Entry, Exit, Firm Dynamics, and Aggregate Fluctuations*

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This version: February 15, 2010

Largely preliminary and incomplete draft submitted to the 2010 SED Meeting – Please do not quote

Abstract

We amend Hopenhayn (1992)'s model of equilibrium industry dynamics by explicitly modeling the firm's investment choice and by introducing aggregate fluctuations. Our main goal is to study the model's implications for the cyclical behavior of entry, exit, and the cross–section of operating firms. We show that the vector of state variables include the size distribution of firms, an infinite–dimensional object. We overcome this obstacle by showing that firms incur in small errors when predicting the evolution of the relevant price by means of a simple forecasting rule. Preliminary results show that the model is able to replicate key features of the cross–section of US manufacturing plants, such as the mean and standard deviation of the investment rate, as well as the average entry rate and the average ratio of entrants' size to incumbents' size. Entry rates are higher in expansion than in recession, while the opposite holds true for exit rates. Entering plants tend to be more productive during recessions than during expansions.

Key words. ***.

JEL Codes: *** .

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1 Introduction

The model of equilibrium industry dynamics introduced by Hopenhayn (1992) has proven to be a very powerful device for the analysis of firm dynamics in stationary economies. See for example Hopenhayn and Rogerson (1993). In this paper we amend its structure in order to allow for investment and for aggregate fluctuations.

Our main goal is to study the model's implications for the cyclical behavior of entry, exit, and the cross–section of operating firms.

Firms produce an homogeneous good by means of capital and labor. Their technologies display decreasing returns to scale. We assume that the demand for firms' output is infinitely elastic, while the supply of labor services has finite elasticity.

Fluctuations are introduced by assuming that firm–level productivity is the product of an idiosyncratic component and an aggregate component. Both are driven by persistent stochastic processes.

At every point in time, firms are heterogenous along three dimensions. One is permanent: the covariance between the innovation to the idiosyncratic component of productivity and the innovation to the common component. The others, the realization of the idiosyncratic productivity shock and the level of the capital stock, are time–varying.

Future cash flows are evaluated by means of a stochastic discount factor, whose realization depends on the realization of the aggregate shock.

Every period, a unit mass of prospective entrants observe a signal about their initial productivity. Only those whose expected value from operating at the efficient scale is greater than the cost of entry start producing. Exit is modeled by assuming that there is a time–invariant opportunity cost to operating.

We show that the vector of state variables consist of the distribution of firms over the three dimensions of heterogeneity, along with the realization of the aggregate shock. Knowledge of the distribution is necessary in order to form expectations about the evolution of the wage rate.

Faced with the daunting task of working with an infinite–dimensional state space, we follow the lead of Krusell and Smith (1998) and assume that firms form expectations by means of a simple forecasting rule. We posit that the wage is an affine function of the wage in the previous period and the aggregate productivity shock in the current and previous period. Our numerical results show that the forecasting error is unbiased, and its mean and standard deviations are tiny.

Preliminary results show that the model is able to replicate key features of the cross–section of US manufacturing plants, such as the mean and standard deviation of the investment rate, as well as the average entry rate and the average ratio of entrants' size to incumbents' size. Entry rates are higher in expansion than in recession, while the opposite holds true for exit rates. Entering plants tend to be more productive during recessions than during expansions.

The dynamics of the cross–section of firms has been the object of interest for a number of recent papers. Several of them shares many features with our environment, but differ in important respects. Veracierto (2002), Khan and Thomas (2007) and Bachman and Bayer (2009) focus on investment and decide not to model entry and exit. Samaniego (2008) studies entry and exit in a general equilibrium model, but limits the analysis to transitional dynamics. Bilbiie, Ghironi, and Melitz (2007) and Lee and Mukoyama (2009) don't allow for investment in physical capital. Campbell (1998) analyzes a vintage capital model.

2 Model

Time is discrete and is indexed by $t = 1, 2, ...$ The horizon is infinite. At time t, a mass $N_t \geq 0$ of price–taking firms produce an homogenous good by means of the production function $y_t = e^{z_t+s_t}(k_t^\alpha l_t^{1-\alpha})^\theta$, where k_t denotes physical capital, l_t is labor, and z_t and s_t denote an aggregate and idiosyncratic random disturbances, respectively. The parameter $\theta \in (0, 1)$ is the span of control.

The common component of productivity z_t is driven by the stochastic process

$$\log z_{t+1} = \rho_z \log z_t + \sigma_z \varepsilon_{z,t+1},$$

where $\varepsilon_{z,t} \sim N(0, 1)$ for all $t \geq 0$. The dynamics of the idiosyncratic component s_t is described by

$$\log s_{t+1} = \rho_s \log s_t + \sigma_s \varepsilon_{s,t+1},$$

with $\varepsilon_{s,t} = \beta\varepsilon_{z,t} + (1 - \beta^2)^{1/2}\eta_t$, $\beta \in [0, 1)$, and $\eta_t \sim N(0, 1)$ for all $t \geq 0$. Notice that this implies $\varepsilon_{s,t} \sim N(0, 1)$ and $cov(\varepsilon_z, \varepsilon_s) = \beta$. Firms differ in the covariance between innovations to idiosyncratic and aggregate shocks.

Firms hire labor services on the spot market at the wage rate $w_t \geq 0$ and discount future cash flows by means of the stochastic discount factor $\frac{1}{R}m(z_{t+1}|z_t)$. Adjusting the capital stock by x requires firms to incur cost $g(x, k)$. Capital depreciates at the rate $\delta \in (0, 1)$.

We assume that the demand for the firm's output is infinitely elastic and normalize its price at 1. The supply of labor is given by the function $L_s(w) = w^{\gamma}$, with $\gamma > 0$.

Firms that quit producing cannot re–enter the market at a later stage and obtain a value $V_o > 0$.

Every period there is a unit mass of prospective entrants, each of which receives a signal q about their productivity, with $q \sim Q(q)$. Conditional on entry, the distribution of the idiosyncratic shock in first period of existence is $B(s'|q)$. Entrepreneurs that decide to enter the industry pay an entry cost $c_e > 0$.

At all $t \geq 0$, the distribution of operating firms over the three dimensions of heterogeneity is denoted by $\Gamma_t(\beta, k, s)$. Finally, let $\omega_t = \{\Gamma_t, z_t\} \in \Omega$ denote the vector of aggregate state variables and $J(\omega_{t+1}|\omega_t)$ its transition operator.

2.1 The incumbent's optimization program

Given the aggregate state ω, capital in place k, and idiosyncratic shock s, the value of an incumbent $V(\omega, k, s)$ is the fixed point of the following functional equation:

$$V(\omega, k, s) = \max[V_o, \max_{x,l} \pi(\omega, k, s, x, l) + \frac{1}{R} \int_{\Omega} \int_{\Re} m(z'|z) V(\omega', k', s') dH(s'|s, z, z') dJ(\omega'|\omega)], \tag{P1}$$

$$\text{s.t. } \pi(\omega, k, s, x, l) = e^{s+z} f(k, l) - wl - x - g(x, k),$$
$$k' = k(1 - \delta) - x.$$

2.2 Entry

The value of a prospective entrant that obtained a signal q when the aggregate state is ω is

$$V_e(\omega, q) = \max_{k'} -k' + \frac{1}{R} \int m(z'|z) V(\omega', k', s') dB(s'|q) dJ(\omega'|\omega) - c_e. \tag{1}$$

She will invest and start operating if and only if $V_e(\omega, q) \geq 0$.

2.3 Equilibrium

For given Γ_0, a competitive recursive equilibrium consists of (i) value functions $V(\omega, k, s)$ and $V_e(\omega, q)$, (ii) policy functions $x(\omega, k, s)$, $l(\omega, k, s)$, $k'(\omega, q)$, and (iii) bounded sequences of wages $\{w_t\}_{t=0}^{\infty}$, of incumbents' measures $\{\Gamma_t\}_{t=1}^{\infty}$, and of entrants'measures $\{\mathcal{E}_t\}_{t=0}^{\infty}$ such that, for all $t \geq 0$,

1. The function $V(\omega, k, s)$ is a fixed point of the operator (P1);

2. The functions $x(\omega, k, s)$ and $l(\omega, k, s)$ solve the optimization problem in (P1);
3. The function $V_e(\omega, q)$ is given by (1);
4. The function $k'(\omega, q)$ solves the optimization problem in (1);
5. The labor market clears: $\int l(\omega_t, k, s) d\Gamma_t(\beta, k, s) = L_s(w_t)$;
6. For all $\beta \in [0, 1)$ and for all Borel sets $\mathcal{S} \times \mathcal{K} \in \Re \times \Re^+$,
$$\mathcal{E}_{t+1}(\beta, \mathcal{S} \times \mathcal{K}) = \int_{\mathcal{S}} \int_{\mathcal{B}_e(\beta, \mathcal{K}, \omega_t)} dQ(q) dB(s'|q),$$
where $\mathcal{B}_e(\beta, \mathcal{K}, \omega_t) = \{q \text{ s.t. } k'(\beta, \omega_t, q) \in \mathcal{K} \text{ and } V_e(\beta, \omega_t, q) \geq 0\}$;
7. For all $\beta \in [0, 1)$ and for all Borel sets $\mathcal{S} \times \mathcal{K} \in \Re \times \Re^+$;
$$\Gamma_{t+1}(\beta, \mathcal{S} \times \mathcal{K}) = \int_{\mathcal{S}} \int_{\mathcal{B}(\beta, \mathcal{K}, \omega_t)} d\Gamma_t(\beta, k, s) dH(s'|s, z_t, z_{t+1}) + \mathcal{E}_{t+1}(\beta, \mathcal{S} \times \mathcal{K}),$$
where $\mathcal{B}(\beta, \mathcal{K}, \omega_t) = \{(k, s) \text{ s.t. } k(1 - \delta) + x'(\omega_t, k, s) \in \mathcal{K}\}$.

3 The State Space

The labor market clearing condition implies that the equilibrium wage can be expressed as a function of the aggregate shock and the distribution $\Gamma(\beta, k, s)$:
$$\log w_t = \frac{\log[(1-\alpha)\theta z_t]}{1 + \gamma[1 - (1-\alpha)\theta]} + \frac{1 - (1-\alpha)\theta}{1 + \gamma[1 - (1-\alpha)\theta]} G_t, \tag{2}$$
with $G_t = \log \left[\int \left(s k^{\alpha\theta} \right)^{\frac{1}{1-(1-\alpha)\theta}} d\Gamma_t(\beta, k, s) \right]$. This shows that the vector of state variables consists of the distribution Γ_t and the aggregate shock z_t.

Faced with the formidable task of approximating an infinitely–dimensional object, we follow Krusell and Smith (1998) and conjecture that G_{t+1} is an affine function of G_t and $\log z_{t+1}$. Then, (2) implies that the equilibrium wage follows the following law of motion:
$$\log w_{t+1} = \beta_0 + \beta_1 \log z_{t+1} + \beta_2 \log z_t + \beta_3 \log w_t. \tag{3}$$

When computing the numerical approximation of the equilibrium allocation, we will impose that firms form expectations about the evolution of the wage assuming that (3) holds true. This means that the aggregate state variables reduce to the wage w and the aggregate shock z.

The values of the parameters $\{\beta_0, \beta_1, \beta_2, \beta_3\}$ are chosen to maximize the accuracy of the prediction rule. See the appendix for an illustration of the algorithm.

4 Calibration

Table 1 lists all the model's parameters. In our benchmark scenario, the discount factor is constant, i.e. $m(z'|z) = 1$. Furthermore, we let $\beta = 0$ for all firms. The latter means that idiosyncratic shocks are uncorrelated with the aggregate shock.

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Persist. idiosync. shock	ρ_s	0.7
Variance idiosync. shock	σ_s	0.2
Persist. aggregate shock	ρ_z	0.8
Variance aggregate shock	σ_z	0.008
Variable cost of investment	c_1	0.05
Labor supply elasticity	γ	3.0
Pareto exponent	ξ	5.0
Pareto lower bound	$\underline{q}$	0.432
Entry cost	c_e	0.15
Value of exit	V_o	0.15

Table 1: Parameter Values.

The distribution of signals for the entrants is Pareto. We posit that $q \geq \underline{q} \geq 0$ and that $Q(q) = (\underline{q}/q)^{\xi}$, $\xi > 1$. The realization of the idiosyncratic shock in the first period of operation follows the process $\log(s) = \rho_s \log(q) + \eta$. Capital adjustment costs are given by $g(x, k) = c_1(x/k)^2$, with $c_1 \geq 0$.

One period is assumed to be one year. Consistently with most macroeconomic studies, we assume that $R = 1.04$ and $\delta = 0.1$. We also set $\alpha = 0.3$ and $\theta = 0.8$. The latter is consistent with Basu and Fernald (1997). The choice of $c_1 = 0.05$ is within the range of estimates obtained by Cooper and Haltiwanger (2006).

The parameters governing the stochastic processes were chosen in such a way that a number of statistics computed using a panel of of simulated data are close to their empirical counterparts.

The values of ρ_z and σ_z were picked so that mean and standard deviation of the Solow residual are close to the figures reported by Cooley and Prescott (1995).

The parameters of the process driving the idiosyncratic shock are chosen to match the mean and standard deviation of the investment rate. Using a balanced panel from the LRD from 1972 to 1988, Cooper and Haltiwanger (2006) obtain estimates for the two moments equal to 0.122 and 0.337, respectively.

Finally, the parameters ξ, c_e, and V_o were chosen to match the relative size of entrants and exiters and the entry rate. The targets are the statistics obtained by Lee and Mukoyama (2009) using the LRD.

Table 2 shows that the model is able to hit all the targets, with the exception of the exiters' relative size.

Statistic	Model	Data
Persist. Solow residual	0.76434	0.95
Std. Dev. Solow residual	0.00743	0.007
Mean investment rate	0.1354	0.122
Std. Dev. investment rate	0.3217	0.337
Entry rate	0.06	0.062
Entrants' relative size	0.5665	0.60
Exiters' relative size	0.1613	0.49

Table 2: Parameter Values.

5 Preliminary Results

In the benchmark calibration, the forecasting rule for the equilibrium wage is

$$\log w_{t+1} = 0.2505 + 0.4762 \log z_{t+1} - 0.2364 \log z_t + 0.7719 \log w_t + \varepsilon_{t+1}$$

The R^2 of the regression is 0.9954 and the absolute value of the forecasting error is never higher than 0.37% of the actual equilibrium price. The correlation between the actual market–clearing wage and its forecast is 0.9977.

The left panel in Figure 1 illustrates the distribution of forecasting errors. The right panel is a scatter plot of equilibrium wages and their respective forecasts.

The forecasting rule implies that the process for the wage is persistent and mean–reverting. A positive aggregate shock increases the demand for labor from both incumbents and entrants. This is why the coefficient of $\log(z_{t+1})$ (β_1) is estimated to be positive. For the same reason, the coefficient of $\log(z_t)$ (β_2) is negative. The larger the aggregate shock in the previous period, the smaller is going to be the expected increment in aggregate productivity, and therefore the lower the increase in the price.

Figure 2 illustrates the unconditional distribution of firm over size, proxyed by capital. The picture, which was constructed by averaging the firm size distribution over a very long simulation, hints that our model is consistent with qualitative features of the data, among which the extreme positive skewness.

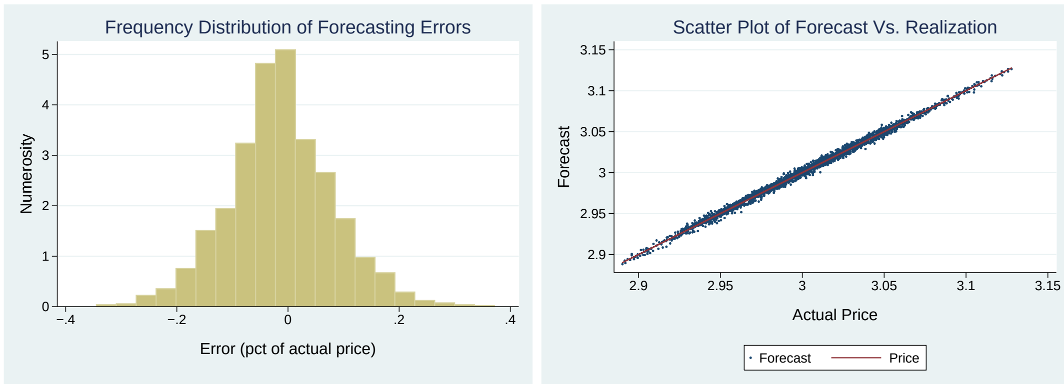

Figure 1: Accuracy of the Forecasting Rule.

Analyzing data from the LRD, Lee and Mukoyama (2009) find that the selection of entry is quantitatively very important. Entering plants tend to be more productive when the industry is in recession than when it is in expansion. Preliminary results hint that our model shares this feature of the data. In figure 3, we portray the (average) cumulative distribution over the idiosyncratic component of productivity. The distribution conditional on recession stochastically dominates that conditional on expansion. (Expansions are defined as periods in which the industry's output is greater than average. The converse for recessions.)

The left panel of Figure 4 displays the cumulative distribution of entry rates over a very long series of simulations. Similarly for exit rates on the right panel. As expected, entry rates tend to be higher in expansions. Exit rates are higher in recession.

TO BE CONTINUED

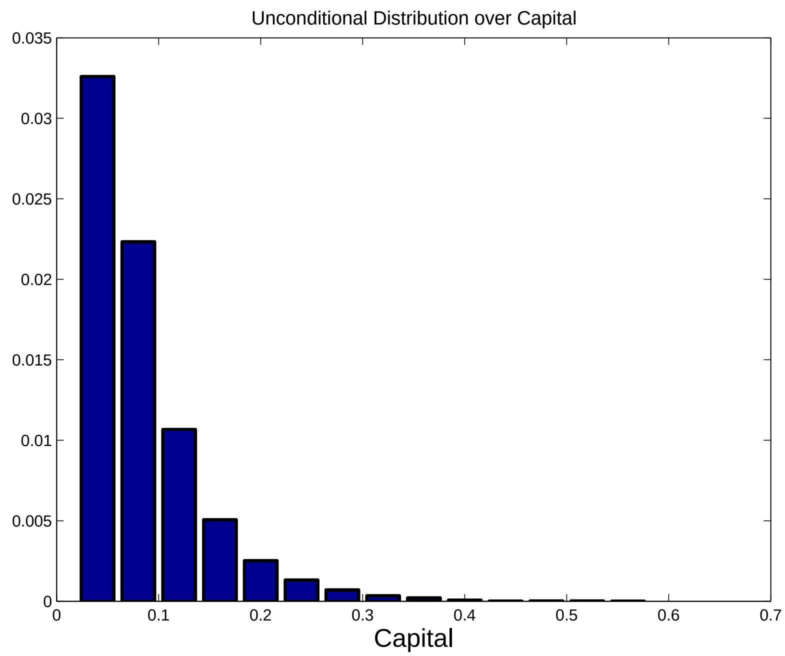

Figure 2: The unconditional distribution of firms over size.

A Numerical Approximation

The solution to the incumbent's optimization problem is approximated by value function iteration. The grid for capital is constructed following the method suggested by McGrattan (1999). The grids and transition matrices for the two shocks are constructed following Tauchen (1986). The grid for the equilibrium wage is centered around the equilibrium wage of the stationary economy.

We start by guessing values for $\left\{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\right\}$ and for the value function. Using a simple variant of the value function iteration algorithm, we compute optimal decision rules for labor, investment, and exit. Given the value function for the incumbent firm, we solve for the optimal entry decision rule and for the entrant's optimal investment policy.

Then, we proceed to simulate the model for a very large number of periods. We end up with time–series $\{w_t^*, z_t\}$, where w_t^* is the actual market–clearing wage.

A revised guess for the vector of parameters $\{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\}$ is obtained by running the regression

$$\log w_{t+1}^* = \beta_0 + \beta_1 \log z_{t+1} + \beta_2 \log z_t + \beta_3 \log w_t^* + \varepsilon_{t+1}$$

We repeat the procedure until convergence of the parameters.

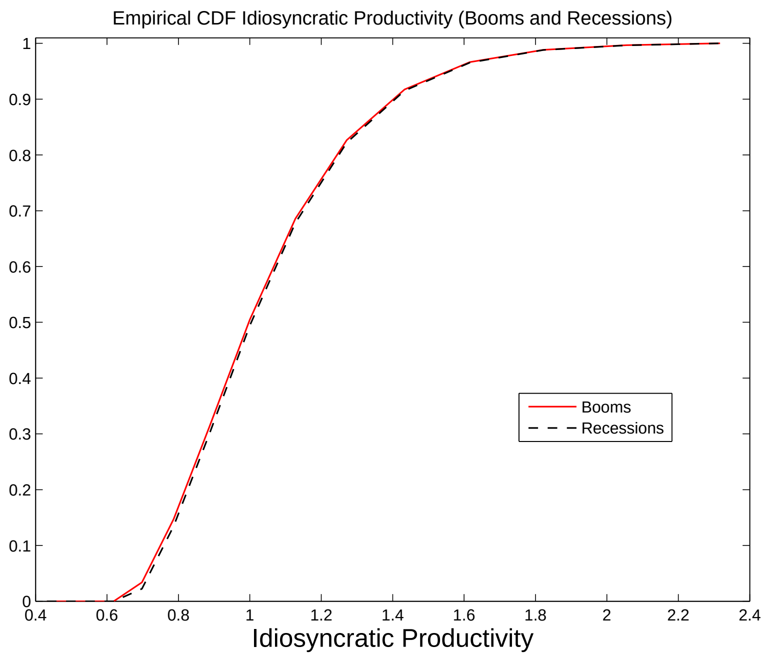

Figure 3: The cdf of the idiosyncratic component in expansion and in recession.

References

Bachman, R., and C. Bayer (2009): "The Cross–Section of Firms over the Business Cycle: New Facts and a DSGE Exploration," Working paper, University of Michigan.

Basu, S., and J. Fernald (1997): "Returns to Scale in US Production: Estimates and Implications," *Journal of Political Economy*, 105(2), 249–283.

Bilbiie, F., F. Ghironi, and M. Melitz (2007): "Endogenous Entry, Product Variety, and Business Cycles," NBER Working Paper # 13646.

Campbell, J. (1998): "Entry, Exit, Embodied Technology, and Business Cycles," *Review of Economic Dynamics*, 1, 371–408.

Cooley, T., and E. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, pp. 1–38. Princeton University Press.

Cooper, R., and J. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 73(3), 611–633.

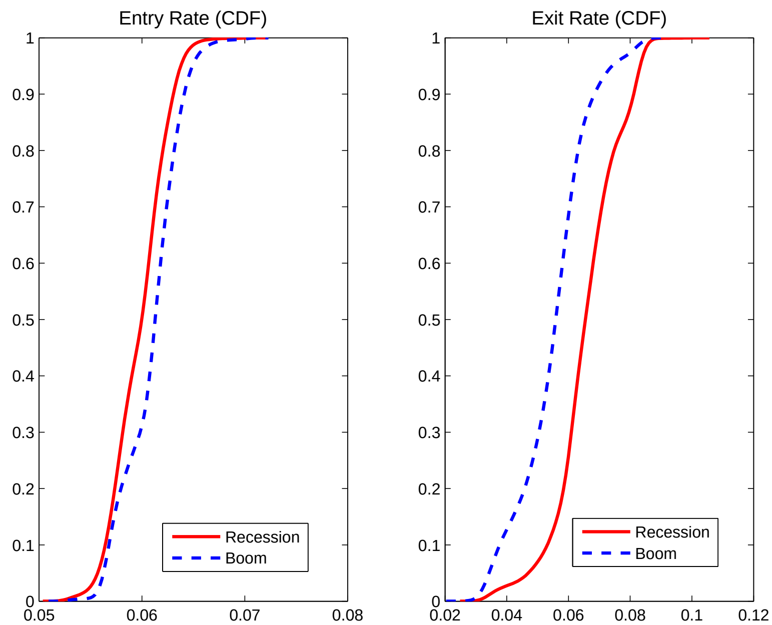

Figure 4: The cdf of entry and exit rate in expansion and in recession.

Hopenhayn, H. (1992): "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127–1150.

Hopenhayn, H., and R. Rogerson (1993): "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 101(5), 915–938.

Khan, A., and J. K. Thomas (2007): "Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies," *American Economic Review*, 97(4), 1165–1188.

Krusell, P., and A. Smith (1998): "Income and wealth heterogeneity in the macroeconomy," *Journal of Political Economy*, 106, 867–896.

Lee, Y., and T. Mukoyama (2009): "Entry, Exit, and Plant–level Dynamics over the Business Cycle," Working paper, University of Virginia.

McGrattan, E. (1999): "Application of weighted residual methods to dynamic economic models," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott. Oxford University Press.

Samaniego, R. (2008): "Entry, Exit and Business Cycle in a General Equilibrium Model," *Review of Economic Dynamics*, 11, 529–541.

Tauchen, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20, 177–181.

Veracierto, M. (2002): "Plant–Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review*, 92(1), 181–197.