A Quantitative Model of Banking Industry Dynamics (Preliminary and Incomplete)

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November 11, 2009

Abstract

We study the relation between commercial bank market structure, business cycles, and borrower default frequencies. The model is parameterized to match a set of key aggregate and cross-sectional statistics for the U.S. banking industry. As in the data, the model generates countercyclical interest rates on loans, bank failure rates, borrower default frequencies, and charge-off rates as well as a procyclical loan supply and entry rates. The model can be used to study bank competition and the benefits/costs of policies to subsidize/mitigate bank entry/exit.

1 Introduction

The objective of this paper is to formulate a simple quantitative structural model of the banking industry consistent with recent data on market concentration. Banks in our environment intermediate between a large number of households who supply funds and a large number of borrowers who demand funds to undertake risky projects. By lending to a large number of borrowers, a given bank diversifies risk that any particular household may not accomplish individually. When mapping the model to data, we attempt to match both aggregate and cross-sectional statistics for the U.S. banking industry.

Our paper is related to the following literature. Our underlying model of banking is based on Allen and Gale [1], section IV of Boyd and De Nicolo [2], and Martinez-Miera and Repullo [11]. As in those models, we do not assume perfect competition since current data suggests that there is a moderate degree of market concentration in the U.S. In particular, to take the model to data we use the Markov Perfect Industry equilibrium concept of Ericson and Pakes [6]. Thus, we depart from quantitative competitive models of banking such as Diaz-Gimenez, et. al. [4].

2 Banking Data Facts

In this section, we document the cyclical behavior of entry and exit rates, bank lending, measures of loan returns and the level of concentration in the U.S. We focus on individual commercial banks in the U.S.[1] The source for the data is the Consolidated Report of Condition and Income (known as Call Reports) that insured banks submit to the Federal Reserve each quarter.[2] We compile a large data set from 1976 to 2008 using data for the last quarter of each year. We follow Kashyap and Stein [10] in constructing consistent time series for our variables of interest.

One clear trend of the commercial banking industry during the last three decades is the continuous drop in the number of banking institutions. In 1980, there were approximately 14,000 institutions and this number has declined at an average of 360 per year, bringing the total number of commercial banks in 2008 to less than 7,100. This trend was a consequence of important changes in regulation that were introduced during the 1980's and 1990's (deposit deregulation in the early 1980's and the relaxation of branching restrictions later).

This decline in the level (or stock) of banks is evidenced by flow measures of exit, entry, and (resulting) net entry. The number of exits (including mergers and failures) and entrants expressed as a fraction of the banking population in the previous year are displayed in Figure 1. We also incorporate real detrended GDP to understand how entry and exit rates move along the cycle.[3] There was an important increase in the fraction of banks that exited starting in the 1980's and the high level continued through the late 1990's due to the aforementioned regulatory changes. The bulk of the decline was due to mergers and acquisitions. However, from 1985 through 1992, failures also contributed significantly to the decline in the number of banks. Figure 2 decomposes the exit rate into mergers and failures as well as the fraction of 'troubled' banks for a subset of the period.[4] The figure also shows that there has been a consistent flow of entry of new banks, cycling around 2%. Since 1995 the net decline in the number of institutions has trended consistently lower (except in 2008) so that the downward trend is leveling off. This recent leveling off of the trend is also documented in Table 6

[1] In some cases, commercial banks are part of a larger bank holding company. For example, in 2008, 1383 commercial banks (20% of the total) were part of a bank holding company. As in Kashyap and Stein [10] we focus on individual commercial banks. As they argue, there are not signficant differences in modelling each unit. The holding company is subject to limited liability protection rules with respect to the losses in any individual bank.

[2] The number of institutions and its evolution over time can be found at http://www2.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10.

Balance Sheet and Income Statements items can be found at https://cdr.ffiec.gov/public/.

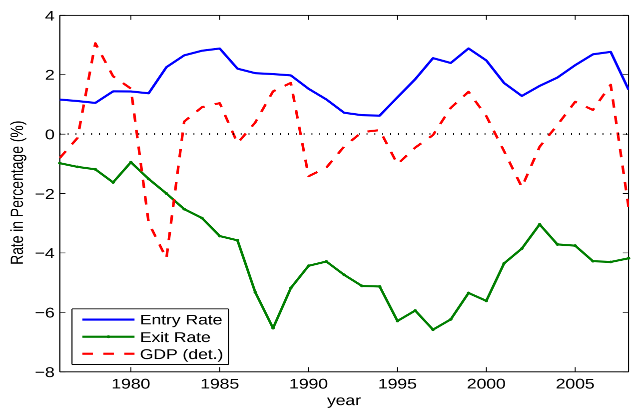
[3] The H-P filter with parameter equal to 6.25 is used to extract the trend from real GDP data.

[4] A troubled bank, as defined by the FDIC, is a commercial bank with CAMEL rating equal to 4 or 5. CAMEL is an acronym for the six components of the regulatory rating system: **C**apital adequacy, **A**sset quality, **M**anagement, **E**arnings, **L**iquidity and market **S**ensitivity (since 1998). Banks are rated from 1 (best) to 5 (worst), and banks with rating 4 or 5 are considered 'troubled' banks (see FDIC Banking Review 2006 Vol 1). This variable is only available since 1990.

of Janicki and Prescott [9].

Figures 1 and 2 also make clear that there was a significant amount of cyclical variation in entry and exit. The correlation of the entry and exit rates with detrended GDP is 0.25 and 0.07 respectively (if we restrict to the post-reform years - after 1990 - the correlations with detrended GDP are 0.87 and 0.11 for entry and exit rates respectively).[5] Since exits can occur as the result of a merger, these correlations hide what we usually think of the cyclical component of exits; failures and 'troubled' banks have a more important cyclical component than mergers. We find that the failure and 'troubled' bank rates are countercyclical while the merger rate displays a procylciclical behavior. Specifically, the correlation with real detrended GDP of the failure and 'troubled' banks rates (since 1990) are -0.39 and -0.49 respectively. On the other hand, the correlation of the merger rate for the same period equals 0.28.

Figure 1: Bank Industry Dynamics and Business Cycle



Note: Data corresponds to commercial banks in the US. Source: FDIC, Consolidated Report of Condition and Income. Entry corresponds to new charters and conversions. Exit correspond to unassisted mergers and failures. GDP (det) correspond to real GDP detrended. The trend is extracted using the H-P filter with parameter 6.25.

[5]The correlation of the failure rate and the merger rate with detrended GDP since 1976 are 0.11 and 0.04 respectively.

Figure 2: Exit Rate Decomposed

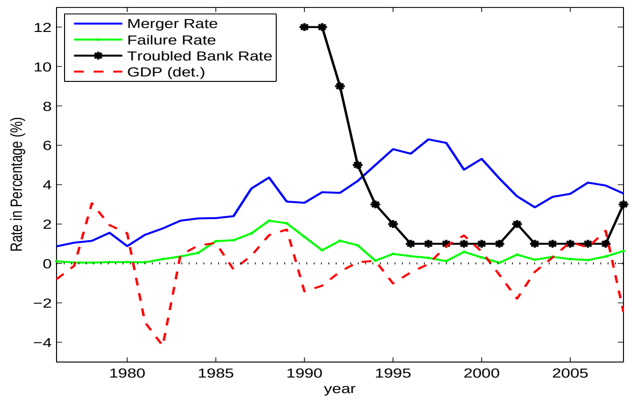


Table 1 shows that there are considerable differences in bank entry and exit (merger and failure) by size. We note that the bulk of entry, exit, mergers and failures correspond to banks that are in the bottom 99% of the distribution. The time series average accounted for by the bottom 99% of banks is close to 99% across all categories. We also observe that, in some periods, all entry, exits and failures correspond to small banks (as measured by the maximum over time).

Table 1: Entry and Exit Statistics by bank size

Fraction of Total x, accounted by:	x Entry	Exit	Exit/Merger	Exit/Failure
Top 4 Banks (avg.)	0.00	0.01	0.02	0.00
Top 1 % Banks (avg.)	0.27	1.07	1.57	2.88
Top 10 % Banks (avg.)	4.87	14.18	16.13	15.36
Bottom 99 % Banks (avg.)	99.73	98.93	98.43	97.12
Top 4 Banks (max.)	0.00	0.23	0.24	0.00
Top 1 % Banks (max.)	2.02	2.97	3.04	25.00
Top 10 % Banks (max.)	20.00	24.05	24.11	50.00
Bottom 99 % Banks (max.)	100.00	100.00	99.64	100.00

Note: Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports. Entry and Exit period extends from 1976 to 2008. Merger and Failure period is 1990 - 2007.

Figure 3 displays how bank lending and deposits move along the cycle. The series for loans and deposits are constructed using the individual commercial bank level data.[6] The stock of loans and deposits have an important cyclical component. We find that both measures of bank activity are highly procyclical with correlations with detrended GDP equal to 0.51 and 0.29 for loans and deposits respectively.

Figure 3: Loans, Deposits and Business Cycle

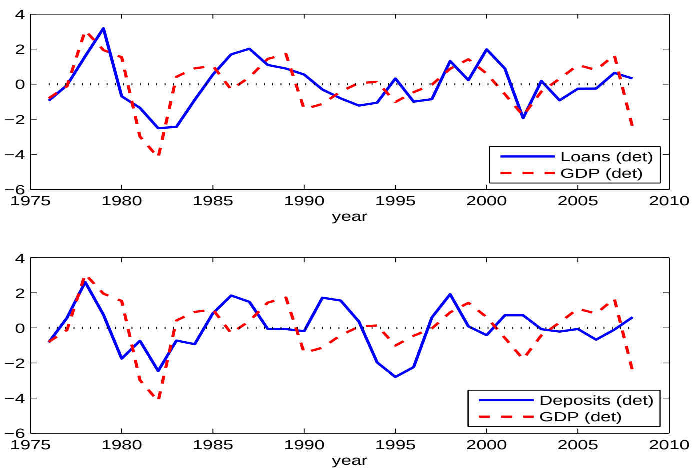


Figure 4 shows the cyclical behavior of the rate of return on loans, the rate of return on deposits, and the 'mark-up' rate (the difference between the two).[7] Rates of return on loans, deposits, and the mark-up display a highly countercyclical behavior. The correlation with detrended GDP equals -0.46, -0.38 and -0.29 respectively.

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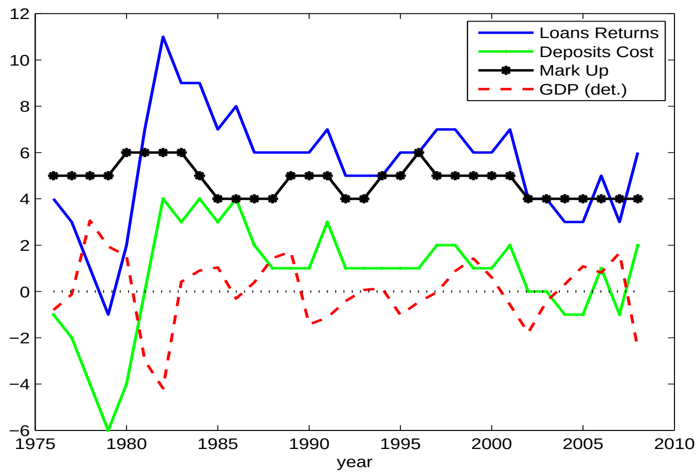
[6]The data for total loans since 1984 come from item RCON2122, total loan and leases net of unearned income. Prior to 1984, Lease Financing Receivables (RCON2165) are not included as part of total loans so the two series need to summed to insure comparability. The Implicit GDP deflator is used to convert nominal variables into real. The H-P filter with parameter equal to 6.25 is used to extract the trend from the data.

[7]The rate of return on loans is defined as total income from loans divided by total loans . The rate of return on deposits is constructed as the total expense on deposits divided by the stock of deposits. Total income from loans is computed from Income Statement data (schedule RI) using variables RIAD4010 and RIAD4059. The stock of deposits is constructed from RCON 2200. Nominal returns are converted to real returns by using a measure of expected inflation derived from the CPI index. In particular, we assume that inflation follows an AR(1) process and use the fitted value for each period as expected inflation.

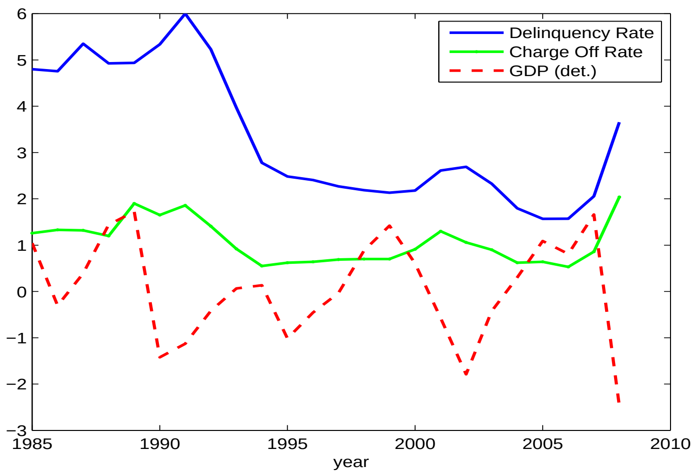
Figure 4: Loans Return, Deposits Cost and Business Cycle



In Figure 5 we show the evolution of loan delinquency rates and charge off rates. Data is only available since 1985.[8] Delinquency rates and charge off rates are countercyclical. The correlation with detrended GDP is -0.15 and -0.29 respectively.

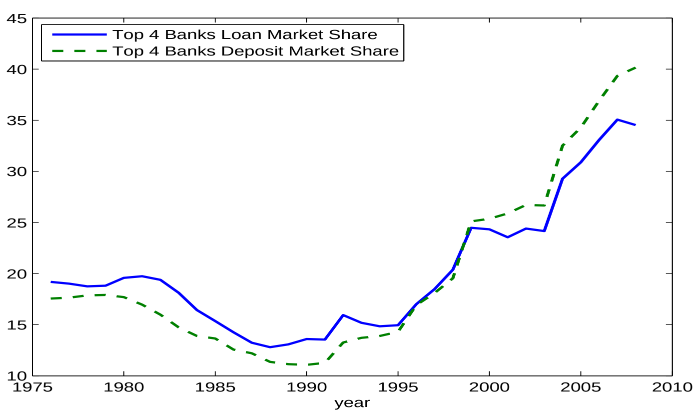
[8]The delinquency rate is the ratio of the dollar amount of a bank's delinquent loans to the dollar amount of total loans outstanding. Charge-off rates are defined as the flow of a bank's net charge-offs (gross charge-offs minus recoveries) during a year divided by the average level of its loans outstanding during the same period.

Figure 5: Loan Delinquency Rates, Charge Off Rates and Business Cycle



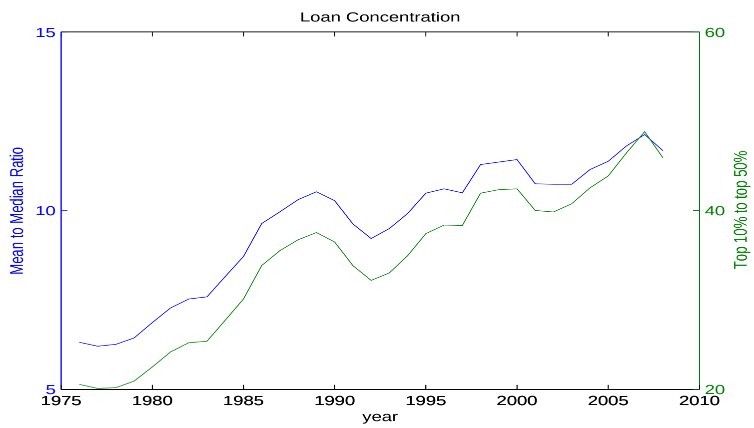
The size distribution of banks has always been skewed but the large number of bank exits (mergers and failures) that we documented above resulted in an unparalleled increase in loan and deposit concentration during the last 35 years. For example, in 1976 the four largest banks held 19 and 18 percent of the banking industry's loans and deposits respectively while by 2008 these shares had grown to 35 and 40 percent. Figure 6 displays the trend in the share of loans and deposits in the hands of the four largest banks since 1976.

Figure 6: Increase in Concentration: Loan and Deposit Market



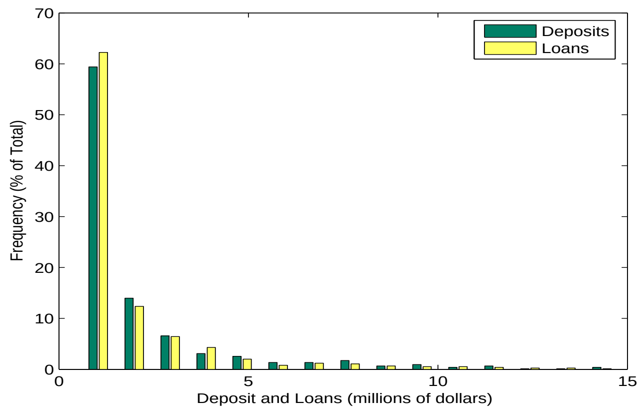
The increase in the degree of concentration is also evident if we look at the mean-to-median ratio and the share of loans of the top 10 percent of banks to the bottom 50 percent. Figure 7 displays the trend in these two measures of concentration in the loan market since the year 1976.

Figure 7: Increase in Concentration: Loan Market



The high degree of concentration in the banking industry is the reflection of the large number of small banks and a few large banks. In Figure 8 we provide the distribution of deposits and loans for the year 2007. Given the large number of banks at the bottom of the distribution we plot only banking institutions with less than 15 million dollars in deposits (93% of the total). Banks with 1 million dollars of deposits and loans account for approximately sixty percent of the total number of banks. However, total deposits and loans in these banks make up only twenty percent of the total loans and deposits in the industry.

Figure 8: Distribution of Bank Deposits and Loans



Note: Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports, Balance Sheet and Income Statement.

In Table 2, we provide measures of deposit and loan concentration for commercial banks in the US for the year 2008.[9] The table shows the high degree of concentration in deposits and loans. It is striking that the the four largest banks (measured by the C_4) hold approximately forty percent of deposits and loans and that the top 1 percent hold 53 and 48 percent of total deposits and loans respectively. We also observe a ratio of mean-to-median of around 7 suggesting sizeable skewness of the distribution. This high degree of inequality is also evident in the Gini coefficient of around 0.9 (recall that perfect inequality corresponds to a measure of 1). The Herfindahl Index and the Normalized Herfindahl Index are are around 4.5-5%. These figures are somewhat lower than what is typically associated with a highly concentrated industry (between 10-20%) due to the fact that the largest four banks have the same market share (approximately 10%) and the large number of firms that have a very small market share (more than 95% of banking institutions have deposit and loan market shares below 1%). In contrast, however, the values of HHI and $NHHI$ when all firms have equal market shares (i.e. 1/N) are 0.13% and $1.17e - 15$% respectively. In summary, taken together these measures suggest the banking industry is less than perfectly competitive.

[9] C_4 refers to the top 4 banks concentration index. The Herfindahl Index (HHI) is a measure of the size of firms in relation to the industry and often indicates the amount of competition among them. It is computed as $\sum_{n=1}^{N} s_n^2$ where s_n is market share of bank n. The Herfindahl Index ranges from $1/N$ to one, where N is the total number of firms in the industry. The normalized Herfindahl Index ($NHHI$) is computed as $\frac{HHI-1/N}{1-1/N}$.

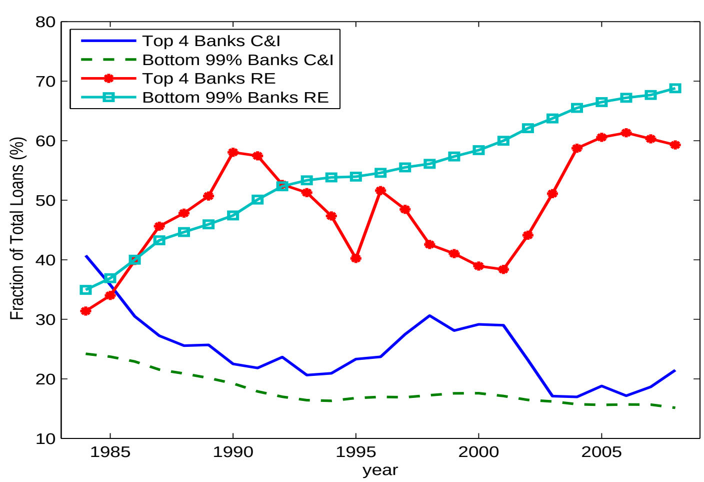
Table 2: Bank Deposit and Loan Concentration (in 2008)

Measure	Deposits	Loans
Percentage of Total in top 4 Banks (C_4)	40.2	34.5
Percentage of Total in top 1% Banks	76.7	74.7
Percentage of Total in top 10% Banks	89.2	89.1
Ratio Mean to Median	10.5	10.6
Ratio Total Top 10% to Total Smallest 50%	36.9	40.7
Gini Coefficient	.91	.90
HHI : Herfindahl Index (%)	4.9	3.8
$NHHI$: Normalized Herfindahl Index (%)	4.9	3.8

Note: Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports. Total Number of Banks 7092. Top 1% banks corresponds to 71 banks. Top 10% banks corresponds to 709 banks. Bottom 50% banks corresponds to 3545 banks.

An important trend in the loan portfolio composition of commercial banks is the increase in the fraction of loans secured by real estate and the decrease in the amount of commercial and industrial lending. In Figure 9, we document this trend for the largest 4 banks and the bottom 99% "small" banks when sorted by total loans. We compute the share of total loans that corresponds to commercial and industrial loans ($C\&I$) and real estate loans (RE) for each bank and then plot the average of these shares for each group and year.

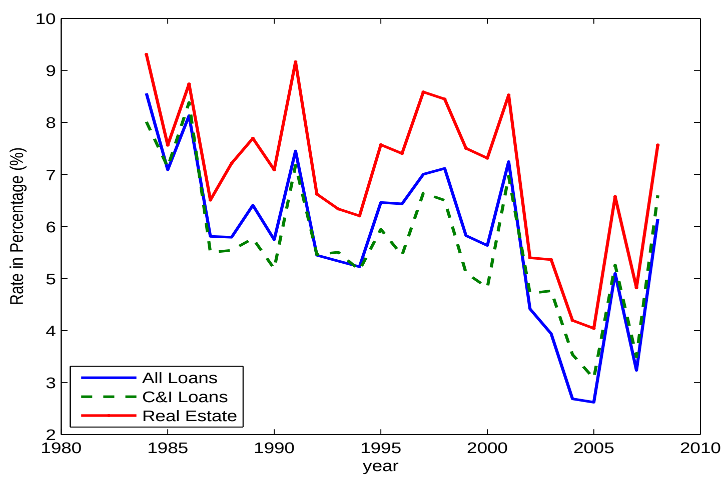
Figure 9: Portfolio Composition of Small and Big Banks



We observe that for both small and large banks, loans secured by real estate have become much more important. In the case of small banks, the share of loans secured by real estate almost doubled during this period (the share went from approximately 35 percent to around 70 percent). A similar trend is observed for the largest banks. The share of real estate loans in their portfolio increased from 40 percent to 60 percent. For this group of banks we also note a faster increase in this share during the last decade. The counterpart of the increase in real estate loans is the decrease in the share of commercial and industrial loans. We note that one of the differences between small and big banks is the portfolio composition. During the entire period loans secured by real estate constitute a more important component for small banks than for big banks. The opposite is true for commercial and industrial loans.

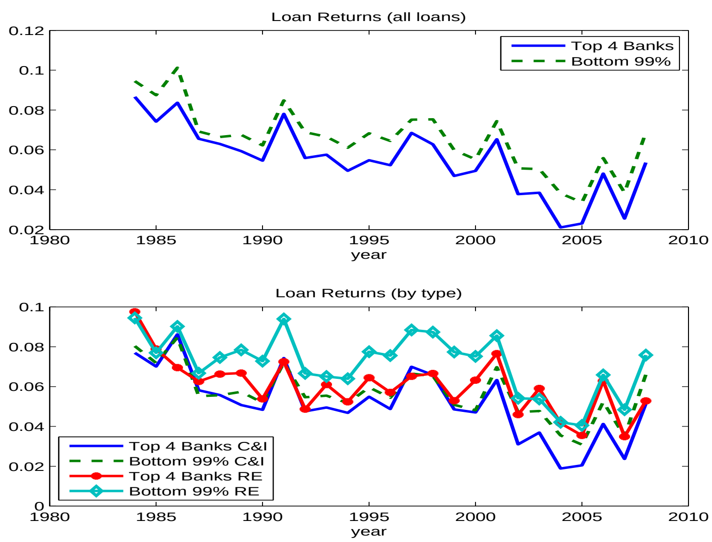
In Figure 10 we show that loan returns display countercyclical behavior even when dissagregated by loan type (commercial and industrial loans and real estate loans). The correlation with detrended GDP equals -0.17 and -0.09 for real estate and commercial and industrial loans respectively.

Figure 10: Loan Returns by Loan Type



In Figure 11, we analyze the evolution of loan returns by loan type and bank size. Small banks have higher returns for both real estate and commercial and industrial loans.

Figure 11: Loan Returns by Loan Type



In Table 3 we report the average over time of the cross-sectional average and standard deviation of loan returns by bank size since 1984.

Table 3: Loan Return and Volatility by Bank Size

Loan Returns (all loans)	Avg.	Std. Dev.	Corr. with GDP
Top 4 Banks (%)	4.93	0.75	-0.10
Top 1% Banks (%)	5.28	1.80	-0.18
Top 10% Banks (%)	5.51	1.56	-0.17
Bottom 99% Banks (%)	5.99	2.34	-0.17

Note: Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports. Values correspond to the time series average of the corresponding cross-sectional average and standard deviation among each group since 1984.

We note that smaller banks have higher returns and higher volatility of returns and a stronger negative correlation with detrended GDP. An important component of loan returns is the fraction of loans that is not repaid. For this reason, in Figures 12 and 13 we present charge-off rates and delinquency rates by bank size over time.

Figure 12: Charge-Off Rates by Bank Size

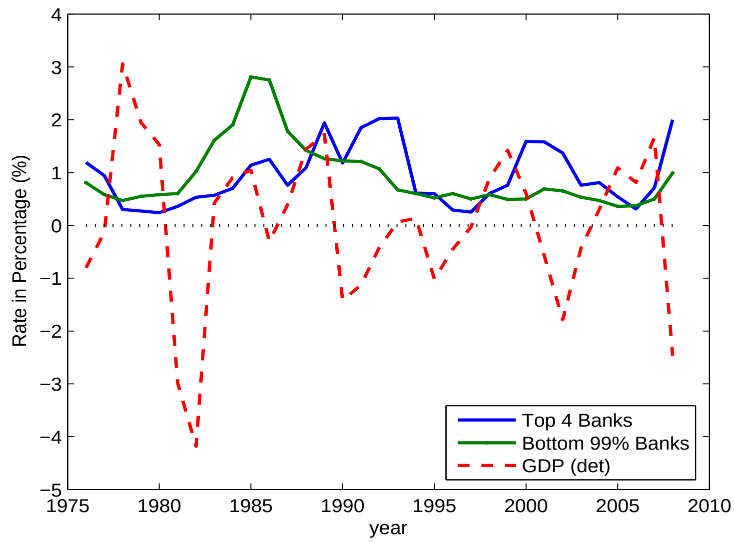
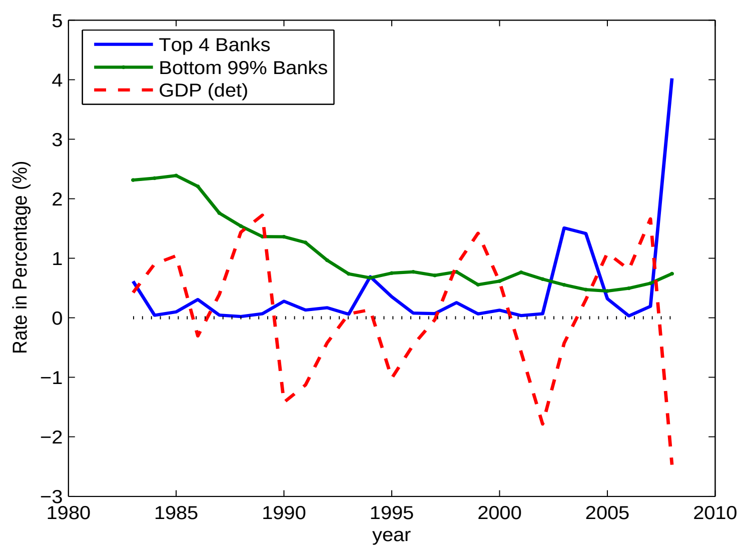


Figure 13: Delinquency Rates by Bank Size



As it was described before, both, charge-off rates and delinquency rates have an important negative cyclical component. From Figure 12 we observe no clear pattern between small and big bank charge-off rates. In the early period of the sample (1976-1988), charge-off rates were higher for small banks than for big banks. However, the opposite is true after 1988. On the contrary, delinquency rates are lower for big banks than for small banks through virtually the entire sample. In Table 4 we present the time series average of the cross-sectional average and standard deviation as well as the correlation with detrended GDP of charge-off rates and delinquency rates.

Table 4: Charge-Offs and Delinquency Rates by Bank Size

	Avg.	Std. Dev.	Corr. with GDP
Charge Off Rate Top 4 Banks (%)	0.94	0.53	-0.18
Charge Off Rate Top 1% Banks (%)	1.08	1.25	-0.27
Charge Off Rate Top 10% Banks (%)	0.84	1.29	-0.33
Charge Off Rate Bottom 99% Banks (%)	0.93	1.55	0.04
Del. Rate Top 4 Banks (%)	0.43	0.54	-0.53
Del. Rate Top 1% Banks (%)	0.57	0.95	-0.37
Del. Rate Top 10% Banks (%)	0.62	1.13	0.02
Del. Rate Bottom 99% Banks (%)	1.07	1.95	0.14

Note: Data corresponds to commercial banks in the US. Source: FDIC, Call and Thrift Financial Reports. Values correspond to the time series average of the corresponding cross-sectional average and standard deviation among each group. Data on delinquency rates is available only since 1983 and corresponds to loans delinquent 90 days or more.

3 Model Environment

Time is infinite. There are two regions $j \in 1, 2$. Each period, a mass B of one period lived ex-ante identical borrowers and a mass $H = 2B$ of one period lived ex-ante identical households (who are potential depositors) are born in each region.[10] There is an aggregate state of the world denoted $z_t \in \{z_b, z_g\}$ which evolves as a Markov process $\theta(z', z) = prob(z_{t+1} = z' | z_t = z)$ with $z_b < z_g$ and a regional specific shock $s_{t+1} \in \{1, 2\}$ that is equally likely, iid over time, and independent of z_{t+1}.

3.1 Borrowers

Borrowers in region j demand bank loans in order to fund a project. The project requires one unit of investment at the beginning of period t and returns at the

[10] The assumption $H = 2B$ is a normalization that simplifies the analysis below.

end of the period

$$\begin{cases} 1 + z_{t+1}R_t^j & \text{with prob } p^j(R_t^j, z_{t+1}, s_{t+1}) \\ 1 - \lambda & \text{with prob } [1 - p^j(R_t^j, z_{t+1}, s_{t+1})]. \end{cases} \tag{1}$$

The success or failure of a borrower's project is independent across borrowers but conditional on the borrower's choice of technology $R_t^j \geq 0$, the aggregate state of the economy at the end of the period z_{t+1} and the regional shock s_{t+1} (the dating convention we use is that a variable which is chosen/realized at the end of the period is dated $t+1$). Borrower gross returns in the successful state depend on the aggregate state $1 + z_{t+1}R_t^j$. Borrower gross returns in the unsuccessful state is $1 - \lambda$. At the beginning of the period when the borrower makes his choice of R_t both z_{t+1} and s_{t+1} have not been realized. As for the likelihood of success or failure, a borrower who chooses to run a project with higher returns has more risk of failure and there is less failure in good times. Specifically, $p^j(R_t^j, z_{t+1}, s_{t+1})$ is assumed to be decreasing in R_t^j and increasing in z_{t+1}. Moreover, we assume that the borrower success probability depends on the regional specific shock s_{t+1}. Specifically, $p^{j=s_{t+1}}(R_t^j, z_{t+1}, s_{t+1}) > p^{j \neq s_{t+1}}(R_t^j, z_{t+1}, s_{t+1})$. Hence in any period, one region has a higher likelihood of success than the other. While borrowers are ex-ante identical, they are ex-post heterogeneous owing to the realizations of the shocks to the return on their project. We envision borrowers either as firms choosing a technology which might not succeed or households choosing a house that might appreciate or depreciate.

There is limited liability on the part of the borrower. If $r_t^{L,j}$ is the interest rate on bank loans that borrowers face in region j, the borrower receives $\max\{z_{t+1}R_t^j - r_t^{L,j}, 0\}$ in the successful state and 0 in the failure state. Specifically, in the unsuccessful state he receives $1 - \lambda$ which must be relinquished to the lender.

Borrowers have an outside option (reservation utility) $\omega_t \in [\underline{\omega}, \overline{\omega}]$ drawn at the beginning of the period from distribution function $\Upsilon(\omega_t)$.

3.2 Depositors

All households are endowed with 1 unit of the good and have preferences denoted $u(c_t)$. All households have access to a risk free storage technology yielding $1 + r$ with $r \geq 0$ at the end of the period. They can also choose to supply their endowment to a bank in their region or to an individual borrower. If the household deposits its endowment with a bank, they receive $r_t^{j,D}$ whether the bank succeeds or fails since we assume deposit insurance. If they match with a borrower, they are subject to the random process in (1). At the end of the period they pay lump sum taxes τ_{t+1}.

3.3 Banks

In any period, there can be at most 3 banks. There is at most one "big" geographically diversified national bank that can extend loans and receive deposits in both regions. There is also at most one "small" regional bank restricted to each geographical area. Since we allow regional specific shocks to the success of borrower projects, small banks may not be well diversified. This assumption can, in principle, generate ex-post differences in loan returns documented in the data section.

We denote loans made by bank n to borrowers in region j by $\ell^j_{n,t}$ and accepted units of deposits by $d^j_{n,t}$. Without an interbank market, if n is a regional bank then $\ell^j_{n,t} \leq d^j_{n,t}$. If n is a national bank then $\ell^1_{n,t} + \ell^2_{n,t} \leq d^1_{n,t} + d^2_{n,t}$.

Active banks in a region play a Cournot game in the loan market $\ell^j_{n,t}$ which determines the equilibrium regional lending rate $r^{L,j}_t$. In principle one could also have banks be Cournot competitors in the deposit market as in Boyd and DeNicolo [2]. However, since we assume that $H > 2B$ there are sufficient funds to cover all possible loans if banks offer the lowest possible deposit rate $r^{D,j}_t \geq r$. In the future, we intend to consider the case where there are insufficient funds.

There is limited liability on the part of banks. A bank that has negative profits can exit, in which case it receives value zero. We assume that if a big bank exits, it must exit both regions.

Entry costs are denoted $\kappa^s \geq 0$ for the creation of small banks and $\kappa^b > \kappa^s$ for the creation of big banks. We assume that each entrant satisfies a zero expected discounted profits condition.

We denote the industry state by

$$\mu_t = \{N^b_t, N^{s,1}_t, N^{s,2}_t\}, \tag{2}$$

where the 3 elements of μ_t are simply the number of *active* banks by type and region: $N^b_t \in \{0,1\}$ corresponds to "big" banks, $N^{s,1}_t \in \{0,1\}$ to "small" banks in region 1 and $N^{s,2}_t \in \{0,1\}$ to "small" banks in region 2 respectively. For example, if in period t, there are only two active banks, one "big" and one "small" in region 2 the distribution will be equal to $\mu_t = \{1,0,1\}$. We denote the total number of active banks in the economy in period t by $N_t = N^b_t + N^{s,1}_t + N^{s,2}_t$. Also, we denote the number of active banks in region j by $N^j = N^b_t + N^{s,j}_t$.

3.4 Information

There is asymmetric information on the part of borrowers and lenders. Only borrowers know the riskiness of the project they choose R and their outside option ω. All other information is observable.

3.5 Timing

At the beginning of period t,

1. Starting from beginning of period state (z_t, μ_t), borrowers draw ω_t.

2. Borrowers in region j choose whether or not to undertake a project of technology R_t^j and banks choose how many loans $\ell_{n,t}^j$ to extend in each region at rate $r_t^{L,j}$ and deposits $d_{n,t}^j$ to accept at rate $r_t^{D,j}$.

3. Return shocks z_{t+1} and s_{t+1} are realized, as well as idiosyncratic borrower shocks.

4. Exit and entry decisions are made in that order. Entry occurs sequentially (one bank after another).

5. Households pay taxes τ_{t+1} and consume.

4 Industry Equilibrium

4.1 Borrower Decision Making

Starting in state z, borrowers take the loan interest rate $r^{L,j}$ as given and choose whether to demand a loan and if so, what technology R^j to operate. Specifically, if an borrower in region j chooses to participate, then given limited liability his problem is to solve:

$$v(r^{L,j}, z) = \max_{R^j} \sum_{z',s'} \frac{\theta(z', z)}{2} p^j(R^j, z', s') \left(z' R^j - r^{L,j}\right). \tag{3}$$

Let $R(r^{L,j}, z)$ denote the borrower's decision rule which that solves (3). Since borrowers are ex-ante identical in each region we omit the superscript j in the decision rule. We assume that the necessary and sufficient conditions for this problem to be well behaved are satisfied.

The borrower chooses to demand a loan if

$$v(r^{L,j}, z) \geq \omega. \tag{4}$$

In an interior solution, the first order condition is given by

$$\sum_{z',s'} \theta(z', z) p^j(R, z', s') z' + \sum_{z',s'} \theta(z', z) \frac{\partial p^j(R, z', s')}{\partial R} \left[z' R - r^{L,j}\right] = 0 \tag{5}$$

The first term is the benefit of choosing a higher return project while the second term is the cost associated with the increased risk of failure.

To understand how bank lending rates influence the borrower's choice of risky projects, one can totally differentiate (5) with respect to r^L

$$\begin{aligned}
0 \quad = \quad & \sum_{z',s'} \theta(z', z) \frac{\partial p^j(R^*, z', s')}{\partial R^*} \frac{dR^*}{dr^{L,j}} z' \\
& + \sum_{z',s'} \theta(z', z) \frac{\partial^2 p^j(R^*, z', s')}{(\partial R^*)^2} \left[z' R^* - r^{L,j}\right] \frac{dR^*}{dr^L} \\
& + \sum_{z',s'} \theta(z', z) \frac{\partial p^j(R^*, z', s')}{\partial R^*} \left[z' \frac{dR^*}{dr^{L,j}} - 1\right]
\end{aligned}$$

where $R^* = R(r^{L,j}, z)$. But then

$$\frac{dR^*}{dr^L} = \frac{\sum_{z',s'} \theta(z', z)\frac{\partial p^j(R^*,z',s')}{\partial R^*}}{\sum_{z',s'} \theta(z', z)\frac{\partial^2 p^j(R^*,z',s')}{(\partial R^*)^2}\left[z'R^* - r^{L,j}\right] + 2\sum_{z',s'} \theta(z', z)\frac{\partial p^j(R^*,z',s')}{\partial R^*}z'} > 0 \tag{6}$$

since both the numerator and the denominator are strictly negative (the denominator is negative by virtue of the sufficient conditions). Thus a higher borrowing rate implies the borrower takes on more risk. Boyd and De Nicolo [2] call $\frac{dR^*}{dr^L} > 0$ in (6) the "risk shifting effect". Risk neutrality and limited liability are important for this result.

An application of the envelope theorem implies

$$\frac{\partial v(r^{L,j}, z)}{\partial r^{L,j}} = -\sum_{z',s'} \theta(z', z)p^j(R, z', s') < 0. \tag{7}$$

Thus, participating borrowers are worse off the higher are borrowing rates. This has implications for the demand for loans determined by the participation constraint. In particular, since the demand for loans is given by

$$L^{d,j}(r^L, z) = E \cdot \int_{\underline{\omega}}^{\overline{\omega}} 1_{\{\omega \leq v(r^{L,j},z)\}} d\Upsilon(\omega), \tag{8}$$

then (7) implies $\frac{\partial L^{d,j}(r^{L,j},z)}{\partial r^L} < 0$.

4.2 Depositor Decision Making

If $r_t^D = r$, then a household would be indifferent between matching with a bank and using the autarkic storage technology so we can assign such households to a bank). If it is to match directly with a borrower, the depositor must compete with banks for the borrower. Hence, in successful states, the household cannot expect to receive more than the bank lending rate $r^{L,j}$ but of course could choose to make a take-it-or-leave-it offer of their unit of a good for a return $\widehat{r} < r^{L,j}$ and hence entice a borrower to match with them rather than a bank. Thus, the household matches with a bank if possible and strictly decides to remain in autarky otherwise provided

$$U \equiv u(1 + r) > \max_{\widehat{r} < r^{L,j}} \sum_{z',s'} \frac{\theta(z', z)}{2} \left[p^j(\widehat{R}, z', s')u(1 + \widehat{r}) + (1 - p^j(\widehat{R}, z', s'))u\,(1 - \lambda) \right] \equiv U^E. \tag{9}$$

If this condition is satisfied, then the total supply of deposits in region j, $D^{s,j}$, is given by

$$D^{s,j} = \sum_{n=1}^{N^j} d_n^j \leq H \tag{10}$$

Condition (10) makes clear the reason for a bank in our environment. By matching with a large number of borrowers, the bank can diversify the risk of project failure and this is valuable to risk averse households. It is the loan side uncertainty counterpart of a bank in Diamond and Dybvig [3].

4.3 Incumbent Bank Decision Making

An incumbent bank n in region j chooses a loan decision rule ℓ_n^j in order to maximize profits and chooses whether to exit x_n^j after the realization of the aggregate shock z' and the regional shock s'.

It is simple to see that no bank would ever accept more total deposits than it makes total loans.[11] Further, the deposit rate $r_t^{D,j} = r$.[12] Simply put, a bank would not pay interest on deposits that it doesn't lend out and with excess supply of funds, households are forced to their reservation value.

Let $\sigma_{-n} = (\ell_{-n}^j, x_{-n}^j, e^j)$ denote the industry state dependent lending, exit, and entry strategies of all other banks. The supply of loans in region j is given by

$$L^{s,j} = \sum_{n \in N^j} \ell_n^j \tag{11}$$

From (8) and (11) we derive the equilibrium loan interest rate $r^{L,j}$.

Limited liability implies a bank will exit if its end-of-period-profits are negative. The end-of-period profits for bank n extending loans ℓ_n^j in region j is given by:

$$\pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) \equiv \Big\{p^j(R, z', s')(1+r^{L,j}) + (1-p^j(R, z', s'))(1-\lambda) - (1+r)\Big\}\ell_n^j.$$

The first two terms represent the net return the bank receives from successful and unsuccessful projects respectively and the last term corresponds to its costs.

The value function of a "big" incumbent bank n at the beginning of the period is then given by

$$\begin{aligned} V_n^b(\mu, z; \sigma_{-n}) = \max_{\{\ell_n^j\}_{j=1,2}} \quad & \textstyle\sum_{z',s'} \frac{\theta(z',z)}{2} \Big[\sum_{j=1,2} \pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) \\ & + \beta V_n^b(\mu', z'; \sigma_{-n}) \Big] (1 - x_n^b) \end{aligned} \tag{12}$$

where

$$x_n^b(\mu, z, z', s'; \sigma_{-n}) = \begin{cases} 1 & \text{if } \sum_{j=1,2} \pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) < 0 \\ 0 & \text{if } \sum_{j=1,2} \pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) \geq 0 \end{cases}, \tag{13}$$

and $\mu' = \{N_x^b + N_e^b, N_x^{s,1} + N_e^{s,1}, N_x^{s,2} + N_e^{s,2}\}$ with $N_x^b + N_e^b = 1$.

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[11] Suppose not and $d_n > \ell_n$. The net cost of doing so is $r_t^{D,j} \geq 0$ while the net gain on $d_n - \ell_n$ is zero, so it is weakly optimal not to do so.

[12] Suppose not and some bank is paying $r_t^D > r$. Then the bank can lower r_t^D, still attract deposits since H=2B and make strictly higher profits, so it is strictly optimal not to do so.

The evolution of the distribution of banks depends on the number of big and small active banks after exit $N_x^b, N_x^{s,j}$ and the number of entrants $N_e^b, N_e^{s,j}$ in each region j. The number of "big" banks in the industry after exit is given by

$$N_x^b = \sum_{n=1}^{N^b} \left(1 - x_n^b(\mu, z, z', s'; \sigma_{-n})\right), \tag{14}$$

and N_e^b the number of "big" entrants to be defined below. Similarly, the number of "small" banks in region j after exit is given by

$$N_x^{s,j} = \sum_{n=1}^{N^{s,j}} \left(1 - x_n^{s,j}(\mu, z, z', s'; \sigma_{-n})\right), \tag{15}$$

and $N_e^{s,j}$ the number of entrants to be defined below.

The value function of a "small" incumbent bank n in region j at the beginning of the period is then given by

$$\begin{aligned} V_n^{s,j}(\mu, z; \sigma_{-n}) = \max_{\ell_n^j} \quad & \textstyle\sum_{z',s'} \frac{\theta(z',z)}{2} \Big[\pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) \\ & +\beta V_n^b(\mu', z'; \sigma_{-n})\Big](1 - x_n^{s,j}) \end{aligned} \tag{16}$$

where

$$x_n^{s,j}(\mu, z, z', s'; \sigma_{-n}) = \begin{cases} 1 & \text{if } \pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) < 0 \\ 0 & \text{if } \pi_{\ell_n}^j(\mu, z, z', s'; \sigma_{-n}) \geq 0 \end{cases}, \tag{17}$$

and $\mu' = \{N_x^b + N_e^b, N_x^{s,1} + N_e^{s,1}, N_x^{s,2} + N_e^{s,2}\}$ with $N_x^{s,j} + N_e^{s,j} = 1$.

4.4 Entrant Bank Decision Making

In each period, new banks can enter the industry by paying the setup cost κ^s to become a "small" bank or κ^b to become a "big" bank. They will enter the industry if the net present value of entry is positive. For example, taking the entry and exit decisions by other banks as given, a potential small entrant in region 1 will choose $e_n^{s,1}\left(\{N_x^b + N_e^b, 0, N_x^{s,2} + N_e^{s,2}\}, z'\right) = 1$ if

$$\beta V_n^{s,1}(\{N_x^b + N_e^b, 1, N_x^{s,2} + N_e^{s,2}\}, z'; \sigma_{-n}) - \kappa_s > 0. \tag{18}$$

4.5 Definition of Equilibrium

A pure strategy Markov Perfect Equilibrium (MPE) is a set of functions $\{v(r^{L,j}, z), R(r^{L,j}, z)\}$ describing borrower behavior, a set of functions $\{V_n^b(\mu, z; \sigma_{-n}), \ell_n^{b,j}(\mu, z, D_n; \sigma_{-n}), V_n^{s,j}(\mu, z; \sigma_{-n}), \ell_n^{s,j}(\mu, z, D_n; \sigma_{-n}), x_n^b(\mu, z, z', s'; \sigma_{-n}), x_n^{s,j}(\mu, z, z', s'; \sigma_{-n}), e_n^b(\mu, z'), e_n^{s,j}(\mu, z')\}$ describing bank behavior, a loan interest rate $r^{L,j}(\mu, z)$ for each region, a deposit interest rate $r^D = r$, an industry state μ, a function describing the number of entrants $N_e^b(\mu, z')$, $N_e^{s,j}(\mu, z')$, and a tax function $\tau(\mu, z, z', s')$ such that:

1. Given a loan interest rate $r^{L,j}$, $v(r^{L,j}, z)$ and $R(r^{L,j}, z)$ are consistent with borrower's optimization in (3) and (4).

2. For any given interest rate $r^{L,j}$, loan demand $L^{d,j}(r^{L,j}, z)$ is given by (8).

3. At $r^D = r$, the household deposit participation constraint (10) is satisfied.

4. Given the loan demand function, the value of the bank $V^b(\mu, z; \sigma_{-n})$, $V^{s,j}(\mu, z; \sigma_{-n})$, the loan decision rules $\ell^b(\mu, z; \sigma_{-n})$, $\ell^{s,j}(\mu, z; \sigma_{-n})$ and exit rules $x^b(\mu, z, z', s'; \sigma_{-n})$, $x^{s,j}(\mu, z, z', s'; \sigma_{-n})$ are consistent with bank optimization in (12), (13), (16) and (17).

5. The entry decision rules $e^b(\mu, z')$ and $e^{s,j}(\mu, z')$ are consistent with bank optimization in (18).

6. The law of motion for the industry state $\mu' = \{N_x^b + N_e^b, N_x^{s,1} + N_e^{s,1}, N_x^{s,2} + N_e^{s,2}\}$ is consistent with entry and exit decision rules.

7. The interest rate $r^{L,j}(\mu, z)$ is such that the loan market clears:

$$L^{d,j}(r^{L,j}, z) = \sum_{n=1}^{N^j} \ell_n^j(\mu, z; \sigma_{-n}).$$

8. Across all states (μ, z, z', s'), taxes cover deposit insurance:

$$\tau(\mu, z, z', s') = -\frac{1}{2H}\Bigg\{ x_n^b(\mu, z, z', s'; \sigma_{-n})\pi_{\ell_n}^b(\mu, z, z', s'; \sigma_{-n}) + \sum_j x_n^{s,j}(\mu, z, z', s'; \sigma_{-n})\pi_{\ell_n}^{s,j}(\mu, z, z', s'; \sigma_{-n}) \Bigg\}$$

5 Parameterization

At this preliminary stage, we parameterize the model to show the main characteristics of equilibrium behavior. In the future, parameters will be chosen to match the key statistics of the U.S. banking industry described in Section 2. A model period is considered to be one year.

First, we consider the stochastic process for aggregate technology shocks $\theta(z', z)$. To calibrate this process, we use the NBER recession dates and create a recession indicator. More specifically, for a given year, the recession indicator takes a value equal to one if two or more quarters in that year were dated as part of a recession. The correlation of this indicator with HP filtered GDP equals -0.87. Then, we identify years where the indicator equals one with our periods of $z = z_b$ and construct a transition matrix. In particular, the maximum likelihood estimate of θ_{kj}, the $(j, k)th$ element of the aggregate state transition matrix, is the ratio of the number of times the economy switched from state j to state k to the number of times the economy was observed to be in state j. We normalize the value of $z_g = 1$ and choose z_b to match the variance of detrended GDP.

We calibrate r^D using data from the banks' balance sheet. The deposit interest rate is computed as the average ratio of total interest expense over total interest bearing deposits for commercial banks in the US from 1980 to 2008.[13]

Next we consider the stochastic process for the borrower's project. For each borrower in region j, let $y^j = \alpha z' + (1-\alpha)\varepsilon_e - bR$ where ε_e is drawn from $N(\mu_\varepsilon + \phi(s'), \sigma_\varepsilon^2)$. The regional shock affects the mean of the idiosyncratic shock through $\phi(s') \in \{-\overline{\phi}, \overline{\phi}\}$. We assume that if $s' = j$, $\phi(s') = \overline{\phi}$ and $\phi(s') = -\overline{\phi}$ otherwise. Borrower's idiosyncratic project uncertainty is iid across agents. Define success to be the event that $y > 0$, so in states with higher z or higher ε_e success is more likely. Then

$$\begin{aligned} p^j(R, z') &= 1 - prob(y \leq 0 | R, z') \\ &= 1 - prob\left(\varepsilon_e \leq \frac{-\alpha z' + bR}{(1-\alpha)}\right) \\ &= 1 - \Phi\left(\frac{-\alpha z' + R}{(1-\alpha)}\right) \end{aligned} \quad (19)$$

where $\Phi(x)$ is a normal cumulative distribution function with mean $(\mu_\varepsilon + \phi(s'))$ and variance σ_ε^2.

For the borrower's outside option, we assume that $\Upsilon(\omega)$ corresponds to the uniform distribution $[\underline{\omega}, \overline{\omega}]$ and set $\underline{\omega} = 0, \overline{\omega} = v(r^{L,j} = 0, z = 1)$. We let consumer's preferences be given by $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and set $\sigma = 2$ a standard value in the macro literature. At this level of risk aversion the consumer participation constraint is satisfied. The mass of borrowers and depositors are normalized to one.

We are left with six more parameters to calibrate: $\{\mu_\varepsilon, \overline{\phi}, \sigma_\varepsilon, \alpha, b, \lambda\}$. We use data for commercial banks and asset returns to pin down these parameters. We link borrowers net return with average real equity return for the post-war era as reported by Mehra [12]. A possible set of targets includes the average default frequency, borrower return, exit rate, loan return and charge-off rate.

Table 5 shows the parameters we use in this example.

[13] Source: FDIC, Call and Thrift Financial Reports, Balance Sheet and Income Statement (http://www2.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10). The nominal interest rate is converted to a real interest rate by using the US GDP deflator (expected inflation is computed as the average of previous four quarters inflation).

Table 5: Model Parameters

Parameter		Value
Deposit Int. Rate (%)	r^D	0.7
Discount Factor	$\beta = \frac{1}{1+r^D}$	0.99
Depositors' Preferences	σ	2
Mean Entrep. Dist.	μ_ε	-0.85
Volatility Entrep. Dist.	σ_ε	0.035
Project slope	b	1.75
Distribution Reservation Value	$\overline{\omega}$	0.17
Weight Aggregate Shock	α	0.65
Regional Shock	$\overline{\phi}$	0.05
Mass of Borrowers	B	1
Mass of Households	H	2
Aggregate Shock in Bad State	z_b	0.959
Aggregate Shock in Good State	z_g	1
Transition Probability	$\theta(z_g, z_g)$	0.85
Transition Probability	$\theta(z_b, z_b)$	0.5
Loss Rate	λ	0.05
Entry Cost Small	κ^s	0.13
Entry Cost Big	κ^b	0.34

6 Results

For the parameter values in Table 5, we find an equilibrium where: (i) if $N^b = 0$ and $z' = z_g$, there is entry by a big bank; (ii) if there is no small bank in one of the regions ($N^{s,j} = 0$ for $j = 1$ and/or $j = 2$) and $z' = z_g$ there is entry by a small bank in that region; (iii) a small bank which experiences a negative regional shock exits if $z = z_g$ and $z' = z_b$.

Table 6 displays some key first moments of the model and a comparison with the data. Since we have not finished calibrating the model, it is not surprising that the model moments are far from the data. We use the following definitions to connect the model to some of the variables we presented in the data section. In particular,

- Cross-Sectional Average Default frequency (measured by the 90 day delinquency rate): $1 - p(R^*, z', s')$
- Cross-Sectional Average Borrower return: $p(R^*, z', s')(z'R^*)$
- Bank exit rate: $\frac{N^b - N^b_x + \sum_j (N^{j,s} - N^{s,j}_x)}{N}$
- Cross-Sectional Average Loan return: $p(R^*, z', s')r^L$
- Cross-Sectional Average Loan Charge-off rate $(1 - p(R^*, z', s'))\lambda$

Table 6: Model and Data Moments

Moment Average (%)	Model	Data
Default Frequency	5.47	1.06
Borrower Return	16.57	8.4
Bank Exit Rate	3.83	3.9
Loan Return	6.19	5.5
Charge-off Rate	0.16	0.93

To understand the equilibrium, we first describe borrower decisions. Figure 14 shows the borrower's optimal choice of project riskiness $R^*(r^{L,j}, z)$ and the inverse demand function associated with $L^d(r^{L,j}, z)$.

Figure 14: Borrower's Project $R(r^{L,j}, z)$ and Loan Inverse Demand $r^{L,j}(L, z)$

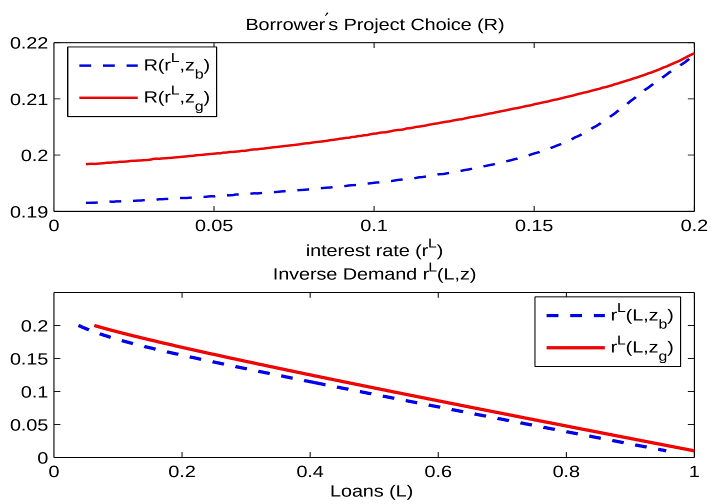

Figure 14 shows that borrower's optimal project R is an increasing function of the loan interest rate $r^{L,j}$. Moreover, given that the value of the borrower is decreasing in $r^{L,j}$, loan demand is a decreasing function of $r^{L,j}$.

In tables (7) to (11) we provide a description of borrower and bank decision rules and their implications for loan supply, loan interest rates, borrower returns and success probabilities. Note that while these are equilibrium functions not every state is necessarily on-the-equilibrium path (starting with Table 12 we evaluate the behavior of the model on-the-equilibrium path)

Table 7 provides the loan decision rules for big and small banks. The first four columns correspond to the loans made by a big bank in region 1 and 2 respectively. The next two columns correspond to the small bank in region 1 and the last two columns to the small bank in region 2. We observe that banks offer more loans when $z = z_g$ than when z_b. Independent of the aggregate shock, big banks offer more loans in regions where they have more market power.

Table 7: Bank Loan Decision Rules $\ell_n^j(\mu, z; \sigma_{-n})$

	Big Region 1		Big Region 2		Small Region 1		Small Region 2	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z = z_b$	$z = z_g$	$z = z_b$	$z = z_g$	$z = z_b$ 1	$z = z_g$	$z = z_b$	$z = z_g$
$\{0, 1, 0\}$	0.000	0.000	0.000	0.000	0.479	0.493	0.000	0.000
$\{0, 1, 1\}$	0.000	0.000	0.000	0.000	0.479	0.493	0.479	0.493
$\{1, 0, 0\}$	0.479	0.493	0.479	0.493	0.000	0.000	0.000	0.000
$\{1, 1, 0\}$	0.305	0.319	0.479	0.493	0.305	0.325	0.000	0.000
$\{1, 1, 1\}$	0.305	0.319	0.305	0.319	0.305	0.325	0.305	0.325

The optimal exit rule implies that active big banks do not exit under any combination of the state space but there is exit for small banks in region j when the economy moves from good times to bad times ($z = z_g$ and $z' = z_b$) and the small bank receives the negative regional shock ($s' \neq j$). That is,

$$x_n^{s,j}(\mu, z, z', s') = \begin{cases} 1 & \text{if } z = z_g,\ z' = z_b \text{ and } s' \neq j \\ 0 & \text{otherwise} \end{cases} . \tag{20}$$

Tables 8, 9 and 10 display aggregate loan supply, the loan interest rate and borrower return as a function of the industry state μ and the aggregate state z (i.e. across market structure and the business cycle) for each region. As discussed above, not all cells in 8, 9 and 10 are on-the-equilibrium path. In particular, the equilibrium path corresponds to ($\mu = \{1, 1, 0\}, z = z_b$) and ($\mu = \{1, 1, 1\}, z = z_g$).

Table 8: Loan Supply $L^{s,j}(\mu, z)$

	$L^{s,1}$		$L^{s,2}$	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z = z_b$	$z = z_g$	$z = z_b$	$z = z_g$
$\{0, 1, 0\}$	0.479	0.493	0.000	0.000
$\{0, 1, 1\}$	0.479	0.493	0.479	0.493
$\{1, 0, 0\}$	0.479	0.493	0.479	0.493
$\{1, 1, 0\}$	0.611	0.643	0.479	0.493
$\{1, 1, 1\}$	0.611	0.643	0.611	0.643

Table 9: Loan Interest Rate $r^{L,j}(\mu, z)$

	$r^{L,1}$		$r^{L,2}$	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z = z_b$	$z = z_g$	$z = z_b$	$z = z_g$
$\{0, 1, 0\}$	0.0829	0.0903	0.0000	0.0000
$\{0, 1, 1\}$	0.0829	0.0903	0.0829	0.0903
$\{1, 0, 0\}$	0.0829	0.0903	0.0829	0.0903
$\{1, 1, 0\}$	0.0598	0.0634	0.0829	0.0903
$\{1, 1, 1\}$	0.0598	0.0634	0.0598	0.0634

Table 10: Borrower's Project $R^j(\mu, z)$

	R^1		R^2	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z = z_b$	$z = z_g$	$z = z_b$	$z = z_g$
$\{0, 1, 0\}$	0.1712	0.1801	0.0000	0.0000
$\{0, 1, 1\}$	0.1712	0.1801	0.1712	0.1801
$\{1, 0, 0\}$	0.1712	0.1801	0.1712	0.1801
$\{1, 1, 0\}$	0.1695	0.1783	0.1712	0.1801
$\{1, 1, 1\}$	0.1695	0.1783	0.1695	0.1783

We observe that, conditional on the aggregate state z, less concentration (cases with $N^b + N^{s,j} = 2$) imply a higher loan supply, a lower loan interest rate $r^{L,j}$ and a lower project return R. This observation is consistent with Proposition 2 of Boyd and DeNicolo [2]. In particular, they consider exogenous increases in N and find that r^L declines to r^D. We also note that, conditional on the number of banks N, the total loan supply is higher in good times ($z = z_g$) than in bad times ($z = z_b$). However, also conditional on the number of banks N, interest rates $r^{L,j}$ are higher in good times than in bad times and by the risk shifting effect (6) the same is true for R^*. This observation is not inconsistent, however, with the countercylical nature of loan returns in the data since the data is generated by states ($\mu = \{1, 1, 0\}, z = z_b$) and ($\mu = \{1, 1, 1\}, z = z_g$) along the equilibrium path.

In Table 11 we show the implications of loan interest rates $r^{L,j}$ and borrower project choice R^j on default frequencies $1 - p^{j=1}(R, z', s')$ across market structure and business cycle in region 1. The table shows that conditional on current state (μ, z), the realization of a bad aggregate shock $z' = z_b$ as well as the realization of a negative regional shock imply a higher default frequency. We also observe that more concentrated industries $N^j = 1$ have a higher default frequency across both aggregate states. Moreover, the highest default frequencies are observed at a turning point from good to bad times (i.e. from $z = z_g$ to $z' = z_b$).

Table 11: Default Freq. across Market Structure and Business Cycle (%)

$1 - p^1(\mu, z, z', s')$

$s' = 1$	$z = z_b$		$z = z_g$	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z' = z_b$	$z' = z_g$	$z' = z_b$	$z' = z_g$
$\{0, 1, 0\}$	0.02	0.00	1.09	0.00
$\{0, 1, 1\}$	0.02	0.00	1.09	0.00
$\{1, 0, 0\}$	0.02	0.00	1.09	0.00
$\{1, 1, 0\}$	0.01	0.00	0.53	0.00
$\{1, 1, 1\}$	0.01	0.00	0.53	0.00
$s' = 2$	$z = z_b$		$z = z_g$	
$\mu(N^b, N^{s,1}, N^{s,2})$	$z' = z_b$	$z' = z_g$	$z' = z_b$	$z' = z_g$
$\{0, 1, 0\}$	23.89	0.20	71.31	5.34
$\{0, 1, 1\}$	23.89	0.20	71.31	5.34
$\{1, 0, 0\}$	23.89	0.20	71.31	5.34
$\{1, 1, 0\}$	16.93	0.09	61.77	3.03
$\{1, 1, 1\}$	16.93	0.09	61.77	3.03

Moving back to moments calculated on-the-equilibrium path, in Table 12 we show how aggregate variables correlate with the technology shock to GDP.[14] We observe that, as in the data, the model generates countercylical loan interest rates, exit rates, default frequencies and charge-off rates. Moreover, the model generates procyclical aggregate loan supply. However, the model generates a positive correlation between the aggregate state and loan returns. This is because we measure loan returns as $p(R^*, z', s')r^L$ and while the first row of Table 12 shows that $corr(z', r^L(\mu', z')) < 0$, this is offset by the fact that success probabilities are procyclical (since default frequencies are countercyclical). Obviously it is *possible* to generate countercyclical loan returns in the model if the relative magnitudes of the correlations are different from what we find in this particular parameterization.

[14] We use the following dating convention in calculating correlations. Since most variables depend on z and z' (e.g. default frequency $1 - p(R(r^L(\mu, z)), z', s')$), we display $corr(z', f(z, z'))$.

Table 12: Business Cycle Correlations

Variable Correlated with GDP Shock	Model	Data
Loan Interest Rate r^L	-1.00	-0.21
Exit Rate	-0.66	-0.26
Loan Supply	1.00	0.51
Default Frequency	-0.83	-0.36
Loan Return	0.76	-0.20
Charge-off rate	-0.93	-0.46

Note: The figure for the loan interest rate in the data is taken from the Prime Bank Loan Rate at http://research.stlouisfed.org/fred2/categories/117corresponds

Table 13 presents a decomposition of the equilibrium moments conditional on the value of the aggregate shock. The table shows that more banks are active in good times (implying more competition). This results in a higher loan supply, a lower interest rate and a lower default frequency. The borrower decision rule R^* implies that, even though the loan interest rate is lower in good times, the optimal borrower return is higher. Loan returns pr^L are higher in good times and while $dR/dr^L > 0$ holds, the slope of the decision rule in the r^L dimension is small (see Figure 14 above). Profit rates are always higher and charge-off rates are much lower in good times than in bad times.

Table 13: Model Moments Across the Business Cycle

Moment Average	$z = z_b$	$z = z_g$
Loan interest rate (r^L)	6.69	6.34
Default frequency	21.40	1.30
Loan supply	1.09	1.29
Borrower return	13.12	17.48
Number of banks	2.00	3.00
Fraction of Periods with one bank	0.00	0.00
Fraction of Periods with two banks	1.00	0.00
Fraction of Periods with three banks	0.00	1.00
Survivor profit rate	4.29	5.51
Survivor loan return rate	5.65	6.29
Survivor charge-off rate	0.65	0.07
Failure profit rate	-1.38	n.a.
Failure loan return rate	2.42	n.a.
Failure charge-off rate	3.09	n.a.

Note: 'survivor' corresponds to every active bank that does not exit after the realization of z', 'failure' corresponds to banks that exit after the realization of z'. The profit rate is defined as π_{ℓ_n}/ℓ_n.

Table 13 also shows that profits of banks that exit (failures) are negative which is covered by means of lump-sum taxation since we assume there is deposit insurance. Exit happens only for the small bank when $N^{s,j} = 1$, $z = z_g$, $z' = z_b$ and $s \neq j$. Thus, taxes are nonzero only in that case. In particular, taxes are equal to -0.0045.

Table 14 shows the relation between the degree of bank competition measured by the number of banks and important first moments for the economy. More competition (i.e. more active banks) implies a higher loan supply, a lower interest rate on loans, and lower bank profit rates. Despite lower interest rates on loans with more competition, the fact that exit occurs when the aggregate state changes from $z = z_g$ to $z' = z_b$ implies a large default frequency $1 - p(R, z_b, s \neq j)$ in that event. Thus, even though the event does not occur frequently, a high default frequency is still associated with loans being made in $z = z_g$ where there are three banks. This counteracts the usual lower default frequency associated with lower interest rates (which induce the borrower to choose less risky projects). Again, it is difficult to disentangle the contribution of concentration and the aggregate shock separately on-the-equilibrium path. Obviously it is *possible* to generate lower default frequencies with more competition depending upon the calibration.

Table 14: Model Moments and Market Concentration

Moment Average	$N = 1$	$N = 2$	$N = 3$
Loan interest rate (r^L)	n.a.	6.69	6.34
Default frequency	n.a.	4.15	5.82
Loan supply	n.a.	1.09	1.29
Borrower return	n.a.	16.02	16.72
Bank profit rate	n.a.	5.48	5.29
Loan return rate	n.a.	6.40	6.15
Charge-off rate	n.a.	0.21	0.15

In Table 15 we present the main differences between small and big banks generated on the equilibrium path by the model.

Table 15: Model Moments by Bank Size

Moment Average	Big	Small
Loan interest rate (r^L)	6.56	6.26
Default frequency	6.42	1.79
Borrower return	16.42	17.20
Bank profit rate	5.20	5.40
Loan return rate	6.19	6.19
Charge-off rate	0.28	0.08

In this equilibrium, big banks enjoy periods as regional monopolists since their diversified loan portfolio allows them to survive the regional shocks that induce small banks to exit. This is reflected in higher interest rates for big banks than that of small banks. We also observe that the average default frequency of big banks is higher than that of small banks. Default frequencies (and all other variables) are calculated for survivor banks only. When the economy transits from good times to bad times ($z = z_g$ and $z' = z_b$), big banks stay active but a large fraction of their borrowers default in the region where $s' \neq j$. At the same time, the survivor small bank faces a low default frequency. This is also reflected in a lower borrower return, lower profit rates and higher charge-off rates for big banks than for small banks. Differences in default frequencies and loan interest rates also generate variation in loan returns across banks of different size. This variation is not evident when looking only at the average loan return. However, we find that in periods where there is exit, loan returns of big banks are lower than those of small banks since it suffers a high fraction of defaults and there were no ex-ante differences in loan interest rates. Moreover, during periods where the big bank is a regional monopolist, its returns are higher than those of the active small bank since the effect of a higher interest rate dominates that of a higher default frequency.

7 Concluding Remarks

Using Call Report data from commercial banks in the U.S. from 1976 to 2008 (the same data employed by Kashyap and Stein [10]) we document that entry and exit by merger are procyclical, exit by failure is countercyclical, total loans and deposits are procyclical, loan returns and markups are countercyclical and delinquency rates and charge-offs are countercyclical. Furthermore, we show that bank concentration has been rising and that the top 4 banks have 35% of loan market share. Finally, we document important differences between small and large banks. For example, we find that small banks have higher returns and higher volatility of returns than large banks and that returns for small banks are more countercyclical than large banks.

We built a model where “big” geographically diversified banks coexist in equilibrium with “small” regional banks that are restricted to a geographical area. Since we allow for regional specific shocks to the success of borrower projects, small banks may not be well diversified. This assumption generates ex-post differences between big and small banks. As documented in the data section, the model generates not only procyclical loan supply but also countercyclical interest rates on loans, bank failure rates, default frequencies, charge-off rates. Since bank failure is paid for by lump sum taxes to fund deposit insurance, the model predicts countercylical taxes. One qualitative difference under the current parameterization is that the model generates procyclical loan returns while in the data loan returns are countercyclical. Also, the model generates differences in loan interest rates, profit rates and default frequencies between banks of different sizes since larger banks are able to diversify across both regions. These differences are also reflected in variation in loan returns between small and big banks (even though for the current parameterization the average is about the same). Finally, in line with the findings of Boyd and De Nicolo [2], in Tables 8 to 11 we show that conditional on the aggregate state, more banks (higher N) imply a higher loan supply, lower interest rate, a lower borrower return and hence a lower default and bank failure probability. The benefit of our model relative to existing literature, however, is that the number of banks is derived endogenously and varies over the business cycle.

There is much work left to do. First, we need to calibrate so that the model moments match the data moments. In the current version of the model with at most two banks per region, it is not surprising that interest rates on loans are higher than in the data. This has implications for the borrower project choice and the resulting higher returns on projects and higher bankruptcy rates in the model than in the data. Second, we intend to consider a larger number of banks to further assess the risk shifting effect and competition. Along this dimension it may be useful to incorporate a competitive fringe. Third, we intend to include deposit competition as well as loan competition similar to Boyd and De Nicolo [2]. Finally, it may be important to allow banks to have a merger option since we see that in the data as well.

References

[1] Allen, F. and D. Gale (2000) *Comparing Financial Systems,* Cambridge, MA: The MIT Press.

[2] Boyd, J. and G. De Nicolo (2005) “The Theory of Bank Risk-Taking and Competition Revisited”, *Journal of Finance*, 60, p. 1329-43.

[3] Diamond, D. and P. Dybvig (1983) ”Bank Runs, Deposit Insurance, and Liquidity”, *Journal of Political Economy*, 91, p. 401–19.

[4] Diaz-Gimenez, J., E. Prescott, F. Alvarez, and T. Fitzgerald (1992) “Banking in Computable General Equilibrium Models”, *Journal of Economic Dynamics and Control*, 16, p.533-60.

[5] Ennis, H. (2001) “On the Size Distribution of Banks”, Federal Reserve Bank of Richmond *Quarterly Review*, 87, p. 1-25.

[6] Ericson, R. and A. Pakes (1995) “Markov-Perfect Industry Dynamics: A Framework for Empirical Work”, *Review of Economic Studies*, 62, p. 53-82.

[7] Gowrisankaran, G. (1999) “A Dynamic Model of Endogenous Horizontal Mergers”, *RAND Journal of Economics,* 30, p. 56-83.

[8] Hopenhayn, H. (1992) “Entry, Exit, and Firm Dynamics in Long-Run Equilibrium”, *Econometrica*, 60, p. 1127-50.

[9] Janicki, H. and E. Prescott (2006) “Changes in the Size Distribution of U.S. Banks: 1960-2005”, Federal Reserve Bank of Richmond *Quarterly Review*, 92, p. 291-316.

[10] Kashyap A. and J. Stein (2000) “What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?” *American Economic Review*, vol. 90(3), p. 407-428.

[11] Martinez-Miera, D. and R. Repullo (2008) “Does Competition Reduce the Risk of Bank Failure?”, CEMFI Working Paper No. 0801.

[12] Mehra, R. (2003) “The Equity Premium: Why is it a Puzzle?,”NBER Working Papers 9512, National Bureau of Economic Research.

[13] Williamson, S. (1987) “Financial Intermediation, Business Failures, and Real Business Cycles”, *Journal of Political Economy*, 95, p. 1196-1216.