On Population Structure and Marriage Dynamics

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Abstract

I develop an equilibrium, two-sided search model of marriage with endogenous population growth to study the interaction between fertility, the age structure of the population and the age at first marriage of men and women.

I show that, given an increase of the desired number of children, age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population produces a "thicker market" for young people, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, young women become relatively less choosy than young men. In equilibrium, women are more likely to marry older men and single men outnumber single women.

In the second part of the paper I extend the model to a finite number of periods and, using fertility data, show that two mechanisms described above may have acted as persistence mechanisms after a fertility shock like the baby boom in the U.S. Specifically, I show that demographic dynamics may account for up to a third of the increase in men's age of marriage since 1977.

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1 Introduction

In this paper, I investigate the relationship between fertility, the age structure of the population and the timing of marriage of men and women. Looking at the evolution of those variables in the U.S., we observe several facts that suggest the existence of such a relationship. First, the median age of marriage of men and women appears to be negatively correlated with the total fertility rate (TFR), although with a delay.[1] Observe in Panel (a) of Figure A.5 that fertility rates were sharply falling from the early sixties to the mid seventies, while the age of marriage of men and women starts to unambiguously increase only when fertility rates become stable.

The relationship between marriage timing and fertility can be reconciled if we take a closer look at the demographic implications of the U.S. "Baby Boom". Notice in Panel (b) of Figure A.5, which shows the evolution of the share of population aged 15-29 years-old on those aged 15-44, together with the TFR, that this measure of population structure mimics fertility rate with a lag of about 20 years. Specifically, this share starts increasing in the early sixties, when the early "boomers" became 15 year-olds and peaks in the mid-seventies, when they turned 30, to then decrease until the mid-nineties. Panel (c) shows the comparison between this measure of population structure and the median age of marriage of men and women. Notice that both measures of marriage timing start increasing approximately at the same time the population starts to become older again, around 1975.

The second fact that links fertility and marriage timing is related to the age gap at marriage between men and women. As shown in Panel (d) of Figure A.5, the difference between the average age at first marriage men and women shows a positive correlation with the total fertility rate. Observe that the age gap increases in the forties and starts to decrease just by the end of the baby boom, around 1963.[2]

In this paper, I will argue that the timing of marriage of men and women is affected by two endogenous sources of frictions. The first one is the age structure of the population, which is a function of past fertility and produces a "thicker market externality" that induces men and women to marry earlier in the case where the population becomes younger. The second source is the ratio of single men and single women, which produces asymmetries in the marriage opportunities of men and women. As I will show in this paper, assuming that there are always equal total quantities of men and women in the economy, the age gap and the sex imbalances are jointly determined by the behavior of the agents. Observe in Panel (d) of Figure A.5 the positive correlation between the sex ratio and the age gap in the time series. Finally, notice in Panel (f) that the evolution in

[1] Throghout this paper I use the median age at first marriage as a measure of marriage timing, because population structure is a key factor in this paper. The U.S. Census Bureau publishes yearly the estimated median age at first marriage since 1947, using an indirect method based on the proportion of people who were ever married within the single age cohort. This methodology prevents the age of marriage from being affected mechanically by population structure. Since the mean age of marriage is conditional on marriage, this measure is subject to those mechanical effects. However, I use means of age at first marriage in order to calculate the age gap.

[2] In contrast to the previous marriage literature, which assumes that the age difference is constant, Ni Brolcháin (2001), has shown that the age gap changes according to the size of the cohorts.

the sex ratio does not appear to follow changes in the relative total quantity of men and women, at least since 1940.[3]

In Section 2 I present the basic, steady state model. Specifically, I develop an overlapping generations general equilibrium model with two-sided search in which agents only differ in their age and are able to marry once in their lifetimes. I assume that they live as adults for two periods and that they obtain utility from the (idiosyncratic) quality of their marriage and from the joy of having children. I also assume that both men and women desire to have a given number of children, which I keep as exogenous. Given desired fertility, population growth, and therefore the age structure of the population, are endogenous. In this model, as people get older they become less attractive to their prospective younger mates. In the case of both sexes, this is because younger spouses know that they will lose the utility of being married relatively early in life when their older spouse dies. In the case of women, the disutility of aging increases because they lose their fecundity relatively early.

Given the above assumptions, I show that, given an increase of the desired number of children, age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population is altered, which reduces the frictions that men and women face in order to meet other younger counterparts, and therefore increases their likelihood to marry younger. The second channel comes from differential fecundity: if the biological constraints for older women are binding, young men become more reluctant to marry older women. Consequently, young women anticipate their shorter fecundity horizon and reduce their reservation match quality for both younger and older men. Therefore, women increase their likelihood of marrying older men, who have a greater willingness to marry than younger men. In the steady state equilibrium, as young women are more likely to marry older men, young men are (relatively) more likely to wait and single men will outnumber single women.

The results above have dynamic implications, which I study in Section 3, where I extend the model to a full life-cycle model with people living a finite numbers of yearly periods. In order to study marriage dynamics, I proceed in two steps. First, I assume that the economy in its steady state and calibrate the model parameters to U.S.1930 demographics, to obtain an initial state Then, since desired fertility is exogenous in this model, I take those parameters from the U.S. time series data and assume that desired fertility reaches a new steady state value in 1977. From that year onwards, marriage timing is exclusively affected by a persistence mechanism that is basically caused by the change of the age structure of the population due to the "baby boom". Therefore, until the age structure of the population reaches its new steady state level (around the year 2050 in the model), people tend to marry younger than in the steady state because they face relatively higher matching rates of young mates.The calibration exercise decline in fertility in the period 1957-1976 may account for more than a 35% of the increase in the age of marriage occurred during the period 1977-2006. However, the slow convergence to the steady state is particularly observed in the case of men. Differently, women reach their steady state age of marriage much sooner, slightly after fertility rates stabilize.

[3]However, total and single sex ratios appear to be correlated between 1900 and 1940.

In order to understand the how much influence the demographic structure may have had in men's marriage slow convergence, I perform two counterfactual experiments. In the first experiment, I assume than fertility rates converge slowly from the 1940 levels to the replacement rate, as if the "baby boom" never occurred. In the second experiment, I assume that even there is even there is an increase in desired fertility from the 1945 to 1965, the inflow of people to the economy is constant , so I am isolating the persistence mechanism. What I find in either case that men would have reached their steady state median at first marriage in the mid eighties, 40 years before than in the benchmark model. In the case of women, the effects are in both experiments negligible.

Review of the literature.

The earliest work devoted to the explanation of the age gap in economics is based on gender roles of traditional societies. Bergstrom and Bagnoli (1993) present a model with incomplete information where women are valued as marriage partners for their ability to bear children and to manage a household, and men are valued for their ability to make money. Women's ability is observable earlier in life than men's ability, which is private knowledge. Therefore, men who expect to be successful delay marriage until they are able to give a signal that allows them to attract more desirable women.

Siow (1998) introduced the issue of the shorter fecundity period of women in the economics literature. In a two-period model with uncertainty over human capital accumulation, and where utility comes exclusively from having children, old and young men (all fertile) compete for young, fertile women. Consequently, some old men, those who successfully obtain a higher wage, are able to marry, displacing some of the young men in the competition over scarce fertile women.

In this paper, different from the works mentioned, search for a partner takes time and there is no role for perceptions about future earnings but, as in Siow (1998), women have a shorter fecundity period than men.[4] However, Siow also explores how differential fecundity affects gender roles, but the analysis is focused on the steady state with no population growth. Here, as stated above, I explore the role of population structure in marriage timing and the transitional dynamics of the model.[5] The observed relationship between sex imbalances and the age gap makes search models particularly useful to analyze this problem.[6]

The present work is also related to previous studies on marriage timing, for example,

[4] In this paper, differential fecundity produces the age gap through a different mechanism than the one in Siow (1998). In Siow's paper, older men compete over scarce young women and provide them with a higher compensation (through their higher wages). Here, young women become less choosy and therefore more likely to marry older men. Therefore, differential fecundity is a sufficient condition for the existence of the age gap.

[5] Another related paper is Tertilt (2005), who also allows for endogenous population growth to study the effects of polygyny in certain African countries. Also in the development literature, Edlund (1999) discusses possible links between son preference in certain Asian countries and marriage patterns (for example spousal age gap, as here).

[6] Seitz (2002) studies of the impact of imbalances of the total sex ratio (like mortality or inmigration) on differences in the marriage timing between whites and blacks. Differently from this paper, she does not allow for marriage across cohorts, and therefore the sex ratio is only affected by exogenous shocks on the total population of men and women.

Caucutt, Guner and Knowles (2002). They study the roles played by wage inequality, human capital accumulation, and returns to experience for the timing of marriage and fertility. They show that highly productive women marry and have children later in life than their less productive colleagues. With a richer structure than this model, they abstract from cross-age marriages and from population growth.

Section 2 of the paper is a completely revised version of Giolito (2004), which is, to the extent of my knowledge, the first attempt to model the age of the individuals, both as a state variable and as a source of agents' heterogeneity, within the search literature studying equilibrium marriage behavior with ex-ante heterogeneous agents and non-transferrable utility (NTU).[7] While here I preserve the main characteristics of that model, the previous version focused exclusively on the steady state with no population growth.[8] More recently, Díaz-Giménez and Giolito (2008) have developed a full life-cycle setting that accounts for differences in fecundity, earnings and mortality profiles between men and women. With a richer structure than in this paper, but focusing only in the steady state and abstracting from population growth, they find that differential fecundity is all that matter to account for age differences at first marriage, and that the distribution of age differences along the life cycle, which shows a pattern which is increasing with age for men, and reducing (in absolute value) for women is also more consistent with a partially random matching search model that with a model where people exclusively meet within their own age group.

Recently, Coles and Francesconi (2007) have analyzed a two-sided search model where men and women differ in their earnings (which evolve stochastically) and in their fitness, which decays deterministically with age. They show that when market opportunities are similar for both sexes, then the equilibrium exhibits 'toy-boy marriages' (old women with young men) and 'toy-girl marriages' (old men with young women). When labor market opportunities are better for men than for women, toy-boy marriages disappear. The main difference with this work is that Coles and Francesconi abstract from population growth and also assume that matching rates are independent of the marriage patterns across ages.[9] Therefore, they abstract from the relationship between the sex ratio and the age gap, which is crucial in this model.

[7] Following Burdett and Coles (1997) and Smith (2006), there is an extensive literature in NTU equilibrium search models of marriage. See Burdett and Coles (1999) for a survey. More recent examples include Bloch and Ryder (2000), Cornelius (2003), Burdett et al. (2004), Wong (2003).and Coles and Francesconi (2007).

[8] More recently, Díaz-Giménez and Giolito (2008) have developed a full life-cycle setting that accounts for differences in fecundity, earnings and mortality profiles between men and women. With a richer structure than in this paper, but focusing only in the steady state and abstracting from population growth, they calibrate the model to U.S. data and show that gender differences in fecundity are all important in accounting for life-cycle marriage profiles, and that differences in earnings matter little.

[9] They assume that the population of unmarried men is always equal to the population of unmarried women.

2 A two-period model of marriage

2.1 Demographics

The economy is inhabited by a continuum of men and women who live for three periods, one as children and two as adults: young (age 1) and old (age 2). Adult men and women derive utility from being married and from having children, do not discount time, and their main economic activity is searching for a mate. In the steady state, the population grows every period at a constant rate n, which is endogenous. Therefore, in any period t, the economy is populated by N_t young and N_{t-1} old agents, where $N_t = N_{t-1}(1+n) = N_0(1+n)^t$. Since half of the agents are male and half are female, I assume for simplicity that $N_0 = 2$.

The population dynamics in this economy determines the steady state age structure of the population. The fraction of young people in this economy is given by

$$q_t = \frac{N_t}{N_t + N_{t-1}} = \frac{N_0(1+n)^t}{N_0(1+n)^{t-1}(2+n)} = \frac{1+n}{2+n}, \tag{1}$$

which is increasing in the population growth rate.

Each period t, equal measures $s_{1,t}^w$ of young single women and $s_{1,t}^m$ of young single men enter the economy, where $s_{1,t}^m = s_{1,t}^w = (1+n)^t$. Consequently, every period there is a measure, $S_t^w = (1+n)^t + s_{2,t}^w$, of single women in the economy, of which $(1+n)^t$ are young and $s_{2,t}^w$ are old. Similarly, there is a measure, $S_t^m = (1+n)^t + s_{2,t}^m$, of single men, of which $(1+n)^t$ are young and $s_{2,t}^m$ are old. Since the measures of single old men and women are endogenous in this model, the total measures of singles, and therefore the ratio of single men to single women (henceforth, the sex ratio), $\phi_t = \frac{S_t^m}{S_t^w}$ are also endogenous.

Since men and women marry in pairs, the total number of brides, $w_{1,t}$ young plus $w_{2,t}$ old, must be equal to the total of grooms, $h_{1.t}$ young plus $h_{2,t}$ old. Since in the steady state the sex ratio is constant, $\phi_t = \phi$, the fact that every period an equal number of young men and women enter the market requires the number of men who exit the marriage market either because they marry, or because they die single to also be equal to the number of women who leave the market. Consequently,

$$w_{1,t} + w_{2,t} + u_t^m = h_{1,t} + h_{2,t} + u_t^w \tag{2}$$

where u_t^m and u_t^w denote the number of men and women who die unmarried, respectively. Notice that condition (2) and the fact that people marry in pairs imply that $u_t^m = u_t^w$.

2.2 Matching technology

I assume that singles of both ages meet randomly at most once every period and decide whether or not to marry. Once married, both men and women remain so until the end of their lives and people whose spouse dies are not allowed to search for a new spouse.[10]

[10] For a model of marriage with endogenous separations, see Brien, Lillard and Stern (2002) and Cornelius (2003). Moreover, Chiappori and Weiss (2006) examine equilibrium divorce incentives where there is a thick market externality in the remarriage market.

Whenever a man and a woman meet, either one can propose marriage to the other. If they both accept, they marry. Otherwise they both stay single.

The total number of meetings between singles is represented, in the spirit of Pissarides (1990), by the following constant returns to scale matching function,

$$\eta_t = \rho \left(S_t^m\right)^{\theta} \left(S_t^w\right)^{1-\theta} \tag{3}$$

where $\theta \in (0,1)$ and $\rho \in (0, \frac{1}{2}]$.[11] As is standard in the search and matching literature, the probabilities of being matched depend on the sex ratio ϕ. Therefore, a single man meets a single woman with probability

$$\lambda^m = \frac{\eta_t}{S_t^m} = \rho \left(\phi\right)^{\theta-1} \tag{4}$$

Similarly, a single woman meets a single man with probability $\lambda^w = \rho \left(\phi\right)^{\theta}$. Since the sex ratio is fully endogenous in this economy, here the matching function also captures the way in which the aggregate effects of the marriage decision feed back into the individual decision problem.

The probability that an individual meets either a young or and old agent of the opposite sex depends on the age structure of the single population. For example, the probability that a single man meets a single young woman is $\lambda^m p_1^w$ and the probability of meeting an old single women is given by $\lambda^m p_2^w = \lambda^m \left(1 - p_1^w\right)$, where $p_a^w = \frac{s_{a,t}^w}{S_t^w}$ is the fraction of single women of age a, which is constant in the steady state. Similarly, from a single women point of view, the probabilities of meeting a young and an old man are given by $\lambda^w p_1^m$ and $\lambda^w \left(1 - p_1^m\right)$, respectively.[12]

2.3 Payoffs

Table 1 define the utility that people derive from their marital status. I assume that married people derive utility both from the match quality and from the number of children they are able to have with a given spouse, and the utility of being single is normalized to zero. I assume that men and women desire to have $k \in \left[\frac{1}{2}, \overline{k}\right]$ children during their lifetime, where each "child" is a pair of twins, one girl and one boy. I also assume that women bear all their children in the same period they marry. Finally, men and women differ in their fecundity horizons. On one hand, while young women are able to bear k children, old women are only capable of bearing at most one child. On the other hand, men of age 1 and of age 2 are able to have k children.

The utility that a woman derives from being married to a specific man depends also on the man's type, assumed to be a random variable with cumulative distribution $G_m\left(y\right)$, support in $[0, \overline{y}]$, and mean μ_y. Similarly, I assume that the utility that a man derives

[11] Given that the sex ratio can be unbalanced in equilibrium, I assume that ρ is small enough to ensure that the matching probabilities are not higher than one. The interval assumed for ρ ensures the uniqueness of a symmetric equilibrium in a two-period economy.

[12] Given that the matching technology is CRS and that $s_{1,t}^m = s_{1,t}^w$, it is straightforward that $\lambda^w p_1^m = \frac{\eta}{S_t^w} \frac{(1+n)^t}{S_t^m} = \lambda^m . p_1^w$

from being married to a specific woman depends on the woman's type, which is a random variable with cumulative distribution $G_w(x)$, support in $[0, \overline{x}]$, and mean μ_x. I further assume that both cumulative distributions are continuous and differentiable, and that their probability density functions are $g_m(y)$ and $g_w(x)$, respectively. Since I assume that singles are homogeneous in their quality, the distributions G_m and G_w are identical across singles, and the realizations that people draw at each meeting are independent.[13]

For example, if a young woman marries a young man, their utilities are $2ky$ and $2kx$, because the marriage will last for two periods. If a young woman marries an old man, they will also receive ky and kx but only for one period. On the other hand, if a young man marries an old woman, their utilities are only $\min\{k, 1\}x$ and $\min\{k, 1\}y$ for one period, respectively. Since old women are less fecund and only able to bear one child, they receive the same utility regardless of the age of their spouse.

The rationale for a functional form where the number of desired children and the type of the spouse are complements can be interpreted as if the joy of a "having a family" is a function of the match quality. In other words, people enjoy having children more with people they care about more.

	Young Spouse	Old Spouse
	Women	
Marry young	$2ky$	ky
Marry old	$\min\{k, 1\}y$	$\min\{k, 1\}y$
Never marry	0	0
	Men	
Marry young	$2kx$	$\min\{k, 1\}x$
Marry old	kx	$\min\{k, 1\}x$
Never marry	0	0

Table 1: Table Payoffs for men and women

Furthermore, since in the model economy people who do not marry miss out on the joys of both companionship and childbearing, I normalize to zero the utility of being single.

In order to ensure existence of an equilibrium, I assume that the maximum value of k is such that the utility of marrying an average spouse and having k children with them is not higher than the joy of finding a perfect match and having only one child. That is, $k\mu_x \leq \overline{x}$ and $k\mu_y \leq \overline{y}$, which implies

$$\overline{k} = \min\left\{\frac{\overline{x}}{\mu_x}, \frac{\overline{y}}{\mu_y}\right\}. \tag{5}$$

2.4 The decision problem of singles

Since people match randomly in our economy, the single men's problem is to choose reservation values for young and old single women. Let $R^m_{a,b}$ denote the steady state

[13] The assumption about the two separate distributions for men and women, as in Burdett and Coles (1997) is made exclusively for analytical tractability and does not affect the results.

reservation value that single men of age a set for single women of age b, and, similarly, let $R^w_{b,a}$ be the reservation values that age b single women set for age a single men. Since I assume that people meet only once per period, and that they derive zero utility from being single, the reservation values of old single men for both young and old single women is zero, trivially. That is, $R^m_{2,1} = R^m_{2,2} = 0$. Similarly, the reservation values of old single women for both old and young single men are also zero, that is, $R^w_{2,1} = R^w_{2,2} = 0$.[14]

2.4.1 Expected utility of marriage for old single people

Denote as $\gamma^j_{a,b}$ the steady state probability that a single agent of age a and gender $j \in \{m, w\}$ marries someone of age b. This probability depends first, on the probability of meeting someone of the opposite sex and age b, $\lambda^j p_b^{-j}$. Given the meeting probability, the probability of marriage depends on mutual acceptance. That is, for example for men,

$$\gamma^m_{a,b} = \lambda^m p^w_b [1 - G_m(R^w_{b,a})][1 - G_w\left(R^m_{a,b}\right)] \tag{6}$$

Let us observe the case of old single people. Since I assume that old people who do not marry obtain zero utility, they accept any marriage offer. For example, if an old single man (woman) meets a young single woman (man) they will marry provided the youngest agent accepts. Therefore,

$$\gamma^m_{2,1} = \lambda^m p^w_1 [1 - G_m(R^w_{1,2})] \text{ and } \gamma^w_{2,1} = \lambda^w p^m_1 [1 - G_w\left(R^m_{1,2}\right)] \tag{7}$$

On the other hand, if an old single man meets an old single woman, they marry with certainty. Without loss of generality, I assume that everybody eventually marries at some point in life. Consequently, those old men and women that either were unmatched or rejected by younger counterparts will face a frictionless technology that automatically matches them among themselves.[15] Therefore $\gamma^m_{2,2} = [1 - \gamma^m_{2,1}]$ and $\gamma^w_{2,2} = [1 - \gamma^w_{2,1}]$.

Given the payoffs described in Table 1, the expected utility of marriage for old single men is

$$\begin{aligned} U^m_2 &= \gamma^m_{2,1} k\mu_x + \left(1 - \gamma^m_{2,1}\right) \min\{k, 1\}\mu_x \\ &= \lambda^m p^w_1 [1 - G_m(R^w_{1,2})] k\mu_x + \left(1 - \lambda^m p^w_1 [1 - G_m(R^w_{1,2})]\right) \min\{k, 1\}\mu_x \end{aligned} \tag{8}$$

For old single women, since they will enjoy the same utility regardless of whom they marry, we have,

$$\begin{aligned} U^w_2 &= \gamma^w_{2,1} \min\{k, 1\}\mu_y + \left(1 - \gamma^w_{2,1}\right) \min\{k, 1\}\mu_y \\ &= \min\{k, 1\}\mu_y. \end{aligned} \tag{9}$$

[14] This rules out the trivial equilibrium where all women reject all marriages because they know that all men will reject them and vice versa.

[15] This assumption is for convenience, and will be relaxed in the next section, when I study the transitional dynamics of the model. Recall that by condition (2), steady state implies that an equal number of old men and women would remain unmatched after period 2.

2.4.2 Expected Utility of Marriage for Young Single People

Since young single men and women have a chance to marry in the future, they choose their reservation values taking into account the next period's prospects. For example, provided that a single young man meets a single young woman (with probability $\lambda^m p_1^w = \lambda^w p_1^m$), they will marry if both of them find each other mutually acceptable. A single young woman will accept a single young man with probability $[1 - G_m(R_{1,1}^w)]$, and he will accept her with probability $[1 - G_w(R_{1,1}^m)]$. Therefore, the probability that a young man marries a young woman (or vice-versa) is

$$\gamma_{1,1}^m = \gamma_{1,1}^w = \lambda^m p_1^w [1 - G_w\left(R_{1,1}^m\right)][1 - G_m(R_{1,2}^w)] \tag{10}$$

On the other hand, if young man or woman meets an old single agent of the opposite sex (with probability $\lambda^m p_2^w$ or $\lambda^w p_2^m$, respectively), the acceptance of the young agent will be sufficient for marriage to occur. Consequently,

$$\gamma_{1,2}^m = \lambda^m p_2^w [1 - G_w\left(R_{1,2}^m\right)] \tag{11}$$

and

$$\gamma_{1,2}^w = \lambda^w p_2^m [1 - G_m\left(R_{1,2}^w\right)], \tag{12}$$

for men and women, respectively.

The expected utility of marriage for young single men, U_1^m, is then,

$$U_1^m = \gamma_{1,1}^m 2kE\left[x \mid x \geq R_{1,1}^m\right] + \gamma_{1,2}^m \min\{k, 1\} E\left[x \mid x \geq R_{1,2}^m\right], \tag{13}$$

and the one for women, U_1^w,

$$U_1^w = \gamma_{1,1}^w 2kE\left[y \mid y \geq R_{1,1}^w\right] + \gamma_{1,2}^w kE\left[y \mid y \geq R_{1,2}^w\right] \tag{14}$$

2.4.3 The young single people's optimization problem

Young single people of gender $j \in \{m, w\}$ choose the reservation values for young and old single agents of the opposite sex, $R_{1,1}^j$ and $R_{1,2}^j$, which maximize their expected lifetime utility. In order to do this, they solve the following problem:

$$V_1^j = \underset{R_{1,1}^j, R_{1,2}^j}{max} \left[U_1^j + \left(1 - \Gamma_1^j\right) U_2^j\right] \tag{15}$$

subject to

$$\Gamma_1^j = \gamma_{1,1}^j + \gamma_{1,2}^j \tag{16}$$

where Γ_1^j stands for the steady state probability that a single individual of gender j marries at age 1.

The solutions to this problem require a young single agent to be indifferent between marrying when young or resampling again the following period. If a young single agent meets an old individual of the opposite sex he/she must compare the utility of a one period marriage with the value of marrying when old. In the case of a young man who meets an old women, this type of marriage becomes less valuable if their desired number of children is greater than the one that an old woman is able to conceive $(k > 1)$. Therefore,

$$\begin{aligned} \min\{k,1\}R^m_{1,2} &= U^m_2 \\ R^m_{1,2} &= \frac{U^m_2}{\min\{k,1\}} \end{aligned} \tag{17}$$

The case of a young woman who meets an old man is different. Since men of all ages are able to have k children, she must compare the utility of marrying young with the value of waiting until the next period and facing the limited fecundity which will be binding in the case where $k > 1$. Then we have

$$kR^w_{1,2} = U^w_2 = \min\{k,1\}\mu_y,$$

hence

$$R^w_{1,2} = \frac{\min\{k,1\}\mu_y}{k}. \tag{18}$$

Now we focus on the reservation values that young singles set for people of their same cohort. Given that a young single man meets a young single woman (or vice-versa), they have to compare between the value of waiting and the utility of a two-period marriage with k children. Therefore,

$$2kR^j_{1,1} = U^j_2,$$

or

$$R^j_{1,1} = \frac{U^j_2}{2k} \tag{19}$$

for $j \in \{m, w\}$.

To obtain the reservation values chosen by the young single men, substituting expression (8) into Equations (17) and (19), respectively, we get

$$R^m_{1,1} = \frac{\mu_x}{2k}\left\{\min\{k,1\} + \lambda^m p^w_1 \left[k - \min\{k,1\}\right]\left[1 - G_m\left(R^w_{1,2}\right)\right]\right\}, \tag{20}$$

and

$$\begin{aligned} R^m_{1,2} &= \mu_x\left\{1 + \lambda^m p^w_1 \left[k - \min\{k,1\}\right]\left[1 - G_m\left(R^w_{1,2}\right)\right]\right\} \\ &= \frac{2k}{\min\{k,1\}} R^m_{1,1}. \end{aligned} \tag{21}$$

Notice that the young single men's reservation values depend positively on the average match quality of single women, μ_x, and on the difference between their desired number of children and the number that old women are capable to bear $(k - \min\{k,1\})$. When the desired number of children is not biologically binding for old women $(k \le 1)$, we have that $R^m_{1,1} = \frac{\mu_x}{2}$ and $R^m_{1,2} = \mu_x$. On the other hand, when $k > 1$, we have that

$$R^m_{1,1} = \frac{\mu_x}{2k}\left\{1 + \lambda^m p^w_1 \left[k - 1\right]\left[1 - G_m\left(R^w_{1,2}\right)\right]\right\}, \tag{22}$$

and

$$R_{1,2}^m = \mu_x \left\{1 + \lambda^m p_1^w \left[k-1\right] \left[1 - G_m \left(R_{1,2}^w\right)\right]\right\}, \tag{23}$$

which are increasing on the probability of meeting a single young woman, $\lambda^m p_1^w$, and decreasing on $R_{1,2}^w$, i.e., the reservation value of young single women for old single men. Naturally, when $R_{1,2}^w$ decreases, that is, when it becomes more likely that young single women accept the proposals of old single men, young men are more willing to wait and therefore become choosier. Finally, notice that while $R_{1,1}^m$ is decreasing in the desired number of children, k, $R_{1,2}^m$ is increasing in k. That is, as the desired number of children increases, young men become choosier with respect to old women and less choosy with young women.

On the other hand, reservation values chosen by young single women are simply,

$$R_{1,1}^w = \frac{\min\{k,1\}}{2k}\mu_y, \tag{24}$$

and

$$R_{1,2}^w = \frac{\min\{k,1\}}{k}\mu_y. \tag{25}$$

Notice that, in this case, when fecundity is binding $(k>1)$, the reservation values that young women set for all men become decreasing functions of their desired number of children.[16]

2.5 Steady state demographics

2.5.1 The number of single men and women

This section presents the endogenous number of single people in the steady state economy as a function of the probabilities of marriage at age 1, Γ_i^j for $j \in \{m,w\}$ and the rate of population growth, n.

Since the population of old single men and women is composed of the unmarried young singles of the previous period, and given that the population of young singles is $s_{1,t}^m = s_{1,t}^w = (1+n)^t$, we have that

$$s_{2,t}^j = s_{1,t-1}^j \left(1-\Gamma_1^j\right) = (1+n)^{t-1}\left(1-\Gamma_1^j\right), \tag{26}$$

Therefore, the total number of singles is

$$S_t^j = s_{1,t}^j + s_{2,t}^j = (1+n)^{t-1}\left(2+n-\Gamma_1^j\right) \tag{27}$$

The age distributions of singles —that is the fraction of young single people of gender $j \in \{m,w\}$, is

$$p_1^j = \frac{s_{1,t}^j}{S_t^j} = \frac{(1+n)}{2+n-\Gamma_1^j}, \tag{28}$$

[16]Notice that the reservation values of men and women are all *continuous* in the value of the parameter k, which allows us to find an equilibrium for each value of k.

Finally, the sex ratio is

$$\phi = \frac{S_t^m}{S_t^w} = \frac{2+n-\Gamma_1^m}{2+n-\Gamma_1^w}. \tag{29}$$

Note that in the steady state the sex ratio and the fractions of young single people are constant. Moreover, the fractions of young singles depend positively on the population growth rate.

2.5.2 Fertility and population growth

To obtain the rate of population growth, recall that a woman that marries at age 1 is capable of having k pairs of twins, and that a woman who does so at age 2 is able to bear only one pair of twins. Given those assumptions, we have that the number of young men and women who enter the market each period are

$$s_{1,t}^m = s_{1,t}^w = k s_{1,t-1}^w \Gamma_1^w + \min\{k,1\} s_{2,t-1}^w. \tag{30}$$

Given that $s_{1,t}^w = (1+n)^t$, substituting (26) in (30) ,and manipulating we have

$$(1+n)^2 = k(1+n)\Gamma_1^w + \min\{k,1\}(1-\Gamma_1^w). \tag{31}$$

Solving the quadratic equation (31) and taking the positive root we have that the rate of population growth in this economy is

$$n = \frac{k\Gamma_1^w}{2} - 1 + \frac{\sqrt{(k\Gamma_1^w)^2 + 4\min\{k,1\}(1-\Gamma_1^w)}}{2}. \tag{32}$$

Notice that women's fecundity is normalized to obtain $n=0$ when $k=1$. The population growth reaches its minimum when $k=0.5$ and $\Gamma_1^w = 1$ $(n=-0.5)$ and its maximum when $k=\overline{k}$ and $\Gamma_1^w = 1$ $\left(n = \overline{k}-1\right)$.

2.6 Steady State Equilibrium

2.6.1 Marriage market clearing

Since men and women marry in pairs, the total number of brides must be equal to the total of grooms in each type of marriage. Therefore, for men of age a and women of age b, for $a,b \in \{1,2\}$, we have

$$s_{a,t}^m \gamma_{a,b}^m = s_{b,t}^w \gamma_{b,a}^w. \tag{33}$$

This condition is trivially satisfied when $a=b=1$, since $s_{1,t}^m = s_{1,t}^w = (1+n)^t$ and, by (10), $\gamma_{1,1}^m = \gamma_{1,1}^w$. In the case of people marrying across cohorts we have,

$$\begin{aligned} s_{1,t}^m \gamma_{1,2}^m &= s_{2,t}^w \gamma_{2,1}^w \\ s_{1,t}^m \lambda_2^m [1 - G_w\left(R_{1,2}^m\right)] &= s_{2,t}^w \lambda^w p_1^m [1 - G_w\left(R_{1,2}^m\right)] \\ s_{1,t}^m \frac{\eta_t}{S_t^m}\frac{s_{2,t}^w}{S_t^w} &= s_{2,t}^w \frac{\eta_t}{S_t^w}\frac{s_{1,t}^m}{S_t^m}. \end{aligned} \tag{34}$$

Similarly, we have that .

$$
\begin{aligned}
s_{2,t}^{m}\gamma_{2,1}^{m} &= s_{1,t}^{w}\gamma_{1,2}^{w} \\
s_{2,t}^{m}\lambda^{m}p_{1}^{w}[1-G_{m}\left(R_{1,2}^{w}\right)] &= s_{1,t}^{w}\lambda^{w}p_{1}^{m}[1-G_{m}\left(R_{1,2}^{w}\right)] \\
s_{2,t}^{m}\frac{\eta_t}{S_t^m}\frac{s_{1,t}^{w}}{S_t^w} &= s_{1,t}^{w}\frac{\eta_t}{S_t^w}\frac{s_{2,t}^{m}}{S_t^m}.
\end{aligned}
\tag{35}
$$

2.6.2 Definition

Given a desired number of children, k, a steady state equilibrium for this economy is a vector of reservation values for young single men, $R_1^m = (R_{1,1}^m, R_{1,2}^m)$, a vector of reservation values for young single women, $R_1^w = (R_{1,1}^w, R_{1,2}^w)$, a probability of marriage for young single men, $\Gamma_1^m = \gamma_{1,1}^m + \gamma_{1,2}^m$, a probability of marriage for young single women, $\Gamma_1^w = \gamma_{1,1}^w + \gamma_{1,2}^w$, and a population growth rate n, such that:

(i) Given R_1^w, Γ_1^m, Γ_1^w and n, R_1^m solves the young single men's decision problem described in expression (15), and is determined by equations (20) and (21) .

(ii) Given R_1^m, Γ_1^m, Γ_1^w and n, R_1^w solves the young single women's decision problem described in expression (15), and is determined by equations (24) and (25) .

(iii) The young men's probability of marriage, Γ_1^m, determined by (10) and (11) , is consistent with the reservation values chosen both by the young single men and the young single women, the probability of marriage of young single women and the rate of population growth.

(iv) The young women's probability of marriage, Γ_1^w, determined by (10) and (12) , is consistent with the reservation values chosen both by the young single men and women, the probability of marriage of young single men and the rate of population growth.

(v) The rate of population growth, n, determined by (32), is consistent with the reservation values and marriage probabilities of single young men and women.[17]

(vi) Marriage market clears, by (34) and (35) .

2.6.3 Existence

Here I show the existence of a steady state equilibrium for this economy. Notice that, by expression (20), $R_{1,1}^m$ is uniquely determined by $R_{1,2}^m$. In addition, the reservation values of women are completely determined by k and μ_y (equations (24) and (25)). Therefore, I have to show the existence of an interior solution of a system of four equations in four unknowns: $R_{1,2}^m$, Γ_1^m, Γ_1^w and n.

[17] Notice that, strictly speaking, the reservation values for old single men, $R_2^m = (R_{2,1}^m, R_{2,2}^m)$, and old single women, $R_2^w = (R_{2,1}^w, R_{2,2}^w)$, are also part of the steady state equilibrium of this economy. However, as shown in Section 2.4, they are trivially zero, and I have omitted them from the definition of the equilibrium for the sake of simplicity.

Theorem 1 (Existence) *A steady state equilibrium exists for this economy, i.e.*

The following result rules out the existence of corner solutions and establishes that in equilibrium marriages across cohorts occur with positive probability.

Corollary 2 *The equilibrium is interior,*

$$R_{1,1}^m, R_{1,2}^m \in (0, \overline{x}),$$
$$\Gamma_1^m \in (0, 1),$$
$$\Gamma_1^w \in (0, 1),$$

and

$$n \in \left(-0.5, \overline{k} - 1\right).$$

Furthermore, in equilibrium all type of marriages occur with positive probability,

$$\gamma_{1,1}^m = \gamma_{1,1}^w > 0$$
$$\gamma_{1,2}^m, \gamma_{2,1}^w > 0$$
$$\gamma_{1,2}^w, \gamma_{2,1}^m > 0.$$

Proof. See Appendix A.1 ■

2.7 Equilibrium characterization

In this section I characterize the equilibrium of the economy, assuming that the distribution of types is identical for men and women, that is $G_m = G_w = G$, denoting its mean by μ. The analysis is divided into two parts. Next, I analyze the case where old women's biological restriction is not binding $(k \leq 1)$. This case is called "symmetric" because the decision problems faced by the single men and the single women are identical. Then, in Section 2.7.2 I analyze the case of differential fecundity, that is, when the desired fertility is higher than the number of children that old women are capable of bearing.

2.7.1 Symmetric case

Consider the case were people desire to have $k \in [0.5, 1]$ children.[18] Consequently, differential fecundity plays no role because the decision problems faced by the single men and the single women are identical. Moreover, in this case the reservation values of single men and single women are identical, and so are the numbers of single men and single women in the economy. Notice that in this case the reservation values of men and women are independent of their desired fertility.

[18]Note that the results depend exclusively on symmetry and not on the negative population growth rates implied by the assumption of $k \leq 1$. This assumption is made merely for expositional reasons and will be relaxed in the next section.

Assumption 1 *$G_m = G_w = G$ with mean μ and density g.*

Assumption 2 *The probability density function g is log-concave.*

Under Assumption 1, by (21) , (20) , (24) and (25) we trivially have

$$R_{1,1}^m = R_{1,1}^w = \frac{\mu}{2}, \tag{36}$$

and

$$R_{1,2}^m = R_{1,2}^w = \mu. \tag{37}$$

Proposition 1 *Under Assumption 1, if people only want to have a number of children that is biologically feasible for old women, $k \in [\frac{1}{2}, 1]$, there is always a steady state equilibrium where men and women marry at age 1 with equal probability, $\Gamma_1^m = \Gamma_1^w = \Gamma_1^s$, and where population is not increasing over time, $n \leq 0$. This implies that the number of single men is equal to the number of single women, $S_t^m = S_t^w$, and therefore the sex ratio is equal to one, $\phi = 1$. Furthermore, the economy is mostly populated by old agents, $q \leq \frac{1}{2}$.*
Proof. *See Appendix A.2* ■

Proposition 2 *Under Assumptions 1 and 2 ,there is a unique steady state equilibrium for $k \in [\frac{1}{2}, 1]$.*
Proof. *See Appendix A.3* ■

Given that men and women face identical problems the existence of an unique equilibrium where men and women marry at the same age seems trivial. However, the interesting feature of the symmetric case comes from comparative statics. As we show below, when men and women are identical, an increase in the desired number of children leads to men and women marrying earlier as population growth increases, despite the constant reservation rules derived above.

Proposition 3 *Under Assumptions 1 and 2, if $k \in [\frac{1}{2}, 1)$, given an increase in the desired number of children:*

1. *Population growth rate increases: $\frac{\partial n}{\partial k} > 0$. This implies that the fraction of young people in the economy increases: $\frac{q}{\partial k} > 0$.*

2. *Men and women marry at age 1 with higher probability: $\frac{\partial \Gamma_1^s}{\partial k} > 0$.*

Proof. *See Appendix A.4* ■

The result above comes from the interaction between young people's preferences for marrying within their cohort and the effect of desired fertility on the age structure of the population. When k increases, population growth increases and therefore the age structure of the economy is affected, with a larger share of young people $q = \frac{1+n}{2+n}$. Therefore young people meet people of the same cohort with higher probability. Since young agents obtain higher utility from marrying other young people (and therefore set lower reservation values for them), they tend to marry younger as age the structure of population is also becoming younger. Notice that this relationship is consistent with cross section data of U.S. states for the year 2000 (see Figure **??**, Panels (a) and (b) for men and women, respectively).

2.7.2 Differential fecundity

Now consider the case where people desire to have $k \in (1, \overline{k}]$ children. If $k > 1$, old women are not able to bear their desired number of children. As shown in Section 2.4.1, if men wait until age 2 they still can marry a young woman and have k children. Recall from (8) that the expected utility that a man obtains from marrying at age 2 is $U_2^m = \gamma_{2,1}^m k\mu + \left(1 - \gamma_{2,1}^m\right)\mu$. On the other hand, if women wait until age 2, they will get the same utility regardless of whom they marry, $U_2^w = \mu$, (9). Since in equilibrium $\gamma_{2,1}^m > 0$ (from Theorem 1), this implies that differential fecundity increases the value of waiting for men. Therefore, men become relatively choosier than women.

Proposition 4 *Under Assumption 1, if people want to have more children than those that old women are able to bear $k \in (1, \overline{k}]$, there is always a steady state equilibrium where young men are choosier than young women, that is $R_{1,1}^m > R_{1,1}^w$ and $R_{1,2}^m > R_{1,2}^w$.*
Proof. *When $k > 1$ we have from* (24) *and* (25) *that $R_{1,1}^w = \frac{\mu}{2k}$ and $R_{1,2}^w = \frac{\mu}{k}$. Substituting $R_{1,2}^w$ in* (20) *and* (21) *we have,*

$$R_{1,1}^m = \frac{\mu}{2k}\left\{1 + \lambda^m p_1^w \left[k-1\right]\left[1 - G_m\left(\frac{\mu}{k}\right)\right]\right\} > R_{1,1}^w,$$

and

$$R_{1,2}^m = \mu\left\{1 + \lambda^m p_1^w \left[k-1\right]\left[1 - G_m\left(\frac{\mu}{k}\right)\right]\right\} > R_{1,2}^w,$$

since $\lambda^m p_1^w > 0$. ■

From simple observation of (24) and (25) it is clear that women's reservation values both for young and old men old men are decreasing in k. From Proposition 4 we know that if differential fecundity plays a role, men are choosier than women. The previous result does not imply that as desired fertility increases men's reservation values will necessarily increase. In fact, as I show below, if desired fertility increases men also reduce their reservation values for young women, but to a lesser extent than women do. However, young men will increase their reservation values for older women, which is the main driving force behind the difference in the timing of marriage.

Proposition 5 *Under Assumptions 1 and 2, then in the neighborhood of the unique steady state equilibrium where $k = 1$, given a small increase in the desired number of children*

1. *Young men become choosier with old women and less choosy with young women: $\frac{\partial R_{12}^m}{\partial k} > 0$ and $\frac{\partial R_{11}^m}{\partial k} < 0$. In the latter case, the decrease in the reservation value is lower than the decrease in young women's reservation value: $\left|\frac{\partial R_{11}^m}{\partial k}\right| < \left|\frac{\partial R_{11}^w}{\partial k}\right|$.*

2. *Women marry young with higher probability: $\frac{\partial \Gamma_1^w}{\partial k} > 0$. In addition, the difference between the probability of marriage at age 1 of women and men, $\Gamma_1^w - \Gamma_1^m$, is positive and increasing in k.*

3. *The ratio of single men to single women, ϕ, is greater than 1 and increasing in k.*
4. *Similarly to Proposition 3, population growth increases, therefore the fraction of young people in the economy, q increases.*

Proof. *See Appendix A.5* ■

Proposition 5 establishes that when people desire to have more children than the number that a woman who marries at age 2 is able to bear, differential fecundity plays a role and single men become choosier than single women. Consequently, single women tend to marry younger than men, and the age gap is an increasing function of the desired number of children. Moreover, it establishes that, as the desired number of children increases, single men increasingly outnumber single women.

The mechanism behind Proposition 5 is the following. If people in the economy want to have a number of children that is feasible for women that marry at age 2, the biological differences between men and women play no role in the marriage market. However, biological gender asymmetries do appear if society want to have more children than those that women of age 2 are able to bear ($k > 1$). Therefore, young men are less willing than young women to accept an older spouse and the value of waiting until age 2 increases for men relative to women. Notice that men also decrease their reservation values for young women (albeit in a lesser extent) because if they wait they may not be able to marry a young woman when old.

The increase in the sex ratio is the necessary counterpart of the increase in the age gap. As young women are more likely to marry old men, in equilibrium more young men have to wait until period 2 in order to marry. Therefore, it must be the case that single men outnumber single women. This result has some implications in light of the recent empirical literature on marriage timing. It is a standard practice in this literature to use the single sex ratio as a measure of relative availability of men and women, ignoring the endogeneity problem caused by the relationship between the sex ratio and the age gap.[19]

Notice that an while and increase in k makes women unambiguously marry younger, it has ambiguous effects on men's timing of marriage. In the case of men there are two countervailing effects: because of differential fecundity, they tend to marry older as k increases, but because of the change in population structure they would marry younger (Proposition 3). These two effects will be analyzed in more detail in the next section when we study the model dynamics. As we will see later, the transitional dynamics of the model is particularly affected by the level of the sex ratio, ϕ_t, which will act as a persistence factor in determining the age gap, and by the evolution of the structure of the population, q_t, which will affect the marriage timing of both men and women.

3 An extended model

In this section I relax the assumption of a two-period life horizon in the model developed in Section 2 The objective of this extension is to analyze in more detail the effects of the

[19] See, for example, Fitzgerald (1991), McLaughlin et al. (1993), and Parrado and Centeno (2002).

results found above on the dynamics of marriage behavior, and specifically to compare the model results with the evolution of the age of marriage in the U.S. starting. from the period known as the "baby boom". Since in this stylized model desired fertility is considered exogenous, the "baby boom" will be modeled as a shock on desired fertility. However, our focus of interest will not be this period itself, but the demographic consequences of it and their potential influence in the marriage behavior of the generations born during and after the "baby boom".

In this section, I study a model economy populated by a continuum of men and a continuum of women. Men and women live for at most J years, which we denote with subindex $j = 15, 16, \ldots, J$. Men and women in this model economy differ in their fecundity, and in their longevity. And they derive utility from being married and having children. From now on I relax the assumption of the constant population growth rate maintained thorough Section 2. For the sake of brevity, in this section I describe only the main assumptions of this model economy. A detailed description of the T-period model economy is contained in Appendix **??**.

3.1 Mortality

In our model economy each period every person faces an exogenous probability of dying until the following period. We denote these probabilities by $\delta^i_{j,t}$, for $i \in \{m, w\}$and we assume that they are gender, age and time dependent. Since people live at most for J periods, $\delta^m_{J,t} = \delta^w_{J,t} = 1$ for any period $t = 1, 2, \ldots$.. These assumptions also imply that the probability that a person of gender i who is a year-old in period t survives until age b is

$$\Psi^i_t(a, b) = \prod_{j=a}^{b-1} \left(1 - \delta^i_{,j,t+j-a}\right) \tag{38}$$

3.2 Matching technology

Costly double sided search for spouses is a distinguishing feature of our model economies. The probabilities of being matched depend on an exogenous parameter that measures the search frictions, and on the ratio of available singles. Let $s^i_{j,t}$ denote the number of j year-old singles of gender i in period t and let $S^i_t = \sum_{j=15}^{J} s^i_{j,t}$ denote the total number of singles of gender i. Then the probability that a single man meets a single b year-old woman is

$$\lambda^m_{b,t} = \rho \left[\min\left(\frac{S^w_t}{S^m_t}, 1\right)\right] \frac{s^w_{b,t}}{S^w_t} \tag{39}$$

where parameter $0 < \rho \leq 1$ measures the search frictions.[20]

In this model economy the decision problem of single men and women are exactly identical. For notational convenience I describe the problem and variables that pertain to

[20] Notice, that this matching function is different from the one in Section 2. In the first part of the paper a differentiable function was required, but keeping this function in the calibration exercise would mean one more free parameter to calibrate. This change does not affect the results at all.

single men only. To obtain the corresponding variables for single women simply substitute the m's for w's and the b's for a's.

3.3 Fecundity

This expected number of children depends on an exogenous, time varying parameter that desired fertility k_t and biological constraints based in men's and women's age. (determined at the time of marriage) and biological constraints. Even though that this is not an easy matter, I will make the following assumptions that are roughly consistent with the literature in the subject.[21, 22]. Define as $c_t^w(a,b)$ the number of children that a woman of age a expects to have if she marries at time t a man of age b. Then,

$$c_t^w(a,b) = \begin{cases} k_t & \text{if } a \leq 30 \text{ and } b \leq 55 \\ \min[k_t, 3] & \text{if } 30 < a \leq 35 \text{ and } b \leq 55 \\ \min[k_t, 2.5] & \text{if } 35 < a \leq 38 \text{ and } b \leq 55 \\ \min[k_t, 1] & \text{if } 41 < a \leq 44 \text{ and } b \leq 55 \\ \min[k_t, 0.5] & \text{if } 44 < a \leq 49 \text{ and } b \leq 55 \\ 0 & \text{if } a > 49 \text{ or } b > 55. \end{cases} \tag{40}$$

In a similar way is defined $c_t^m(a,b)$. Therefore, I assume that men are fecund until age 55 and women married after than age 30 have not enough time to have more than three children (this restriction will be only binding during the peak of the baby boom).[23]

3.4 Payoffs from marriage

The period utility of marriage that a single men a obtains from marrying in period t with someone of age b and a given marriage quality is $zc_t^m(a,b)$, where z is an independent and identically distributed realization from $G(z)$, the same for men and women (Assumption 1). The period utility of single people is constant endowment of A..

The expected values of marriage. The value that an a year-old groom expects to obtain

[21] According to Wood and Weinstein (1988), women's probability of any conception changes rapidly after age 40, as a result of large changes in the ovarian function. Between ages 25 and 40, tthis probability of women is remarkably constant. This finding suggests that any reduction in the physiological capacity to bear children between ages 25 and 40 is attributable to an elevation in intra-uterine mortality rather than to a decline in the ability to conceive. But, even accounting for intra-uterine loss, the pattern of effective fecundability remains fairly flat between ages 20 and 35.

[22] According to Hassan and Killick (2003), the effect of men's age on fecundity remains uncertain. Experiments that study the effect of age on male fecundity have been criticized on methodological grounds for using age at conception and for not taking into account confounding factors such as the age of the female or coital frequency. Hassan and Killick study male infecundity by comparing the time to pregnancy —measured from the onset of the attempts to achieve pregnancy— for men of different age groups. They found that male aging leads to a significant increase in the time to pregnancy, especially after ages 45 to 50.

[23] Here "2.5 children" should be interpreted as "two children and a 50% probability of a third child".

from marrying a b year-old bride who has drawn realization z and has proposed to him is

$$u_t^m(a,b,z) = zc_t^m(a,b) \sum_{\ell=0}^{D} \beta^{\ell} \Psi_t^m(a,a+\ell) \Psi_t^w(b,b+\ell) \tag{41}$$

where $D = \min\{J-a-15, J-b-15\}$ and $\beta \in (0,1)$ is the discount factor.

3.5 The decision problem of singles

The probabilities of marriage. The probability that an a year-old bachelor marries a b year-old single woman is

$$\gamma_t^m(a,b) = \lambda_{b,t}^m \{1 - G[R_t^m(a,b)]\} \{1 - G[R_t^w(b,a)]\} \tag{42}$$

where $R_t^m(a,b)$ denotes the reservation value that a single man who is a year-old in period t set for b year-old single women, and $R_t^w(a,b)$ is the reservation value that b year-old single women set for a year-old bachelors.

Consequently, the probability that a single a year-old bachelor marries a woman of any age is

$$\Gamma_{a,t}^m = \sum_{b=15}^{J} \gamma_t^m(a,b) = \sum_{b=15}^{J} \lambda_{b,t}^m \{1 - G[R_t^m(a,b)]\} \{1 - G[R_t^w(b,a)]\} \tag{43}$$

which naturally depends on the reservation values of both men and women.

The expected values of marrying in the current period. The value that an a year-old single expects to obtain for marrying in the current period t, before any matches have taken place is

$$EU_{a,t}^m = \sum_{b=15}^{J} \gamma_t^m(a,b)\, Eu_t^m[a,b,z \mid z \geq R_t^m(a,b)] \tag{44}$$

The expected values of remaining single. Therefore, the expected value of remaining single one more period for a single a year-old man is then,

$$V_{a,t}^m = EU_t^m + \left(1 - \Gamma_{a,t}^m\right) \left\{A + \left(1 - \delta_{a,t}^m\right) \beta E_t \left[V_{a+1,t+1}^j\right]\right\} \tag{45}$$

where the first term in the curly brackets shows the expected value of getting married before a match takes place and the second term the value of remaining single for one more period. To avoid agents to massively delay (anticipate) marriage in anticipation of the beginning (end) of the baby boom, have to put some restrictions in the formation of expectations. Therefore,

$$E_t\left[V_{a+1,t+1}^j\right] = \pi \left[V_{a+1,t+1}^j\right] + (1-\pi)\left[V_{a+1,t}^j\right], \tag{46}$$

In order to avoid sudden jumps in marriage due to agents anticipating their own future desired fertility during the baby boom, I assume that the parameter π in expression (46) is equal to zero before 1965. In other words, I assume that during the baby boom people form their expectations for the next period by observing the environment faced by singles who are one year older in the current period. Between 1965 and 1973, I assume that π increases linearly and from 1974 onwards I assume $\pi = 1$, relaxing the previous assumption.

Reservation values. The optimal reservation values that a year-old men and b year-old women set for each other in each period can be found solving the system of $2J^2 \times T$ equations in $2J^2 \times T$ unknowns that results from equating the J^2 expressions (41) and (**??**) for the men and the corresponding J^2 women's equations for periods $t = 1, 2, 3...T$.. Formally, the $\{R_t^i(a,b))\}$ are the values of z that solve

$$u_t^m(a, b, R_t^m(a,b)) = \left\{A + \left(1 - \delta_{a,t}^m\right) \beta E_t \left[V_{a+1,t+1}^m\right]\right\} \tag{47}$$

and

$$u_t^w(a, b, R_t^w(a,b)) = \left\{A + \left(1 - \delta_{a,t}^w\right) \beta E_t \left[V_{a+1,t+1}^w\right]\right\}. \tag{48}$$

3.6 Fertility and Population Dynamics

Every period, a number of young single men and women of age 1, $s_{15,t}^m = N_{15,t}^m$ and $s_{15,t}^w = N_{15,t}^m$ enter the market. Since now agents live 15 years as children, the population entering the market at period t is born at period $t-15$. Therefore equation (30) is replaced by

$$N_{15,t}^i = \varpi_{t-15}^i \Psi_{t-15}^i(0,15) \left\{ \sum_{a=1}^{J} s_{a,t-15}^w \sum_{b=15}^{J} \gamma_{t-15}^w(a,b)\, c_{t-15}^w(a,b) \right\}, \tag{49}$$

where the expression between curly bracket is the number of children born in period $t-15$, ϖ_{t-15}^i is the fraction of children of gender $i \in \{m, w\}$ and $\Psi_{t-15}^i(0,15)$ is their survival probability up to age 15.[24] The stock of agents of older ages $\{N_{,j,t}^i\}$, is given by

$$N_{j,t}^i = \left(1 - \delta_{,j,t-1}^i\right) N_{j-1,t-1}^i \text{ for } 16 \geq j \leq J. \tag{50}$$

3.7 Equilibrium

An equilibrium for this economy is a measure of people, $\{N_{j,t}^i\}$, a measure of singles, $\{s_{j,t}^i\}$, and matrices of the optimal reservation values that singles set for each other, $\{R_t^m(a,b), R_t^w(a,b)\}$, for gender $i \in \{m, w\}$, ages $a, b, j \in \{15, 16, \ldots, J\}$ and period t, such that:

(i) Measures $\{N_{j,t}^i\}$ satisfy expressions (50) and (49).

[24] For the easiness of exposition, Equation (49) implies that all children are born in the first year of marriage. Given that this assumption may affect results significantly in periods of rapidly changing fertility, when solving the problem I actually restricted women to have one children per period in up to four consecutive periods.

(ii) Measure $\{s^i_{j,t}\}$ satisfies

$$s^i_{15,t} = N^i_{15,t} \text{ and} \tag{51}$$
$$s^i_{j+1,t} = s^i_{j,t-1}\left(1-\delta_{i,j}\right)\left(1-\Gamma^i_{j,t}\right) \text{ for } j < 15 \leq J \tag{52}$$

where the $\Gamma^i_{j,t}$are defined in expression (43)

(iii) The reservation values $\{R^m_t(a,b), R^w_t(a,b)\}$ solve the decision problems of singles described in expressions (47) and (48).

4 Calibration

To calibrate the model economy we must choose the duration of the model period an initial period, a functional form for the distribution function of the match values, $G(z)$, and a value for every parameter in the model economy. These parameters are the maximum life-time, J; the fecundity profiles; the mortality probabilities, $\delta^i_{j,t}$; the fraction of people born of gender i, ϖ^i_t, the search friction parameter ρ, the discount factor, β; and the parameters that characterize the payoffs, the desired number of children per period, k_t , and the period utility of remaining single, A. I also have to assume

The model period. I assume the period in the model is yearly. The model starts in 1930 but the first period where the stocks of men and women entering the market are actually "produced" by the model is 1945. Between 1930 and 1945 the initial stocks of men and women are taken from fertility data.

The time discount factor. We choose $\beta = 0.96$. This choice is standard in the literature and it implies that the yearly discount rate in our model economies is four percent.

The distribution of the match values. I assume that the distribution of match values, $G(z)$, is the logarithm of normally distributed function with mean 0 and standard deviation σ.

The maximum life-time. I assume that $J = 60.$, and therefore that all men and women lives at most until age 75.

The mortality probabilities. The mortality probabilities are death rates taken from the Human Mortality Database for the years 1933-2006.[25,26]

The fraction of people born by gender. I use the sex ratios at birth for 1940-2002 from Mathews and Hamilton. (2005).

The fecundity profiles. The assumptions on fecundity profiles are detailed in expression (40).These values are chosen so that the fecundity profiles in the model economy are roughly consistent with the findings of the literature on the subject.[27]

Data sources. I use data on the median age of marriage published yearly by the U.S. Census Bureau. Data the age structure of the population is taken from ,the Interim State

[25] The Human Mortality Database is compiled by the University of California, Berkeley (USA) and the Max Planck Institute for Demographic Research (Germany). This dataset is available at www.mortality.org.

[26] I use the 2006 mortality rates for 2007 onwards.

[27] For example Hassan and Killick (2003) and Wood and Weinstein (1988), discussed above.

	1930 Census	Model
Median age at first marriage for men	24.27	24.33
Median age at first marriage for women	21.41	22.57
Mean Age difference at first marriage (years)	2.86	2.81
Fraction ever married men *	0.66	0.66
Fraction ever married women *	0.73	0.72
Sex ratio for never married people	1.32	1.23
Ratio 15-29 /15-44 years-old *	0.54	0.54

* Calibration targets.

Table 2: The Initial (steady state) Economy and the 1930 U.S. Census

Population Projections from the U.S. Census Bureau and Fertility data form the NCHS, as stated before. Data of several decennial U.S. Census is obtained form IPUMS.

To complete the calibration of the benchmark model economy we are left with three free parameters and a vector of the number of desired children per period, k_t. We proceed in two steps, first we calibrate our economy

The free parameters:are the standard deviation of the distribution of match values, σ; the matching function parameter, ρ and the period utility of remaining single,A. I will proceed in two steps. First, to obtain, an initial state for this economy, I assume that the economy is in the steady state and choose the numerical values of these three free parameters using three targets from the 1930 U.S. census. These targets are the fraction of ever married men, the fraction of ever married women and the ratio 15-29 / 15-44 year-olds. For this initial state, I choose $k_1 = 3.1$, the average total fertility rate during the period 1920-29. Therefore, I choose the values of σ, ρ, and A that minimize the sum of the squared differences between outcomes in the model economy and our United States targets. The values the parameters that deliver this result are $\sigma = 0.495$, $\rho = 0.38$, and $A = 1.31$. The comparison between data and the model economy is shown in Table 2. Observe that with the chosen parameters women marry around one year later than in the data. We will observe the same pattern when comparing the model dynamics with the time series.

The desired number of children. Once I obtained the parameters for the initial state, the next step is to construct the vector of desired number of children per period, k_t in order to analyze the dynamic of agents' behavior. As stated before, I use the Total Fertility Rates (TFR) for the period 1940- 2003, and for crude birth rates from 1920-1939 all provided by the NCHS.[28] Specifically I pick the estimated parameters of the following

[28] The total fertility rate computed by the National Center of Health Statistics is the sum of the birth rates of mothers in 5-year age groups multiplied by five. The birth rates are the numbers of live births per 1,000 women in a given age group. Beginning in 1970, the total fertility rate excludes the children born by nonresidents. The National Center of Health Statistics data is available at www.cdc.gov/nchs/data/statab/t991x07.pdf.Data on birth rates for the period 1909-2000 is available from www.cdc.gov/nchs/data/statab/t001x01.pdf.

OLS regression, starting in 1930,

$$TFR_t = \mu_0 + \sum_{i=1}^{6} \mu_i (t)^i + \varepsilon_t \tag{53}$$

In addition, I will assume that the economy reaches eventually a steady state where population growth is equal to zero. Therefore, using the parameters chosen previously obtain the values of k_t that, in the steady state produces the replacement rate, $k_T = 2.3$. Given the previous choices, the values of k_t are the following:

$$k_t = \begin{cases} \alpha_0 + \sum_{i=1}^{6} \mu_i (t)^i & \text{if } 1930 \leq t \leq 1965 \\ k_T + (k_{1970} - k_T) e^{-0.5t} & \text{if } 1966 \leq t \leq 1976 \\ k_T & \text{if } t \geq 1977 \end{cases} \tag{54}$$

where the expression in the second line of (54) ensures that desired fertility converges smoothly to their steady state value (in 1977).

Observe finally that the parameter α_0 in expression (54) is different from the constant term μ_0 in (53) .Since the fertility rate in the model depends on marriage probabilities, α_0 become a fourth free parameter chosen to minimize the sum square differences between the TFR in the data and those produced by the model. Given the previous choices, the evolution of the TFR in the data and the model is displayed in Panel (b) of Figure A.5, where the reader can notice two vertical dashed lines. The first one shows the peak on the total fertility rate, in 1957, and the second the minimum level of the TFR after falling for around two decades, in 1977. From the late seventies and trough the eighties fertility rates recover to reach a rate close to two children per woman, which in this model is taken as the new steady state. Recall that the model's parameters for desired fertility became constant precisely in 1977.

5 Findings

5.1 The Benchmark Model Economy

As stated before, the objective of the extended model developed in the previous section is to study a potential causality effect from demographic changes on marital behavior. Specifically, I would like to analyze if the demographic change originated by the baby boom that took place mainly in the 1950's and early 1960's had any impact in the meeting rates and therefore in the marriage timing of the generations born during and after the baby boom, that is, those that were likely to be in the marriage market from the 1970's to the 1990's. Table 3 and Panels (a) and (b) of Figure A.5 show the evolution of the median age at first marriage in the model and U.S. data, for men and women respectively. Panel (b) of Figure A.5 shows a comparison between the model and the data for a measure of the structure of the population,the fraction of 15-29 over 15-44 years old (from now on, "the fraction of young people"). Notice two vertical dashed lines: our demographic measure a

Year	Men (1)		Women (2)	
	Data	**Model**	**Data**	**Model**
1957	22.6	23.3	20.3	21.4
1977	24.0	24.4	21.6	23.0
2006	27.1	25.5	25.3	24.4
2050		26.1		24.5
△ 1957-2006	4.5	2.1	5.0	3.1
△ 1957-1977	1.4	1.0	1.3	1.7
△ 1977-2006	3.1	1.1	3.7	1.4
△ 1977-2050		1.7		1.5

Table 3: Median age at first marriage: Benchmark and U.S. data

maximum in 1976 (when the first baby boomers, born since 1946, are about to turn 30 years old) and minimum in 1995 (when those born at the end of the baby boom turns 30). Notice that the model economy shows a new maximum before 2020, which indicates that the cohort of children of early baby boomers (born around 1980), will turn up 30 years-old. Therefore, given I assume our parameters of desired fertility reach their steady state in 1977, we the ratio of young people in the marriage market would reach its own steady state value (0.5) around the year 2040.

Panel (c) shows that men's age in the model follows closely the evolution of the data from 1940 to the mid 80's. Then, while men's age continues increasing up to around 27 years old in 2005, age our benchmark economy increases at a much slower space. Since the inverse relationship between fertility an age of marriage is well-known, we are particularly interested in the increase of the age of marriage occurred after 1977 when the fertility in the data reached its historical minimum. As shown in Column 1 of Table 3, men's median age at first marriage increased by 3.1 years .between 1977 and 2006. In our benchmark economy, the increase during the same period is of 1.1 years, 35% of the total variation. Panel (a) of Figure A.5, shows that men's age continue increasing in the model to reach its steady state around 2050. Recall that, given the fertility parameter of the model reach its steady state value in 1977, the increase of 1.7 years between that year and the steady state is purely due to the transitional dynamics of the model. The comparison for women is shown Panel (d) of Figure A.5 and Column 2 of Table 3. Notice women in the model are more than one year older than in the data in the initial state (see Table 2 above), and of course that initial divergence make the evolution of women's age at first marriage in the model to follow the data less closely than in the case of men. However, the increase in the period 1977-2006 is 1.4 years a 37% of the change in the data.

The last two panels of Figure A.5 displays comparison for variables for which there is not much data available. The evolution of the average age gap at first marriage (which obviously is different than the gap between the two medians, as discussed above) is shown in Panel (e) from 1940 to 1995.[29] Finally, Panel (f) displays the evolution of the ratio

[29] Unfortunately, there is not consistent series of the age at first marriage. The U.S census stop asking

single men/ single women, in the model an in the U.S. Census (only decennial data is available is available). Even though in this case a comparison is difficult, the evolution of this variable in the model is consistent with that in the data. However, in the model economy the sex ratio increases less than in the data, what is another way to express that the model falls short in reflecting the increase in the age difference during the baby boom.

In Figure A.5 I explore the benchmark economy in more detail. Panel (a) shows the comparative evolution of the TFR and the fraction of young people. The first dashed line the year when the TFR reach it maximum, 1957 and the second dashed line is in 1976, when on the one hand, the TFR reaches its post "baby boom" minimum and on the other hand, the fraction of young people reaches its maximum. Remember that, as the same time, from 1977 onwards, he desired fertility parameter reaches its steady state value and so all changes come from model dynamics. In Panels (b) and (c) I compare the median age at first marriage with the TFR and population structure, respectively. Notice that , after reaching a minimum age in 1957, women's age start rapidly to recover, but in the case of men, most of the increase occurs after fertility reaches its minimum (Panel (b)) or the fraction of young people reaches its maximum (Panel (c)). Panel (d) shows that the high correlation between the age difference and the single sex ratio, which illustrates Proposition 5 and. despite the data limitations, it seems to be consistent with the evolution shown in Panel (e) of Figure A.5.

Panels (e) and (f) of Figure A.5 show the evolution of two model variables that may be helpful in order to understand the mechanism of the model: The probability of meeting someone of age 25 $\lambda^i_{25,t}$ in Equation (39) , and the expected utility of marriage (or value of being single) at age 25 $EU^i_{25,t}$ from Equation (44) . Notice that the value of marriage reaches its maximum in 1957 and the matching probabilities in 1976, and the increase in the value of marriage is relatively more important for women than for men.

5.2 Counterfactual Experiments

In this section I perform two counterfactual experiments. In the first experiment, I assume than fertility rates converge slowly from the 1940 levels to the replacement rate, as if the "baby boom" never occurred. In the second experiment, I assume that even there is even there is an increase in desired fertility from the 1945 to 1965, the inflow of people to the economy is constant , so I am isolating the persistence mechanism.

5.2.1 "No Baby Boom"

The results of this counterfactual experiment are shown in Figure A.5, Panels (a) through (f). Notice in Panel (c) that the women age of marriage in the benchmark model converge with the series without baby boom relatively rapidly, around 1990 (when fertility rates stabilize). However, this is not the case of men's median age at first marriage, whose convergence do not occur until around 2050.

about the age at the first marriage in 1980. So the data used in Panel (e), just to be taken as reference, is constructed using 1930-1980 U.S. Census from (except for the 1950 Census, when this question was not asked) and the Marriage data Files (NCHS) from 1980 to 1995.

5.2.2 Constant Inflow

Here I suppose that every period the inflow rate 15 year-olds to the economy is constant, regardless of what happen with fertility rates. As shown in Figure A.5, in this case, the experiment shows the same evolution than the benchmark in the case of fertility (Panel (a)), and utility of marriage (Panel (e)). The big difference is that the age structure of the population is practically constant (Panel (b)) [30]. Observe in panel (d) that the sex ratio in the counterfactual economy continue increasincreasing after the benchmark reached its maximum, in a similar fashion than the fraction of young people in the benchmark economy (Panel (b)). By assuming that the inflow of people is constant, the the are is not an increase of the fraction of young people begninning in the mid-sixties. The entering of those bigger cohorts of young people, helped to balance the sex ratio that already had increased during the baby boom. But, if we assume no such change in the population structure, men's age at first marriage have had adjusted much rapidly, starting to increase in 1957 and not in the mid seventies, as it finally ocurred (see Panel (c)). This experiment suggest that the reason behind the slow adjustment of men's age of marriage is closely related with the changes in population structure caused by the baby boom.

6 Conclusion

In this paper, I develop an equilibrium, two-sided search model of marriage with endogenous population growth to study the interaction between fertility, the age structure of the population and the age at first marriage of men and women. I show that, given an increase of the desired number of children (which is an exogenous parameter in this model), age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population acts as a "thicker market" externality, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, young women become relatively less choosy than young men. In equilibrium, women are more likely to marry older men and single men outnumber single women.

The results above have dynamic implications that are consistent with observed patterns in the U.S. data. In the case of a temporary increase of desired fertility (a "baby boom"), the fraction of young people and the single sex ratio will act as persistence mechanisms, affecting marriage timing. First, until the age structure of the population reaches its original steady state level, men and women marry younger than in the steady state. Second, as the initial increase in the age gap produces an imbalance in the sex ratio, women face a higher matching rate than men, and therefore the age gap persists even if the desired number of children is too low for differential fecundity to arise.

Even though that this stylized model is not able to account for all the factors that may have influenced the sharp increase in the age of marriage since the mid-seventies, it provides a link, demographic composition, to reconcile the evolution of fertility rates with time series of the age of marriage. Moreover, the strong correlation observed between the

[30] In fact the inflow is constant but in the initial state we where not in the steady state.

sex ratio and the age gap in the data suggests that this model's predictions about the age difference are consistent with observed data. Specifically, it helps to understand why differential fecundity may be the main explanation for the age gap with current fertility rates (around 2 children per woman in the U.S. and lower rates in several European countries). Given that these current fertility rates do not seem to create much asymmetry between men and women, what this model suggests is that today's age gap may be determined by higher fertility rates in the past that created an imbalance in the marriage market, which may still be in a slow process of adjustment.

The marriage market model developed in this article can be easily extended to a richer environment that includes human capital accumulation, labor supply and fertility choice in order to make a quantitative analysis of the dynamics of marriage in the last few decades. The results above suggest that the baby boom may have affected the marriage behavior of baby boomers themselves through identifiable, persistence factors, whose study requires going beyond standard, comparative static analysis.

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A Technical Appendix

A.1 Existence

I need to show that the following functional relationship is a fixed point

$$\Psi\left(R_{1,2}^{m},\Gamma_{1}^{m},\Gamma_{1}^{w},n\right)\longrightarrow\begin{pmatrix}R_{1,2}^{m}=\Psi_{1}\left(R_{1,2}^{m},\Gamma_{1}^{m},\Gamma_{1}^{w},n\right)\\ \Gamma_{1}^{m}=\Psi_{2}\left(R_{1,2}^{m},\Gamma_{1}^{m},\Gamma_{1}^{w},n\right)\\ \Gamma_{1}^{w}=\Psi_{3}\left(R_{1,2}^{m},\Gamma_{1}^{m},\Gamma_{1}^{w},n\right)\\ n=\Psi_{4}\left(R_{1,2}^{m},\Gamma_{1}^{m},\Gamma_{1}^{w},n\right)\end{pmatrix}$$

where

$$\Psi_{1}=\mu_{x}+\left\{\rho\left(\frac{2+n-\Gamma_{1}^{m}}{2+n-\Gamma_{1}^{w}}\right)^{\theta-1}\left(\frac{1+n}{2+n-\Gamma_{1}^{w}}\right)\left[1-G_{m}\left(\frac{\min\{k,1\}}{k}\mu_{y}\right)\right]\left[k-\min\{k,1\}\right]\right\}\mu_{x} \tag{55}$$

$$\Psi_{2}=\rho\left(\frac{2+n-\Gamma_{1}^{m}}{2+n-\Gamma_{1}^{w}}\right)^{\theta-1}\left\{\left(\frac{1+n}{2+n-\Gamma_{1}^{w}}\right)\left[1-G_{w}\left(\frac{\min\{k,1\}R_{1,2}^{m}}{2k}\right)\right]\left[1-G_{m}\left(\frac{\min\{k,1\}\mu_{y}}{2k}\right)\right]\right. \tag{56}$$

$$\left.+\left(\frac{1-\Gamma_{1}^{w}}{2+n-\Gamma_{1}^{w}}\right)\left[1-G_{w}\left(R_{1,2}^{m}\right)\right]\right\}$$

$$\Psi_{3}=\rho\left(\frac{2+n-\Gamma_{1}^{m}}{2+n-\Gamma_{1}^{w}}\right)^{\theta}\left\{\left(\frac{1+n}{2+n-\Gamma_{1}^{m}}\right)\left[1-G_{w}\left(\frac{\min\{k,1\}R_{1,2}^{m}}{2k}\right)\right]\left[1-G_{m}\left(\frac{\min\{k,1\}\mu_{y}}{2k}\right)\right]\right. \tag{57}$$

$$\left.+\left(\frac{1-\Gamma_{1}^{m}}{2+n-\Gamma_{1}^{m}}\right)\left[1-G_{m}\left(\frac{\min\{k,1\}}{k}\mu_{y}\right)\right]\right\}$$

$$\Psi_{4}=\frac{k\Gamma_{1}^{w}}{2}-1+\frac{\sqrt{\left(k\Gamma_{1}^{w}\right)^{2}+4\min\{k,1\}\left(1-\Gamma_{1}^{w}\right)}}{2} \tag{58}$$

Define a set $\Omega=[0,\overline{x}]\times[0,1]\times[0,1]\times\left[-0.5,(\overline{k}-1)\right]$. The first step is to show that $\Psi(.)$ is continuous. The continuity of $\Psi(.)$ is ensured because of the continuity of $G_w(.)$and $G_m(.)$ and the boundaries of the ratio single men/single women. The next step is to show that $\Psi(.)$ maps from Ω to itself. In order to show this, I pick a point $\chi_0=\left(R_0^m,\Gamma_0^m,\Gamma_0^w,n_0\right)$, where $R_0^m\in[0,\overline{x}]$, $\Gamma_1^m\in[0,1]$, $\Gamma_1^w\in[0,1]$ and $n_0\in[-0.5,(k-1)]$. I have to show that $\Psi_{10}\left(R_0^m,\Gamma_0^m,\Gamma_0^w,n_0\right)\in[0,\overline{x}]$, $\Psi_{20}\left(R_0^m,\Gamma_1^m,\Gamma_1^w,n_0\right)\in[0,1]$, $\Psi_{30}\left(R_0^m,\Gamma_0^m,\Gamma_0^w,n_0\right)\in[0,1]$ and $\Psi_{40}\left(R_0^m,\Gamma_0^m,\Gamma_0^w,n_0\right)\in\left[-0.5,\left(\overline{k}-1\right)\right]$.

Consider first the case of Ψ_{10}. By simply observation of equation (55), we know that $\Psi_{10}\geq\mu_x$, so we only have to look for the upper bound of Ψ_{10}, which occurs at $k=\overline{k}$. By definition, $\lambda_{10}^m=\rho\left(\frac{2+n_0-\Gamma_0^m}{2+n_0-\Gamma_0^w}\right)^{\theta-1}\left(\frac{1+n_0}{2+n-\Gamma_0^w}\right)\leq 1$ and $G_m\left(\frac{\mu_y}{k}\right)\in(0,1)$. Therefore, evaluating Ψ_{10} at $\overline{k} = \frac{\overline{x}}{\mu_x}$ (by condition (5)), we have

$$\begin{aligned}\Psi_{10} &= \mu_x + \left\{ \lambda_{10}^m \left[1 - G_m \left(\frac{\mu_y}{\overline{k}} \right) \right] \left[\overline{k} - 1 \right] \right\} \mu_x \\ &= \mu_x + \left\{ \lambda_{10}^m \left[1 - G_m \left(\frac{\mu_y}{\overline{k}} \right) \right] \left[\overline{x} - \mu_x \right] \right\} < \overline{x},\end{aligned}$$

which implies that $\Psi_1 = R_{1,2}^m \in [\mu_x, \overline{x})$.

Consider now Ψ_{20}. Denote $p_{w0} = \left(\frac{1+n_0}{2+n-\Gamma_0^w} \right) > 0$. Given that by definition $\rho \left(\frac{2+n_0-\Gamma_0^m}{2+n_0-\Gamma_0^w} \right)^{\theta-1} \leq 1$, that $G_m \left(\frac{\min\{k,1\}\mu_y}{2k} \right) \in (0,1)$ and also that $G_m \left(\frac{\min\{k,1\}\mu_y}{2k} \right) \in (0,1)$, whe have,

$$\begin{aligned}\Psi_{20} = \rho \left(\frac{2+n-\Gamma_1^m}{2+n-\Gamma_1^w} \right)^{\theta-1} & \left\{ p_{w0} \left[1 - G_w \left(\frac{\min\{k,1\} R_0^m}{2k} \right) \right] \left[1 - G_m \left(\frac{\min\{k,1\}\mu_y}{2k} \right) \right] \right. \\ & \left. + (1-p_{w0}) \left[1 - G_w \left(R_0^m \right) \right] \right\} \in (0,1). \qquad (59)\end{aligned}$$

Therefore $\Psi_2 = \Gamma_1^m \in (0,1)$. This trivially implies $\gamma_{1,1}^m = \gamma_{1,1}^w > 0$.

A similar argument can be used to show that $\Psi_3 = \Gamma_1^w \in (0,1)$. Notice that the fact that Γ_1^m and Γ_1^w are both less than one imply that $\left(\frac{1-\Gamma_1^m}{2+n-\Gamma_1^m} \right) > 0$ and $\left(\frac{1-\Gamma_1^w}{2+n-\Gamma_1^w} \right) > 0$. In addition, $R_{1,2}^m < \overline{x}$, as shown above. Therefore, we have that $\gamma_{1,2}^m > 0$ and $\gamma_{1,2}^w > 0$, which, by (34) and (35), imply $\gamma_{2,1}^w > 0$ and $\gamma_{2,1}^m > 0$.

Finally, in Section 2.5.2 I have already shown that $n \in \left[-0.5, \overline{k} - 1 \right]$. Given the interior solutions of Γ_1^m and Γ_1^w we have that $\Psi_4 \in \left(-0.5, \overline{k} - 1 \right)$. That concludes the proof.

A.2 Proof of Proposition 1 (symmetric equilibrium)

I have to show that when $G_m = G_w = G$ and $k \in [\frac{1}{2}, 1)$, the point $s = (\mu, \Gamma_s, \Gamma_s, n_s)$ is a solution of $\Psi(.)$.[31] Plugging s into $\Psi(.)$, making $k < 1$ and manipulating we get

$$\begin{aligned}\Psi_2(\mu, \Gamma_s, \Gamma_s, n_s) &= \rho \left(\frac{2+n_s-\Gamma_s}{2+n_s-\Gamma_s} \right)^{\theta-1} \left\{ \left(\frac{1+n_s}{2+n_s-\Gamma_s} \right) \left[1 - G\left(\frac{\mu}{2} \right) \right]^2 + \left(\frac{1-\Gamma_s}{2+n_s-\Gamma_s} \right) \left[1 - G(\mu) \right] \right\} \\ &= \left(\frac{1+n_s}{2+n_s-\Gamma_s} \right) \left[1 - G\left(\frac{\mu}{2} \right) \right]^2 + \left(\frac{1-\Gamma_s}{2+n_s-\Gamma_s} \right) \left[1 - G(\mu) \right] \\ &= \Psi_3(\mu, \Gamma_s, \Gamma_s, n_s) \\ \Psi_4(\mu, \Gamma_s, \Gamma_s, n_s) &= \frac{k\Gamma_1^s}{2} - 1 + \sqrt{\frac{\left(k\Gamma_1^s\right)^2}{4} + k\left(1 - \Gamma_1^s\right)}.\end{aligned}$$

Therefore s is an equilibrium of $\Psi(.)$ and satisfies

$$\begin{aligned}\Gamma_1^m &= \Gamma_1^w \in (0,1) \\ n &\in (-0.5, 0).\end{aligned}$$

[31] Recall that, when $k \leq 1$, $R_{1,2}^m = R_{1,2}^w = \mu$, by (36).Similarly, $R_{1,1}^m = R_{1,1}^w = \frac{\mu}{2}$.

Given that $\Gamma_1^m = \Gamma_1^w$, it is straightforward that the number of single old men equal the number of old single women, $s_{2,t}^m = s_{2,t}^w = (1+n)^{t-1}(1-\Gamma_1^s)$ (by (26)), which makes $S_t^m = S_t^w$. Furthermore, since $n < 0$, the fraction of young people in the population, $q_1 = \frac{1+n}{2+n} < \frac{1}{2}$ (by (1)).

A.3 Proof of Proposition 2 (uniqueness of the symmetric equilibrium)

Now I have to show that the equilibrium found in Proposition 1 is unique for each value of $k \in \left[\frac{1}{2}, 1\right]$. Define

$$\digamma\left(R_{1,1}^m, \Gamma_1^m, \Gamma_1^w, n\right) = \begin{pmatrix} \digamma_1\left(R_{1,1}^m, \Gamma_1^m, \Gamma_1^w, n\right) \\ \digamma_2\left(R_{1,1}^m, \Gamma_1^m, \Gamma_1^w, n\right) \\ \digamma_3\left(R_{1,1}^m, \Gamma_1^m, \Gamma_1^w, n\right) \\ \digamma_4\left(R_{1,1}^m, \Gamma_1^m, \Gamma_1^w, n\right) \end{pmatrix} \tag{60}$$

where $\digamma_1 = \left[\frac{\min\{k,1\}}{2k}\Psi_1 - R_{1,1}^m\right]$, $\digamma_2 = [\Psi_2 - \Gamma_1^m]$, $\digamma_3 = [\Psi_3 - \Gamma_1^w]$ and $\digamma_4$ is given by equation (31).[32] In Proposition 1 whe found that the point $s = (\Gamma_s, \Gamma_s, n_s)$ is a solution of $\Psi(.)$.To show the uniqueness of s using the Implicit Function Theorem, we need continuity in all the parcial derivatives of $\digamma(.)$. We also need that Jacobian determinant of $\digamma(.)$ with respect to the endogenous variables is nonzero when evaluated at s. We know that the system continuous and differentiable for $k \in [\frac{1}{2}, 1)$, being their partial derivatives also continuous because g is differentiable.[33]

Given that under symmetry $R_{1,1}^m = \frac{\mu}{2}$ (by (**??**)), we can eliminate one equation $(\digamma_1)$ and one unknown $\left(R_{1,1}^m\right)$ from the system $\digamma(.)$.Therefore, evaluating our system at point s we have,

$$\begin{pmatrix} \digamma_2\left(\Gamma_1^s, \Gamma_1^s, n_s\right) \\ \digamma_3\left(\Gamma_1^s, \Gamma_1^s, n_s\right) \\ \digamma_4\left(\Gamma_1^s, \Gamma_1^s, n_s\right) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

and we need that,

$$|J| = Det\begin{bmatrix} \digamma_{22} & \digamma_{23} & \digamma_{24} \\ \digamma_{32} & \digamma_{32} & \digamma_{33} \\ \digamma_{42} & \digamma_{43} & \digamma_{44} \end{bmatrix} \neq 0. \tag{61}$$

Calculating the partial derivatives of $\digamma(.)$ and manipulating, we get

$$|J| = \frac{1}{\left(2+n_s-\Gamma_1^s\right)^3}\left(2+n_s-\rho\left[1-G(\mu)\right]\right)\left\{\left(2+n_s-\Gamma_1^s\right)^2\left(2+2n-k\Gamma_1^s\right)+\right.$$
$$\left.\left(2+n_s(4+k+2n)-k(1+2n)\Gamma_1^s\right)\rho\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)+G(\mu)\right)\right\}.$$

[32] Notice that, in order to make the exposition of the following results easier, now the system is expressed in terms of $R_{1,1}^m$.

[33] The system $\digamma(.)$ is not differentiable at $k = 1$, that is, the right hand side and the left hand side derivatives with respect to k are not equal. However, $\digamma(.)$ is continuous in k and differentiable for the intervals $k \in [\frac{1}{2}, 1)$ and $k \in (1, \overline{k}]$.

Given that by assumption $\rho \leq \frac{1}{2}$ and that always $n_s \geq -\frac{1}{2}$, we have that $(2 + n_s - \rho [1 - G(\mu)]) > 0$. Therefore $|J|$ will be positive if the expresion under curly brackets, $\{x_1 + x_2 * x_3\} > 0$, where $x_1 = (2 + n_s - \Gamma_1^s)^2 (2 + 2n - k\Gamma_1^s)$, $x_2 = (2 + n_s (4 + k + 2n) - k (1 + 2n) \Gamma_1^s) \rho$ and $x_3 = \left(\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu)\right)$.We clearly have that $x_1 > 0$ $x_2 > 0$. In the case of x_3, the log-concavity of g implies

$$\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) \leq G(\mu) \tag{62}$$

and therefore $x_3 \leq 0$ and $x_2 * x_3 \leq 0$.[34] However, given that $|x_3| \leq G(\mu)$. it is sufficient that $\rho \leq \frac{1}{2}$ to have $|x_1| > |x_2| * |x_3|$. Therefore $|J| > 0$ which implies that the equilibrium is unique.

A.4 Proof of Proposition 3 (comparative statics of the symmetric equilibrium).

In Proposition 2 we found that $|J| > 0$, which allows us to define Γ_1^s and n_s as implicit functions of k in a neighborhood of any point s that is a solution of the system $\digamma(.)$ defined in (60). Given that $\digamma(.)$ has a unique solution for each value of $k \in [\frac{1}{2}, 1)$, the following results hold for the whole interval.

Now I turn to comparative statics. Applying Cramer's rule, the derivatives of the endogenous variables in $\digamma(.)$ with respect to k are $\frac{\partial \Gamma_1^m}{\partial k} = \frac{\partial \Gamma_1^w}{\partial k} = \frac{|J_2|}{|J|}$, and $\frac{\partial n}{\partial k} = \frac{|J_4|}{|J|}$, where where $|J|$ is defined in (61) and $|J_2|$ and $|J_4|$ are obtained by replacing $-\digamma_{ik} = -\frac{\partial \digamma_i}{\partial k}$ for $i = \{2, 3, 4\}$ in the respective column of $|J|$. Therefore we have,

$$|J_2| = -\frac{(1 - \Gamma_1^s)}{(2 + n_s - \Gamma_1^s)^3} (1 + n\Gamma_1^s)(2 + n_s - \rho [1 - G(\mu)]) \rho \left(\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu)\right)$$

Given that $\left(\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu)\right) \leq 0$ (by (62)), we have that $|J_1| > 0$. Therefore

$$\frac{\partial \Gamma_1^s}{\partial k} > 0, \text{ for } k \in [\frac{1}{2}, 1).$$

Similarly,

$$|J_4| = \frac{(2 + n_s - \rho [1 - G(\mu)])}{(2 + n_s - \Gamma_1^s)^3} \left\{(2 + n_s - \Gamma_1^s)^2 + (1 + n_s) \rho \left(\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu)\right)\right\}$$

We know that $(2 + n_s - \Gamma_1^s) > 1 + n_s \geq 0.5$ (because of $\Gamma_1^s < 1$ and the boundaries of n) and that $\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu) \leq G(\mu)$. Given that $\rho \leq \frac{1}{2}$, it must follow that $|J_4| > 0$. Therefore

$$\frac{\partial n}{\partial k} > 0, \text{ for } k \in [\frac{1}{2}, 1).$$

Finally, since $q_1 = \frac{1+n}{2+n}$, it must follow that $\frac{\partial q_1}{\partial k} > 0$.

[34] Log-concave probability densisities have the "New is better than used" (NBU) property: $[1 - F(x + y)] \leq [1 - F(x)][1 - F(y)]$. Making $x = y = \frac{\mu}{2}$, this implies that $\left(\left[2 - F\left(\frac{\mu}{2}\right)\right] F\left(\frac{\mu}{2}\right)\right) \leq F(\mu)$. See An (2003).

A.5 Proof of Proposition 5 (comparative statics of the differential fecundity equilibrium).

To show the comparative statics with differential fecundity we use the system defined in (60). Given than now $k \in (1, \overline{k}]$, $\digamma_1$ is defined by (20) and we work with the full system of four equations in four unknowns. For tractability I pick a point s' defined by the unique solution of the system at $k = 1$. Since the right-hand side derivatives of $\digamma(.)$ are defined and are continuous for $k = 1$, we are able to make local comparative statics for the neighborhood of s' (for $k = 1 + \epsilon$, for ϵ small). First of all, we have to show that the Jacobian of the full system, $|H|$ is nonzero when evaluated at s'. Calculating the partial derivatives of $\digamma(.)$ and manipulating, we get

$$|H| = \frac{\left(2 - \Gamma_1^s - \rho\left[1 - G(\mu)\right]\right)\left\{\left(2 - \Gamma_1^s\right)^2 + \rho\left(\left[2 - G\left(\frac{\mu}{2}\right)\right] G\left(\frac{\mu}{2}\right) + G(\mu)\right)\right\}}{\left(2 - \Gamma_1^s\right)^2} > 0$$

Therefore, we can define $R^m_{1,1}$, Γ^m_1, Γ^w_1 and n as implicit functions of k in a neighborhood of s'.Applying Cramer's rule, the derivatives of the endogenous variables in $\digamma(.)$ with respect to k are $\frac{\partial R^m_{1,1}}{\partial k} = \frac{|H_1|}{|H|}$, $\frac{\partial \Gamma^m_1}{\partial k} = \frac{|H_2|}{|H|}$, $\frac{\partial \Gamma^w_1}{\partial k} = \frac{|H_3|}{|H|}$ and $\frac{\partial n}{\partial k} = \frac{|H_4|}{|H|}$, where $|H_1|$, $|H_2|$, $|H_3|$ and $|H_4|$ are obtained by replacing $-\digamma_{ik} = -\frac{\partial \digamma_i}{\partial k}$ for $i = \{1, 2, 3, 4\}$ in the respective column of $|H|$.

Now I will show that in a neighborhood of s' we have that, given an increase in k:

1. *Young men are less choosy with young women and choosier with old women:* $\frac{\partial R^m_{11}}{\partial k} < 0$ and $\frac{\partial R^m_{12}}{\partial k} > 0$.

   To show that men's reservation values of men for young women are decreasing in k we need that,

   $$\frac{|H_1|}{|H|} = -\frac{\mu}{2}\frac{\left(2 - \Gamma_1^s - \rho\left[1 - G(\mu)\right]\right)}{\left(2 - \Gamma_1^s\right)} < 0$$

   In addition, since $\frac{\left(2-\Gamma_1^s-\rho[1-G(\mu)]\right)}{\left(2-\Gamma_1^s\right)} < 1$, we also have $\left|\frac{\partial R^m_{11}}{\partial k}\right| < \left|\frac{\partial R^w_{11}}{\partial k}\right| = \frac{\mu}{2}$ (evaluated at $k = 1$). On the other hand, we have that young men become choosier with respect to old women. By (21), $R^m_{12} = 2kR^m_{11}$ when $k \geq 1$. Therefore,

   $$\begin{aligned} \frac{\partial R^m_{12}}{\partial k} &= 2\left[R^m_{1,1} + \frac{\partial R^m_{11}}{\partial k}\right] \\ &= 2\left[\frac{\mu}{2}\left(1 - \frac{\left(2 - \Gamma_1^s - \rho\left[1 - G(\mu)\right]\right)}{\left(2 - \Gamma_1^s\right)}\right)\right] > 0 \end{aligned}$$

2. *Probability of marriage*

   (a) *Women marry young with higher probability.*

To show that $\frac{\partial \Gamma_1^w}{\partial k} > 0$ is enough to show that $|H_3| > 0$. Therefore,

$$|H_3| = \frac{1}{(2-\Gamma_1^s)^3}\left\{-\left(2-\Gamma_1^s-\rho\left[1-G(\mu)\right]\right)\left(1-\Gamma_1^s\right)\left(\left[2-G\left(\frac{\mu}{2}\right)\right]G\left(\frac{\mu}{2}\right)-G(\mu)\right)\right.$$
$$+\mu\left(2-\Gamma_1^s\right)^2\left[1-G\left(\frac{\mu}{2}\right)\right]\left(2-\Gamma_1^s-\rho\left[1-G(\mu)\right]\right)\left(4-2\Gamma_1^s-\rho\left[1-G(\mu)\right]\right)g\left(\frac{\mu}{2}\right)$$
$$+2\left(2-\Gamma_1^s\right)\left(1-\Gamma_1^s\right)\mu f(\mu)\left\{\left[\left(2-\Gamma_1^s\right)^2\left[2-\Gamma_1^s-(1-\theta)\rho+\rho^2\theta\right]-\left(1+\theta\left(3-2\Gamma_1^s\right)\right)\rho^2\right]\right.$$
$$+\left[\left(2-\Gamma_1^s\right)(1-\theta)\rho F(\mu)+\rho^2 G(\mu)^2\left(1+\theta\left(1-\Gamma_1^s\right)\right)\right]$$
$$\left.\left.+\left(2-\Gamma_1^s-\rho\left[1-G(\mu)\right]\right)\left[2-G\left(\frac{\mu}{2}\right)\right](1-\theta)G\left(\frac{\mu}{2}\right)\right\}\right\}$$

We know by (62) that the first line is positive. Recalling that $\rho \leq \frac{1}{2}$, it is also clear that the second, fourth and fifth lines are also positive. In the the third line we have $-1 < \left(1+\theta\left(3-2\Gamma_1^s\right)\right)\rho^2 < 0$. However, at $k=1$ we have $\phi = 1$ which implies $\Gamma_1^s \leq \rho$ and $\left[2-\Gamma_1^s-(1-\theta)\rho+\rho^2\theta\right] \geq 1$. Therefore, the third line is also positive and $|H_3| > 0$, which implies $\frac{\partial \Gamma_1^w}{\partial k} > 0$ given that $|H| > 0$.

(b) *The difference between the probability of marriage at age 1 of women and men, $\Gamma_1^w - \Gamma_1^m$, is positive and increasing in k: $\frac{\partial \Gamma_1^w}{\partial k} > \frac{\partial \Gamma_1^m}{\partial k}$.*

Here we need $\frac{|H_3|-|H_2|}{|H|} > 0$

$$\frac{|H_3|-|H_2|}{|H|} = \frac{\left(1-\Gamma_1^s\right)\rho\mu\left(2-\Gamma_1^s+\rho\left[1-G(\mu)\right]\right)g(\mu)}{\left(2-\Gamma_1^s\right)\left(2-\Gamma_1^s-\rho\left[1-G(\mu)\right]\right)} > 0$$

3. *The ratio of single men to single women, ϕ, is greater than 1 and increasing in k.*

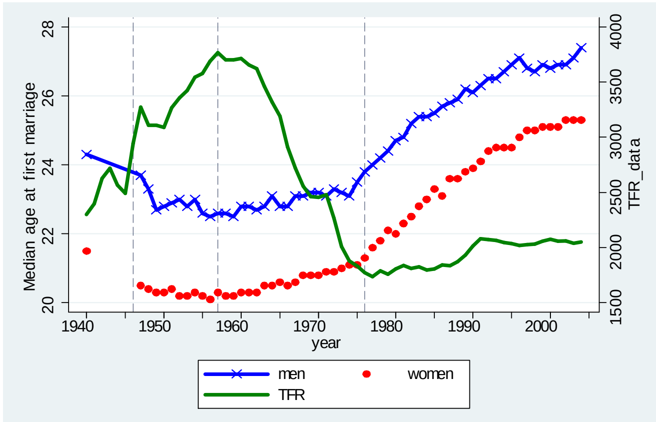
   From (26) we know that $s_{2,t}^m = s_{1,t-1}^m\left(1-\Gamma_1^m\right)$ and $s_{2,t}^w = s_{1,t-1}^w\left(1-\Gamma_1^w\right)$. Since $s_{1,t}^m = s_{1,t}^w$, $s_{2,t}^m > s_{2,t}^w$ because $\Gamma_1^w > \Gamma_1^m$. Therefore $S_t^m > S_t^w$ and $\phi > 1$. Furthermore, since $\frac{\partial \Gamma_1^w}{\partial k} > \frac{\partial \Gamma_1^m}{\partial k}$, we have $\frac{\partial \phi}{\partial k} > 0$.

4. *Population growth and the fraction of young people in the economy increases.*For $\frac{\partial n}{\partial k} > 0$ we need

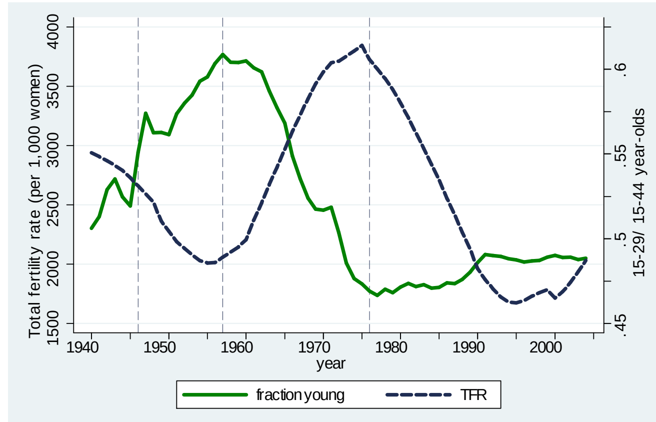
$$\frac{|H_4|}{|H|} = \frac{\Gamma_1^s}{\left(2-\Gamma_1^s\right)} > 0$$

   It is straightforward that here we also have $\frac{\partial q_1}{\partial k} > 0$, as in Proposition 3.

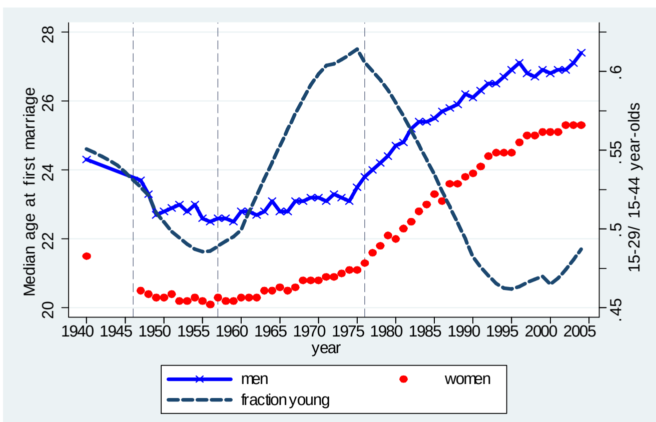
Figure 1: Marriage and fertility indicators in the U.S.



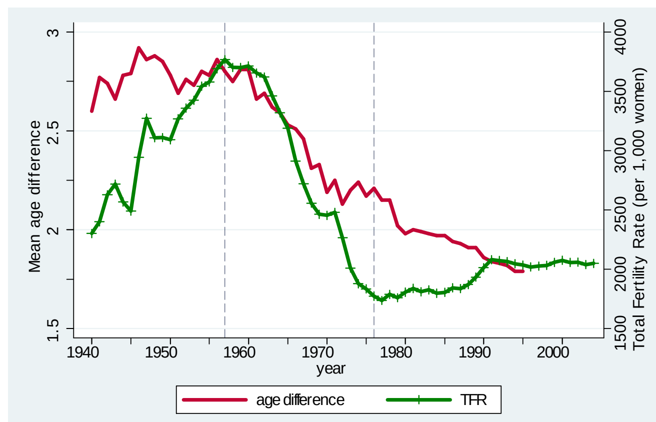
(a) Median age at first marriage and TFR.



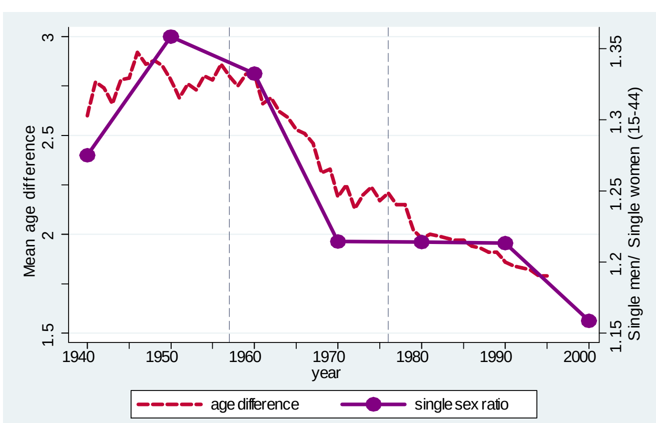
(b) Population structure and TFR.



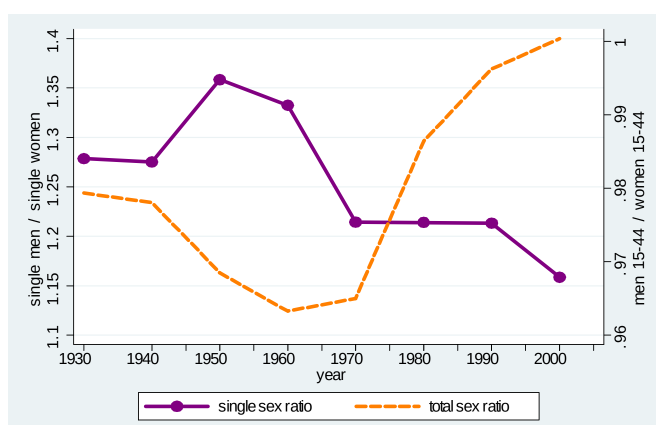
(c) Population structure and age of marriage.



(d) Mean age difference and TFR

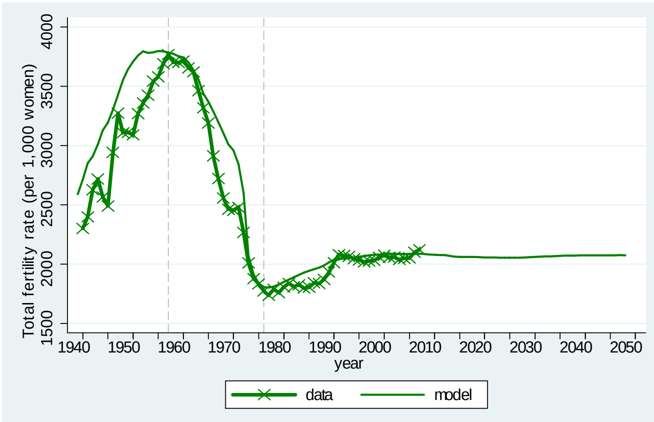


(e) Mean age Difference and Sex Ratio

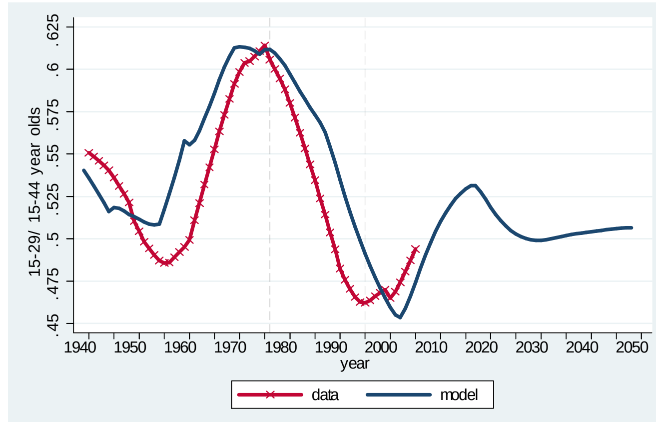


(d) Single and total sex ratio (ages 15-44)

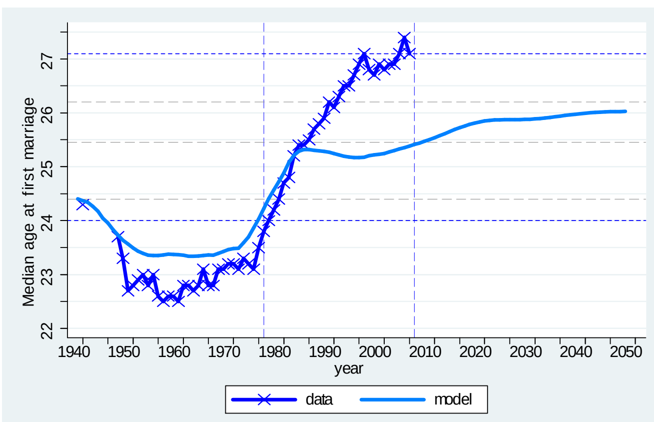
Figure 2: The Benchmark Economy: Comparison with U.S. data



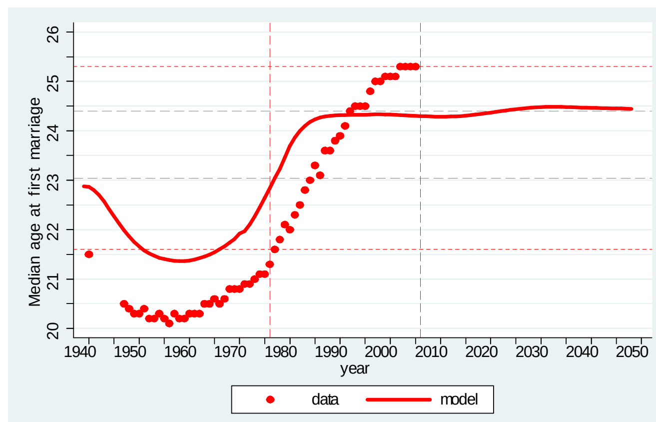
(a) Total Fertility rate



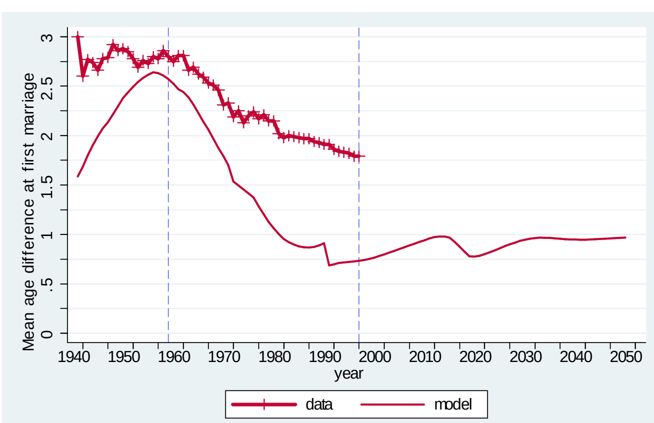
(b) Ratio 15-29 /15-44 year-olds



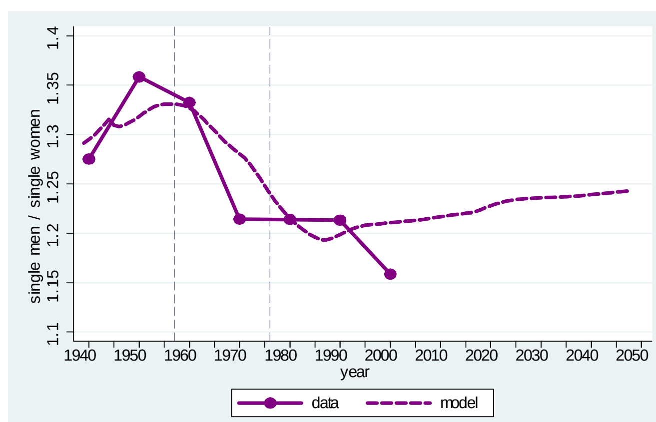
(c) Median age at first marriage (men)



(d) Median age at first marriage (women)

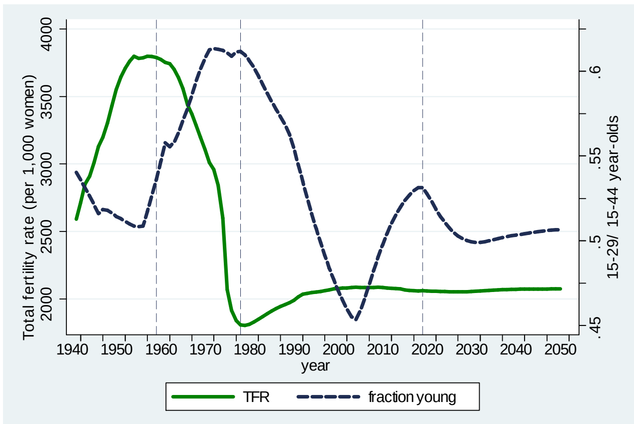


(e) Mean age difference at first marriage

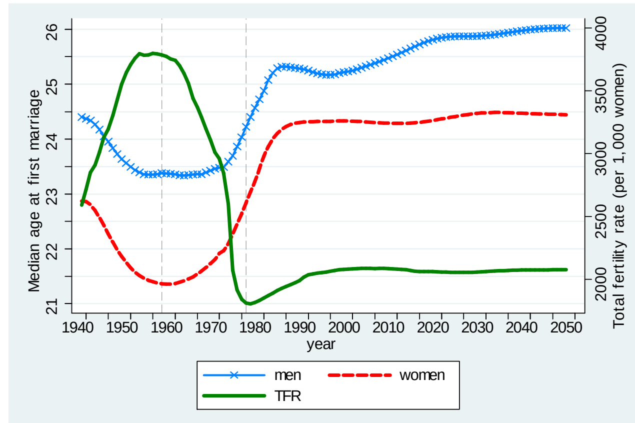


(f) Ratio single men / single women (15-44)

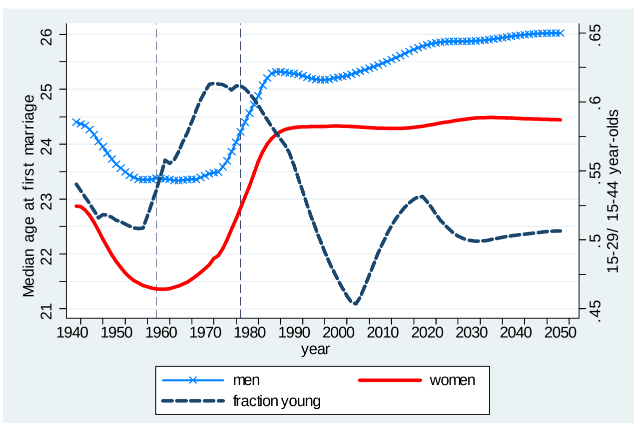
Figure 3: The Benchmark Economy.



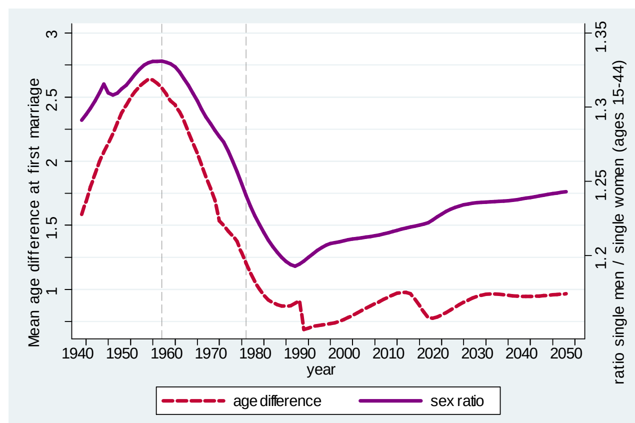
(a) Population structure and TFR.



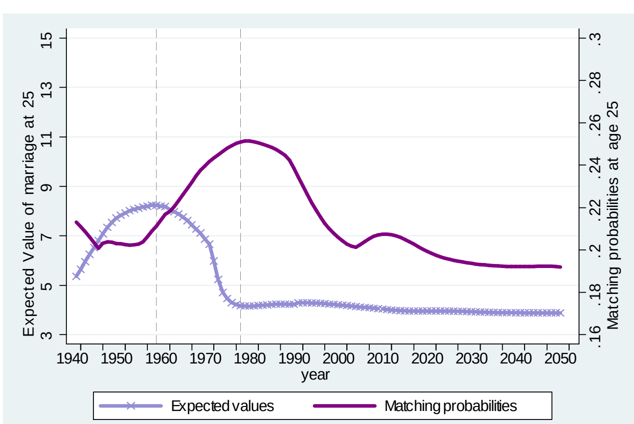
(b) Median age at first marriage and TFR.



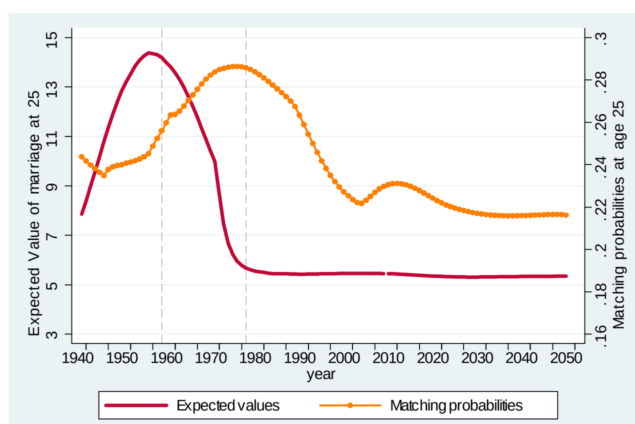
(c) Population structure and age of marriage.



(d) Mean age difference and sex ratio

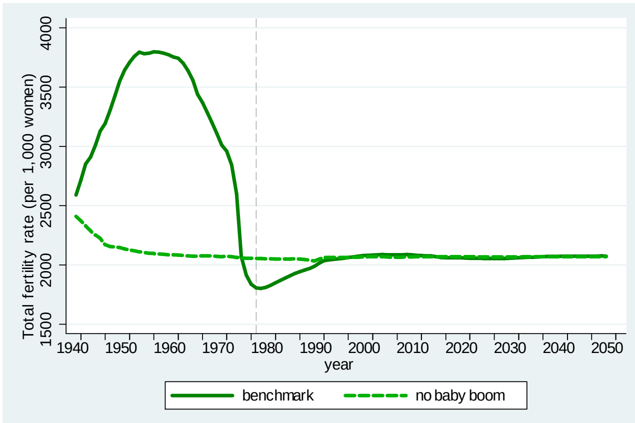


(e) Value of being single and matching probabilities at age 25 (men)

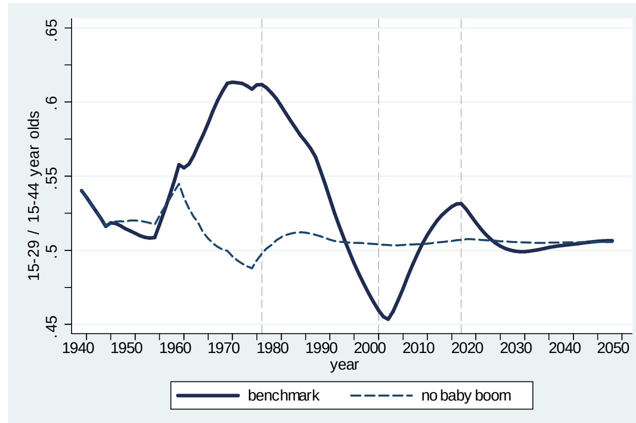


(f) Value of being single and matching probabilities at age 25 (women)

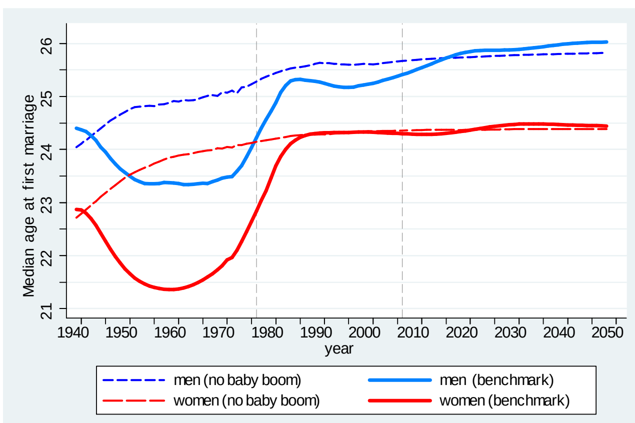
Figure 4: Counterfactual Economy I: No baby boom.



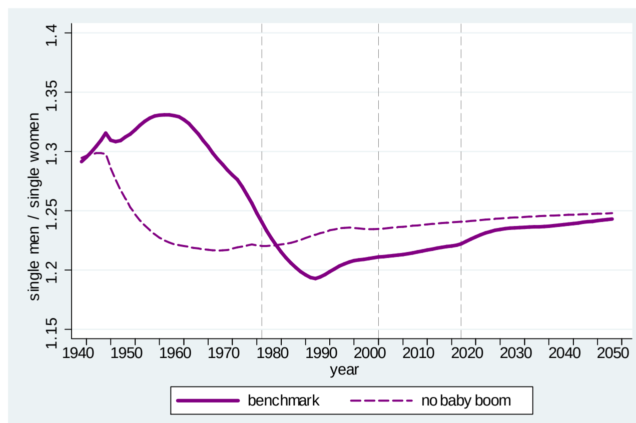
(a) Total Fertility Rate



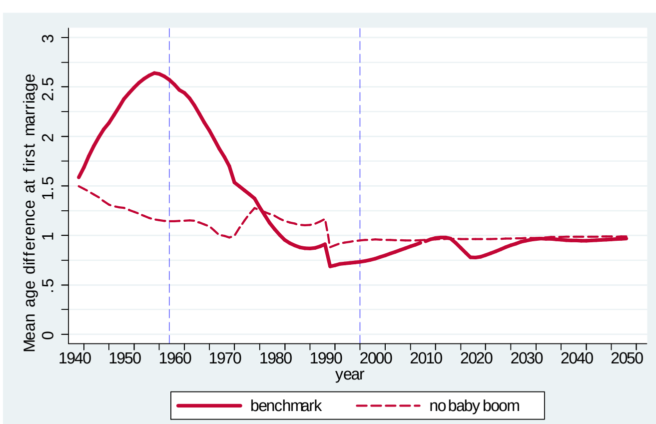
(b) Ratio 15-29 /15-44 year-olds



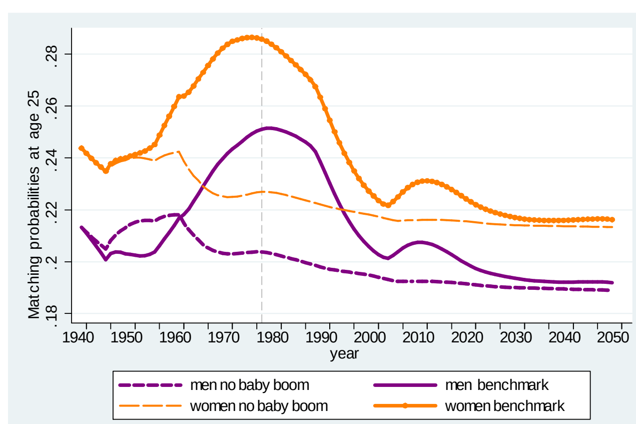
(c) Median age at first marriage



(c) Single men / Single women (ages 15-44)

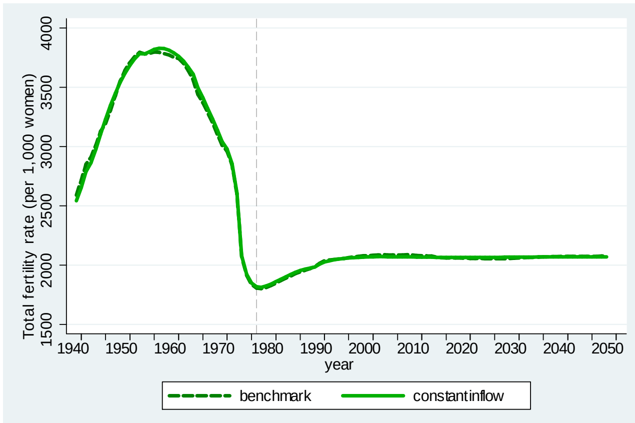


(e) Mean age difference at first marriage

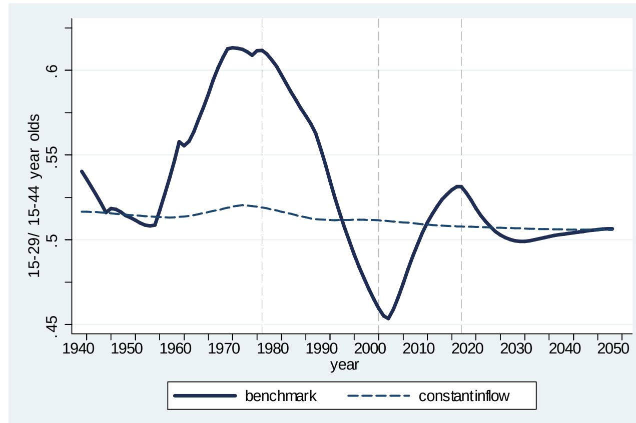


(f) Matching probabilities at age 25

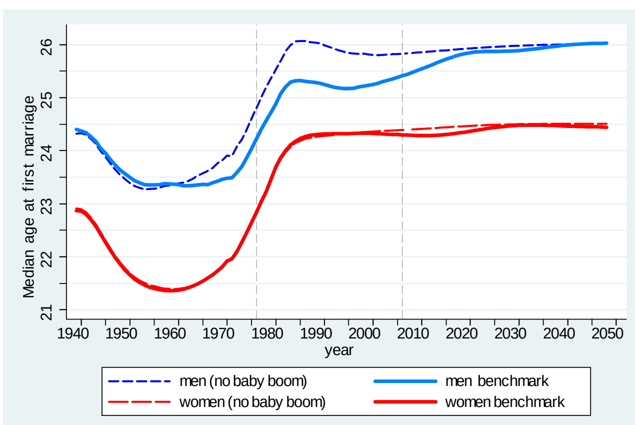
Figure 5: Counterfactual Economy II: Constant population inflow.



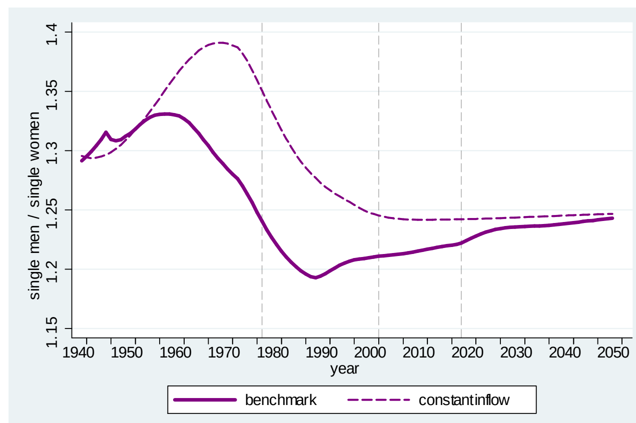
(a) Total Fertility Rate



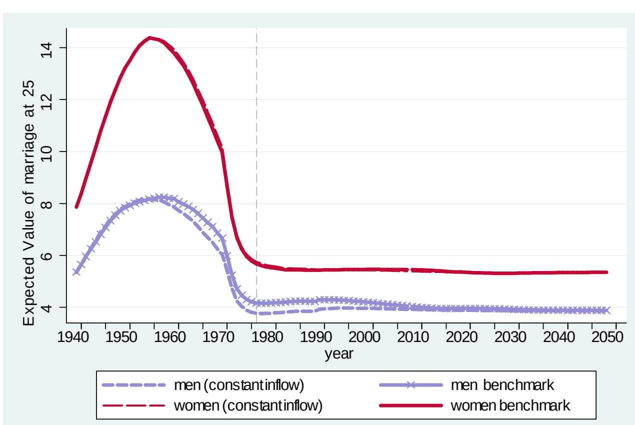
(b) Ratio 15-29 /15-44 year-olds



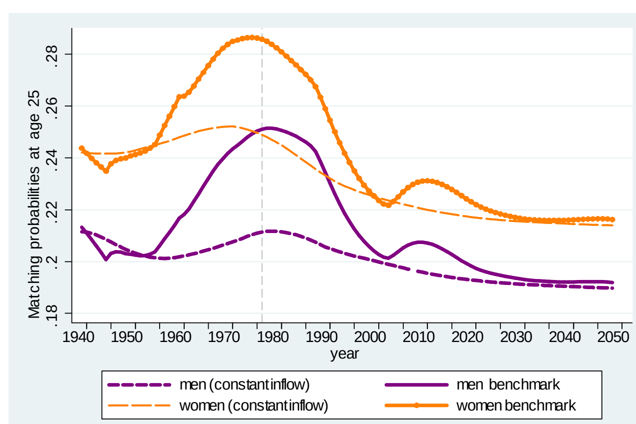
(c) Median age at first marriage



(d) Sex Ratio single men / single women



(e) Value of being single at age 25.



(f) Matching probabilities at age 25.