Optimal Fiscal Policy over the Business Cycle Revisited

Martin Gervais[1] Alessandro Mennuni[2]

[1]Southampton and IFS

[2]Southampton and EUI

Federal Reserve Bank of St Louis

January 2010

The Question

- How should fiscal policy be set over the business cycle?
- Can we rationalize a counter-cyclical fiscal policy in a stochastic neoclassical growth model?
- What are the statistical properties of fiscal policy instruments in the long run?

American Recovery and Reinvestment Act 2009

- Increase in spending (3.5% of GDP)
  - 1/4 of which for infrastructures (.85% of GDP)
- Tax cuts (2.0% of GDP)
  - Largely labor income taxes
- Financed by debt

Note: See Alesina and Ardagna (2009) for 90 other episodes

Prescription from Chari et al. ’94 (CCK)

In the period of a negative productivity shock

1. Government Spending: exogenous (in their model)
2. Labor income taxes: depends on preferences
3. Capital income taxes: *increase substantially*
4. Debt: *decreases*

Main Features of the Model

1. Business cycle driven by technology shocks
2. Government infrastructures are an input (not chosen) into production
3. Investment becomes productive within the period
4. The government optimizes fiscal policy (taxes, debt and new infrastructures)
5. Complete and incomplete markets (without state contingent debt)

Main Findings

In the period of a negative shock

1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. State contingent debt *decreases!*

Main Findings

In the period of a negative shock

1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. State contingent debt *decreases!*

Without state contingent debt

1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. Debt increases

Related Literature

- Complete markets
  - Chari, Christiano, and Kehoe, 1994
- Incomplete markets without capital income taxes
  - Aiyagari, Marcet, Sargent, and Seppälä, 2002
  - Scott, 2007
  - Marcet and Scott, 2009
- Incomplete markets with pre-announced capital income taxes
  - Farhi, 2009

Household Problem

- Preferences

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)U\Big(c(s^t),l(s^t)\Big)$$

- Budget Constraint

$$c(s^t)+k(s^t)+\sum_{s_{t+1}}q(s^t,s_{t+1})b(s^t,s_{t+1})$$
$$=w(s^t)l(s^t)+r(s^t)k(s^t)+k(s^{t-1})+b(s^t)$$

Microfoundation

Euler Equation

- Conventional timing

$$U_c(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1}) \left(1 + r(s^{t+1})\right)$$

- Our timing

$$U_c(s^t)\left(1 - r(s^t)\right) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1})$$

Technology and Feasibility

- Production

$$y(s^t) = A(s^t)k^g(s^t)^\gamma k(s^t)^\alpha l(s^t)^{1-\alpha}$$

where $\gamma + \alpha < 1$

- Feasibility

$$\begin{aligned} &c(s^t) + c^g(s^t) + k(s^t) + k^g(s^t) \\ &\quad = y(s^t) - \delta k(s^t) - \delta^g k^g(s^t) + k(s^{t-1}) + k^g(s^{t-1}) \end{aligned}$$

Fiscal Policy Instruments

- Proportional labor income tax: $\tau^w(s^t)$
- Proportional capital income tax: $\tau^k(s^t)$
- State-contingent debt: $b(s^t, s_{t+1})$
- Investment in infrastructures:
  $i^g(s^t) = k^g(s^t) - k^g(s^{t-1}) + \delta^g k^g(s^t)$

Note: Complete tax system with no redundant instruments

Prices and Taxes

- Before-tax prices

$$\hat{r}(s^t) = f_k(s^t) - \delta$$
$$\hat{w}(s^t) = f_l(s^t)$$

- After-tax prices

$$r(s^t) = \left[1 - \tau^k(s^t)\right] \hat{r}(s^t)$$
$$w(s^t) = \left[1 - \tau^w(s^t)\right] \hat{w}(s^t)$$

Ramsey Problem

- Objective function

$$\max_{c,l,k,g_{(t,s^t)}} \sum_{t,s^t} \beta^t \pi(s^t) U\left(c(s^t), l(s^t)\right)$$

- Subject to feasibility and

  the implementability constraint

$$\sum_{t,s^t} \beta^t \pi(s^t) \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)\right] = A_0$$

  where $A_0 = U_c(s_0)(k_{-1} + b_{-1})$

Note: GBC holds

Optimal Policy: Government Infrastructures

Optimality requires that for any s^t

$$f_k(s^t) - \delta = f_{k^g}(s^t) - \delta^g$$

- If $\delta^g = \delta$, then k^g/k is constant
- If $\delta^g < \delta$, then k^g/k is counter-cyclical

Optimal Policy: Labor Income Tax Rate

The tax rate on labor income is given by

$$\tau^w(s^t) = \frac{\lambda\left(H_l(s^t) - H_c(s^t)\right)}{1 + \lambda + \lambda H_l(s^t)}$$

where

$$H_c(s^t) = \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}$$

$$H_l(s^t) = \frac{U_{l,l}(s^t)l(s^t) + U_{l,c}(s^t)c(s^t)}{U_l(s^t)}$$

and λ is the multiplier on the implementability constraint

Optimal Policy: Labor Income Tax Rate

- If $U(c,l) = u(c) + v(l)$ and both u and v are CES (in consumption and *labor*)

  then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$

Optimal Policy: Labor Income Tax Rate

- If $U(c,l) = u(c) + v(l)$ and both u and v are CES (in consumption and *labor*)

  then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$

- If $U(c,l) = (1-\sigma)^{-1} c^{1-\sigma}(1-l)^{\nu(1-\sigma)}$ and $l(s^t) > l(\tilde{s}^t)$

  then $\tau^w(s^t) > \tau^w(\tilde{s}^t)$ if and only if

$$1 + \lambda(1-\sigma)(1+\nu) > 0$$

Optimal Policy: Capital Income Tax Rate

The tax rate on capital income determined by

$$\frac{1-r(s^t)}{1-\hat{r}(s^t)} = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\left(1+\lambda+\lambda H_c(s^t)\right)U_c(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\left(1+\lambda+\lambda H_c(s^{t+1})\right)U_c(s^{t+1})}$$

Recall that

$$H_c(s^t) = \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}$$

Optimal Policy: Capital Income Tax Rate

- If $U(c, l) = u(c) + v(l)$ and u is CES

  then the capital income tax rate is zero from date 1 on

Optimal Policy: Capital Income Tax Rate

- If $U(c,l) = u(c) + v(l)$ and u is CES

  then the capital income tax rate is zero from date 1 on

- If $U(c,l) = (1-\sigma)^{-1} c^{1-\sigma}(1-l)^{\nu(1-\sigma)}$ ($\sigma > 1$, $\nu > 0$) and labor is pro-cyclical

  then the capital income tax rate is *'likely'* to be pro-cyclical

$$H_c(s^t) = -\sigma - \nu(1-\sigma)\frac{l(s^t)}{1-l(s^t)} \text{ is increasing in } l$$

Numerical Simulation

- Preferences:

$$U(c,l) = \frac{c^{1-\sigma}\eta(1-l)^{\nu(1-\sigma)}}{1-\sigma}$$

- Technology:

$$y = Ak^{g\gamma}k^{\alpha}l^{1-\alpha}$$

- Some parameters:
  - $\gamma = .03$
  - $\delta^g = 0.065 < 0.1 = \delta$
  - λ such that $b/y = .4$

Simulation Results

In the period of a 2% negative shock:

- Government spending: 0.26 % of GDP
- Labor income tax revenue: -0.40 % of GDP
- Capital income tax revenue: -0.04 % of GDP

Simulation Results

In the period of a 2% negative shock:

- Government spending: 0.26 % of GDP
- Labor income tax revenue: -0.40 % of GDP
- Capital income tax revenue: -0.04 % of GDP
- Government debt: *-4.32* % of GDP

$\Rightarrow$ New debt issued goes down in bad times!

Ruling out State Contingent Debt

- Budget constraint with state contingent debt

$$c(s^t) + k(s^t) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})$$
$$= w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s^t)$$

- Budget constraint without state contingent debt

$$c(s^t) + k(s^t) + q(s^t) b(s^t)$$
$$= w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s^{t-1})$$

Ruling out State Contingent Debt

- State by state implementability constraint

$$c(s^t) + (k(s^t) + b(s^t))\beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)}$$

$$= -\frac{U_l(s^t)}{U_c(s^t)} l(s^t) + k(s^{t-1}) + b(s^{t-1})$$

$$w(s^t) = -\frac{U_l(s^t)}{U_c(s^t)}$$

$$1 - r(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)}$$

A Lagrangian – Marcet and Marimon

$$L = \min_{\lambda_{(t,s^t)}} \max_{c,l,k,g,b_{(t,s^t)}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \Big\{ U\big(c(s^t), l(s^t)\big) + \lambda(s^t) \left(c(s^t) + \frac{U_l(s^t)}{U_c(s^t)} l(s^t) - k(s^{t-1}) - b(s^{t-1}) \right) U_c(s^t) + \lambda(s^{t-1}) \big(k(s^{t-1}) + b(s^{t-1})\big) U_c(s^t) \Big\}$$

subject to

$$c(s^t) + c^g(s^t) + k(s^t) + k^g(s^t) = y(s^t) - \delta k(s^t) - \delta^g k^g(s^t) + k(s^{t-1}) + k^g(s^{t-1})$$

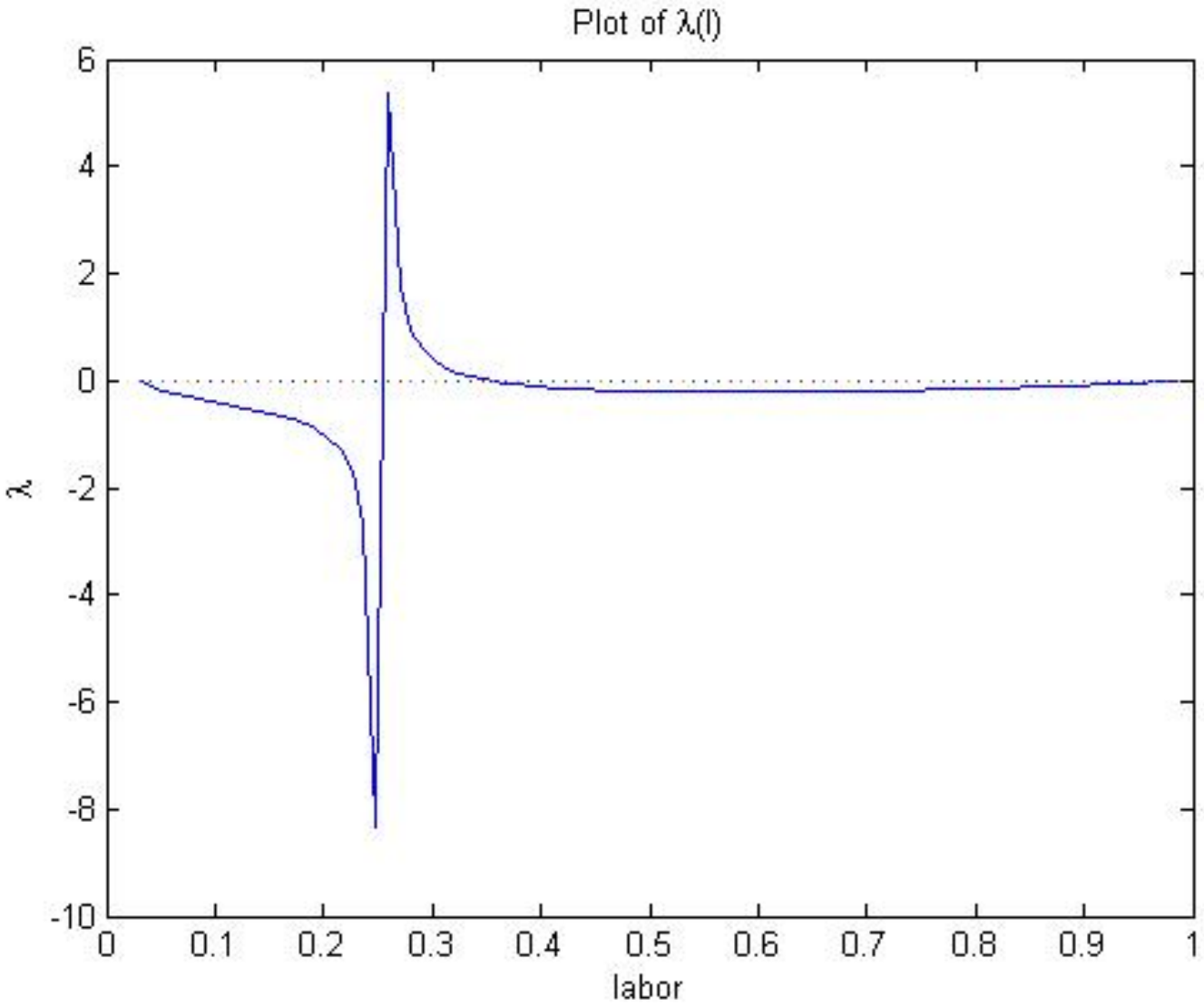
Evolution of the Multiplier λ

- Non-negative adjusted Martingale

$$\lambda(s^t) = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1}) \lambda(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1})}$$

- Under a quasi-linear per-period utility (with 'natural debt limits'): $\lambda \to 0$

Tradeoff in a Simple Economy



Some (embarrassingly few) Results

- If $\delta^g < \delta$, then k^g/k is counter-cyclical
- If $U(c,l) = c + v(l)$ then the capital income tax rate is zero
- If $U(c,l) = c + v(l)$ then the Ramsey allocation converges to a first best allocation

Numerical Simulation

- Preferences:

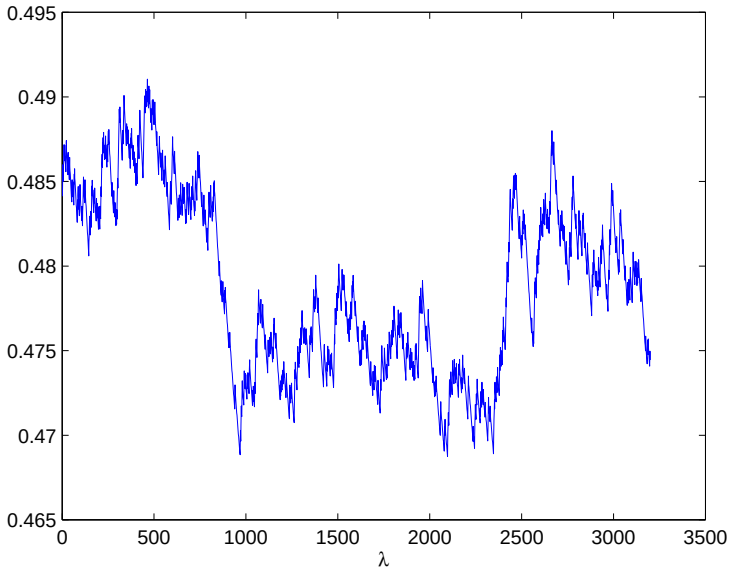
$$U(c, l) = \ln c + \eta \ln(1 - l)$$

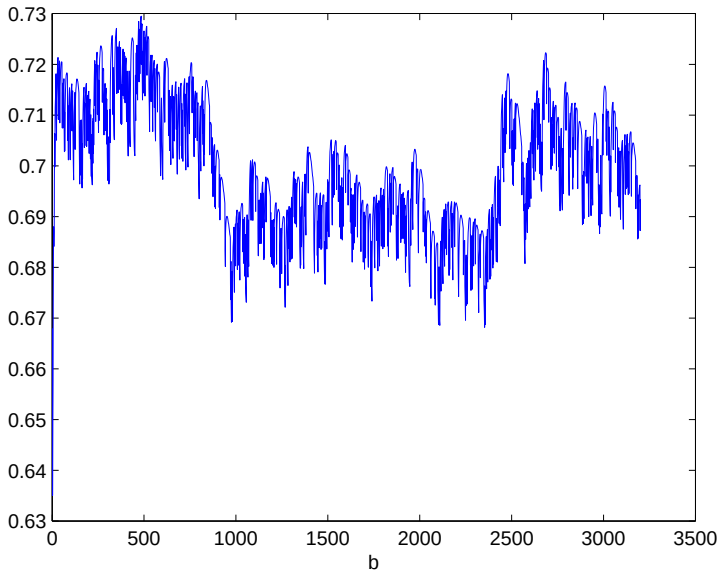
- Technology:

$$y = Ak^{g^{\gamma}} k^{\alpha} l^{1-\alpha}$$

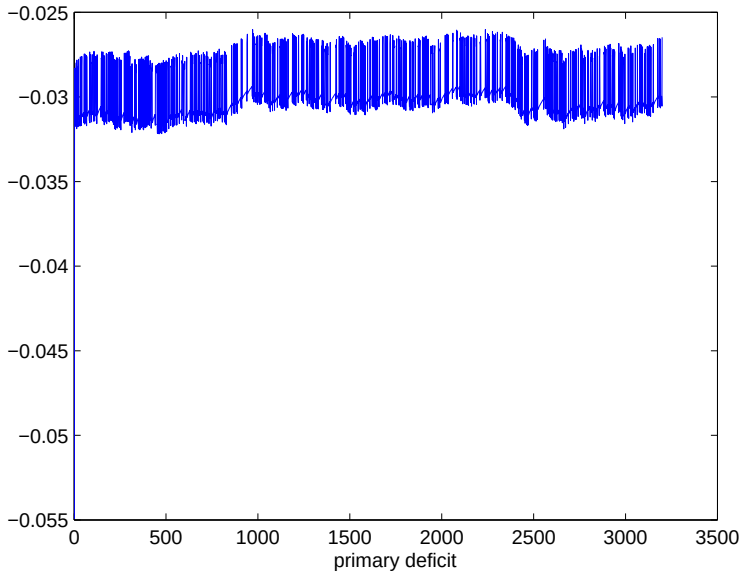
- Some parameters:
  - $\gamma = 0$ (Exogenous government spending)
  - $A \sim$ 2-state Markov matrix

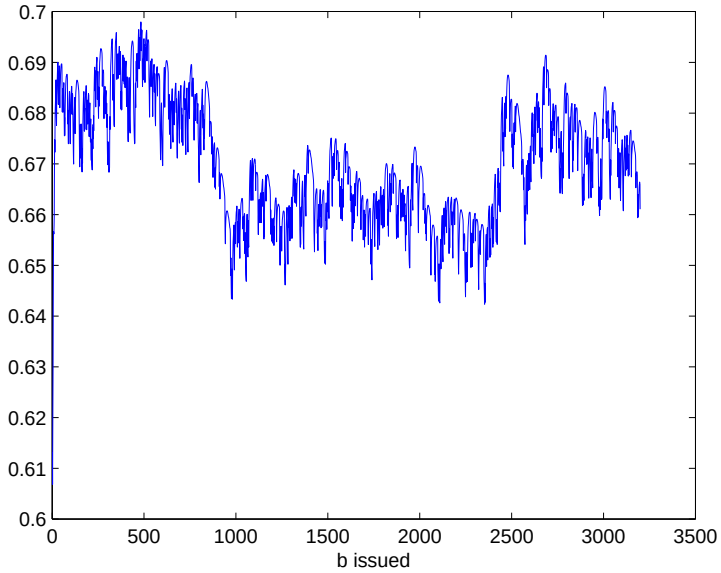
Simulation: λ and Debt



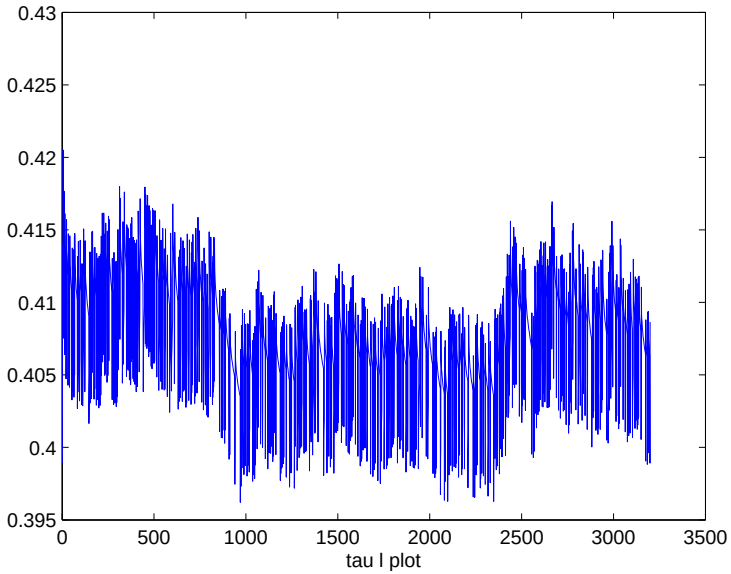


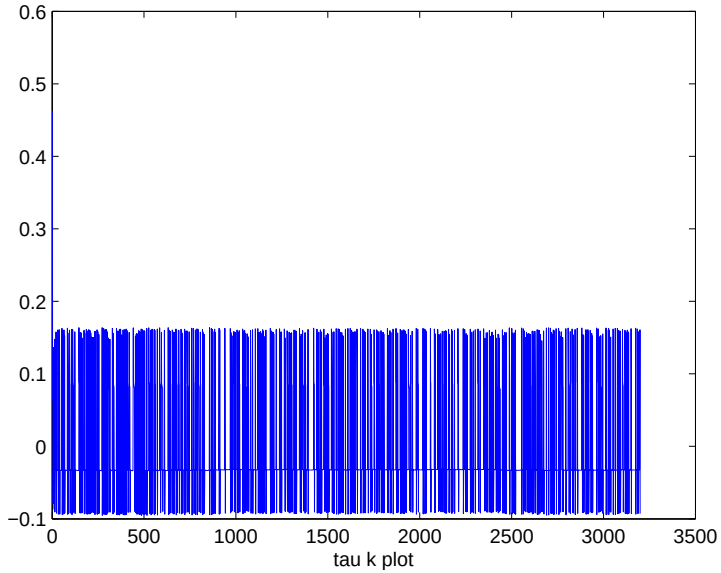
Simulation: Primary Deficit and Debt Issued





Simulation: τ^w and τ^k





Quantitative Results with IM ($\rho = .5$)

In the period of a 2% negative shock:

- Government spending: 0.0% of GDP
- Labor income tax revenue: -1.02% of GDP
  - change in labor tax rate -.004
- Capital income tax revenue: -0.21% of GDP
  - change in capital tax rate -1.26
- *Increase* in government debt: 1.17% of GDP

Quantitative Results with CM ($\rho = .5$)

In the period of a 2% negative shock:

- Government spending: 0.0% of GDP
- Labor income tax revenue: -0.69% of GDP
  - change in labor tax rate -.001
- Capital income tax revenue: 0.0% of GDP
  - change in capital tax rate 0.0
- *Decrease* in government debt: -0.86% of GDP

Conclusion

- Business cycle model with elastic capital-supply and endogenous government infrastructures
- Study complete and incomplete markets
- Provide a rationale for some kind of stimulus package within the neoclassical framework
- Plausible (and implementable) policy implications
- Found an ‘optimal’ amount of debt/gdp in the long run

Microfoundation

- Period t composed of n sub-periods
- First sub-period budget constraint

$$c(s^t,1)+k(s^t,1)+\sum_{s_{t+1}} q(s^t,s_{t+1})b(s^t,s_{t+1})$$
$$= w(s^t,1)l(s^t,1)+\big(1+r(s^t,1)\big)k(s^{t-1})+b(s^t)$$

- Sub-periods 2 to n

$$c(s^t,i)+k(s^t,i)$$
$$= w(s^t,i)l(s^t,i)+\big(1+r(s^t,i)\big)k(s^t,i-1),\ i=2,...,n$$

Microfoundation

- Summing up the sub-periods budget constraints

$$\sum_{i=1}^{n} c(s^t, i) + k(s^t, n) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})$$

$$= \sum_{i=1}^{n} \left[w(s^t, i) l(s^t, i) + r(s^t, i) k(s^t, i-1) \right] + k(s^{t-1}) + b(s^t)$$

- Conventional assumption

$$\sum_{i=1}^{n} \left[(r(s^t, i)) k(s^t, i-1) \right] = r(s^t) k(s^{t-1})$$

- Opposite extreme

$$\sum_{i=1}^{n} \left[(r(s^t, i)) k(s^t, i-1) \right] = r(s^t) k(s^t)$$