

Consumption and Home Production over the Life Cycle*

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Abstract

We incorporate home production in a dynamic general equilibrium model of consumption and saving with illiquid housing and a collateralized borrowing constraint. We show that the model is capable of explaining life-cycle patterns of households' time use and consumption of different categories. Specifically, households' market hours and home hours are fairly stable early in the life cycle. At age 50, market hours start to decline sharply while home hours begin to increase. Households' consumption of the market good, home input, and housing services all exhibit hump shapes over the life cycle, with the market good having the most pronounced hump, followed by the home input, and then housing services. A plausibly parameterized version of our model predicts that the interaction of the labor efficiency profile and the availability of home production technology explains households' time use over the life cycle. The resulting income profiles and the endogenous borrowing constraint account for the initial hump in all three consumption goods. The consumption profiles in the second half of the life cycle are mostly driven by the complementarity of home hours, home input, and housing in home production. Policy changes on retirement age or collateralized borrowing constraint have important implications on household house tenure, labor supply, and consumption decisions.

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1 Introduction

This paper explains a number of salient features of households life-cycle behavior regarding the demand for housing, consumption, and leisure. Recent literature emerged indicates that all of these decisions are interrelated and that in order to explain the patterns in one component of household decisions requires analyzing the other components as well. Further, the natural environment to most clearly exposit these decisions is to think of the household as a joint consumption-production unit. Thus, our modeling strategy incorporates home production in a standard life-cycle model that includes precautionary savings motives and endogenous labor supply decisions. Home production uses housing, certain types of market produced goods that serve as intermediate inputs, and home hours to produce final home goods. The home goods, in turn, are substitutable with market goods that can be directly purchased and consumed. The richness of our framework enables us to broadly match the life-cycle behavior of consumption of both market and home produced goods, the time allocation decisions of households, and the amount of housing services consumed at various points in the life cycle. Our model also has important implications for government policies particularly those that affect older households.

The paper is motivated by four observations. The first is the well established finding that total household consumption, durable as well as nondurable, exhibits a strong hump shape over the life cycle even after adjusting for economic growth and household size.¹ The second less noted feature is the substantial heterogeneity across life-cycle profiles of individual consumption expenditures. Using recent Consumption Expenditure Survey, we confirm the findings in Aguiar and Hurst (2009) that most of the decline in nondurable expenditure in the second half of a household's life cycle is driven by categories related to households' expenditure on food away from home, transportation to and from work, and clothing/personal care. The other categories that are more closely related to households' home production including food at home, household operation, as well as utilities, by contrast, do not decline nearly as much. Third is the "housing puzzle." Old age homeowners' house values do not decline much if at all toward the end of their life cycle, and this puzzle pertains to renters as well.² Standard models imply

¹See, among many others, Attanasio and Weber (1995), Hubbard, Skinner, and Zeldes (1994), Carroll (1997), Attanasio, Banks, Meghir and Weber (1999), Angeletos, Laibson, Repetto, Tobacman, and Weinberg (2001) Fernandez-Villaverde and Krueger (2002), Gourinchas and Parker (2002), Borwning and Ejrnaes (2006), and Bullard and feigenbaum (2007). The existing explanations include liquidity constraints, myopia, consideration of leisure, marriage and children, and changing preferences with demographics.

²This latter observation poses challenges to the traditional theory of bequest as well adjustment cost of older homeowners' housing consumption behavior. For more discussion on the housing puzzle, see Venti and Wise (2002) and Davidoff (2006).

that the housing stock should be consumed aggressively late in life. And fourth, one observes that households labor supply also exhibits strong life cycle patterns with market hours and home hours being fairly stable early in the life cycle. At age 50, market hours start to decline sharply and home hours begin to increase after age 55.

We show that including home production helps explain all four observations. In explaining the observed heterogeneity across the three broadly defined consumption categories, housing services, market produced consumption goods, and home produced consumption goods, in the second half of households' life cycle, our theory complements Aguiar and Hurst (2008). The main difference is that Aguiar and Hurst (2008) directly follow Becker (1965) and emphasizes the intra-temporal substitution between time used in the consumption of a particular consumer good and its corresponding expenditure.³ By contrast, we separate non-market time into leisure and home hours where home hours comprise time spent on activities at home to produce goods and services that can also be purchased in the market and, for the most part, are not enjoyably produced.⁴ That is, as discussed in Baxter and Rotz (2009), home production activities are associated with disutility whereas leisure activities provide utility. This modeling strategy allows us to highlight the substitution of home-produced goods and market goods. Finally, our model generates predictions for the life-cycle dynamics of other observable characteristics such as labor supply and wealth accumulation beyond just consumption.

Our model has three key features that help account for these four observations. First, given the prominent role housing plays in our model, we explicitly characterize it along both the extensive margin of owning versus renting and the intensive margin of house size. We assume that housing adjustment is costly for homeowners but costless for renters. This assumption helps separate homeownership from renting, which in turn allows us to directly measure the predictions of our model concerning owners and renters' consumption and labor supply against those from the data. Second, households are subject to collateralized borrowing constraints; they can borrow only up to a fraction of their house value. This second feature together with the standard assumption regarding the labor efficiency profile along with uninsurable labor income risk are largely responsible for the increasing consumption profiles in the early part of a households' life cycle as in Fernandez-Villaverde and Krueger (2010). Third, at each point in time, households divide their time between market hours, hours in home production, and leisure. Home

³Earlier works along similar lines include, among others, Abbot and Ashenfelter (1976), Barnett (1979), and Chang (2000).

⁴Our definition, thus, follows Reid (1934, pg. 11) who defines home production as "those unpaid activities which are carried on, by and for the members, which activities might be replaced by market goods, or paid services, if circumstances such as income, market conditions, and personal inclinations permit the service being delegated to someone outside the household group." We will come back to this point in our empirical section.

production uses labor effort, housing, and an intermediate good which we call a home good as inputs to produce a composite good. As households age, their market labor efficiency declines and they assign more and more of their time to home production. As a result, consumption of the market good declines drastically. Consumption of the home good and housing services also decline, but given the complementarity between the home good and housing services with hours in home production, the decline is much more muted. For homeowners, housing adjustment costs further contribute to the slow decline in the homeownership rate and the value of the housing stock owned by each household.

We calibrate our model to key economic moments calculated off households' life-cycle time use and consumption. The profiles themselves are constructed according to the American Time Use Survey and the Consumer Expenditure Survey. We show that the model does a good job not only at matching our target moments, but also the consumption and labor profiles. Further analysis indicates that the introduction of home production provides a key role in explaining the different consumption profiles. Finally, we use our calibrated model to conduct policy experiments motivated ongoing policy debate and the current economic environment. In the first experiment, we delay the age at which pensions become available from 62 to 67, and in the second we restrict households collateralized borrowing constraint from 80 percent loan to value ratio to 70 percent. We show that these experiments have important implications on households house tenure, labor supply, and consumption decisions.

The paper brings together two strands of literature, the home production literature and the consumption literature. The introduction of home production in an otherwise standard dynamic general equilibrium models has proven to be useful in understanding a variety of macroeconomic issues ranging from domestic and international business cycles, fiscal policies, and asset equilibrium puzzles.⁵ A key way in which models with home production differ from standard dynamic general equilibrium models (that do not include home production) is that home production allows households to substitute between additional margins, both in labor supply and in total consumption. While many recent studies have utilized time use data in order to understand household production (Aguiar and Hurst (2007) and Ramey (2008)), much less attention has been paid to the other factors involved in the production of home goods.⁶ The consumption literature, on the other hand, with the notable exception of work by Aguiar and Hurst (2008)

⁵See papers by Baxter and Jermann (1999), Benhabib, et al. (1991), Bils, et al. (2009), Campbell and Ludvigson (2001), Canova and Ubide (1998), Gomme, et al. (2001), Greenwood and Hercowitz (1991), Greenwood, et al. (1995), and McGratten, et al. (1997).

⁶See Rupert, Rogerson, and Wright (2000) and papers cited there. One exception is Heathcote (2002) who studies the effect of home production and retirement and consumption.

and Fernandez-Villaverde and Krueger (2010), has focused largely on aggregate non-durable consumption, and much less on the substantial heterogeneity across life-style profiles of individual consumption expenditures and even less on the relation between housing and non-housing consumption. Both heterogeneity and housing decisions are empirically and theoretically relevant for understanding life cycle behavior. Further, the link between consumption and labor supply, and labor supply in home production in particular, has important implications for policies that affect households' retirement.

The rest of the paper is organized as follows. In section 2, we present our empirical analysis where we construct the households' life cycle profiles of time use and consumption. In section 3, we present the model. We discuss our calibration strategy in section 4 and results in section 5. We present policy experiments in section 6. Section 7 concludes.

2 Empirical Observations on Life-cycle Behavior

In this section, we present our empirical findings on time use and consumption over the life cycle. We first study time-use profiles using data from the American Time Use Survey, then we study consumption profiles of market goods, home input, and housing services using data from the Consumption Expenditure Survey. We separate households into owners and renters and deal explicitly with problems of household size, cohort effect, and survey effect using the synthetic cohort strategy widely used in the literature.⁷ It is worth noting that many of our empirical results have already been documented in the literature. Our contribution lies in merging and reclassifying the two data sets according to the macro literature on home production. Our differentiation between homeowners and renters is also novel and important in explaining older age households' "housing puzzle."

2.1 Life Cycle Profiles of Time Use

The American Time Use Survey (ATUS), carried out by the Bureau of Census under the contract with the Bureau of Labor Statistics, measures the amount of time people spend performing various activities, such as work, child care, housework, watching television, volunteering, and socializing (see Table 1). The data is strictly cross sectional as respondents are interviewed only once. Households are top coded at age 80.⁸ The survey started in 2003 with the most recent one ending in 2007.

⁷We follow most closely the linear regression strategy in Gourinchas and Parker (2002) and Aguiar and Hurst (2008).

⁸All households between age 80 and 84 are assigned age 80 and those that are 85 and above are assigned age 85.

We include in our sample the 2005 to 2007 ATUS since the ATUS started reporting household house tenure in 2005. We focus on households whose head is between the ages of 25 and 80 (inclusive) but exclude those whose head is either in school or in the military at the time of the survey. Our final sample consists of 30,720 households, about evenly split across the three survey years. Close to 75 percent of households in our sample own homes and we include both male and female respondents in our sample.

We follow the tradition of Reid (1934) and separate non-market time into pure leisure and home hours where home hours comprise time spent on activities of home to produce goods and services that can also be purchased in the market and are, for the most part, not enjoyable to produce. According to Robinson and Godbey (1999, Table 0), in a 1985 enjoyment of activities survey, households ranked sex, play sports, fishing, art and music, and bars and lounges at the top and yardwork, cleaning house/dishes, laundry, child health, and car repair shop at the bottom. Indeed, empirical studies on home production or homemaking, as in earlier literature, typically classify food preparation, cleaning the house, care of family members living in the household, and shopping and managing the household as home production. Some also include gardening, care of others who are not in the household, and entertaining children.⁹ We thus define market hours as the time the head of the household spends working, job searching and commuting. We define home hours as time spent doing house work, shopping, pet care, car care, personal care, child care, adult care, care service, professional service, personal care service, and house work service. We treat the remaining time as leisure.

For those households who were interviewed on Saturday or Sunday (holidays are viewed the same as Sunday in the ATUS), we approximate their weekday hours by the average hours of those interviewed on weekdays, in the same year, of the same education, and gender.¹⁰ We adjust all hours by family size, cohort, and survey year effects. We also control for families that have young children (under the age of 6). Following Aguirra and Hurst (2008), we identify life cycle from cohort effects by using the multiple cross sections in our model and use cross sectional differences in family size and interview year, respectively, to identify family size and interview year effects. Specifically, we estimate the following equations,

$$(1) \quad H_{it}^k = \beta_0^k + \beta_{age}^k AGE_{it} + \beta_{interviewyear}^k INTERVIEWYEAR_{it} + \beta_{familysize}^k FAMSIZE_{it} + \beta_{familywithyoungchildren}^k YOUNGKIDS_{it} + \varepsilon_{it}^k,$$

where H_{it}^k represents time use in category k (market work or home work), AGE_{it} is a

⁹See Ramey (2008) for a thorough discussion.

¹⁰Deleting those interviewed on Saturday, Sunday or holidays will result in loss of close to half of the data.

vector of 56 one-year age dummies, INTERVIEW_{it} is a vector of 2 one-year interview dummies, and FAMSIZE_{it} is a vector of family structure dummies that include 11 family size dummies, 1 to 10 and 11 and above. YOUNGKIDS_{it} is a dummy indicating whether the family has any children under the age of 6. The coefficients on the constant β_0^k together with age dummies, β_{age}^k , capture the impact of life cycle conditional on cohort, family size, and interview effects.¹¹

Figure 2 charts the share of market hours and home hours by age for the household head. As can be seen, market hours by homeowners and renters track each other pretty well with homeowners supplying slightly more market hours earlier in the life cycle and slightly more home hours later in the life cycle than renters. In particular, both homeowners and renters spend roughly 30 percent of their time working until age 50, then they start reducing their market work sharply. By age 70, households spend less than 5 percent of their time working. Home hours hold steady until age 55 for homeowners and renters. Then, they begin to increase and noticeably more so for homeowners.

2.2 Life Cycle Profiles of Consumption Expenditure

The Consumer Expenditure Survey (CEX), also carried out by the Bureau of Census under the contract with the Bureau of Labor Statistics, collects household demographic characteristics and consumption information. The data is a rotating panel with each household being interviewed from 2 to 5 quarters, and every quarter 25 percent of the sample is replaced by new households. The short-panel dimension of CEX makes the direct use of panel techniques nearly infeasible. We, thus, pool the data and treat it as one cross-section.

Consistent with our time-use data, we include in our consumption sample the 2003 to 2006 CEX data. We delete the fifth interviews from the sample as they are identical to the first interview. For those who appeared in all of the four remaining interviews, we add their quarterly consumption together to arrive at an annual estimate. For the rest of the households, we first calculate a quarterly average consumption for each category and then multiply it by four to obtain an annual estimate.

We include in our market good food away from home, alcohol, tobacco, apparel, other lodging, fees and admissions for entertainment, and related equipment such as, televisions, radios, sound, pets, toys, and playground equipment, reading, and personal care. We also include education expenses and out-of-pocket medical expenses in the market good, but our results are robust to the exclusion of these categories. We include in our

¹¹The variance of the estimate at each age k is thus, $\text{var}(\beta_0^k) + \text{var}(\beta_{age}^k) + 2\text{cov}(\beta_0^k, \beta_{age}^k)$. Later in the simulation exercise, we draw households profiles over the life cycle that are two-standard deviations away from the mean profile calculated based on the formular.

home input food at home, household operations, household furnishing and equipment, utilities, fuels, and public services. We prorate transportation expenses by travel time for home production or market production and leisure we obtained from the American Time Use Survey discussed in the previous subsection. For housing expenditures, we use rental payments for renters, and we use homeowners reported house value of owned residence because we believe homeowners typically have a better idea of how much their house is worth than how much their house can be rented for. Using reported rental value for homeowners does not change our results qualitatively.

We delete from our sample households who reported zero or negative consumption of the market good plus the home input, renters who reported less than \$300 in annual rent, and homeowners who reported less than \$1000 in house value. All consumption data are adjusted by their respective 2000 Chained Consumer Price Index. Our final sample consists of 48,048 households and about 68 percent are home owners.

We use the same strategy outlined in the previous subsection to identify life-cycle profiles of the three consumption categories with the exception that we take the log of our consumption data. The results are presented in Figure 3 in log deviations from age 25. As one can see, for homeowners, the market good, home good, and housing services all move up substantially from age 25 to age 50. The hump in housing services, however, is the most pronounced. The increase is about 50 percent as opposed to about 30 percent in market and home goods. Starting in a household's mid 50s, both consumption of the market good and the purchases of home inputs begin to decline with market goods experiencing a more significant drop. The decline in home inputs is much more muted. Housing services, by comparison, decline very slightly starting in age 60. For renters, the market good starts declining from age 25 and expenditures on home inputs is relatively flat till age 50 at which time it starts to decline. Rental housing services is mostly flat with a slight drop occurring beginning in household's late 50s.

3 Model

We consider a model that is a modification of Fernandez-Villaverde and Krueger (2002), Gervais (2002), Heathcote (2002), and Yang (2009), among others. In particular, it is a discrete-time overlapping generations economy with an infinitely-lived government. The government taxes labor income and provides pensions to retirees. The model has several key features. First, consumers value leisure and a composite consumption good that consists of a market good and a home produced good. We model home production technology along the lines of Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), and Rupert, Rogerson, and Wright (2000). Second, households face

uninsurable idiosyncratic shocks to their labor efficiency. Finally, we restrict intertemporal trade by borrowing constraints collateralized by housing.

3.1 Technology and Timing

There is only one type of market good produced according to the aggregate market production function

$$(2) \quad F^m(K, N^m) = K^\alpha (N^m)^{1-\alpha},$$

where K is the aggregate market capital stock and N^m is the aggregate market labor input. The final good can be directly consumed, invested in physical capital, or used as an intermediate input in home production.¹² Physical capital, durable goods, and housing depreciate at rates δ^k , δ^d and δ^h , respectively.

Home production requires an intermediate home input, housing, and labor. In particular, the home technology has the following CES functional form,¹³

$$(3) \quad c_h = f^H(d, h, n_h) = \{\omega_2[\omega_1 d^{1-\frac{1}{\varsigma_1}} + (1-\omega_1)(h + (1-\delta)s)^{1-\frac{1}{\varsigma_1}}]^{\frac{1-\frac{1}{\varsigma_2}}{1-\frac{1}{\varsigma_1}}} + (1-\omega_2)(n_h)^{1-\frac{1}{\varsigma_2}}\}^{\frac{1}{1-\frac{1}{\varsigma_2}}},$$

where d denotes the intermediate home input, h denotes the housing stock owned by homeowners, s denotes the rental stock of renters, and n_h the labor input in home production. The parameter δ captures the discount of rental housing in home production, ω_1 and ω_2 control the weights associated with housing, and composite home capital in home production, ς_1 governs the intra-class substitutability between home input and housing, and ς_2 governs the inter-class elasticity of substitution between the composite home input and home time in home production.¹⁴ Note that a household can either be a homeowner or renter, but not both. Therefore, h and s cannot simultaneously take positive values.

¹²For simplicity, we have combined both nondurable expenditures such as raw food with consumer durables such as appliances into a composite durable good used in home production. We term this composite good home input. An interesting extension would be to treat these separately especially for modelling the cyclicity of consumption.

¹³Note that we use small case letter to represent the home production technology as home production takes place at household level.

¹⁴Following Sato (1967), we justify our aggregation by the fact that intra-class elasticity (between home input and housing) is potentially higher than the inter-class elasticity (between home input and home hours or housing and home hours) since home input and housing are more similar in technological characteristics.

3.2 Financial Institutions

Following Gervais (2002), we assume there exists a two-period-lived financial institution that pools households capital to supply mortgages and purchase rental housing. They purchase final goods and use them as housing services which they rent out to renters for use in home production. Specifically, at the end of first period, the intermediary accepts deposits and buys residential capital. In the second period, it repays deposits with interest at rate r . Residential capital is rented to agents at a price η per unit. At the end of the second period, the financial institution sells the undepreciated residential stock to a new agency. The no-arbitrage condition implies that the rental rate on housing is given by

$$(4) \quad \eta = r + \delta^h.$$

3.3 Demographics

During each model period, a continuum of consumers is born. They immediately begin working and consuming. Each consumer retires at $t = T_r$ and dies by the end of age T . Each consumer faces a positive probability of survival, given by λ_t , where $0 \leq \lambda_t \leq 1$.

The probability of survival is exogenous and independent of other household characteristics. Since the demographic patterns are stable, agents at age t make up a constant fraction of the population at any point in time. Annuity markets are assumed to be absent and accidental bequests are distributed to all households in the economy. We discuss the exact distribution rules in the calibration section.

3.4 Consumer's Maximization Problem

3.4.1 Preferences

Individuals derive utility from consumption of a composite good c that consists of a market produced nondurable good, c_m , and a home produced good, c_h , and leisure, l . Preferences are assumed to be time separable, with a constant discount factor β . The momentary utility function from consumption is of the constant relative-risk aversion class given by

$$(5) \quad U(c, l) = \frac{[\omega_4 c^{1-\frac{1}{\varsigma_4}} + (1 - \omega_4) l^{1-\frac{1}{\varsigma_4}}]^{\frac{1-\gamma}{1-\frac{1}{\varsigma_4}}} - 1}{1 - \gamma},$$

where

$$(6) \quad c = [\omega_3 c_m^{1-\frac{1}{\varsigma_3}} + (1 - \omega_3) c_h^{1-\frac{1}{\varsigma_3}}]^{\frac{1}{1-\frac{1}{\varsigma_3}}}.$$

ω_4 represents the relative weight of the composite consumption good in utility, ς_4 represents the degree of substitution between the composite consumption good and leisure, γ denotes the risk aversion parameter, ω_3 denotes the relative weight of the market good in the composite consumption good, and ς_3 measures the degree of substitution between the market good and the home produced good.

3.4.2 Labor Productivity

In the market economy, all consumers of the same birth cohort face the same exogenous age-efficiency profile, e_t . Each worker i also faces stochastic productivity shocks ε_t^i , which follows a Markov process

$$(7) \quad \ln \varepsilon_t^i = \rho_\varepsilon \ln \varepsilon_{t-1}^i + v_t^i, \quad v_t^i \sim N(0, \sigma_\varepsilon^2).$$

This Markov process, Q_ε , is the same for all households and there is no uncertainty over the aggregate labor endowment. The total productivity of a worker at age t is given by the product of the worker's age- t productivity shock and age- t deterministic efficiency index: $e_t \varepsilon_t^i$.

After age T_r , households retire and receive Social Security income. We assume the Social Security benefits are uniform across all eligible households.¹⁵

3.4.3 Transaction Costs and Housing

Due to the heterogeneity and spatial fixity of housing, both potential buyers and sellers in the housing market are forced to spend a considerable amount of time and resources to acquire information about the value of a specific housing unit. As a consequence, there are both implicit and explicit search costs associated with moving. These include the opportunity costs of time associated with search, brokerage and agent fees, recording fees, legal fees, and origination fees. Moreover, households have to physically move to a new house, which entails moving costs and psychological costs of changing neighborhoods (Smith, Rosen and Fallis (1988)).

We consider non-convex transaction costs in the housing stock. The specification of

¹⁵ A more realistic assumption is that the Social Security benefit is a concave function of the accumulated lifetime contributions. Under this assumption, however, the accumulated contribution becomes a state variable, which increases the computation time dramatically.

the transaction costs is

$$(8) \quad \varphi(h, h') = \begin{cases} 0, & \text{if } h' \in [(1 - \mu_1)h, (1 + \mu_2)h]; \\ \rho_1 h + \rho_2 h', & \text{otherwise,} \end{cases}$$

where h' denotes next period's housing stock. This formulation of transaction costs allow households to change their level of housing consumption without moving by undertaking housing renovation up to a fraction μ_2 of the value of the house, or by allowing depreciation up to a fraction μ_1 of the value of the house. If the house depreciates by more than a fraction μ_1 of the value, or appreciates by more than a fraction μ_2 of the value, we assume that the house has been sold. In that case, the household must pay the transaction costs as a fraction ρ_1 of its selling value. Buying a property incurs a fraction ρ_2 of its buying value. Finally, an owner occupied property has to exceed a minimum size $\underline{h}$.

3.4.4 Borrowing Constraints

We assume that only collateralized credit is available and that the borrowing rate, mortgage rate, and deposit interest rate are all equal. This implies that mortgages and deposits are perfect substitutes. We use a' to denote the net asset position. To buy a house, a household must satisfy a minimum down payment requirement equal to a fraction θ of the value of the house. Housing also serves as collateral for loans (through home equity loans or refinancing) up to a fraction $(1 - \theta)$. Therefore, at any given period the household's financial assets must satisfy¹⁶

$$(9) \quad a' \geq -(1 - \theta)h'.$$

In addition, to rule out negative bequests, net worth is bounded below by 0 according to

$$(10) \quad (1 + r)a' + (1 - \delta^h)h' + (1 - \delta^d)d' \geq 0.$$

Finally, as mentioned there is a minimum house size $\underline{h}$ ($\underline{h} > 0$) that can be purchased, i.e.,

$$(11) \quad h' \geq \underline{h}.$$

¹⁶For a household without a house, the borrowing constraint reduces to the standard form $a' \geq 0$.

3.4.5 Renting Shock

In a model where households differ only by age, income, and wealth, rich households tend to be homeowners and poor tend to be renters. In the US, a fraction of rich households are renters. As reported by the Bureau of Labor Statistics in 2000, 12% of households whose income is in the top quintile are renters, and 25% of those whose income is in the fourth quintile are renters. The existence of high-income renters may be due to heterogeneity in house prices, job mobility, preferences, or family composition. For example, a high-income household that lives in a city where house prices are much higher than the national average might not be able to afford to buy a house. Additionally, an individual who dislikes the responsibility of owning a home or is likely to move to another city for work might choose to be a renter.

To capture factors other than age, income, and wealth that affect household's renting/owning decision, we assume households face renting shocks. A household who receives a renting shock is not allowed to own and can only rent. Let q_t denote the probability of receiving a renting shock at age t . The shock is exogenous and independent of other household characteristics.

3.4.6 Time Line

Before we describe the household's optimization problem, we present the time line for their decisions (Figure 1). At the beginning of each period, after they observe their current idiosyncratic labor shocks, the next period's rental shock, and receive a bequest, the household makes its labor supply decision and rents capital to firms. At this point, market production takes place. Home production also takes place using labor, the home input, and housing. After production, households receive factor payments and make their consumption and asset allocation decisions. At the end of the period, market capital, housing and home capital (home input) depreciate and uncertainty about early death is revealed. Accidental bequests from those that die early are distributed to new agents next period to first satisfy an exogenous beginning of period asset position, and if funds are leftover, they are distributed to the other agents in the economy. Households also makes rental versus owning decisions if they did not receive a rental shock, and in the event of owning they choose their house size at the end of the period.

3.4.7 The Household's Recursive Problem

In a stationary equilibrium, the interest rate is constant at r as is the wage rate w per efficiency unit of labor. The household's state variables are given by $(m, t, a, h, d, \varepsilon)$, which denote the agent's rental shock for next period (m), current age (t), financial

assets (a), undepreciated housing stock (h), home input from previous period (d), and labor productivity of the current period (ε). If $m = 1$, then this household is not allowed to own a house next period. If $h = 0$, then the household is a renter for this period. We have

$$V(m, t, a, h, d, \varepsilon) = \max_{\{c_m, d', s, a', n_m, n_h\}} \left\{ U(c, 1 - n_m - n_h) + \beta \lambda_t EV(m', t + 1, a', h', d', \varepsilon') \right\}$$

subject to

$$\begin{aligned} c_m + a' + d' + \varphi(h, h') + \eta s 1(h = 0) + h' 1(m = 0) &= b_t + (1 + r)a + (1 - \delta^h)h + (1 - \delta^d)d \\ &+ (1 - \tau)[e_t \varepsilon w n_m] 1(t < T^r) + pen(t) 1(t \geq T^r), \\ a' &\geq -(1 - \theta)h' \\ (1 + r)a' + (1 - \delta^h)h' + (1 - \delta^d)d' &\geq 0, \\ h' &\geq \underline{h}, \\ (12) \quad c_m \geq 0, \quad s &\geq 0, \quad a' \geq 0, \quad d' \geq 0, \quad 0 \leq n_m, n_h \leq 1, \end{aligned}$$

where $1(\cdot)$ is an index function that takes a value of 1 if the statement inside the parenthesis is true and 0 otherwise, τ denotes the social security income tax before retirement, w denotes the wage, and pen is pension after retirement. Note that we let pension depend on the age of the household. In particular, if a household retires after age 65, it receives the full pension payment. If he retires at age 63-64, he receives 80 percent of the full pension, and if he retires at age 61-62, he receives 40 percent of the full pension.¹⁷ In any sub-period, an agent's resources depend on asset holdings, a , labor endowment, $e_t \varepsilon$, or pension, pen , housing stock, h , home input d , and received bequests, b . Note that, agents receive pension only after retirement. The composite consumption good c is defined as in equation (6), the home produced good is defined as in equation (3).

3.5 Definition of Stationary Equilibrium

A formal definition of a stationary equilibrium which includes market clearing conditions is provided in Appendix A. The model is solved numerically. Appendix B describes the computation algorithm in greater detail.

¹⁷The rule of our pension distribution follows the general practice of the Social Security Administration. In practice, however, early retirees receive a permanently lower pension while in our model the reduction is only up to age 65. As a result, the per period reduction in pension payment before age 65 in our model doubles the permanent percentage reduction in practice, i.e., 40 percent versus 20 percent.

4 Calibration

4.1 Parameter Calibration

The model period is two years.¹⁸ At age 25, each person enters into the model. The retirement age T_r is 63, and the maximum life expectancy T is 100. Figure 4, panel b, shows the λ_t s, the vector of 2-year conditional survival probabilities. We use the mortality probabilities for people born in 1960, weighted by gender from the Social Security Administration life tables.¹⁹

The term α is the share of income that goes to the nonresidential stock of capital and is set at 0.240. This capital share is lower than in many real business cycle calibrations because housing is not part of our model's capital stock. We set δ^k to 0.10 and δ^h to 0.02. The rate, r is the interest rate on capital net of depreciation and is set to 0.04. Appendix C explains the rationale behind these choices in greater detail. We assume that the intermediate home input depreciates completely in two years given that household appliance and equipment accounts for less than 0.10 of total home input.

The deterministic age-profile of the unconditional mean of labor productivity, e_t , taken from French (2005), is shown in the top panel of Figure 4. The labor-efficiency profile is hump-shaped, with a peak at age 50. The persistence ρ_y and variance σ_y^2 of the stochastic productivity process are 0.977 and 0.014, respectively. The variance of the initial distribution of productivity estimated from the Survey of Consumer Finances (SCF) is 0.189. For simplicity, we assume the labor efficiency profile for home production to be constant.

We calibrate the Social Security tax τ to 0.096 to match the average payroll tax. Accidental bequests are first distributed to new agents at age 23-26 to reproduce the distribution of capital endowments for households at age 23-26 according to SCF 2001, 2004, and 2007.^{20,21} The rest, if there is any, is distributed evenly to all agents alive, which endogenously determines b_t .

The down payment rate θ is set to be 0.20, which is commonly used in the housing literature. The probability of receiving a renting shock is from Li and Yao (2007) where they calibrated to average households migration rates for non-housing related reasons and is shown in the bottom panel of Figure 4. Gruber and Martin (2003) estimate the

¹⁸Given the model period, we adjust parameters in the model accordingly. We report parameters at annual frequency, unless stated otherwise.

¹⁹Since our model is gender neutral, we use a gender-weighted mortality probability.

²⁰The purpose of this procedure is to generate some heterogeneity at the beginning of the life cycle. Otherwise, income will be the only factor to determine the renting/owning decision for agents at age 25.

²¹Since the model does not allow negative wealth, negative wealth holdings in the data are treated as zero. Most households start with wealth endowments close to zero.

reallocation cost of tax and agency costs from the CEX and find the median household spends 7 percent of a house's value to sell it and 2.5 percent to purchase. In our simulation, we therefore choose transaction costs from sales to be $\rho_1 = 0.07$ and from purchases to be $\rho_2 = 0.025$. Davidoff (2006) shows that homeowners over age 75, compared with younger owners of similar homes, spend about 0.8 percent of home value less per year on routine maintenance. We choose a big range and set $\mu_1 = \delta_h$, $\mu_2 = 4\delta_h$. That is to say, households can change their level of housing consumption by allowing depreciation or renovation as alternatives to moving.

We take the risk aversion parameter, γ , to be 1.5, from Attanasio, Banks, Meghir and Weber (1999), and Gourinchas and Parker (2002), who estimate it from consumption data. This value is in the commonly used 1 to 5 range in the macro literature. The initial distribution over state variables (wealth, house size and house tenure) for households of age 25 is calculated using Survey of Consumer Finances (2001, 2004, and 2007) for households whose heads are between age 23 and 26. The remaining parameters are chosen so that the model-generated data match a given set of aggregate targets generated from the ATUS and the CEX as shown in Table 4. The moments basically involve various expenditure and asset to income ratios as well as moments pertaining to the use of time. The calibration of all parameters are presented in Table 3.

For households' life-cycle labor supply and consumption demand, eight parameters are of particular importance and they are the ω_i 's and ζ_i 's ($i = 1, 2, 3, 4$). In particular, the ω_i 's govern household consumption/labor share and the ζ_i 's govern the elasticity of substitution between different consumption goods and consumption and leisure. According to our calibration, the home input and housing are Hicksian substitutes in the production of the composite home good. The composite home good and home hours exhibit strong complementarity in home production. Market good and home good, on the other hand, are substitutes.²² Finally, the final composite consumption good of market good and home good is substitutable with leisure in households' utility. The existing literature on home production has largely lumped home hour and leisure together into non-market hour, making the comparison with our estimates difficult. There are nevertheless some supporting evidence from the literature. For example, Abbott and Ashenfelter (1976) find that housing, transportation, and other services tend to be complementary with non-market time but durables tend to be substitutable with non-market time. Barnett (1979) estimates a model of joint goods and leisure consumption and finds non-weakly-separable substitution between consumption and leisure. Green-

²²The direct partial elasticities of substitution between home input and home hour and housing and home hour can be derived using formulas provided in Sato (1976). Using the economy wide average consumption, we calculate the elasticity of substitution between home input and home hour is 0.817 and the elasticity of substitution between housing and home hour is 0.869.

wood and Hercowitz (1991) argue that to generate comovement in investments in durable goods in the market and at home one needs to have complementarity between durable goods and time in home production.²³ The finding that home input and housing are complements with home hours in home production explains why after a household transits from a two-earner family into a one-earner family, home capital typically increased as documented in Baxter and Rotz (2009). After we present household optimal decision rules and life-cycle profiles, we will conduct additional analysis to understand the identification of these parameters.

4.2 Optimal Labor Supply and Consumption Decision Rules

Under our benchmark calibration, the home input depreciates completely in two years. Thus, a household at the beginning of a period is described by its rental shock m which determines its house tenure in the next period, age t , financial assets a , and labor productivity of the current period ε . A homeowner is further characterized by his house value (after depreciation) h , which is chosen in the previous period. A household's house tenure decision is driven by the household's wealth, current labor income shock, and age. Everything else the same, richer households are more likely to become homeowners than poor households and the wealth threshold for becoming homeowners declines with household age. The added incentive for older agents to own a house derives from their having more time for home production, which implies that owning a house has added value to them.

In Figures 5 and 6, we plot households labor supply and consumption decisions against their cash-on-hand (liquid wealth plus housing equity net of transaction cost for owners) for three age groups (ages 30 (young), 50 (middle age), and 70 (old)). The owners' decisions are charted on the left and the renters on the right. All households have received the same idiosyncratic labor shock ($\varepsilon = 3$) and the owners do not have to move next period for non-housing reasons ($m = 0$). Furthermore, since the current house value for owners is determined in the last period, to facilitate comparison, we pick those owners with a house value at around 88 (which is about 1.4 times of the annual economy-wide average).

As depicted in Figure 5, at each given level of cash-on-hand middle-aged households work more than the young households and the old households work the least. This feature of the model applies to both homeowners and renters, and the pattern stems largely from the labor efficiency profile as the middle-aged have the highest market labor efficiency followed by the young and then the old. The resulting home hours and leisure

²³Chang (2002) argues that adjustment cost in capital accumulation can account for the comovement in investments.

have the reverse pattern. Not surprisingly, the richer a household is, the less it works in the market, and the more it enjoys leisure. The wealth effect on home production is ambiguous. On the one hand, wealthier households are less likely to engage in costly home production. On the other hand, wealthier households tend to own bigger houses which would encourage more home hours for home production. In our simulation, for the particular households we charted, the latter effect slightly dominates.

Figure 6. depicts households' optimal consumption. Note that homeowners' housing stock/house value is of the current period which is decided in the previous period. Consistent with their labor supply decisions, middle-aged households, homeowners as well as renters, spend more on market consumption than young households. Compared with the young households, given their high income level, the middle-aged households compensate their low supply of home hours with the purchase of more home input. In terms of housing, the middle aged renters rent larger households than the young. Old households spend less on all consumption goods when their wealth is low. As soon as their wealth passes a certain threshold, however, they spend more on consumption than the middle-aged and the young with the same wealth and same house tenure. This is because old households discount their future utility much more than young and middle-aged households. Finally, consumption of all three categories increases monotonically with wealth.

5 Numerical Results

This section studies the implications of the model economy for homeownership rates by age, and for the life-cycle profiles of consumption, hours and wealth for both homeowners and renters. We contrast the implied life-cycle patterns with those constructed in the data as discussed in section 2.

5.1 Homeownership

Figure 7 shows the fraction of homeowners at each age, the model prediction against the data. Our model prediction tracks the data profile pretty well with only the youngest household (age 25) and the oldest (age 80) falling outside of the two-standard deviation error band. In the model, most young agents rent while accumulating financial assets. As time goes by, more households have accumulated sufficient funds for down payments to become homeowners. Homeownership rates continue to be very high late in life.

In our model, renting has several advantages over owning. First, since there is no minimum size in rental units, relatively poor households can rent relatively small units

rather than buy a large one. Second, renters can adjust housing without paying transaction costs. On the other hand, owning might dominate renting. Owned housing can be used as collateral and relaxes borrowing constraints. Also, owner-occupied housing is more productive than rental properties in home production with an efficiency gap of 15 percent. For young agents, who face future income shocks and on average receive lower income than middle-aged agents, renting is more attractive than owning. Once agents have accumulated a down payment and most uncertainty in income has been revealed, they choose to own. The gradual process of acquiring enough wealth to purchase a home also has implications regarding the distribution of agents who continue to rent over their life cycle. Apart from those that draw a rent shock, they tend to be households that have drawn a low labor productivity profile. Thus over time there is a selection effect regarding the productivity of agents who rent. Also, the desire to own a house has implications for the life cycle profile of asset accumulation. As in Fernandez-Villaverde and Kruger (2002) most of a households initial wealth accumulation is in the form of houses.

5.2 Life-cycle Profiles of Hours

Figure 8 shows the life-cycle profiles of the average fraction of time spent in working and home production in the model. Notice that our model does a good job of capturing the life-cycle profiles of hours spent in market work, home production, and leisure for both homeowners and renters. Young agents, all starting with little wealth, work relatively intensively to buy goods, to accumulate precautionary assets, and to save for future house purchases. As agents age, they spend more time at home and decrease market hours. This is largely driven by the labor efficiency profile which peaks at around age 55. The distribution of Social Security benefits starting at age 62 provides additional incentive for households to reduce their labor supply after age 62. Note that the increase of resources after age 62 from Social Security benefits induces more reduction in market hours for renters in the model. This occurs because in our model pension distribution is only a function of age. Therefore, for old age renters whose labor productivity tends to be low, pension payment may exceed their labor income substantially.

Homeowners spend more time in home production than renters in the same age group. This stems from the fact that under our parameterization, time and houses are complements. Homeowners on average have more housing capital than renters, and thus spend more time at home.

5.3 Life-cycle Profiles of Consumption

The left panel of figure 9 shows, in percentage deviation from the corresponding value at age 25, the life-cycle profiles of average demand for market consumption, housing and market input for home-owners. Again our model prediction falls within the two-standard deviation error band of the data estimation. Over the life cycle, average market goods consumption for homeowners is hump-shaped and peaks at age 55. Market goods consumption at age 55 is about 30 percent more than that at age 25. After the peak, market goods consumption decreases dramatically with age. Market goods consumption at age 80 is about 50 percent less than that at age 25. Facing an increasing future income profile, young agents would like to borrow to finance their current consumption but they are borrowing-constrained. This explains why early in life consumption increases as income does. As households age, they start to decrease their market consumption due to the fact that the mortality rates are increasing along the life cycle.

The demand for housing in the model reproduces the empirically observed profiles, increasing early in life and downsizing slowly later in life. Households begin their economic lives without any housing stock. During the early part of their lives, because of the existence of borrowing constraints and the role of housing as collateral, they forego non-housing consumption and build housing stock quickly. Agents build up their highest housing stock at age 60. The elderly decrease their housing stock quite slowly, due to transaction costs and the increasing home hours which are complements to home input and housing in home production. Old households are less likely to move and incur the accompanying transactions costs than young households, because they can only live in the new house for a relatively short period of time. Home input and housing generally track each other over the life cycle as both are complements with home hours in home production. However, the transaction cost in housing adjustment for homeowners makes the track less than perfect. Using detailed American Time Use Survey, Baxter and Rotz (2009) find that when a wife leaves the labor force home hours rise as expected but durables also rise which is unexpected according to standard home production models. This pattern, however, is entirely consistent with our model as home input, of which durable home appliances are a subset, is a complement with home hours with the direct partial elasticity of substitution at 0.817 (see our discussion in the calibration section).

The right panel of figure 9 shows the life-cycle profiles of average market consumption and demand for housing and home input for renters. Note that the life-cycle profiles of renters are strongly affected by selection effects. As renters age, more and more of them become homeowners. Those who remain renters tend to be relatively poor with fewer resources. Since rental housing is costless to adjust, utility optimization implies that the ratio of housing to home input is constant for renters, thus the profile for durables

coincides with the one for housing. Average market goods consumption for renters does not vary much before age 65, but decreases very dramatically after retirement.

5.4 Parameter Identification

As discussed earlier, the four elasticity parameters (ζ_i , $i = 1, 2, 3, 4$) play crucial roles in determining households' supply of labor to different activities and consumption of different goods. To further understand the importance and identification of these parameters, we conduct the following experiments. In each experiment, we change one of the elasticity parameter by either doubling it if the parameter is less than 1 or by cutting it by half if the parameter is greater than 1 while keeping the values of the other parameters unchanged. We then compute the same target moments and compare them with those from the benchmark analysis. The results are presented in Table 5.

The identification of ζ_1 , the elasticity of substitution between home input and housing in home production comes from the relative expenditure share between home input and housing for homeowners.²⁴ When we reduce the elasticity between home input and housing in the home production by half making the two complements instead of substitutes, the aggregate relative expenditure of home input to housing stock increased from 0.20 to 0.22. The increase mostly comes from older households not reducing their purchase of home input for home production purposes. In our model, older households generally have too much house because it is costly to move to a smaller house. The substitutability of home input implies that they will have a lower ratio of home input to housing than they would in the case of complements. Complementarity between these inputs makes it difficult for the model to match the ratio of .20.

The parameter ζ_2 , the elasticity of substitution between the composite home input and home hours, is strongly identified by households' supply of home hours at old age as well as the fraction of homeowners. When we force the composite home input to be a substitute with the home hours, households accumulate much more housing and they reduce their supply of home hours to near zero. As households now desire bigger houses, some opt to be renters trading off the efficiency gain of owned housing in home production with the size of the house as many of them are constrained by the downpayment requirement. The end result is that too few households become homeowners.

When we reduce ζ_3 by half, the market good and the home good are less substitutable in households' utility. In other words, households now desire more home good for a given amount of market goods than before. As a result, home hours go up and market hours

²⁴Since there is no transaction cost in housing for renters, their supply of home input and rental housing track each other with the relative expenditure of the two goods constant at 0.19.

come down. At the same time, the expenditure for the other inputs for home production, housing stock as well as home input, also go up, implying that sufficient substitutability is needed if we are to match the relative importance of home hours to market hours and to not overshoot the housing stock and consumption of the home input.

Finally, we identify the elasticity of substitution between final composite consumption and leisure from the demand for leisure relative to consumption. When leisure becomes a strong complement with overall consumption, households demand for leisure increases substantially and consumption of all goods comes down significantly.

5.5 Life-cycle Profiles of Wealth Composition

Figure 10 displays the evolution of the wealth portfolio over the life cycle for home owners and renters calculated using the same synthetic cohort method. Figure 11 displays the same profiles generated by our model simulation. Although our model does not match the quantitative feature of the profiles exactly, it does match the qualitative features.²⁵

Young households, who start with little wealth and expect to have much higher earnings in the future, do not hold much wealth. Early in life, households borrow as much as possible to buy houses, and thus save in the form of housing. As time progresses, agents have accumulated stocks of houses and start to increase their holding of financial assets. The profile of financial assets and housing assets intersect in the early 40s. Financial wealth holding peaks at age 55. Afterwards, households start to use their assets to finance consumption. At very old ages, homeowners borrow against their homes and take on debt.

Renters hold less financial assets than homeowners. Compared with the data, the financial assets profiles for both owners and renters have humps that are more pronounced. Since we abstract from bequest motives, health expenditure uncertainty or other shocks, old agents in our model do not have bequest or precautionary saving motives and run down their assets much more quickly than in the data.²⁶

6 Policy Experiments

We conduct two policy experiments in this section that help us determine the important factors in driving households' labor supply as well as consumption decisions over the life

²⁵This is a well-known challenge in the literature. See Fernandez and Krueger (2010) for more discussion on this.

²⁶The risk of incurring substantial medical expenses such as out-of-pocket medical expenses and uninsurable nursing home expenses might generate precautionary savings and affect the wealth profile (De Nardi, French and Jones (2005)). The effect of medical costs on the life-cycle consumption and saving in an environment with housing is left for future research.

cycle. In the first experiment, we postpone the time at which pension becomes available from age 62 to age 67. This experiment is motivated by policy debates surrounding the insolvency of the nation’s social security system. Some experts have been advocating an increase in Social Security retirement ages as a way to cut cost and even out the years of potential benefit receipt across generations.

In the second experiment, we relax the downpayment requirement from 20 percent in the benchmark to the minimum that is allowed in our environment in order to satisfy the no debt at death requirement, which is about 11 percent.²⁷ The relaxed borrowing constraint has been credited with expanding the nation’s homeownership rate and we would like to study the consumption implication of this financial development in our new consumption model where housing serves as an input to home production rather than a direct consumption good.²⁸

In both experiments, we keep all parameters the same including pension payments after retirement. The interest rate r , wage rate w and payroll tax rate τ are chosen to clear the capital and labor markets and to balance the government budget constraint. The results are reported in Table 6 where we compare homeownership rates, consumption and labor supply among homeowners and renters with the benchmark economy.

6.1 Postponing Pension

One of the most talked about policy reform that aimed at solving the nation’s social security crisis is to delay retirement age. The most direct impact of postponing the age when the pension becomes available from age 62 to age 67 is on households’ saving and labor supply. Households now have to work and save more for the five years they were previously receiving pension. With more savings, the economy wide capital output ratio rises, the equilibrium interest rate moves down from 4 percent to 3.8 percent and the wage rate rises by about 3 percent. The payroll tax rate comes down substantially because the tax base is now larger. As reported in Table 6, both homeowners and renters supply more market hours. In particular, homeowners increase their market hours by 3.3 percent while renters increase their market hours by 2.6 percent. The difference in

²⁷The two constraints (9) and (10) require that

$$(1 + r)(1 - \theta) \leq (1 - \delta^h),$$

or

$$\theta \geq 1 - \frac{1 - \delta^h}{1 + r}.$$

Given our calibration and at two-year frequency, this amounts to $\theta \geq 0.89$.

²⁸Since housing prices are not endogenized in the model, we are silent on the implication of this relaxation in borrowing constraint on house prices.

the change of market hours between homeowners and renters are largely due to the fact that homeowners tend to be more productive.

Housholds, especially young households, hold most of their assets in housing. As a result, homeownership rate rises by 0.6 percentage point. Both homeowners and renters now have bigger houses. The increase in renters' houses is larger partly because renters do not have to purchase the whole house and, therefore, are not subject to the collateral constraint. Bigger houses drive up home hours as they are complements in home production. On balance, homeowners supply slightly less home hours as the downward pressure resulting from higher wage dominates the upward pressure from larger houses. The home hours supplied by renters do not change.

Since households are now richer, they also consume more home input and market good. Specifically, homeowners increase their consumption of home input by 0.3 percent and renters by 1.6 percent.

6.2 Lax Borrowing Constraint

Prior to the onset of the current mortgage crisis, the nation has enjoyed a period of relative easy credit supply. In the mortgage market, the required mortgage loan to value ratio has gone up from the traditional 80 percent to 90 percent or even higher. It is far beyond the scope of this paper to analyze the linkage between the lax borrowing constraint and the financial crisis. We nevertheless ask, what are the consumption implications of this lax borrowing constraint in our unique economic environment where housing does not enter directly into households' utility but rather serves as an input for home production?

As can be seen from Table 6, when we raise the mortgage loan to value ratio from 80 percent to 89 percent, the maximum allowed in our model economy, households' collateralized borrowing constraint is also greatly relaxed. As a result, households reduce their precautionary saving. The economy wide capital output ratio comes down. The equilibrium interest rate increases and the wage rate decreases.

Since owned houses are more productive in home production, more households become homeowners and the homeownership rate rises substantially (15 percent). As a matter of fact, the remaining renters are mostly those that have received a rental shock and thus cannot be a homeowner. Compared with the benchmark, the new homeowners tend to be less efficient and less wealthy. That together with the lower wage leads to homeowners work fewer hours, 1.345 percent fewer in the new economy. The composition effect also leads to smaller average home size owned by homeowners (7.414 percent smaller). Given the complementarity between housing and home hours, home hours

also declines by -0.676 percent. Smaller houses, on the one hand, pushes up supply of home input for home production given the substitutability between housing and home input. Fewer home hours, on the other hand, reduces demand for home input given their complementarity. In equilibrium, the latter force dominates and homeowners purchase 7.044 percent less home input.

Changes in renters' behavior are largely driven by the new composition of the renters. Under the new equilibrium, the new renters are "forced" renters, i.e., households that have received a rental shock. Therefore, compared with the benchmark economy, these renters tend to be more efficient and wealthier. As reported in Table 6, in the new equilibrium, renters work substantially more than benchmark and they also tend to rent larger houses. Larger houses, in turn, requires more home hours in home production. Furthermore, renters also purchases more home input for home production accomodate the increased home hours.

7 Conclusions

We extend a standard life-cycle model of consumption by including housing decisions that involve an important external margin concerning home ownership along with the addition of an explicit home production environment. The model, thus, explicitly distinguishes between market and non-market related labor supply and consumption variables. We show that such a model can account for the observed life-cycle patterns in households' time use as well as consumption of different categories. In particular, the labor efficiency profile together with the availability of households' retirement funds imply that households have incentives to drastically reduce their labor supply at around age 50. As they reduce their market hours, households allocate more of their time to home production and leisure. Because home production requires complementary inputs of home hours, housing, and home goods homeowners after retirement enjoy relatively more home hours than renters. On the consumption front, households initially increase their consumption of market goods, home goods, and housing as they accumulate more assets to relax their borrowing constraint. Toward retirement age, as households reduce their market hours, the cost of home production is lower. Consequently, consumption of market goods declines since households substitute home produced domestic goods for market goods. Home goods and housing also decline slightly as households approach the end of their life cycle but the decline is partially offset by the requirement of home production. Further, the explicit modeling of the housing decision allows us to discriminate between renters and home owners and to show that our framework is capable of matching both types of households behavior. That ability allows us to investigate the

effects that lax collateral constraints have on economic behavior and the model also allows us to explore the effects of altering pension schemes. Policy analysis indicates changing retirement age or the collateralized borrowing constraint has important implications on households' house tenure decision, labor supply, and consumption as well as for steady-state capital accumulation, interest rates, and wages. **Further there are important distributional implications of each of these policies which we need to flesh out.**

Appendix A. Definition of the Stationary Equilibrium

We focus on the stationary equilibrium of the economy where factor prices and agent distribution over state space are constant over time. Each agent's state is denoted by x . Let S denote the aggregate housing stock available for renting, H denote the aggregate owner-occupied housing stock, D the aggregate stock of home input, C_m the aggregate consumption of market good, I_h the aggregate investment on housing, I_d the aggregate investment on home input, I_k the aggregate investment on physical capital, and T^c the total transaction costs for trading housing, N_m aggregate market hours supplied, N_h aggregate home hours supplied.

Definition 1. A stationary equilibrium is given by government policies tax rate τ , and pension pen ; an interest rate r and a wage rate w ; value functions $V(x)$; allocations $c_m(x)$, $a'(x)$, $h'(x)$, $d'(x)$, $s(x)$, $n_m(x)$, $n_h(x)$; bequest $b(x)$; and a constant distribution of people over the state variables x , $v(x)$, such that the following conditions hold:

- (i) Given the government policies, the interest rate, the wage, and the expected bequest, the value functions and allocations solve the above described maximization problem for a household with state variables x .
- (ii) $v(\cdot)$ is the invariant distribution of households over the state variables.
- (iii) The price of each factor is equal to its marginal product.

$$\begin{aligned} r &= F_1^m(K, N^m) - \delta^k, \\ w &= F_2^m(K, N^m). \end{aligned}$$

- (iv) The expected bequest is consistent with the actual bequest left

$$\int bv(dx) + \int_{t=0} a(1+r)v(dx) = \int (1-\lambda_t)[(1+r)a' + (1-\delta^h)h' + (1-\delta^d)d']v(dx).$$

(v) All markets clear.

$$\begin{aligned}
H &= \int h'v(dx), \\
S &= \int_{h'=0} sv(dx), \\
D &= \int d'v(dx), \\
K &= \int a'v(dx) - S, \\
C_m &= \int c_mv(dx), \\
T^c &= \int \varphi(h, h')v(dx), \\
N_m &= \int \varepsilon e_t n_m v(dx), \\
I_h &= \int [h' - (1 - \delta^h)h]v(dx), \\
I_d &= \int [d' - (1 - \delta^d)d]v(dx), \\
I_k &= K' - (1 - \delta^k)K \\
F^m(K, N_m) &= C_m + I_k + I_h + I_d + T^c.
\end{aligned}$$

(vi) Government budget is balanced at each period

$$\tau \int \varepsilon e_t w n_m v(dx) = \int_{t \geq T^r} pen(t)v(dx).$$

Appendix B: Computation of the Model

Due non-convex transaction costs on housing and collateralized borrowing constraint, we can not use either Euler equation approximation or policy function iteration. Hence we solve the model using value function approximation.

To compute the steady state of our model, we first discretize the income process into 5 points. The state space for owner-occupied housing and asset holdings are discretized into unevenly spaced grids. The upper bounds on the grids are chosen to be large enough so that they do not constitute a constraint on the optimization problem. We chose 20 grid points for each of the asset variables. The choice variables are searched over 100 grid points for housing and asset, and continuous for other variables. We use linear approximation to approximate valuation functions for the points not on the state grids.

We solve for the steady state equilibrium as follows:

1. Make an initial guess of interest rate r , the wage rate w and pension.
2. Guess the size of accidental bequests.
3. Set value function after the last period to be 0 and solve the value function for the last period of life for each of the points of the grid. This yields policy functions and value function at the last period.
4. By backward induction, repeat step 3 until the first period in life.
5. Compute the associated stationary distribution of households by forward induction using the policy functions starting from the known distribution over types of age.
6. Check whether the associated accidental bequests are consistent with the initial guess. If so, continue to step 7. If not, go back to step 2 and update accidental bequests.
7. Check whether market clearing conditions hold, and whether government budget is balanced. If so, an equilibrium is found. If not, go to step 1 and update initial guessed.

Appendix C: Calibration

We use data from the National Income and Product Accounts and the Fixed Assets Tables for the years 1957-2007. In order to properly calibrate a model with two assets and without government taxes and expenditures, we make some imputations.

In measuring capital income share, we first remove income from the housing sector and the government sector from National Income accounts. Then we define private labor income, Y_{pl} , as compensation of employees, unambiguous capital income (UCI) as rental income, corporate profits and net interest, and ambiguous capital income (ACI) as other income excluding employee compensation, UCI , and depreciation. Thus total private non-housing income Y_p is the sum of Y_{pl} , ACI , UCI , and depreciation. Private capital income Y_{pk} is defined as $UCI + dep_{UCI} + \alpha \cdot (ACI + dep_{ACI}) = \alpha \cdot Y_p$. In other words, we allocate the ambiguous components of capital income and its depreciation according to the share of capital income in measured total output. The share of capital is calibrated as

$$(13) \quad \alpha = (UCI + dep_{UCI}) / (Y_p - ACI - dep_{ACI}).$$

We compute an average share of capital $\alpha = 0.240$.

The variable I_k is total private nonresidential investment, K is private fixed non-residential assets. We calculate the average capital-output ratio $\frac{K}{Y} = 1.61$, and the investment-capital ratio $\frac{I_k}{K} = 0.10$. Given that the paper abstracts from population as well as technology growth, we set the depreciation rate for non-housing capital δ^k at 0.10. The implied real interest rate is thus $r = \alpha \frac{Y}{K} - \delta^k = 0.05$. Note that this rate is somewhat higher than the 0.027 and 0.040 range typically used in the literature. Given that capital stock is measured with considerable error (Gomme and Rupert 2007), we decide to follow the literature and set our equilibrium real rate of interest at 0.04, holding the nonhousing capital depreciation rate at 0.10, this implies a capital-output ratio of 1.71, slightly higher than the calculated 1.61. We set the depreciation rate on housing capital δ^h to 0.02, well within the range of those used in the literature.

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Table 1. Market Hours versus Home Hours

Category	Definition
Market hour	Working (including work related as well as income-generating other activities), Job search, Job interview Commuting
Home hour	House work (interior cleaning; laundry; sewing; repair and maintainaning textile; storing interior household items; food and drink preparation, presentation and cleaning; interior arrangement, decoration and repairs; building and repairing furniture; heating and cooling; interior maintenance; exterior cleaning; exterior repair, improvement, and cleaning; lawn, garden, and houseplant care; ponds, pools, and hot tubs; appliance and tool set-up, repair, and maintenance (by self); financial management; household and personal organization and planning; household and personal mail and messages; home security; and related travel) House work service (using interior cleaning services; using meal preparation services using clothing repair and cleaning services; waiting associated with using household service; using and waiting associated with home maint/repair/décor/ construction services; using and waiting associated with lawn and garden services; telephone calls to/from household services providers; and all related travel) Shopping (grocery, gas, food, and others, waiting associated with shopping, security procedures related to consumer purchases; telephone calls to and from sales person; and all related travel) Pet care (care for animals and pet, using and waiting for veterinary services, using and waiting for pet services; and travel related to these services) Car care (vehicle repair and maintenance and related services including travel) Child care (physical care for children; reading, playing and talking with children; planning and attending children' activities; doing homework, meeting and school conferences; medical care and associated services to children) Adult care (physical care and related services to adult; house work, animal care, vehicle repair and maintenance; financial management; medical care and services; and related travel) Shop search (comparison shopping and research purchases) Child care service (paid child care services and related travel) Professional service (financial; legal; health and care; real estate related; related telephone and travel)

Table 2. Market Consumption versus Home Input

Category	Definition
Market good	Food away from home
	Alcoholic beverages
	Tobacco
	Apparel and services
	Other lodging
	Fees and admissions for entertainment
	Televisions, radios, sound and other entertainment related equipment
	Pets, toys, and playground equipment
	Reading
	Personal care
	Education
	Out of pocket medical expense
	Transportation expenses prorated by travel time for leisure or market related activities
Home input	Food at home
	Household operations
	Household furnishing and equipment
	Utilities, fuels, and public services
	Transportation expenses prorated by travel time for home production

Table 3. Calibration

Parameters		Calibrations
Demographics		
T	maximum life expectancy	90
T_r	retirement age	63
Technology		
α	capital share in National Income	0.240
δ^k	annual depreciation rate of capital	0.100
δ^h	annual depreciation rate of housing	0.020
δ^d	bi-annual depreciation rate of home input	1.00
Endowment		
ρ_y	AR(1) coefficient of 2-year income process	0.850
σ_y^2	innovation of 2-year income process	0.300
Government policy		
τ	Social Security tax	0.096
pen^*	Social Security benefit	0.267
Housing market		
$\underline{h}^*$	minimum housing size	1.260
θ	down payment rate	0.200
ρ_1	transaction costs of selling a house	0.070
ρ_2	transaction costs of buying a house	0.025
μ_1	maximum depreciation	1
μ_2	maximum renovation	3
δ^*	discount of rental housing in home production	0.150
Preference		
γ	risk aversion coefficient	1.500
ς_1^*	substitutability between home input and housing	1.359
ω_1^*	weight on home input	0.700
ς_2^*	substitutability between home input and housing composite and time	0.772
ω_2^*	weight on durable and housing composition	0.859
ς_3^*	substitutability between market and home goods	2.463
ω_3^*	weight on market goods	0.194
ς_4^*	substitutability between composite good and leisure	1.296
ω_4^*	weight on the composite good	0.306
β^*	discount factor	0.956

Note: Parameters with * are estimated to match the targeted moments in Table 4. The other parameters are calibrated according to the data or the literature. See the main text for the life cycle survival probability, age-efficiency ability profile, and renting shock.

Table 4. Calibration Results

Moments	Model	Data
capital output ratio (K/Y)	1.710	1.700
home ownership	0.726	0.682
social security tax rate	0.096	0.096
renter nonhousing wealth/owner nonhousing wealth	0.162	0.170
renters		
average expenditure on home input goods/income	0.372	0.370
average housing value/income	1.939	1.926
average share of home hours	0.132	0.130
average share of market hours	0.236	0.230
owners		
average expenditure on home input goods/income	0.653	0.661
average housing value/income	3.156	3.181
average share of home hours	0.148	0.150
average share of market hours	0.223	0.220

Table 5. Identification Analysis

Moments	Benchmark	$\zeta_1(0.679)$	$\zeta_2(1.543)$	$\zeta_3(1.231)$	$\zeta_4(0.648)$
capital output ratio (K/Y)	1.710	1.721	1.744	4.474	3.110
home ownership	0.726	0.700	0.649	0.701	0.432
renter wealth/owner wealth (nonhousing)	0.162	0.152	0.602	0.149	0.124
renters					
average exp. home input/income	0.372	0.387	2.000	0.456	0.174
average housing value/income	1.939	2.016	10.407	2.377	0.907
average share of home hours	0.132	0.143	0.003	0.150	0.054
average share of market hours	0.236	0.231	0.498	0.218	0.076
owners					
average exp. on home input/income	0.653	0.710	2.359	0.928	0.238
average housing value/income	3.156	3.178	8.597	4.247	1.550
average share of home hours	0.148	0.160	0.003	0.188	0.043
average share of market hours	0.223	0.218	0.377	0.184	0.046

Table 6. Policy Experiments

	Benchmark	No SS before 67 (% relative to benchmark)	Downpayment at 11% (% relative to benchmark)
r (interest rate)	0.040	0.038 (5.00)	0.041 (1.383)
w (wage rate)	0.724	0.726 (2.762)	0.723 (-0.119)
τ (labor tax rate)	0.096	0.076 (-20.833)	0.097 (1.042)
K/Y (capital/output)	1.710	1.732 (1.050)	1.707 (-0.175)
SS/Y (social security/output)	0.073	0.058 (-20.638)	0.074 (1.370)
homeownership	0.726	0.732 (0.826)	0.837 (15.289)
renter wealth/owner wealth (nonhousing)	0.162	0.177 (9.260)	0.343 (117.284)
renters			
average exp. home input/income	0.372	0.378 (1.613)	0.414 (11.290)
average housing value/income	1.939	2.032 (4.796)	2.130 (9.850)
average share of home hours	0.132	0.132 (0.076)	0.130 (1.515)
average share of market hours	0.236	0.242 (2.542)	0.257 (8.898)
owners			
average exp. home input/income	0.653	0.655 (0.306)	0.607 (-7.044)
average housing value/income	3.156	3.259 (3.264)	2.922 (-7.414)
average share of home hours	0.148	0.147(-0.676)	0.147 (-0.676)
average share of market hours	0.223	0.230 (3.139)	0.220 (-1.345)
welfare	0	-0.07%	-0.77%

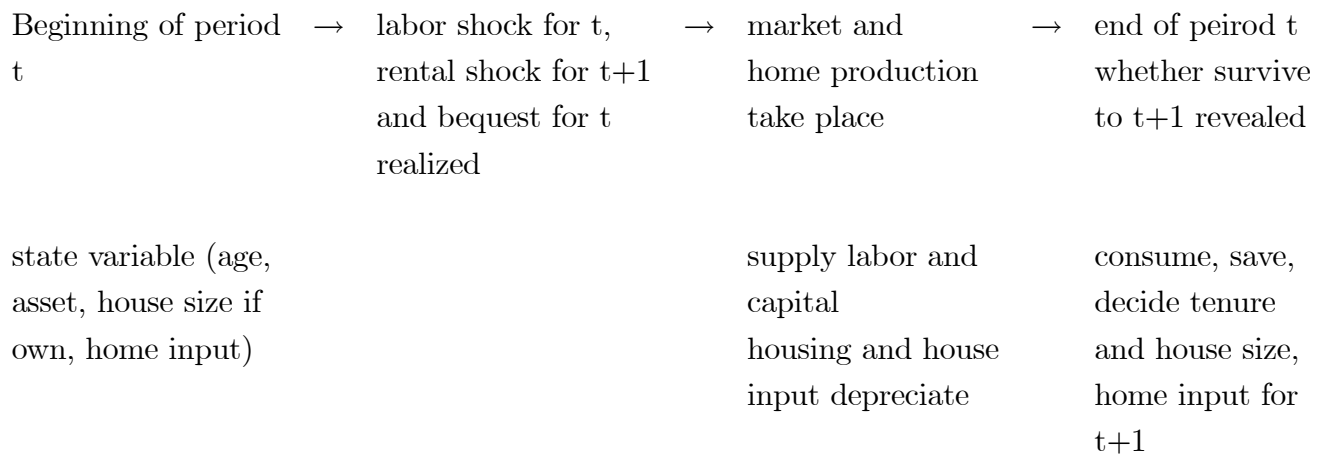


Figure 1. Households Time Line of Decisions

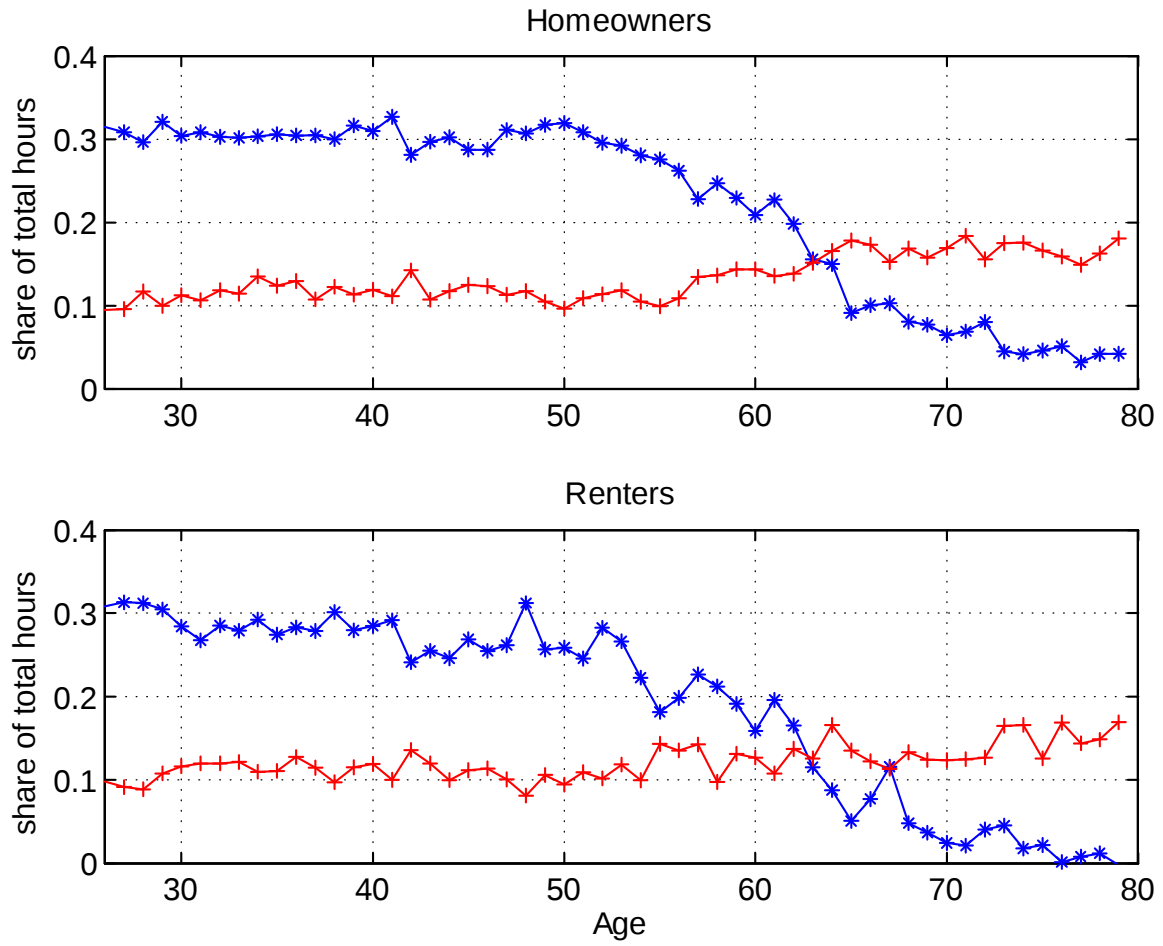


Figure 2. Supply of Hours by Homeowners and Renters (data source: American Time Use Survey 2005-2007. share of market hours: -*; share of home hours: -+)

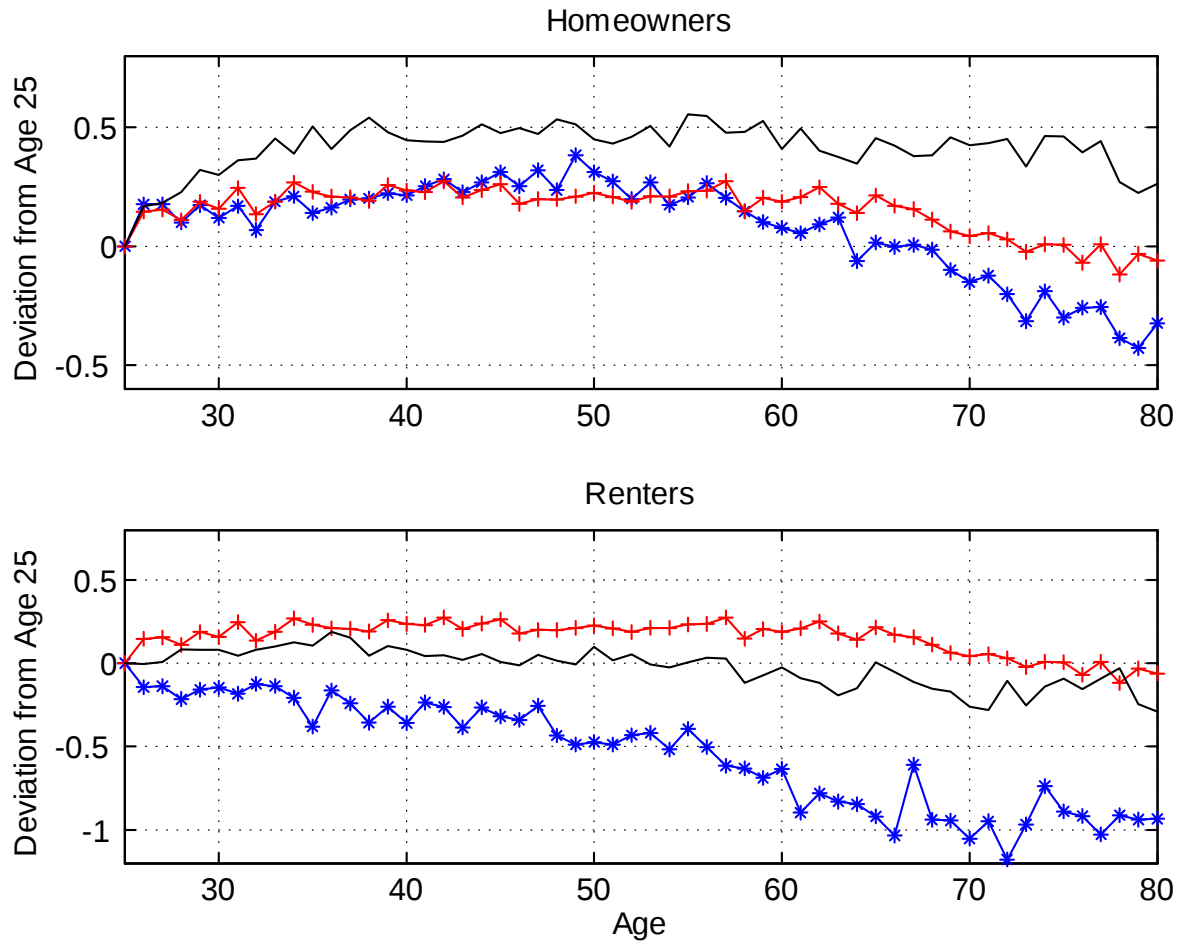


Figure 3. Consumption by Homeowners and Renters (data source: Consumer Expenditure Survey 2003-2006; market consumption: -*; home input: -+; housing: -)

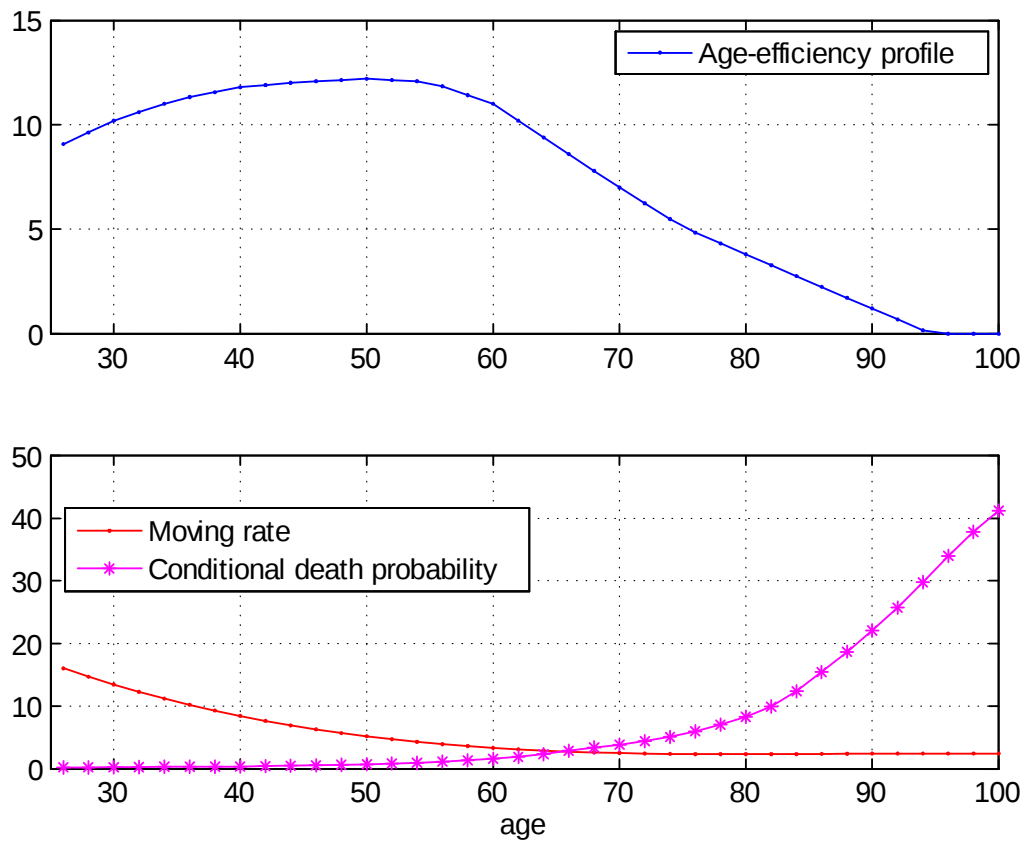


Figure 4. Exogenous profiles

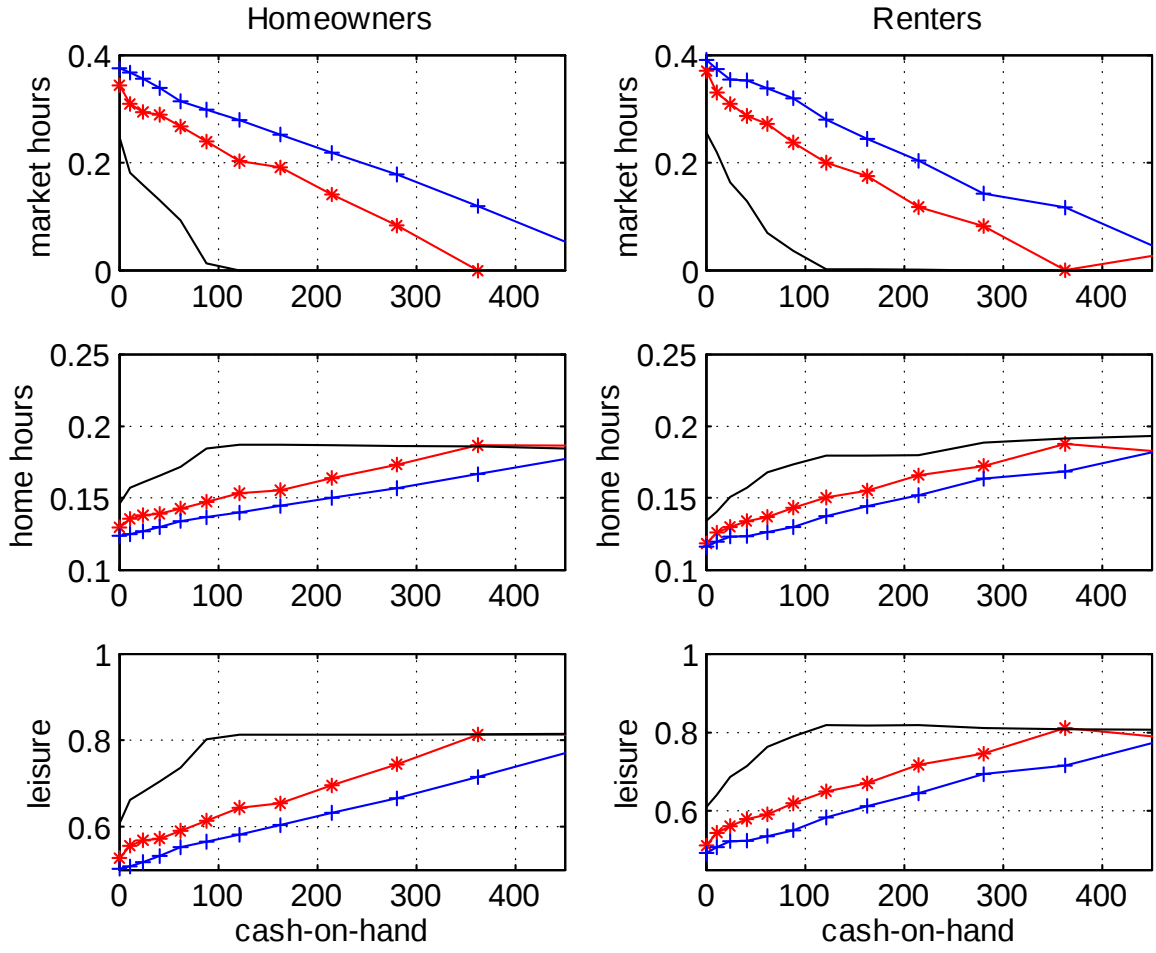


Figure 5. Policy Rules – Hours Supplied by Homeowners and Renters (30 yr olds: -*; 50 yr olds: -+; 70 yr olds: -)

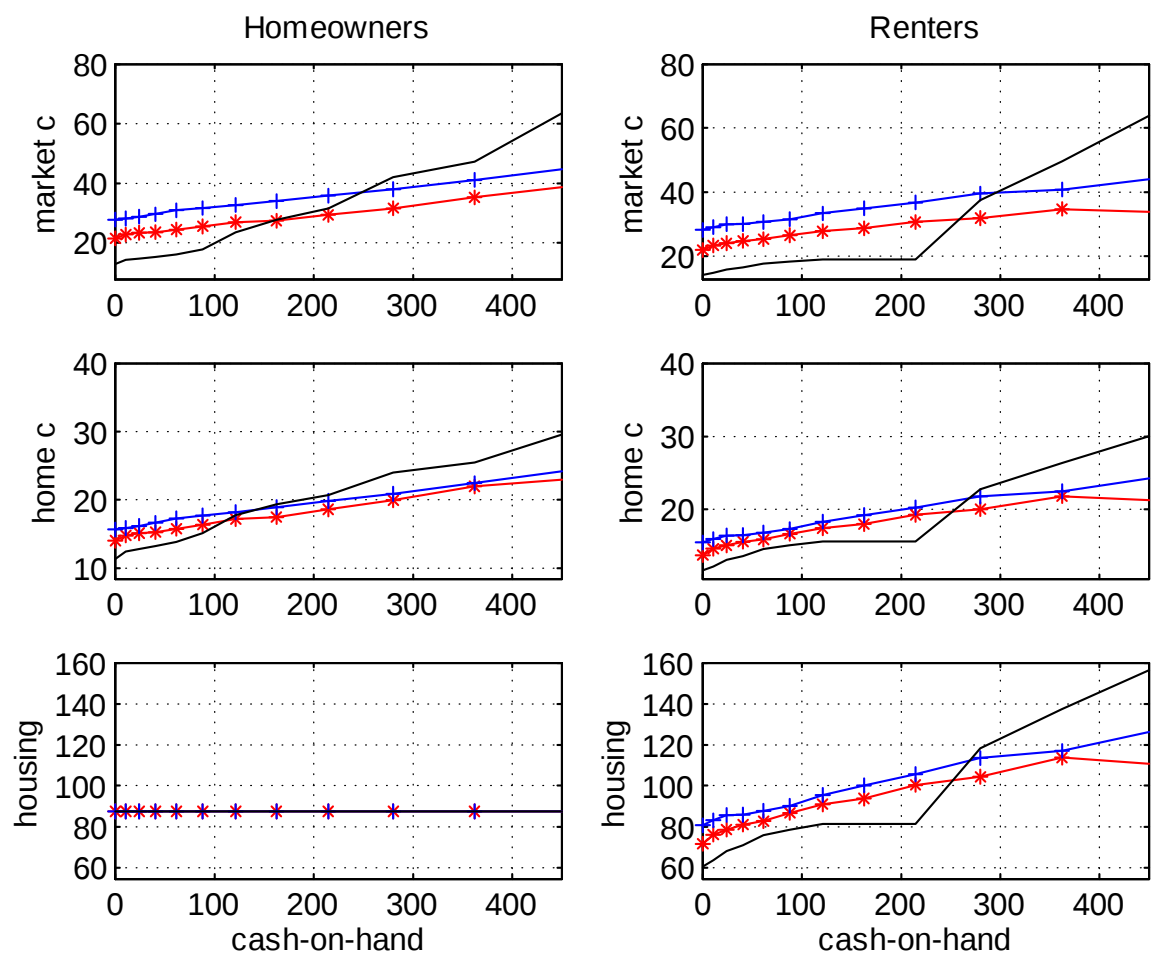


Figure 6. Policy Rules – Consumption by Homeowners and Renters (30 yr olds: -*; 50 yr olds: -+; 70 yr olds: -)

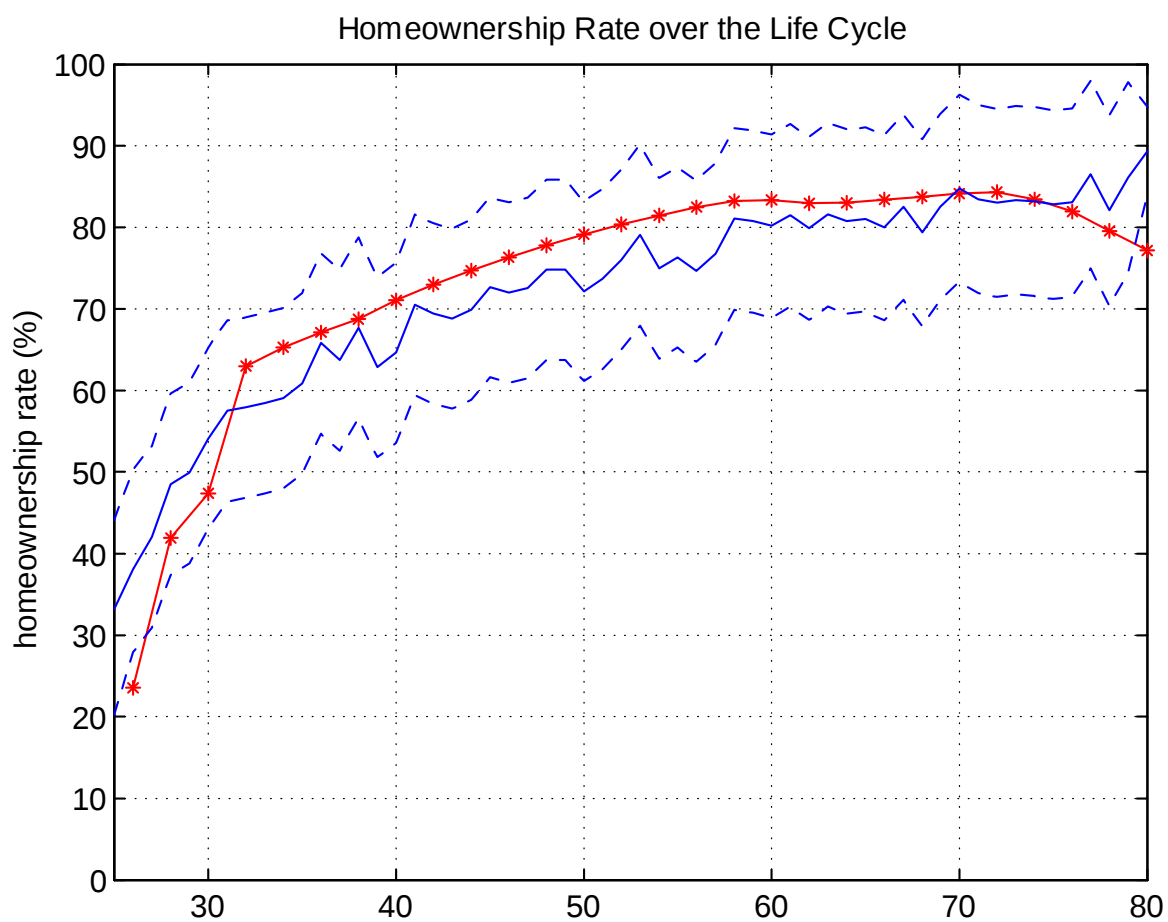


Figure 7. Homeownership Rates over the Life-cycle (data: —; model: -*; the dotted lines are two-standard deviations calculated off the data)

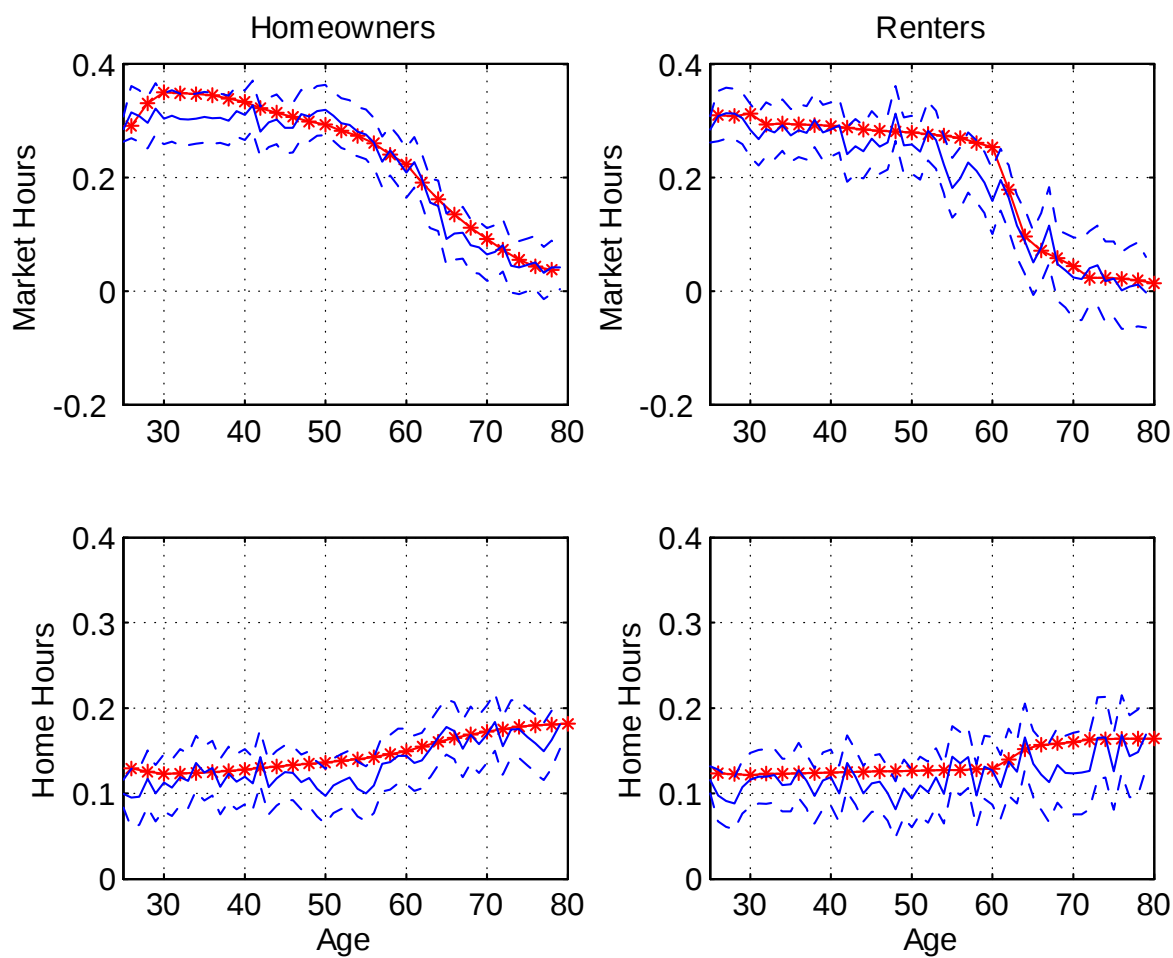


Figure 8. Hours over the Life-cycle (data: —; model: -*; the dotted lines are two-standard deviations calculated off the data)

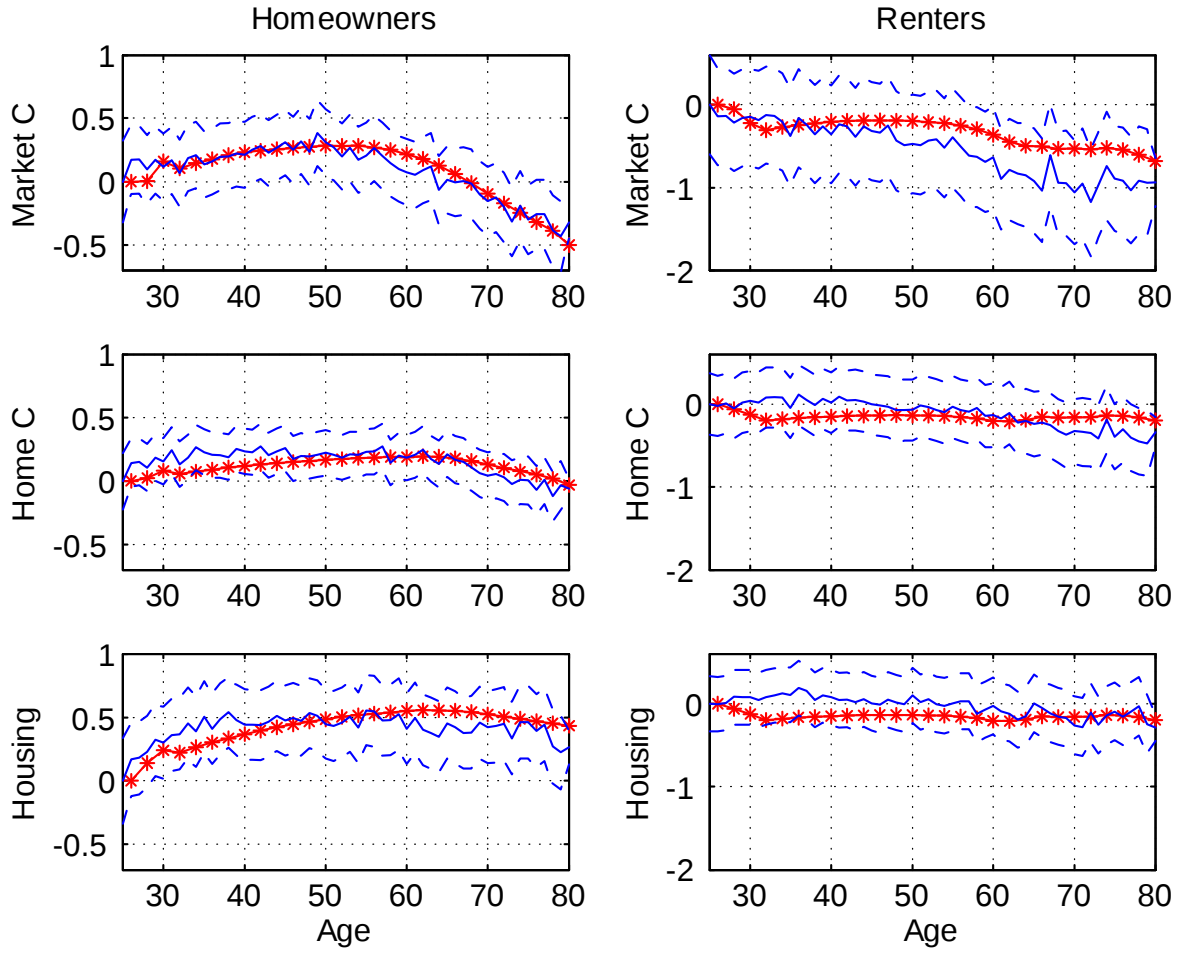


Figure 9. Consumption over the Life-cycle (data: —; model: -*; the dotted lines are two-standard deviations calculated off the data)

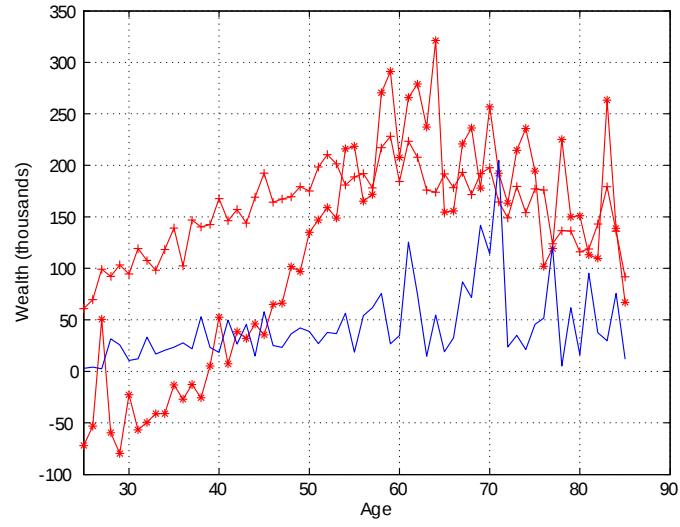


Figure 10. Wealth Profiles over the Life Cycle (SCF 2001, 2004, and 2007; owners' financial wealth: -*; owners' housing asset: -+; renters financial wealth: -)

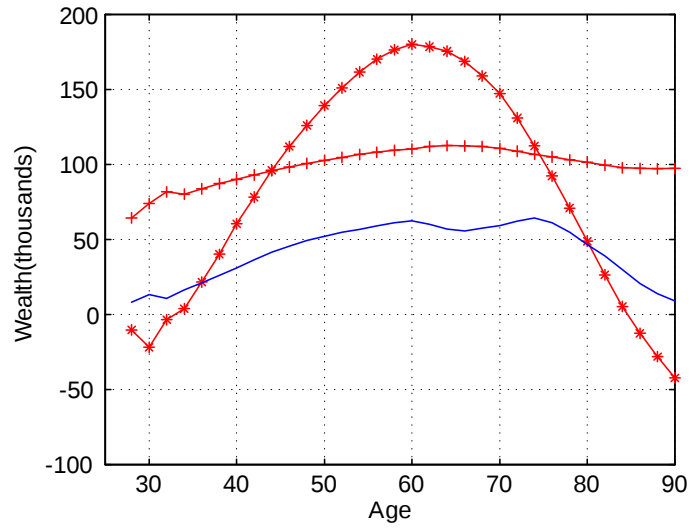


Figure 11. Wealth Profiles over the Life Cycle (model; owners' financial wealth: -*; owners' housing asset: -+; renters financial wealth: -)