SEARCH WITH MULTILATERAL BARGAINING

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Abstract. Wage determination plays a key role in labor market search models and recent evidence suggests workers' and firms' alternative employment opportunities matter. This paper develops the first search model that, to the best of my knowledge, incorporates multilateral bargaining between multiple firms and multiple workers. Despite superficially differing from the search literature in many dimensions, the framework developed nests much of it as special cases. The question of when decentralized markets deliver efficient levels of unemployment and vacancy creation is then reexamined. With multilateral bargaining too few vacancies are created unless firms have all the bargaining power (can make take-it-or-leave-it offers). However, vacancy creation is nonetheless approximately efficient in local labor markets where people are suitable for many positions and positions are suitable for many people. To the extent that this describes unskilled labor markets, any intervention to encourage vacancy creation might be better targeted elsewhere.

Key Words: Search, matching, unemployment, multilateral bargaining, outside option, inefficiency, networks.

1. Introduction

Wage determination plays a key role in search models and recent evidence suggests that workers and firms alternative employment opportunities matter. Nonetheless, most of the search literature assumes bilateral bargaining between a worker and a firm, non-negotiable posted wages or perfect competition between identical vacancies and unemployed people. This paper is the first, to the best of my knowledge, to incorporate general multilateral bargaining. Further, the generality of the framework is such that it is able to nest much of the search literature. With multilateral bargaining between heterogenous workers and firms it is shown that too few vacancies are created in equilibrium, unless firms have all the bargaining power. This result mediates

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between efficient vacancy creation in the Mortensen-Pissarides framework where vacancy creation is generically inefficient (Hosios (1990)) and models of wage posting and competitive search where vacancy creation is efficient (Shimer (1996), Moen (1997)). Like the Mortensen-Pissarides class of models vacancy creation is only efficient for a specific level of vacancy bargaining power but this level of bargaining power corresponds to firms making take-it-or-leave it offers which is similar in spirit to firms posting wages. Further analysis shows that although there is always too little vacancy creation, vacancy creation is approximately efficiency in markets where people are suitable for many positions and positions are suitable for many people. Efforts to encourage vacancy creation should then be concentrated in other markets.

Recent evidence suggests that workers and firms alternative employment opportunities matter for wage determination. Cahuc, Postel-Vinay and Robin (2006) find that competition between firms for workers is quantitatively more important than bilateral bargaining power for raising workers wages above their reservation level. Hall and Krueger (2009) find direct survey evidence for workers' alternative employment opportunities featuring in their wage negotiations. However, no search models have, as of yet, fully incorporated multilateral bargaining. In many models there is bilateral bargaining[1] and in competitive search models non-negotiable wages are posted.[2] Whilst in some search models there are intermediate forms of wage determination, bargaining between many parties on one side of the market and one party on the other side or prefect competition within local labor markets,[3] general multilateral bargaining has not been modeled. In this paper I allow for multilateral bargaining between many firms and many workers with any arbitrary gains from trade between worker-firm pairs. Alternative employment opportunities bound parties' payoffs from below, establishing 'outside options' and the remaining surplus is bargained over bilaterally.

With multilateral bargaining over wages the efficiency of unemployment is reexamined. Given the frictions present in the labor market is vacancy creation efficient? It is shown that there is weakly too little vacancy creation unless firms have all the bargaining power (Proposition 1). This contrasts with the Hosios (1990) condition which shows (absent any wage competition) that an intermediate level of bargaining power is required for efficient vacancy creation. With

[1]Many urn and ball models, such as Butters (1977), many matching function models, as in Pissarides (1990), and some stock and flow models, such as Coles and Smith (1998) (assumption 1), have bilateral bargaining.
[2]See, for example, Shimer (1996), Moen (1997), Acemoglu and Shimer (1999) and Guerrieri (2008).
[3]In Cahuc, Postel-Vinay and Robin (2006) and Coles and Smith (1998) (assumption 2), multiple parties on one side of the market bargain with a single party on the other. In Shimer (2007) there is prefect competition within local labor market between many identical buyers and sellers.

multilateral bargaining a new vacancy must compete to 'steal' a worker from another vacancy. This competition drives the worker's wage up. The negative externalities imposed on other vacancies when a new vacancy is created are then appropriate by the workers as higher wages. This negative externality does not then distort vacancy creation decisions. In the directed search literature, unlike the multilateral bargaining framework or the standard matching framework, efficient vacancy creation arises endogenously (Shimer (1996), Moen (1997)). However, posting wages is similar to firms being able to make take-it-or-leave it offers, which would be modeled in the multilateral bargaining framework as firms having all the bargaining power. This is precisely the condition required for efficient vacancy creation with multilateral bargaining. Nonetheless, the multilateral bargaining framework is fundamentally different from wage posting. For example, Burdett and Mortensen's (1998) directed on-the-job search model has posted wages that are not renegotiation proof.[4] In the multilateral bargaining framework wages are renegotiation proof by construction.

As there is too little vacancy creation in the multilateral bargaining framework there is a role for market intervention that encourages vacancy creation. Should then this intervention be targeted at any particular type of local labor market? When unemployed people and vacancies have many potential employment opportunities and when these opportunities generate similar surpluses, vacancy creation is approximately efficient (Proposition 2). Market intervention should then be targeted at local labor markets where the surpluses generated by different matches are fairly different and unemployed people and vacancies do not have many alternative employment opportunities. If low-skill workers are highly substitutable for one another the role for labor market intervention in these markets is then likely to be limited. In contrast if high-skill workers are less substitutable for one another there is a greater potential benefit from labor market intervention in these markets.

Although superficially the multilateral bargaining framework is very different from much of the literature, there is multilateral bargaining, agents are non-zero measure and any (heterogenous) gains from trade are permissable, it can be shown that the multilateral bargaining framework nests much of the literature. Microfounded models of search typically work in similar ways. They restrict the matches that are possible and do so in such a way as to ensure wage determination

[4]Coles (2001) discusses this problem with Burdett and Mortensen (1998) and shows how reputation can be used to help firms commit to their posted wages. Shimer (2006) develops a model of bilateral bargaining rather than wage posting which also overcomes this inconsistency.

is relatively straightforward.[5] This paper unites these models by permitting any restrictions on which people can be employed by which jobs, modeling this as a network and allowing for a generalized form of multilateral bargaining. For example in Shimer's (2007) mismatch model, and in contrast with the multilateral bargaining framework, there are local labor markets, a continuum of identical workers and firms, perfect competition to determine wages within local labor markets and the model is set in continuous time. Nonetheless, in the limit, the multilateral bargaining framework is able to replicate the mismatch model as a special case (See Section 3).

To further examine the likelihood that vacancy creation will be efficient two extensions are considered. When the exact structure of the local labor market is uncertain and unemployed workers must pay to participate there need not exist *any* level of vacancy bargaining power consistent with efficient entry (Proposition 3). Similarly, when a fully dynamic version of the model is considered there need not exist any level of vacancy bargaining power consistent with efficient vacancy creation. This results is similar in spirit to Guerrieri (2008), but driven by a very different force. In the multilateral bargaining framework the result is driven by non-zero measure vacancies making dynamically strategic decisions. In Guerrieri (2008) agents are zero measure. Indeed, restrictions on vacancies information sets are found that make vacancies behave non-strategically and these restrictions are necessary and sufficient for restoring the efficient entry requirements of the one period model, despite the presence of dynamic and static externalities (Proposition 4).

The paper proceeds as follows. The model is introduced in Section 2 before its generality and ability to nest other models is discussed in Section 3. Section 4 considers the efficiency of entry decisions in decentralized markets before the implication for policy are addressed in Section 5. Section 6 extends the model and Section 7 concludes.

2. Networked Market Model

2.1. **Players and timing.** For convenience a table listing the notation used is given in Appendix A. There are a set of $\mathbf{N}$ people, indexed i, with finite cardinality ($|\mathbf{N}| < \infty$). There are a

[5]In models of mismatch unfilled vacancies in one local labor market can coexist with unemployment in another because matches across these markets are not permitted (Lagos (2000), Shimer, (2007)). In stock and flow models only some people and jobs are compatible leading the flow of newly separated positions and workers to match to the stock of unemployed people and positions respectively (Coles (1994), Coles and Smith (1998), Ebrahimy and Shimer (2008)). In urn and ball models people can only apply to a limited number of positions and fail to coordinate on which of these positions to apply to (Butters (1977), Hall (1979), Pissarides (1979)). In social network models workers are not aware of all available jobs and information is transmitted over social networks (Calvo Amengol and Jackson (2004), Calvo Amengol and Zenou (2005), Calvo Amengol and Jackson (2007)).

set of possible job positions $\mathbf{P}_t$ in period t, indexed j, also with finite cardinality ($|\mathbf{P}_t| < \infty$, $\forall t$). At the beginning of period t a subset of people are employed $\mathbf{E}_{t-1} \subseteq \mathbf{N}$ and the rest are unemployed $\mathbf{U}_{t-1}$. Similarly, a subset of positions are filled $\mathbf{J}_{t-1} \subseteq \mathbf{P}_t$. A network draw L_t is then made identifying which people can be employed by which positions and the surpluses these matches would generate. For example a link $l_{i,j,t} \in L_t$ indicates that should position j employ worker i in period t a joint surplus of $l_{i,j,t}$ will be generated. Encoded into the network draw is the possible separation of matched employed people to their jobs. Whilst this separation, and continued matches, are publicly observable the rest of the network draw is not currently observable. Knowing the probability distribution that governed the network draw ($\mathbb{L}_t$) and the realized separations, potential vacancies V_t' must decide whether to enter the market or not. Once entry decisions have been made the network is common knowledge for all market participants. Matches and payoffs are then determined.

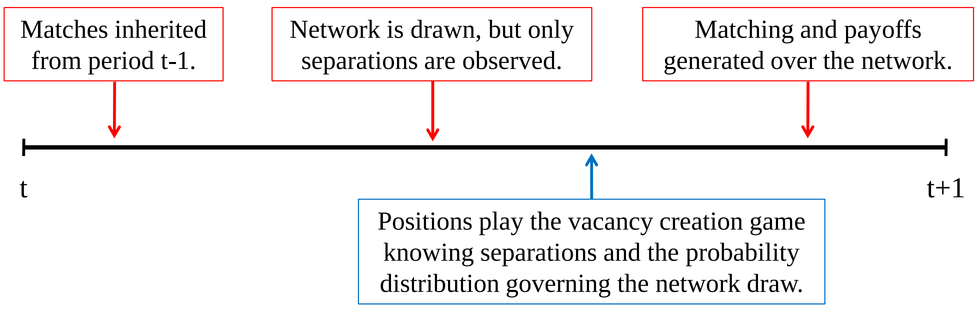
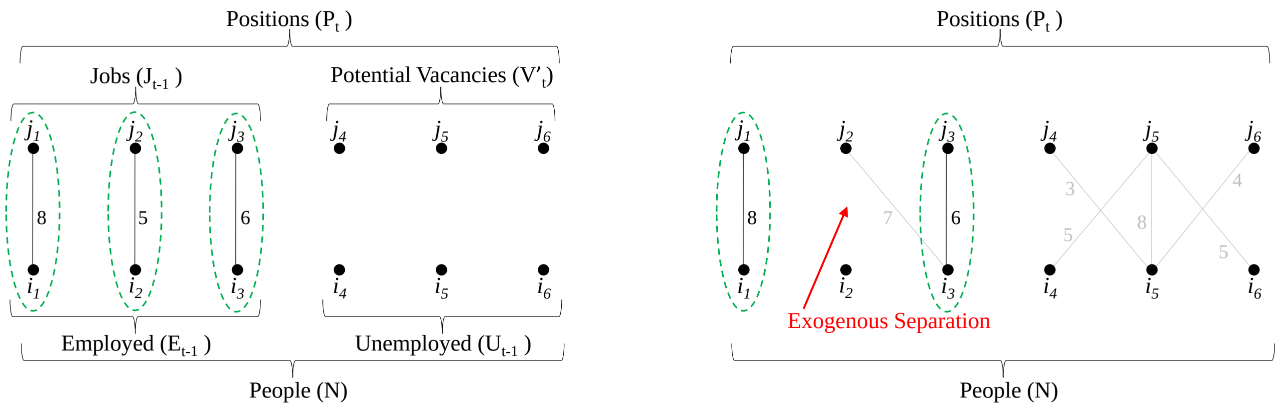


FIGURE 1. Timing

2.2. **Network Draw.** Every person can potentially be employed by only a subset of positions. Equivalently, each position can only potentially employ a subset of people. Only positions that are in the market can employ anyone. Position must be in the market to employ anyone. The people a position might employ are represented by links in a bipartite network (such that people are only linked to positions and positions are only linked to people).

The period t network $L_t = \{l_{i,j,t}\}$ is drawn from the set of all possible bi-partite networks and governed by the probability distribution $\mathbb{L}_t$. A link $l_{i,j,t}$ represents that, if position j is in the market, they can employ person i generating a joint surplus $l_{i,j,t} \in (0, \bar{l})$.[6] Initially the period t network draw is unobservable with the exception of separations and continued employment relationships.

[6]It is assumed that $\bar{l} < \infty$ such that the value of any possible employment relationship is bounded from above. This is a non-restrictive technical assumption. Further, as a link $l_{i,j,t} \leq 0$ could be removed from the network without loss of generality the assumption that $l_{i,j,t} > 0$ is also non-restrictive.



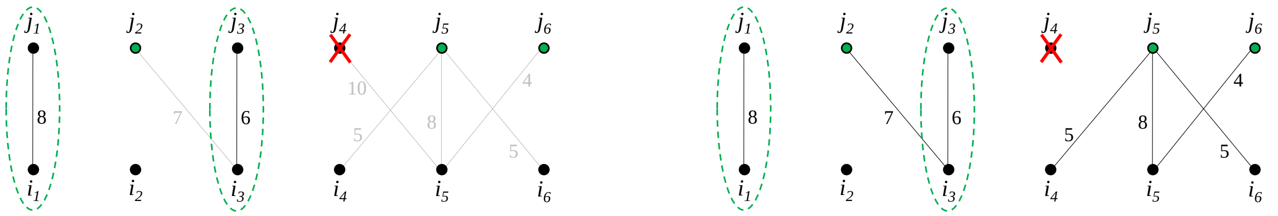
(A) Existing matches (B) Separation

FIGURE 2. Matches and associated bargained outcomes are inherited from the previous period. The network draw is made, but unobserved other than exogenous separation which is common knowledge.

Example 1. *Figure 2a shows the existing matches in the market in period $t-1$. The network draw is then made. This determines whether any current matches are separated. In figure 2b i_2 and j_2 are separated from each other whilst the other matches remain. The rest of the network draw, represented by the greyed out links, is unobserved before vacancy creation decisions have been made.*

2.3. **Vacancy Creation.**

Although a position must create a vacancy to learn the network draw the probability distribution governing the network draw $\mathbb{L}_t$ is common knowledge. The set of possible vacancies $\mathbf{V}'_t$, including those just separated from an employed worker, decide whether to (re)enter the market at cost c based on $\mathbb{L}_t$ and observed separations. These decisions are modeled as a simultaneous move game.[7]



(A) Vacancy creation (B) Effective networked market revealed

FIGURE 3. Vacancy creation decisions are made. Those vacancies that are created enter the market and learn the network draw.

[7]Filled vacancies enter the labor market at no cost. However, it is straightforward to restrict the network draw such that they are only ever linked to their current worker.

Example 2. *Continuing form Example 1 vacancy creation decisions are made based on the probability distribution governing the network draw and observed separations and continued matches. Newly separated vacancies can also reenter the market. Figure 3a shows that all potential vacancies, included the newly separated vacancy j_2 enter the market at a cost c, except for j_4. Having entered the market, the effective network draw (L_t^E) connecting market participants is revealed as shown in Figure 3b.*

2.4. **Matches and wages.** Employment matches and wages are determined over the effective network L_t^E. Formally the match $\mu(L_t^E)$ is a function from the set of all positions $\mathbf{P}_t$ and people $\mathbf{N}$ into itself, $\mu(L_t^E) : \mathbf{P}_t \cup \mathbf{N} \to \mathbf{P}_t \cup \mathbf{N}$ satisfying the following properties: $i \in \mathbf{N}$ is matched to a position or no one represented by them being matched to themselves ($\mu_t(i) \in \{\mathbf{P}_t \cup i\}$), $j \in \mathbf{P}_t$ is matched to a person or themselves ($\mu_t(j) \in \{\mathbf{N} \cup j\}$), i is matched to j ($\mu_t(i) = j$) if and only if j is matched to i ($\mu_t(j) = i$), and if i and j are matched ($\mu_t(i) = j$) then they must be linked in the effective network ($l_{i,j,t} \in L_t^E$). Where obvious, notation will be abused and the dependence of the match on the effective network L_t^E suppressed.

These matches are determined by multilateral bargaining, with the exception of existing employment contracts that are stable to deviations. The results of this subsection are replicated from Elliott (2009). To identify who is employed where it is (only) required that no person-position pair could do better by instead matching with each other. This relatively minimal requirement identifies a generically unique match μ_t^*, which maximizes the total gains from trade that can be generated over L_t^E.[8]

$$\mu_t^* \equiv \underset{\mu(L_t^E) \in M(L_t^E)}{\text{argmax}} \left\{ \sum_{i \in \mathbf{N}} l^E_{i\mu(i)} \right\},$$

where $M(L_t^E)$ is the set of all possible matches on the effective network L_t^E which connects market participants.

The network provides alternative employment opportunities for people and created positions. These alternative opportunities generate interrelated and endogenous outside options that guarantee a minimum payoff for each party. These 'outside options' follow directly from the pairwise

[8]Although this is a cooperative approach to the bargaining problem, two non-cooperative models that generate consistent results, but in more restrictive environments, are Cahuc, Postel-Vinay and Robin (2006) and Corominas-Bosch (2004). Echenique, Katz and Yariv (2009) conducted an experiment that analyzed the matches reached in a decentralized market by placing very little structure on interactions between the participants of the experiment. Pairwise stability was found to be important in determining the outcomes reached in the decentralized markets, although the non-transferable utility case was considered so there was no role for wages.

stability requirement. Shapley and Shubik (1972) show that there will be no pairwise profitable deviations if and only if there are no profitable coalitional deviation. Thus by requiring no pairwise deviation there will be no profitable coalitional deviations and the outcomes considered are in the core. Shapley and Shubik (1972) also show that there are a lattice of pairwise stable (core) payoffs. A subset of these payoffs, following in spirit much of the search literature, are considered. It is assumed that all matched parties receive at least their minimum pairwise stable payoff and that remaining gains from trade are bargained over and split according to positions' relative bargaining power β.[9]

For example, if person i is matched to position j ($\mu_t(i)^* = j$) in period t then

$$\pi_{i,t} \quad = \quad \underline{\pi}_{i,t}(L_t^E) + (1-\beta)[l_{i,j,t} - \underline{\pi}_{i,t}(L_t^E) - \underline{\pi}_{j,t}(L_t^E)]$$

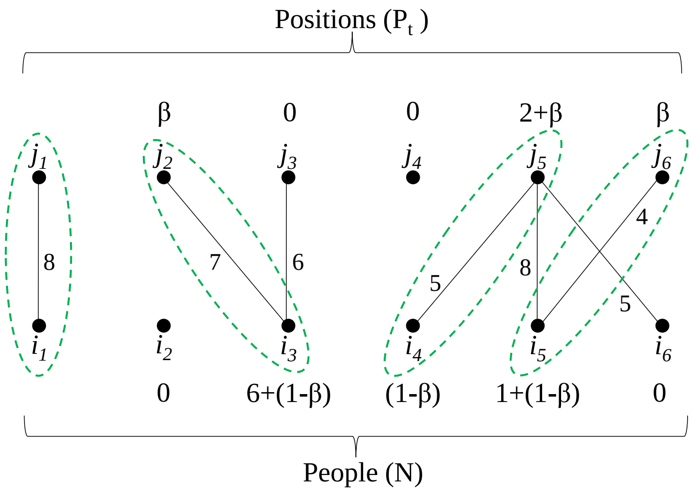
where $\underline{\pi}_{i,t}(L_t^E)$ is i's minimum possible payoff (outside option) in period t and $\underline{\pi}_{j,t}(L_t^E)$ is j's outside option in period t. It is instructive to consider the matches and payoffs proposed in the context of the following example:

Example 3. *Continuing from Example 2, the following matches and payoffs are implemented, where $\beta \in [0,1]$ is vacancies relative bargaining power.*

Position j_1 and person i_1 honor their existing contract as neither has a binding outside option. In contrast j_3 and i_3 renegotiate employment terms as i_3 has the opportunity to be employed by j_2 at a higher wage than they currently receive. Thus i_3 engages in on the job bargaining and ultimately switches job to match with position j_2.

Matches are determined by the set of people-position pairs that generate the highest possible total surplus on the effective network. The dotted ovals in Figure 4a show the matches. To see how wages are determined the two effective network components engaging in multilateral bargaining can be considered separately. Person i_3 is linked to both j_2 and j_3. As i_3 can be employed by only one position, j_2 and j_3 must 'compete' to employ them. Were they to receive a wage of less than 6 from j_2 they could remain employed at j_3. This leaves remaining gains from trade generated

[9]The network is able to provide outside options despite the Diamond paradox. Diamond (1971) found that when workers sequentially search for vacancies, the gains from trade are homogenous and vacancies have all the bargaining power workers are able to extract no value from their different options even for discount factors arbitrarily close to one. An important difference between this set up and that of Diamond's is the ability of workers to simultaneously consider matching to multiple vacancies.



(A) Matches and payoffs

FIGURE 4. Matches and payoffs are determined over the network. Existing employment contracts are honored unless deviations are profitable.

by i_3 and j_2 of 1. These are split according to the bargaining power parameter β. Determining wages for the other matched people is more complicated, but the same logic can be applied.[10]

Consider the minimum possible payoff that the other people can receive- their outside options. Person i_4 has only one link which they are matched over and so they have no outside option and can receive a minimum payoff of zero. Position j_5 is matched to i_4 and if i_4 receives zero they will receive 3. Consider now i_5. They are matched to j_6, but are also linked to j_5. As j_5 is receiving a payoff of 3, i_5 must receive a minimum payoff of 1 to accept employment with j_6. As i_6 is unmatched they receive a minimum payoff of zero. The only person to receive an outside option is i_5 and their outside option has value 1. Using a similar logic it can be shown that the only position to receive an outside option is j_5 and their outside option is 2. These outside options are allocated to i_5 and j_5 and then remaining gains from trade between matched pairs split according to the bargaining power parameter β.

Appendix C identifies an algorithm that can be used to find which links are traded over and which links are used to establish each party's outside option in any bi-partite network. Each party uses at most one link to generate their 'outside option'. If link l_{ij} is used by i to generate their outside option, $\nu(i) = j$ is i's outside trade partner. A party always has a profitable deviation trading with their outside trade partner if they receive a payoff below their outside

[10]The discussion below implicitly uses the lattice structure of core payoffs: There are extreme points of the core in which all people (positions) receive their highest possible payoff whilst positions (people) receive their lowest possible payoffs.

option payoff.[11] Further, a parties' outside trade partner is who they would trade with were their trade partner removed from the network.[12]

A person i's *outside option chain* starts with their link to their outside trade partner ($\nu(i)$), then follows the link from $\nu(i)$ to $\nu(i)$'s trade partner, then follows the link from $\nu(i)$'s trade partner to their outside trade partner and so on alternating between outside option links and trade links. It ends when the next link in the sequence is not present. Denote the set of outside option links in i's outside option chain by $L_i^{E,oo}$ and the set of traded over links $L^i_{E,tr}$.[13]

Lemma 1 replicates Proposition 1 in Elliott (2009). It formalizes how person i's period t payoff can be found from a decomposed network.[14]

Lemma 1. *Person i's outside option in period t is*

$$\underline{\pi}_{i,t} = \sum_{l \in L_i^{E,oo}} l - \sum_{l \in L_i^{E,tr}} l$$

The proof of Lemma 1 uses the result that all parties maximum core payoffs $\overline{\pi}$ are equal to their Vickrey payoff- their marginal contribution to the grand coalition (See Demange (1982) and Leonard (1983)). The outside option links in i's outside option chain ($L_i^{E,oo}$) correspond to those links that become traded over when their trade partner $\mu(i)^*$ is removed and the new match that maximizes the gains from trade is implemented. Those links traded over in i's outside option chain $L_i^{E,tr}$ correspond to those links no longer traded over once $\mu(i)^*$ is removed. The difference in the values of these links, $l_{i\mu(i)^*} + \sum_{l \in L_i^{E,tr}} l - \sum_{l \in L_i^{E,oo}} l$, is $\mu(i)^*$'s marginal contribution to the grand coalition and thus their maximum possible core payoff. Person i's outside option (minimum core payoff) is then the surplus remaining from trade with $\mu(i)^*$ after $\mu(i)^*$ has extracted their maximum core payoff. This follows from the lattice structure of the core.

From Lemma 1 it is easy to identify exactly how each link in a network affects all parties' outside options and therefore all parties' payoffs. These affects are summarized in table 1. Consider a fixed effective network L_t^E and suppose that on this network person i is matched to position j ($\mu(i)^* = j$). The payoff i receives is affected by marginal changes in the gains from trade

[11]See Elliott (2009).
[12]See Appendix C.
[13]It is shown in Elliott (2009) that outside option chains never cycle. This is because the gains from trade must be higher when all parties trade over their trade links than when all parties trade over their outside option links.
[14]The outside options of all other people and all vacancies can be found in an equivalent way.

between different people and positions in the effective network (the value of links in the effective network), in the following way:

link	$\Delta\pi_i$	Sign
$l_{i,j}$	$(1-\beta)$	+'ve
i's outside option link	β	+'ve
j's outside trade link	$-(1-\beta)$	−'ve
other oo links in i and not j's oo chain	β	+'ve
other oo links in j and not i's oo chain	$-(1-\beta)$	−'ve
oo link in both i and j's oo chains	$2\beta-1$	+'ve iff $\beta \geq \frac{1}{2}$
traded link in i and not j's oo chain	$-\beta$	−'ve
traded link in j and not i's oo chain	$(1-\beta)$	+'ve
traded link in both i and j's oo chains	$1-2\beta$	+'ve iff $\beta \leq \frac{1}{2}$

Table 1. The marginal change in i's payoff following changes in value of the links in the network.

3. Generality of the Multilateral Bargaining Framework

Many different labor market frictions can be captured within the multilateral bargaining framework by placing different restrictions on the network draw through $\mathbb{L}_t$. In this way a useful organizing framework for different search models is provided. Indeed this section will show how many different search model can be captured within the multilateral bargaining framework. First it will be shown in detail how Shimer's (2007) mismatch model can be nested. Three further classes of models, urn and ball models, stock and flow models and matching functions models are then considered. A model from each class is selected and approximately replicated. Some further models are then discussed briefly before it is shown that a variety of labor market features can be also modeled by placing different restrictions on the network draw.

Whilst the multilateral bargaining framework is able to accurately capture many other models, and in particular the frictions they are modeling, there are also ways in which it naturally differs from much of the search literature. The multilateral bargaining framework most naturally has a finite number of people and positions rather than continuums. These differences provide a useful comparison with other models in section 4.

3.1. **Labor mobility - Mismatch.** The Shimer (2007) mismatch model is superficially pretty different from the multilateral bargaining framework. In the mismatch model separate local labor markets are modeled as opposed to a single networked labor market, outcomes in each such labor market are competitive rather than determined by a multilateral bargaining process and the mismatch model is set in continuous time instead of discrete time. Nonetheless it will be shown that, in the limit, the mismatch model is a special case of the multilateral bargaining model.

The mismatch model is driven by the friction that people have limited mobility across labor markets. Some labor markets can then have an excess of vacancies whilst others have unemployed workers. This friction is modeled by random stochastic reallocations of workers and jobs across local labor markets. There are $\mathbf{\Upsilon}$ distinct labor markets and $\mathbf{N}$ people. Both $\mathbf{\Upsilon}$ and $\mathbf{N}$ are sent to infinity and at the same rate. Within each labor markets there is perfect competition. When there are more vacancies than workers all workers are employed and they extract all the gains from trade. When there are more workers than vacancies all vacancies are filled and they extract all the gains from trade. Workers receive only their reservation wage. When there are equal numbers of vacancies and workers it is assumed that all workers and vacancies are matched but that workers still receive only their reservation wage.

Workers move between local labor markets exogenously according to a Poisson process with arrival rate q. When this shock hits a worker is equally likely to move to every local labor market. This Poisson process induces an approximate Poisson distribution of unemployed people across labor markets at every instant in time, with parameter $\frac{|\mathbf{N}|}{|\mathbf{\Upsilon}|}$ (the mean number of people per labor market). Thus the fraction of markets with k_i workers is given by $\frac{\frac{|\mathbf{N}|}{|\mathbf{\Upsilon}|}^{k_i} e^{\frac{|\mathbf{N}|}{|\mathbf{\Upsilon}|}}}{k_i!}$. An independent exogenous Poisson process with arrival rate ρ governs the destruction of positions. At any point a firm can choose to create a new job by paying $c > 0$. When a new job is created it is assigned randomly, with equal probability, to all markets. New jobs are then created whenever it is profitable for them to be created such that a zero profit condition is maintained.

Job creation decisions reflect the probability that the job will be created in a market with unemployed people. Entry into any other market would yields no return. The job birth-death process then induces an approximate Poisson distribution of jobs (including vacancies and filled positions) across labor markets at every instant in time, with parameter $\frac{|\mathbf{J}|+|\mathbf{V}|}{|\mathbf{\Upsilon}|}$ (the mean number of jobs per labor market) and the fraction of markets with k_j jobs is $\frac{\frac{|\mathbf{J}|+|\mathbf{V}|}{|\mathbf{\Upsilon}|}^{k_j} e^{\frac{|\mathbf{J}|+|\mathbf{V}|}{|\mathbf{\Upsilon}|}}}{k_j!}$.

As the Poisson distributions of workers and jobs across markets are independent the fraction of markets with k_i workers and k_j jobs is: $\frac{\frac{|\mathbf{J}|+|\mathbf{V}|}{|\boldsymbol{\Upsilon}|}^{k_j} \frac{|\mathbf{N}|}{|\boldsymbol{\Upsilon}|}^{k_i} e^{\frac{|\mathbf{J}|+|\mathbf{V}|+|\mathbf{N}|}{|\boldsymbol{\Upsilon}|}}}{k_j! k_i!}$. The distributions governing the number of unemployed people and unfilled vacancies are then straightforward to calculate.[15]

To replicate the mismatch model in the multilateral bargaining framework it is necessary to replicate the presence of local labor markets, the perfectly competitive wage determination and to ensure that the distribution of vacancies and unemployed people across labor markets follow (approximate) independent Poisson distributions. Replicating the presence of many local labor markets and the perfectly competitive wage determination is relatively straight forward. The productivity of every possible match is set to 1, $l_{ij} \in \{0, 1\}$, and the probability distribution governing labor market draws $\mathbb{L}_t$ set such that positive probability weight is placed only on networks where there are $\boldsymbol{\Upsilon}$ completely connected components. Multilateral bargaining over these completely connected components is then consistent with wage determination in the mismatch model.[16]

Lemma 2. *Suppose there are gains from trade of* 1 *between all connected workers and jobs and that within a network component all people are connected to all jobs. Under multilateral bargaining workers receive a surplus of* 1 *when there are more workers than jobs in the network component, a surplus of* 0 *when there are more jobs that workers and a surplus of* β *when there are equal numbers of workers and jobs.*

Proof. Suppose there are more workers than jobs, the case of more jobs than workers is analogous. To maximize the surplus generated all jobs must be filled. Further, each job will be linked to an unemployed worker. The surplus received by the employed worker must be 0 otherwise an unemployed worker would take their job at a lower wage. When there are equal numbers of workers and jobs, no worker or firm has an outside option: All workers (firms) can simultaneously receive a minimum surplus of 0 without any one having a profitable deviation. Thus workers receive a surplus of β. $\square$

From Lemma 2, setting vacancies bargaining power $\beta = 1$ the wages generated by the mismatch model are replicated in the multilateral bargaining framework.

[15]Lagos (2000) endogenizes mismatch in the context of the taxi cab market. Taxis decide to locate in one of two markets. There are different expected levels of profitability *per passenger* across markets. Taxis then endogenously distribute themselves to equalize profits across market leaving an excess of taxis in some locations and an excess of passengers in others.

[16]The alternating public offer bargaining game of Corominas-Bosch (2004) generates the same bargaining outcomes.

To replicate the distribution of workers and jobs across labor markets conditions further restrictions will first be placed on $\mathbb{L}_t$ and the number of workers, potential vacancies and labor markets will be sent to infinity. Let the network draw be highly persistent with only the following changes in each period: (i) each worker independently looses all their current links with probability q and is reassigned to any network component with equal probability across components; and (ii) independently each job looses all their links with probability ρ and is reassigned to any network component with equal probability across components. However, reassigned vacancies do not automatically enter the labor market. Without knowing which local labor market they have been reassigned to vacancies must decide whether to pay the vacancy creation cost and re-enter. Indeed, any vacancy not in the labor market can pay c to enter the labor market in a period and learn which local labor market they will be in. In the limit, as the number of people, number of potential vacancies and the number of local labor markets are sent to infinity at the same rate, the number of people and vacancies across labor markets will (approximately) follow independent Poisson distributions.[17]

3.2. Miscoordination - Urn and ball models.

Urn and ball models suppose that unemployed people (balls) randomly apply to (are placed in) vacancies (urns).[18] Frictions in these models arise due to a lack of coordination: Multiple unemployed people might apply to one vacancy whilst none apply to an otherwise identical vacancy.[19]

There are a set unemployed people $\mathbf{U}$ and a set of firms $\mathbf{P}$, a subset of which have vacancies $\mathbf{V}$. All vacancies and unemployed people are identical. Unemployed people randomly apply to a single vacancy with uniform probability.[20] If only one person applies to a vacancy they are hired at wage w. When more than one unemployed person applies to the same vacancy one of the applicants is selected at random and receives the wage w. The probability a vacancy receives no applications is then $\left(\frac{|\mathbf{V}|-1}{|\mathbf{V}|}\right)^{|\mathbf{N}|}$. As a vacancy is filled whenever it receives at least one application and as all vacancies are identical, the expected number of matches is given by $|\mathbf{V}|\left(1-\left(\frac{|\mathbf{V}|-1}{|\mathbf{V}|}\right)^{|\mathbf{N}|}\right)$. For a large number of vacancies this approximates a Poisson distribution.

There are multiple ways the multilateral bargaining framework can model this process. Most directly restrict $\mathbb{L}_t$ such that: (i) links matched over in the previous period are removed with

[17]In the Shimer model the homogenous match surplus generated by all matches in a given period varies over time through productivity shocks. This too can be easily incorporated into $\mathbb{L}$.

[18]For early examples of this approach see Butters (1977), Hall (1979) and Pissarides (1979).

[19]See Petrongolo and Pissarides (2001) for a review of these models.

[20]This can be motivated as the symmetric mixed strategy Nash equilibrium. All pure strategy Nash Equilibria require a high degree of coordination.

constant probability and separated parties then treated as unmatched; (ii) each unemployed person has a single link to a vacancy of value 1; and (iii) links between any unemployed person and any vacancy are equally likely. Links are then equivalent to applications in the urn and ball model: A vacancy will be filled whenever it has at least one link and the probability distribution governing the number of links each vacancy has is identical. Suppose that positions have all the bargaining power ($\beta = 1$), then workers will always also receive a constant wage from employment, their reservation wage.[21]

3.3. **Incompatibilities - Stock and flow models.** When a vacancy or unemployed person enters the job market they seek out profitable employment opportunities. If none exist they remain in the market but are unlikely to find a profitable trade partner among those they have already considered and rejected. The existing stock of vacancies (unemployed people) therefore match to the flow of unemployed people (vacancies) into the market. This dynamic is captured by stock and flow models.

In Coles and Smith (1998) only some pairs of workers and vacancies are compatible, when compatible a worker and vacancy generate a surplus of (1), newly separated workers enter the market according to a Poisson process and new vacancies are created according to a Poisson process. The compatibility and entry structure ensures that the flow of workers (vacancies) only matches to the stock of vacancies (workers). When a worker is compatible with a single vacancy they bargain over wages and agree $w = \beta \in [0, 1]$. When a worker is compatible with multiple vacancies these vacancies compete to employ the worker, driving up their share of the surplus to 1. The converse is true when a vacancy is compatible with many workers.[22]

The underlying friction in the Coles and Smith (1998) model, and other stock and flow models, is a fundamental incompatibility between certain positions and people. These compatibilities can be modeled as a network. In the multilateral bargaining framework let matched parties have links only to their trade partner and remove these links with a constant probability each period. Let all unemployed workers and vacancies have no links, with the exception of newly separated vacancies and people. Suppose that newly unemployed people have a link of value 1 to each old vacancy with constant probability and that newly separated vacancies have a link of value 1 to

[21] The construction here generates somewhat different wages from the urn ball model- unemployed people extract none of the surplus. However, the urn ball model could be captured exactly by restricting the network draw in a more abstract way: Only the links matched over, for any given network draw above, could be formed.

[22] This is one of two wage determination processes considered. Other stock and flow models include Coles (1994), Gregg and Petrongolo (1997), and Coles and Muthoo (1998). Ebrahimy and Shimer (2008) have a finite person stock and flow model.

each old unemployed person with constant probability. Finally, set the cost of vacancy creation to zero so that all vacancies are always created. In the limit, sending the length of time between periods to zero, the Coles and Smith (1998) model is then replicated.[23]

3.4. **Matching Function Models.** Consider a discrete time constant returns to scale Cobb Douglas matching function model.[24] Each period there are $m(U,V) = \alpha U^{\gamma} V^{1-\gamma}$ matches such that the probability an unemployed person is matched is $\frac{m(U,V)}{U} = \alpha \left(\frac{V}{U}\right)^{1-\gamma}$. The (expected) delay matching represents frictions in the labor market. To captured this model in the multilateral bargaining framework restrict $\mathbb{L}_t$ such that: (i) the surplus of any match is the same ($l_{ij} \in \{0,1\}$): (ii) the probability a link present in the previous period is absent in the next is a constant (exogenous separation); (iii) each node (person and position) has degree one or zero; (iv) the total number of links is $\alpha U^{\gamma} V^{1-\gamma}$; and (v) there is anonymity regarding unmatched parties' network draws.[25]

3.5. **Other models.** The multilateral bargaining framework is able to capture many further models of job search frictions. Calvo-Armengol and Zenou (2005) model job information being passed through social networks. Unemployed workers can only be employed by vacancies they have learned about and employed workers pass job information to their unemployed social contacts. Information (or the lack thereof) about job opportunities is the friction being modeled. This information can be represented directly as a network between vacancies and unemployed people who know about these vacancies (different from the social network connecting people).

Cahuc, Postel-Vinay and Robin (2006) model multilateral non-cooperative bargaining for employed workers who receive an alternative job opportunity. In the model the two firms compete via an ascending wage auction to determine who will employ the worker. The winner of the auction then bargains bilaterally with the worker, but with the auction price as the disagreement point for the worker. These outcomes are also a special case of wage determination in the multilateral bargaining framework (where network components are restricted to include (at most) two firms and single worker).[26]

[23]Adjusting the separation probability such that in expectation the same number of separations occur in fixed period of time.

[24]See Rogerson Shimer and Wright (2005) and chapter 1 of Pissarides (2000).

[25]Each person has an equivalent ex-ante position in the network to every other person and each potential job has an equivalent ex-ante position in the network to every other potential job.

[26]The non-cooperative mechanism in Cahuc, Postel-Vinay and Robin (2006) therefore provides some non-cooperative justification for cooperative wage determination employed.

In Appendix D it is shown more precisely how models of job search over social networks and Cahuc, Postel-Vinay and Robin's (2006) multilateral non-cooperative bargaining model can be captured in the multilateral bargaining framework.

3.6. **Other labor market features.** Whilst the Mortensen-Pissarides matching framework is appealing because, amongst other things, it can be easily extended to capture additional features of the labor market the multilateral bargaining framework can be easily restricted to capture additional features of the labor market. This section considers a couple of these features.

On-the-job search for people but not positions can be included by restricting the probability distribution $\mathbb{L}_t$. Vacancies that were filled in period $t-1$ can have at most a single link to the person they employ whilst employed people can have links connecting them to alternative employment opportunities. Employed people can therefore switch jobs or use these employment opportunities to affect their wages.[27] Alternatively firms could also be allowed to search for alternative workers for one period reflecting a period of probationary employment.[28]

The accumulation of job specific capital over time can be modeled by an increase in the expected value of an employment link over time, whilst the accumulation of general human capital from working can be modeled as an increase in the expected value of all links to an employed worker. Similarly, unemployed workers might loose general human capital over time.

4. Efficient Vacancy Creation

This section considers the efficiency of equilibrium vacancy creation decisions. First a simple one period model where only vacancy creation decisions are made will be considered. Even here externalities are prevalent and it will be shown that all the bargaining power must be transferred to vacancies for social and private incentives to be aligned. This condition is not satisfied endogenously.

4.1. **Efficiency Benchmark.** The multilateral bargaining framework is intended to capture a variety of frictions in the labor market. Although directly removing or limiting these frictions can improve efficiency, a social planner may not be able to control them. For example a social

[27]On-the-job search is an important part of the labor market. Fallick and Fleishman (2004) argue that 50% of vacancies are filled by job to job transitions. The survey analysis reported by Hall and Krueger (2009) suggests that for at least some job to job transitions a person's current employment position affects the wage they receive when they move job.

[28]Modeling this in the network draw might be important as it could change firms hiring behavior and in particular how long they search for a worker and how good they require a worker to be (Pries and Rogerson (2005))

planner cannot, at least instantaneously, make a good but unemployed lumberjack a good match to a computer technician vacancy. Following the search literature, a more subtle efficiency question will be posed: Subject to the exogenous frictions are agents making constrained efficient decisions - the best decisions they could make from a social perspective? If so, policy can focus on removing these frictions, to the extent that this is possible and has no role to the extent this is not possible. If not, policy may be able to improve the functioning of the labor market in other ways.

What are the best decisions agents can make from a social perspective? There are two key elements to this criterion. First, agents can only make decisions under their control. They take the frictions represented by the network draws as given. Second, agents must be able to make the required decisions. Each decision made by an agent must be measurable with respect to their information set. Thus the choices of a constrained social planner are considered where it is assumed that the social planner maximizes the expected discounted surplus but is constrained in two ways: (i) to take the network draws as given; *and* (ii) to make decisions on behalf of an agent that depend only on the information available to that agent.[29]

More specifically constrained efficient vacancy creation decisions $\mathbf{V}^*$ are given by

$$\mathbf{V}^* = \underset{\mathbf{V}}{\operatorname{argmax}}\, \mathbb{E}_{\mathbb{L}} \left[\sum_{i \in N} l_{i,\mu(i)^*} \right] - |\mathbf{V}|c$$

where the expectation is take over the network draw probability distribution, which is common knowledge, and matches $\mu(i)^*$ maximize the gains form trade generated on the formed effective network as defined in section 2.[30]

4.2. **Equilibrium Vacancy Creation.** Vacancy creation decisions impose externalities on other vacancies and people. This is illustrated in example 4.

Example 4. *Suppose that there are two villages* 1 *and* 2. *In village* 1 *there is a highly skilled baker* i_1 *but only an old bakery* j_2. *If the baker from village* 1 *is employed by the bakery in village* 1 *then a surplus of one is generated. In village* 2 *there is a low skilled baker* i_2 *but a new bakery* j_2. *If the baker from village* 2 *is employed by the bakery in village* 2 *then a surplus of one is also*

[29]This is like the social planner being able to give vacancies rules to follow, but not being able to impart additional information to vacancies. In the literature constrained social planner problems are often considered as the benchmark against which to judge decentralized markets. See for example Shimer and Acemoglu (1999) and Guerrieri (2008).

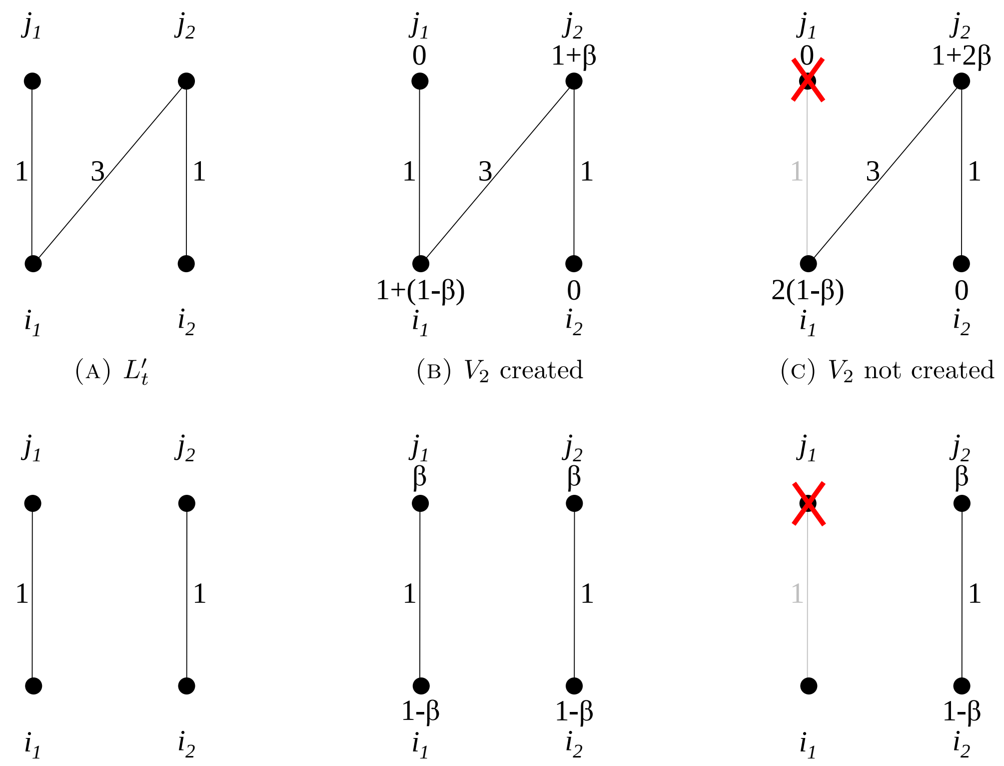
[30]Although the social planner does not choose the match, this is the match they would choose.

Party	$\mathbb{E}[Payoff\ \vert j_1\ enters]$	$\mathbb{E}[Payoff\ \vert j_1\ doesn't\ enter]$
i_1	$\frac{3}{2} - \beta$	$1 - \beta$
i_2	$\frac{1-\beta}{2}$	$\frac{1-\beta}{2}$
j_1	$\frac{\beta}{2} - c$	0
j_2	$\frac{1+2\beta}{2} - c$	$\frac{1+3\beta}{2} - c$
Total	$\frac{5}{2} - 2c$	$2 - c$

TABLE 2. The impact of V_A entering on payoffs.

generated. The low skilled baker i_2 would be unproductive in the bad bakery j_1 and generate no surplus. It is not known whether i_1 will be able to work with new oven in j_2. With probability 50% they can and a surplus of 3 would be generated. If not no surplus would be generated (See Figures 5a and 5d).

Each bakery can create a vacancy at cost c. Suppose that the new bakery j_2 always enters the market. Before j_1 learns whether i_1 can be employed by j_2, they must decide whether to also enter the market or not. The payoff j_1 will receive from entering the market will depend on whether i_1 can be employed by j_2 or not. These payoffs are shown in Figure 5 and Table 2.



(A) L'_t (B) V_2 created (C) V_2 not created

(D) L''_t (E) V_2 created (F) V_2 not created

FIGURE 5. Network Draw

It is immediate from Table 2 that the entry decision of bakery j_1 affects the bakers' expected payoffs and the payoff of the other bakery j_2. The payoff of the other bakery j_2 decreases, whilst the payoffs of bakers i_1 and i_2 weakly increase if j_1 enters. Indeed it is shown more generally in Lemma 3 that all vacancies are made weakly worse off following the entry of another vacancy but that all unemployed people are made weakly better off.

It can be seen that for $c \leq \frac{1}{2}$, j_1 will increase the expected net surplus by entering. However, j_1 will only enter the market if $c \leq \frac{\beta}{2}$. Thus j_1 will make the socially optimal entry decisions that maximize the expected net surplus when $\beta = 1$.

The external effects of vacancy creation present in example 1 can be generalized:

Lemma 3. *In the one period model:*

(1) *The presence of an additional person in the labor market weakly increases the payoffs of all vacancies and weakly decreases the payoffs of all people in that period.*
(2) *The presence of an additional position in the labor market weakly increases the payoffs of all people and weakly decreases the payoffs of all positions in that period.*

As evident from Table 1 the externalities imposed by the creation of a vacancy need not be local and do not diminish with distance. The result that vacancies face the correct incentives to enter only when $\beta = 1$ also generalizes:

Proposition 1. *In the one period model constrained efficient vacancy creation decisions, corresponding to $\mathbf{V}^*$, are a pure strategy Nash Equilibrium of the vacancy creation game if $\beta = 1$ and generically no other value of β aligns social and private incentives.*

Proposition 1 can be strengthened in the following way:

Corollary 1. *There will never be too many vacancies created in the following senses:*

(i) *In any Nash equilibrium of the vacancy creation game the ex-post removal of a vacancy will weakly reduce efficiency.*
(ii) *There will never be more vacancies created in any Nash equilibrium (in the strong set order) than are efficient.*

To gain further intuition for these results it is helpful to compare them to a standard matching framework (Hosios (1990)) and then to directed search models (Shimer (1996), Moen (1997)). In the standard matching model there are two externalities influencing vacancy creation decisions. The business stealing affect where vacancy creation hurts other positions and can lead to over-entry, and a non-appropriability of consumer surplus affect, where vacancy creation benefits people can lead to under-entry. It is these two affects that the Hosios condition trades off against each other to require an intermediate level of bargaining power for efficient vacancy creation. However, with multilateral bargaining the business stealing affect is always negated. For a new vacancy to steal a worker from another position they must compete for that worker and drive their wages to the point where the other position no longer wants to employ that worker. This competition over wages ensures that workers appropriate the loss suffered by the other position. As the new vacancy does not appropriate the other position's losses this does not factor into their vacancy creation decision. Thus the non-appropriability of consumer surplus affect always weakly dominates. Giving vacancies all the bargaining power internalizes this non-appropriability affect and $\mathbf{V}^*$ is then a Nash equilibrium of the one period vacancy creation game.[31][32]

The Hosios condition can be interpreted to mean that efficient vacancy creation cannot be expected. There is no reason to expect the bargaining power to be distributed in precisely the correct way to align private and social incentives. However, Shimer (1996), Moen (1997) and the directed search literature more generally, show that when firms can commit to posted wages that efficient vacancy creation can arise *endogenously.* Firms choose a wage to post and enter the corresponding submarket in which other firms post the same wage. All else equal such an entry decision would impose negative externalities upon the other firms in this submarket and positive externalities on the unemployed workers in this submarket. However, unemployed people are free to move between submarkets and in equilibrium will do so to maintain a constant expected payoff across these markets. This reallocation of unemployed people counteracts the externalities such that private and social incentives for vacancy creation are aligned. One interpretation of this result is that when firms can post wages the bargaining power of firms is determined endogenously and this endogenous value is precisely the value required by the Hosios condition for efficient vacancy creation.

[31]This result is consistent with Proposition 2 in Shimer (2007).

[32]Some alternative intuition for Proposition 1 can be gained from noting that the bargaining process employed over the network is incompatible with parties receiving more that their marginal contribution. Were a vacancy j able to enter a market and take more than their marginal contribution all other vacancies and unemployed people must be better off without the additional vacancy. They could therefore profit by forming a coalition that excluded j. Thus vacancies can never extract more than their marginal contribution to the grand coalition when entering and so there cannot be excess entry for any β. See also Lemma 1.

The key feature of the directed search literature is the ability of unemployed workers to choose which submarkets to search in. In contrast, in the multilateral bargaining the positions workers can be employed by are given exogenously and this captures the frictions in the labor market. Further, consistent with the Hosios condition Proposition 1 shows that when bargaining is multilateral efficient vacancy creation does not arise endogenously. However, one way in which job posting behavior could be proxied in the multilateral bargaining set up is to allow firms to make take-it-or-leave-it offers to workers. This is equivalent to transferring all the bargaining power to firms and setting $\beta = 1$. Under this restriction job creation will be efficient, but for very different reasons that in the directed search literature. Efficient job creation requires an exogenous level of bargaining power, but this level of bargaining power is consistent with firms being able to make take-it-or-leave it offers. Note that even when vacancies make take-it-or-leave-it some bargaining occurs in effect - the wage offers made by firms will take into accounts workers alternative employment opportunities.

Whilst directed search models are not typically renegotiation proof- they require a firm to be able to commit to their posted wage (Coles (2001)) - there are no such problems in the multilateral bargaining framework. By construction wages are renegotiation proof. Further, suppose that firms could make ex-ante position specific investments that increased the productivity of a position before it is filled and regardless of who fills it. Acemoglu and Shimer (1999) show that directed search models can generate efficient investment as well as job creation, whilst the standard matching framework cannot achieve efficient for any level of bargaining power. This striking result can be replicated in multilateral bargaining framework. Efficiency in investments and job creation can again be achieved when firms have all the bargaining power. It has already been shown that efficient job creation occurs when firms have all the bargaining power and this is also the distribution of bargaining power required to provide firms with the correct investment incentives.

Corollary 2. *Suppose firms can make ex-ante position specific investments in the multilateral bargaining framework. If firms have bargaining power ($\beta = 1$) constrained efficient vacancy creation and efficient specific investments are a pure strategy Nash Equilibrium of the vacancy creation game.*

5. Policy

Proposition 1 shows that efficient vacancy creation cannot be expected in decentralized markets unless vacancies have all the bargaining power. Should the government then seek to intervene indiscriminately in labor markets to encourage vacancy creation or are the returns to intervening in some labor markets likely to be higher than in others- should intervention be targeted?

This section shows that in local labor market where potential vacancies and unemployed people have many employment opportunities, such that the network draw is dense, and these potential matches are roughly as productive as each other, that decentralized vacancy creation is approximately efficient. This suggests that intervention should be targeted in other local labor markets.

A network draw is *dense*, when for every possible realized network and every possible set of vacancies $\mathbf{V}$ that could be created:

(i) $|\mathbf{V}| = |\mathbf{U}|$ and everyone can be matched; or
(ii) $|\mathbf{U}| > |\mathbf{V}|$, every vacancy can be matched and every vacancy could still be matched if any one unemployed person were removed; or
(iii) $|\mathbf{V}| > |\mathbf{U}|$, every person can be matched and every person could still be matched if any one vacancy was removed,

Hall's marriage theorem and the network decomposition from section 2 can be used to further examine what it means for a network to be dense. Hall's marriage theorem identifies a necessary and sufficient condition for all unemployed people to be matched. Let $\eta(i, L) \subseteq \mathbf{V}$ be the set all vacancies an unemployed person is connected to in the network L. Hall's marriage theorem then states that all unemployed people $\mathbf{U}$ can be simultaneously matched to vacancies in $\mathbf{V}$ if and only if for any subset of unemployed people, $\mathbf{\Phi_U} \subseteq \mathbf{U}$, $|\eta(\mathbf{\Phi_U}, L)| \geq |\mathbf{\Phi_U}|$. Applying Hall's marriage theorem to both sides of the market and considering the network decomposition, dense networks are characterized in Lemma 4.

Lemma 4. *The following three statements are equivalent:*

(i) *The network draw is dense.*
(ii) *For every possible realized network and every possible set of vacancies V that could be created,*

(a) *if* $|\mathbf{V}| = |\mathbf{U}|$ *then for every subset of* $\mathbf{V}$ *the neighbors of* $\mathbf{V}$ *are such that* $|\eta(\mathbf{V})| \geq |\mathbf{V}|$, *or equivalently for every* $\mathbf{\Phi_U} \subseteq \mathbf{U}$ *the neighbors of* $\mathbf{\Phi_U}$ *are such that* $|\eta(\mathbf{\Phi_U})| \geq |\mathbf{\Phi_U}|$,

(b) *if* $|\mathbf{U}| > |\mathbf{V}|$ *then for every* $\mathbf{\Phi_V} \subseteq \mathbf{V}$, $|\eta(\mathbf{\Phi_V})| \geq |\mathbf{V}| + 1$, *and*

(c) *if* $|\mathbf{V}| > |\mathbf{U}|$ *then for every* $\mathbf{\Phi_U} \subseteq \mathbf{U}$, $|\eta(\mathbf{\Phi_U})| \geq |\mathbf{\Phi_U}| + 1$.

(iii) *For every possible realized network and every possible set of vacancies* $\mathbf{V}$ *that could be created, resetting the value of all links to unity, the decomposed network is a also a decomposed network for the complete network when all links have value unity.*

Lemma 4 helps to identify what it means for a network to be dense. Part i is proved by applying Hall's marriage theorem and part (ii) by utilizing properties of the decomposed network.

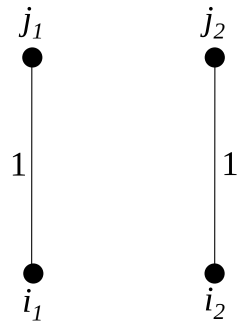
Vacancy creation is *approximately efficient* when the set of vacancies created is efficient ($\mathbf{V} = \mathbf{V}^*$) or there are $|\mathbf{V}^* - 1|$ vacancies created and all vacancies that are created would have been created under efficient vacancy creation: $\mathbf{V} \subset \mathbf{V}^*$.

Proposition 2. *If the network draw is dense and the value of all links is homogeneous then all pure strategy Nash equilibria of the vacancy creation game are approximately efficient for all levels of vacancy bargaining power* β.

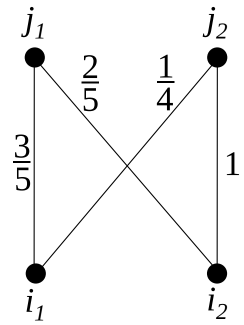
Proposition 2 is not tight in the sense that the conditions of a dense network draw and homogeneous gains from trade are sufficient for approximately efficient vacancy creation but are not necessary. Indeed Proposition 2 results from a discontinuity in vacancies' payoffs as the number of created vacancies changes. This discontinuity means that (generically) vacancies will receive strictly positive profits. Further, this means that Proposition 2 is robust to small amount of heterogeneity and to a non-dense network draw as long as the drawn networks are dense with sufficiently high probability. Nonetheless, the following example shows that the interaction of the density and homogeneous link values conditions is important for guaranteeing there exists an equilibrium with approximately efficient vacancy creation.

Example 5. *Suppose that there are two unemployed people* i_1, i_2 *and two potential vacancies* j_1, j_2 *and that the network draw is deterministic with homogeneous potential gains from trade as shown in Figure 6a. When the cost of link formation is* $c = \frac{1}{2}$, *then it is efficient for both* j_1 *and* j_2 *to enter. However, if* $\beta < \frac{1}{2}$ *neither* j_1 *nor* j_2 *will enter.*

Suppose now that there is a dense deterministic network draw but without homogenous gains from trade as shown in Figure 6b. Again, when the cost of link formation is $c = \frac{1}{2}$, *then it is efficient for both* j_1 *and* j_2 *to enter. However, if* $\beta < \frac{1}{3}$, *neither* j_1 *nor* j_2 *will enter.*



(A) Homogenous gains from trade



(B) Dense network draw

FIGURE 6. A homogenous gains from trade network with inefficient vacancy creation and a dense network with inefficient vacancy creation.

Proposition 2 and Example 5 suggest that labor market interventions to increase vacancy creation should be targeted at labor markets with heterogeneous gains from trade and where possible matches are limited (networks are relatively sparse). It more seems likely that local labor markets with these characteristics will be for highly skilled workers than for low skilled workers. Proposition 2 and Example 5 then suggest that labor market interventions encouraging job creation should be targeted at labor markets for skilled workers rather than labor markets for unskilled workers.

Less surprisingly intervention that increases the 'thickness' of labor markets and directly remove the frictions associated with the network draw is also beneficial.

Corollary 3. *Consider a probability distribution over networks $\mathbb{L}'$ such that for any network draw the set of links present is a superset of the links drawn under probability distribution $\mathbb{L}$. Then, for a given level of vacancy creation, the surplus generated by the network $\mathbb{L}'$ is weakly greater than the surplus generated by the network $\mathbb{L}$.*

6. EXTENDING THE MODEL

Whilst the one period model provides a useful benchmark, interesting policy insights and helped to relate efficiency in directed search models and the Hosios condition to one another, two further impediments to efficient vacancy creation are now considered. It is shown that there may not exist any bargaining power (β) that aligns social and private incentives when either people have to pay to participate in the labor market or dynamic decisions are modeled.

6.1. **Search costs for people.** Suppose that people have to pay some non-trivial cost $c_{\mathbf{N}}$ to search and enter the labor market.[33] The entry game now becomes a simultaneous move game of both vacancy creation and people entering the labor market. This makes an important difference to the efficiency of entry decisions.

Proposition 3. *If people and vacancies simultaneously decide whether to enter the labor market in a one period model at costs $c_{\mathbf{N}}$ and $c_{\mathbf{P}}$ respectively, then*

(i) *when the network draw is deterministic (such that $\mathbb{L}_t$ is places all weight on a single draw)*
  (a) *efficient entry decisions are a Nash equilibrium if $\beta = \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$, and*
  (b) *$\forall\beta \neq \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$ there exist networks in which efficient entry decisions are not a Nash equilibrium.*

(ii) *If there are multiple possible network draws then there may not exist a β for which efficient entry decisions are a Nash equilibrium.*

Proposition 3 identifies a key difference between the ability of the market to achieve efficient vacancy creation, even in the one period model, in the presence of uncertainty. Whenever there is uncertainty about the labor market prior to entry decisions efficient entry may not be possible for any level of β. Further Proposition 3 shows that the policy implications of Section 5 should only be applied either in tandem with a policy that removes people's participation costs, or where these participation decisions are trivial.

6.2. **Dynamic Efficiency.** Moving to a dynamic setting, constrained efficient vacancy creation requires that each vacancy chose a (contingent) sequence of vacancy creation decisions, measurable with respect to their information set in each period, that maximizes their expected discounted contributions to total surplus net of entry costs ($\{\mathbf{V}_\tau^*\}_{\tau=0}^{\infty}$).[34] The dynamic constrained efficient vacancy creation decisions need not correspond to the sequence of static constrained efficient vacancy creation decisions. A vacancy's entry decision today may yield a profitable

[33]Pries and Rogerson (2009) argue that participation costs vary stochastically and are important. Many people face non-trivial participation decisions.

[34]As there are a finite number of vacancies that can enter the labor market each period the social planner's choice set is compact (the product of compact sets is compact). Further as there is discounting $\delta < 1$, a finite number of people $|N|$ and the surplus generated by any match is bounded by $\bar{l}$, the total surplus generated by any match in any period is bounded by $N\bar{l}$ and $\delta^\tau \sum_{i=1}^{\mathbf{N}} l_{i,\mu_\tau(i)} \to 0$ as $\tau \to \infty$ for any possible network draw. It can then be shown that the social planner's payoff function is continuous in the product set topology. Thus there exists a solution to the social planner' problem and the efficiency benchmark is well defined.

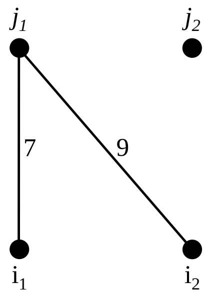
Enters	*Total Surplus*	*j_1's Payoff*	*j_2's Payoff*
j_1	$9+9-15=3$	$7+2\beta+9\beta-15=11\beta-8$	0
j_2	$20-15=5$	0	$20\beta-15$
j_1, j_2	$9+20-15-15=-1$	$7+2\beta-15$	$11\beta-15$
$\emptyset$	0	0	0

TABLE 3. Entry contingent payoffs and surpluses.

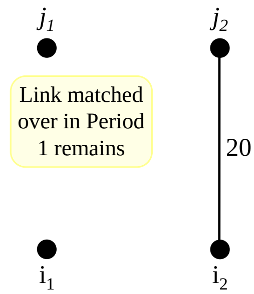
long term match whilst delay for a period and then entry yields a substantially more profitable long term match. Static constrained efficient vacancy creation might then prescribe entry in the first period whilst dynamic constrained efficient vacancy creation does not.[35]

The following example shows that there may not exist a level of bargaining power for which constrained efficient vacancy creation is a perfect equilibrium.

Example 6. *Let the cost of entry be* 15*, there are two periods with no discounting, two potential vacancies, two unemployed people and the network draw is deterministic. These network draws are shown in Figure 7.*



(A) L_1



(B) L_2

FIGURE 7. Period one and two network draws

As shown in Figure 7b, although the period 2 *network draw is deterministic it depends on the period* 1 *match insofar as matched parties remained matched and generate the same surplus. Vacancy* j_1 *has no benefit from entering in period* 1 *and* j_2 *has no benefit from entering in period* 2 *- if they are not already in the market and matched they will have no links. Thus each vacancy has only one entry decision to make. The total surplus, profits of* j_1 *and* j_2 *are shown for the different possible entry decisions below:*

As can be seen from Table 3 constrained efficient vacancy creation requires that no one enter in period 1 *and only vacancy* j_2 *enter in period* 2*. However, this is not a subgame perfect*

[35]Note that the value of multiple periods of employment resulting from entry now can be captured in the static model through the link value.

equilibrium. If $\beta > \frac{8}{11}$*,* j_1 *could enter in period* 1*. Vacancy* j_2 *would then receive a negative payoff were they to enter in period* 2 *and so would remain out of the market. This deviation from efficient vacancy creation yields positive payoffs for* j_2*. Indeed, this is unique subgame perfect equilibrium for* $\beta > \frac{8}{11}$*. When* $\beta < \frac{8}{11}$ *the unique subgame perfect equilibrium is for no vacancy creation in either period.*

Although example 6 is similar in spirit to Guerrieri (2008) the driving force behind the inefficiency is different. In Guerrieri (2008) people have private information about their disutility of working in a given position that is realized only at the time of bilateral bargaining with that position. This private information means that workers are able to extract ex-ante information rents and this distorts vacancy creation away from the first best in the static model, but not the constrained optimal. However, in the dynamic model workers value from remaining unmatched becomes endogenous and depends on vacancy creation. This externality is not internalized by the directed (competitive) search and a social planned can improve on decentralized outcomes by manipulating peoples' values from being unmatched. Reducing these values relaxes incentive compatibility constraints which can be beneficial to the social planner.

In the multilateral bargaining framework inefficiency is driven by the strategic effects of non zero measure agents who can influence market outcomes to their own benefit. In Example 6 j_1 is able to profit from influencing j_2's future entry decisions through their current entry decision. If j_1 were zero measure, as vacancies and people are in Guerrieri (2008), they would not be able to have these strategic effects. As number of potential vacancies in many local labor markets does not seem to be well approximated by infinity, Proposition 4 emphasizes that relaxing the zero measure assumption can result in dynamic externalities that inhibit efficient vacancy creation.

Although actors in the labor market are unlikely to have a negligible affect on outcomes because they are approximately zero measure it might be more reasonable to assume that their impact on the market is sufficiently uncertain for dynamic externalities to have no impact on vacancy creation decisions in practice. In Example 6 constrained efficient vacancy creation is not possible because j_1 is able to correctly and accurately anticipate how their current vacancy creation decision will affect the future vacancy creation decision of j_2. If j_2 did not know hat happened in period 1, then they could not condition their entry strategy on this information. j_1 would then be unable to influence future vacancy creation decisions by their current vacancy creation decision and there would be a (perfect) equilibrium in which only j_2 entered. How limited must future potential vacancies' information be for social and private incentives to again be aligned?[36] Proposition 4 addresses this question.

Proposition 4. *There is a perfect equilibrium of the dynamic vacancy creation game implementing constrained efficient vacancy creation for all probability distributions $\mathbb{L}$ if and only if no position's information set $\mathcal{I}_t$ is measurable with respect to any other position's previous entry decisions.*

The proof of Proposition 4 is in Appendix B. Corollary 4 follows from the construction of this proof.

Corollary 4. *If no position's information set $\mathcal{I}_t$ is measurable with respect to any other position's previous entry decisions, then there is a* sequential *equilibrium of the dynamic vacancy creation game implementing constrained efficient vacancy creation* for $\beta = 1$ *and constrained efficient vacancy creation* need not *correspond to creating vacancies that maximize the expected net total surplus within each period.*

Proposition 4 and Corollary 4 have two implications of particular note. First, despite the continued presence of dynamic and static externalities vacancies' private and social incentives can be aligned by setting $\beta = 1$ when each vacancy is sufficiently unsure about how their decisions today will impact on future market conditions and entry decisions. Second, conditions are identified under which dynamic strategic effects can be ignored. This helps place the common assumption in the literature that vacancies are zero measure in context.

Appendix AM-1 develops a model to illustrate the condition that "*entry decisions are not measurable with respect to any other position's previous entry decisions*" might entail. In this model vacancies' information sets are restricted in three ways. Firstly a network draw is considered such that all potential vacancies enter the market at most once- once a vacancy has been created and not filled or a job has been separated from their worker and left unfilled they are destroyed. This prevents potential vacancies having any information about current matches they gained from previous participation in the labor market. Second, the network draw is split into a draw between potential vacancies and people and a draw governing the separation of employed people from jobs. Only the distribution governing the draw between potential vacancies and people is

[36]This approach follows some of the recent networks literature that limits parties information about the network structure. For example, Fainmesser (2009) considers the impact of network structure on the ability of buyers and sellers to cooperate and maintain trust when their information about the network structure is limited.

know by potential vacancies at the start of a period. Vacancies do not observe the probability distribution governing separation and so are unable to infer anything about which vacancies and people are currently matched from this. Third, although the aggregate unemployment rate is observed, it is assumed that a change in employment status of a single person cannot be discerned. In this set up constrained efficient vacancy creation requires vacancies to be created that internalize dynamic externalities. Constrained efficient vacancy creation decisions do not then (typically) correspond to the vacancy creation decisions each period that would maximize the static expected net surplus. It follows immediately from Corollary 4 that for this information structure and $\beta = 1$ there is a sequential equilibrium implementing constrained efficient vacancy creation.

7. Conclusions

Wage determination is central to the search literature.[37] However, despite recent evidence suggesting that alternative employment opportunities affect wages (Hall and Krueger (2009), Cahuc Postel-Vinay and Robin (2006)), multilateral bargaining has not been considered. This paper introduces multilateral bargaining. Parties' alternative employment opportunities bound their payoffs from below and remaining surpluses are bargained over in the standard bilateral way. Further, frictions in the labor market are capture through restrictions on the possible matches which are modeled as a network. In this way bargained outcomes are characterized for any possible matches and any possible gains form trade between any vacancies and any person.

With multilateral bargaining too few vacancies are created in equilibrium. In many search models worker-vacancy pairs are first selected and then bilateral bargaining determines wages. This precludes competition between vacancies to employ workers in the space of wages. Allowing for this competition negates the negative externalities that could otherwise lead to excessive vacancy creation and too few vacancies are created in equilibrium unless vacancies have all the bargaining power. This is equivalent to letting vacancies make take-it-or-leave-it offers, which is similar in spirit to wage posting. Efficiency is then generated in situations meeting the assumptions of the competitive search literature, whilst in other situations there is too little vacancy creation. Further, in labor markets with dense network draws and similar surpluses across matches vacancy creation is approximately efficient. There is then little role for policy intervention to create vacancies in such markets.

[37]Hosios (1990), Shimer (1996), Moen (1997), Hall and Milgrom (2008), Hall (2009), Gertle and Trigari (2009), etc.

An important difference between the multilateral bargaining framework and other models of search is that agents are not by assumption zero measure. In a dynamic setting strategic incentives can then prevent efficient vacancy creation. Proposition 4 identifies necessary and sufficient restrictions on agents information sets such that efficient vacancy creation is a subgame perfect equilibrium when vacancies having all the bargaining power. This is a non-trivial extension of Proposition 1. There are dynamic externalities and vacancies must make forward looking decisions that differ from the static efficient entry decisions. Further, this result provides an alternative interpretation of the zero measure assumption made in much of the literature.

The multilateral bargaining framework also permits questions to be addressed that other search models are not well equipped to answer. For example, the advent of internet job search can be modeled as generating more employment opportunities and captured through denser network draws. Modeling internet search in this way Elliott and Stein (2010) examine why the Beverage curve has not shifted inwards despite the improvement in search technology.

Appendix A. Notation

Table 4 lists some of frequently used notation.

Symbol	Meaning
$\mathbf{N}$	Finite set of all people.
$\mathbf{U}_t$	People who are unemployed at the end of period t.
$\mathbf{E}_t$	People who are employed at the end of period t.
$\mathbf{P}_t$	Finite set of all possible positions in period t.
$\mathbf{V}'_t$	Potential vacancies, after separation, in period t.
$\mathbf{V}_t$	Vacancies created in period t.
$\mathbf{J}_t$	Jobs filled at the end of period t.
$\mathbf{\Phi}$	Coalition (subset) of people and positions.
Ω_t	Social planner's value function.
L_t	Network drawn in period t.
$l_{i,j,t}$	Surplus from person i being employed by position j in period t above the one-period value of remaining unmatched.
$\mathbb{L}_t$	Distribution period t network draw is made from.
$\mathbb{L}$	Vector of the $\mathbb{L}_t$ probability distributions.
L_t^E	Effective network draw over which employment and wages are determined.
$\mathcal{I}_{i,t}$	Information known by person i in period t.
μ_t	Feasible employment outcomes in period t (possible match).
μ_t^*	Employment outcomes in period t (matches).
$\mu_t(i)^*$	Job person i is employed by in period t.
β	Vacancies' bargaining power relative to peoples'
c	Cost of creating a vacancy.
π_t	payoffs in period t.
δ	Discount factor.

Table 4. Frequently used notation

Appendix B. Proofs

B.1. **Lemma 1.** *Person i's outside option in period t (minimum core payoff) is*

$$\underline{\pi}_{i,t} = \sum_{l \in L_i^{E,oo}} l - \sum_{l \in L_i^{E,tr}} l$$

Proof. Denote the matches on the initial network by $\mu(L^E)$ and the matches on the network without $\mu(i)^*$ by $\mu(L^E - \mu(i)^*)^*$. The network decomposition algorithm finds i's outside trade partner $\nu(i)$ by removing $\mu(i)^*$ from the network and considering the rematches that maximize the gains from trade. It is shown in Elliott (2009) (Lemma 1) that this rematching corresponds exactly to breaking matches over the links $L_i^{E,tr}$ and $l_{i\mu(i)^*}$ and then rematching over the links $L_i^{E,oo}$. Thus:

$$
\begin{aligned}
l_{i,\mu(i)^*} + \sum_{l \in L_i^{E,tr}} l - \sum_{l \in L_i^{E,oo}} l &= \sum_{k \in \mathbf{N}} \left(l_{k,\mu(k,L^E)^*} - l_{k,\mu(k,L^E-\mu(i)^*)^*} \right) \\
&= \overline{\pi}_{\mu(i)^*}
\end{aligned}
$$

The final line follows from a party's maximum core payoff equalling their marginal contribution to the grand coalition (see Demange (1982) and Leonard (1983)). Thus

$$
\begin{aligned}
\underline{\pi}_i &= l_{i,\mu(i)^*} - \overline{\pi}_{\mu(i)^*} \\
&= \sum_{l \in L_i^{E,oo}} l - \sum_{l \in L_i^{E,tr}} l
\end{aligned}
$$

□

B.2. **Lemma 3.** In the one period model:

(i) *The presence of an additional person in the labor market weakly increases the payoffs of all vacancies and weakly decreases the payoffs of all people in that period.*
(ii) *The presence of an additional position in the labor market weakly increases the payoffs of all people and weakly decreases the payoffs of all positions in that period.*

Proof. This proof will utilize the multi-unit auction mechanism analyzed by Demange, Gale and Sotomayor (1986). This mechanism finds the position optimal point of the core and thus the outside options of people.

Consider the position optimal point of a network L^E as found by the multi-unit auction mechanism analyzed in Demange, Gale and Sotomayor (1986). In their mechanism the wages of people who are over-demanded by firms at a given wage vector have their wage bid up until they are no longer over-demanded. Suppose now that person i is removed from the network. Any wage vector at which trade with a person was previously over demanded, at any point in the multi-unit auction mechanism, would still result in trade with that person being over demanded. Thus the same wages can be reached at an interim stage of this mechanism as was reached at the end of the mechanism on the network L^E. Continuing the multi-unit auction mechanism from this point people's wages are bid up further. Thus the position optimal (outside option) payoff of all people is weakly greater on the network $L^E - i$ than L^E and the position optimal payoff of all positions is weakly lower on the network $L^E - i$ than L^E.

The person optimal point of a network can also be found from the multi-unit auction mechanism but with the role of people and positions reversed such that people bid up positions' payoffs (and bid down their wages) when employment with them is over-demanded. Consider the payoff of positions at the person optimal point on $L^E - i$. Suppose now that person i was added to this network. Any price vector at which employment with a position was previously over demanded on $L^E - i$, at any point in the multi-unit auction mechanism, would still result in employment with that position being over demanded. Thus the same prices can be reached at an interim stage of the multi-unit auction mechanism on network L^E as were reached at the end of the multi-unit mechanism on the network $L^E - i$. Continuing the multi-unit auction mechanism from this point positions payoffs can only be bid up further. Thus the person optimal (outside option) payoff of all positions is weakly greater on the network L^E than on $L^E - i$ and the person optimal payoff of all people is weakly lower on the network L^E than $L^E - i$.

It has been shown that on network $L^E - i$ remaining people's outside option (minimum core) payoffs and their maximum core payoffs are both weakly higher than on L^E. It therefore follows that for any β, people's payoffs are weakly higher on $L - i$ than L. Conversely on $L^E - i$ vacancies' outside option (minimum core) payoffs and their maximum core payoffs are both weakly lower than on L^E, such that for any β vacancies' payoffs are weakly lower on $L^E - i$ than L^E.

The second part of the statement follows immediately from symmetry. □

B.3. **Proposition 1.** *In the one period model constrained efficient vacancy creation decisions, corresponding to* $\mathbf{V}^*$*, are a pure strategy Nash Equilibrium of the vacancy creation game if* $\beta = 1$ *and no other value of* β *aligns social and private incentives.*

Proof. First note that in the one period model that there is no dynamic / strategic value form being matched or remaining unmatched and so l_{ij} can be interpreted as the one period surplus generated by i and j. Consider a vacancy created by static efficient vacancy creation $j' \in \mathbf{V^E}$. Denote by $TS(V, L)$ the total surplus generated when vacancies V enter and the network L is drawn. $\mathbb{E}[TS(\mathbf{V^E}) - TS(\mathbf{V^E} \backslash j')] \geq c$ as efficient vacancy creation maximizes the net gains from trade and otherwise higher net gains from trade could be achieved by the removal of j' form $\mathbf{V^E}$. Consider now a vacancy j'' not created by the static efficient vacancy creation $j'' \notin \mathbf{V^E}$. $\mathbb{E}[TS(\mathbf{V^E} \cup j'';) - TS(\mathbf{V^E})] \leq c$ otherwise higher net gains from trade could be achieved by adding j'' to $\mathbf{V^E}$.

As when vacancies have all the bargaining power $\beta = 1$ they receive their Vickrey payoff (Demange (1982), Leonard (1983)): $\pi_j(\beta = 1, \mathbf{V}) = TS(\mathbf{V}) - TS(\mathbf{V} \backslash j)$. Thus the static efficient network is a Nash equilibrium for $\beta = 1$: vacancies $j' \in \mathbf{V^E}$ receive a payoff $\pi_{j'}(\beta = 1, \mathbf{V^E}) = TS(\mathbf{V^E}) - TS(\mathbf{V^E} \backslash j') \geq c$ and so are playing best responses by entering whilst all vacancies $j'' \notin \mathbf{V^E}$ would receive a payoff $\pi_{j''}(\beta = 1, \mathbf{V^E}) = TS(\mathbf{V^E}) - TS(\mathbf{V^E} \backslash j'') \leq c$ were they to enter and so are playing best responses by not entering. The core is a lattice that generically admits a convex set of payoffs for each player. Thus for $\beta < 1$, $\pi_j(\beta, \mathbf{V}) < TS(\mathbf{V}) - TS(\mathbf{V} \backslash j)$ and private and social incentives are not aligned. □

B.4. **Corollary 1.** *There will never be too many vacancies created in the following senses:*

(i) *In any Nash equilibrium of the vacancy creation game the ex-post removal of a vacancy will weakly reduce efficiency.*
(ii) *There will be never be more vacancies created in any Nash equilibrium (in the strong set order) than are efficient.*

Proof. **Part i:** By Lemma 1, conditional on any other set of vacancy creation decisions, vacancy's payoffs is equal to their marginal contribution to the grand coalition when they have all the bargaining power ($\beta = 1$). As vacancies' payoffs are weakly decreasing in β, and $\beta \in [0, 1]$, their payoffs are always weakly lower than their marginal contribution to total surplus given the other vacancy creation decisions. Thus in *any* Nash equilibrium all created vacancies generate marginal contributions to the total surplus weakly greater than they cost. It follows that removing any such vacancy weakly reduces the expected net surplus generated.

Part ii: Consider a Nash equilibrium with vacancies $V^E \cup V'$. Consider a vacancy $j' \in V' \backslash V^E$. From Lemma 3, Lemma 1, and the definition of the expected surplus maximizing entry decisions $\pi_{j'}(V^E \cup V') \leq \pi_{j'}(V^E \cup j') \leq TS(V^E \cup j') - TS(V^E) < c$. Thus j' cannot profitably enter and will not enter in a Nash equilibrium. □

B.5. **Corollary 2.** *Suppose firms can make ex-ante position specific investments in the multilateral bargaining framework. If firms have bargaining power ($\beta = 1$) constrained efficient vacancy creation and efficient specific investments are a pure strategy Nash Equilibrium of the vacancy creation game.*

Proof. Position specific investments are socially efficient when they increase the surplus generated more than they cost. The increase in surplus generated by a position specific investment is translated one to one into an increase in a vacancy's marginal contribution to the grand coalition (the additional surplus would be lost without the vacancy). From the proof of Proposition 1 when $\beta = 1$ vacancies payoffs are their marginal contributions to the grand coalition. Thus when position specific investments can be made and $\beta = 1$ vacancies' social and private incentives are aligned. □

B.6. **Lemma 4.** *The following three statements are equivalent:*

(i) *The network draw is dense.*
(ii) *For every possible realized network and every possible set of vacancies V that could be created,*
   (a) *if $|\mathbf{V}| = |\mathbf{N}|$ then for every subset of $\mathbf{V}$ the neighbors of $\mathbf{V}$ are such that $|\eta(\mathbf{V})| \geq |\mathbf{V}|$, or equivalently for every $\mathbf{\Phi_U} \subseteq \mathbf{U}$ the neighbors of $\mathbf{\Phi_U}$ are such that $|\eta(\mathbf{\Phi_U})| \geq |\mathbf{\Phi_U}|$,*
   (b) *if $|\mathbf{N}| > |\mathbf{V}|$ then for every $\mathbf{\Phi_V} \subseteq \mathbf{V}$, $|\eta(\mathbf{\Phi_V})| \geq |\mathbf{V}| + 1$, and*
   (c) *if $|\mathbf{V}| > |\mathbf{N}|$ then for every $\mathbf{\Phi_U} \subseteq \mathbf{U}$, $|\eta(\mathbf{\Phi_U})| \geq |\mathbf{\Phi_U}| + 1$.*
(iii) *For every possible realized network and every possible set of vacancies $\mathbf{V}$ that could be created, resetting the value of all links to unity, the decomposed network is a also a decomposed network for the complete network when all links have value unity.*

Proof. **1 $\iff$ 2:** By Hall's marriage theorem all unemployed people $i \in \mathbf{U}$ can be simultaneously matched to vacancies $j \in \mathbf{V}$ if and only if $\forall \mathbf{\Phi_U} \subseteq \mathbf{U}$, $|\eta(\mathbf{\Phi_U}, L)| \geq |\mathbf{\Phi_U}|$. Equivalence between statements i and ii is then immediate for $|\mathbf{V}| = |\mathbf{N}|$. When $|\mathbf{N}| > |\mathbf{V}|$ then 1 requires that all vacancies can be matched to someone in $\mathbf{U}/i$, $\forall i \in U$. Applying Hall's theorem, this is possible if and only if $\forall \mathbf{\Phi_V} \subseteq V$ and $\forall i \in \mathbf{U}$, $|\eta(\mathbf{\Phi_V}, L - i)| \geq |\mathbf{\Phi_V}|$, where $L - i$ indicates the network left once i is removed. This condition is equivalent to requiring that $\forall \mathbf{\Phi_V} \subseteq \mathbf{V}$, $|\eta(\mathbf{\Phi_V}, L - i)| \geq |\mathbf{\Phi_V}| + 1$. The argument for $|\mathbf{N}| < |\mathbf{V}|$ is equivalent.

1 $\iff$ 3: Whenever values are homogeneous and $|\mathbf{N}| = |\mathbf{V}|$ the decomposed complete network consists of $|\mathbf{N}| = |\mathbf{V}|$ trade links and no outside option links. Clearly all parties are matched if and only if the decomposed network has $|\mathbf{N}| = |\mathbf{V}|$ trade links. Further, whenever there are homogenous gains from trade and a decomposed network has $|\mathbf{N}| = |V|$ trade links, all additional links are redundant-there can be no outside option links: Without any additional links $\pi_i = \pi_{i'}$, $\forall i, i' \in \mathbf{N}$ and $\pi_j = \pi_{j'}$, $\forall j, j' \in \mathbf{V}$ and thus $\forall \{i, j\} \in \mathbf{N} \times \mathbf{V}$, $\pi_i + \pi_j = 1$. Adding any additional links cannot then permit a profitable deviation and so these links are redundant. All networks with homogenous gains from trade and $|\mathbf{N}| = |V|$ that match all unemployed people and all vacancies then have a network decomposition consistent with the complete network.

Whenever values are homogeneous and $|\mathbf{N}| > |\mathbf{V}|$ the decomposed complete network consists of $|\mathbf{V}|$ trade links and $|\mathbf{V}|$ vacancy outside option links. Outside option links represent who a party would rematch to were their trade partner removed form the network. Thus on the complete network all vacancies would be rematched if their trade partner was removed. Further, as rematching occurs along outside option chains and outside option chains do not cycle (Elliott (2009)), all vacancies will remain matched or be rematched whenever any unemployed person is removed from the network. Further, if a vacancy did not have an outside option link, they would not rematch were their trade partner removed from the network. Thus in all networks where $|\mathbf{N}| > |\mathbf{V}|$, all vacancies can be matched even when any unemployed person is removed if and only if all vacancies have an outside option link in the decomposed network for the network where the gains from trade have been reset to unity for each present link. Further, this decomposed network is a network decomposition for the complete network with homogeneous gains from trade. The argument for $|\mathbf{N}| < |\mathbf{V}|$ is equivalent. □

B.7. **Proposition 2.** *If the network draw is dense and the value of all links is homogeneous then all pure strategy Nash equilibria of the vacancy creation game are approximately efficient for all levels of vacancy bargaining power β.*

Proof. Without loss of generality the value of links is normalized to unity. If the network draw is dense and the value of links are homogenous, then by part *iii*) of Lemma 4 the network behaves as if it were the complete network. By Lemma 2 vacancies payoffs from entering are then 0 if $|\mathbf{V}| > |\mathbf{N}|$, β if $|\mathbf{V}| = |\mathbf{N}|$ and 1 if $|\mathbf{V}| < |\mathbf{N}|$. Trivially, if $1 < c$ then no vacancies are created and this is efficient. If $1 > c$ then efficient vacancy creation requires $|\mathbf{V}| = |\mathbf{N}|$. Thus there will then be efficient vacancy creation when $\beta > c$ and approximately efficient vacancy creation when $\beta < c$ in all pure strategy Nash equilibria (with $|\mathbf{N}| - 1$ vacancies created vacancies receive a payoff of 1). □

B.8. **Proposition 3.** *If people and potential vacancies simultaneously decide whether to enter the labor market at costs $c_{\mathbf{N}}$ and $c_{\mathbf{P}}$ respectively, then*

- (i) *when the network draw is deterministic (such that $\mathbb{L}_t$ is places all weight on a single draw)*
  - (a) *expected net surplus maximizing entry decisions are a Nash equilibrium if $\beta = \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$, and*
  - (b) *$\forall\beta \neq \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$ there exist networks in which net surplus maximizing entry decisions are not a Nash equilibrium.*
- (ii) *If there are multiple possible network draws then there may not exist a β for which expected net surplus maximizing entry decisions are a Nash equilibrium.*

Proof. **Part i)a):** Denote the set of vacancies and people that must enter the market to maximize the expected net surplus by $\mathbf{V}^*$ and $\mathbf{N}^*$ respectively. When the network draw is deterministic these people and vacancies must all be matched. It therefore follows from Section C that every vacancy's outside option chain terminates at another vacancy and similarly every person's outside option chain terminates at another person.

Suppose now that a person $i \in \mathbf{N}^*$ deviated and did not enter the market. Expected net surplus maximization requires that $TS(\mathbf{V}^* \cup \mathbf{N}^*) - TS(\mathbf{V}^* \cup \mathbf{N}^*\backslash i) \geq c_{\mathbf{N}}$. It is shown in section C that following the removal of i from the network, rematching will occur along $\mu(i)$'s outside option chain. This rematching will therefore leave a vacancy j' unmatched. Efficiency of the initial match therefore requires that $TS(\mathbf{V}^* \cup \mathbf{N}^*) - TS(\mathbf{V}^* \cup \mathbf{N}^*\backslash i) \geq c_{\mathbf{N}} + c_{\mathbf{P}}$. In Lemma 1 it is shown that i's maximum core payoff is their marginal contribution to the grand coalition: $\overline{\pi}_i = TS(\mathbf{V}^* \cup \mathbf{N}^*) - TS(\mathbf{V}^* \cup \mathbf{N}^* \backslash i)$. Expected net surplus maximization thus requires:

$$\begin{aligned} TS(\mathbf{V}^* \cup \mathbf{N}^*) - TS(\mathbf{V}^* \cup \mathbf{N}^* \backslash i) &\geq c_{\mathbf{N}} + c_{\mathbf{P}} \\ \overline{\pi}_i &\geq c_{\mathbf{N}} + c_{\mathbf{P}} \\ (1-\beta)\overline{\pi}_i + \beta\underline{\pi}_i - c_{\mathbf{N}} &\geq (1-\beta)(c_{\mathbf{N}} + c_{\mathbf{P}}) + \beta\underline{\pi}_i - c_{\mathbf{N}} \\ \pi_i - c_{\mathbf{N}} &\geq (1-\beta)(c_{\mathbf{N}} + c_{\mathbf{P}}) + \beta\underline{\pi}_i - c_{\mathbf{N}} \end{aligned}$$

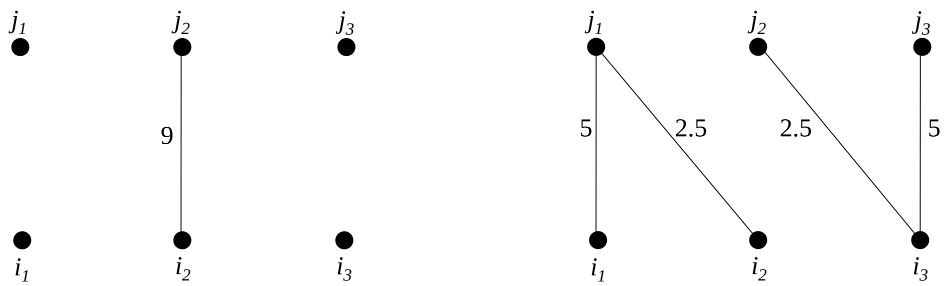
Thus when $(1-\beta)(c_{\mathbf{N}} + c_{\mathbf{P}}) + \beta\underline{\pi}_i - c_{\mathbf{N}} \geq 0$, $\pi_i \geq c_{\mathbf{N}}$ and entry is profitable for i. Thus entry is profitable for i when $\beta \leq \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}} + \frac{\beta\underline{\pi}_i}{c_{\mathbf{N}}+c_{\mathbf{P}}}$ and hence whenever $\beta \leq \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$.

An equivalent argument establishes that entry is profitable for $j \in \mathbf{V}^*$ whenever $\beta \geq \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$. It follows that entry for all parties $k \in \mathbf{V}^* \cup \mathbf{N}^*$ is profitable whenever $\beta = \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}}$.

Consider now that a party $k \notin \mathbf{V}^* \cup \mathbf{N}^*$. By efficiency $TS(\mathbf{V}^* \cup \mathbf{N}^* \cup k) - TS(\mathbf{V}^* \cup \mathbf{N}^*) \leq c_k$, where $c_k = c_{\mathbf{N}}$ for $k \in \mathbf{N}$ and $c_k = c_{\mathbf{P}}$ for $k \in \mathbf{P}$. Thus $\overline{\pi}_k \leq c_K$ and $\pi_k \leq 0$. Party k therefore plays a best response by not entering. It follows that vacancies $\mathbf{V}^*$ being created and people $\mathbf{N}^*$ entering is an equilibrium.

Part i)b): This proof is by counter example. Consider a draw L_t that comprises a single link $L_t = l_{i,j,t}$ and let $c_{\mathbf{N}} = c_{\mathbf{P}} = \frac{l_{i,j,t}-\varepsilon}{2}$. Suppose without loss of generality that $\beta > \frac{c_{\mathbf{P}}}{c_{\mathbf{N}}+c_{\mathbf{P}}} = \frac{1}{2}$. For all $\varepsilon > 0$ efficient entry requires that both i and j enter. Consider now the entry decision by i. $\pi_i = (1-\beta)l_{i,j,t} - c_{\mathbf{N}} = (\frac{1}{2} - \beta)l_{i,j,t} + \frac{\varepsilon}{2}$. Thus $\forall\beta > \frac{1}{2}$ there exists a $\varepsilon > 0$ such that $\pi_i < 0$ and entry is unprofitable.

Part ii): This proof is by counter example. Suppose the network draw is L'_t with probability $\frac{1}{2}$ and L''_t with probability $\frac{1}{2}$. These networks are shown in Figure 8.



(A) L'_t (probability $\frac{1}{2}$) (B) L''_t (probability $\frac{1}{2}$)

FIGURE 8. Possible network draws

Let the cost of entry be $c_{\mathbf{N}} = c_{\mathbf{P}} = 1$. The expected net total surplus is maximized by all Vacancies and all Unemployed people entering the labor market. This generates an expected net total surplus of 3.5.[38] However there is no value of β for which entry is profitable for both i_1 and j_3. $\pi_{i_1} = \frac{1}{2}(1-\beta)(5-2.5) - c_{\mathbf{N}}$ and $\pi_{j_3} = \frac{1}{2}\beta(5-2.5) - c_{\mathbf{P}}$. Thus for $\pi_{i_1} \geq 0$, $\beta \leq \frac{1}{5}$ but for $\pi_{j_3} \geq 0$, $\beta \geq \frac{4}{5}$.

□

[38]Were just j_2 and i_2 to enter the market the expected net total surplus would be 2.5 and were j_1, j_2, i_2 and i_3 to enter the net total surplus would be 3.

B.9. **Proposition 4.** *There is a perfect equilibrium of the dynamic vacancy creation game implementing constrained efficient vacancy creation for all probability distributions $\mathbb{L}$ if and only if no position's information set $\mathcal{I}_t$ is measurable with respect to any other position's previous entry decisions.*

Proof. Only if:

The *only if* part of the statement is that: The existence of a constrained efficient Perfect equilibrium for all $\mathbb{L}$ implies that no position's information set $\mathcal{I}_t$ is measurable with respect to any other position's previous entry decisions. The contrapositive of this statement is: at least one position's information set $\mathcal{I}_t$ being measurable with respect to any other position's previous entry decisions implies that there exists a $\mathbb{L}$ for which there is no efficient perfect equilibrium. This is proved by example 6.

If:

The *if* part of the statement is that: No position's information set $\mathcal{I}_t$ being measurable with respect to any other position's previous entry decisions implies that there exists a constrained efficient perfect equilibrium for all $\mathbb{L}$. Consider a potential vacancy j's future entry decisions when all other vacancy creation decisions, including future vacancy creation decisions, are fixed. For $\beta = 1$, j will then receive their marginal contribution to the total surplus generated each period (See Dmange (1982) and Leonard (1983)). Thus j will choose a contingent sequence of vacancy creation decisions that maximize their expected discounted contributions to the grand coalition taking all other vacancy creation decisions as fixed: j's private incentives to enter are aligned with social incentives. Suppose now that all vacancies make constrained efficient vacancy creation decisions. Whilst, by the above argument this is a Nash equilibrium it may not be a perfect Nash equilibrium if one position's vacancy creation decision can influence another position's current or future entry decision. However, as the information available to potential vacancies is restricted and their entry decisions are not measurable with respect to any one position's previous entry decisions, no one vacancy can hope to influence the vacancy creation decisions of other positions. Constrained efficient vacancy creation decisions are then a perfect equilibrium.

□

B.10. **Corollary 4.** *If no position's information set $\mathcal{I}_t$ is measurable with respect to any other position's previous entry decisions, then there is a* sequential *equilibrium of the dynamic vacancy creation game implementing constrained efficient vacancy creation* for $\beta = 1$ *and constrained efficient vacancy creation* need not *correspond to creating vacancies that maximize the expected net total surplus within each period.*

Proof. Suppose that all vacancies are making constrained efficient vacancy creation decisions. From the proof of Proposition 4 this is a perfect equilibrium when $\beta = 1$ and entry decisions maximize positions expected discounted stream of future marginal contributions to the grand coalition. These entry decisions need not maximize the expected total surplus within a given period (see Example 6). Now suppose each vacancy makes a mistake and deviates from their constrained efficient entry decision in period t with probability ϵ. Let $\epsilon \to 0$. Let each vacancy, in period t, believe play has been constrained efficient as long as this is consistent with their information $\mathcal{I}_{i,t}$. Assign any beliefs consistent with the trembles ϵ to other observations of $\mathcal{I}_{i,t}$. These beliefs are consistent by construction and support constrained efficient entry decisions as a sequential equilibrium. □

Appendix C. Network Decomposition Algorithm

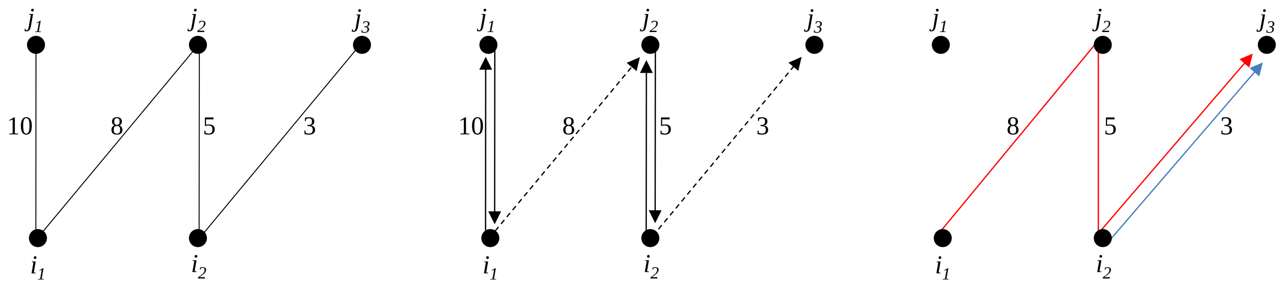
The network decomposition algorithm, reproduced from Elliott (2009), decomposes *any* possible bipartite network into a directed network that identifies which links are traded over and which links are used by which parties to establish their outside options:[39]

(i) Identify trade partners. The matches on the effective network that are in the core must maximize the gains from trade. In the non-generic case any such match can be selected.
(ii) Identify outside trade partners. To find person i's outside trade partner ($\nu(i)$) remove their trade partner $\mu(i)^*$ from the effective network and identify the match that maximizes the gains from trade on this new network.[40] i is now matched to their outside trade partner. If i is left unmatched they have no outside trade partner. Repeat for all people and positions.
(iii) Construct decomposed network. This graph consists of two types of directed links. Links to trade partners and links to outside trade partners. For each person and position construct a solid link to trade partners and dashed directed links to outside trade partners.

Links other than those to a trade partner or to an outside trade partner are redundant. It is immediate from the algorithm that a party's outside trade partner is who they would trade with if their trade partner had not entered the market.

The following relatively simple example shows how to apply this decomposition, identify outside option chains and use these chains from find parties endogenous outside options. More involved examples are presented in Elliott (2009).

Example 7. *Consider the network shown in Figure 9a.*



(A) Effective network (B) Outside options (C) Outside option chains

Figure 9. Directed network decomposition

Matching j_1 to i_1 and j_2 to i_2 maximizes the possible gains from trade. This is shown below in match $\mu(L^E)^*$. Each unemployed person's outside trade partner is found by removing their trade partner from

[39]If the only interest was in finding the minimum core payoffs and thus outside options of parties, they could be found directly from the matching literature. The multi-unit auction mechanism analyzed by Demange, Gale and Sotomayor (1986) and discussed in Roth and Sotomayor (1990) provides a way of identifying the position and person optimal points of the core. These payoffs correspond to people and positions' outside option payoffs respectively. The purpose of the network decomposition is to identify the impact of the network structure on payoffs. Kranton and Minehart (2000a) use a different methodology to find how links in a network with homogeneous gains from trade matter for different parties' payoffs.

[40]In the non generic case any such match can be selected. Although the directed network representation will not be unique the same outside option payoffs will ultimately be derived.

the network and considering who they would be rematched to in order to maximize the gains form trade over this new network. For example to find i_1*'s outside trade partner we remove their trade partner* j_1 *from the network. On this reduced network* $(L^E - j_1)$ *match* $\mu(L^E - v_j)^*$ *maximizes the gains from trade by matching* i_1 *to* j_2*.* j_2 *is therefore* i_1*'s outside trade partner. Outside trade partners for the other unemployed people are found in a similar way. These outside trade partners are represented by dashed links in Figure 9b whilst trade partners are shown by the solid links. Vacancies' outside trade partners can be found in a similar way to generate the full directed network representation of L. In this example no vacancy has an outside trade partner. If* j_1*'s trade partner* i_1 *were removed from the network* j_1 *would not rematch and ii* j_2*'s trade partner* i_2 *were removed from the network* j_2 *would not rematch. In Figure 9b parties benefit from a directed link that originates from them. For example* i_1 *has a directed trade link to* j_1 *and a directed non-trade (outside option) link to* j_2*.*

$$\mu(L)^* = \begin{matrix} \{j_1, i_1\} \\ \{j_2, i_2\} \\ \{j_3, \emptyset\} \end{matrix} \quad \mu(L - v_1)^* = \begin{matrix} \{\boldsymbol{j_2}, \boldsymbol{i_1}\} \\ \{j_3, i_2\} \end{matrix} \quad \mu(L - j_2)^* = \begin{matrix} \{j_1, i_1\} \\ \{\boldsymbol{j_3}, \boldsymbol{i_2}\} \end{matrix}$$

Unemployed peoples' outside option chains can then be found from Figure 9b.[41] *This is done by following the set of directed links originating at them and alternating between outside option links and trade links. These outside option chains are highlighted in Figure 9c. Unemployed person* i_1*'s outside option chain is in red and* i_2*'s is in blue.*

Each unemployed person's outside option can then be found by alternately adding and then subtracting the value of the links in their outside option chain:

$$\underline{\pi}^U = \begin{pmatrix} l_{1,2} - l_{2,2} + l_{2,3} \\ l_{2,3} \end{pmatrix} = \begin{pmatrix} 8 - 5 + 3 = 6 \\ 3 \end{pmatrix}$$

This representation of unemployed people's outside option payoffs identifies exactly how each link in the network affects their outside option payoff.

Appendix D. Nesting other models

D.0.1. *Calvo Armengol and Zenou (2005).* Calvo-Armengol and Zenou (2005) model job information being passed through social networks between people (and not between people and vacancies). Employed workers pass job information along to their unemployed social contacts. Information (or the lack thereof) about job opportunities is the friction in this model that gives rise to the matching function.[42]

There are a continuum of workers and a continuum of firms. Each worker-firm match generates the same surplus. People have k randomly drawn social contacts each period. Vacant jobs are advertised with identical predetermined wages w and this information reaches a single person. This person is equally likely to be any person. If they are employed they pass the information on at random to one unemployed social contact (someone in their social network). If they are unemployed they hold the information for themselves. At the end of this information transmission stage unemployed people have accumulated information about vacancies both directly and through their social contacts. Unemployed workers then select at random one vacancy to apply to from those they have information about. If they have no information about any vacancies they cannot apply to any jobs. The number of matches

[41] There are no vacancy outside option chains as no vacancy has an outside trade partner.

[42] For other models of unemployment with vacancy information transmission over social networks see Calvo-Armengol and Jackson (2004), (2007) and Calvo-Armengol and Zenou (2005).

is then simply the number of unemployed people who have information about one or more vacancies. From this process it is relatively straight forward to determine the combined probability ($\rho(\mathbf{U}, \mathbf{V}, k)$) with which an individual hears about a job either directly or through their social contacts.

To represent this model in the multilateral bargaining framework let the productivity of any person-job pair be the constant l. Set $c = 0$ such that all vacancies always enter the market. Let $|\mathbf{P}| > |\mathbf{N}|$, and consider networks draws such that:

(i) each unemployed person has one link with probability $\rho(\mathbf{U}, \mathbf{V}, k)$ and no links with probability $1 - \rho(\mathbf{U}, \mathbf{V}, k)$,
(ii) this link is to a vacancy
(iii) each vacancy has no more than one link, and
(iv) equal weight is placed on all such networks.

As all vacancies always enter the market, $|\mathbf{P}| > |\mathbf{N}|$ ensures that $|\mathbf{V}| > |\mathbf{U}|$ such that each unemployed person can link to a different vacancy. Thus each unemployed person has probability $\rho(\mathbf{U}, \mathbf{V}, k)$ of being matched as in Calvo-Armengol and Zenou (2005). Further, setting l and $\beta \in [0, 1]$ such that $(1 - \beta)l = w$ ensures that people receive the same wage as in Calvo-Armengol and Zenou (2005).

D.1. **Cahuc, Postel-Vinay and Robin (2006).** Cahuc, Postel-Vinay and Robin (2006) model between employer competition for workers. Firms compete for workers and this affects their wages. Quantitatively they find that between firm competition affects wages. Competition for workers is modeled in the follow way. Bargaining between unemployed workers and a prospective employer is modeled as a bilateral alternating offer bargaining game.[43] When an employed worker receives a new employment opportunity the worker may decide to engage in multilateral bargaining. If they do this an auction game is triggered in which the two firms bid up the workers wage until one firm drops out. In equilibrium a firm will drop out when the wage of worker is sufficiently high that they do not want to employ them at that wage. The wage at which this happens then becomes the disagreement point for bilateral alternating offer bargaining with the remaining firm. Thus workers by engaging in on the job search may increase their wage in their current position or switch to a preferred job.

The wages bargained to in Cahuc, Postel-Vinay and Robin (2006) are precisely the same wages that are reached in equivalent situations in the multilateral bargaining framework. Matched workers engage in re-bargaining precisely when doing so is valuable to them and this re-bargaining generates the same outcomes proposed in Cahuc, Postel-Vinay and Robin (2006). The firm that values the worker more highly will ultimately employ them, but the other firm provides an outside option that in effect determines the disagreement point of the worker in the wage negotiations. Splitting remaining gains from trade according to the bargaining power parameter β is then equivalent running the alternating offer bargaining game where the relative discount factors of the worker and firm can be mapped into the bargaining power parameter. This provides some non-cooperative justification for the cooperative bargaining solution considered.[44]

[43]See Rubinstein (1982).
[44]The alternating public offer bargaining over a network game of Corominas-Bosch (2004) provides further justification. See Elliott (2009).

References

Acemoglu, D., and R. Shimer (1999): "Holdups and efficiency with search frictions," *International Economic Review*, pp. 827–849.

Burdett, K., and D. Mortensen (1998): "Wage differentials, employer size, and unemployment," *International Economic Review*, pp. 257–273.

Butters, G. (1977): "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies*, 44(3), 465–491.

Cahuc, P., F. Postel-Vinay, and J. Robin (2006): "Wage Bargaining with on-the-job search: Theory and Evidence," *Econometrica*, pp. 323–364.

Calvo-Armengol, A., and M. Jackson (2004): "The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94(3), 426–454.

Calvó-Armengol, A., and M. Jackson (2007): "Networks in labor markets: Wage and employment dynamics and inequality," *Journal of Economic Theory*, 132(1), 27–46.

Calvó-Armengol, A., and Y. Zenou (2005): "Job-matching, social network and word-of-mouth communication," *Journal of Urban Economics*, 57, 500–522.

Coles, M. (2001): "Equilibrium wage dispersion, firm size, and growth," *Review of Economic Dynamics*, 4(1), 159–187.

Coles, M., and C. for Economic Policy Research (Great Britain) (1994): *Understanding the matching function: The role of newspapers and job agencies*. Centre for Economic Policy Research.

Coles, M., and A. Muthoo (1998): "Strategic bargaining and competitive bidding in a dynamic market equilibrium," *Review of Economic Studies*, pp. 235–260.

Coles, M., and E. Smith (1998): "Marketplaces and matching," *International Economic Review*, pp. 239–254.

Corominas-Bosch, M. (2004): "Bargaining in a network of buyers and sellers," *Journal of Economic Theory*, 115(1), 35–77.

Démange, G. (1982): "Strategyproofness in the assignment market game," *Labratorie dEconometrie de lEcole Polytechnique, Paris*.

Demange, G., D. Gale, and M. Sotomayor (1986): "Multi-Item Auctions," *The Journal of Political Economy*, 94(4), 863.

Diamond, P. (1971): "A model of price adjustment," *Journal of Economic Theory*, 3(2), 156–168.

Ebrahimy, E., and R. Shimer (2008): "Stock-Flow Matching," *unpublished paper*.

Echenique, F., G. Katz, and L. Yariv (2009): "An Experimental Study of Decentralized Matching," .

Elliott, M. (2009): "Inefficiencies in Trade Networks with Heterogeneous Gains from Trade," .

Fainmesser, I. (2009): "Community Structure and Market Outcomes: Towards a Theory of Repeated Games in Networks," .

Fallick, B., and C. Fleischman (2004): "Employer-to-employer flows in the US labor market: The complete picture of gross worker flows," .

Gertler, M., and A. Trigari (2009): "Unemployment fluctuations with staggered Nash wage bargaining," *Journal of Political Economy*, 117(1), 38–86.

Gregg, P., and B. Petrongolo (1997): *Random or non-random matching?: implications for the use of the UV curve as a measure of matching performance*. University of Oxford, Institute of Economics and Statistics, Centre for Economic Performance.

Guerrieri, V. (2008): "Inefficient Unemployment Dynamics under Asymmetric Information," *Journal of Political Economy*, 116(4), 667–708.

Hall, R. (1979): "A theory of the natural unemployment rate and the duration of employment," *Journal of Monetary Economics*, pp. 153–169.

Hall, R. (2009): "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," *Journal of Political Economy*, 117(2), 281323.

Hall, R., and P. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review 98(4) (September 2008): 1653-1674.*, 98(4), 1653–1674.

Hall, R. E., and A. B. Krueger (2009): "Wage Formation between Newly Hired Workers and Employers: Survey Evidence," *IZA Discussion Paper.*

Hopenhayn, H., and R. Rogerson (1993): "Job turnover and policy evaluation: A general equilibrium analysis," *Journal of Political Economy*, pp. 915–938.

Hosios, A. (1990): "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies*, pp. 279–298.

Kranton, R., and D. Minehart (2000a): "Competition for goods in buyer-seller networks," *Review of Economic Design*, 5(3), 301–331.

——— (2000b): "Networks versus Vertical Integration," *RAND Journal of Economics*, 31(3), 570–601.

Kranton, R., and D. Minehart (2001): "A Theory of Buyer-Seller Networks," *American Economic Review*, 91(3), 485–508.

Lagos, R. (2000): "An Alternative Approach to Search Frictions," *Journal of Political Economy*, 108(5), 851–873.

Leonard, H. (1983): "Elicitation of honest preferences for the assignment of individuals to positions," *The Journal of Political Economy*, pp. 461–479.

Moen, E. (1997): "Competitive search equilibrium," *Journal of Political Economy*, 105(2), 385–411.

Mortensen, D. (1982): "Property rights and efficiency in mating, racing, and related games," *The American Economic Review*, pp. 968–979.

Petrongolo, B., and C. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

Pissarides, C. (1979): "Job matchings with state employment agencies and random search," *The Economic Journal*, pp. 818–833.

Pissarides, C. (1990): *Equilibrium unemployment theory.* Basil Blackwell Cambridge, Mass.

——— (2000): *Equilibrium unemployment theory, second edition.* The MIT Press Cambridge, Massachusetts London, England.

Pries, M., and R. Rogerson (2005): "Hiring policies, labor market institutions, and labor market flows," *Journal of Political Economy*, 113(4), 811–839.

——— (2009): "Search frictions and labor market participation," *European Economic Review*, 53(5), 568–587.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey, Forthcoming," *Journal of Economic Literature.*

Roth, A., and M. Sotomayor (1990): *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.* Cambridge University Press.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50(1), 97–109.

Shapley, L., and M. Shubik (1972): "The assignment game," *International Journal of Game Theory*, 1, 111–130.

Shimer, R. (1996): "Essays in Search Theory," *PhD Thesis.*

——— (2006): "On-the-job search and strategic bargaining," *European Economic Review*, 50(4), 811–830.

——— (2007): "Mismatch," *American Economic Review*, 97(4), 1074–1101.