

# Uncertainty, Financial Frictions, and Investment Dynamics

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## Abstract

The canonical framework used to price risky debt implies that the payoff structure of levered equity resembles the payoff of a call option, while the bondholders face a payoff structure that is equivalent to that of an investor writing a put option. As a result, an increase in the payoff uncertainty benefits equity holders at the expense of bondholders, a feature of the debt contract with two potentially important implications for real economic activity: First, to the extent that firms face significant frictions in financial markets, an increase in the default-risk premium implies a higher cost of capital and hence a decrease in investment. Second, a reduction in the supply of credit stemming from an increase in uncertainty hampers the efficient reallocation of capital and causes an endogenous decline in total factor productivity (TFP) that amplifies the economic downturn. This paper analyzes—both empirically and theoretically—how fluctuations in uncertainty interact with financial market imperfections in determining economic outcomes. Using both aggregate time-series and firm-level data, we find strong evidence supporting the notion that financial frictions play a major role in shaping the uncertainty-investment nexus. We then develop a tractable general equilibrium model in which individual firms face time-varying uncertainty and imperfect capital markets when issuing risky bonds and equity to finance investment projects. We calibrate the uncertainty process using micro-level estimates of shocks to the firms' profits and show that the combination of uncertainty shocks and financial frictions can generate fluctuations in economic activity that are observationally equivalent to the TFP-driven business cycles.

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# 1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic returns such as labor income, sales growth, and equity valuations is one of the stylized facts of business cycle fluctuations.<sup>1</sup> The swings in the dispersion of economic returns imply that households and businesses make their decisions in an environment of time-varying uncertainty. In macroeconomics, irreversible investment provides the traditional mechanism through which changes in uncertainty affect economic activity; see for example, Bernanke [1983]; Dixit and Pindyck [1994]; Caballero and Pindyck [1996]; and more recently, Bloom [2009] and Bloom et al. [2009]. However, as shown by Abel [1983], Abel and Eberly [1993], Veracierto [2002], and Bachmann and Bayer [2009], the effect of uncertainty on aggregate investment in the presence of irreversibilities can be theoretically ambiguous, depending importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting. As a result, the literature on irreversible investment lacks a clear consensus regarding the direction of the impact on economic activity from an increase in uncertainty.

Imperfections in financial markets—owing to agency problems between borrowers and the lenders—provide an alternative, though not necessarily an exclusive, channel through which fluctuations in uncertainty can affect economic outcomes. In the standard framework used to price corporate debt (e.g., Merton [1974]), the payoff structure of levered equity—under limited liability—resembles the payoff of a call option, while the bondholders face the payoff structure that is equivalent to that of an investor writing a put option. An increase in the riskiness of the firm’s assets thus benefits equity holders at the expense of bondholders, implying a rise in the default-risk premium to compensate bondholders for increased uncertainty.

The link between uncertainty and yield spreads on corporate debt, however, does not necessarily have any direct implications for capital formation. Under the assumption of frictionless financial markets, firms can avoid paying a high default-risk premium in credit markets by issuing new equity or relying on retained earnings to fund their capital expenditures. However, internal funds may be scarce or altogether unavailable, while issuing new equity in the presence of capital market imperfection may be subject to significant agency costs.<sup>2</sup> In such an environment, the firm’s choice of capital structure involves a nontrivial trade-off between debt and equity financing.

The aim of this paper is to investigate—both empirically and theoretically—the relation-

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<sup>1</sup>See, for example, Storesletten et al. [2004]; Campbell and Taksler [2003]; Eisfeldt and Rampini [2006]; and Bloom et al. [2009].

<sup>2</sup>As shown by Myers [2000], for example, if the management of a firm cannot commit itself to report truthfully its earnings to outside investors, the shares of the firm will be sold at discount, thereby raising the effective cost of capital for the firm.

ship between uncertainty and investment in the context of imperfect financial markets. Our first step is to construct a proxy for idiosyncratic time-varying economic uncertainty using daily firm-level stock returns for the U.S. nonfinancial corporate sector. We use the common (time-varying) component of this uncertainty measure to examine the dynamic interaction between output, investment, uncertainty, and credit spreads on corporate debt—an indicator of the degree of financial frictions—within a structural vector autoregression (VAR) framework. We then extend the analysis by constructing a new panel data set that combines information on prices of individual corporate bonds trading in the secondary market with our estimates of firm-specific uncertainty and the issuers’ income and balance sheet information. The panel aspect of this data set enables us to examine the relationship between credit spreads and uncertainty at the firm level and to provide new evidence on the interplay of uncertainty and financial conditions and its effect on the firm-level investment.

Our empirical results indicate that conditions in the corporate debt markets—as summarized by the level of credit spreads—are an important conduit through which fluctuations in uncertainty are propagated to the real economy. Unanticipated increases in uncertainty lead to a significant widening of credit spreads, a drop in output and a protracted decline in business fixed investment. Results from our panel-data analysis confirm our aggregate time-series findings: Conditional on the firm’s leverage, profitability, and other indicators of credit quality, our firm-specific measure of uncertainty is an important determinant—both economically and statistically—of credit spreads on the firm’s outstanding bonds. According to our results, an increase in uncertainty of 10 percentage points boosts credit spreads about 15 basis points, an economically substantial effect given the extent of the observed variation in uncertainty.

We also find that conditional on the investment fundamentals—that is, proxies for the marginal product of capital—the long-run elasticity of investment demand with respect to uncertainty lies in the range between -0.70 and -0.40, implying that a 10 percentage point increase in uncertainty leads to a decline in the investment rate between 2.0 and 3.5 percentage points. However, once the information content of credit spreads is taken into account, the impact of uncertainty on investment ceases to be statistically or economically significant. Capital formation, in contrast, remains highly sensitive to the firm-specific financial conditions, with a 100 basis points rise in credit spreads leading to a drop in the investment rate of more than a full percentage point in the long run. All told, the confluence of our aggregate and firm-level results strongly supports the notion that the impact of uncertainty on investment is influenced importantly by the presence of significant financial market frictions.

To provide a theoretical context for our empirical findings, we construct a tractable bond-contracting model of the type analyzed by Bernanke et al. [1999], Cooley and Quadrini

[2001], Hennessy and Whited [2007], and Philippon [2009]. We embed this contracting framework into a standard capital accumulation problem, in which firms employ a production technology that is subject to a persistent idiosyncratic shock, the variance of which is allowed to vary over time according to a stochastic law of motion. The firms make investment decisions subject to a full range of choices regarding their capital structure—internal funds, debt, and equity financing—in an environment where external funds are costly because of frictions in financial markets.<sup>3</sup> Our theoretical analysis complements that of Christiano et al. [2009], who emphasize the effect of changes in uncertainty on macroeconomic outcomes using a DSGE model with financial frictions derived from the financial accelerator model developed by Bernanke et al. [1999]. In contrast, to the framework used by Bernanke et al. [1999] and Christiano et al. [2009], we adopt a dynamic contracting environment, which allows for a meaningful degree of heterogeneity in the joint distribution of productivity and net worth across firms in the economy.

The model simulations accord well with our empirical results in a number of dimension. An increase in uncertainty causes corporate bond prices to fall and credit spreads to widen immediately as investors seek greater protection against the increased downside risk. The rise in private yields pushes up the effective cost of capital, because the firms cannot costlessly replace debt with new equity to finance their investment projects. As a result, aggregate investment falls in response to an increase in uncertainty. Moreover, the “credit crunch” resulting from an increase in uncertainty generates countercyclical fluctuations in the measured total factor productivity (TFP), which lead to a contraction not only in investment but also in consumption, a result that stands in contrast to that of Christiano et al. [2009], where the uncertainty-induced drop in investment is accompanied by a consumption boom.

In our model, fluctuations in TFP reflect capital reallocation—the movement of capital from less- to more-productive firms—due to the persistent differences in the productivity of heterogeneous firms. The interaction of financial frictions with the time-varying uncertainty limits the effectiveness of this reallocation mechanism: In periods of elevated uncertainty, firms with a high marginal product of capital, partly because of their smaller size, find it difficult to expand productive capacity because of the limited amount of collateral that can be pledged in the debt markets. Thus, capital reallocation in our model is countercyclical, consistent with the evidence reported by Eisfeldt and Rampini [2006], who motivate the procyclical nature of capital reallocation by assuming countercyclical capital adjustment

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<sup>3</sup>Cooley and Quadrini [2001], Hennessy and Whited [2007], and Philippon [2009] consider similar contracting frameworks, though only in partial equilibrium. Bernanke et al. [1999] do allow for general equilibrium feedback effects but consider only debt financing. In our setting, the combination of persistent idiosyncratic productivity shocks and a debt-renegotiation problem delivers a considerably richer set of dynamic implications, because the joint distribution of productivity shocks and the condition of the firms’ balance sheets become the state variables of the model.

costs. In our model, by contrast, the presence of financial market frictions generates an endogenous increase in the cost of reallocation during the uncertainty-induced economic downturns, leading to the type of productivity dynamics emphasized by Kiyotaki [1998].<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 contains our empirical findings; the first part focuses on the aggregate time-series evidence regarding the role that financial frictions play in shaping the dynamic response of the economy to fluctuations in uncertainty; the second part buttresses our time-series results with extensive firm-level evidence on the link between uncertainty, credit spreads, and capital formation. The theoretical model is presented in Section 3. Section 4 describes the calibration of the model’s parameters, while Section 5 presents our simulation exercises and discusses the model’s implications against the background of our earlier empirical findings. Section 6 concludes.

## 2 Empirical Evidence

This section documents the empirical evidence in support of the hypothesis that financial frictions act as an important conduit in transmitting fluctuations in uncertainty to the macroeconomy. First, we describe the methodology used to construct our benchmark estimate of time-varying uncertainty. We then analyze the interaction between uncertainty, financial conditions, and the real economy using aggregate time-series data. Finally, we explore this interaction using a newly constructed firm-level panel data set.

### 2.1 Measuring Time-Varying Economic Uncertainty

We utilize high-frequency (i.e., daily) firm-level equity returns to construct our benchmark estimate of time-varying economic uncertainty. The advantage of using equity valuations to measure uncertainty is that asset prices should, in principle, encompass all aspects of the firm’s environment that investors view as important. Specifically, from the Center for Research in Security Prices (CRSP) data base, we extracted daily stock returns for all U.S. nonfinancial corporations with at least 1,250 trading days (essentially five years) of data. This selection criterion yielded a panel of 10,729 firms over the period from July 1, 1963 (1963Q3) to December 31, 2009 (2009Q4).<sup>5</sup>

Our benchmark estimate of uncertainty is based on the following three-step procedure. In the first step, we remove the systematic component of (excess) equity returns using the

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<sup>4</sup>More recently, the role of resource misallocation in shaping productivity dynamics has been analyzed by Kleenow and Hsieh [2009] and Basu et al. [2009].

<sup>5</sup>To ensure that our results were not driven by a small number of extreme observations, we eliminated all observations with a daily absolute return in excess of 100 percent.

standard Fama and French [1992] 3-factor model:

$$(r_{it_n} - r_{t_n}^f) = \alpha_i + \beta_i^M (r_{t_n}^M - r_{t_n}^f) + \beta_i^{SMB} SMB_{t_n} + \beta_i^{HML} HML_{t_n} + u_{it_n}, \quad (1)$$

where  $i$  indexes firms and  $t_n$ ,  $n = 1, \dots, N$ , indexes trading days in quarter  $t$ . In equation (1),  $r_{it_n}$  denotes the (total) log return of firm  $i$ ;  $r_{t_n}^f$  is the continuously-compounded 3-month Treasury yield (i.e., the risk-free rate);  $r_{t_n}^M$  is the value-weighted (total) log return for the market as a whole; and  $SMB_{t_n}$  and  $HML_{t_n}$  are the Fama-French “risk” factors.

In the second step, we calculate the quarterly standard deviation of daily abnormal returns for each firm  $i$ :

$$\sigma_{it} = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (\hat{u}_{it_n} - \hat{\bar{u}}_{it})^2}, \quad (2)$$

where  $\hat{u}_{it_n}$  denotes the OLS residual from equation (1) and  $\hat{\bar{u}}_{it} = N^{-1} \sum_{n=1}^N \hat{u}_{it_n}$  is the sample mean of daily abnormal returns in quarter  $t$ . Thus,  $\sigma_{it}$  is an estimate of time-varying equity volatility for firm  $i$ , a measure that abstracts from the common risk factors that drive differences in expected returns across firms. To gain a sense of heterogeneity in this measure of idiosyncratic uncertainty, Figure 1 shows the histogram of firm-specific means of  $\sigma_{it}$ . Two features of the distribution are worth noting: First, at almost 61 percent (annualized), the average level of idiosyncratic uncertainty is quite high. And second, there is considerable dispersion in the average level of uncertainty across firms, with the range running from about 13 to more than 320 percent.

In the final step, we assume that the firm-specific uncertainty in equation (2) follows an autoregressive process of the form:

$$\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}, \quad |\rho| < 1 \text{ and } \epsilon_{it} \sim N(0, \omega^2). \quad (3)$$

In equation (3),  $\gamma_i$  denotes a firm fixed effect intended to control for the cross-sectional heterogeneity in  $\sigma_{it}$  depicted in Figure 1, whereas the firm-specific term  $\delta_i t$  captures the secular upward trend in the idiosyncratic risk of publicly-traded U.S. firms documented by Campbell et al. [2001]. Our benchmark estimate of time-varying macroeconomic uncertainty is the sequence of time fixed effects  $v_t$ ,  $t = 1, \dots, T$ , which captures shocks to idiosyncratic volatility that are common to all firms. We estimate the parameters of equation (3) by OLS, which yields an estimate of  $\rho = 0.423$ , an indication that idiosyncratic equity volatility tends to be fairly persistent.<sup>6</sup> The specification also fits the data quite well, explaining almost 75 percent of the variation in the dependent variable.

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<sup>6</sup>Because the average firm is in the panel for almost 60 quarters, the bias of the OLS estimator, owing to the presence of a lagged dependent variable and firm fixed effects, is negligible (see, for example, Nickell [1981]).

Figure 2 shows our benchmark estimate of time-varying uncertainty derived from the estimated time fixed effects in equation (3).<sup>7</sup> The figure also plots the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield, an indicator of conditions in the corporate debt markets. Clearly evident is the fact that both series are countercyclical, typically rising sharply before recessions.

## 2.2 Aggregate Time-Series Evidence

In this section, we investigate the interaction between our benchmark estimate of economic uncertainty, business financial conditions, and real economic activity. We begin by estimating several variants of the following aggregate investment regression:

$$\Delta i_t = \alpha + \beta_1 v_{t-1} + \beta_2 s_{t-1} + \sum_{k=1}^K \gamma'_k \mathbf{z}_{t-k} + \epsilon_t, \quad (4)$$

where  $i_t$  denotes the logarithm of real business fixed investment,  $v_t$  is our benchmark estimate of economic uncertainty,  $s_t$  is the 10-year BBB-Treasury credit spread, and  $\mathbf{z}_t$  is a vector of variables that includes output growth and real long-term interest rates, which are included to control for the standard accelerator and the user cost of capital effects.

Results of this exercise are presented in Table 1. According to column 1, uncertainty is an important predictor—both statistically and economically—of future growth in capital expenditures: An increase in uncertainty of 10 percentage points in quarter  $t$  leads to a decline in the growth of investment of about 2 percentage points (at an annual rate) in the subsequent quarter.<sup>8</sup> Columns 2–3 indicate that the credit spread is a highly significant predictor of future investment growth. The magnitude of the coefficient on the BBB-Treasury spread implies that a 100 basis points jump in spreads leads to a decline in the growth rate of investment of about 3 percentage points (at an annual rate) in the following quarter. Moreover, controlling for financial conditions in the corporate bond market, noticeably attenuates the effect of uncertainty on investment.

The next set of empirical exercises explores the dynamic interaction between uncertainty, credit spreads, and the real economy. In particular, we estimate a VAR consisting of the following six endogenous variables: the logarithm of real GDP ( $y_t$ ); the logarithm of real business fixed investment ( $i_t$ ); the logarithm of the GDP price deflator ( $p_t$ ); the (nominal) effective federal funds rate ( $f_t$ ) as an indicator of the stance of monetary policy; the 10-year BBB-Treasury credit spreads ( $s_t$ ); and our benchmark estimate of time-varying

<sup>7</sup>To ease the interpretation, the estimates of  $v_t$ ,  $t = 1, \dots, T$ , have been re-scaled and expressed in annualized percent.

<sup>8</sup>As shown in Figure 2, fluctuations in uncertainty of such magnitude have been quite common over the past 45 years; the standard deviation of our benchmark estimate of uncertainty is about 13 percentage points.

uncertainty ( $v_t$ ). This VAR is estimated over the 1963Q3–2009Q4 period using four lags of each endogenous variable and, in addition to a constant term, also includes dummy variables for 1987Q4 and 2008Q4 as two additional exogenous regressors.<sup>9</sup>

We focus on the implications of two types of shocks: uncertainty and financial (i.e., shocks to credit spreads). To identify these disturbances, we employ standard recursive ordering techniques. The order of endogenous variables in the first identification scheme  $(y_t, i_t, p_t, v_t, s_t, f_t)$  implies that shocks to uncertainty have a contemporaneous effect on credit spreads, while financial disturbances affect uncertainty with a lag. In contrast, the second identification scheme, which uses the ordering  $(y_t, i_t, p_t, s_t, v_t, f_t)$ , implies the opposite. These two sets of identifying assumptions provide a useful framework for evaluating the importance of financial conditions in propagating shocks to uncertainty to the broader economy.

Figure 3 plots the impulse response functions of selected variables to uncertainty and financial shocks orthogonalized using the first identification scheme. Given these identifying assumptions, an unanticipated increase in uncertainty causes an immediate widening of corporate credit spreads. Moreover, this uncertainty shock has significant adverse consequences for the real economy. Output declines almost immediately, reaching a trough about a year after the initial spike in uncertainty. The response of investment is considerably more pronounced and protracted, as capital spending falls steadily, bottoming out almost a full percentage point below the trend five quarters after the shock. A financial shock, which causes an increase of about 25 basis points in the 10-year BBB-Treasury spread, similarly leads to a significant contraction in economic activity. However, the shock to credit spreads has no discernible effect on uncertainty.

Figure 4 shows the implications of uncertainty and financial shocks orthogonalized using a scheme in which credit spreads are ordered before uncertainty. Using these identifying assumptions, an unanticipated increase in uncertainty has no statistically discernible effect on the real economy. Financial shocks, in contrast, have significant and long-lasting effects on both output and investment. A one standard deviation shock to the 10-year BBB-Treasury spread is associated with an immediate jump in uncertainty, a substantial decline in real GDP, and a protracted fall in business fixed investment. Indeed, the magnitude and the shape of the impulse response functions of both output and investment are very similar to those shown in Figure 3.

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<sup>9</sup>The inclusion of these two dummy variables is motivated by the fact that the volatility spike in 1987Q4 and the surge in uncertainty and credit spreads during the period of acute financial turmoil in late 2008 appear to be well outside historical norms. Indeed, standard regression diagnostics indicate that these two observations exert an unduly large influence on the estimated coefficients, especially in the uncertainty and credit spread equations. By including these two dummy variables in the VAR, we ensure that our results are not driven by a small number of extreme observations. Nonetheless, the results reported in the paper are robust to the exclusion of these two dummies from the VAR.

In summary, the time-series evidence presented above implies that an increase in uncertainty leads to an economically and statistically significant widening of credit spreads on corporate bonds, a drop in output, and a protracted decline in business fixed investment. The evidence also suggests that changes in credit conditions are an important part of the transmission mechanism propagating uncertainty shocks to the real economy. Indeed, once shocks to uncertainty are orthogonalized with respect to the contemporaneous information from the corporate bond market, uncertainty shocks have no statistically significant effect on economic activity.

## 2.3 Firm-Level Evidence

In this section, we utilize a new firm-level data set to provide additional evidence regarding the role of financial market frictions as a determinant of investment dynamics in response to fluctuations in economic uncertainty. Following Leahy and Whited [1996], our empirical strategy involves regressing investment on the firm-specific estimate of idiosyncratic uncertainty, while controlling for the fundamental determinants of investment spending.

Given our focus on the interaction between uncertainty and financial frictions, our regression specification also includes credit spreads at the level of an *individual* firm. To that purpose, we constructed a panel data set of almost 1,000 publicly-traded nonfinancial firms covered by CRSP and S&P's Compustat over the 1973–2009 period. The distinguishing characteristic of these large U.S. corporations is that a significant portion of their outstanding liabilities is in the form of long-term bonds that are actively traded in the secondary market. We use the secondary market prices of individual securities to construct firm-level credit spreads, which are then matched to the issuer's income and balance sheet data. The description of the bond-level data set and the details regarding the construction of credit spreads are contained in Appendix A.

### 2.3.1 Uncertainty, Credit Spreads, and Investment

The first empirical exercise using our firm-level data examines the link between credit spreads and uncertainty. In particular, we use OLS to estimate the following (reduced-form) bond-pricing equation:

$$\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R_{it}^E + \beta_3 [\Pi/A]_{it} + \beta_4 \log [D/E]_{i,t-1} + \theta' \mathbf{x}_{it}[k] + \epsilon_{it}[k], \quad (5)$$

where  $\log s_{it}[k]$  is the credit spread on a bond issue  $k$  in period  $t$ , a security that is a liability of firm  $i$ .<sup>10</sup> In addition to our estimate of idiosyncratic uncertainty given by equation (2),

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<sup>10</sup>Although our data on credit spreads are at a monthly frequency, the requisite income and balance sheet information from Compustat is available only at a quarterly frequency. In addition, the firms' fiscal

credit spreads are allowed to depend on the firm’s repayment prospects, as measured by the firm’s realized quarterly return on equity  $R_{it}^E$  and the ratio of operating income to assets  $[\Pi/A]_{it}$ , while the ratio of the book value of total liabilities to the market value of the firm’s equity—denoted by  $[D/E]_{it}$ —captures the strength of the firm’s balance sheet. The vector  $\mathbf{x}_{it}[k]$  contains variables capturing bond- or firm-specific characteristics that could influence bond yields through either liquidity or term premiums, including the bond’s duration, the amount outstanding, the bond’s (fixed) coupon rate, and an indicator variable that equals one if the bond is callable and zero otherwise.<sup>11</sup>

Table 2 contains these estimation results. According to column 1, an increase in uncertainty leads to a significant widening of credit spreads—the elasticity estimate of 0.843 implies that an increase in uncertainty of 10 percentage points in quarter  $t$  will boost credit spreads more than 50 basis points. The coefficients on the remaining key variables are also economically and statistically highly significant and have their expected signs: Strong profitability performance, as evidenced by a high realized return on equity or an increase in the ratio of operating income to assets, is associated with a narrowing of credit spreads, whereas an increase in the debt-to-equity ratio leads to a rise in credit spreads.

These results are robust to the inclusion of fixed credit rating effects (column 2) and to the inclusion of fixed industry effects (column 3). The specification in column 4 also controls for macroeconomic developments by including a full set of time dummies in the regression. Although the magnitude of the coefficient on uncertainty diminishes appreciably in this specification, the impact of uncertainty on credit spreads remains statistically significant and economically important: A 10 percentage point increase in uncertainty is associated with a rise in credit spreads of about 15 basis points. These results provide compelling evidence that fluctuations in uncertainty influence business financing conditions by significantly altering the level of credit spreads in the corporate bond market.

We now turn to the link between investment, uncertainty, and credit spreads. Our empirical investment equation is given by the following regression specification:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it}, \quad (6)$$

where  $[I/K]_{it}$  denotes the investment rate of firm  $i$  in period  $t$  (i.e., the ratio of capital expenditures in period  $t$  to the capital stock at beginning of the period);  $\sigma_{it}$  is our estimate of idiosyncratic uncertainty;  $s_{it}$  is the credit spread on the portfolio of bonds issued by

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years/quarters end at different months of the year. The timing of our firm-level data reflects these differences, as our observations occur at different months but are spaced at regular quarterly (i.e., three-month) intervals.

<sup>11</sup>Specification (5) is similar to those used by Bharath and Shumway [2008] to predict credit default swap and corporate bond yield spreads. As in their paper, our main explanatory variables—volatility, expected profitability, and leverage—correspond to the “naïve” constituents of the distance-to-default, which, according to the Merton [1974] model, should be a sufficient statistic for default.

firm  $i$ ; and  $Z_{it}$  is a proxy for the marginal product of capital, a variable that measures firm  $i$ 's future investment opportunities.<sup>12</sup> In addition to uncertainty, credit spreads, and investment fundamentals, the regression equation (6) includes a fixed firm effect  $\eta_i$  and a fixed time effect  $\lambda_t$ —the former controls for systematic differences in the average investment rate across firms, while the latter captures a common investment component reflecting macroeconomic factors, which can influence firm-level investment through either output or interest rates.<sup>13</sup>

We measure the investment fundamentals  $Z_{it}$  using either the current sales-to-capital ratio  $[Y/K]_{it}$  or the operating-income-to-capital ratio  $[\Pi/K]_{it}$ , two proxies for the marginal product of capital used by Gilchrist and Himmelberg [1998]. Taking logs of  $[Y/K]_{it}$  is straightforward, but because operating income may be negative, we use  $\log(c + [\Pi/K]_{it})$ —where  $c$  is chosen so that  $(c + [\Pi/K]_{it}) > 0$  for all  $i$  and  $t$ —when relying on the operating income to measure the firm's investment opportunities.<sup>14</sup> As an alternative forward-looking measure of investment fundamentals, we also consider Tobin's  $Q$ , denoted by  $Q_{it}$ .

Result in columns 1–3 of Table 3 indicate a significant role for uncertainty in the investment process. Regardless of the measure of investment fundamentals, the coefficient on uncertainty is statistically highly significant and lies in the range between -0.17 and -0.09. These estimated elasticities of investment demand with respect to uncertainty imply that a 10 percentage point increase in uncertainty depresses the investment rate between one-half and three-quarters of a percentage point. However, once the credit spreads are included in the regression, columns 4–6, uncertainty ceases to be—either statistically or economically—an important determinant of investment spending. The coefficients on credit spreads, in contrast, are statistically highly significant and economically large, with a 100 basis points rise in credit spreads implying a drop in the investment rate of the same magnitude.

As a robustness check, we also considered a dynamic specification of the form:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta_1 \log Z_{it} + \theta_2 \log[I/K]_{i,t-1} + \eta_i + \lambda_t + \epsilon_{it}.$$

In this case, we eliminated fixed firm effects using the forward orthogonal deviations trans-

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<sup>12</sup>The frequency of data on capital expenditures and capital stock is annual, but the data are recorded at different months of the year, reflecting the differences in the fiscal years across firms. As a result, the uncertainty measure  $\sigma_{it}$  in equation (6) is calculated using daily abnormal returns over the 250 trading days of the firm's fiscal year, and the credit spread is the average of the monthly credit spreads calculated over the 12 months of the firm's fiscal year. For the firms that have more than one bond issue trading in the secondary market in a given period, we calculate the portfolio spread by computing a weighted average of credit spreads on the firm's outstanding bonds, with weights equal to the market value of the issue.

<sup>13</sup>The log-log nature of regression (6) reflects the fact that the firm-level investment rates, uncertainty, and credit spreads are highly positively skewed, a feature of the data that is significantly ameliorated through the use of a logarithmic transformation.

<sup>14</sup>In principle, the estimated elasticities may depend on the constant  $c$ . In practice, however, reasonable variation in  $c$  has no effect on the estimated elasticities.

formation and estimated the resulting specification by GMM (cf. Arellano [2003]).<sup>15</sup> Although the lagged investment rate has significant explanatory power, the results regarding the effect of both uncertainty and credit spreads on investment are virtually the same as those reported in Table 3. That is, the adverse effect of increased uncertainty on investment spending is completely attenuated once the information content of credit spreads is taken into account. In contrast, the impact of the change in this measure of financial frictions is statistically and economically highly significant.

In summary, our aggregate time-series and firm-level panel analysis shows that the uncertainty-investment nexus is strongly influenced by conditions in the corporate bond market. In particular, increases in economic uncertainty are associated with a substantial widening of corporate credit spreads, which, in turn, leads to a significant contraction in economic activity. To the extent that credit spreads provide a useful barometer of the degree of frictions in the financial system, our empirical evidence indicates that financial frictions are an important conduit through which shocks in economic uncertainty are propagated to the real economy.

### 3 Structural Model

We now consider a general equilibrium framework in which fluctuations in economic uncertainty influence bond prices and investment in a manner consistent with our empirical findings. The model includes many of the salient features employed in the literature that allows for departures from the Modigliani and Miller [1958] paradigm of perfect capital markets, departures that imply a significant role for financial conditions in the determination of macroeconomic outcomes. In particular, firms use both internal and external sources of funds to finance capital expenditures. However, the presence of capital market imperfections implies that external funds command a premium and that this external finance premium increases in response to a rise in uncertainty. As a result, an increase in uncertainty causes a widening of corporate bond spreads and a reduction in aggregate investment.

Similar to the financial accelerator mechanisms emphasized by Kiyotaki and Moore [1997], Bernanke et al. [1999], Christiano et al. [2009], and Hall [2010], the effect of uncertainty on economic activity is amplified through the endogenous movements in the price of assets that serve as collateral for future borrowing. In contrast to those models of business cycle fluctuations, our framework allows for meaningful heterogeneity in the technology and financial conditions of firms in the economy. In such an environment, an increase in uncertainty impedes the amount of reallocation of factor inputs from less productive firms with high net worth to more productive firms with low net worth. As a result, fluctua-

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<sup>15</sup>The complete set of results is available from the authors upon request.

tions in uncertainty cause movements in aggregate TFP, which further amplify the effect of uncertainty on economic activity.

### 3.1 Preferences, Technology, and Shocks

We consider a model with four types of economic agents: (i) a representative household; (ii) a continuum of firms producing final goods; (iii) a continuum of firms producing capital goods; and (iv) bond (i.e., financial) specialists. The representative household lives forever and maximizes the expected sum of discounted utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t),$$

where  $c_t$  and  $h_t$  are consumption of final goods and hours worked, respectively. The utility function  $u(\cdot, \cdot)$  is strictly increasing and concave in both arguments and  $0 < \beta < 1$  denotes a subjective time preference parameter. The representative household earns a competitive market wage  $w_t$  by working  $h_t$  hours and saves by purchasing equity shares of firms that produce final goods.

Firms in the final-goods sector combine capital and labor using a decreasing returns-to-scale (DRS) Cobb-Douglas technology to produce output, which can be used for consumption or as an intermediate input in the production of new capital goods. The DRS production technology is subject to a persistent idiosyncratic productivity shock—denoted by  $z_t$ —that evolves according to

$$\log z_{t+1} = \rho_z \log z_t + \log \epsilon_{t+1}; \quad |\rho_z| < 1 \quad \text{and} \quad \log \epsilon_{t+1} \sim N(-0.5\sigma_t^2, \sigma_t^2). \quad (7)$$

The assumptions underlying the production technology can be summarized by a function

$$y_t = z_t^\nu (k_t^\alpha h_t^{1-\alpha})^\gamma, \quad (8)$$

where  $0 < \alpha < 1$  is the value-added share of capital and  $0 < \gamma < 1$  is the DRS parameter. The normalization parameter  $\nu = 1 - (1 - \alpha)\gamma$  ensures that the firm's profit function  $\pi(z_t, k_t) = z_t \pi(k_t)$  is linear in  $z_t$ .<sup>16</sup>

Because the producers of final goods employ a DRS technology, they earn strictly posi-

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<sup>16</sup>The profit function can be derived from the following static optimization problem:

$$\max_{h_t} \{z_t^\nu (k_t^\alpha h_t^{1-\alpha})^\gamma - w_t h_t\}.$$

In contrast to the quasi-fixed nature of capital, labor hours are freely adjustable within a given time period, making the profit function convex in  $\tilde{z}_t \equiv z_t^\nu$ . The parameter  $\nu$  then nullifies this convexity, making the profit function linear in  $z_t$ .

tive profits. To keep the model tractable, we do not explicitly model the firm's endogenous entry/exit decision. Rather, following Carlstrom and Fuerst [1997], Bernanke et al. [1999], and Cooley and Quadrini [2001], we assume that a constant fraction of final-goods producers exogenously exits the market in each period and that the same number of new firms enters the market within the same period, a stochastic overlapping generation structure that provides a convenient way to motivate the use of leverage by firms in the steady state without introducing a corporate income tax shield.

The capital-goods producers combine existing capital and final goods to produce new capital using a constant returns-to-scale (CRS) technology. The newly-produced capital is homogeneous and is sold at a competitive market price  $Q_t$  to the firms engaged in the production of final goods; the price  $Q_t$  denotes the price of capital goods relative to the price of final goods, the numeraire of the economy. Because of the CRS technology, the producers of capital goods earn zero profits in equilibrium, and that sector can be represented by a single firm.

The role of bond investors is to provide debt financing to firms engaged in the production of final goods. A CRS technology is available to any bond investor, and the financial industry is assumed to be competitive. As a result, bond investors earn zero profits in equilibrium. Risk pooling across different bond contracts is not allowed by assumption, and the zero-profit condition must be satisfied for each bond contract.

To model time-varying economic uncertainty, we assume that the level of idiosyncratic uncertainty associated with the production technology in the final-goods industry evolves over time according to a persistent Markov process. Specifically, we assume that  $\sigma_t$  in equation (7) follows a Markov Chain process with  $N$  states and a transition matrix  $p(\sigma_t, \sigma_{t+1})$ . In our setup, a shock to the level of uncertainty corresponds to an aggregate shock that alters the level of uncertainty faced by all firms engaged in the production of final goods. Because  $\epsilon_{t+1}$  is distributed log-normally with  $E(\epsilon_{t+1}|\sigma_t) = \exp[0.5\sigma_t^2 + E(\log \epsilon_{t+1}|\sigma_t)] = 1$ , fluctuations in uncertainty do not change the conditional expectation of the productivity shock  $z$ ; that is, an increase in uncertainty represents a mean-preserving spread to the conditional distribution of profits. As a result, fluctuations in uncertainty in our model do not have any direct implication for investment dynamics under the standard neoclassical assumptions.

Finally, we adopt a timing convention in which  $\sigma_t$  determines the distribution of  $\epsilon$ 's in the subsequent period. An increase in uncertainty in period  $t$ , therefore, represents "news" to the economic agents regarding the distribution of profits in period  $t + 1$ . To streamline notation, we let  $s_t$  denote the vector of aggregate state variables:

$$s_t = (\sigma_t, \mu_t, K_t)',$$

where  $\mu_t$  denotes the joint distribution of idiosyncratic shocks and net worth of the final-goods producers in period  $t$ , and  $K_t$  is the aggregate stock of capital. In what follows, we omit time subscripts unless clearly needed.

### 3.2 The Firm's Problem

To finance investment projects, firms producing final goods use a combination of internal and external funds, where the sources of external funds are debt and equity. Relative to internal funds, external funds command a premium, either because of the direct cost of issuing equity, or in the case of debt, because of the costs associated with default and bankruptcy. We first present the assumptions under which bond financing and default occurs in equilibrium. We then turn to equity issuance and the dynamic programming problem of the firm.

The net worth  $n$  of a firm engaged in the production of final goods is defined as

$$n \equiv z\pi(k, s) + Q(s)(1 - \delta)k - b, \quad (9)$$

where  $Q(s)(1 - \delta)k$  is the resale value of installed capital  $k$ , and  $b$  is the face value of the bond issued by the firm in the previous period;  $0 < \delta < 1$  denotes the depreciation rate of physical capital. Because we only consider one-period discount bonds, the market value of debt coincides with the face value of debt as long as the issuer does not default on its payment obligation. In the beginning of each period, all economic agents in the model observe the realization of the idiosyncratic productivity shock  $z$  and the uncertainty shock  $\sigma$ . Based on this information, the bond contract specifies the face value of the issue and the price. The price of the bond incorporates all information, including conditions under which a firm will default and how much of the face value will be recovered upon default in each state of the world.

Suppose that the firm issues a bond in the amount of  $b'$  at price  $q$ , yielding the total amount of debt financing  $qb'$ . Using these and other sources of funds, the firm purchases capital to be used in production. In the subsequent period—after observing the realization of shocks—the firm decides whether or not to fulfill its debt obligation. If the firm decides not to default, it pays the face value of the debt  $b'$  to the lender and makes its production and financial decisions for the next period. If the firm chooses to default, it enters a debt-renegotiation process with the investor. The renegotiation process is conducted under limited liability by assuming that there exists a lower bound to the net worth of the firm—denoted by  $\bar{n}$ —below which the firm cannot promise to pay back any outstanding liability. We also assume that the firm does not have an outside option other than the current

equity value of the firm, unless the net worth of the firm falls below  $\bar{n}$ .<sup>17</sup>

These two assumptions imply that the firm defaults if and only if the realized net worth is lower than the default boundary. Given the price of capital, its capital stock, and the amount of debt outstanding, the firm defaults if and only if the realized technology is lower than a threshold level  $\bar{z}$ , which is defined as the level that makes the firm's net worth equal to the following default boundary:

$$\bar{n} = \bar{z}(k', b', s')\pi(k', s') + Q(s')(1 - \delta)k' - b'. \quad (10)$$

Equation (10), in turn, defines a threshold level

$$\bar{\epsilon} \equiv \bar{\epsilon}(k', b', z, s') = \exp[\log \bar{z}(k', b', z, s') - \rho \log(z)],$$

such that the firm defaults if and only if  $\epsilon' < \bar{\epsilon}$ .

Under the limited liability, the new level of debt renegotiated by the firm and the investor—denoted by  $b^R$ —cannot exceed the upper bound of debt  $\bar{b}(k', z', s')$  that is consistent with the lower bound of the net worth:

$$b^R \leq \bar{b}(k', z', s') \equiv z'\pi(k', s') + Q(s')(1 - \delta)k' - \bar{n}.$$

The maximum recovery for the bondholder is the level of debt that brings the firm's net worth back to the lower bound of  $\bar{n}$ . We assume that default entails a dead-weight loss, captured by bankruptcy costs that are assumed to be proportional to the face value of the debt outstanding. Thus the actual recovery in the case of default is given by  $b^R - \chi b'$ , where the parameter  $0 < \chi < 1$  governs the magnitude of the bankruptcy costs and hence the degree of frictions in the bond market. As in the costly-state verification framework of Townsend [1979], the presence of bankruptcy costs captures the efficiency loss stemming from the moral hazard problems inherent in the process of credit intermediation. Note that the loss given default depends on the firm-specific as well as aggregate economic conditions.

To simplify the analysis, we assume that the firm does not have any bargaining power during the renegotiation process. Under our assumptions, it is then straightforward to show that the amount of renegotiated debt is set equal to the upper bound of the amount of debt

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<sup>17</sup>This type of bond contract is similar to that of Merton [1974], Cooley and Quadrini [2001], and Hennessy and Whited [2007]. However, in our setup, a default occurs when the net worth of the firm  $n$  hits the lower bound  $\bar{n}$ , whereas in the aforementioned literature, a default occurs when the value of equity  $V$  hits the lower bound  $\bar{V}$ . If the technology shock follows an i.i.d. process and the analysis is conducted in a partial equilibrium, the two assumptions are equivalent. However, if the technology shock is persistent or the firm's value function has other arguments, such as aggregate state variables, the two assumptions are no longer equivalent. The decision to use a lower bound for the net worth to determine the default threshold is a simplifying assumption that allows us to avoid the computationally intensive task of inverting the value function to compute the default boundary  $\bar{n}(z, s)$  in each iteration of the dynamic programming routine.

recovered—that is,  $b^R = \bar{b}(k', z', s')$ . Therefore, the recovery rate  $\mathcal{R}$  in the case of default is given by

$$\mathcal{R}(k', b', z', s') = \frac{\bar{b}(k', z', s')}{b'} - \chi.$$

The price of the bond is then equal to its discounted expected return:

$$q(k', b', z, s) = \frac{1}{1 + r(s)} E \left[ 1 + \int_{\epsilon' < \bar{\epsilon}} [\mathcal{R}(k', b', z', s') - 1] dH(\epsilon' | \sigma) \middle| z, s \right], \quad (11)$$

where  $r(s)$  denotes the risk-free rate, and  $H(\cdot)$  denotes the CDF of the log-normal distribution. Letting  $\bar{\theta}(k', b', z, s') = \frac{1}{\sigma} \left[ \log \bar{\epsilon}(k', b', z, s') + \frac{\sigma^2}{2} \right]$  and using the properties of the log-normal distribution, we can rewrite the price of the bond as

$$\begin{aligned} q(k', b', z, s) &= \frac{1}{1 + r(s)} E \left[ 1 - \Phi(\bar{\theta}(k', b', z, s')) \right. \\ &\quad + \Phi(\bar{\theta}(k', b', z, s') - \sigma) \frac{z^\rho \pi(k', s')}{b'} \\ &\quad \left. + \Phi(\bar{\theta}(k', b', z, s')) \left( \frac{Q(s')(1 - \delta)k' - \bar{n}}{b'} - \chi \right) \middle| z, s \right]; \end{aligned}$$

where  $\Phi(\cdot)$  denotes the standard normal CDF.

The asset-pricing equation (11) was derived under the assumption of risk-neutral bond investors who discount future returns using the risk-free rate. Because corporate bond default rates are countercyclical, while recoveries tend to be procyclical, corporate bonds may be a bad consumption hedge and therefore may command a risk premium in addition to the compensation for the expected loss. To allow for the possibility that bond prices also reflect the market price of risk, determined by the covariation of returns with household consumption growth, we also consider the following alternative bond-pricing formula:

$$q(k', b', z, s) = E \left[ m(s, s') \left( 1 + \int_{\epsilon' < \bar{\epsilon}} [\mathcal{R}(k', b', z', s') - 1] dH(\epsilon' | \sigma) \right) \middle| z, s \right], \quad (12)$$

where  $m(s, s') = \beta u_c(s')/u_c(s)$  is the pricing kernel of the representative household. The pricing formula (12) can be derived under the alternative assumption that the household directly holds shares of the financial intermediaries.

To motivate the issuance of debt in equilibrium, we assume that the firms in our model economy face a constant probability of exit, denoted by  $\eta$ . This exogenous exit rate implies that the effective discount rate used by the firm is equal to  $(1 - \eta)m(s, s')$ . As a result, the risk-free rate  $1/E[m(s, s')|s]$  is less than  $1/(1 - \eta)E[m(s, s')|s]$ , the inverse of the firm's discount factor, and the firms are induced to hold a positive amount of debt in equilibrium. The exogenous exit shock occurs after the firm makes the payment decision on its existing

debts ( $b$ ), but before making its investment ( $k'$ ) and borrowing decision ( $b'$ ) for the current period. As a result, the exit shock does not directly affect the returns of bond investors.

At the margin, the firms will only issue debt if equity issuance is also costly. We therefore assume the existence of a lower bound on dividends—denoted by  $\bar{d}$ —and a function governing the cost of issuing equity.<sup>18</sup> Specifically, the functional form of the per-unit cost of issuing equity is given by

$$\lambda(e) = \lambda_1 + \frac{\lambda_2}{2}e; \quad \lambda_1, \lambda_2 > 0,$$

where  $e$  is the amount of equity issued by the firm.

Given our setup, the firm's problem can be expressed recursively. Let  $d$  denote the firm's dividend:

$$d = z\pi(k, s) - Q(s)[k' - (1 - \delta)k] - b + qb' + e. \quad (13)$$

The value of the firm then solves the following dynamic programming problem:

$$\begin{aligned} V(n, z, s) &= \min_{\phi} \max_{k', b', d, e} \left\{ d + \phi(d - \bar{d}) - [1 + \lambda(e)]e \right. \\ &\quad \left. + (1 - \eta)E \left[ m(s, s') \max \{ V(n', z', s'), V(\bar{n}, z', s') \} \middle| z, s \right] \right\} \\ \text{s.t.} & \\ n' &= z'\pi(k', s') + Q(s')(1 - \delta)k' - b'; \\ \mu' &= \Gamma(\mu, K, \sigma, \sigma'); \end{aligned} \quad (14)$$

where  $\phi$  is the Lagrange multiplier associated with the dividend constraint  $d \geq \bar{d}$ , and  $\Gamma$  is the aggregate law of motion for the joint distribution of the idiosyncratic productivity shock and net worth. The firm's continuation value is truncated by the default payoff and can be expressed as

$$\begin{aligned} E \left[ m(s, s') \max \left\{ V(n', z', s'), V(\bar{n}, z', s') \right\} \middle| z, s \right] = \\ E \left[ m(s, s') \left( \int_{\epsilon' < \bar{\epsilon}} V(\bar{n}, z'(\epsilon'), s') d\Phi(\epsilon') + \int_{\epsilon' \geq \bar{\epsilon}} V(n', z'(\epsilon'), s') d\Phi(\epsilon') \right) \middle| z, s \right]. \end{aligned}$$

The first-order condition for equity issuance equates the shadow value of dividends to the marginal cost of issuance:

$$1 + \phi = 1 + \lambda(e) + \lambda'(e)e,$$

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<sup>18</sup>This modeling device is present in Cooley and Quadrini [2001], Gomes et al. [2006], Hennessy and Whited [2007], and Jermann and Quadrini [2007]. More generally, costly equity issuance can be motivated by the existence of moral hazard or informational asymmetries between insiders (managers) and outside equity investors.

which implies that  $\phi > 0$  when  $e > 0$ . In other words, it is never optimal for the firm to pay out more than the dividend bound  $\bar{d}$  while issuing equity. Because equity financing is costly, a dollar of issuance reduces the value of existing shares more than a dollar, where the additional discount is given by  $\lambda'(e)e$ . The optimality of the firm's financial policy requires the firm to be indifferent between debt and equity finance. Accordingly, the first-order condition for debt issuance implies that

$$q(k', b', z, s) + q_b(k', b', z, s)b' = E \left[ m(s, s') \int_{\epsilon' \geq \bar{\epsilon}} \left( \frac{1 + \lambda(\epsilon') + \lambda'(\epsilon')\epsilon'}{1 + \lambda(e) + \lambda'(e)e} \right) d\Phi(\epsilon') \middle| z, s \right], \quad (15)$$

where the term  $q_b(k', b', z, s)b'$  captures the effect of increased leverage on borrowing costs.

The optimality conditions for capital accumulation imply the following Euler equation for investment:

$$Q(s) = q_k(k', b', z, s)b' + (1 - \eta)E \left[ m(s, s') \int_{\epsilon' \geq \bar{\epsilon}} \frac{1 + \phi'}{1 + \phi} [z' \pi_k(k', s') + (1 - \delta)Q(s')] d\Phi(\epsilon') \middle| z, s \right]. \quad (16)$$

This Euler equation has several non-neoclassical features. First, for any given level of borrowing  $b'$ , an increase in capital raises the amount of available collateral and lowers the threshold level of technology at which default occurs, effects captured by the term  $q_k(k', b', z, s)b' > 0$ . Second, the firm discounts the future cash-flows using the stochastic discount factor  $(1 + \phi')/(1 + \phi)$ , which is determined by the trade-off between debt and equity financing. Lastly, the expected marginal benefit of investment is truncated by the default boundary  $\bar{\epsilon}(k', b', z, s')$ , a consequence of introducing strategic default into the firm's optimization problem.

### 3.3 Market Clearing

The solution of the firm's problem described above determines the demand for capital at the level of an individual firm. Aggregate demand for capital can then be constructed by aggregating the individual demand functions:

$$I(s) = \int k'(n, z, s) di - (1 - \delta)K(s_{-1}),$$

where  $i$  indexes the continuum of firms in the final-good sector, and the argument  $n$  of function  $k'(\cdot, \cdot, \cdot)$  pertains to the post-renegotiation value of the firm's net worth.

The price of capital is determined by the supply and demand for capital at the aggregate level. The capital-goods sector, employing a CRS technology, takes the un-depreciated capital  $K$  and final goods  $I$  as inputs to produce new capital  $K'$ . The new capital is

sold to the producers of final goods at a unit price  $Q(s)$ . Capital accumulation is subject to adjustment costs  $\xi(I(s)/K(s_{-1}))K(s_{-1})$ , where the function  $\xi(\cdot)$  is strictly convex. The zero-profit condition and market clearing imply that the unit price of capital in our economy must satisfy<sup>19</sup>

$$Q(s) = 1 + \xi'(I(s)/K(s_{-1})).$$

The efficiency conditions for the representative household can be summarized by a complete set of asset-pricing equations for the continuum of firms producing final goods and a first-order condition linking the marginal disutility of hours to the valuation of marginal consumption. Using the zero-profit condition from the capital-goods sector and imposing the stock market clearing conditions, the market clearing condition for consumption goods implies the following aggregate resource constraint:

$$\begin{aligned} C(s) = & Y(s) - I(s) - \xi(I(s)/K(s_{-1}))K(s_{-1}) \\ & - \int \left[ \mathbf{1}(n(i, s) \leq \bar{n})\chi b(i, s) + \mathbf{1}(e(i, s) \geq 0)\lambda(e(i, s))e(i, s) \right] di, \end{aligned}$$

where  $Y(s) = \int y(i)di$  and  $\mathbf{1}(\cdot)$  denotes the indicator function that equals one if the argument is true and zero otherwise.<sup>20</sup> Compared with a frictionless real business cycle model, this constraint has two non-standard terms: the bankruptcy costs and equity issuance costs, which represent the loss of resources due to capital market imperfections. Because these costs are small relative to aggregate output, financial frictions modify the macroeconomic equilibrium primarily by altering the first-order conditions of the agents, rather than by directly affecting the available resources.

To fully solve the problem, economic agents need to understand how the aggregate state variables evolve over time. One of the aggregate state variables is the joint distribution of net worth and technology across heterogeneous firms. The exact law of motion for this joint

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<sup>19</sup>The optimization problem of the capital-goods sector—normalized by  $K$  given the CRS technology—can be formulated as

$$\max_{I(s)/K(s_{-1})} \left\{ Q(s) \left[ \frac{I(s)}{K(s_{-1})} + (1 - \delta) \right] - \frac{I(s)}{K(s_{-1})} - Q(s)(1 - \delta) - \xi \left( \frac{I(s)}{K(s_{-1})} \right) \right\}.$$

Because of the capital adjustment costs, the value of existing capital depends separately on the joint distribution  $\mu$  of net worth and technology, as well as on the current aggregate capital stock  $K$ , as indicated by the vector of aggregate state variables  $s = (\sigma, \mu, K)'$ .

<sup>20</sup>We assume that in the aggregate resource constraint there is no loss of output due to the exogenous exit of firms. That is, “death shocks” are realized after the firms produce output, and we assume that an entrant who replaces an exiting firm inherits all of its real and financial characteristics. The entry/exit process is thus fully frictionless and plays no role in the model, other than creating a wedge between the internal rate of discounting and the risk-free rate.

distribution is given by

$$\mu(N_0, Z_0) = \int_{N_0 \times Z_0} \left[ \int_{N \times Z} \mathbf{1}(n' = \max \{ \bar{n}, [z' \pi(k'(n, z, s), s')] \right. \\ \left. + Q(s')(1 - \delta)k'(n, z, s) - b'(n, z, s)] \} G(z'|z, \sigma) d\mu) \right] dn' dz', \quad (17)$$

where  $N \subseteq \mathbb{R}$ ,  $Z \subseteq \mathbb{R}_{++}$ , and  $\mu$  is a measure on the measurable space  $(N \times Z, \mathcal{N} \times \mathcal{Z})$ , where  $\mathcal{N}$  and  $\mathcal{Z}$  denote Borel sigma algebras generated by the subsets of  $N$  and  $Z$ , respectively. Note that  $\mu(N_0, Z_0)$  measures the proportion of firms with the net worth and technology in  $N_0 \times Z_0$  next period, where  $N_0 \in \mathcal{N}$  and  $Z_0 \in \mathcal{Z}$ . In equilibrium, this measure depends on (i) the firms' investment and debt policy functions  $k'(n, z, s)$  and  $b'(n, z, s)$ ; (ii) the transition function  $G(z'|z, \sigma)$  of the idiosyncratic productivity shock  $z$ ; and (iii) the aggregate market clearing conditions.<sup>21</sup>

Given the infinite dimensionality of our state space, it is not feasible to solve the problem exactly. Following the literature on computable general equilibrium with heterogeneous agents (e.g., Ríos-Rull [1995], Krusell and Smith [1998], and Khan and Thomas [2008]), we adopt the assumption of bounded rationality—that is, the agents concern themselves with only a finite number of moments of the distribution and use them in log-linear functional forms to forecast equilibrium prices. For computational purposes, agents in our model carry with them only the first moments of the distribution of net worth and capital as state variables. Agents use these state variables to forecast the three prices needed to solve their optimization problems: the marginal utility of the representative consumer ( $u_c(s)$ ); real wage ( $w(s)$ ); and the price of capital ( $Q(s)$ ). The approximate laws of motion are given by the following system of linear regressions:

$$\log \mathbf{y} = \mathbf{C}(\sigma, \sigma_{-1}) + \mathbf{B} \log \mathbf{y}_{-1} + \mathbf{e}, \quad (18)$$

where the vector  $\mathbf{y}$  includes the marginal utility of consumption  $u_c(s)$ , the aggregate net worth  $N(s)$ , and the aggregate capital stock  $K'(s)$ . The matrix of regression coefficients  $\mathbf{B}$  is of the form

$$\mathbf{B} = \begin{bmatrix} 0 & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix},$$

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<sup>21</sup>Note that the distribution of net worth tomorrow also depends on the price of capital tomorrow—that is,  $Q(s')$ —because the collateral value of capital tomorrow depends on  $Q(s')$ . However,  $Q(s')$  depends on the distribution of net worth and technology tomorrow, because the demand for capital tomorrow will depend on that distribution—hence the fixed point problem. Consequently, the aggregate law of motion in the firm's problem (14) is given by  $\mu' = \Gamma(\mu, K, \sigma, \sigma')$ , where we explicitly express the dependency of  $\mu'$  on  $\sigma'$  and  $\sigma$ .

where the first column of zeros reflects the fact that the marginal utility of consumption is not a state variable. In the formulation of the aggregate laws of motion, we also allow the matrix of constants  $\mathbf{C}$  in equation (18) to depend not only on the current realization of uncertainty, but also on its value in the previous period. Specifically, the system includes four distinct constant terms, corresponding to the four possible transitions for the uncertainty regime (i.e., “low-to-high,” “low-to-low,” etc.).<sup>22</sup>

## 4 Calibration

We let the time period  $t$  in our model correspond to one year—specifying the model at an annual frequency reduces computational time substantially. For the most part, our calibration relies on parameter values that are standard in the literature. However, there are a number of parameters that are specific to our model, the calibration of which we discuss below.

To calibrate the curvature of the profit function of firms engaged in the production of final goods and the parameters governing the stochastic uncertainty process, we utilize the S&P’s Compustat (quarterly) database. Specifically, we selected from the Compustat database all U.S. nonfinancial firms with at least 20 quarters of data on sales and capital over the period 1976Q1 to 2009Q4, a procedure yielding an unbalanced panel of 9,469 firms for a total of 540,409 firm/quarter observations.<sup>23</sup> To calibrate  $\gamma$ , the DRS parameter in equation (8), we use this panel to estimate the following revenue function:

$$\log Y_{it} = \beta \log K_{it} + \eta_i + \lambda_t + u_{it}, \quad (19)$$

where  $Y_{it}$  denotes (real) sales of firm  $i$  in quarter  $t$ ,  $K_{it}$  is firm  $i$ ’s (real) capital stock at the beginning of the quarter, and the error term  $u_{it}$  represents the empirical counterpart of the productivity shock  $\log z_t$  in our model. In our regression analysis, we include a firm fixed effect  $\eta_i$  to control for any unobservable (time-invariant) differences in the revenue process of individual firms, while the time fixed effect  $\lambda_t$  captures shocks affecting the profitability of all firms. Equation (19) is estimated by OLS yielding  $\hat{\beta} = 0.618$ , with the 95-percent confidence interval of  $[0.606, 0.630]$ . We calibrate  $\alpha$ , the share of capital in the

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<sup>22</sup>Importantly, this allows us to obtain a much better goodness-of-fit for the approximate aggregate laws of motion relative to the specification that allows constant terms to differ only across the two uncertainty regimes. In the latter case, the goodness-of-fit statistics for the laws of motions—as measured by the  $R^2$ —were about 0.70 for most of endogenous aggregates. However, by allowing for regime switching, we obtained much better goodness-of-fit statistics:  $R^2 = 0.951$  for the marginal utility of consumption and  $R^2 = 0.998$  for the aggregate net worth. One exception was the law of motion for aggregate capital, where  $R^2 = 0.910$  indicates a relatively poor fit.

<sup>23</sup>Prior to 1976, most firms did not report their capital stock data (i.e., net property, plant, and equipment) on the quarterly basis. To ensure that our results were not driven by a small number of extreme observations, we dropped from the sample all observations with the sales-to-capital ratio below 0.01 and above 20.

Cobb-Douglas production function (8) to be 0.30, which together with our estimate of  $\beta$  implies that  $\gamma = 0.84$ , an estimate of decreasing returns that is within the range of values estimated in the literature.

We use the residuals from the estimation of the revenue function (19) to calibrate the process for the idiosyncratic productivity shock. First, the persistence of the productivity process is obtained by estimating the following pooled regression

$$\hat{u}_{it} = \rho_z \hat{u}_{i,t-1} + \epsilon_{it},$$

which yields (at a quarterly frequency)  $\hat{\rho}_z = 0.77$ , implying the persistence of the process at an annual frequency of  $0.77^4 = 0.35$ ; in our calibration, we set  $\rho_z = 0.40$ . Second, if  $\epsilon_{it}$  is distributed normally, then  $\sqrt{\pi/2}|\hat{\epsilon}_{it}|$  is an unbiased estimator of the standard deviation of  $\epsilon_{it}$ . Figure 5 depicts the distribution of firm-specific means of this measure of idiosyncratic uncertainty. Although somewhat more diffused, the distribution of uncertainty derived from the revenue shocks is quite similar to that based on the equity valuations shown in Figure 1; indeed, based on either estimate, the level of uncertainty faced by an average nonfinancial firm is in the range between 50 to 60 percent.

To obtain a corresponding measure of time-varying uncertainty, we estimate the following panel regression:

$$\log \left[ \sqrt{\frac{\pi}{2}} |\hat{\epsilon}_{it}| \right] = \gamma_i + v_t + \zeta_{it}, \quad \zeta_{it} \sim N(0, \omega^2),$$

where  $\gamma_i$  and  $v_t$  denote fixed firm and time effects, respectively. In keeping with our earlier approach, a measure of uncertainty based on the shocks to the revenue function—shown in Figure 6—corresponds to the estimated sequence  $\hat{v}_t$ ,  $t = 1, \dots, T$ , which captures common movements in the idiosyncratic uncertainty regarding the profitability prospects in the non-financial corporate sector.<sup>24</sup> Note that like its counterpart based on equity valuations, this estimate of uncertainty is countercyclical, typically rising before an onset of an economic downturn.

In our simulations, the uncertainty process for  $\sigma_t$  is assumed to evolve according to a two-state Markov chain, with the two states corresponding to the “low” and “high” uncertainty regimes. To calibrate the Markov chain, we first estimate an AR(1) process for our measure of uncertainty based on the revenue shocks and then use the approach of Tauchen [1986] to discretize the process. Estimating  $\hat{v}_t = \mu + \rho_\sigma \hat{v}_{t-1} + e_t$ , yields an estimate of the autoregressive parameter  $\rho_\sigma = 0.82$ , with the 95 percent confidence interval of [0.72, 0.92]. We set the level of uncertainty corresponding to the low uncertainty regime—

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<sup>24</sup>To ease the interpretation, the estimates of fixed time effects  $v_t$ ,  $t = 1, \dots, T$ , have been re-scaled, seasonally adjusted using the X11 filter, and expressed in annualized percent.

denoted by  $\sigma_L$ —to 35 percent and that in the high uncertainty regime—denoted by  $\sigma_H$ —to 55 percent; the steady-state level of dispersion  $\bar{\sigma}$  is calibrated to 45 percent. The values for  $\sigma_L$  and  $\sigma_H$  correspond approximately to the 5th and 95th percentiles of the distribution of our uncertainty measures, whereas the value of  $\bar{\sigma}$  is slightly below the median of the distribution. The probability that the uncertainty regime in period  $t + 1$  will be the same as in period  $t$  is set to 0.70, implying an AR(1) representation (at an annual frequency) with  $\rho_\sigma = 0.82^4 = 0.45$ .

We calibrate the degree of financial frictions in the bond market—the bankruptcy cost parameter  $\chi$ —to match the median credit spread of 160 basis points for the 10-year BBB-Treasury spread over the 1976–2009 period. Accordingly, we set  $\chi = 0.12$ , a value consistent with that used by Bernanke et al. [1999] and the micro-level evidence of Levin et al. [2004] and one that implies a relatively modest degree of welfare loss from bankruptcy. In calibrating the survival probability, we follow Carlstrom and Fuerst [1997] and let  $1 - \eta = 0.95^4 = 0.80$ . The parametric form of per-unit cost of issuing equity  $\lambda(e) = \lambda_1 + \frac{\lambda_2}{2}e$  implies that the marginal cost of issuing shares equals  $1 + \lambda_1 + \lambda_2 e$ . We set  $\lambda_1 = 0.15$  and  $\lambda_2 = 0.50$ , values that generate a substantial price discount on newly issued equity and imply that equity is not a preferable source of external finance unless the firm is facing a substantial default-risk premium in the bond market.<sup>25</sup> This calibration generates a share of equity in total external financing of 11 percent, a proportion that is roughly in line with the average share of 8 percent reported by Bolton and Scharfstein [1996] for the U.S. corporate sector.

We consider two representations for the preferences of the representative household—a utility function that is separable in the marginal utilities of consumption and leisure and one that is not. The latter case is motivated by the fact that the driving force of economic fluctuations in our model works directly through the investment demand rather than through the resource constraint of the economy, as would be the case if the business cycles were due entirely to the fluctuations in TFP.<sup>26</sup> For our baseline case, we assume a log utility of consumption and linear disutility for hours:  $u(c, 1 - h) = \log c + \psi(1 - h)$ . In the non-separable case, we follow Greenwood et al. [1988] (GHH hereafter) and let  $u(c, 1 - h) = \log[c - (\psi/\theta)h^\theta]$ , with the Frisch elasticity of labor supply  $1/(\theta - 1) = 1.7$ .<sup>27</sup>

<sup>25</sup>Our calibration of the cost of equity issuance falls in the range of empirical estimates found in the corporate finance literature. For example, the estimates of the cost for seasoned equity offerings range between 8 to 10 percent of the amount issued—where these costs include both the underwriting fee and the initial price discount—while the costs of private equity investment in public equity can be as high as 20 percent of the amount sold; see, for example, Hertz and Smith [1993], Wu [2004], and Chen et al. [2010].

<sup>26</sup>In such an environment, the separability of marginal utility of consumption from that of leisure has important consequences for the comovement of key macroeconomic aggregates; see, for example, Barro and King [1984] and Greenwood et al. [1988].

<sup>27</sup>In the baseline case, the first-order condition with respect to hours worked is given by  $w(s) = \psi/u_c(s) = \psi c(s)$ , which implies that the labor supply function does not have to be approximated separately from the

The subjective discounting factor  $\beta$  is set equal to  $0.99^4 = 0.96$ , so that the annual risk-free rate is equal to 4 percent in the steady state.

The annual depreciation rate of physical capital  $\delta$  is set to 18 percent, a value consistent with the firm-level Compustat data. We employ the following standard quadratic specification for the capital adjustment cost function:  $\xi(I/K) = \frac{\vartheta}{2}(I/K - \delta)^2$ . There is no clear consensus in the literature regarding the value of the adjustment cost parameter  $\vartheta$ , with the range of published estimates running from 0.13 to 20. Early work by Summers [1981] and Hayashi [1982] in particular has found a substantial degree of adjustment costs in the investment process, while the more recent work indicates that this friction is likely to be less important. For example, using a large firm-level panel, Gilchrist and Himmelberg [1995] estimated  $\vartheta$  to be around 3; using a simulation-based estimation method and plant-level data, Cooper and Haltiwanger [2006] estimated  $\vartheta$  to be 0.13, when allowing for only convex adjustment costs in the capital adjustment process. In light of this evidence, we set  $\vartheta = 1$ .<sup>28</sup>

## 5 Simulation Results

### 5.1 Bond Pricing and Investment Policy

To examine the key features of our model, we first solve the model for the bond-pricing and investment policy functions by abstracting from the aggregate variation in idiosyncratic uncertainty. Figure 7 shows the resulting bond-pricing function, in which the bond price ( $q$ ) is shown as a function of the firm's capital assets ( $k'$ ) and debt outstanding ( $b'$ ), both of which are expressed relative to their steady-state values. The pricing surface has two distinct regions: A plateau in which the firm's leverage ratio  $b'/k'$  is sufficiently low so that the default probability is essentially zero and the price of debt is insensitive to the changes in the firm's financial condition; and a downward-sloping region, in which the firm faces increasing marginal cost of borrowing, and the price drops sharply in response to an increase in leverage.

Figure 8 depicts the firm's optimal investment policy as a function of the technology level ( $z$ ) and net worth ( $n$ ), with both arguments scaled by their respective steady-state levels. The firm's investment policy also exhibits a significant nonlinearity. In particular,

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marginal utility of consumption in order to determine hours worked. As a result,  $h(s)$  can be omitted from the vector  $\mathbf{y}$  that describes the evolution of the aggregate state variables. In the case of the GHH preferences, in contrast, the first-order condition for hours worked is given by  $h(s) = [\psi w(s)]^{1/(1-\theta)}$ , which implies that the marginal utility of consumption and the labor supply function must be approximated separately.

<sup>28</sup>We find that the model dynamics are not very sensitive to the value of  $\vartheta$ , at least within the range of 0.13 to 3.0. On the one hand, a higher value of  $\vartheta$  makes investment smoother by inducing a greater friction in the capital adjustment process; on the other hand, higher capital adjustment costs make investment more volatile, because the value of collateral assets becomes more sensitive to cyclical fluctuations. The two effects tend to cancel each other out, leaving the overall investment dynamics in our model largely unchanged.

at low levels of net worth, investment—for a given technology level—is highly responsive to the movements in the firm’s net worth.

These nonlinear aspects of bond prices and investment imply that financial conditions can exert a powerful influence on the response of aggregate investment to uncertainty shocks. In Figure 9, we overlay the distribution of the debt-to-capital ratio of our model economy with the bond-pricing functions corresponding to the two uncertainty regimes.<sup>29</sup> The overall distribution—across firms and time—of the debt-to-capital ratio is bimodal, reflecting the convolution of the distributions corresponding to the two uncertainty regimes. As shown in the figure, an increase in uncertainty affects aggregate investment by boosting the firms’ borrowing costs, as evidenced by the downward shift in the bond-pricing function when the economy switches from a low to a high uncertainty regime. Faced with a significantly higher cost of debt finance, firms in the model begin to deleverage and cut back on capital expenditures aggressively. This process results in a negative comovement between uncertainty and leverage and a positive comovement between uncertainty and credit spreads, two key features of economic fluctuations driven by the interaction of uncertainty with financial market frictions.

## 5.2 Uncertainty, Credit Spreads, and Economic Fluctuations

We now report our main simulation results. We simulate the model for 400 periods (i.e., years), assuming that there are always 10,000 firms in the economy. The simulation is designed so that the average level of technology in the economy is equal to one at any point in time, while the dispersion of technology fluctuates over time according to the two-state Markov chain process. We iterate the procedure until the aggregate laws of motion converge.

The solid line in Figure 10 shows the model-implied credit spread computed using the bond-pricing formula in equation (11), that is, under the physical measure. The shaded vertical bars indicate periods in which the economy is in the high uncertainty regime.<sup>30</sup> According to our model, periods of heightened uncertainty are associated with elevated credit spreads. In the transition from the state of low uncertainty to that of high uncertainty, credit spreads jump about 150 basis points and then tend to increase another 50 basis points or so as the high uncertainty regime persists. This further worsening of credit conditions reflects the endogenous interaction between the real and financial sides of the economy. In

<sup>29</sup>The functions shown in Figure 9 are computed in a partial equilibrium setting, so that the risk-free rate is not affected by the change in the level of uncertainty.

<sup>30</sup>In all the figures that show the simulated time path of the model’s key financial and macroeconomic variables, we plot only 100 “years” of data, corresponding to the period from  $t_{151}$  to  $t_{250}$ . This range was chosen because it yielded visually appealing figures in terms of the frequency and duration of uncertainty regimes—plotting all 400 periods rendered the figures virtually unintelligible. However, all the statistics based on the simulated data are computed using the full sample of 400 observations.

particular, the transition to the state of high uncertainty depresses investment because the resulting increase in the downside risk leads to a widening of credit spreads. The initial drop in investment has adverse implications for future profits, which raises the likelihood of subsequent defaults and causes credit spreads to widen further—the financial accelerator mechanism.

Figure 11 displays the evolution of the key macroeconomic aggregates of the model, expressed in deviations from their steady-state values. As shown by the black line, the transition from a low to a high uncertainty regime is associated with an immediate drop in aggregate investment; conversely, the transition from a high to a low uncertainty regime generates an investment boom. The size of these fluctuations is quite substantial, ranging between 10 to 15 percent of the steady-state level of investment. Moreover, the endogenous movements in credit spreads amplify the initial swings in investment because of the nonlinearity of the firm’s investment policy function in the financially-sensitive region.

This link between fluctuations in uncertainty, credit spreads, and aggregate investment depends importantly on the extent to which the quality of the firms’ balance sheets makes their investment decisions sensitive to the changes in financial conditions. In our model, most firms face economically significant degree of financial frictions. In fact, only 5 percent of firms in the economy, on average, operate in the region of flat marginal costs of debt financing, a seemingly small proportion and one that suggests that our calibration may be overstating the severity of frictions in the corporate bond market. However, the degree of frictions—that is, the bankruptcy cost parameter  $\chi$ —was calibrated to match the median BBB-Treasury credit spread over the past three and a half decades, and, as shown in Figure 10, at 164 basis points, the time-series average of model-implied credit spreads closely matches its intended target of 160 basis points.

Figure 11 also shows that our model implies a high degree of comovement between aggregate consumption, investment, and output. In Table 4, we compare some standard business cycle statistics to their model-implied counterparts; the baseline case corresponds to the model with the log utility of consumption and linear disutility of hours worked, whereas the GHH model denotes the specification with non-separable preferences. The top panel of the table shows that our model successfully replicates the relative volatilities of the key macroeconomic aggregates. In particular, in both specifications, the model-implied investment is two to three times more variable than output, a result that accords well with the U.S. historical experience. Similarly, the relative volatility of the model-implied consumption appears to be roughly in line with that observed in the actual data.

The bottom panel of the table summarizes the comovement properties of the model. As shown in the middle column, our baseline case implies a strong comovement among the main endogenous quantities. With the exception of hours worked, the correlation coeffi-

cients based on the simulated data from the baseline economy closely match their empirical counterparts, a rather remarkable result given that all the comovements are generated in the absence of shocks to the aggregate resource constraint. The fact that hours are countercyclical reflects the source of fluctuations in our model. Unlike the TFP shocks, which affect the agents' behavior by altering the supply side of the economy, uncertainty shocks impact investment and thus are akin to the "investment efficiency shocks" of Greenwood et al. [1988, 1997] or "news shocks" of Jaimovich and Rebelo [2009].

As emphasized by Barro and King [1984] and Greenwood et al. [1988], a shock that increases the rate of return on investment will also cause an increase in the labor supply through the intertemporal substitution of leisure. In the absence of shocks to TFP, however, this effect implies a reduction in consumption, resulting in a negative correlation between consumption and hours worked, an anomaly that can be rectified by introducing the non-separability of consumption and leisure in the utility of the representative household.<sup>31</sup> Indeed, in the model with the GHH preferences, the correlation between output and hours is positive and of the same order of magnitude as that seen in the data; in addition, the correlation coefficient between consumption and investment accords much better with its empirical counterpart.

Importantly, our model also delivers a positive correlation between the measured TFP and output fluctuations that is very close to the one found in the U.S. data.<sup>32</sup> Although the average level of productivity in our model is not changing in response to the fluctuations in uncertainty, resources are nonetheless being reallocated from high productivity firms with low net worth to low productivity firms with high net worth because of the constraints imposed on capital formation by the increased severity of financial frictions. The resulting decline in TFP occurs simultaneously with the collapse in investment; moreover the size of the movement in TFP is correlated with the magnitude of the drop in investment. Thus from a measurement perspective, the economy has experienced a shock that, in the aggregate, appears similar to the shock to the investment-specific technological change. In principle, such shocks should be accounted for in the quality-adjusted prices of capital goods. The standard quality adjustment procedures, however, do not measure reallocation benefits, and the productivity declines associated with the drop in investment appear in the measured TFP.

The results in Table 5 indicate that our model also captures quite well the cyclical co-

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<sup>31</sup>Note that that utility function  $u(c - (\psi/\theta)h^\theta)$  provides the representative household with a powerful incentive to coordinate the level of consumption with hours worked. In this context, the object of smoothing becomes the difference between consumption and the disutility of hours worked, and the utility function penalizes opposite movements in consumption and hours.

<sup>32</sup>For this comparison, we constructed a TFP series for both the data and the model economies using the conventional formula:  $TFP = \exp[\log(Y) - 0.3\log(K) - 0.7\log(L)]$ . Correcting for the effects of a DRS technology matters very little for our conclusion.

movements between investment, the key financial variables—credit spreads and borrowers’ net worth—and uncertainty. In particular, both model specifications deliver a strong negative correlation between investment growth and credit spreads, though the magnitude of this negative comovement accords somewhat better with the actual data in the baseline case. In contrast, the GHH model is able to match much more closely the observed positive correlation between the changes in the net worth of the U.S. nonfinancial corporate sector and the growth rate of business fixed investment.<sup>33</sup> In addition, the correlation between uncertainty and investment growth in the specification with the GHH preferences is essentially identical to the one found in the data.

### 5.3 Capital Reallocation

As shown by Eisfeldt and Rampini [2006], the benefits of capital reallocation increase during economic downturns, while the actual amount of capital reallocation declines during recessions. Because the driving force of cyclical fluctuations in our model is the time-varying volatility of productivity shocks, the benefits of capital reallocation are countercyclical. To see whether or not the capital reallocation itself is procyclical, we construct the following reallocation measure:

$$\text{RAC}_t = \sum_n w_{nt} (|i_{nt}| - i_{nt}), \quad (20)$$

where  $i_{nt}$  is the investment of firm  $n$  in period  $t$  and  $w_{nt} = k_{nt} / \sum k_{nt}$  is the corresponding weight. Equation (20) measures reallocation as the amount of capital that is redeployed among heterogeneous firms—that is, the amount of capital that changes ownership across firms.<sup>34</sup> When constructing aggregate investment in the general equilibrium setting, reallocated capital drops out of the aggregation because one firm’s investment is exactly offset by another firm’s disinvestment; the reallocation measure given in equation (20) then recovers the quantity of redeployed capital omitted from the construction of aggregate investment.

Table 6 examines the cyclical properties of this reallocation measure for both the baseline and GHH model specifications. According to our results, capital reallocation in both specifications is strongly procyclical, as evidenced by the positive correlation of RAC with the key economic aggregates.<sup>35</sup> More importantly, the amount of capital reallocation moves

<sup>33</sup>The net worth data come from the Federal Reserve’s Z.1 statistical releases, “Flow of Funds Accounts of the United States.” The nominal net worth series was deflated by the implicit GDP price deflator.

<sup>34</sup>Consider the following two cases. Assume that there only two firms in the economy. In the first case, both firms increase their stock of capital by one unit. In the second case, one firm increases capital by 3 units, while the other firm disinvests itself of one unit of capital. In both cases, the aggregate investment is 2 units. However, the reallocation measure in the first case is zero, whereas in the second case it is  $[|3| + |-1| - (3 - 1)] = 2$ .

<sup>35</sup>The correlation between RAC and output shown in the table is somewhat higher than that reported by Eisfeldt and Rampini [2006], whose reallocation measure is based on the firm-level Compustat data. This difference, however, seems natural—in our model, there is only one shock that affects the dispersion

closely with the fluctuations in TFP, with the correlation coefficients in the range of 0.75 to 0.90. A similar degree of comovement was obtained by Eisfeldt and Rampini [2006] by assuming countercyclical capital adjustment costs. Our setup, in contrast, uses financial market frictions to effectively endogenize the countercyclical nature of capital adjustment costs—the intensification of financial frictions during an economic downturn limits the firm’s investment relative to its fundamentals as measured by the changes in productivity.<sup>36</sup>

## 5.4 Uncertainty and the Risk Premium

As documented in the corporate finance literature (e.g., Elton et al. [2001] and Huang and Huang [2003]), traditional debt-contracting models imply counterfactually low credit spreads—and hence significant risk premiums—given the observed probabilities of default and actual recovery rates. Our results regarding the pricing of corporate debt thus far were based entirely on the assumption of risk neutrality on the part of financial intermediaries. In this section, we examine the behavior of the bond risk premium by simulating the model with the alternative bond-pricing formula—equation (12)—which assumes that corporate debt claims are priced by risk-averse intermediaries.

Figure 12 shows the result of this exercise. It decomposes the time-path of the model-implied average credit spread computed using the risk-neutral pricing formula (12) into two components: the credit spread based on the physical measure (the black line) and the risk premium (the shaded yellow region).<sup>37</sup> Three observations are immediate: First, the use of the risk-neutral measure in the pricing equation does increase the average credit spread by inducing a risk premium component in the prices of corporate bonds. Second, the risk premium component accounts for a relatively small portion of credit spreads. And lastly, for practical purposes, the risk premium is present only during regimes of high uncertainty.

Table 7 reports the time averages of actual and model-implied credit spreads. The average risk premium in our baseline specification is only 10 basis points. However, conditional on the economy being in the high uncertainty regime, the premium is about 20 basis points, which accounts for most of the average risk premium. In fact, the risk premium is essentially zero in the low uncertainty regime. Thus, the model is unable to generate the risk premium that could systematically account for a significant portion of credit spreads.

One implication of this result is that when our model is calibrated to match the average

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of productivity. As emphasized by Eisfeldt and Rampini [2006], new investment is driven by the changes in aggregate productivity, while the amount of capital reallocation is influenced mainly by the differences in productivity across firms. In reality, however, there are likely multiple shocks that affect macroeconomic outcomes, with some of these shocks having a direct impact on the aggregate productivity, thereby dampening the degree of comovement seen in the data.

<sup>36</sup>Khan and Thomas [2008] reach a similar conclusion in a dynamic general equilibrium setup with borrowing constraints and a partial investment irreversibility.

<sup>37</sup>Results shown Figure 12 are based on the baseline specification of the model.

level of credit spreads in the U.S. economy, the model-implied default rates exceed those found in the data by a considerable margin. According to the entries in Table 8, the unconditional model-implied default rates are, on average, about 2 percentage points higher than the actual average default rate on nonfinancial corporate bonds during the 1989–2009 period. Although the model is unable to deliver empirically realistic default rates, it does predict that defaults should rise in periods of heightened uncertainty. According to Figure 13, this is indeed the case—our benchmark estimate of uncertainty is highly positively correlated with the realized bond defaults. In addition, the average actual bond default rate during the last three recessions is considerably closer to the average model-implied default rate, conditional on the economy being in the high uncertainty regime.

The inability of our model to match more closely the observed average default rate may be due in part to the more general failure of the expected utility theory to provide empirically realistic risk premiums. Figure 14 depicts the model-implied excess returns on equity and bonds, along with the aggregate investment, for our baseline specification. Both excess return series are strongly procyclical—stocks and corporate bonds tend to do significantly better than the risk-free bonds during the economic expansions, while doing much worse during the recessions. Nevertheless, the average excess return on both assets fails to price this procyclicality, an indication that the average risk premium in both markets is too low.<sup>38</sup>

## 5.5 External Financing Patterns

Our final set of results relates to the cyclical pattern of external financing. If equity financing was costless in the model, firms could avoid the increased cost of bond finance associated with the elevated uncertainty by deleveraging their balance sheets. Indeed, as shown in Figure 15, the firms in our model do increase their reliance on equity financing during the periods of heightened uncertainty. The extent of this substitution, however, is limited. In our model economy, equity, on average, accounts for about 11 percent of all external funds. Despite the sharp increase in the volume and the frequency of equity financing, the share of equity financing rises only about 10 percentage points in the high uncertainty regimes. In effect, the rising marginal cost of equity issuance limits the amount of equity issued, as

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<sup>38</sup>In an attempt to resolve this type of pricing anomalies, Gomes and Schmid [2009] incorporate the Epstein-Zin preferences into a dynamic general equilibrium setting with costly state verification, while Chen et al. [2009] embed the standard Merton framework into a partial equilibrium framework with habit formation. Both approaches make some important progress in helping to resolve the “credit spread” puzzle. However, Gomes and Schmid [2009] allow for only the extensive margin of investment and abstract from the endogenous labor supply decision, an approach that makes it difficult to analyze the business cycle implications of their model. Chen et al. [2009] show that it is necessary to introduce a procyclical default boundary—essentially a shorter distance-to-default in recessions—to generate a sufficiently large risk premium component of corporate credit spreads.

the firms attempt to equalize the marginal costs of the two sources of external funds. Given our calibration, the firms maintain equity, as a share of total external finance, at about 6 percent in the low uncertainty regime, a proportion that rises to about 16 percent in the high uncertainty regime.

According to Figure 15, the model-implied share of equity finance is countercyclical, while the leverage is procyclical. Although the latter result is consistent with the data, the model-implied pattern of equity financing runs contrary to the observed cyclical behavior of equity issuance.<sup>39</sup> This counterfactual result likely reflects the reduced-form nature of the costs governing equity issuance—an increase in uncertainty affects the marginal cost of corporate debt but has no impact on the marginal cost of equity finance. Thus, a more realistic description of the external financing patterns would allow for the fact that the firm trying to issue equity in an environment of elevated economic uncertainty may face moral hazard or asymmetric information problems similar to those encountered when trying to place debt with the bond investors.

## 6 Conclusion

According to the standard macroeconomic theory, investment irreversibilities are the main channel through which fluctuations in uncertainty affect capital formation. In this paper, we exploit the implications of uncertainty for the cost of external debt finance by developing a general equilibrium framework in which financial market frictions provide the link between uncertainty and the aggregate investment cycle. The notion that conditions in the financial markets are an important conduit through which fluctuations in uncertainty are transmitted through to the real economy is strongly supported by our empirical evidence. According to both the macro and micro data, increases in uncertainty lead to the widening of spreads on corporate bonds and protracted declines in investment and output.

The quantitative general equilibrium structure of our model implies that empirically realistic increases in uncertainty can replicate the negative comovement between credit spreads and investment, the positive comovement between net worth and investment, while also accounting for many of the salient characteristics of the business cycle fluctuations. By allowing for heterogeneity in productivity and net worth across firms, the model also implies an important reallocation mechanism for the economy as a whole, a mechanism that generates a procyclical reallocation of factor inputs and, as a result, procyclical movements in the measured TFP. Overall, our simulations demonstrate that fluctuations in economic

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<sup>39</sup>Choe et al. [1993] and Bayless and Chaplinsky [1996] provide microeconomic evidence regarding the procyclical nature of equity financing. In contrast, Jermann and Quadrini [2006] show that aggregate equity issuance is countercyclical. Covas and Den Haan [2007] show this dichotomy reflects the disproportionate influence of very large firms.

uncertainty have important consequences for macroeconomic outcomes in an environment that allows for the departures from the Modigliani-Miller paradigm of frictionless financial markets.

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Table 1: Uncertainty, Credit Spreads, and Investment  
(Aggregate Time-Series Data)

| Explanatory Variable                                 | Specification     |                   |                   |
|------------------------------------------------------|-------------------|-------------------|-------------------|
|                                                      | (1)               | (2)               | (3)               |
| Uncertainty: $\sigma_{t-1}$                          | -0.044<br>(0.020) | -0.031<br>(0.016) | -0.026<br>(0.016) |
| Credit spread: $s_{t-1}$                             | -                 | -0.776<br>(0.317) | -0.860<br>(0.309) |
| Sum of coefficients: output growth <sup>a</sup>      | 1.168<br>(0.263)  | 0.980<br>(0.260)  | 0.944<br>(0.276)  |
| Sum of coefficients: real interest rate <sup>b</sup> | -                 | -                 | -0.052<br>(0.063) |
| L-R effect: uncertainty                              | -0.053<br>(0.025) | -0.032<br>(0.016) | -0.027<br>(0.016) |
| L-R effect: credit spread                            | -                 | -0.805<br>(0.321) | -0.888<br>(0.309) |
| Adj. $R^2$                                           | 0.442             | 0.480             | 0.466             |
| Pr > $E_p$ <sup>c</sup>                              | 0.000             | 0.026             | 0.082             |

NOTE: Sample period: 1963Q1–2009Q4 ( $T = 184$ ). Dependent variable is  $\Delta i_t$ , the log-difference of (real) business fixed investment in quarter  $t$ . Each specification includes a constant, three lags of  $\Delta i_t$  (not reported), and is estimated by OLS. Robust (HC<sub>3</sub>) asymptotic standard errors are reported in parentheses.

<sup>a</sup>Estimated sum of coefficients on the log-difference of (real) nonfarm business sector output at lags 1 through 3.

<sup>b</sup>Estimated sum of coefficients on the real 10-year Treasury yield at lags 1 through 4.

<sup>c</sup> $p$ -value for the Doornik and Hansen [2008] test of the normality of OLS residuals.

Table 2: Uncertainty and Credit Spreads  
(Firm-Level Panel Data)

| Explanatory Variable               | Specification               |                             |                             |                             |
|------------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
|                                    | (1)                         | (2)                         | (3)                         | (4)                         |
| $\log \sigma_{it}$                 | 0.876<br>(0.027)            | 0.594<br>(0.018)            | 0.616<br>(0.018)            | 0.238<br>(0.019)            |
| $R_{it}^E$                         | -0.351<br>(0.033)           | -0.441<br>(0.023)           | -0.430<br>(0.023)           | -0.233<br>(0.016)           |
| $[\Pi/A]_{it}$                     | -4.598<br>(0.939)           | -2.222<br>(0.558)           | -1.927<br>(0.511)           | -0.915<br>(0.432)           |
| $\log[D/E]_{i,t-1}$                | 0.223<br>(0.043)<br>(0.032) | 0.063<br>(0.020)<br>(0.020) | 0.067<br>(0.018)<br>(0.019) | 0.123<br>(0.015)<br>(0.017) |
| Adj. $R^2$                         | 0.472                       | 0.621                       | 0.629                       | 0.785                       |
| Credit Rating Effects <sup>a</sup> | -                           | (0.000)                     | (0.000)                     | (0.000)                     |
| Industry Effects <sup>b</sup>      | -                           | -                           | (0.000)                     | (0.000)                     |
| Time Effects <sup>c</sup>          | -                           | -                           | -                           | (0.000)                     |

NOTE: Sample period: bond-level monthly data from January 1973 to December 2009 at a quarterly frequency (No. of firms/bonds = 944/5072; Obs. = 88,447). Dependent variable is  $\log(s_{it}[k])$ , the logarithm of the credit spread of bond  $k$  in month  $t$  (issued by firm  $i$ ). All specifications include a constant (not reported) and are estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

<sup>a</sup> $p$ -value for the test of the null hypothesis of the absence of fixed credit rating effects.

<sup>b</sup> $p$ -value for the test of the null hypothesis of the absence of fixed industry effects.

<sup>c</sup> $p$ -value for the test of the null hypothesis of the absence of time fixed effects.

Table 3: Uncertainty, Credit Spreads, and Investment  
(Firm-Level Panel Data)

| Explanatory Variable | Specification     |                   |                   |                   |                   |                   |
|----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                      | (1)               | (2)               | (3)               | (4)               | (5)               | (6)               |
| $\log \sigma_{it}$   | -0.172<br>(0.039) | -0.086<br>(0.036) | -0.147<br>(0.037) | -0.060<br>(0.037) | 0.008<br>(0.034)  | -0.062<br>(0.036) |
| $\log s_{it}$        | -                 | -                 | -                 | -0.167<br>(0.019) | -0.149<br>(0.018) | -0.130<br>(0.021) |
| $\log[Y/K]_{it}$     | 0.572<br>(0.046)  | -                 | -                 | 0.549<br>(0.045)  | -                 | -                 |
| $\log[\Pi/K]_{it}$   | -                 | 1.292<br>(0.081)  | -                 | -                 | 1.207<br>(0.081)  | -                 |
| $\log Q_{i,t-1}$     | -                 | -                 | 0.710<br>(0.045)  | -                 | -                 | 0.641<br>(0.046)  |
| $R^2$ (within)       | 0.289             | 0.260             | 0.251             | 0.309             | 0.277             | 0.262             |

NOTE: Sample period: firm-level monthly data from January 1973 to December 2009 at an annual frequency (No. of firms = 905; Obs. = 8,367). Dependent variable is  $\log[I/K]_{it}$ , the logarithm of the (real) investment rate of firm  $i$  in year  $t$ . All specifications include time fixed effects (not reported) and firm fixed effect, which are eliminated using the within transformation. The resulting specification is estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses. Parameter estimates for  $\log[\Pi/K]_{it}$  and the associated standard errors are adjusted for the fact that  $\log[\Pi/K]_{it}$  is computed as  $\log(0.5 + [\Pi/K]_{it})$ .

Table 4: Descriptive Business Cycle Statistics  
(Actual vs. Model-Implied)

| Variable                         | Relative Standard Deviation <sup>a</sup> |                |           |
|----------------------------------|------------------------------------------|----------------|-----------|
|                                  | Actual Data                              | Baseline Model | GHH Model |
| Consumption                      | 0.816                                    | 0.704          | 0.863     |
| Investment                       | 3.145                                    | 3.315          | 2.207     |
| Hours                            | 1.168                                    | 1.120          | 0.671     |
| <i>Memo: <math>STD(Y)</math></i> | 0.014                                    | 0.019          | 0.025     |

| Variable                             | Correlation with Output |                |           |
|--------------------------------------|-------------------------|----------------|-----------|
|                                      | Actual Data             | Baseline Model | GHH Model |
| Consumption                          | 0.908                   | 0.859          | 0.950     |
| Investment                           | 0.824                   | 0.771          | 0.746     |
| Hours                                | 0.895                   | -0.391         | 0.989     |
| Measured TFP                         | 0.826                   | 0.811          | 0.815     |
| <i>Memo: <math>Corr(C, I)</math></i> | 0.675                   | 0.375          | 0.526     |

NOTE: Sample period for the actual annual data: 1954–2008 ( $T = 55$ ). Actual data are in logs and have been detrended using the Hodrick-Prescott filter with  $\lambda = 6.25$ ; see Ravn and Uhlig [2002] for details.

<sup>a</sup>Scaled by the standard deviation of detrended output.

Table 5: Cyclical Properties of Aggregate Investment  
(Actual vs. Model-Implied)

| Variable         | Correlation with Investment Growth |                |           |
|------------------|------------------------------------|----------------|-----------|
|                  | Actual Data                        | Baseline Model | GHH Model |
| Credit spread    | -0.597                             | -0.536         | -0.345    |
| Net worth growth | 0.390                              | 0.498          | 0.417     |
| Uncertainty      | -0.360                             | -0.592         | -0.360    |

NOTE: Sample period for the actual quarterly data: 1963Q4–2009Q4 ( $T = 185$ ). Entries for the actual data are the correlations between the log-difference of real business fixed investment, the level of the 10-year BBB-Treasury spread, the log-difference of real net worth for the nonfinancial corporate sector, and our benchmark estimate of time-varying uncertainty.

Table 6: Cyclical Properties of Capital Reallocation

| Variable     | Model Specification |       |
|--------------|---------------------|-------|
|              | Baseline            | GHH   |
| Output       | 0.752               | 0.814 |
| Consumption  | 0.782               | 0.796 |
| Investment   | 0.370               | 0.482 |
| Measured TFP | 0.752               | 0.903 |

NOTE: Entries in the table denote the correlation coefficients of capital reallocation with the specified variable. Capital reallocation is defined as the sum of all investment flows associated with the redeployment of the existing capital stock (see text for details).

Table 7: Uncertainty, Credit Spreads, and the Risk Premium

| Credit Spread                      | Data <sup>a</sup> | Model Specification |              |         |
|------------------------------------|-------------------|---------------------|--------------|---------|
|                                    |                   | Baseline-PHM        | Baseline-RNM | GHH-PHM |
| Overall                            | 161               | 164                 | 174          | 134     |
| Conditional on $\sigma = \sigma_L$ | -                 | 94                  | 96           | 82      |
| Conditional on $\sigma = \sigma_H$ | -                 | 256                 | 277          | 134     |

NOTE: Entries in the table denote the average level of credit spreads (in basis points), conditional on the uncertainty regime for various model specifications (see text for details). PHM = physical measure; RNM = risk-neutral measure;  $\sigma = \sigma_L$  corresponds to the low uncertainty regime; and  $\sigma = \sigma_H$  corresponds to the high uncertainty regime.

<sup>a</sup>10-year BBB-Treasury credit spread.

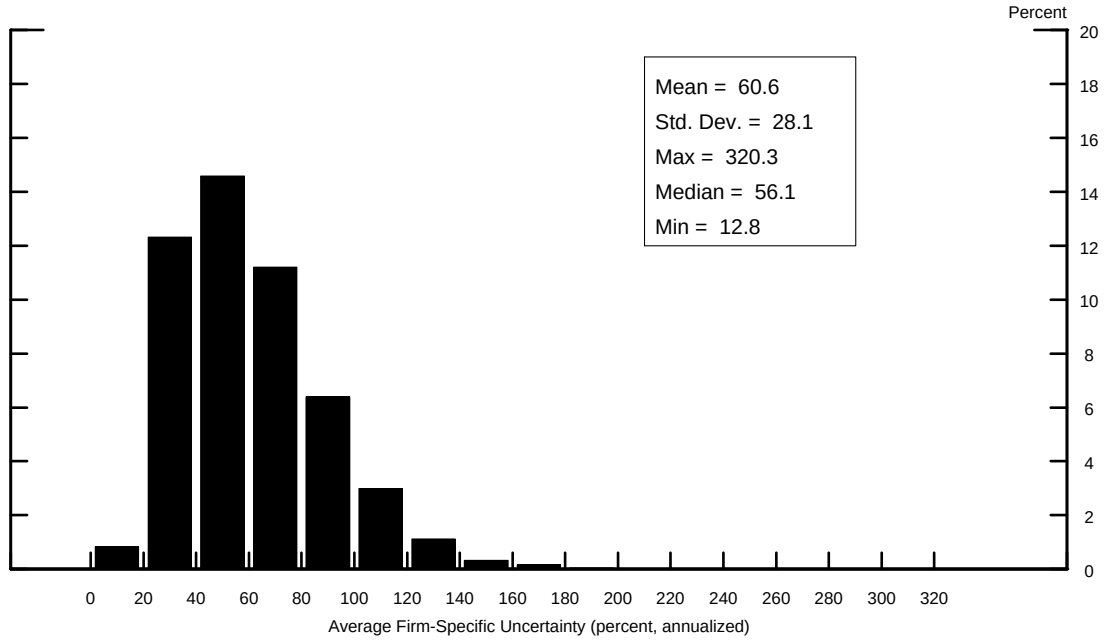
Table 8: Uncertainty and Default Rates

| Default Rate                       | Data <sup>a</sup> | Model Specification |              |         |
|------------------------------------|-------------------|---------------------|--------------|---------|
|                                    |                   | Baseline-PHM        | Baseline-RNM | GHH-PHM |
| Overall                            | 0.43              | 2.37                | 2.40         | 2.26    |
| Conditional on $\sigma = \sigma_L$ | -                 | 1.88                | 1.88         | 1.90    |
| Conditional on $\sigma = \sigma_H$ | -                 | 3.00                | 3.04         | 2.71    |

NOTE: Entries in the table denote the realized default rates (in percent), conditional on the uncertainty regime for various model specifications (see text for details). PHM = physical measure; RNM = risk-neutral measure;  $\sigma = \sigma_L$  corresponds to a low uncertainty regime; and  $\sigma = \sigma_H$  corresponds to a high uncertainty regime.

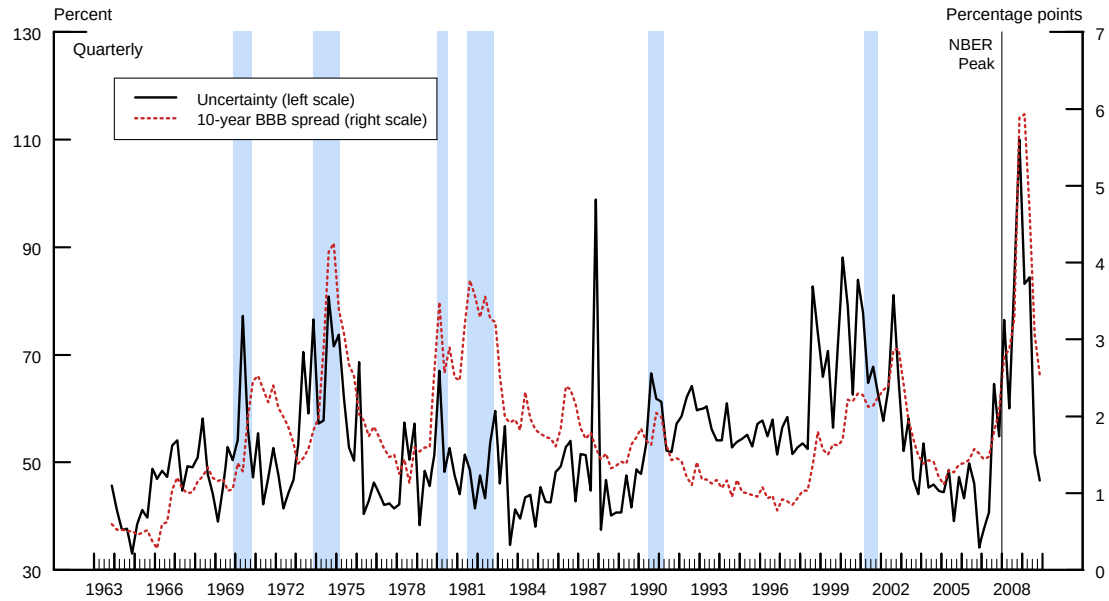
<sup>a</sup>Quarterly nonfinancial bond default rate.

Figure 1: Distribution of Uncertainty Based on Equity Valuations



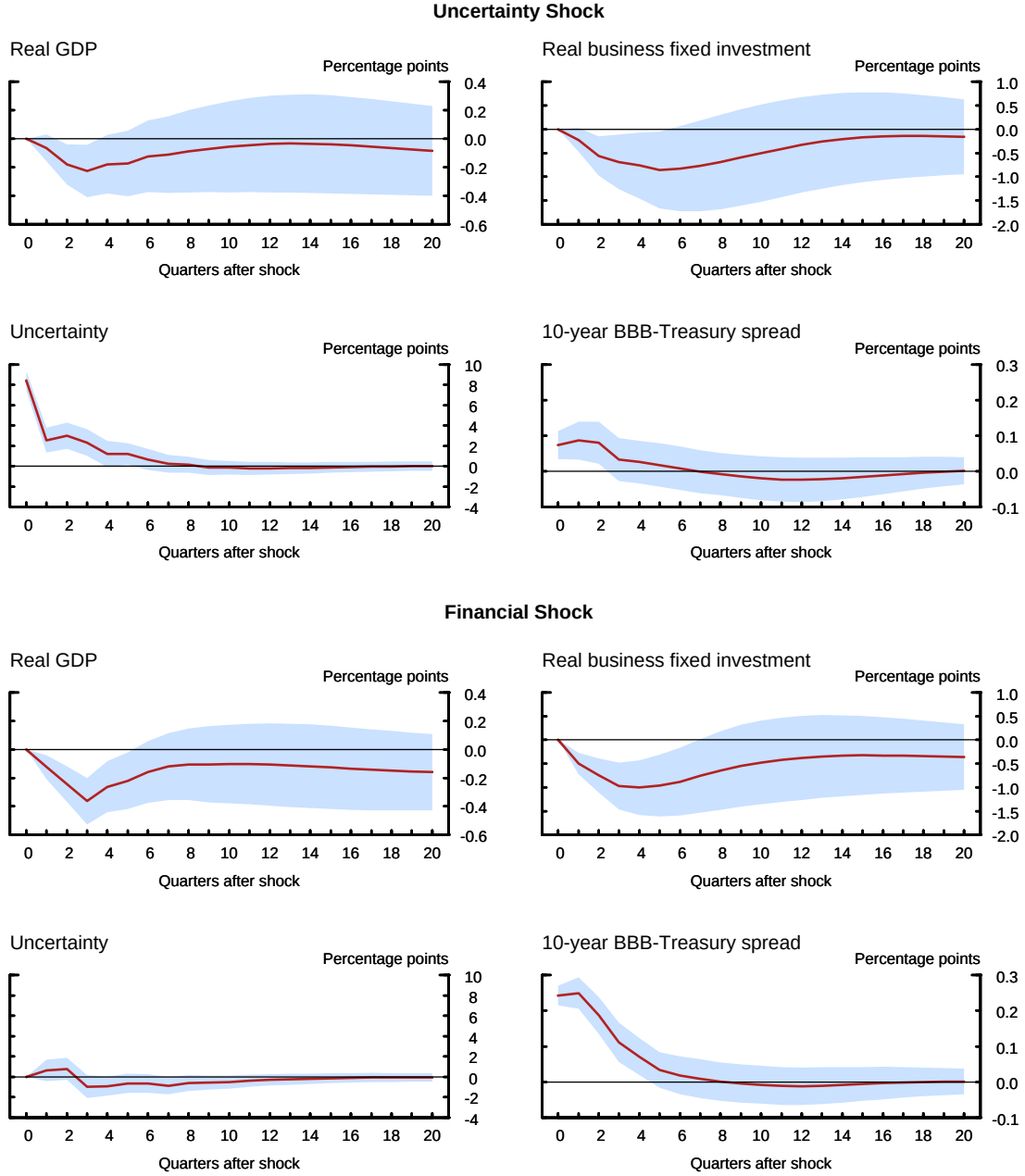
NOTE: The figure depicts the distribution of  $\bar{\sigma}_i = T_i^{-1} \sum_{t=1}^{T_i} \sigma_{it}$ , the firm-specific mean of  $\sigma_{it}$ , the standard deviation of daily abnormal stock returns for firm  $i$  in quarter  $t$ . Daily abnormal returns are estimated using the 3-factor Fama-French model for a sample of 10,729 U.S. nonfinancial corporations over the period from July 1, 1963 (1963Q3) to December 31, 2009 (2009Q4).

Figure 2: Uncertainty and Corporate Credit Spreads



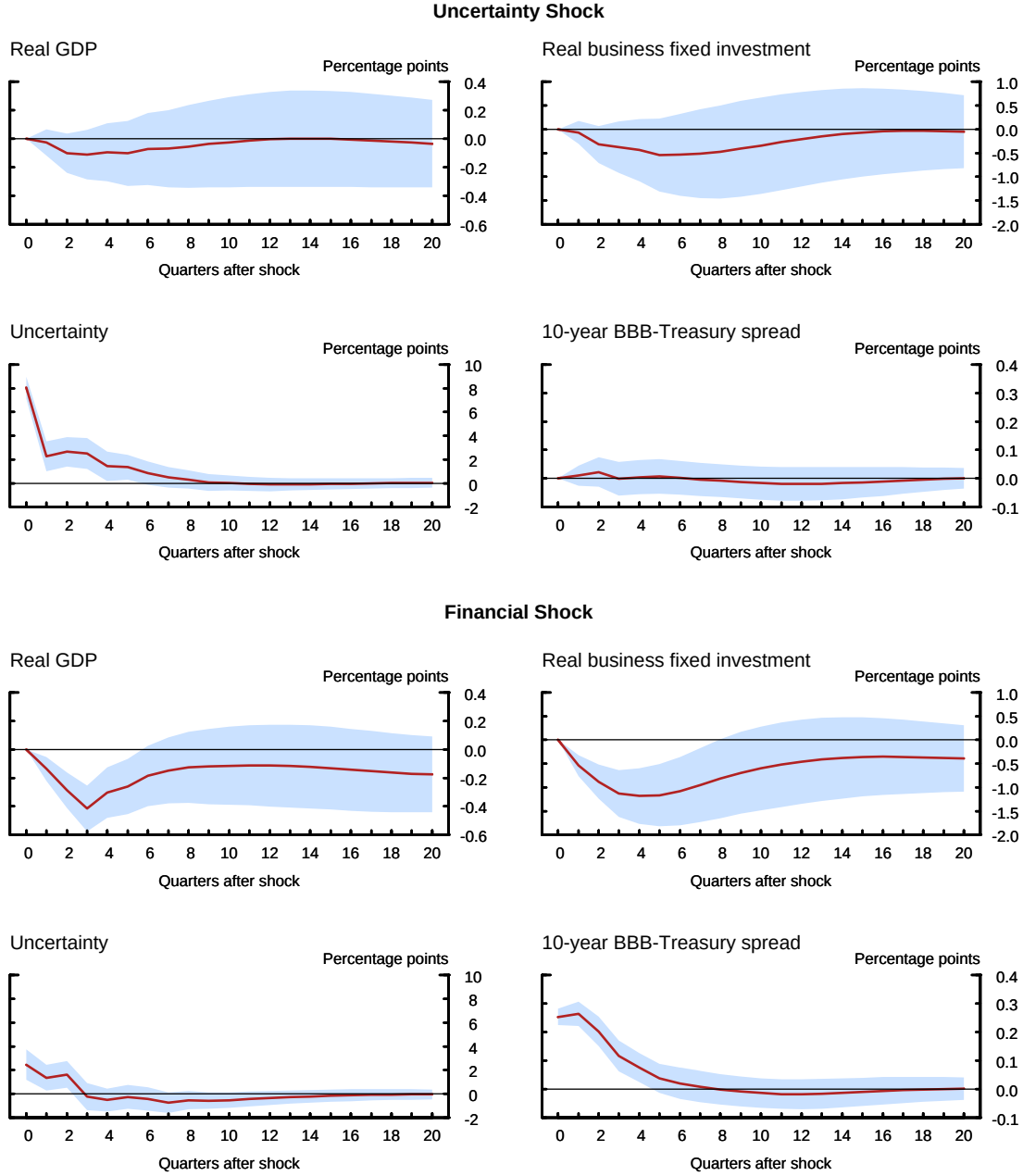
NOTE: Sample period: 1963Q4–2009Q4. The solid line depicts our benchmark estimate of time-varying uncertainty based on equity valuations (see text for details). The dotted line depicts the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote NBER-dated recessions.

Figure 3: Dynamic Implications of Uncertainty and Financial Shocks  
(Identification Scheme I)



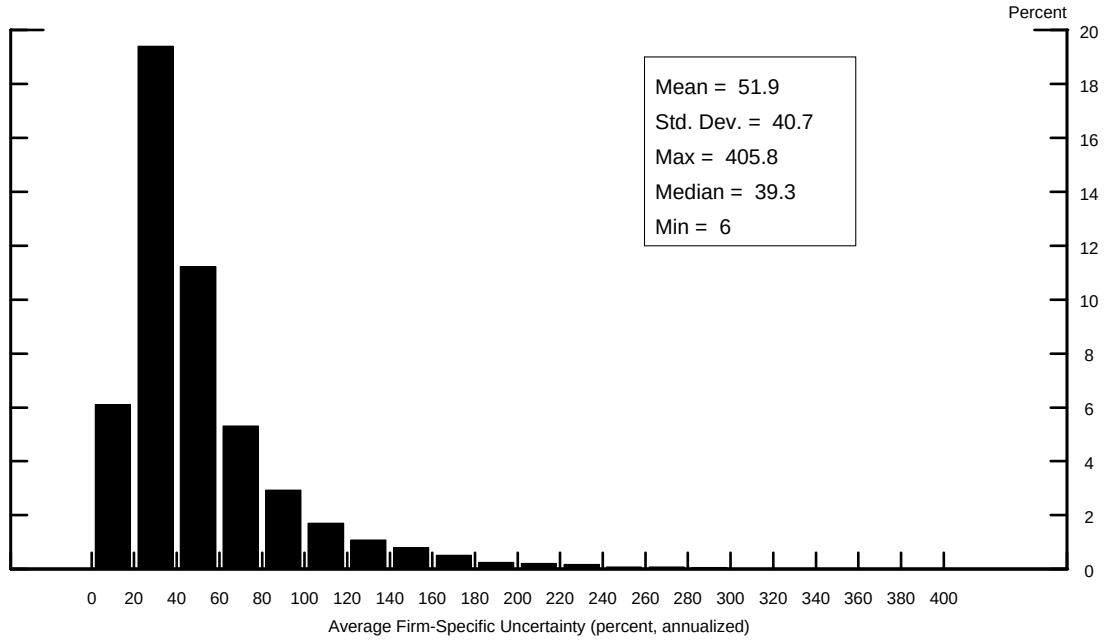
NOTE: The top four panels depict the impulse response functions to an orthogonalized one standard deviation shock to our benchmark estimate of time-varying uncertainty. The bottom four panels depict the impulse response functions to an orthogonalized one standard deviation shock to the 10-year BBB-Treasury spread. Identification scheme I corresponds to the following recursive ordering of the VAR system:  $(y_t, i_t, p_t, s_t, v_t, f_t)$ . Shaded bands represent 95-percent confidence intervals based on 2,000 bootstrap replications.

Figure 4: Dynamic Implications of Uncertainty and Financial Shocks  
(Identification Scheme II)



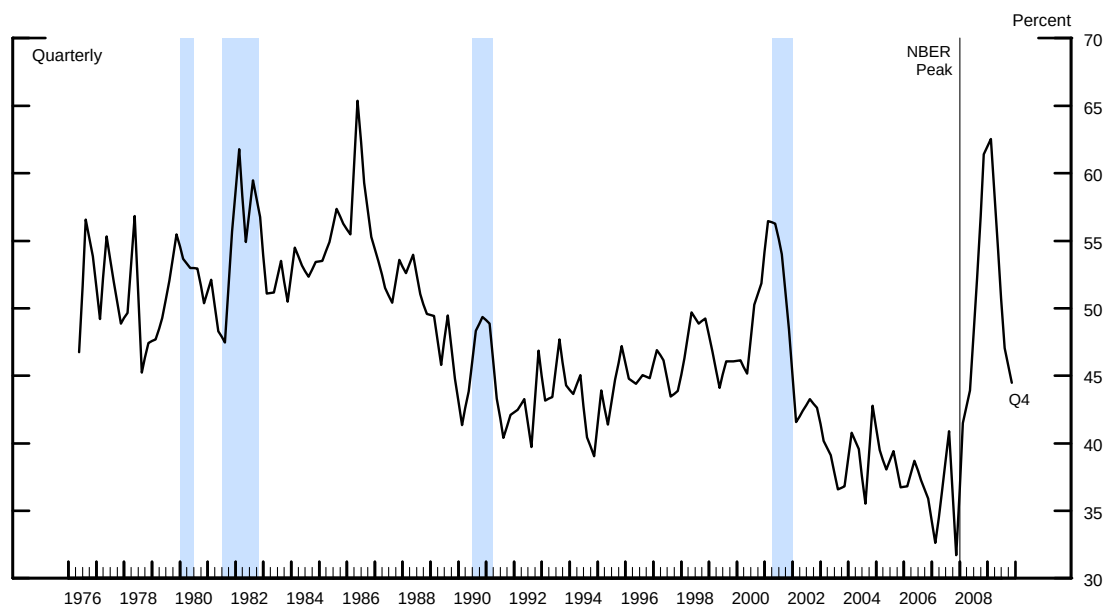
NOTE: The top four panels depict the impulse response functions to an orthogonalized one standard deviation shock to our benchmark estimate of time-varying uncertainty. The bottom four panels depict the impulse response functions to an orthogonalized one standard deviation shock to the 10-year BBB-Treasury spread. Identification scheme II corresponds to the following recursive ordering of the VAR system:  $(y_t, i_t, p_t, v_t, s_t, f_t)$ . Shaded bands represent 95-percent confidence intervals based on 2,000 bootstrap replications.

Figure 5: Distribution of Uncertainty Based on Revenue Shocks



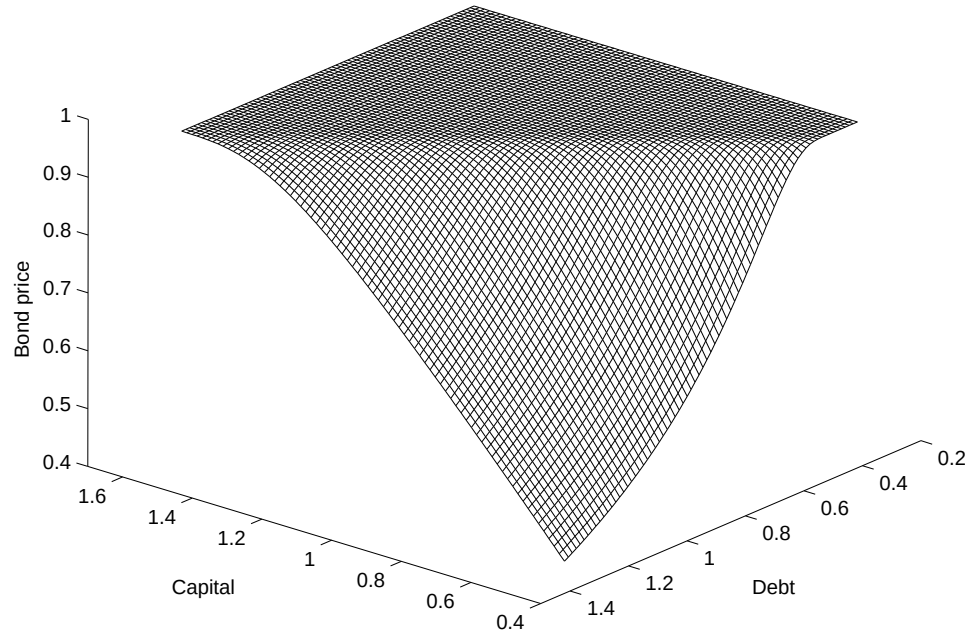
NOTE: The figure depicts the distribution of firm-specific means of  $\sqrt{\pi/2}|\hat{\epsilon}_{it}|$ , an estimate of the standard deviation of shocks to the firm's revenue function (see text for details). Quarterly revenue shocks are estimated for a sample of 9,469 U.S. nonfinancial corporations over the period from 1976Q1 to 2009Q4.

Figure 6: Uncertainty Based on Revenue Shocks



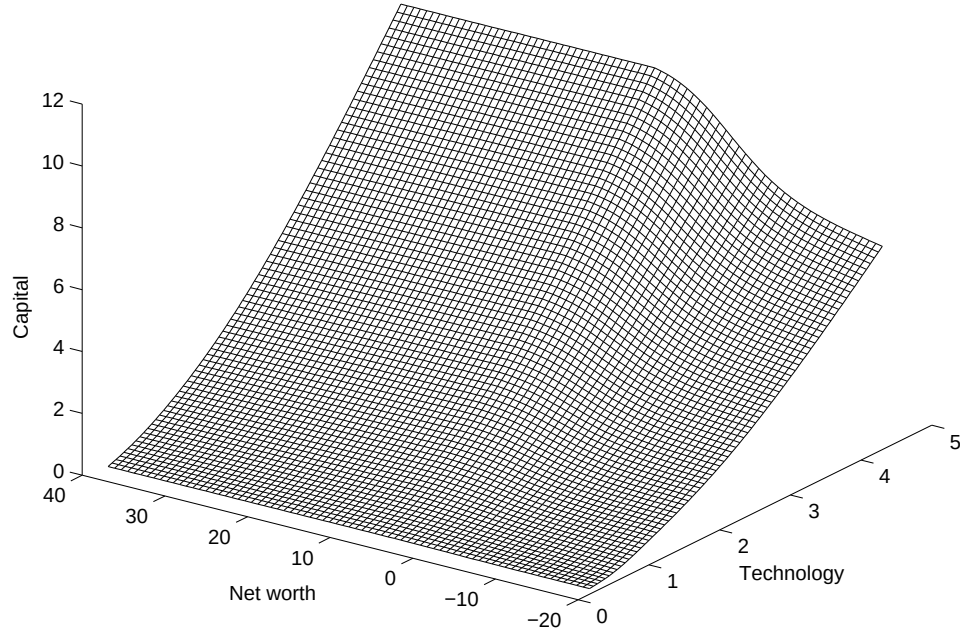
NOTE: Sample period: 1976Q1–2009Q4. The figure depicts an estimate of time-varying uncertainty based on shocks to the firm’s revenue function (see text for details). The shaded vertical bars denote NBER-dated recessions.

Figure 7: Bond-Pricing Policy Function



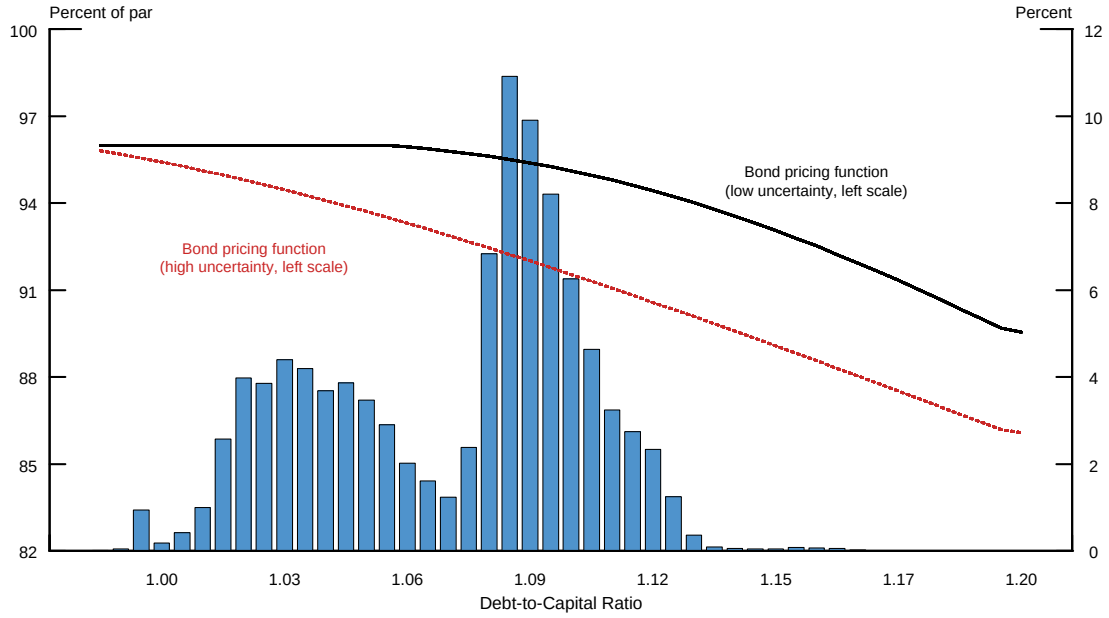
NOTE: The figure depicts the model-implied optimal price of the bond as a function of capital assets and debt outstanding, both of which are expressed relative to their steady-state values. The bond-pricing function is computed under the assumption of no aggregate shock to idiosyncratic uncertainty (see text for details).

Figure 8: Investment Policy Function



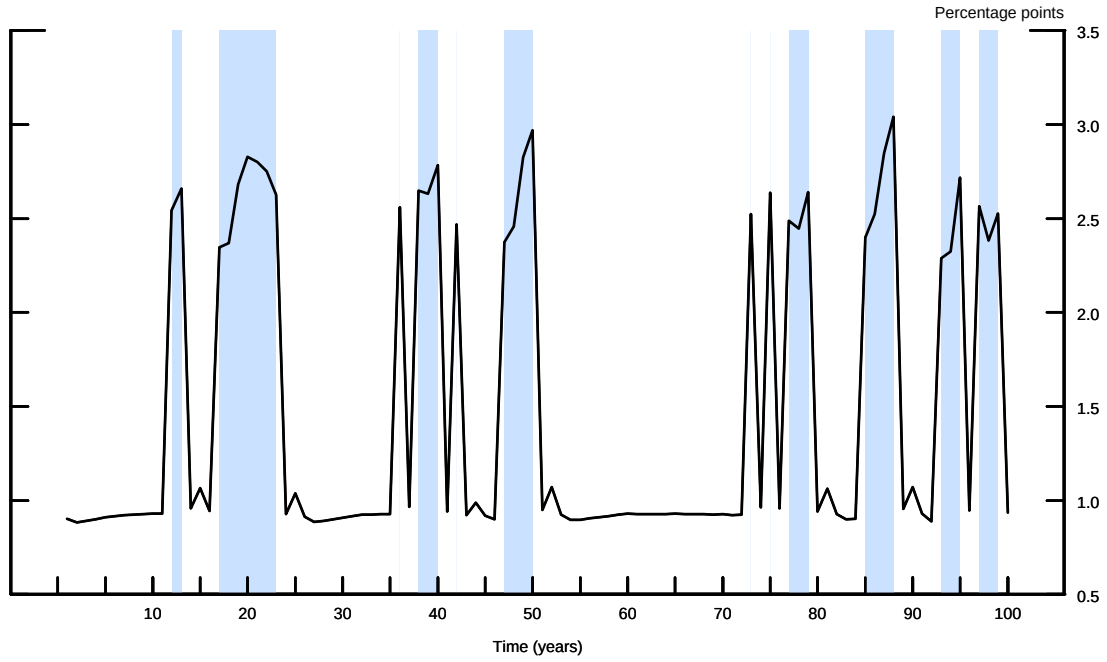
NOTE: The figure depicts the the model-implied optimal investment policy as a function of net worth and the level of technology, both of which are expressed relative to their steady-state values. The investment policy function is computed under the assumption of no aggregate shock to idiosyncratic uncertainty (see text for details).

Figure 9: Uncertainty and the Bond-Pricing Policy Function



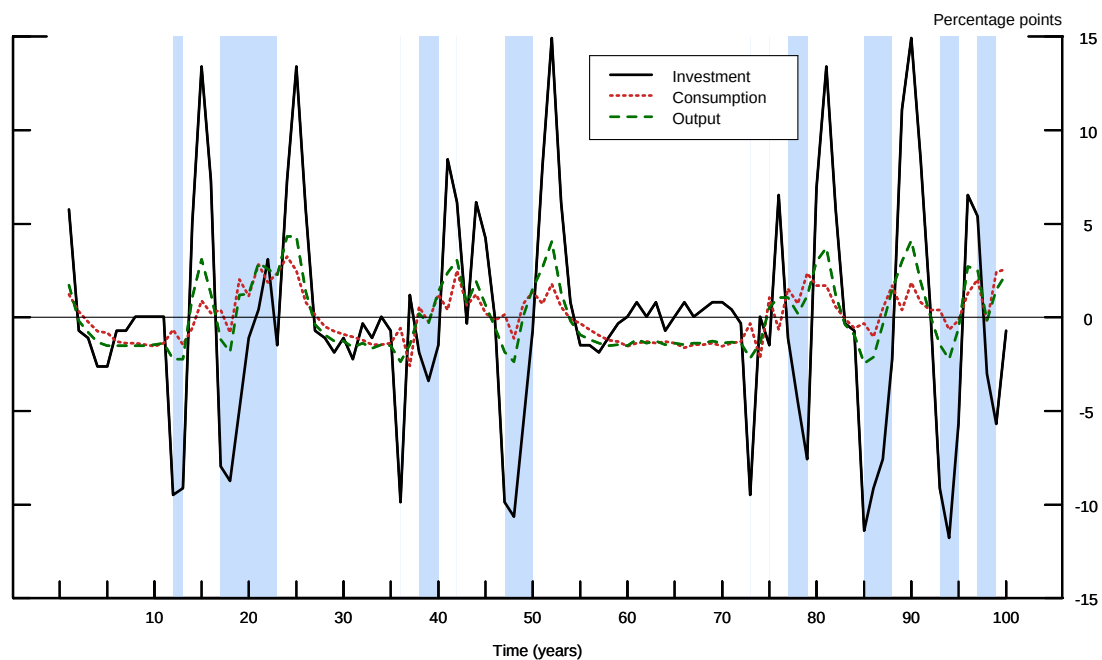
NOTE: The histogram depicts the model-implied distribution—across firms and time—of the debt-to-capital ratio ( $b/k$ ). The solid line depicts the optimal bond-pricing policy as a function of the debt-to-capital ratio in the low uncertainty regime ( $\sigma_L = 0.35$ ), and the dotted line depicts the bond-pricing policy function in the high uncertainty regime ( $\sigma_H = 0.55$ ).

Figure 10: Uncertainty and Credit Spreads



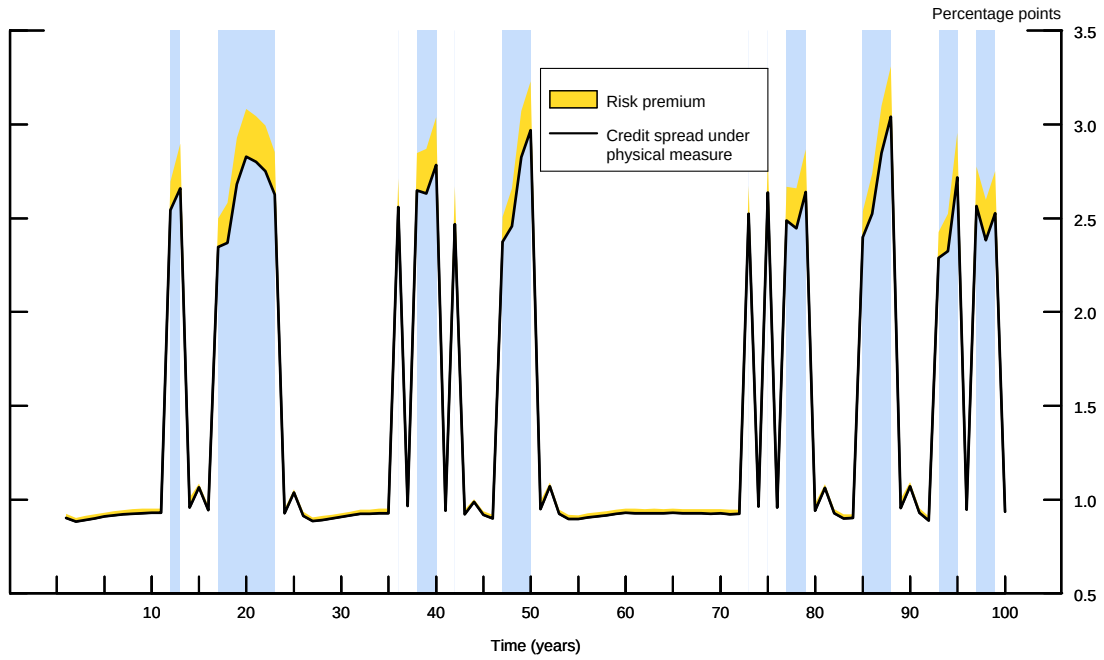
NOTE: The figure shows the simulated time path of the average credit spread based on bond prices computed under the physical measure (see text for details). The shaded vertical bars correspond to periods of high uncertainty.

Figure 11: Uncertainty and Real Economic Activity



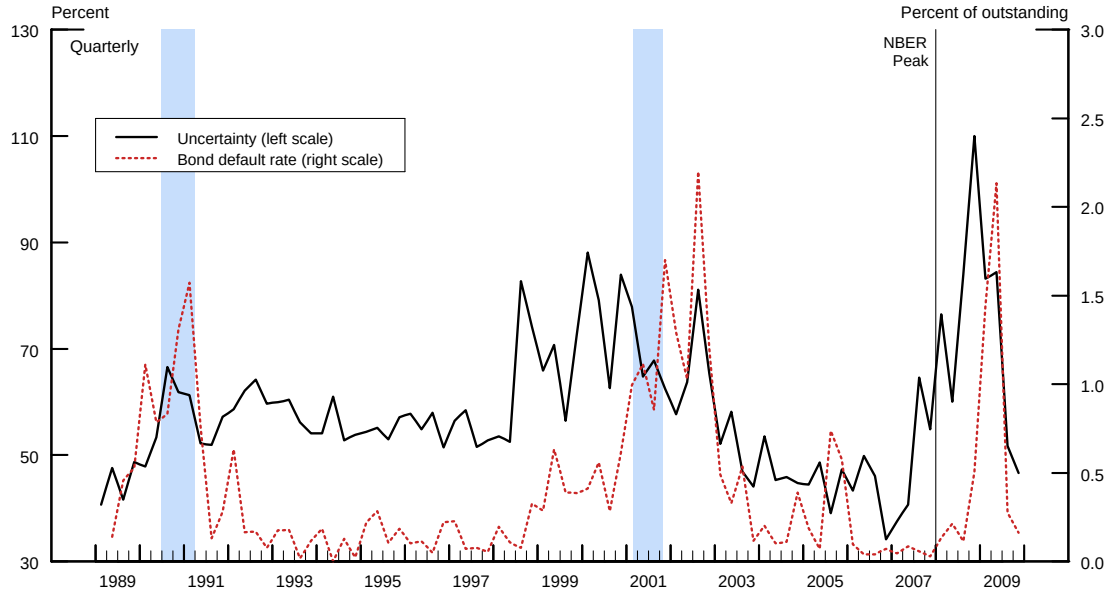
NOTE: The figure shows the simulated time path of aggregate investment (solid line), consumption (dotted line), and output (dashed line). All three series are expressed in percentage-point deviations from their steady-state values. The shaded vertical bars correspond to periods of high uncertainty.

Figure 12: Uncertainty and the Risk Premium



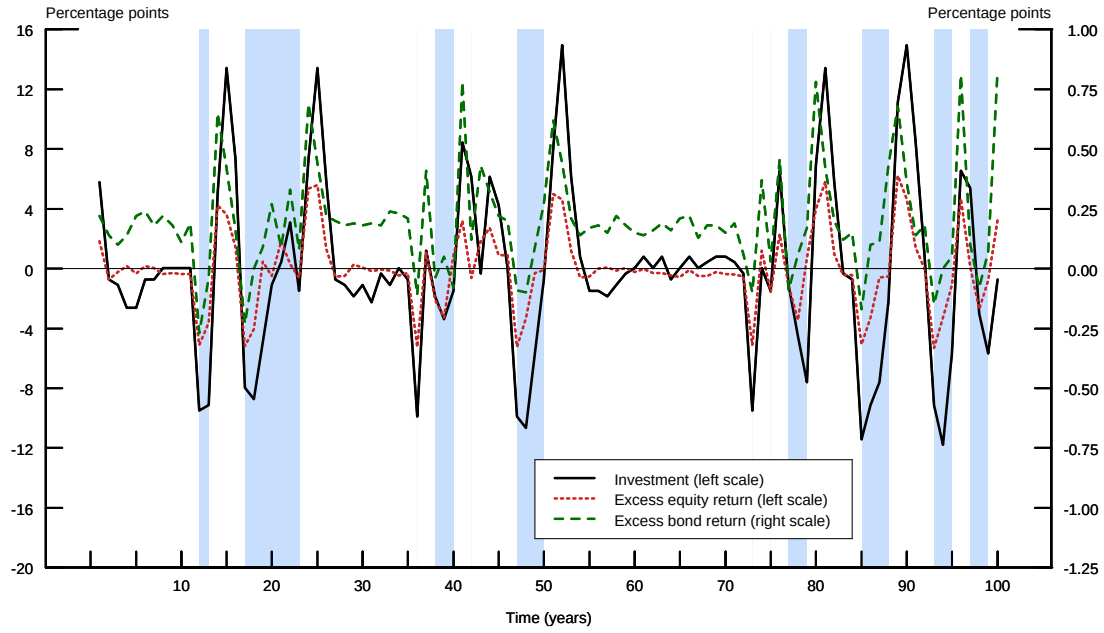
NOTE: The figure shows the simulated time path of the average credit spread based on bond prices computed under the risk-neutral measure; the solid line corresponds to the average credit spread computed under the physical measure, whereas the shaded yellow region represents the model-implied risk premium (see text for details). The shaded vertical bars correspond to periods of high uncertainty.

Figure 13: Uncertainty and Corporate Bond Defaults



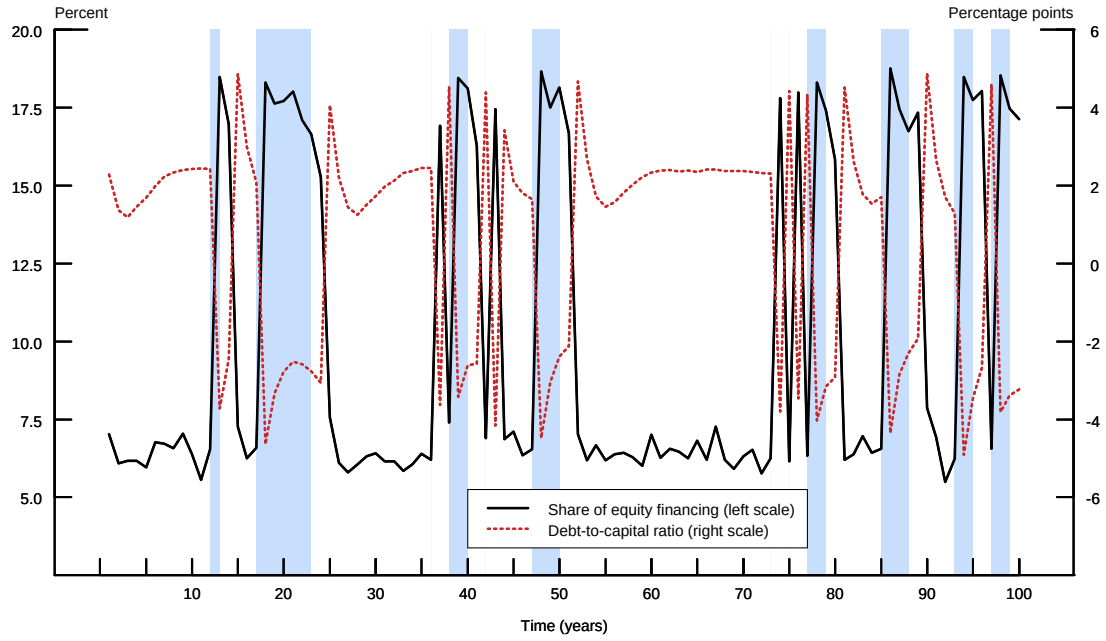
NOTE: Sample period: 1989Q4–2009Q4. The solid line depicts our benchmark estimate of time-varying uncertainty based on equity valuations (see text for details). The dotted line depicts the nonfinancial bond default rate in quarter  $t$ , calculated as the sum of defaults during the quarter, divided by the amount outstanding at the beginning of the quarter. The shaded vertical bars represent the NBER-dated recessions.

Figure 14: Uncertainty, Investment, and Financial Asset Returns



NOTE: The figure shows the simulated time path of aggregate investment (solid line), the excess return on equity (dotted line), and the excess return on corporate bonds (dashed line). All three series are expressed in percentage-point deviations from their steady-state values. The shaded vertical bars correspond to periods of high uncertainty.

Figure 15: Uncertainty and External Financing Patterns



NOTE: The solid line shows the model-implied share of equity financing, and the dotted line depicts the debt-to-capital ratio, expressed in percentage-point deviations from its steady-state value. The shaded vertical bars correspond to periods of high uncertainty.

# Appendices

## A Data Sources and Methods

The key information underlying the firm-level analysis comes from a large sample of fixed income securities issued by U.S. nonfinancial corporations. Specifically, from the Lehman/Warga (LW) and Merrill Lynch (ML) databases, we obtained month-end prices of outstanding long-term corporate bonds that are actively traded in the secondary market.<sup>40</sup> To guarantee that we are measuring borrowing costs of different firms at the same point in their capital structure, we restricted our sample to senior unsecured issues with a fixed coupon schedule only. For such securities, we spliced their month-end prices across the two data sources.

The micro-level aspect of our data set allows us to construct credit spreads that are not subject to the maturity/duration bias. In particular, we construct for each individual bond issue a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. For example, consider a corporate bond  $k$  issued by firm  $i$  that at time  $t$  is promising a sequence of cash-flows  $\{C(s) : s = 1, 2, \dots, S\}$ , consisting of the regular coupon payments and the repayment of the principle at maturity. The price of this bond in period  $t$  is given by

$$P_{it}[k] = \sum_{s=1}^S C(s)D(t_s),$$

where  $D(t) = e^{-r_t t}$  is the discount function in period  $t$ . To calculate the price of a corresponding risk-free security—denoted by  $P_t^f[k]$ —we discount the promised cash-flow sequence  $\{C(s) : s = 1, 2, \dots, S\}$  using continuously-compounded zero-coupon Treasury yields in period  $t$ , obtained from the daily estimates of the U.S. Treasury yield curve reported by Gürkaynak et al. [2007]. The resulting price  $P_t^f[k]$  can then be used to calculate the yield—denoted by  $y_t^f[k]$ —of a hypothetical Treasury security with exactly the same cash-flows as the underlying corporate bond. The credit spread  $s_{it}[k] = y_{it}[k] - y_t^f[k]$ , where  $y_{it}[k]$  denotes the yield of the corporate bond  $k$ , is thus free of the “duration mismatch” that would occur were the spreads computed simply by matching the corporate yield to the estimated yield of a zero-coupon Treasury security of the same maturity.

To ensure that our results are not driven by a small number of extreme observations, we eliminated all bond/month observations with credit spreads below 5 basis points and with spreads greater than 3,500 basis points. In addition, we dropped from our sample very small corporate issues—those with a par value of less than \$1 million—and all observations with a

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<sup>40</sup>These two data sources are used to construct benchmark corporate bond indexes used by the market participants. Specifically, they contain secondary market prices for a significant fraction of dollar-denominated bonds publicly issued in the U.S. corporate cash market. The ML database is a proprietary data source of daily bond prices that starts in 1997. Focused on the most liquid securities in the secondary market, bonds in the ML database must have a remaining term-to-maturity of at least two years, a fixed coupon schedule, and a minimum amount outstanding of \$100 million for below investment-grade and \$150 million for investment-grade issuers. By contrast, the LW database of month-end bond prices has a somewhat broader coverage and is available from 1973 through mid-1998 (see Warga [1991] for details).

remaining term-to-maturity of less than one year or more than 30 years; calculating spreads for maturities of less than one year and more than 30 years would involve extrapolating the Treasury yield curve beyond its support.<sup>41</sup> These selection criteria yielded a sample of 5,378 individual securities between January 1973 and December 2009. We matched these corporate securities with their issuer’s quarterly and annual income and balance sheet data from Compustat and daily data on equity valuations from CRSP, yielding a matched sample of 944 firms.

Table A-1 contains summary statistics for the key characteristics of bonds in our sample. Note that a typical firm has only a few senior unsecured issues outstanding at any point in time—the median firm, for example, has two such issues trading at any given month. This distribution, however, exhibits a significant positive skew, as some firms can have as many as 74 different senior unsecured bond issues trading in the market at a point in time. The distribution of the real market values of these issues is similarly skewed, with the range running from \$1.2 million to more than \$5.6 billion. Not surprisingly, the maturity of these debt instruments is fairly long, with the average maturity at issue of about 13 years. Because corporate bonds typically generate significant cash flow in the form of regular coupon payments, the effective duration is considerably shorter, with both the average and the median duration of about 6 years.

According to the S&P credit ratings, our sample spans the entire spectrum of credit quality, from “single D” to “triple A.” At “BBB1,” however, the median bond/month observation is still solidly in the investment-grade category. Turning to returns, the (nominal) coupon rate on these bonds averaged 7.31 percent during our sample period, while the average total nominal return, as measured by the nominal effective yield, was 7.82 percent per annum. Reflecting the wide range of credit quality, the distribution of nominal yields is quite wide, with the minimum of 0.66 percent and the maximum of more than 44 percent. Relative to Treasuries, an average bond in our sample generated a return of about 202 basis points above the comparable risk-free rate, with the standard deviation of 284 basis points.

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<sup>41</sup>We also eliminated a small number of putable bonds from our sample.

Table A-1: Summary Statistics of Corporate Bond Characteristics

| Bond Characteristic                       | Mean  | SD    | Min  | P50   | Max   |
|-------------------------------------------|-------|-------|------|-------|-------|
| # of bonds per firm/month                 | 2.83  | 3.46  | 1.00 | 2.00  | 74.0  |
| Mkt. value of issue <sup>a</sup> (\$mil.) | 310.1 | 315.6 | 1.22 | 231.0 | 5,628 |
| Maturity at issue (years)                 | 13.3  | 9.5   | 1.0  | 10.0  | 50.0  |
| Term to maturity (years)                  | 11.4  | 8.6   | 1.0  | 8.2   | 30.0  |
| Duration (years)                          | 6.50  | 3.20  | 0.91 | 6.10  | 15.6  |
| Credit rating (S&P)                       | -     | -     | D    | BBB1  | AAA   |
| Coupon rate (pct.)                        | 7.31  | 1.95  | 1.95 | 7.00  | 17.5  |
| Nominal effective yield (pct.)            | 7.82  | 3.24  | 0.66 | 7.25  | 44.3  |
| Credit spread (bps.)                      | 202   | 284   | 5    | 116   | 3,499 |

*Panel Dimensions*Obs. = 345,785     $N = 5,378$  bonds

Min. Tenure = 1    Median Tenure = 53    Max. Tenure = 327

NOTE: Sample period: Monthly bond-level data from January 1973 to December 2009 for a sample of 944 nonfinancial firms. Sample statistics are based on trimmed data (see text for details).

<sup>a</sup>Market value of the outstanding issue deflated by the CPI (1982–84 = 100).