On the Optimality of a Dominant Unit of Account*

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June 2009

Abstract

We develop a theory that gives rise to an endogenous unit of account. Agents enter into non-contingent contracts with a variety of business partners. Trade unfolds sequentially and is subject to random matching. By using a unified unit of account, agents can lower their exposure to relative price risk, avoid costly default, and create more total surplus. We discuss the use of a unified unit of account in intertemporal trade and the robustness of a unit of account when there is aggregate price-level uncertainty.

*Preliminary and incomplete. Financial support from the National Science Foundation (grant SES-0519265) and the Alfred P. Sloan Foundation is gratefully acknowledged. Doepke: Department of Economics, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208 (e-mail: doepke@northwestern.edu). Schneider: Department of Economics, Stanford University, Landau Economics Building, 579 Serra Mall, Stanford, CA 94305 (e-mail: schneidr@stanford.edu).

1 Introduction

An important function of money is to serve as a unit of account. Within countries, contracts tend to be denominated in a common unit. Most often, the medium of exchange also serves as the unit of account. However, the function of unit of account is logically separate from money's role as a medium of exchange. In addition, there are many examples of the use of a unit of account that is different from the medium of exchange. In some cases, the currency of another country can serve as a unit of account. One example of this is the use of the U.S. dollar in foreign trade relationships not involving the United States. There are even examples of units of accounts that do not correspond to the medium of exchange of any country. Well-known example include the livre tournois in France, which was used as a unit of account for many years even when no corresponding coin was in circulation, and the ECU (European Currency Unit), which was based on a basket of European currencies and served as a unit of account in European Trade before the introduction of the Euro.

Most models of money focus on the medium-of-exchange role of money. In contrast, this paper develops a theory of the emergence of a unified unit of account. In our view, a key function of a unified unit of account is to lower relative-price risk in agents' balance sheets. Production often takes place in chains, where producers enter into various contracts with both their suppliers and their customers. We describe an environment where such contracts are non-contingent, and where default is costly. In addition, contracting is non-synchronized, in the sense that agents meet their business partners sequentially and do not know everybody who they will interact with when contracting starts. We show that in such an environment, the adoption of a unified unit of account can serve to minimize relative-price risk, leading to more opportunities for trade and less need for collateral.

Our work is related to a small literature on the optimality of nominal contracts. Jovanovic and Ueda (1997) consider a static moral hazard problem in which nominal output is observed before the price level (and therefore real output) is revealed. In addition, contracts are not negotiation proof, so that principal and

agent have an incentive to renegotiate after nominal output is observed. In particular, in the optimal renegotiation-proof solution the principal offers full insurance to the agent once nominal output is known. This implies that the real wage depends on nominal output, so that the contract can be interpreted as a nominal contract. Freeman and Tabellini (1998) consider an overlapping-generations economy with spatially separated agents in which fiat money serves as a medium of exchange. The authors show that under several particular circumstances it is then also optimal to use fiat money as a unit of account. Unlike these papers, in our theory neither delays in observation of prices or the use of money as a medium of exchange plays any role. Instead, we emphasize the use of a unified unit of account to lower the relative-price risk that economic agents carry on their balance sheets, leading to more possibilities for exchange and less need for collateral.

2 The Model

2.1 Environment

The model economy extends over two dates, 0 and 1. The economy is populated by two groups of agents, farmers and artisans. There is a finite set Φ of different types of farmers enumerated by $A, B, \ldots$ as well as a finite number n of different types of artisans enumerated by $1, 2, \ldots n$. There is a continuum of agents of each type; the mass of type i agents is denoted μ_i. An individual agent is identified by a pair (i, ι), where $\iota \in [0, \mu_i]$. The size of the set Φ and the number of artisans will be specified below when we study specific examples.

Each agent is endowed with one unit of labor at date 0. Each type has a technology that uses labor to make a good that is specific to him. We label a good by the type who produces it, that is, there are farm goods $A, B, \ldots$ and artisanal goods $1, 2, \ldots$. An artisan makes his specific artisanal good one-for-one from labor. A farmer can use one unit of labor to produce $1 + \theta$ units of his specific farm good. Instead of making his specific good, an agent may also use labor one-for-one to produce a numeraire good 0. Labor allocated to making the specific good

is denoted h and must be either zero or one; the remaining time $1 - h$ is enjoyed as leisure. In some examples below, we also assume that an agent of type i is endowed with k_i units of good zero that can be stored at no cost until date 1.

The key difference between farmers and artisans is in how their specific goods are sold. Farm goods can be sold in a spot market that opens at date 1. In the spot market, farm good i can be exchanged into good 0 at price p_i. The price p_i is stochastic and given exogenously. The vector of all farm good prices is denoted by p. In contrast, artisans produce customized goods for a particular customer. At date 0, before production takes place, every artisan (i, ι) is matched to one potential customer. If the artisan agrees to make and supply his artisanal product to the matched customer at date 1, no other agent can obtain utility from that product at date 1. In particular, an artisan cannot produce artisanal products for himself.

The details of the matching process are described below. For now, we summarize the outcome of matching by a function c, where $c(i, \iota)$ denotes the index pair of agent (i, ι)'s customer. Since matching will be one-to-one, there is an inverse function s such that $s(i, \iota)$ identifies an agent's potential supplier of an artisanal good.

The role of artisans and artisanal goods in the model is to generate a need for credit. Indeed, an artisan must commit to work at date 0 to produce the customized (nontradable) good at date 1. Moreover, the customer does not have sufficient tradable goods at date 0 to pay for the customized good up front. Exchange must thus be arranged through a system of forward contracts. The role of farmers and farm goods in the model is twofold. First, farm goods have quoted prices, so that contract can be denominated in farm goods. In contrast, the customized artisanal goods are not traded in spot markets and contracts cannot be denominated in them. Second, farmers are naturally exposed to aggregate risk induced by price volatility.[1] Price risk will matter because default on forward contracts is costly. Our analysis will consider how exchange can be organized in the economy to optimally deal with price risk.

[1]Price volatility is the only source of aggregate risk in our model. In addition, there is idiosyncratic risk from the matching process, to be discussed below.

The utility function of a type i agent (farmer or artisan) is given by:

$$u(x_0, x, h) = x_0 + (1 + \theta) \sum_{l=1}^{n} x_l - h.$$

Here x_0 is consumption of numeraire good and x_l is consumption of the customized good produced by artisan l. Since every agent will be matched to one supplier, a feasible allocation has an agent (i, ι) consume a positive amount of at most one artisan good, namely that produced by his supplier $s(i, \iota)$. Given that the utility cost of production is one, assuming $\theta > 0$ ensures that producing artisan goods in a match is optimal.

Since all agents in the model are risk neutral, a concern with price risk will arise exclusively from costs of default. We have also assumed that no agent consumes farm goods. One interpretation is that farm goods are monies, and that farmers actually produce other goods that they sell to agents outside the model in exchange for money. As a result, their revenue is in money. The exogenous farm prices would then represent exchange rates between different monies and good 0.

A feasible allocation must respect the technology of the agents as well as the outcomes of the matching process. The latter generates a network that links suppliers and customers at date 0, and prohibits trades of artisanal goods except through those links. Formally, a feasible allocation is a function that maps agents into choices of labor on the specific good, consumption of the good provided by their supplier as well as consumption of good zero:

$$(i, \iota) \to (h, x_{s(i,\iota)}, x_0).$$

An allocation is individually rational if every agent (i, ι) prefers his consumption bundle to the outside option that he can generate himself without entering into any contracts. For a farmer the best outside option is to sell in the spot market, so his individual rationality condition is

$$(1 + \theta)x_{s(i,\iota)} + x_0 - h \geq (1 + \theta)E[p_i] - 1. \tag{1}$$

An artisan cannot sell the customized good in the spot market. His outside option is thus to either produce good zero or to remain idle (which are equivalent in utility terms). His individuality rationality condition is thus

$$(1+\theta)x_{s(i,\iota)} + x_0 - h \geq 0. \tag{2}$$

2.2 Matching along a Highway

We now describe the way artisans meet customers at date 0. To visualize the matching process, imagine that all agents are located along a highway. All farmers are located at the west end of the highway, in location 0. Artisan types are located along the highway in ascending order, with types 1 located furthest west next to the farmers, types 2 next in an eastward direction, followed by types 3 and so on. The type n artisans are located at the eastern end of the highway. The same number of agents lives at each location. In particular, the total number of all farmers is equal to the total number of every type of artisan: For all artisan types $i = 1, .., n$, we have

$$\sum_{f \in \Phi} \mu_f = \mu_i \tag{3}$$

An artisan of type i who lives in location i can find a customer only in location $i-1$, immediately to the west of where he lives. In particular, a type 1 artisan can find customers only among the farmers, and an artisan of type $i > 2$ can find customers only among the artisans of type $i-1$. It follows that an agent can find a supplier of artisanal goods only in location $i+1$ immediately to the east of where he lives: a farmer can find a supplier only among the type 1 artisans, and an artisan of type i can find a supplier only among the artisans of type $i+1$.

The simplest highway can be depicted as a graph where the arrow means "can produce for":

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2 \tag{4}$$

Here there are two types of farmers located at the west end of the highway, and

two types of artisans.

In order to trade, agents have to travel to interact with agents up and down the highway (their customers and suppliers). The matching process has two stages, and every agent will meet exactly one of each of his immediate neighbors. In particular, every artisan $i < n$ will meet exactly one potential supplier and one potential customer. However, it is uncertain whether he first meets first the supplier or the customer. This setup captures the idea that bilateral contracts have to be signed before the full set of trading partners is known. In particular, parties have to decide on a unit of account before they know the denomination of contracts that have already been entered by future trading partners.

In the first stage of matching, agents meet any neighbor with equal probability. In particular, an artisan of type $i < n$ who lives in location i may match with an artisan of type $i+1$ (a potential supplier), or with an agent living in location $i-1$ (a potential customer). Both events occur with probability one half. For $i = 1$, the potential supplier is a farmer. A farmer matches with a type 1 artisan with probability one half, otherwise he does not match. The type of farmer matched to a type 1 artisan is independent across individual artisans with probabilities given by the farmer types' population shares. At the other end of the highway, a type n artisan matches with a type $n-1$ artisan with probability one half, otherwise he does not match.

For concreteness consider the example (4): after the first stage of matching, one half of type 1 artisans are matched with type 2 artisans, and there are $\mu_A/2$ and $\mu_B/2$ matches of type 1 artisans with type A and type B farmers, respectively. Moreover, the numbers of unmatched type A, type B and type 2 agents are $\mu_A/2$, $\mu_B/2$ and one half, respectively.

In the second stage of matching, artisans of type $i < n$ meet with a neighbor who lives in the opposite direction from the location of the first stage match. In addition, all farmers and type n artisans who were not matched in stage 1 will be matched (to a type 1 artisan and a type $n-1$ artisan, respectively), in stage 2. For example, a type 1 artisan who matched with a type 2 artisan (a supplier) in stage 1 will for sure meet a previously unmatched farmer (a customer) in stage 2. Again the probability of different farmer types follows from the population

shares.

2.3 No-default Contracts

We now show how allocations can be implemented via bilateral contracts. A contract consists of an artisan's promise to produce the customer's good, together with the customer's contract payment for the good. We impose three restrictions on contract payments. First, the payment must be non-contingent and denominated in units of farm goods (which are traded in spot markets and therefore have quoted prices). For a given match of an agent and his supplier, a contract payment can thus be summarized by a vector of farm goods promises $\pi_{(i,\iota),s(i,\iota)}$. Payment vectors satisfy $\pi_{i,j} = -\pi_{j,i}$.

Second, contracts must be fulfilled; there cannot be default. A farmer enters into contract only with one artisan. The payment promised by farmer (i, ι) who allocates $h = 1$ to his farm good i must be covered by his revenue from selling the farm good as well as the endowment of good zero:

$$\pi'_{(i,\iota),s(i,\iota)}p \leq p_i + k_i \quad \forall p$$

An artisan may enter into contracts with two neighbors, a customer and a supplier. The payment he promises to his supplier must be covered by the payment has been promised by his customer, plus his endowment of good zero:

$$\pi'_{(i,\iota),s((i,\iota))}p \leq \pi'_{c(i,\iota),(i,\iota)}p + k_i \quad \forall p.$$

The third restriction on contracts is that payments must be fixed at the stage of matching when the parties first meet. Let $m(i, \iota)$ denote the agent whom (i, ι) meets in the first stage. The size of the payment between an agent and his first match cannot depend on the identity of the second match or any other feature of the final network summarized by the supplier function s. However, to ensure that no defaults occur, contracts fixed at stage 1 that lead to default by any agent are canceled, with payments reduced to zero.

Formally, we assume that the payment between agent i, ι and his first match consists of two parts. First, there is a dummy variable that is either zero or one which governs whether any payment between the parties occurs. Second, there is the nonzero payment $\tilde{\pi}_{(i,\iota),m(i,\iota)}$ that becomes relevant if trade occurs. The restriction is that $\tilde{\pi}$ may depend only on the *types* of (i, ι) and $m(i, \iota)$, and not on their identities. We thus have

$$\pi_{(i,\iota),m(i,\iota)} = \delta_{(i,\iota),m(i,\iota)} \quad \tilde{\pi}_{(i,\iota),m(i,\iota)} \tag{5}$$

How the third restriction affects contracts can be seen by referring back to the example (4). In the first stage, half of the type 1 artisan match with a type 2 artisans. If the contract payment between these parties is positive (which is needed for types 1 to consume good 2), then it has to be fixed without knowledge of whether type 1 will next match with A or B.

Every contract naturally gives rise to an allocation: the payments determine agent consumption of good zero, and the contract also specifies the transfer of an artisanal good. We say that a feasible allocation can be implemented by a system of bilateral contracts if it is individually rational, and the contracts satisfy the three constraints described above: payments are non-contingent, there is no default, and the fixing of payments respects the information available during matching.

3 Units of Account in Economies without Collateral

We now go through a series of increasingly complex examples to demonstrate how the use of a dominant unit of account can facilitate exchange in our economy. All these examples share the assumption that the variability of prices is sufficiently large to require either the use of an appropriate unit of account or of collateral. Formally, the requirement is that the minimum price $\underline{p}_i$ of any farm good satisfies:

$$\underline{p}_i(1+\theta) < 1.$$

Also recall that the mean price of every farm good equals one:

$$E(p_i) = 1 \quad \forall i.$$

3.1 One Farmer, One Artisan: Insurance

Consider the simplest possible network consisting of a single type of farmer and a single type of artisan:

$$A \longleftarrow 1,$$

with measure one of each type. The assumption $\theta > 0$ guarantees that it is socially optimal for production to take place, i.e., each farmer A should produce good A, and each artisan should produce artisan good 1 for the farmer that this artisan is matched with. The question is which contracts can implement this outcome. We consider the case in which there is no collateral:

$$k_i = 0 \quad \forall i.$$

The matching is trivial in this example, as there is no uncertainty about which types will match. A contract can be described by a single number $\pi = \pi_{(A,\iota),s(A,\iota)}$, which is the payment from each farmer to the artisan who supplies him with good 1. Given that there is only one farm good, the contract has to be denominated in good A. No-default for the farmers requires that:

$$p_A(1+\theta)h_A \geq \pi p_A \forall p_A.$$

The price p_A cancels, and we have $h_A = 1$ in the desired allocation, so that the constraint can be written as:

$$\pi \leq 1 + \theta.$$

The contract also has to observe individual rationality constraints. For farmer A, the constraint is given by:

$$1 + \theta \geq E(\pi p_A) = \pi_{(i,\iota),s(i,\iota)}.$$

or:

$$1 + \theta \geq \pi.$$

Here the left-hand side is the utility from consuming the artisan good that the farmer derives from participating in exchange, whereas the right-hand side is the cost of the contract payment. For the artisan 1, we require:

$$E(\pi p_A) = \pi \geq 1,$$

i.e., the expected payment has to outweigh the disutility of building the artisan good. Combining the constraint, we see that the social optimum can be implemented with any contract that satisfies:

$$1 \leq \pi \leq 1 + \theta.$$

Trade is possible for any contract in this range, where the exact position within the range determines the distribution of social surplus between A and 1.

What is significant about this example for our purposes are the implications for which agents are subject to price risk. Given that the contract is denominated in terms of good A, the farmer is fully insured against fluctuations in the price of good A, and all price risk is carried by the artisan 1. In the example, this outcome was unavoidable, given that only good A was available to denominate contracts. Nevertheless, the underlying result is more general: to maximize opportunities for exchange, it is generally necessary for farmers to pass on their price risk to their trading partners. To see this, consider (for the sake of argument) a variation of the environment in which there is also a second farm good B that, in principle, could be used to denominate contracts. We would like to show that it is not possible to support a socially efficient allocation with good B as the unit of account. In particular, if B were the unit of account, the no-default constraint for the farmer A would read:

$$p_A(1 + \theta)h_A \geq \pi p_B \forall p_A, p_B.$$

Plugging in maximum labor supply $h_A = 1$ and the worst-case scenario for

prices, the binding constraint is:

$$\underline{p}_A(1+\theta) \geq \pi \bar{p}_B.$$

Given the assumption

$$\underline{p}_i(1+\theta) < 1,$$

the no-default constraint implies:

$$\pi < 1.$$

But then such a low payment would violate the individual rationality constraint of artisan 1, so that no exchange is possible.

In summary, what this simple example establishes is that, in the absence of collateral, trading connections between farmers and type-1 artisans have to be using the good specific to the farmer as the unit of account, because otherwise the no-default constraint and the individual rationality constraints cannot be satisfied simultaneously. Put differently, the artisan has to insure the farmer against price risk. In the next examples, we will see where this implication leads in larger networks.

3.2 One Farmer, Two Artisans: Chains with Common Unit of Account

Let us now consider the previous example with an additional element in the chain, namely artisans of type 2 who can produce customized goods for type-1 artisans:

$$A \longleftarrow 1 \longleftarrow 2.$$

Once again, we consider the environment without collateral. It is socially optimal for all agents to produce, and as before there is no uncertainty about matching. The required contracts can now be summarized by two numbers, namely a payment $\pi_{(A,\iota),s(A,\iota)}$ from each farmer to the artisan who supplies him with good 1, and a payment $\pi_{(1,\iota),s(1,\iota)}$ from each type-1 artisan to the artisan who supplies him

with good 2.

Despite the elongation of the chain, the no-default and individual-rationality constraint of the farmer and the individual-rationality constraint for artisan 1 for trade with farmer A are unchanged. Thus, the previous derivations are equally valid for this case, implying that in any contract implementing the social optimum payment $\pi_{(A,\iota),s(A,\iota)}$ has to be denominated in farm good A and has to satisfy:

$$1 \leq \pi_{(A,\iota),s(A,\iota)} \leq 1 + \theta.$$

Let us now consider how the payment $\pi_{(1,\iota),s(1,\iota)}$ to artisan 2 can be designed to maximize exchange. Assuming that the contract is denominated in good A, the no-default constraint for artisan 1 and the individual-rationality constraints for artisans 1 and 2 concerning this payment are given by:

$$\pi_{(A,\iota),s(A,\iota)} \geq \pi_{(1,\iota),s(1,\iota)}, \tag{6}$$

$$1 + \theta \geq \pi_{(1,\iota),s(1,\iota)}, \tag{7}$$

$$\pi_{(1,\iota),s(1,\iota)} \geq 1. \tag{8}$$

Summarizing these findings, the socially optimal exchange is possible if good A is the unit of account for all trades, and if contract payments satisfy:

$$0 \leq \pi_{(1,\iota),s(1,\iota)} \leq \pi_{(A,\iota),s(A,\iota)} \leq 1 + \theta.$$

Moreover, enabling trade between 1 and 2 *requires* the use of the same unit of account that is used between A and 1. If, to the contrary, the contract $\pi_{(1,\iota),s(1,\iota)}$ was denominated in a different farm good B, the no-default constraint would be

$$p_A \pi_{(A,\iota),s(A,\iota)} \geq p_B \pi_{(1,\iota),s(1,\iota)} \quad \forall p_A, p_B,$$

which would imply:

$$\pi_{(1,\iota),s(1,\iota)} < 1$$

and thus violate individual rationality for artisan 2.

What this example shows is that not just direct trades between farmers and ad-

jacent artisans, but entire chains of trades starting at a specific farmer have to be denominated in terms of the farmer's goods to be default-free in the absence of collateral. The price risk is passed on to the end of the chain, in this case to artisan 2. Notice in particular that good A is used in trades between artisans 1 and 2, even though neither of these two participants consumes or produces good A. Thus, this model of a chain provides a first example of the propagation of a unit of account in the economy.

3.3 Two Farmers, Two Artisans: A Dominant Unit of Account

We now further expand our network to introduce non-trivial matching.

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2.$$

Thus, there are now two different types of farmers, A and B, which opens the possibility of denominating contracts in either one of these units. As long as there is no collateral, it will no longer be possible to achieve the best feasible allocation with a single unit of account.

In this economy, matching is no longer trivial. In particular, when artisans of type 1 and 2 meet in the morning, they do not yet know which type of farmer artisan 1 is going to meet at night. As we will see, this friction prevents the implementation of the first best.

The same reasoning as before implies that in all meetings between farmers and type-1 artisans in the morning, the parties will decide to contract in the farmer's units, while obeying the constraints:

$$1 \leq \pi_{(A,\iota),s(A,\iota)} \leq 1+\theta,$$
$$1 \leq \pi_{(B,\iota),s(B,\iota)} \leq 1+\theta.$$

Moreover, when one of the type-1 artisans involved in these matches meets a type-2 artisan at night, they can decide to denominate in the same unit and

thereby create a full chain with a common unit of account as in the previous example.

The new feature of this environment arise from morning meetings between artisans of type 1 and 2. In these meetings, it is not yet known who type 1 will meet later on. These agents will agree on a contract in either good A or good B subject to the constraint:

$$1 \leq \pi_{(1,\iota),s(1,\iota)} \leq 1 + \theta.$$

If they agree on unit A and later on artisan 1 indeed meets a farmer of type A, a full chain can be created, and exchange takes place. If, in contrast, artisan 1 meets a farmer who produces the other good, no exchange can take place, and the existing contract with artisan 2 has to be canceled before production takes place.

The optimal denomination of contracts now depends on the relative number of the two types of farmers. If there were equal numbers of A and B ($\mu_A = \mu_B$), denomination in each good is equally likely to be successful. Thus, the total surplus created in the economy would be the same regardless of whether morning meetings between 1 and 2 are denominated in good A, good B, or a randomization between the two. The situation is different if the two groups of farmers are unequal in size. For example, if $\mu_A > \mu_B$, type-1 agents are more likely to meet farmers of type A, and denomination in good A becomes dominant, because there is a lower probability of canceled contracts. In this case, the only contracts that are denominated in good B are those in chains initiated by a morning meeting between a type-B farmer and a type-1 artisan.

In summary, in this example multiple units of account are used in the economy. However, apart from the symmetric case with $\mu_A = \mu_B$, there will be a dominant unit of account for the majority of trades in the economy.

3.4 Two Farmers, Four Artisans: A Dominant Unit of Account in a Symmetric Environment

In the previous example, a dominant unit of account arose only if there is one largest group of farmers. The point of our final example in this section is to demonstrate that a dominant unit of account arises endogenously *even in a symmetric environment* when the production chains are even longer. Consider, therefore, the following network:

$$\left(\begin{array}{c} A \\ B \end{array} \right) \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4.$$

We return to the assumption that $\mu_A = \mu_B$. Consider, first, what could be achieved in this economy if one of the farm goods serves as a dominant unit of account. Without loss of generality, let this good be A. We are interested in equilibria in which all trades are denominated in good A, except those that start with a morning meeting between a farmer of type B and a type-1 artisan. Under this outcome, one-half of all agents will be involved in full-length chains starting at a type-A farmer and ending at a type-4 artisan. In addition, one quarter of type-1 artisans will have a contract denominated in good B with a type-B farmer. The same outcome could be achieved by using good B as the dominant unit of account.

We now would like to address the question of what could be achieved if, rather than using a dominant unit of account, morning meetings not involving a farmer were denoted in a random unit of account. In the previous example under symmetry, the outcome was equivalent to the one under a dominant unit of account. This is no longer the case here.

To see this, we need to show that fewer relationships can be sustained under this arrangement compared to the dominant unit of account. This is indeed the case, because there is now additional potential for mismatch. Under the random unit of account, in the morning some type-2 agents deal with type-1 agents in terms of good A, while others choose good B. The same is true for morning meetings between type-3 artisans and type-4 artisans. If now type-2 and type-3 artisans

meet at night, some contracts cannot be entered, because the units used by these agents do not match. This breakdown of contracting is *in addition* to the breakdown of contracting because of a failed link between a type-1 agent and a farmer and a type-1 and type-2 artisan, which are the only sources of mismatch under a dominant unit of account. Thus, adopting a dominant unit of account maximizes opportunities for exchange.

This example clarifies the role of random matching for the emergence of a dominant unit of account. We believe it is an important feature of the real world that contracting takes place sequentially, and often future contract partners are not known when initial contracting begins. By adopting a dominant unit of account, in such environments agents can avoid relative price risk even in the face of uncertainty over their future contracting opportunities.

3.5 Optimality of a Dominant Unit of Account

We now proof the optimality of a dominant unit of account with a more general matching process. In this version of the model, ex ante there are only two types of agents, farmers and artisans, but there is no distinction between different types of artisans. There is mass one of farmers and mass n of artisans, where we assume $n \geq 3$. The set of agents is denoted I, and in this section we refer to an agent by a single index $i \in I$. These agents are matched in two stages (as before), where the final outcome of the matching process is a *trading network*. A trading network is described by a function that matches and agent's name i into a triple

$$\{s(i), c(i), \eta(i)\},$$

where $s(i)$ is the supplier of agent i, $c(i)$ is the customer, and $\eta(i) \in \{0, 1\}$ indicates whether the agent first meets a customer or supplier. For artisans, $\eta(i) = 1$ means the artisans meets a customer at night. An artisan with $\eta(i) = 0$ may either meet a supplier at night or does not meet anybody at night, which corresponds to the artisans at the end of each chain in the examples above. A farmer with $\eta(i) = 1$ meets his supplier in the morning, and a farmer with $\eta(i) = 0$ meets his supplier at night. The failure to meet a supplier or customer is denoted by

$s(i) = 0$ or $c(i) = 0$, respectively. The trading network is required to be internally consistent (if $s(i) \in I$, then $c(s(i)) = i$; if $s(i) \in I$ and $\eta(i) = 1$, then $\eta(s(i)) = 0$; etc.).

In addition, we require that the trading network is such that it partitions all agents into chains, where each chain starts with a farmer and ends with an artisan as in the examples above. Formally, we require that all farmers match with a supplier, that all artisans match with a customer, that the mass of farmers with $\eta(i) = 0$ equals the mass of artisans with $\eta(i) = 1$ and $s(i) = 0$ (i.e., equal numbers of artisans and farmers are unmatched in the morning), and that the mass of artisans with $s(i) = 0$ equals the mass of farmers. These restrictions imply that the chains generated by the trading network are as in the examples above.

A *matching process* is now defined by a probability distribution over trading networks and by the information that is available to agents at the first stage of matching (in the morning). We model this information as a signal $\sigma(i)$, where agent i and the agent $m(i)$ he matches with in the morning receive the same signal. In the examples above, we considered the special case where all feasible trading networks consists of chains of equal length, where equal numbers of farmers find suppliers in the morning and the evening, and where agents know their position in their chain before contracting in the morning.

No-default contracts satisfy restrictions analogous to those in Section 2.3. In particular, the payment $\tilde{\pi}_{i,m(i)}$ may only depend on the information $\sigma(i)$ that is available in the morning.

We focus attention on matching processes such that all artisans face the same probability of ending up at any given position (where the definition of position includes whether one first meets a customer or supplier) in any given chain, and where all chains include at least three artisans. Also, we focus on the case in which the only information available to an artisan during a morning meeting is whether the partner is an artisan or a specific type of farmer. Thus, in the morning there are $\Phi + 1$ types of matches: those between one of the Φ types of farmers and an artisan and those between two artisans. We assume that half of farmers match in the morning and the other half matches in the evening.

We now would like to establish that the use of the good of the largest of group of

farmers as a dominant unit of account maximizes social surplus. Without loss of generality, assume that $\mu_A = \max_{i \in \Phi}\{\mu_i\}$. A payment is denominated in A if the vector $\pi_{i,j}$ has only one non-zero element that is denominated in A.

Proposition 1 (Optimality of Dominant Unit of Account)

1. *The maximum social surplus can be implemented by a system of bilateral contracts such that:*
   - *All payments between artisans are denominated in A.*
   - *All payments between farmers and artisans are denominated in units of the farmer's goods.*
2. *If $\mu_A > \mu_i \forall i \in \Phi, i \neq A$, then denominating some or all contracts between artisans in a different unit leads to strictly lower social surplus.*

Proof: Every agent who produces in equilibrium generates an expected surplus of θ. Thus, social surplus can be measured by the mass of agents who produce. In the conjectured system of bilateral contracts, payments can be arranged such that all farmers and mass $\mu_A N + (1-\mu_A)/2$ of artisans produce. On the one hand, every agent in a chain that is headed by a type-A farmer produces. On the other hand, artisans who are matched in the morning to a farmer type other than A also produce. In all other chains, artisans who were not matched in the morning to a farmer do not produce. Indeed, the mismatch between the general unit of account A and the unit used by the leading artisan-farmer pair do not allow a selection of payments that respect individual rationality and absence of default.

Now consider outcomes under an alternative choice of units. First, consider any system in which some farmers do not denominate in their own unit. For a contract satisfying individual rationality constraints, such farmers would default in some states of the world for sure, which is not allowed. Thus, all chains headed by such farmers break down. This shows that to maximize social surplus, payments between farmers and artisans have to be denominated in units of the farmer's goods.

For the second step, assume (as implied by the first step) that all farmers contract in their own units. Now consider any alternative choice of units in artisan-artisan matches in the morning. In the evening, the customer of this pair faces a probability of matching either a farmer or an artisan already matched to a farmer. In those cases, a link can be formed (that is, the supplier produces) if the artisan pair denominates in the units of the farmer they are matched to. If they denominate in unit A, the match breaks down with conditional probability $1 - \mu_A$, whereas if an alternative unit i is chosen the conditional breakdown probability is $1 - \mu_i > 1 - \mu_A$. Thus, in this type of matches a different choice of units leads to less social surplus. Alternatively, the matched artisans may link up to another pair of artisans. In this case, breakdown occurs if the two pairs of artisans contract in different units. Since in the conjectured system all artisan pairs contract in the same A units, no such breakdowns occur, and another choice of units cannot increase surplus. □

4 Units of Account and the Allocation of Collateral

So far, we have considered an environment in which no agent has access to collateral. If collateral is present in the economy, at least some agents have the capacity to bear relative price risk. The optimal choice of a unit of account therefore becomes a function of the distribution of collateral in the economy. Conversely, the optimal distribution of collateral depends on which unit of account is used.

As a first pass of this issue, we consider the question of which combination of accounting units and a distribution of collateral can implement a first best allocation while minimizing the use of collateral. Clearly, with unlimited collateral any choice of accounting units can implement the first best: we merely have to compute the relative price risk of each agent and allocate collateral accordingly. With a limited amount of collateral, it turns out to be optimal to use collateral at the beginning of each chain. Consider once again our last example of a network

of the form:

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4.$$

One way of implementing the first best is to use farm good A as the *only* unit of account, and endow each type-B farmer with sufficient collateral to prevent default. If, in contrast, no dominant unit of account is used, default is possible for a larger set of agents. Implementing the first best would therefore require additional collateral.

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