025/360$ and $\theta = 1/2$.

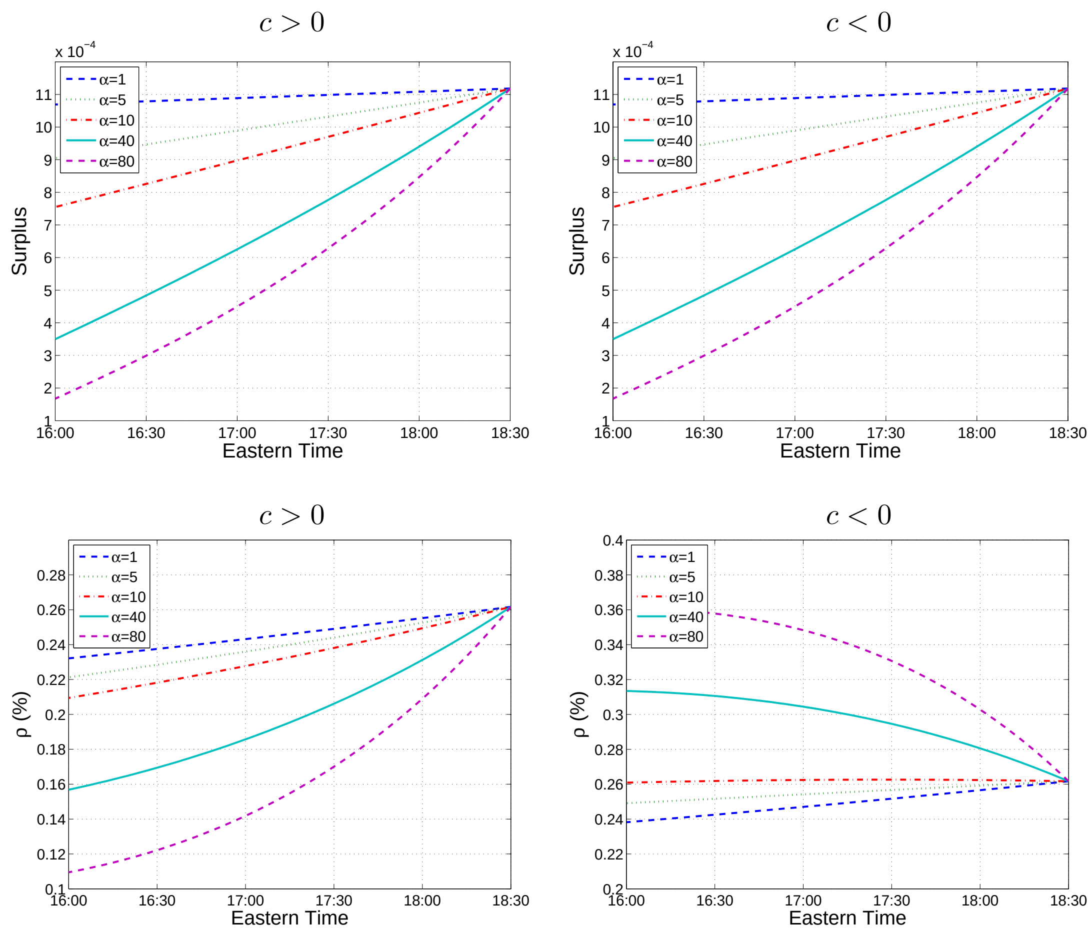

Figure 3: Surplus (top row) and fed funds rate (bottom row) for different values of the frequency of meetings α when $c > 0$ (left column) and when $c < 0$ (right column). Parameter values: $T = 2.5/24$, $\Delta = 22/24$, $n_2(T) = 0.6$ (0.3), $n_0(T) = 0.3$ (0.6), $r = 0.04/365$, $i^d = 0.0036/360$, $i^o = 0.0425/360$, $i^r = i^e = 0.0025/360$, $\theta = 1/2$ and $P^r = 0.001$.

A Appendix

Proof of Lemma 2. The value functions $\{V_k(\tau)\}_{k=0}^{2}$ and (9) imply

$$S(\tau) = \Delta(\tau) + \mathbb{E}\left\{\mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha}\phi(\tau - \tau_\alpha) S(\tau - \tau_\alpha)\right\} \tag{24}$$

where

$$\begin{aligned}\Delta(\tau) &\equiv 2\Delta_1(\tau) - \Delta_2(\tau) - \Delta_0(\tau) = \frac{2v_1 - v_2 - v_0}{r+\alpha} + e^{-(r+\alpha)\tau} S(0) \\ \phi(\tau - \tau_\alpha) &\equiv 1 - \left[\theta n_2(\tau - \tau_\alpha) + (1-\theta) n_0(\tau - \tau_\alpha)\right],\end{aligned}$$

where $q \equiv 2v_1 - v_2 - v_0$. Note that $\dot{\Delta}_k(\tau) = v_k - (r+\alpha)\Delta_k(\tau)$. The expression (24) can be manipulated to give

$$S(\tau) = \Delta(\tau) + \frac{\alpha}{r+\alpha}\int_0^\tau \phi(z) S(z)(r+\alpha) e^{-(r+\alpha)(\tau - z)} dz.$$

Differentiate and substitute the original expression for $S(\tau)$ to get

$$\dot{S}(\tau) = \dot{\Delta}(\tau) + \alpha\phi(\tau) S(\tau) - (r+\alpha)\left[S(\tau) - \Delta(\tau)\right]$$

which can be rewritten as

$$\dot{S}(\tau) + p(\tau) S(\tau) = q$$

where

$$p(\tau) \equiv r + \alpha\left[1 - \phi(\tau)\right].$$

The solution to this differential equation is

$$S(\tau) = e^{-\int_0^\tau \{r+\alpha[1-\phi(s)]\} ds} S(0) + q\int_0^\tau e^{-\int_z^\tau \{r+\alpha[1-\phi(s)]\} ds} dz. \tag{25}$$

This expression can be integrated to yield (20), where $\xi(\tau) = \int_0^\tau \frac{e^{(r+\theta\alpha c)z}}{K - \frac{1}{c}e^{\alpha c z}} dz$ is explicitly given by (21). ■

Proof of Proposition 1. *To be written.*

Proof of Lemma 3. The expression for $R(\tau)$ is in (12). The value functions (18)–(19) can be manipulated to give

$$V_{10} = \Delta_1(\tau) - \Delta_0(\tau) + \alpha\int_0^\tau V_{10}(z) e^{-(r+\alpha)(\tau - z)} dz - \theta\alpha\int_0^\tau n_2(z) S(z) e^{-(r+\alpha)(\tau - z)} dz$$

Differentiating yields

$$\dot{V}_{10}(\tau) + rV_{10}(\tau) = v_1 - v_0 - \theta\alpha n_2(\tau) S(\tau)$$

The solution to this differential equation is

$$V_{10}(\tau) = [U(1) - U(0)] e^{-r\tau} + \int_0^\tau [v_1 - v_0 - \theta\alpha n_2(z) S(z)] e^{-(\tau - z)} dz$$

This expression can be solved explicitly to yield (23). ■

Lemma 4 *The value function $J_k(x,\tau)$ can be written as in (3), where $V_k(\tau)$ is given by (4).*

Proof of Lemma 4. (i) Let $\hat{J}(k,x,\tau) \equiv J_k(x,\tau)$, and let the RHS of (1) define a mapping T. Think of (1) as $\hat{J} = T\hat{J}$, and show that T has a unique fixed point in the appropriate space of functions.

(ii) Substitute (3) in (1) to get

$$V_k(\tau) + e^{-r(\tau+\Delta)}x = \mathbb{E}\left\{\int_0^{\min(\tau_\alpha,\tau)} e^{-rz} v_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau}\left[V_k(0) + e^{-r\Delta}x\right]\right.$$

$$\left. + \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \int \left[V_{k-b_{ss'}(\tau-\tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau-\tau_\alpha+\Delta)}(x + R_{s's}(\tau - \tau_\alpha))\right] dF\left(s', \tau - \tau_\alpha\right)\right\}.$$

Notice that the x cancels and we arrive at (4), so (3) with $V_k(\tau)$ given by (4) satisfies (1). ■

Proof of Proposition 2. For $q = 0$, $R(\tau)$ is still given by (22), but with $S(\tau)$ given by (26), and

$$V_{10}(\tau) = \begin{cases} [U(1) - U(0)] e^{-r\tau} + \frac{v_1 - v_0}{r}(1 - e^{-r\tau}) - \frac{Kc}{Kc-1}\left(1 - e^{-\theta\alpha c\tau}\right) e^{-r\tau} S(0) & \text{if } c \neq 0 \\ [U(1) - U(0)] e^{-r\tau} + \frac{v_1 - v_0}{r}(1 - e^{-r\tau}) - \frac{\theta}{a}\tau e^{-r\tau} S(0) & \text{if } c = 0. \end{cases}$$

From (13), notice that

$$\frac{\partial \rho(\tau)}{\partial x} = \frac{1}{\tau + \Delta} \frac{1}{R(\tau)} \frac{\partial R(\tau)}{\partial x},$$

for $x = \theta$, r, $U(0)$.

(i) Differentiate (22) to obtain

$$\frac{\partial R(\tau)}{\partial r} = \Delta R(\tau) - e^{r(\tau+\Delta)} \frac{v_1 - v_0}{r^2}\left(1 - r\tau - e^{-r\tau}\right) > 0,$$

since $1 - r\tau - e^{-r\tau} \leq 0$. Thus, $\frac{\partial \rho(\tau)}{\partial r} > 0$.

(ii) Differentiate (22) to obtain

$$\frac{\partial R(\tau)}{\partial \theta} = \begin{cases} -e^{r(\tau+\Delta)} \left\{ S(0) e^{-(r+\theta\alpha c)\tau} \frac{c}{Kc-1} \alpha\tau \left[(1-\theta) Kc + \theta e^{\alpha c \tau} \right] + S(\tau) \right\} & < 0 \text{ if } c \neq 0 \\ -e^{r(\tau+\Delta)} \left[S(0) e^{-r\tau} \frac{Kc}{a} \tau + S(\tau) \right] & < 0 \text{ if } c = 0 \end{cases}$$

for $\tau > 0$, hence $\frac{\partial \rho(\tau)}{\partial \theta} < 0$.

(iii) Differentiate (22) to obtain

$$\frac{\partial R(\tau)}{\partial U(0)} = \begin{cases} e^{r\Delta} \frac{1}{Kc-1} \left[1 - \left(\theta e^{(1-\theta)\alpha c \tau} + (1-\theta) \frac{n_2(T)}{n_0(T)} e^{\alpha c (T - \theta\tau)} \right) \right] & \text{if } c \neq 0 \\ -e^{r\Delta} \left[(1-\theta) + 2\theta \frac{\tau}{a} \right] & \text{if } c = 0. \end{cases}$$

Note that $\frac{\partial R(\tau)}{\partial U(0)} < 0$ for all c, since $\theta e^{(1-\theta)\alpha c\tau} + (1-\theta) \frac{n_2(T)}{n_0(T)} e^{\alpha c(T-\theta\tau)} \gtrless 1$ if $c \gtrless 0$. Thus, $\frac{\partial \rho(\tau)}{\partial U(0)} < 0$. ■

Proof of Proposition 3. For $q = 0$, (25) becomes

$$S(\tau) = e^{-\int_0^\tau \{r + \alpha[\theta n_2(s) + (1-\theta) n_0(s)]\} ds} S(0) . \tag{26}$$

We maintain the assumption that $S(0) > 0$ for the rest of the proof, which implies $S(\tau) > 0$ for all τ.

(i) Differentiate (26) to obtain $\frac{\partial S(\tau)}{\partial r} = -\tau S(\tau)$, which is clearly negative for $\tau > 0$.

(ii) Differentiate (26) to obtain

$$\frac{\partial S(\tau)}{\partial \tau} = -\left\{ r + \alpha \left[\theta n_2(\tau) + (1-\theta) n_0(\tau) \right] \right\} S(\tau) < 0.$$

(iii) Differentiate (26) to obtain $\frac{\partial S(\tau)}{\partial \theta} = -\alpha c \tau S(\tau)$, where $\alpha c \tau S(\tau)$ has the sign of $-c$ for $\tau > 0$.

(iv) Differentiate (26) to obtain $\frac{\partial S(\tau)}{\partial U(0)} = -\frac{S(\tau)}{S(0)} < 0$. ■

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