Optimal Patent Policy with Recurrent Innovators

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Motivation

- In models of optimal patent policy with cumulative innovation, typically innovators are short lived, even if the planner is long lived
- One natural question in patent policy is whether or not dominance by one firm is a "bad" idea
- In order to answer this question, we modify the standard setting to allow for long lived innovators, each responsible for many innovations.

Setup

- One product
- Two agents (called "innovators") who improve the quality of the product
- The planner chooses how to allocate the opportunity to sell the object between the two innovators; the innovators get value from selling the object
- As in the model of a sequence of innovations by different firms in Hopenhayn, et al (2006), we denote the current promised expected discounted duration of market leadership by d
- Suppose that "ideas" arrive exogenously, at Poisson rate λ to each of two innovators
- In other words, ideas arrive at rate 2λ and are equally likely to fall on any of two innovators
- For comparison: In Hopenhayn, et al. the arrival is uniform across a continuum of innovators, i.e. no innovator comes twice

Innovation

- Once an idea arrives, the innovator makes a private choice of an innovation level Δ
- Δ is an improvement to the state of the art of the project
- Cost of innovation is $c(\Delta)$, increasing and convex
- Expected discounted increment to social value from an innovation is $R(\Delta(d)) = R(d)$, increasing, concave.

Private returns

- Δ measures the increment of private value for a firm who can exclusively market the state of the art product including Δ
- Private returns are $d\Delta - c(\Delta)$
- Important assumption: value of Δ is independent of prior innovations, except to the extent that the planner's choice of d depends on the past.

Example: Quality Ladder

- One customer, inelastic demand given by $q - p$
- q is quality, increases by Δ with every innovation
- p is the price

Case 1: Exclusive Patents with Expiration

- Suppose that, whenever the firm that is NOT the previous innovator implements and idea, the duration promise for the previous innovator goes to zero
- Whenever this occurs, the previous state of the art goes into the public domain, and is sold at marginal cost.
- Shi and Riis (2009) discuss cases where this may be optimal
- Hopenhayn, et al. (2006) have sufficient conditions for exclusivity to be optimal in the non-recurring case
- Exclusive patents mirrors the market structure in O'Donoghue, et al (1998)
- Quality ladder example: current leader charges $p = \sum \Delta_i$, where the sum is taken over all the leader's innovations since the other firm innovated

Planner's Problem

- At any instant, if an innovation by the previous innovator arrives, the planner makes the new promised duration d_1
- If the other innovator has an idea, he is implemented with probability p. If implemented, he is promised duration d_2
- If nothing arrives, the planner may change the duration promise by $\dot{d}$

Dynamic Program

$$
\begin{aligned}
rV(d) &= \max_{d_1, d_2, \dot{d}, p} \left\{ \begin{array}{c} \lambda\left(R(d_1) + V(d_1) - V(d)\right) + \\ \lambda p\left(R(d_2) + V(d_2) - V(d)\right) + \\ V'(d)\dot{d} \end{array} \right\} \\
& \quad s.t. \\
rd &= 1 + \lambda(d_1 - d) - \lambda p d + \dot{d}
\end{aligned}
$$

First Order Conditions

$$\begin{aligned} R'(d_1) &= -V'(d_1) + \mu(d) \\ R'(d_2) &= -V'(d_2) \\ R(d_2) + V(d_2) - V(d) &= -\mu(d)d \\ V'(d) &= \mu(d) \end{aligned}$$

- Note that the last (for $\dot{d}$) coincides with the envelope condition for $V(d)$ when $\dot{d} = 0$
- The next to last (for p) is independent of p, so p is always at the boundary.

Complete Characterization

- Suppose, as in the quality ladder example
  - $R'(1/r) = 0$; the level of innovation is first best when the duration promise is $1/r$.
  - There are no static distortions, so duration promises should always be at least $1/(r+\lambda)$
- Then

Lemma

If $\lim_{\Delta \downarrow 0} c(\Delta) \leq \bar{F}$, *then* $d_1(d) = 1/(\lambda + r)$. *Otherwise* $d_1(d) > 1/(\lambda + r)$ *solves*

$\max_{d_1, p} d_1 \in [0, 1/r]$

s.t.

$rd = 1 + \lambda(d_1 - d) - \lambda p d$

What does this imply in the latter case?

Evolution of

d

Corollary

$d_1 > d$, with d reaching $1/r$ for a finite number of consecutive ideas for one firm

- Every laggard idea is implemented until $d_1 = 1/r$
- One firm monopolizes innovation in finite time

Case 2: Complete Exclusion Rights

- Suppose that an implemented innovation leads to rights to exclude others from producing *any* version of the product, so that the currently active firm makes $\sum \Delta_i$, where the sum is taken over the entire history of improvements
- In that case, d takes on a different meaning: even after you are overtaken, you may re-overtake later, and all of your contributions will help you to make profits in that situation
- Suppose that, as in the example above, there are no static distortions, so all of the time is given away

Recursive representation

- Slight change in notation: state variable d is the promise to innovator 1.
- d_1 is the duration promise to innovator 1 when an innovation arrives from innovator 1
- d_2 is the duration promise *to innovator* 1 when an innovation arrives from innovator 2.

Value function

$$
\begin{aligned}
rV(d) &= \max_{d_1, d_2, \dot{d}, x} \left\{ \begin{array}{c} \lambda\left(R(d_1) + V(d_1) - V(d)\right) + \\ \lambda\left(R(\frac{1}{r} - d_2) + V(d_2) - V(d)\right) + \\ V'(d)\dot{d} \end{array} \right\} \\
& \quad s.t. \\
rd &= x + \lambda(d_1 - d) - \lambda(d_2 - d) + \dot{d}
\end{aligned}
$$

Results

- Once again, $\dot{d} = 0$
- V concave
- First order conditions:

$$\begin{aligned} V'(d) &= \mu(d) \\ R'(d_1) &= -V'(d_1) + \mu(d) \\ R'(d_2) &= -V'(d_2) + \mu(d) \end{aligned}$$

- By concavity, $d_2 < d < d_1$
- In numerical examples eventual monopolization is sometimes optimal, but never for small values of fixed costs

Message

- May be two regimes
- One (with sufficient fixed costs) has very much "state dependent" rewards; history of innovations matters
- Strong force toward monopolization; leaders get strong incentives to innovate, at the expense of followers
- Alternative regime implements all ideas in a state-independent way (all are treated symmetrically)