Aggregate Fluctuations with Adverse Selection in Credit Markets*

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Abstract

We describe a novel channel through which credit frictions driven by adverse selection problems cause financial shocks to lead to fluctuations in real activity. When firms have private information about their default risk, firms with high probability of default have stronger incentives to run up debt while firms with low probability of default have stronger incentives to internally finance their investments. This asymmetric behavior leads to an adverse selection problem in credit markets. With adverse selection, we show that an otherwise neutral shock to dispersion of firm riskiness can lead to large changes in aggregate productivity, cross sectional dispersion of firm productivity, and credit spreads that are qualitatively in accordance with the data. We then show that a calibrated version of the model performs well regarding real and financial moments in the data.

PRELIMINARY AND INCOMPLETE

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1 Introduction

Shocks to financial market activity seem to matter for real economic activity. Two recent recessions in the United States have been accompanied by downward shocks to the level of activity in financial markets. The financial crisis of 2007 is a stark example since a relatively high decrease in output coincided with a collapse in asset backed securities markets, runs on money market funds and near zero yield on treasury funds. These recent events together with historical correlations between real and financial activity suggest that a channel for shocks to financial activity to have direct consequences for real business cycles is needed to understand economic fluctuations. Standard models of business cycles (Kydland and Prescott (1982),Long Jr and Plosser (1983)) have no financial frictions and hence changes in financial activity have no effect on real activity. Since the early literature on business cycles, many papers have tried to include financial frictions in real business cycle to address the role of financial factors in accounting for business cycles. [1] However, as some recent studies argue, these models do not perform well along some key dimensions. In particular, in most existing models of financial frictions, financial shocks imply large distortions to the so-called investment wedge (see Chari et al. (2006)). The data does not seem to support the view that the investment wedge worsens significantly in downturns. This observation casts some doubts on the existing models of financial frictions.

In this paper, we propose a new channel through which frictions in credit markets cause shocks to financial markets to lead to fluctuations in real activity. Specifically, we demonstrate that modelling financial frictions as adverse selection in debt markets a la Stiglitz and Weiss (1981), survives the Chari et al. (2006) critique about various wedges.

Models of adverse selection have been used theoretically to address various anomalies in credit markets[2]. Moreover, a growing empirical literature suggests asymmetric information

[1] There is an exhausting list of papers that use financial frictions to address aggregate fluctuations, including Bernanke and Gertler (1989), Bernanke et al. (1999), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Kiyotaki and Moore (2008), among others.

[2] Examples include Myers and Majluf (1984), Stiglitz and Weiss (1981), and Garleanu and Pedersen (2004)

leads to adverse selection problems in credit markets and justifies the use of such a friction. In particular, Drucker and Mayer (2008) and Downing et al. (2009) provide strong evidence for existence of adverse selection in syndicated loan markets.

Given the underlying friction, we show how small changes in the cross-sectional dispersion of firm level productivity or default risk together with adverse selection can cause large movements in macroeconomic aggregates and credit spreads. In particular, an increase in the cross-sectional dispersion worsens the adverse selection problem in credit markets and causes aggregate output and investment to fall and credit spreads to rise. It is well documented that the cross-sectional dispersion of firm-level productivity and default rates increases during recessions[3]. In our model, shocks to the cross-sectional dispersion induce fluctuations in aggregate output and countercyclical movements in spreads.

To illustrate the main mechanism of the model, consider an entrepreneur who has access to a risky production technology. The entrepreneur has the option of borrowing funds via debt contracts from an intermediary. When the riskiness of each entrepreneur is observable to intermediaries, intermediaries will offer an interest rate on debt so that the risk adjusted cost of external borrowing for the entrepreneur is the risk free rate of return. Since there are no internal frictions inside the firm, the entrepreneur will borrow funds to equate the return on capital with the risk free rate of return. Therefore, the level of investment is independent of cross-sectional distribution of default risk.

When the riskiness of each entrepreneur is unobservable to intermediaries, intermediaries offer an interest rate that equals the risk free rate of return adjusted for an average riskiness of all projects funded through debt. Hence, effective cost of external borrowing differs across entrepreneurs according to their risk type. Specifically, high risk types face a lower effective cost of external borrowing than low risk types. As a result, high risk types have a stronger incentive to borrow than low risk types. We show that this asymmetry of incentives creates a

[3]See, for example, Eisfeldt and Rampini (2006), Bloom et al. (2009), and Gilchrist et al. (2009) for evidence based on firm-level volatility, Storesletten et al. (2004) for evidence from labor income, and Campbell et al. (2001) for evidence from stock returns.

distortion between external and internal rates of return and the size of this distortion depends on the cross-sectional distribution of default risk. In particular, when dispersion in default risk is low, the dispersion across types in effective interest rate is low so that asymmetry in incentives, and hence the distortion, is small. Therefore, aggregate investment is close to aggregate investment when types are observable. These effects are reversed when dispersion of default risk is high. Notice that since there is no internal friction inside the firm, there are no distortions to the investment/consumption margin. This feature of the model is consistent with the evidence given by Chari et al. (2006).

The outline of paper is the following: In section 2, we present a two period example and analyze the key mechanism of the model. In section 3, we set up the benchmark model with public information and characterize the equilibrium. In section 4, we extend the benchmark model to a private information environment. Section 5, discusses future work and concludes.

2 A Two Period Example

In this section we focus on a stylized two period example in order to illustrate how an adverse selection problem in credit markets can act as a propagation mechanism for changes in the cross-sectional dispersion of entrepreneurial riskiness.

2.1 Environment

Consider an economy that last for two periods, $t = 1, 2$, and is populated by a continuum of entrepreneurs of measure 1 and a continuum of potential lenders of measure $\bar{l}$.

Each lender has an endowment of one indivisible unit of a consumption good at the beginning of the first period. Lenders have access to a risk free investment technology which converts one unit of period 1 consumption good into $\bar{R} > 1$ units of period 2 consumption good. Lenders preferences over consumption in period 1 and 2 are given by

$$W(c_1, c_2) = c_1 + c_2.$$

Each entrepreneur has an endowment at the beginning of the first period, $\bar{k}$. An entrepreneur is indexed by two characteristics: a riskiness or default risk type, π and a productivity, A. An entrepreneur of type (π, A) survives from period 1 to 2 with probability π. In the event that an entrepreneur does not survive, we assign the entrepreneur a continuation utility normalized to 0.[4]

In period 1, the entrepreneur has access to a production technology which converts k units of goods in period 1 into $Af(k)$ units of goods in period 2 if the entrepreneur survives. We assume that $f(\cdot)$ is strictly increasing, strictly concave, twice differentiable and satisfies Inada conditions. We assume that for all (π, A), $\pi A = 1$. Moreover, we assume that $\pi \sim U[\underline{\pi}, \bar{\pi}]$ and we let $G(\pi)$ denote this distribution.

Each entrepreneur's preferences over consumption in periods 1 and 2 are given by

$$U(c_1, c_2) = u(c_1) + \pi c_2.$$

While we assume risk neutral preferences over period 2 consumption to illustrate the key features of the economy in the two period example, we generalize this assumption in the infinite horizon economy described below.

Before undertaking investment in production, the entrepreneur may borrow funds from lenders. Throughout the paper, we restrict attention to one period debt contracts. The restriction to one period contracting is obviously without loss of generality in this example. We can show that a version of Modigliani and Miller (1958) holds in this example and throughout the rest of the paper, so that entrepreneurs are indifferent between debt and

[4]Normalizing the continuation utility in the event of death to 0 is not important per se. It is important that this continuation utility is independent of the entrepreneur's default risk type.

equity financing. We focus on debt contracts. We also assume that an entrepreneur cannot save funds and that there is an ad hoc borrowing limit, given by $\bar{l}$.

In period 1, the entrepreneur must determine how many funds to borrow, how much to invest in production, and how much to consume in period 1. Therefore, facing an interest rate, $(1+r)$ to be repaid if the entrepreneur survives, the entrepreneur's problem is given by

$$\max_{c_1,l,k_1} u(c_1) + \pi[Af(k_1) - (1+r)l_1] \tag{1}$$

subject to

$$c_1 + k_1 \leq \bar{k} + l_1$$
$$0 \leq l_1 \leq \bar{l}$$

where c_1 denotes consumption in period 1, l_1 denotes borrowing in period 1, and k_1 denotes investment in period 1.

2.2 Equilibrium under Full Information

Assume that the entrepreneur's riskiness type, π is observable by all potential lenders at the time of signing debt contracts. Potential lenders simultaneously offer an interest rate on an unsecured loan of size 1. If a lender's loan is selected by an entrepreneur, the lender's expected profit is

$$\pi(1+r) - \bar{R}.$$

We assume that lenders engage in a form of Bertrand competition. Moreover, we allow for joint deviations by a positive measure of lenders. Hence, the equilibrium will require lenders to offer the lowest interest rate yields zero expected profits.[5]

[5] Under the usual Nash equilibrium concept, multiple equilibria will arise because lenders are capacity constrained and can coordinate on arbitrary interest rates. In this case, a deviation by a single lender can have no effect on an entrepreneur's actions. We allow for joint deviations by lenders in order to rule out this type of multiplicity.

Under full information, an equilibrium consists of allocations for each entrepreneur, $(c_1(\pi), l_1(\pi), k_1(\pi))$ and a profile of interest rates, $r(\pi)$, such that given $r(\pi)$, $(c_1(\pi), l_1(\pi), k_1(\pi))$ solves a type π entrepreneur's problem and $r(\pi)$ is the lowest interest rate that allows lenders to break even.

We characterize the unique equilibrium of this economy by analyzing the entrepreneur's problem. At an interior solution, we have the following first order conditions:

$$\begin{aligned} u'(c_0) &= \pi A f'(k_1) = f'(k_1) \\ u'(c_0) &= \pi(1+r) \end{aligned}$$

Given that the entrepreneur's type is observable by lenders, it is immediate that $\pi(1+r) = \bar{R}$. Therefore, at an interior solution to the entrepreneur's problem, we have

$$f'(k_1^*) = \bar{R}.$$

We assume that

$$f'(\bar{k} + \bar{l}) < \bar{R} = f'(k_1^*) < f'\left(\bar{k}\right).$$

This assumption implies that total funds of potential lenders, $\bar{l}$, are sufficient for optimal investment but an entrepreneur's initial endowment is insufficient for optimal investment. Given this assumption, we can show that facing the equilibrium interest rate, the solution to the entrepreneur's problem is, in fact, interior.

Under full information, each entrepreneur, independent of type, is borrowing the same amount and investing k_1^*. Entrepreneur's of different types face different interest rates, $(1+r) = \bar{R}/\pi$, so that riskier types (ie: lower π types) face higher interest rates. Given this and the fact that $\pi A = 1$, each entrepreneur's problem is given by the following:

$$\max_{c_1, l, k_1} u(c_1) + f(k_1) - \bar{R} l_1$$

subject to

$$c_1 + k_1 \leq \bar{k} + l_1$$
$$0 \leq l_1 \leq \bar{l}$$

This implies that consumption, investment and borrowing by each entreprenuer is the same. The fact that investment is independent of type results from indifference between borrowing and investment, i.e., $f'(k) = \bar{R}$. However, consumption and borrowing are independent of type only because entrepreneurs are risk-neutral over consumption in period 2. With risk aversion over period 2 consumption, period 1 consumption and borrowing would vary with an entrepreneur's type. This completes the characterization of equilibrium.

It is clear from the above analysis that aggregate investment is independent of the distribution of types, $G(\pi)$ or $\underline{\pi}, \bar{\pi}$. Moreover, aggregate ouput in second period is given by:

$$\int_{\underline{\pi}}^{\bar{\pi}} \pi A f(k_1^*) dG(\pi) = f(k_1^*) \int_{\underline{\pi}}^{\bar{\pi}} dG(\pi) = f(k_1^*)$$

and therefore, independent of $\underline{\pi}, \bar{\pi}$. Hence, any change in cross-sectional dispersion of enrepreneur's riskiness is neutral with respect to aggregate equilibrium allocations. We can summarize the above in the following proposition:

Proposition 1 *In the example with full information, the level of investment is given by* $k_1^* = (f')^{-1}(\bar{R})$. *Moreover, investment, aggregate consumption, and aggregate output in the second period are independent of* G, *the distribution of types.*

2.3 Equilibrium under Private Information

Suppose now that the entrepreneur's type is unobservable when debt contracts are signed. Lenders must now make inferences about which types of entrepreneurs will accept a loan for any given interest rate. Let $B(r)$ denote the set of entrepreneurial types that will accept an interest rate of r. In this case, the break-even constraint for lenders is

$$\int_{B(r)} \pi(1+r)dG(\pi) = \bar{R}. \tag{2}$$

The equilibrium definition is as in the Full Information example, only now the interest rate must be the lowest interest rate that satisfies (2) and is equal for all types.

First, we show that for any interest rate the optimal debt policy for entrepreneurs is characterized by two cutoffs $\pi_h(r)$ and $\pi_l(r)$ with $\pi_l(r) \leq \pi_h(r)$. High risk types with $\pi \leq \pi_l(r)$, borrow $\bar{l}$, intermediate risk types with $\pi \in (\pi_l(r), \pi_h(r))$ borrow some amount $\hat{l}_1(\pi, r) \in (0, \bar{l})$ and low risk types with $\pi \geq \pi_h(r)$ borrow nothing. When the borrowing constraint is not binding $\hat{l}_1(\pi, r)$ is strictly decreasing in π.

To see this, consider an auxiliary problem taking the debt level, l_1 as given:

$$\max_{c_1, k_1} u(c_1) + \pi[Af(k_1)]$$

subject to

$$c_1 + k_1 \leq \bar{k} + l_1$$

This auxiliary problem yields consumption and investment functions $\hat{c}_1(l_1)$ and $\hat{k}_1(l_1)$ as a function of the debt level, l_1. These functions allow us to define thresholds, $\pi_l(r)$ and $\pi_h(r)$ given by

$$\begin{aligned} f'(\hat{k}_1(0)) &= \pi_h(r)(1+r) \\ f'(\hat{k}_1(\bar{l})) &= \pi_l(r)(1+r). \end{aligned}$$

When $\pi > \pi_h(r)$, the unit cost of borrowing, $\pi(1+r)$, is greater than the return on investment even in autarky (i.e. with zero borrowing). Hence, these types find it optimal to use only internal funds for investment. Similarly, high risk types have an incentive to run up debt, since the unit cost of funds is lower than the return on investment even if they borrow $\bar{l}$.

Therefore, for $\pi > \pi_h(r)$, $l_1 = 0$, $c_1 = \hat{c}_1(0), k_1 = \hat{k}_1(0)$ and for $\pi < \pi_l(r)$, $l_1 = \bar{l}$, $c_1 = \hat{c}_1(\bar{l})$ and $k_1 = \hat{k}_1(\bar{l})$.

Much as in the full information benchmark, it is easy to show that for $\pi \in [\pi_l(r), \pi_h(r)]$, l_1, the solution to (1) is interior and satisfies

$$f'(k_1(l_1)) = \pi(1+r). \tag{3}$$

Call the solution to (3) $\hat{l}_1(\pi, r)$. For $\pi \in [\pi_l(r), \pi_h(r)]$, $c_1 = \hat{c}_1(\hat{l}_1(\pi, r))$, $k_1 = \hat{k}_1(\hat{l}_1(\pi, r))$. Notice that in the full information benchmark, all entrepreneurs borrow the same amount, l^{FI}. By assumption, $l^{FI} \in (0, \bar{l})$ and satisfies:

$$f'(\hat{k}_1(l^{FI})) = \bar{R}.$$

Therefore, if $\pi_h(r) < \bar{\pi}$, there exists a $\pi \in (\pi_l(r), \pi_h(r))$ such that $\hat{l}_1(\pi, r) = l^{FI}$. All types with higher risk consume more and invest more relative to full information; all types with lower risk consume and invest less relative to full information. We have proved the following propositions:

Proposition 2 *For any interest rate r, optimal decision by entrepreneurs is given by*

$$l_1^*(\pi, r) = \begin{cases} \bar{l} & \text{if } \underline{\pi} \leq \pi \leq \max\{\frac{f'(\hat{k}_1(\bar{l}))}{1+r}, \underline{\pi}\} \\ \hat{l}_1(\pi, r) & \text{if } \max\{\frac{f'(\hat{k}_1(\bar{l}))}{1+r}, \underline{\pi}\} \leq \pi \leq \min\{\frac{f'(\hat{k}_1(0))}{1+r}, \bar{\pi}\} \\ 0 & \text{if } \min\{\frac{f'(\hat{k}_1(0))}{1+r}, \bar{\pi}\} \leq \pi \leq \bar{\pi} \end{cases}$$

We now characterize lenders' expected profits given entrepreneurs' optimal decisions. There are three relevant cases to consider. If $\frac{f'(\hat{k}_1(0))}{\bar{\pi}} > 1+r$, then $\pi_h(r) = \bar{\pi}$ implying that the interest rate is low enough to induce entrepreneurs of all types to borrow. If $\frac{f'(\hat{k}_1(0))}{\underline{\pi}} \leq 1+r$, then $\pi_h(r) = \underline{\pi}$ implying that the interest rate is high enough to discourage all entreprenuers from borrowing. If $\frac{f'(\hat{k}_1(0))}{\bar{\pi}} \leq 1+r \leq \frac{f'(\hat{k}_1(0))}{\underline{\pi}}$, then expected marginal profits are given by

$$
\begin{aligned}
\frac{(1+r)\int_{\underline{\pi}}^{\pi_h(r)} \pi dG(\pi)}{G(\pi_h(r))} - \bar{R} &= \frac{(1+r)(\pi_h(r)+\underline{\pi})}{2} - \bar{R} \\
&= \frac{f'(\hat{k}_1(0))}{2} + (1+r)\frac{\underline{\pi}}{2} - \bar{R}.
\end{aligned}
$$

Hence, lenders' expected profits per unit loan can be written as

$$
\Pi(r) = \begin{array}{ccc}
\frac{\bar{\pi}+\underline{\pi}}{2}(1+r) - \bar{R} & if & \frac{f'(\hat{k}_1(0))}{\bar{\pi}} > 1+r \\
\frac{f'(\hat{k}_1(0))}{2} + (1+r)\frac{\underline{\pi}}{2} - \bar{R} & if & \frac{f'(\hat{k}_1(0))}{\bar{\pi}} \leq 1+r \leq \frac{f'(\hat{k}_1(0))}{\underline{\pi}} \\
-\bar{R} & if & \frac{f'(\hat{k}_1(0))}{\underline{\pi}} \leq 1+r
\end{array}
$$

Observe that $\Pi(r)$ is continuous and monotone increasing for $0 \leq 1+r \leq \frac{f'(\hat{k}_1(0))}{\underline{\pi}}$, $\Pi(-1) = -\bar{R}$, and

$$
\Pi\left(\frac{f'(\hat{k}_1(0))}{\underline{\pi}} - 1\right) = f'(\hat{k}_1(0)) - \bar{R}.
$$

From Inada conditions on $u(\cdot)$, $f'(\hat{k}_1(0)) > f'(\bar{k})$ and by assumption $f'(\bar{k}) > \bar{R}$ so that $\Pi\left(\frac{f'(\hat{k}_1(0))}{\underline{\pi}} - 1\right) > 0$. Therefore, there must exist $r^* \in (-1, \frac{f'(\hat{k}_1(0))}{\underline{\pi}} - 1)$ such that $\Pi(r^*) = 0$. This completes the characterization of the equilibrium under private information.

Proposition 3 *If $f'(\hat{k}_1(0)) \geq \bar{R}$, equilibrium interest rate exist and is given by*

$$
1 + r^* = \begin{cases} 2\bar{R} & if\ \bar{\pi} \leq \frac{f'(\hat{k}_1(0))}{2\bar{R}} \\ \frac{2\bar{R} - f'(\hat{k}_1(0))}{\underline{\pi}} & otherwise \end{cases}
$$

This proposition immediately shows that equilibrium interest rate is weakly increasing in dispersion $\bar{\pi} - \underline{\pi}$.

2.4 Comparative Statics under Private Information

In contrast to the full information example, equilibrium allocations under private information depend on the distribution of entrepreneurs' types. In particular, aggregate investment in period 1 and output in period 2 depend on $\underline{\pi}$, and $\bar{\pi}$. In order to analyze the effect of changes in cross-sectional dispersion of firm riskiness, we let $\underline{\pi}(\varepsilon) = \frac{1}{2} - \varepsilon$ and $\bar{\pi}(\varepsilon) = \frac{1}{2} + \varepsilon$ and we vary $\varepsilon \in \left[0, \frac{1}{2}\right]$. The previous proposition yields an immediate corollary:

Corollary 1 *Equilibrium interest rate $r^*(\varepsilon)$ is an increasing function of ε. Moreover, it is strictly monotone and convex for $\varepsilon \geq \frac{f'(\hat{k}_1(0))}{2\bar{R}} - \frac{1}{2}$.*

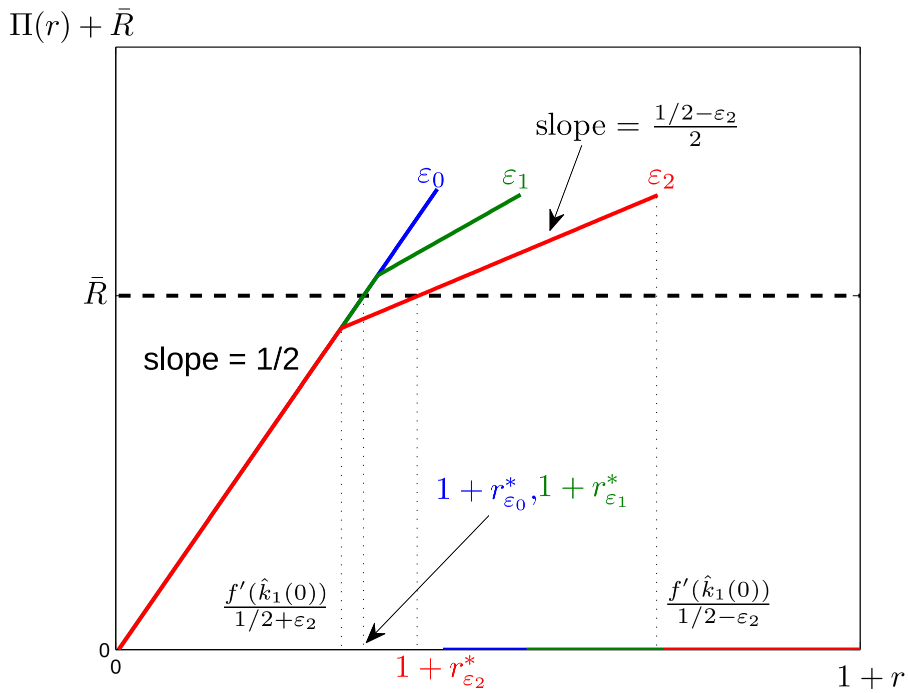

Figure 1: Determination of Equilibrium interest rate in the two period example.

Figure 1, illustrates the determination of the interest rate in equilibrium. The figure depicts revenue per unit loan or $\Pi(r) + \bar{R}$ as a function of the interest rate or $1 + r$. When $\varepsilon = 0$, as in the figure at ε_0, or with full information about the entrepreneur's type, an entrepreneur will borrow at any interest rate below $2f'(\hat{k}_1(0))$. An increase in the interest rate in that region increases revenue by slope $\frac{1}{2}$. Therefore, as depicted, the equilibrium interest rate is $2\bar{R}$.

When $\varepsilon > 0$ and small, as depicted for ε_1, we claim that there is no lemons problem. When all entrepreneurs borrow from lenders, the zero profit condition implies that in equilibrium $1 + r = 2\bar{R}$. Given $1 + r = 2\bar{R}$, we can calculate

$$\pi_h(2\bar{R} - 1) = \frac{f'(\hat{k}_1(0))}{2\bar{R}}$$

If ε is such that $\frac{1}{2} + \varepsilon \leq \pi_h(2\bar{R} - 1)$, then all entrepreneur types borrow given $1 + r = 2\bar{R}$ and therefore, $1 + r = 2\bar{R}$ constitutes an equilibrium.

When, $\varepsilon > \pi_h(2\bar{R} - 1) - \frac{1}{2}$, as for ε_2 in the figure, some types use internal financing only in equilibrium. Hence, the equilibrium interest rate must belong to the second region where $\pi_h(r) < \bar{\pi}$ and increases from $2\bar{R}$. When there is a lemons problem, the interest rate is increasing in the cross-sectional dispersion of entrepreneur type or default risk.

To see the effect of increase in dispersion on aggregate variables, we consider the following numerical example, where $u(c) = c^\alpha$, $f(k) = k^\alpha$. Figure 2, shows aggregate output in second period, $\int f(k_2^*(\pi))dG(\pi)$ as a function of ε.

When ε is close to zero, an increase in ε leads to an increase in output. In this case, the equilibrium interest rate $1 + r$ is $2\bar{R}$ and therefore

$$\int f'(k_2(\pi))dG(\pi) = \bar{R}$$

Since $f\left((f')^{-1}(\cdot)\right)$ is a convex function, an increase in dispersion of π's and therefore in dispersion of $k_2(\pi)$'s, causes output to increase.

However as ε passes the threshold $\pi_h(2\bar{R} - 1) - \frac{1}{2}$, low default risk types do not borrow and that leads to a decrease in output. Eventually, since the equilibrium interest rate $1 + r$ converges to ∞ as $\varepsilon \to \frac{1}{2}$, entrepreneurs of all types choose to not borrow and therefore output converges to $f(\hat{k}_2(0))$, the autarky level of investment. Figure 3 shows the effect of

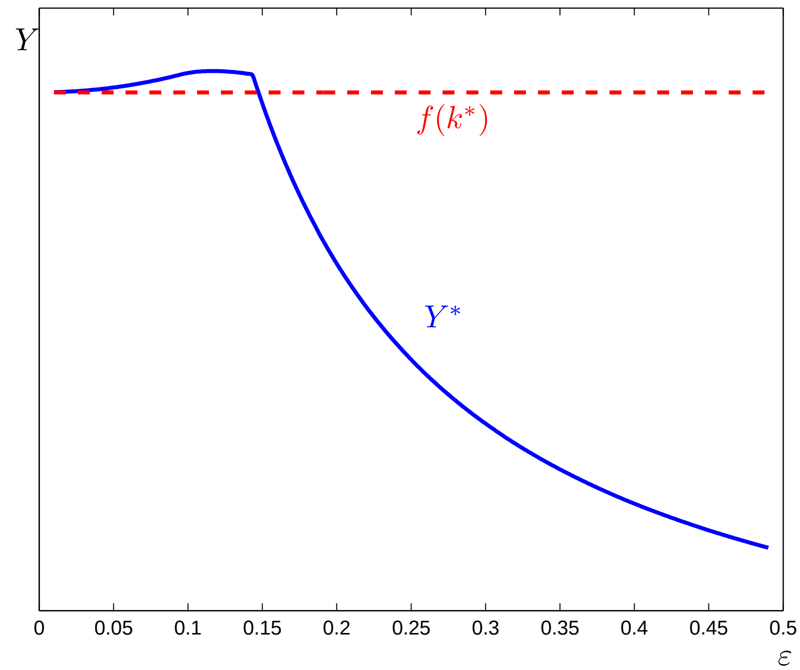

Figure 2: Equilibrium output as a function of dispersion in the two period example.

changes in dispersion on the types of entrepreneurs who borrow from intermediaries.

Figure 3 plots $\underline{\pi}, \bar{\pi}, \pi_h(r^*)$, and $\pi_l(r^*)$ for various values of ε. For low values of ε, all entrepreneurs borrow and none of the borrowing constraints are binding. In this numerical example, when $\varepsilon \approx 0.1$, while all types of entrepreneurs continue to borrow, the lowest types run up their debt. For higher values of ε, low default risk types stop borrowing from intermediaries. As ε converges to $\frac{1}{2}$, almost no types of entrepreneurs borrow from intermediaries. Figures 4 depicts the equilibrium interest rate as a function of the level of dispersion.

3 The Benchmark Model

In order to quantify the model and its predictions, we extend the model to an infinite horizon. In this section we generalize the two period example under full information to an

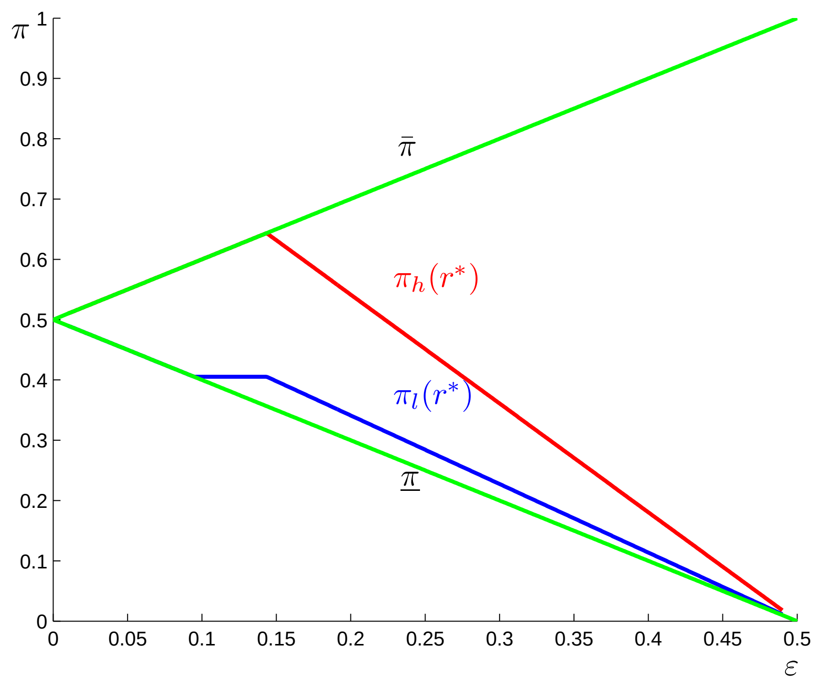

Figure 3: Equilibrium borrowing thresholds as a function of dispersion in the two period example.

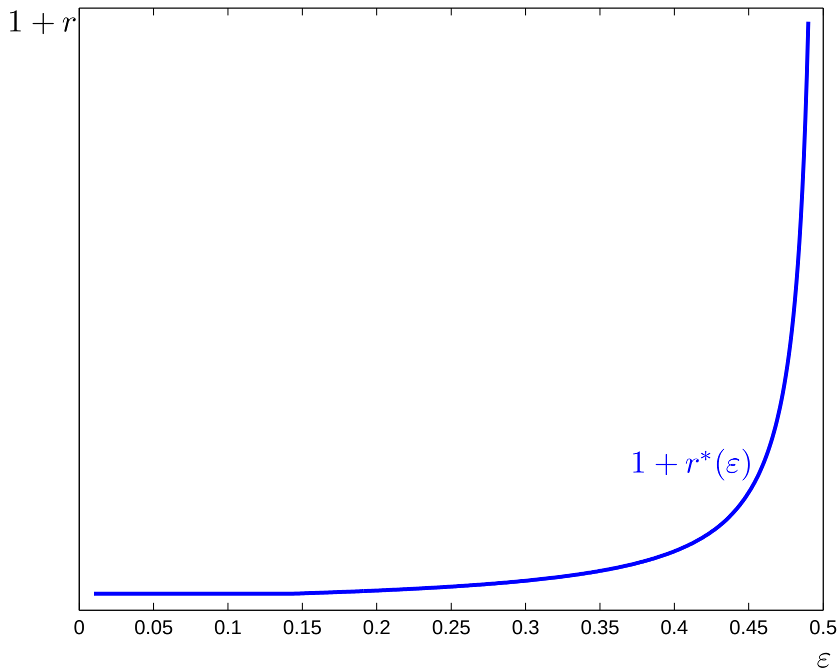

Figure 4: Equilibrium interest rate as a function of dispersion in the two period example.

infinite horizon and show that as in the example, aggregate output is independent of the cross-sectional dispersion of entrepreneurial default risk.

3.1 Environment

Consider a small open economy that lasts forever with time indexed by $t = 0, 1, 2, \ldots$. The economy is comprised of two types of agents: entrepreneurs and intermediaries.

In each period t, a continuum of entrepreneurs of measure λ is born, and each newborn entrepreneur has initial net worth, W_t, and riskiness type π_t drawn from a distribution $\Gamma_0(W_t, \pi_t)$. In period t, an entrepreneur of type π_t has access to a production technology which converts k units of period t consumption into $\frac{1}{\pi_t} f(k)$ units of period $t+1$ consumption upon survival. The entrepreneur survives from period t to period $t+1$ with probability π_t. We normalize the continuation utility upon death to be 0. We assume that π_t is i.i.d. across entrepreneurs within and across periods and drawn from distribution $G(\pi)$. In order to have a fixed measure of entrepreneurs in each period, we assume that λ satisfies

$$\lambda = 1 - \int \pi dG(\pi).$$

Let $\pi_t^s = (\pi_{t+1}, \pi_{t+2}, \ldots, \pi_s)$ denote the history of an entrepreneur's default risk up to period s and let $\mu_s(\cdot|\pi_t)$ denote the probability measure over all possible π_t^s conditional on π_t. Then, an entrepreneur born in date t with type π_t has preferences over infinite streams of consumption given by

$$\sum_{s=t}^{\infty} \beta^{s-t} \int \left(\prod_{j=t}^{s-1} \pi_j \right) u(c_s(\pi_t^s)) d\mu_s\left(\pi_t^s|\pi_t\right).$$

Notice that $\mu_s(A_{t+1} \times \cdots \times A_s|\pi_t) = G(A_{t+1}) \times \cdots \times G(A_s)$ since the π's are i.i.d.

Consider a surviving or new-born entrepreneur in period t with net worth W_t and future default risk π_t.The entrepreneur can consume in period t,invest in the production technology, and borrow funds from intermediaries at rate $(1 + r_t)$. Hence, in period t, the entrepreneur's flow budget constraint is given by

$$c_t + k_{t+1} - l_{t+1} \leq W_t.$$

In any subsequent period $s > t$, conditional on survival, the entrepreneur's flow budget constraint is given by

$$c_s + k_{s+1} - l_{s+1} \leq \frac{1}{\pi_{s-1}} f(k_s) - (1 + r_{s-1}) l_s.$$

We define net worth in any period, $W_s = \frac{1}{\pi_{s-1}} f(k_s) - (1 + r_{s-1}) l_s$, and we can show that the entrepreneur's problem can be written recursively with state variables W_s and π_s.

In each period, a continuum of intermediaries of measure 1 is born and lives for two periods. Each intermediary may freely borrow and lend at a risk-free rate $\bar{R}$ in international markets and may lend to entrepreneurs using one period debt contracts. We assume that each intermediary offers an interest rate targeted at all entrepreneurs of type π and commits to satisfy the resulting entrepreneurs' demand. We assume that an intermediary cannot use the size of the loan to screen potential borrowers.[6] Each intermediary is risk-neutral.

Within a period, the timing of actions is as follows:

1. At the start of period t, entrepreneurs surviving from period $t-1$ yield production $\frac{1}{\pi_{t-1}} f(k_t)$ and repay debt $(1+r_{t-1})l_t$ to intermediaries born in $t-1$. Hence, period t surviving entrepreneurs have net worth of $W_t = \frac{1}{\pi_{t-1}} f(k_t) - (1+r_{t-1})l_t$.

2. Intermediaries from $t-1$ consume any profits and die. New intermediaries are born.

3. New entrepreneurs are born with wealth W_t; all entrepreneurs learn next-period default risk, π_t.

4. Intermediaries born in period t simultaneously choose their asset position in international markets and offer an interest rate $(1+r_t)$ on unsecured debt.

5. Facing an interest rate, $1+r_t$ on one period debt, the entrepreneur decides how much to consume, how much capital to install for next period production, and how much debt to incur (c_t, k_{t+1}, l_{t+1}).

6. With probability π_t, the entrepreneur survives to period $t+1$.

Entrepreneurs Given this timing, we can write the entrepreneur's decision problem recursively as

$$V(W,\pi) = \max_{c,k',l'} u(c) + \beta\pi \int V\left(\frac{1}{\pi} f(k') - (1+r)l', \pi'\right) dG(\pi') \tag{4}$$

[6] Restricting attention to linear debt contracts is a potentially important assumption which we impose for now to allow for a more tractable characterization of equilibria. Allowing intermediaries to use the size of the loan to screen potential borrowers may lead to problems of non-existence and inefficiency of equilibria as in Rothschild and Stiglitz (1976). We intend to analyze a more general set of contracts that allows for such screening in future work.

subject to

$$\begin{aligned} c + k' - l' &\leq W \\ 0 \leq l' &\leq \bar{l}. \end{aligned}$$

Intermediaries Each intermediary in period t chooses an asset position in international markets and an interest rate at which it commits to provide funds to entrepreneurs of type π_t. We assume that intermediaries engage in a form of Bertrand competition so that all entrepreneurs of type π_t, if they choose to borrow, borrow from the intermediary offering the lowest interest rate. If an intermediary's interest rate is chosen by some entrepreneurs of type π_t, the intermediary's expected profit is

$$\Pi_t(r_t) = s_t - a_{t+1} + \pi_t(1 + r_t)a_{t+1} - \bar{R}s_t$$

where a_{t+1} is total lending by the intermediary to entrepreneurs of type π_t, s_t is the intermediary's level of borrowing from the international market, and r_t is the offered interest rate. We assume that intermediaries in any period have no initial endowment. Hence, they must finance total lending in period t so that conditional on an accepted offer, the intermediary's international borrowing must satisfy $s_t \geq a_{t+1}$.

Bertrand competition along with the borrowing constraint implies an immediate no-arbitrage condition: $s_t = a_{t+1}$ and $\pi_t(1 + r_t) = \bar{R}$.

3.2 Stationary Recursive Competitive Equilibrium

A stationary recursive competitive equilibrium under full information consists of offered interest rates $r(\pi)$ and aggregate savings by intermediaries, $a(\pi)$, and policy functions for entrepreneurs, $c(W, \pi; r), k'(W, \pi; r), l'(W, \pi; r)$ together with a distribution of entrepreneurial wealth and default risk, $\Psi(W, \pi)$ and a value function for entrepreneurs, $V(W, \pi)$ such that

1. Intermediaries optimally choose $r(\pi)$ and $a(\pi)$;

2. Entrepreneur policy functions solve the entrepreneur's decision problem, (4);

3. Market clearing: $a(\pi) = \int_W l'(W,\pi;r)d\Psi(W,\pi)$ for all π;

4. $\Psi(W,\pi)$ is the stationary distribution: for any set A,

$$\Psi(A) = \int_{(W,\pi)} \mathbf{1}\left[\frac{1}{\pi}f(k'(W,\pi;q)) - (1+r)l'(W,\pi;q) \in A\right]\pi d\Psi(W,\pi) + \lambda\Gamma_0(A).$$

3.3 Characterization of Equilibrium

As in the two period example, we characterize equilibria of this economy by imposing an interest rate condition from the intermediary's problem and analyzing the optimal policies of the entrepreneur.

From the intermediary's problem, the offered interest rate must satisfy $\pi(1+r) = \bar{R}$. Hence, the entrepreneur's problem is the following

$$V(W,\pi) = \max_{c,k',l} u(c) + \beta\pi \int V\left(\frac{1}{\pi}f(k') - \frac{\bar{R}}{\pi}l', \pi'\right) dG(\pi') \tag{5}$$

subject to

$$\begin{aligned} c + k' - l' &\leq W \\ 0 \leq l' &\leq \bar{l}. \end{aligned}$$

In the appendix, we drive conditions so that for any (W,π), the borrowing constraints are not binding. Then, necessary and sufficient conditions for optimality are given by

$$\begin{aligned} u'(c) &= \beta f'(k') \int V_W\left(\frac{1}{\pi}f(k') - \frac{\bar{R}}{\pi}l', \pi'\right) dG(\pi'), \\ u'(c) &= \beta\bar{R} \int V_W\left(\frac{1}{\pi}f(k') - \frac{\bar{R}}{\pi}l', \pi'\right) dG(\pi'). \end{aligned}$$

This implies that $f'(k'(W,\pi)) = \bar{R}$, so that for all (W,π), $k'(W,\pi) = k^* = (f')^{-1}(\bar{R})$. Therefore, we have proved the following proposition.

Proposition 4 *Assuming appropriate conditions defined in the appendix, aggregate output is independent of the cross-sectional distribution of entrepreneurs' default risk in the stationary recursive competitive equilibrium under full information.*

To finish the characterization, similar to Aiyagari (1994), we first solve for optimal entrepreneurial borrowing from

$$u'(W - k^* + l'(W,\pi)) = \beta f'(k^*) \int V_W \left(\frac{1}{\pi} f(k^*) - \frac{\bar{R}}{\pi} l'(W,\pi), \pi' \right) dG(\pi').$$

Given $l'(W,\pi)$, we can find a stationary distribution that satisfies the following:

$$\Psi(A) = \int_{(W,\pi)} \mathbf{1} \left[\frac{1}{\pi} f(k^*) - \frac{\bar{R}}{\pi} l'(W,\pi) \in A \right] \pi d\Psi(W,\pi) + \lambda \Gamma_0(A)$$

for any set Lebesgue-measurable set, A.

4 The Model with Private Information

As in the two period example, when the entrepreneur's type is unobservable when debt contracts are signed, intermediaries must make inferences about which types of entrepreneurs will choose to borrow at any given interest rate. Let $B_t(r_t)$ denote the set of entrepreneurial types, (W_t, π_t), that will accept an interest rate of r_t. In this case, the break-even constraint for intermediaries is

$$\int_{B_t(r_t)} \pi_t (1 + r_t) l'(W_t, \pi_t; r_t) d\Psi_t(W_t, \pi_t) = \bar{R} \int_{B_t(r_t)} l'(W_t, \pi_t; r_t) d\Psi_t(W_t, \pi_t). \tag{6}$$

The equilibrium definition is as in the Benchmark model, only now the interest rate must be the lowest interest rate that satisfies (6) and is equal for all types.

5 Conclusion and Future Work

In this paper, we argue that shocks to dispersion in productivity or default risk can cause aggregate fluctuations. We provide a novel channel for "dispersion shocks" to have aggregate effects and that is adverse selection in credit markets. When information about default risk is private, an increase in dispersion causes an increase in credit spreads. Facing higher interest rates, firms with low default risk decide not to use external financing leading to a drop in output.

Future work entails completing the theory and a quantitative analysis. Regarding the theory, we want to finish characterizing the equilibrium and specifically, show that the infinite horizon model with private information satisfies the same properties as the two period example.

As for the quantitative analysis, we want to calibrate the model to US data. Our goal is to study how well our model performs in generating aggregate variables similar to those we observe during macroeconomic downturns that coincide with abrupt changes in financial markets. To do so, we need to add "dispersion shocks" to the dynamic model. Computing such a model is not a simple task to undertake, since the model would have both aggregate shocks and incomplete markets causing the distribution of wealth to become a state variable. We plan to employ methods similar to Krusell and Smith (1998) in order to compute the equilibrium. Once we calculate the equilibrium, we can perform various policy analyses, in particular, the type of policies that were put in place during the recent financial crisis and study their effect on macroeconomic variables.

6 Appendix

6.1 Properties of the Benchmark Model

In this section we study the properties of the value function for entreprenuer's problem in the benchmark model. We first provide properties on the primitives of the model, so that borrowing is always positive and we calculate the natural debt limit. Moreover, we discuss, using techniques in Stokey and Lucas (1989), properties of the value function as well policy function.

To this end, we define the following relaxed problem:

$$v(W,\pi) = \max_{c,l'} u(c) + \beta \int V(\frac{1}{\pi}f(k^*) - \frac{f'(k^*)}{\pi}l', \pi')dG(\pi') \tag{7}$$

subject to

$$c + k^* - l' \leq W$$

The following lemma provides a sufficient condition for borrowing to be positive:

Lemma 1 *Suppose that:*

$$u'(\frac{1}{\underline{\pi}}f(k^*) - k^*) \geq \beta f'(k^*)u'(f(k^*) - k^*) \tag{8}$$

Then, the solution to the problem (7), satisfies $l'(W,\pi) \geq 0$.

The sketch of the proof is as follows. If the above constraint is satisfied, then we can use an iteration argument to show that borrowing is always positive. First consider a two period problem with the assumption that $W \leq \frac{1}{\pi}f(k^*)$, in this case we know that in second period there is no borrowing and therefore $l' = 0$, and given the following inequality

$$u'(W - k^*) \geq \beta f'(k^*)u'(f(k^*) - k^*))$$

we can deduce that borrowing in the current period $l_2(W,\pi)$ has to be non-negative. Call the value function $v_2(W,\pi)$, Now consider the three period problem:

$$\begin{aligned} v_3(W,\pi) &= \max_{c,l} u(c) + \beta\pi \int v_2(\frac{1}{\pi}f(k^*) - \frac{\bar{R}}{\pi}l', \pi')dG(\pi') \\ &\quad \text{s.t. } c + k^* - l' \leq W \end{aligned}$$

In this case, by envelope theorem, we know that

$$v_{2W}(W,\pi') = u'(W - k^* + l_2(W,\pi')) \leq u'(W - k^*)$$

Hence, if $W \leq \frac{1}{\underline{\pi}}f(k^*)$, we have

$$\begin{aligned} u'(W-k^*) &\geq \beta f'(k^*)u'(f(k^*)-k^*))s > \beta f'(k^*) \int u'\left(\frac{1}{\pi}f(k^*) - k^*\right) dG(\pi') \\ &\geq \beta f'(k^*) \int u'\left(\frac{1}{\pi}f(k^*) - k^* + l_2(\frac{1}{\pi}f(k^*),\pi')\right) dG(\pi') \\ &= \beta f'(k^*) \int v_2(\frac{1}{\pi}f(k^*),\pi')dG(\pi') \end{aligned}$$

The above inequality again implies that in the borrowing in the current period, $l_3(W,\pi)$ must be non-negative. In this fashion, we can argue that for any length of time T, $l_T(W,\pi) \geq 0$ and therefore, since by contraction mapping theorem

$$\lim_{T\to\infty} l_T(W,\pi) = l'(W,\pi)$$

and therefore, we must have $l(W,\pi) \geq 0$. QED.

Notice that (8) is sufficient and possibly not the weakest assumption possible. Most likely, the necessary and sufficient condition for positive borrowing is weaker than (8), since we are making an extreme assumption that the lowest type, $\underline{\pi}$, borrows nothing.

Next we derive the natural debt limit using a method similar to Aiyagari (1994). Then we show that if we set the borrowing constraint equal to the natural debt limit and show that $l'(W,\pi)$ from the relaxed problem (7) satisfied the borrowing constraint.

To calculate the Natural Debt Limit, we write the budget constraint from the entrepreneur's sequence problem as follows:

$$\begin{aligned} c_t - l_t &\leq \frac{1}{\pi_{t-1}} f(k^*) - k^* - \frac{\bar{R}}{\pi_{t-1}} l_{t-1} \\ c_{t+1} - l_{t+1} &\leq \frac{1}{\pi_t} f(k^*) - k^* - \frac{\bar{R}}{\pi_t} l_t \\ c_{t+2} - l_{t+2} &\leq \frac{1}{\pi_{t+1}} f(k^*) - k^* - \frac{\bar{R}}{\pi_{t+1}} l_{t+1} \end{aligned}$$

Multiply the second by $\frac{\pi_t}{\bar{R}}$ and the third by $\frac{\pi_t \pi_{t+1}}{\bar{R}^2}$, sum to get

$$c_t + \frac{\pi_t}{\bar{R}} c_{t+1} + \frac{\pi_t \pi_{t+1}}{\bar{R}^2} c_{t+2} - \frac{\pi_t \pi_{t+1}}{\bar{R}^2} l_{t+2} \leq \frac{f(k^*)}{\pi_{t-1}} \left[1 + \frac{\pi_{t-1}}{\bar{R}} + \frac{\pi_{t-1}\pi_t}{\bar{R}^2}\right] - k^* \left[1 + \frac{\pi_t}{\bar{R}} + \frac{\pi_t \pi_{t+1}}{\bar{R}^2}\right] - \frac{\bar{R}}{\pi_{t-1}} l_{t-1}$$

Some notation:

$$\mu_{t+s} = \Pi_{j=0}^{s} \pi_{t+j-1}, \quad \mu_{t-1} = 1$$

Now, impose a No-Ponzi condition:

$$\lim_{S \to \infty} \frac{\mu_{t+S}}{\bar{R}^S} l_{t+S} = 0$$

Summing constraints to infinity, and using NPG we get

$$\begin{aligned} \frac{1}{\pi_{t-1}} \Sigma_{j=0}^{\infty} c_{t+j} \frac{\mu_{t+j}}{\bar{R}^j} &\leq \frac{f(k^*)}{\pi_{t-1}} \Sigma_{j=0}^{\infty} \frac{\mu_{t+j-1}}{\bar{R}^j} - \frac{k^*}{\pi_{t-1}} \Sigma_{j=0}^{\infty} \frac{\mu_{t+j}}{\bar{R}^j} - \frac{\bar{R}}{\pi_{t-1}} l_{t-1} \\ \Sigma_{j=0}^{\infty} c_{t+j} \frac{\mu_{t+j}}{\bar{R}^j} &\leq f(k^*) \Sigma_{j=0}^{\infty} \frac{\mu_{t+j-1}}{\bar{R}^j} - k^* \Sigma_{j=0}^{\infty} \frac{\mu_{t+j}}{\bar{R}^j} - \bar{R} l_{t-1} \end{aligned}$$

For "Natural" Debt limit, set $c_{t+j} = 0$:

$$\bar{R} l_{t-1} \leq f(k^*) \Sigma_{j=0}^{\infty} \frac{\mu_{t+j-1}}{\bar{R}^j} - k^* \Sigma_{j=0}^{\infty} \frac{\mu_{t+j}}{\bar{R}^j}$$

Now, if we want debt conditional on survival to be repaid for sure, we need the above condition to be satisfied at every history. For that, do some algebra:

$$\begin{aligned} l_{t-1} &\leq \frac{1}{\bar{R}} f(k^*) + \Sigma_{j=0}^{\infty} \mu_{t+j} \frac{f(k^*)}{\bar{R}^{j+2}} - \frac{k^*}{\bar{R}^{j+1}} \\ &= \frac{1}{\bar{R}} \left[f(k^*) + \Sigma_{j=0}^{\infty} \frac{\mu_{t+j}}{\bar{R}^{j+1}} \left(f(k^*) - \bar{R} k^* \right) \right] \end{aligned}$$

Noting that $\frac{f(k^*)}{k^*} > f'(k^*) = \bar{R}$ by mean value theorem and strict concavity of $f(\cdot)$, the lowest "Natural" debt limit is achieved by the continuation history where $\pi_{t+j} = \underline{\pi}$ for all j. Hence, the "Natural Debt Limit" is given by

$$\begin{aligned} & \frac{1}{\bar{R}} \left[f(k^*) + \left(f(k^*) - \bar{R} k^* \right) \Sigma_{j=0}^{\infty} \frac{\underline{\pi}^{j+1}}{\bar{R}^{j+1}} \right] \\ = & \frac{1}{f'(k^*)} \left[f(k^*) + \left(f(k^*) - k^* f'(k^*) \right) \frac{\underline{\pi}}{f'(k^*) - \underline{\pi}} \right] \end{aligned}$$

Therefore,

Lemma 2 *The solution to the relaxed problem (7) satisfies,*

$$l'(W, \pi) \leq \frac{1}{f'(k^*)} \left[f(k^*) + \left(f(k^*) - k^* f'(k^*) \right) \frac{\underline{\pi}}{f'(k^*) - \underline{\pi}} \right]$$

TO BE COMPLETED.

Next we turn to the properties of the value function in the relaxed problem:

Proposition 5 *Suppose that $u(c)$ is strictly concave, st. increasing and C^1. Then, the value function in the relaxed problem (7), satisfies:*

1. *$v(W, \pi)$ is strictly concave and differentiable in W.*

2. *$v(W, \pi)$ is strictly increasing in W.*

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