Modeling Inflation Expectations

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Inflation Expectations as Observables

- This paper uses inflation expectations as an observable in the estimation of a DSGE model, along with a standard set of macro variables.
    - 4-quarter ahead SPF GDP deflator forecasts (Erceg and Levin, JME '03).
    - 4-quarter ahead SPF CPI forecasts.

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We ask the following questions:

1. Can a 'standard' CEE/SW-type model, which can supposedly explain macro variables reasonably well, also describe the evolution of inflation expectations?
2. If we augment the model with a plausible feature, namely imperfect info about the inflation target on the part of the agents, do we achieve a better fit on both counts – macro variables and expectations?

We find that the answer to both question is: No

What For?

1. Model comparison: Inflation expectations help discriminate across models.

    - The motivation for learning/imperfect information models partly lies in modeling agents' expectation in a more realistic way than RE/perfect information models.

    - Expectations can discriminate among these classes of models (not used in previous literature – see Milani).

    - How to model a time-varying inflation target.

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  - How to model a time-varying inflation target.

- A stated rationale for imperfect information/learning models is that they offer a more plausible mechanism for expectation formation – one where the agents are not omniscent about all aspects of the economy but need to learn about some of its features. given these premises, it seems reasonable to assume that these models should be better at describing the evolution of expectation – hence expectations are important discriminating evidence.
- In particular, this paper is motivated by the literature about a TV inflation targeting on the part of the central bank. There are models w/ perfect info (SW '03), and others with imperfect info (Erceg and Levin). Finding out which of the two better describes infl exp seems a good way to fnd out which is more realistic.

What For?

2. Term structure modeling: There are attempts to use DSGE models to explain the terms structure (Swanson and Rudebusch '08).

   - This paper makes no attempt to explain the term structure directly: we use a linear model.

   - But the model comparison exercise done here can be helpful indirectly: Models that have a hard time generating observed inflation expecations may not be too helpful in understanding the term structure of interest rates.

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Why do we care about infl. expectations? Because they are allegedly important in determining the term structure of interest rates.

How should you model expectation formation in these models?

If a model does poorly in describing infl. exp., is likely to do poorly for the term structure as well.

What For?

3. Signal extraction: Agents in the economy have more information than the econometrician.
   - This information can be exploited for forecasting . . .
   - . . . and shocks identification.
   - There are some attempts to combine factor and DSGE models with the goal of incorporating as much of the available data as possible (Giannone, Monti and Reichlin '08, Biovin and Giannoni '08).
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The econometrician has a relatively poor signal about the state of the economy. Agents have a richer information set. Measuring expectations is a way to exploit such information set.

Moreover, using expectations as observables can lead to different 'histories' for what has happened: What may appear to be a shock from the perspective of the econometrician may not be at all an innovation from the perspective of the agents.

Issues with Modeling Inflation Expectations

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- Data revisions
- Information synchronization
- Heterogeneous expectations

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We use SPF forecasts as a measure of inflation expectations 4-quarters ahead – this is the measure most commonly used in the literature. We plan to use Blue Chip forecasts as well in future drafts.

Several issues arise in modeling inflation expectation. We try to address the issue of data revisions by showing the robustness of the results when we CPI as a measure of inflation (as opposed to the more commonly used GDP deflator): CPI non-seasonally adjusted is never revised; CPI seasonally adjusted has revisions, but these are fairly small compared to those for the GDP deflator.

Also, SPF forecasts are generated in the middle of the quarter. SPF forecasters therefore have partial information about the state of the economy in the current quarter. We deal with this issue by checking the robustness of the results to different assumptions regarding the timing of the agents' information set.

DSGE Model

- Model is a variant of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003,2007).

- Stochastic growth model $+ \ldots$

real rigidites	**nominal rigidites**
investment adjustment costs	price stickiness (ζ_p)
variable capital utilization	wage stickiness (ζ_w)
	partial indexation to lagged inflation
$+$ habit persistence	

- 5 shocks: Neutral technology (unit root), investment specific technology, leisure, mark-up, government spending + 2 policy shocks.

DSGE Model

The model is the same as in Del Negro and Schorfheide JME '08 – equilibrium conditions can be found there.

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Monetary Policy and Agents' Information Set

- Policy rule:

$$R_t = \rho_r R_{t-1} + (1-\rho_r)(\psi_1(\pi_t - \pi_t^*) + \psi_2 \widehat{y}_t) - \sigma_r \epsilon_{R,t},$$

where $\widehat{y}_t$ are deviations of output from the stochastic st.st., $\epsilon_{R,t}$ is i.i.d. and:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_P \epsilon_{P,t}.$$

- Perfect Information (PI): Agents observe π_t^*.

- Imperfect Information (II) (Erceg and Levin, JME '03): Agents only observe the Taylor rule residual
$R_t - \rho_r R_{t-1} - (1-\rho_r)(\psi_1(\pi_t - \pi_t^*) + \psi_2 \widehat{y}_t) = (1-\rho_r)\psi_1 \tilde{\pi}_t$ where:

$$\tilde{\pi}_t = \pi_t^* + \frac{\sigma_r}{(1-\rho_r)\psi_1} \epsilon_{R,t}.$$

Signal Extraction under Imperfect Information

- Measurement equation:

$$\tilde{\pi}_t = \pi_t^* + \sigma_T \epsilon_{R,t},$$

where $\sigma_T = \dfrac{\sigma_r}{(1-\rho_r)\psi_1}$.

- Transition equation:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_P \epsilon_{P,t}.$$

Signal Extraction under Imperfect Information – continued

- Solution:

$$\pi^*_{t+1|t} = \rho_{\pi^*}\pi^*_{t|t-1} + \rho_{\pi^*}K\left(\tilde{\pi}_t - \pi^*_{t|t-1}\right).$$

- $\rho_{\pi^*}K = \rho_{\pi^*}\frac{\sigma_T^2 V(\sigma_P/\sigma_T,\rho_*)}{\sigma_T^2+\sigma_T^2 V(\sigma_P/\sigma_R,\rho_{\pi^*})}$ is the Kalman gain, a positive function of the signal-to-noise ratio $\frac{\sigma_P}{\sigma_T}$ and ρ_{π^*}.

- . . . where $\sigma_T^2 V(\sigma_P/\sigma_T, \rho_*)$ defines the uncertainty regarding the state π^*_t.

- From the agents' perspective there is only one policy innovation – $\tilde{\epsilon}_t = \tilde{\pi}_t - \pi^*_{t|t-1}$ – whose variance is given by $\sigma_T^2 + \sigma_T^2 V(\sigma_P/\sigma_R, \rho_{\pi^*})$.

Measurement

- Output growth (log differences, quarter-to-quarter, in %)
- Hours worked (log, in %)
- Labor Share (log, in %)
- Inflation (annualized, in %) – GDP deflator/CPI
- Nominal interest rate (annualized, in %)
- SPF Inflation expectations (annualized, in %) – GDP deflator/CPI

- 97 quarters of data spanning the Volcker-Greenspan period: 1984Q2 to 2008Q2.

SPF Inflation Expectations

- Baseline estimation uses 4 quarters-ahead GDP deflator median SPF forecasts (following Erceg and Levin, JME '03).

- The survey is sent out at the end of the first month of each quarter (after advance GDP report) and response deadlines are in the middle month of each quarter.

- Two issues:

  1. Data revisions: Robustness check also use CPI forecasts.

  2. Timing of information: Forecasters have 'some' information about the current quarter. We consider the two extremes: i) information only on past quarter; ii) information on current quarter.

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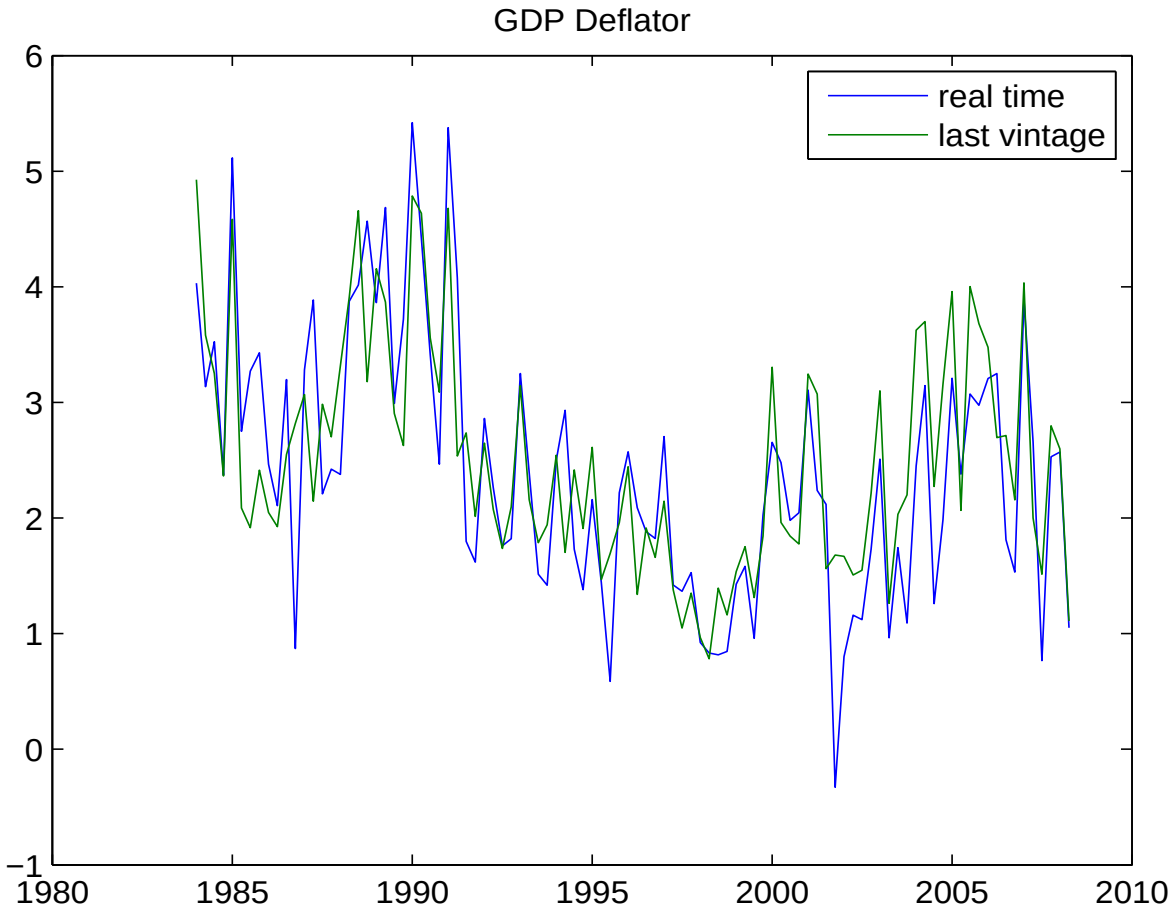
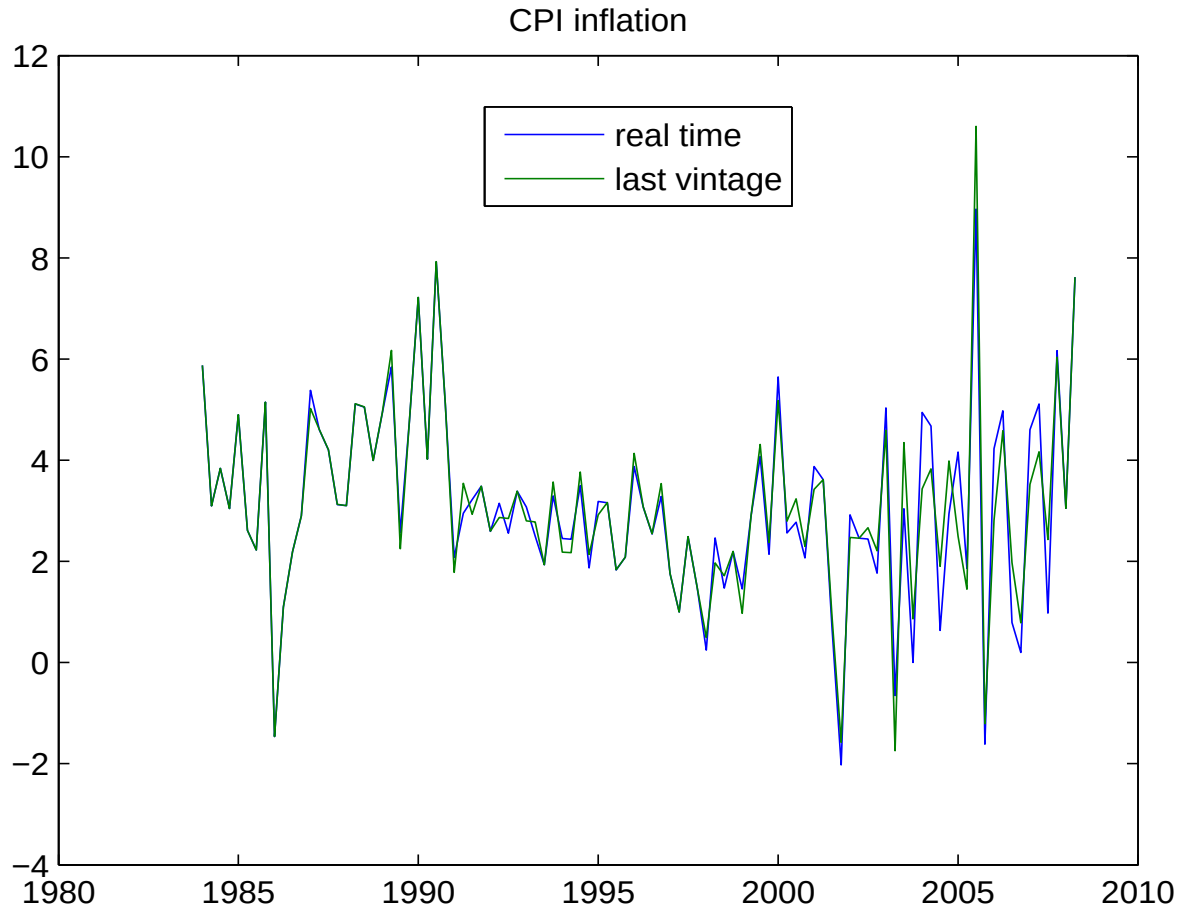
In the baseline estimation we assume that forecasters have current quarter information.

GDP Deflator: Real Time vs Last Vintage



CPI: Real Time vs Last Vintage



Priors on Policy Parameters

Parameter	Support	Density	Mean	Std	5%	95%
ψ_1	$\mathbb{R}^+$	Gamma	2.00	0.25	1.592	2.409
ψ_2	$\mathbb{R}^+$	Gamma	0.20	0.10	0.047	0.347
ρ_r	[0,1)	Beta	0.50	0.200	0.170	0.825
π^*	$\mathbb{R}$	Normal	4.30	2.50	0.165	8.398
σ_r	$\mathbb{R}^+$	InvGamma	0.150	4.00	0.080	0.298
ρ_{π^*}	[0,1)	Beta	0.950	0.025	0.913	0.989
Baseline Prior						
σ_{π^*}	$\mathbb{R}^+$	InvGamma	0.050	8.000	0.032	0.078
Signal-to-Noise Ratio Prior						
$\sigma_{NR} = \frac{\sigma_P}{\sigma_T}$	$\mathbb{R}^+$	Gamma 0.180	0.150	0.001	0.380	

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- Priors for the responses to inflation (ψ_1) and the output gap (ψ_2) in the policy rule, persistence (ρ_r), and steady state inflation target (π^*) are as in Del Negro and Schorfheide '08.
- Prior on variance of i.i.d. policy shocks σ_r is a bit lower than usual because we have additional source of policy shocks, π_t^*. (Prior standard deviations are chosen so that overall variance of endogenous variables is roughly close to that observed in the presample 1959Q3-1984Q1)
- Key priors are those on persistence and standard deviation of the innovation to π_t^* process (they determine the Kalman gain in the II model). We follow Erceg and Levin and make the process followed by π_t^* very persistent (ρ_{π^*} prior centered at .95 with small standard deviation). In the Bechmark prior the prior on σ_{π^*} is independent from all other parameters, and is centered at .05, and is fairly loose.
- An alternative prior ("Signal-to-Noise Ratio Prior") places a prior directly on the Signal-to-Noise ratio (and hence induces dependence between σ_{π^*}, σ_r, ψ_1 and ρ_r) and is centered at the value that delivers a Kalman gain of approximately .13, the value calibrated by Erceg and Levin.

Priors on Nominal Rigidities Parameters

Parameter	Support	Density	Mean	Std	5%	95%
Low Rigidities (Baseline)						
ζ_p	[0,1)	Beta	0.450	0.100	0.285	0.614
ζ_w	[0,1)	Beta	0.450	0.100	0.285	0.614
High Rigidities						
ζ_p	[0,1)	Beta	0.750	0.100	0.590	0.913
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- To check robustness to the degree of nominal rigidities in the economy we consider two priors (as in Del Negro and Schorfheide '08): "Low Rigidities" (loosely calibrated at Bils and Klenow values of average duration less than 2 quarters), and "High Rigidities" (duration about 4 quarters)

Priors on "Endogenous Propagation and Steady State" Parameters

Parameter	Support	Density	Mean	Std	5%	95%
α	[0,1)	Beta	0.330	0.020	0.297	0.362
s'	$\mathbb{R}^+$	Gamma	4	1.500	1.558	6.250
h	[0,1)	Beta	0.700	0.050	0.619	0.782
a''	$\mathbb{R}^+$	Gamma	0.200	0.100	0.049	0.349
ν_l	$\mathbb{R}^+$	Gamma	2	0.75	0.779	3.121
r^*	$\mathbb{R}^+$	Gamma	1.5	1	0.106	2.883
γ	$\mathbb{R}^+$	Gamma	1.650	1	0.204	3.073
g^*	$\mathbb{R}^+$	Gamma	0.300	0.100	0.143	0.459
ι_p	[0,1)	Beta	0.5	0.280	0.089	0.969
ι_w	[0,1)	Beta	0.5	0.280	0.051	0.932

$\delta = 0.025,\ \lambda_f = 0.150,\ \lambda_w = 0.300$

Priors on ρs and σs

Parameter	Support	Density	Mean	Std	5%	95%
ρ_z	[0,1)	Beta	0.400	0.250	0.000	0.764
ρ_ϕ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_{λ_f}	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_μ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_b	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_g	[0,1)	Beta	0.750	0.150	0.530	0.982
σ_z	$\mathbb{R}^+$	InvGamma	0.200	4.000	0.160	0.596
σ_ϕ	$\mathbb{R}^+$	InvGamma	2.500	4.000	1.616	5.941
σ_{λ_f}	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.105	0.395
σ_μ	$\mathbb{R}^+$	InvGamma	0.500	4.000	0.397	1.490
σ_g	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.261	0.984

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- Priors on "Endogenous Propagation and Steady State" chosen as in Del Negro and Schorfheide '08.
- Priors on standard deviations and autocorrelations are chosen so that overall variance and autocorrelations of endogenous variables is roughly close to that observed in the presample 1959Q3-1984Q1.

Prior Implications for Moments of the Endogenous Variables

Variables	St. Dev.			Autocorr.		
	II	PI	*Data*	II	PI	*Data*
OutputGrowth	3.33	3.46	*4.33*	0.39	0.36	*0.28*
LaborSupply	2.84	2.86	*3.20*	0.93	0.92	*0.96*
LaborShare	1.37	1.38	*2.24*	0.86	0.86	*0.95*
Inflation	3.42	3.68	*2.77*	0.73	0.75	*0.88*
InterestRate	4.55	5.97	*4.30*	0.86	0.66	*0.87*
Exp. Inflation	1.49	1.69		0.90	0.90	

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Exp. Inflation	1.49	1.69		0.90	0.90	

Note: II: imperfect information ; PI: perfect information. The pre-sample statistics (column *Data*) are in italics. These statistics are computed over the sample 1959Q3-1984Q1. Inflation expectations are not available during most of the pre-sample. The in-sample mean, standard deviation, and first-order autocorrelation of inflation expectations are 2.85, 1.21, and 0.86, respectively.

Statistics for endogenous variables fairly similar across models – if anything, PI model is slightly favored by choice of prior.

Model Comparison

	Imperfect Information	Perfect Information	Fixed π^*
Dataset w/o Expectations	-715.465	-728.498	-725.465
Dataset w Expectations	-816.232	-801.934	-816.914

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Note: The Table shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target ("Fixed π^*"). For all models we use the Baseline prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. We assume that the expectations are generated using current quarter information.

For the dataset without expectations the II model sizably outperforms the other two: difference in log marginal likelihood is larger than 10. This finding is consistent with Erceg and Levin, who claim that the model with imprefect information has more realistic properties in terms of the dynamics of inflation and output than the perfect information model.

When SPF inflation expectations are included among the observables, the Perfect Information model with time-varying π^* performs significantly better than both the "Fixed π^*" and, most importantly, the II model. In the remainder of the presentation we will try to provide an intuition for this result, and assess its robustness.

In-sample *a-priori* RMSEs

Variable	II Model w/o Exp	II Model w Exp	PI Model w/o Exp	PI Model w Exp
Output Growth	3.67	7.70	3.39	3.74
Labor Supply	1.06	2.22	0.99	1.09
Labor Share	0.82	0.89	0.79	0.80
Inflation	1.83	1.51	1.82	1.33
Interest Rate	2.21	2.70	2.57	2.74
Exp. Inflation		0.78		0.66

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Exp. Inflation		0.78		0.66

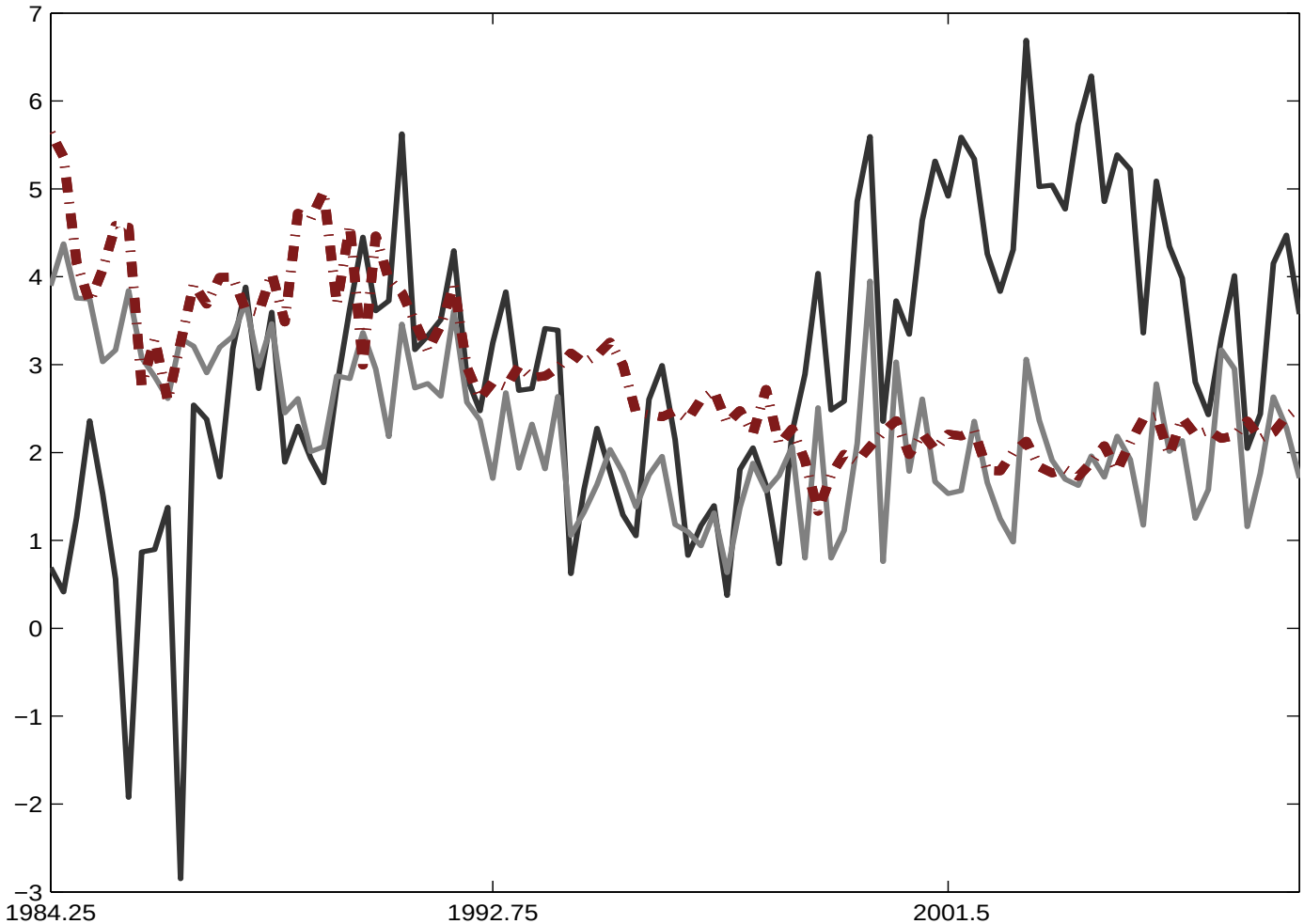
Note: The table shows the median in-sample forecast errors obtained from the Kalman filter. All models (II: imperfect information, and PI: perfect information) use the same prior (*Baseline* prior) for the DSGE parameters. The difference between the *w/o Exp* (without Inflation Expectations) and *w Exp* (with Inflation Expectations) columns is that for the latter inflation expectations are used as an additional observable in the measurement equation.

The marginal likelihood is the integral of the likelihood with the respect to the prior. Therefore a look at in-sample forecast errors (computed using the prior distribution for the parameters) may provide some insights for the model comparison results.

For the dataset without expectations the ranking of the forecast errors between the II and PI models is not all too clear: the II model does better in terms of interest rates forecasts, but worse for output growth. The other RMSEs are very close to each other. When inflation expectations are included, we observe that: 1) for both models the RMSEs increase, with the exceptions of that for inflation. The deterioration of in-sample fit is much larger for the II model, however; 2) the PI model now performs better than the II model in all dimensions, except for interest rates where the RMSEs are basically the same.

Inflation Expectations: Data vs Model Prediction

(i) Parameters drawn from prior distribution; (ii) Inflation Expectations not included among observables



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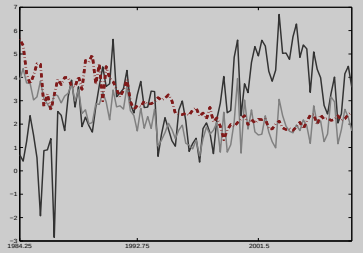

Note: The figure plots SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted), together with the projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid). The projections are computed using time $t-1$ information and are generated i) using the prior distribution for the parameters, and ii) without including inflation expectations among the observables.

The inflation forecasts generated by the II model are very much at odds with the data, especially in the early and late part of the sample. Those generated from the PI model are far from being fully consistent with the SPF forecasts, but are at least in the ballpark.

Why are forecasts generated by the II model so grossly inconsistent with the SPF forecasts?

Interest Rates: Data vs Model Prediction

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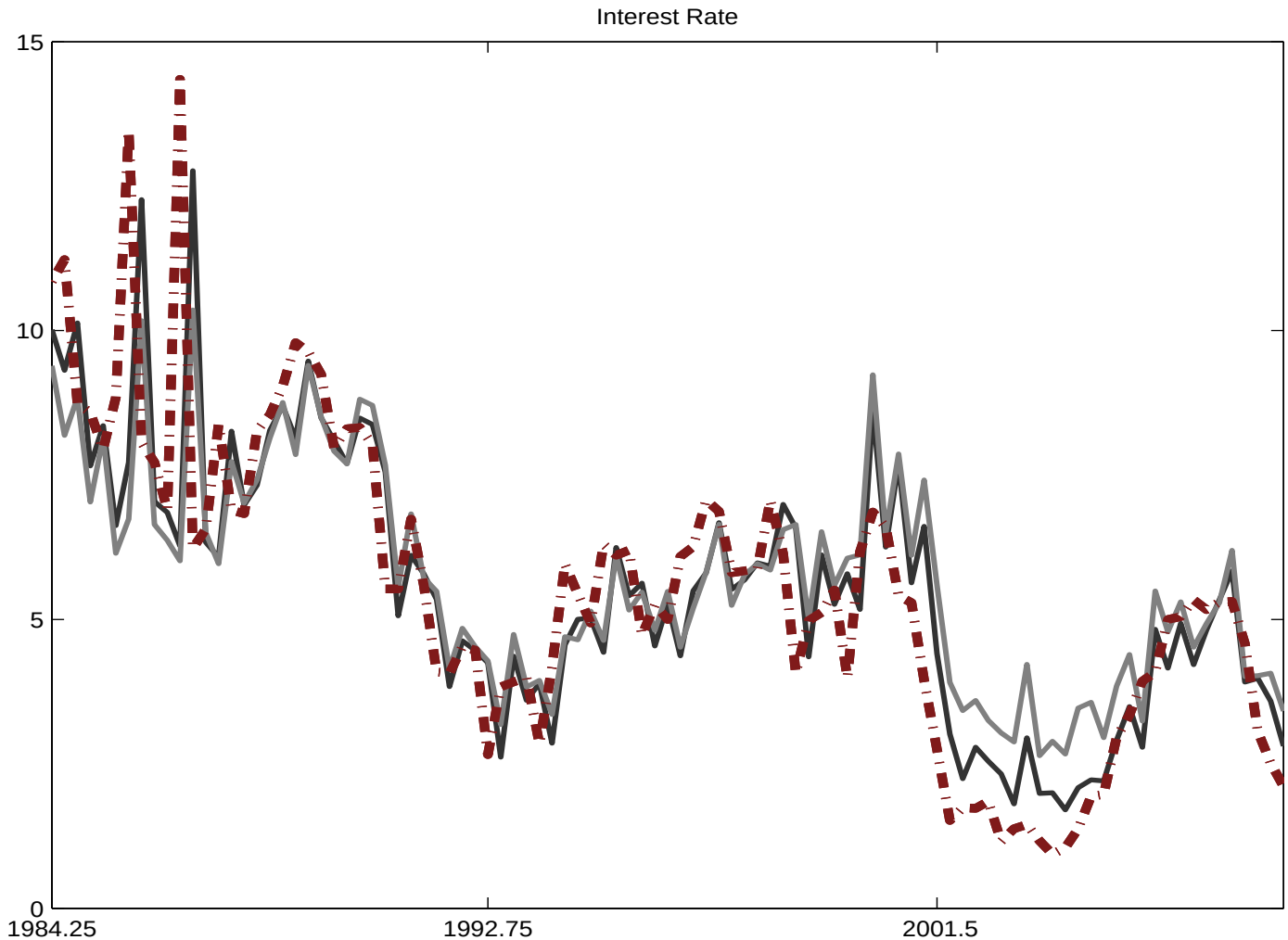
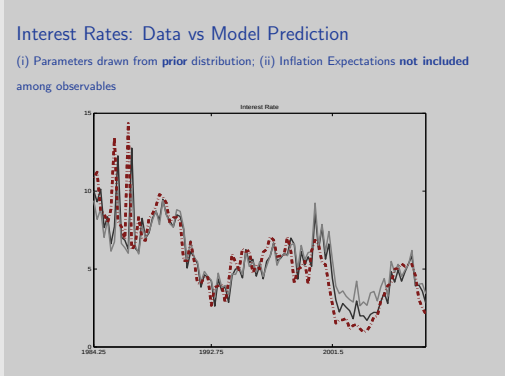


Interest Rates: Data vs Model Prediction (i) Parameters drawn from **prior** distribution; (ii) Inflation Expectations **not included** among observables

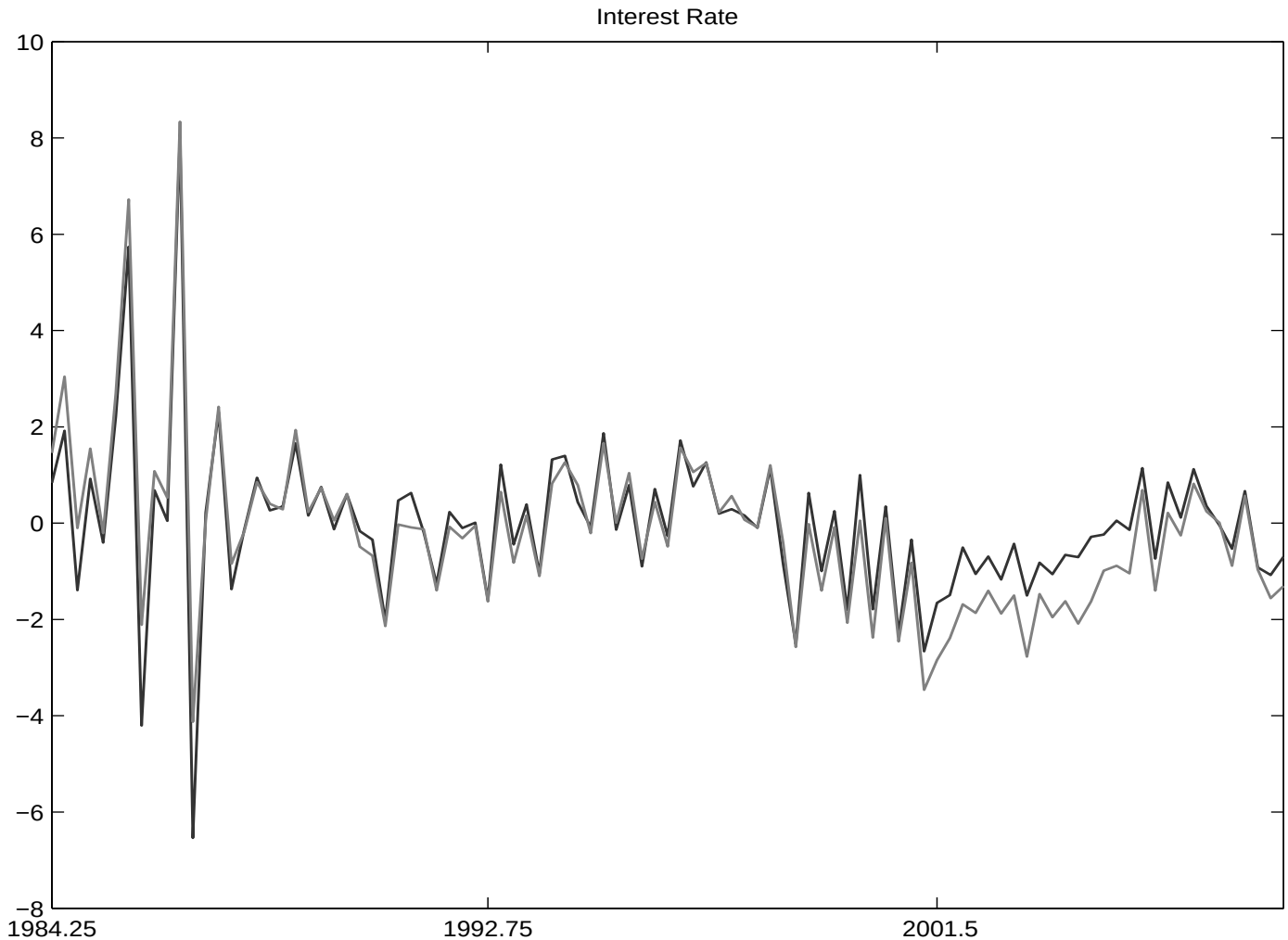


Note: The figure plots the interest rate (red dashed-and-dotted), together with the interest rate projections generated by the II model (black solid) and the PI model (gray solid). The projections are computed using time $t-1$ information and are generated i) using the prior distribution for the parameters, and ii) without including inflation expectations among the observables.

The interest rates projections from the II and PI models are not very different from each other. Both models produce forecast errors, and these appear to be persistent, especially in the early and late part of the sample. The forecast errors from the PI model appear to be larger (consistently with the RMSEs results). However, the effect of *persistent* errors on the inflation forecasts are quite different for the two models: for the II model, a persistent positive (negative) error in predicting the interest rate is an indication that the Central Bank has lowered (raised) its target, and has a dramatic impact on expectations. But in reality expectations do not change that much ("curse of anchored expectations" for the II model).

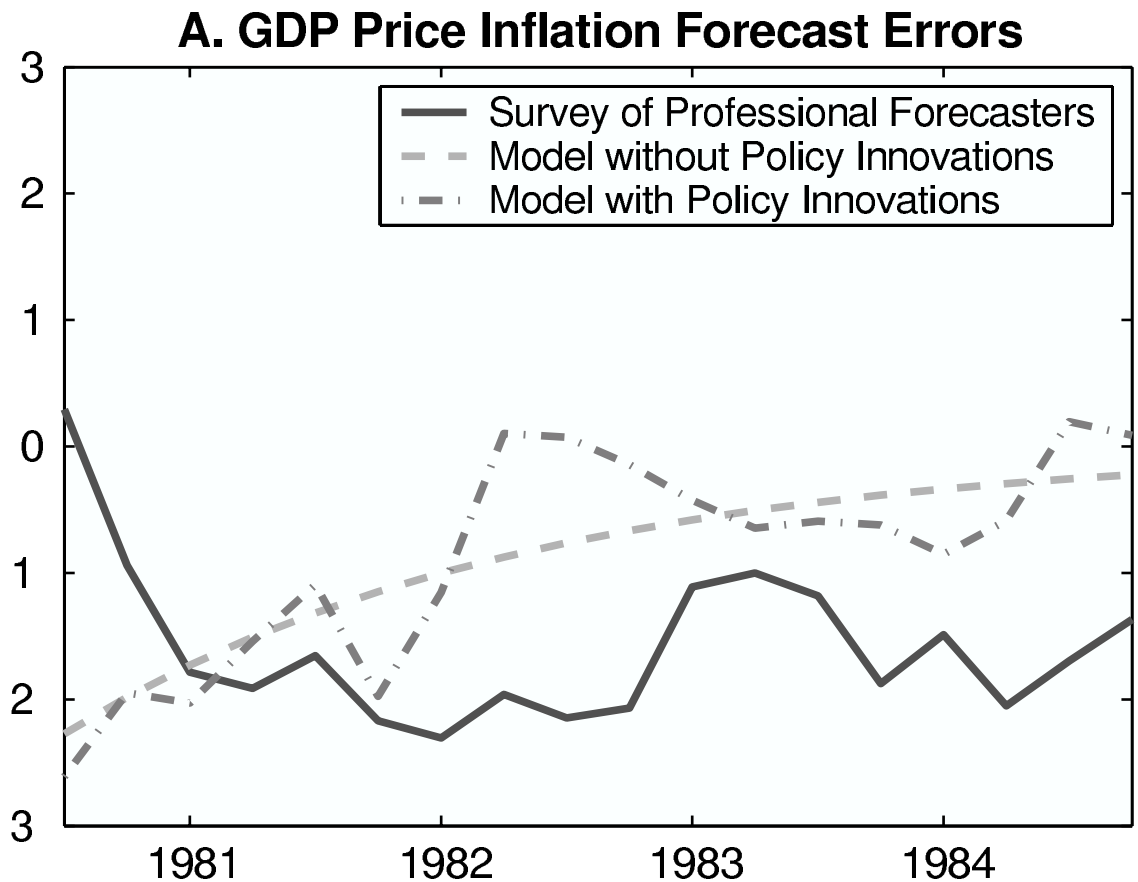
Interest Rates Prediction Error

(i) Parameters drawn from **prior** distribution; (ii) Inflation Expectations **not included** among observables



Inflation Expectations Errors from Erceg and Levin, JME '03

C.J. Erceg, A.T. Levin / Journal of Monetary Economics 50 (2003) 915–944



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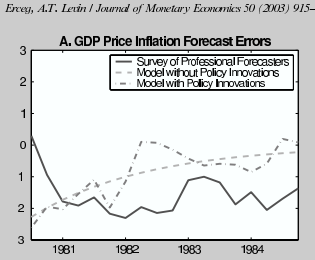
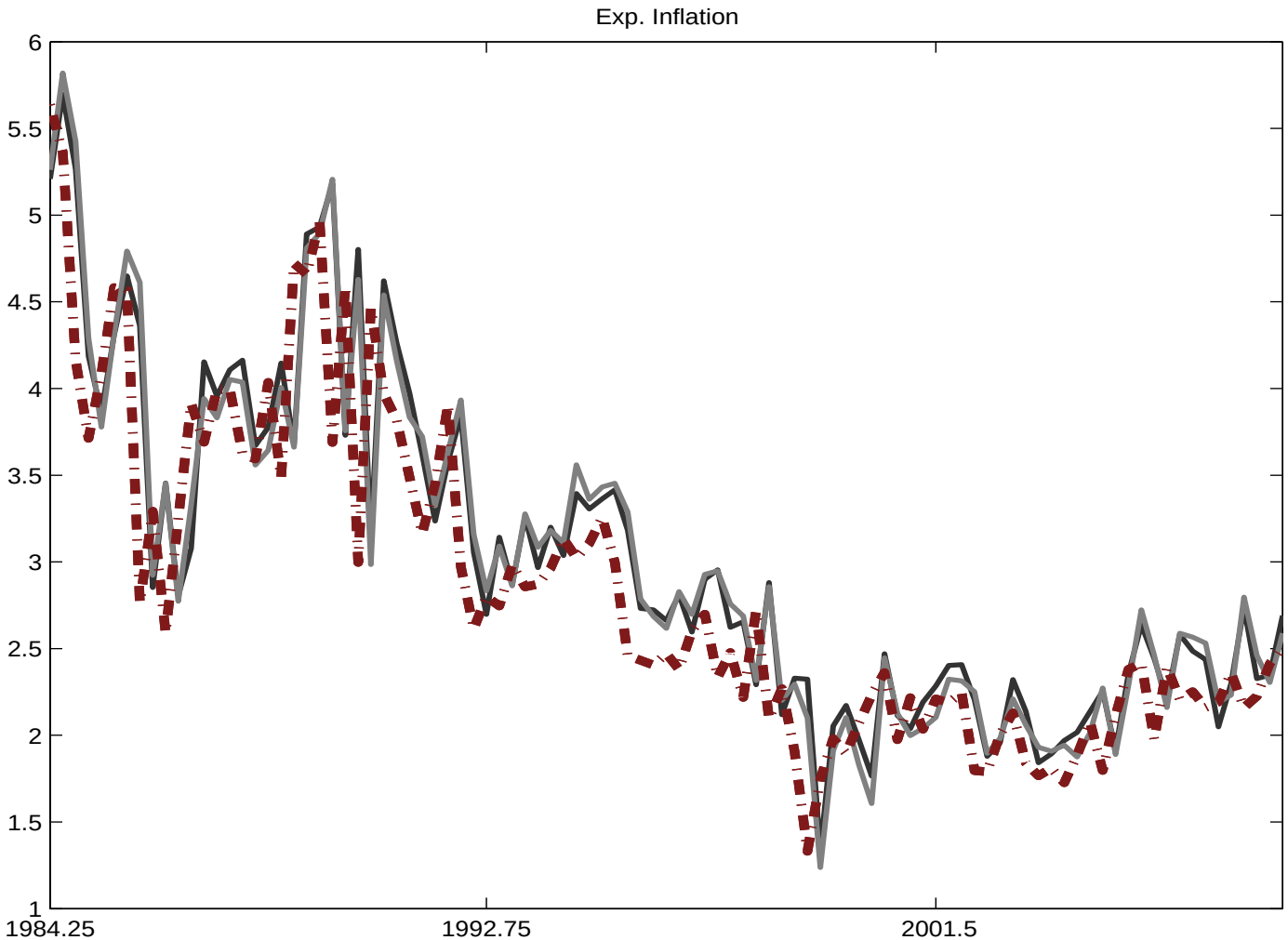


Note that the same problem – expectations from the model being quite different from SPF expectations – seems to be present in Erceg and Levin as well.

Inflation Expectations: Data vs Model Prediction

(i) Parameters drawn from **prior** distribution; (ii) Inflation Expectations **included** among observables



Inflation Expectations: Data vs Model Prediction (i) Parameters drawn from **prior** distribution; (ii) Inflation Expectations **included** among observables

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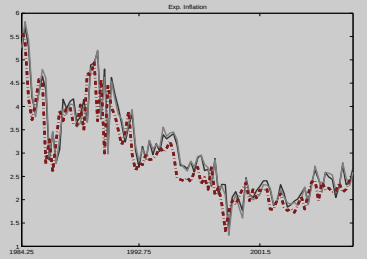
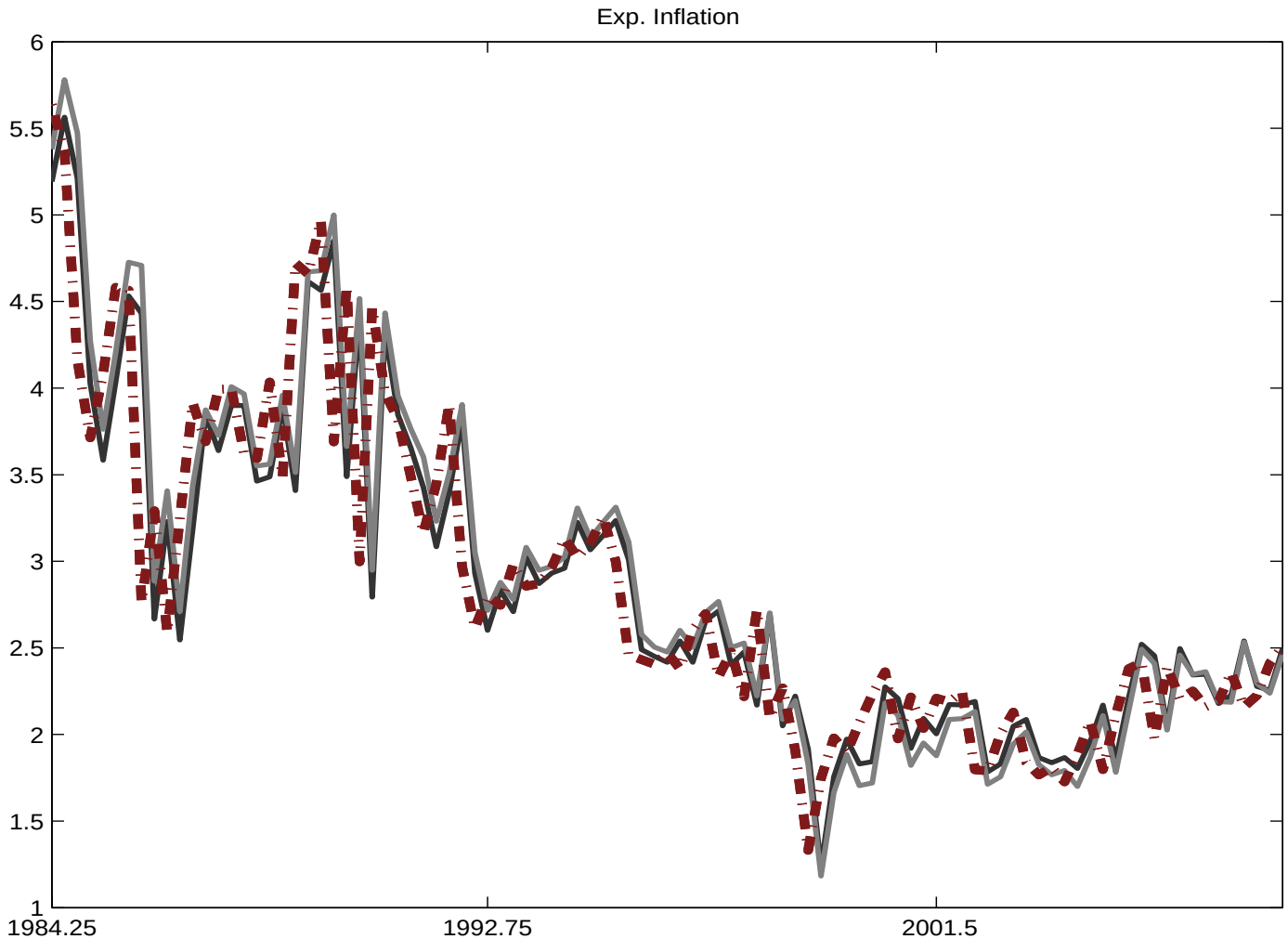
(i) Parameters drawn from **prior** distribution; (ii) Inflation Expectations **included** among observables



When SPF expectations are included among the observables, the II model's one-period ahead projection are much more in line with those from the data, even using the prior. However, in trying to correctly predict SPF forecasts the II model generates worse forecasts in all other dimensions (except inflation), as we have seen from the RMSEs table.

Inflation Expectations: Data vs Model Prediction

(i) Parameters drawn from **posterior** distribution; (ii) Inflation Expectations **included** among observables



Inflation Expectations: Data vs Model Prediction (i) Parameters drawn from **posterior** distribution; (ii) Inflation Expectations **included** among observables

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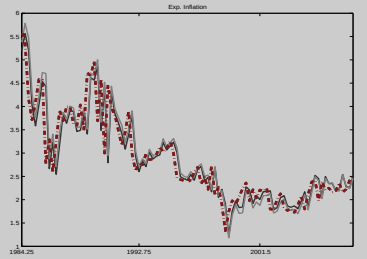

When we use the posterior estimates the II model does even better in fitting the SPF data... This figure shows how little information there is in in-sample posterior RMSEs in terms of assessing whether a model can fit a given variable!

Posterior Estimates – II Model

Parameter	Prior Mean	Prior Stdd	Post Mean	5% Quintile	95% Quintile
Dataset w Expectations					
ζ_p	0.450	0.100	0.504	0.416	0.583
ι_p	0.500	0.280	0.570	0.267	0.942
ζ_w	0.450	0.100	0.160	0.118	0.202
ι_w	0.500	0.280	0.519	0.136	0.973
ρ_r	0.500	0.200	0.380	0.259	0.507
$\rho_{\pi^\star}$	0.950	0.025	0.910	0.879	0.943
σ_r	0.150	4.000	0.687	0.599	0.778
σ_{π^*}	0.050	8.000	0.067	0.038	0.094

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ρ_r	0.500	0.200	0.380	0.259	0.507
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σ_{π^*}	0.050	8.000	0.067	0.038	0.094

Note: The Table shows the posterior mean, the 5th and 95th percentiles of the posterior distribution of selected parameters for the II model.

The posterior estimates in the II model indicate that the best fitting parameter configurations are those where the "learning" component is virtually shut down by making the σ_r (standard deviation of temporary shocks) very large, therefore making the signal to noise ratio small. The persistence of the policy rule is also small in the model. This is needed to generate large forecast errors, consistently with the large values for σ_r. Finally, note that the model does not like nominal rigidities.

Posterior Estimates – PI Model

Parameter	Prior Mean	Prior Stdd	Post Mean	5% Quintile	95% Quintile
Dataset w Exp					
ζ_p	0.450	0.100	0.682	0.602	0.765
ι_p	0.500	0.280	0.268	0.037	0.469
ζ_w	0.450	0.100	0.553	0.430	0.674
ι_w	0.500	0.280	0.299	0.001	0.585
ρ_r	0.500	0.200	0.727	0.680	0.774
$\rho_{\pi^\star}$	0.950	0.025	0.966	0.942	0.990
σ_r	0.150	4.000	0.189	0.085	0.295
σ_{π^*}	0.050	8.000	0.070	0.046	0.091

Posterior Estimates – PI Model

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σ_{π^*}	0.050	8.000	0.070	0.046	0.091

Note: The Table shows the posterior mean, the 5th and 95th percentiles of the posterior distribution of selected parameters for the PI model.

The posterior estimates in the PI model show that the estimates of σ_r and ρ_r are consistent with those in the literature. The posterior means for the nominal rigidities parameters are somewhat larger than the prior means.

Variance Decomposition – II Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Dataset w Expectations							
Output Growth	0.27	0.28	0.14	0.23	0.05	0.00	0.01
Labor Supply	0.01	0.86	0.10	0.01	0.01	0.00	0.00
Labor Share	0.04	0.03	0.00	0.02	0.89	0.00	0.01
Inflation	0.10	0.25	0.39	0.06	0.08	0.02	0.07
Interest Rate	0.09	0.16	0.54	0.06	0.06	0.00	0.08
Exp. Inflation	0.04	0.18	0.70	0.00	0.00	0.04	0.01

Variance Decomposition – II Model

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Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Dataset w Expectations							
Output Growth	0.27	0.28	0.14	0.23	0.05	0.00	0.01
Labor Supply	0.01	0.86	0.10	0.01	0.01	0.00	0.00
Labor Share	0.04	0.03	0.00	0.02	0.89	0.00	0.01
Inflation	0.10	0.25	0.39	0.06	0.08	0.02	0.07
Interest Rate	0.09	0.16	0.54	0.06	0.06	0.00	0.08
Exp. Inflation	0.04	0.18	0.70	0.00	0.00	0.04	0.01

Note: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the II model.

Note that the π^* explains virtually nothing.

Variance Decomposition – PI Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Dataset w Expectations							
Output Growth	0.29	0.06	0.24	0.17	0.06	0.01	0.14
Labor Supply	0.04	0.07	0.73	0.03	0.03	0.01	0.05
Labor Share	0.12	0.20	0.01	0.01	0.52	0.01	0.10
Inflation	0.03	0.07	0.22	0.00	0.09	0.42	0.09
Interest Rate	0.03	0.05	0.35	0.00	0.05	0.19	0.28
Exp. Inflation	0.03	0.00	0.34	0.00	0.00	0.59	0.01

Variance Decomposition – PI Model

Variance Decomposition – PI Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Dataset w Expectations							
Output Growth	0.29	0.06	0.24	0.17	0.06	0.01	0.14
Labor Supply	0.04	0.07	0.73	0.03	0.03	0.01	0.05
Labor Share	0.12	0.20	0.01	0.01	0.52	0.01	0.10
Inflation	0.03	0.07	0.22	0.00	0.09	0.42	0.09
Interest Rate	0.03	0.05	0.35	0.00	0.05	0.19	0.28
Exp. Inflation	0.03	0.00	0.34	0.00	0.00	0.59	0.01

Note: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the II model.

The π_t^* is the main driver of inflation expectations in the PI model.

Model Comparison: Robustness to Priors

	Imperfect Information	Perfect Information
Baseline Prior	-816.232	-801.934
Sticky Prior	-826.696	-797.390
Signal-to-Noise Ratio Prior	-816.890	-802.362

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	Imperfect Information	Perfect Information
Baseline Prior	-816.232	-801.934
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Signal-to-Noise Ratio Prior	-816.890	-802.362

Note: The Table shows the log marginal likelihood for the II and PI models under different priors: "High Nominal Rigidities" prior, and the "Signal-to-Noise Ratio" prior.

Consistently with what we have learned from the posterior estimates, the "High Nominal Rigidities" prior favors the PI model and penalizes the II model. Using the "Signal-to-Noise Ratio" prior makes little difference.

Posterior Estimates – Signal-to-Noise Ratio Prior

Parameter	Prior Mean	Prior Stdd	Post Mean	5% Quintile	95% Quintile
II Model					
ρ_r	0.500	0.200	0.359	0.236	0.489
$\rho_{\pi^\star}$	0.950	0.025	0.886	0.845	0.925
σ_r	0.150	4.000	0.702	0.616	0.794
σ_{SNR}	0.180	0.150	0.161	0.079	0.255
PI Model					
ρ_r	0.500	0.200	0.730	0.685	0.774
$\rho_{\pi^\star}$	0.950	0.025	0.962	0.935	0.989
σ_r	0.150	4.000	0.198	0.100	0.313
σ_{SNR}	0.180	0.150	0.224	0.106	0.335

Posterior Estimates – Signal-to-Noise Ratio Prior

Posterior Estimates – Signal-to-Noise Ratio Prior

Parameter	Prior Mean	Prior Stdd	Post Mean	5% Quintile	95% Quintile
II Model					
ρ_r	0.500	0.200	0.359	0.236	0.489
$\rho_{\pi^\star}$	0.950	0.025	0.886	0.845	0.925
σ_r	0.150	4.000	0.702	0.616	0.794
σ_{SNR}	0.180	0.150	0.161	0.079	0.255
PI Model					
ρ_r	0.500	0.200	0.730	0.685	0.774
$\rho_{\pi^\star}$	0.950	0.025	0.962	0.935	0.989
σ_r	0.150	4.000	0.198	0.100	0.313
σ_{SNR}	0.180	0.150	0.224	0.106	0.335

Note: The Table shows the posterior mean, the 5th and 95th percentiles of the posterior distribution of selected parameters for the II and PI model under the Signal-to-Noise Ratio prior.

For the II model estimates of ρ_r and σ_r are similar to those under the Baseline prior. Estimates of $\rho_{\pi^\star}$ are lower (low $\rho_{\pi^\star}$ reduces the Kalman gain). For the PI model estimates are in line with those under the Baseline prior.

Model Comparison: Robustness to Data Sets and Timing Assumptions

	Imperfect Information	Perfect Information
GDP Defl.	-816.232	-801.934
Lagged Information	-808.287	-794.616
CPI	-863.304	-780.086
1980Q1 Sample	-1053.270	-1052.465

Model Comparison: Robustness to Data Sets and Timing Assumptions

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	Imperfect Information	Perfect Information
GDP Defl.	-816.232	-801.934
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CPI	-863.304	-780.086
1980Q1 Sample	-1053.270	-1052.465

Note: The Table shows the log marginal likelihood for the II and PI models under different timing assumptions ("Lagged Information" specification), inflation measure ("SPF CPI "), and sample ("1980Q1 Sample").

Results are robust to timing assumptions and choice of the inflation measure. For the sample starting in 1980Q1 the two models are comparable: issue worth investigating more.

Model Comparison: Measurement Error

	Imperfect Information	Perfect Information
No Meas. Error	-816.232	-801.934
i.i.d. Meas. Error	-798.115	-796.441
AR(1) Meas. Error	-779.813	-797.69

Model Comparison: Measurement Error

Model Comparison: Measurement Error

	Imperfect Information	Perfect Information
No Meas. Error	-816.232	-801.934
i.i.d. Meas. Error	-798.115	-796.441
AR(1) Meas. Error	-779.813	-797.69

Note: The Table shows the log marginal likelihood for the II and PI models for the Baseline specification, which has no measurement error, and for specifications where the measurement error is i.i.d. ("i.i.d. Meas. Error") or follows and AR(1) process ("AR(1) Meas. Error "). The PI model is still superior to the II model when the measurement error is i.i.d. – although the two models are fairly close. The II model fits better than the PI model under AR(1) measurement error: we conjecture that the measurement error largely "takes care" of inflation expectations for the PI model.

Variance Decomposition – AR(1) Measurement Error, II Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Output Growth	0.36	0.31	0.12	0.11	0.07	0.00	0.00	0.02
Labor Supply	0.02	0.69	0.20	0.01	0.05	0.00	0.00	0.00
Labor Share	0.03	0.03	0.00	0.01	0.91	0.00	0.00	0.01
Inflation	0.12	0.26	0.21	0.03	0.13	0.07	0.00	0.15
Interest Rate	0.12	0.21	0.39	0.03	0.09	0.01	0.00	0.13
Exp. Inflation	0.04	0.02	0.24	0.00	0.00	0.11	0.52	0.01

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Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Output Growth	0.36	0.31	0.12	0.11	0.07	0.00	0.00	0.02
Labor Supply	0.02	0.69	0.20	0.01	0.05	0.00	0.00	0.00
Labor Share	0.03	0.03	0.00	0.01	0.91	0.00	0.00	0.01
Inflation	0.12	0.26	0.21	0.03	0.13	0.07	0.00	0.15
Interest Rate	0.12	0.21	0.39	0.03	0.09	0.01	0.00	0.13
Exp. Inflation	0.04	0.02	0.24	0.00	0.00	0.11	0.52	0.01

Note: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the II model with AR(1) measurement error.

Variance Decomposition – AR(1) Measurement Error, PI Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Output Growth	0.28	0.07	0.23	0.14	0.07	0.00	0.00	0.16
Labor Supply	0.06	0.10	0.64	0.03	0.05	0.01	0.00	0.07
Labor Share	0.12	0.20	0.01	0.01	0.54	0.01	0.00	0.10
Inflation	0.06	0.13	0.17	0.00	0.16	0.24	0.00	0.17
Interest Rate	0.04	0.07	0.26	0.00	0.07	0.08	0.00	0.44
Exp. Inflation	0.07	0.00	0.33	0.00	0.01	0.44	0.06	0.02

Variance Decomposition – AR(1) Measurement Error, PI Model

Variance Decomposition – AR(1) Measurement Error, PI Model

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Output Growth	0.28	0.07	0.23	0.14	0.07	0.00	0.00	0.16
Labor Supply	0.06	0.10	0.64	0.03	0.05	0.01	0.00	0.07
Labor Share	0.12	0.20	0.01	0.01	0.54	0.01	0.00	0.10
Inflation	0.06	0.13	0.17	0.00	0.16	0.24	0.00	0.17
Interest Rate	0.04	0.07	0.26	0.00	0.07	0.08	0.00	0.44
Exp. Inflation	0.07	0.00	0.33	0.00	0.01	0.44	0.06	0.02

Note: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the II model with AR(1) measurement error.

Not surprisingly, we find that the measurement error is the most important source of variation for Expectations in the II model. For the PI model the contribution of measurement error to the variance of expectations is negligible; changes in π_t^* are the most important source of variations.

Conclusions

- Inflation expectations are very helpful in discriminating among models.
- The imperfect information model we consider is unable to produce inflation expectations similar to those observed in the data.
- Perfect information model is better – but only in relative terms. And arguably it is not a very plausible model of expectations formation.
- Future research:
  - Swanson et al. law of motion for inflation target
  - Learning

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- Perfect information model is better – but only in relative terms. And arguably it is not a very plausible model of expectations formation.
- Future research:
  - Swanson et al. law of motion for inflation target
  - Learning

Although the PI model does marginally better than the II model, our results should not be interpreted as stating that the run-of-the-mill perfect information model with time varying target is the right approach for modeling expectations. As we have seen, even for this model introducing expectations among the observables worsens the fit in all other dimensions other than inflation.

In our ongoing research we plan to consider a model where the evolution of π_t^* is not purely exogenous (as in Gurkainak-Sack-Swanson) and models with learning. See if any of those are better models to describe the evolution of inflation expecations.