

A model of credit risk without commitment*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper studies an economy with credit risk in which, as in Bizer and DeMarzo (1992), borrowers cannot commit to exclusive contracts with lenders. In contrast with Bizer and DeMarzo (1992), we study a framework with multiple borrowing periods. In particular, we remove the exclusive-contract assumption from a baseline model of credit risk à la Eaton and Gersovitz (1981), similar to those used in quantitative studies of household bankruptcy, corporate bankruptcy, and sovereign default. We compare equilibrium allocations with and without commitment to exclusive contracts. We show that borrowing levels may be lower without commitment. This is the case because when commitment is not assumed, an increase in current borrowing levels deteriorates future borrowing conditions. This does not occur when commitment is assumed. This finding stands in sharp contrast with the results in previous work that study environment where current borrowing does not affect future borrowing opportunities. We also show that borrowing levels tend to be lower without commitment if the borrowers' discount factor is higher or debt needs to be rolled over more often, and when the endogenous default probability is lower.

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1 Introduction

This paper proposes an extension of a baseline model à la Eaton and Gersovitz (1981) that allows us to study credit risk when borrowers cannot commit to exclusive contracts with lenders.¹ That is, we assume that in each period a borrower can deal with more than one lender. Borrowers receive a stochastic endowment stream. Each period, they first decide whether to default on previously issued debt and later they decide to borrow or save using one-period non-contingent bonds. These assets are priced in a competitive market inhabited by a large number of identical, risk neutral lenders. Lenders have perfect information regarding the borrower’s endowment. The cost of defaulting is modeled as an endowment loss. The main difference between this paper and recent quantitative studies of default is that we do not assume that borrowers can commit to exclusive contracts with lenders (i.e., they cannot commit to a current-period borrowing level).

We compare the equilibrium allocation in an economy with commitment to exclusive contracts with the one obtained when borrowers cannot commit to exclusive contracts.² We show that borrowing levels may be lower without commitment.

Our finding stands in sharp contrast with the results in previous studies. It is well known that credit risk may lead to a “debt dilution” problem. Debt dilution occurs when new debt reduces the expected recovery rate of existing creditors. For instance, this is the case when new debt issuances increase the probability of default on debt held by existing creditors.³ The conventional wisdom in the debt dilution literature is that, because of debt dilution and because borrowers

¹Frameworks similar to the one we use has been used in recent studies of household and corporate bankruptcy (see, for example, Arellano et al. (2007), Athreya (2002), Athreya et al. (2007a,b), Chatterjee et al. (2005), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2007), and Sánchez (2008)) and of sovereign default (see, for example, Aguiar and Gopinath (2006), Arellano (2008), Arellano and Ramanarayanan (2006), Bai and Zhang (2006), Bi (2006), Cuadra and Saprizza (2006a,b), D’Erasmus (2007), Eyigungor (2006), Hatchondo et al. (2006a,b, 2007), Lizarazo (2005, 2006), Mendoza and Yue (2007), and Yue (2005)).

²As explained in previous studies (see, for example, Bizer and DeMarzo (1992)), “commitment to exclusive contracts” can be achieved using clauses that make loan terms contingent on future borrowing. Such clauses are likely to introduce inefficiencies unless they are fully state contingent, and implementing fully-state-contingent clauses would be problematic (if possible). This could explain why many loan contracts do not contain such clauses.

³For example, Bolton and Jeanne (2005) explain that “debt dilution is difficult to avoid in sovereign lending, as there is no obvious way of structuring legally binding seniority agreements nor of enforcing priority of repayment following a sovereign debt default.” Bizer and DeMarzo (1992) argue that debt dilution may occur even when seniority agreements are in place.

cannot commit to stop borrowing, equilibrium debt levels are higher than what they would be if the government could commit—see, for example, Sachs and Cohen (1982), Kletzer (1984), Bizer and DeMarzo (1992), Detragiache (1994), Tirole (2002), and Bi (2006). This in turn implies that defaults are more frequent, which results in higher interest rates. For example, Bizer and DeMarzo (1992) explain that “In the subgame-perfect equilibrium [of their sequential banking game without commitment to exclusive contracts], interest rates charged on loans are higher than when the borrower can commit to obtaining only one loan. Although interest rates are higher, borrowing is also greater, and the probability of default is greater as well.” “The results apply to markets for consumer, corporate, and international debt.” The excessive borrowing predicted in these studies has received considerable attention both in academic and policy discussions, and have lead to the consideration of policies that are both costly and difficult to implement—see, for example, Borensztein et al. (2004) and the references therein. These policies intend to reduce observed borrowing levels in order to avoid overborrowing. Since in our model there may be “underborrowing” (borrowing may be lower without commitment), proposed policies that constraint the borrowers’ ability to issue debt may actually be detrimental to welfare.

In order to explain the difference between our result and previous findings, let us discuss first the mechanism that generates excessive borrowing in the debt dilution literature. First, recall that in environments with credit risk, equilibrium bond prices compensate lenders for the default probability—bond prices are lower when the default probability is higher. Furthermore, since the default probability increases with issuance, bond prices decrease with issuance. If a borrower can commit to an exclusive contract, he knows that by choosing to sell fewer bonds to the lender he is dealing with, he can obtain a higher bond price from this lender. That is, a cost of borrowing an extra dollar is that this decreases the price at which a borrower is selling his bonds. On the contrary when commitment is not assumed, if a borrower chooses to borrow one dollar less from a lender, this lender may not want to offer a higher bond price because this lender knows that borrowers can always continue borrowing from other lenders. Thus, when exclusive contracts are not assumed, the cost of borrowing an extra dollar mentioned above is not present—and thus, the marginal cost of issuing debt is lower. Consequently, the equilibrium issuance level is higher. This is the mechanism behind the overborrowing predicted in previous studies.

The mechanism that may lead borrowers to borrow less when they cannot commit to exclusive contracts arise because issuing an extra dollar of debt may affect the price of future issuances. If a borrower cannot commit to exclusive contracts, the price at which lenders would buy his bonds depends on the issuance level expected by them. In order to infer the issuance level that will be chosen by a borrower, rational lenders use all relevant information, which includes the borrower's current debt level. As the one in previous studies, the equilibrium analyzed in this paper is such that when initial debt is higher, the equilibrium issuance level is higher. Thus, when lenders observe higher debt levels, they anticipate higher issuance levels, and therefore, they offer lower bond prices. Consequently, an increase in the issuance volume today increases future issuance volumes expected by the lenders and deteriorate future borrowing conditions. That is, there is a cost of borrowing an extra dollar given by the effect of this extra borrowing in future borrowing conditions. In contrast, when commitment to exclusive contracts is assumed, lenders do not need to infer the issuance level using the initial debt level because it suffices to observe the level to which a borrower is committed. Consequently, bond prices depend only on the issuance level to which a borrower commits and do not depend on the initial debt level. Thus, the marginal cost of borrowing given by the deterioration of future borrowing conditions is not present when commitment to exclusive contracts is assumed. As a result, equilibrium issuance levels may be higher when commitment is assumed. This is not possible in previous studies where concerns about future borrowing conditions are not present (a description of the related literature is presented below).

The previous discussion shows how removing the exclusivity assumption from a standard credit risk model results in borrowers behaving as borrowing from credit lines: without commitment to an exclusive contract, a borrower does not affect the interest rate in his current contract by changing the amount he borrows. Credit card contracts exhibit this credit-line aspect. This observation (and tractability) led earlier work on household bankruptcy to assume that bond prices do not depend on issuance volumes (see, for example, Athreya (2002) and Li and Sarte (2006)).⁴ But this approach was criticized because it was seen as inconsistent with borrowing

⁴Recent work by Mateos-Planas (2007), Mateos-Planas and Ríos-Rull (2007) and Drozd and Nosal (2007) present models of household bankruptcy with credit lines.

levels being observable. For instance, Chatterjee et al. (2007) explain that in Athreya (2002) “financial intermediaries charge the same interest rate on loans of different sizes even though a large loan induces a higher probability of default than a small loan. As a result, small borrowers end up subsidizing large borrowers, a form of crosssubsidization that is not sustainable with free entry of intermediaries.” Following Chatterjee et al. (2007), more recent work (also in the sovereign debt and corporate debt literatures) study models in which a borrower affect the interest rate in his current borrowing contract by changing the amount he borrows (see, for example, Arellano (2008), Chatterjee et al. (2005) and Livshits et al. (2007)). In contrast, when exclusive contracts are not assumed, financial intermediaries charge the same interest rate on loans of different sizes even though a large loan induces a higher probability of default than a small loan—this occurs because if a borrower wants a small loan from a lender, this lender anticipates that the borrower will ask for another loan. Thus, a borrower cannot obtain a lower interest rate by borrowing less. But there is not crosssubsidization because different borrowers are offered different credit lines.

We also show that without commitment, borrowing levels tend to be lower in environments where borrowers’ discount factor is higher or debt needs to be rollover more often. In these environments, concerns about future borrowing conditions are more important. Equilibrium borrowing levels also tend to be lower when the default probability is lower. If the default probability is higher, the expected impact of current borrowing on future borrowing conditions is less important because if a borrower is more likely to default the expected effect of an increase in current borrowing on future borrowing is less important. These findings indicate that borrowers who face a high default probability may benefit from issuing debt with shorter maturity because this may help reduce the overborrowing problem discussed in the debt dilution literature—for instance, this could help explain why emerging economies issue short term debt; see Broner et al. (2007) for a recent discussion of this issue. Furthermore, these findings also indicate that legislation that lower the frequency with witch credit card issuers can change borrowing conditions can reduce welfare because, as explained above, concerns about future borrowing conditions may moderate the overborrowing problem described in the debt dilution literature.

1.1 Related literature

In order to compare the model presented in this paper with the ones in previous studies of debt dilution, it is useful to make a clear distinction between multiple borrowing periods and multiple borrowing opportunities within a period. Models used to study lack of commitment to exclusive contracts within a period consider multiple borrowing opportunities within this period. This is independent on whether multiple borrowing periods are considered. Our model is a model with multiple borrowing periods in which a borrower cannot commit to exclusive contracts within a period. Previous studies assume either that there is a unique borrowing period or that borrowers can commit to exclusive contracts in each period.

Most studies on debt dilution rely on models with a unique borrowing period (and multiple borrowing subperiods within this period). Consequently, these studies do not consider the concerns about future borrowing conditions highlighted in this paper. Thus, in these studies the marginal cost of borrowing is lower without commitment. This leads these studies to conclude that debt levels are higher when borrowers cannot commit to exclusive contracts—see, for example, Bizer and DeMarzo (1992), and Detragiache (1994).

Other studies on debt dilution assume bonds of different maturities and focus on the lack of commitment to the issuance levels in future borrowing periods—see Sachs and Cohen (1982), Bi (2006), and the references therein. In contrast with our model, these studies assume that each period, borrowers can commit to exclusive borrowing contracts for the period. Consequently, in these studies lenders do not need to form expectations about current issuance levels using information about past issuances. Therefore, the concerns about the effect of current issuances on future borrowing conditions highlighted in this paper do not play a role in these studies. As explained before, this biases the results in these studies towards concluding that when borrowers cannot commit to stop borrowing, debt levels are higher.⁵

⁵In models with debt of different maturities, there is an effect of current issuances in future borrowing opportunities. But this effect is different from the one highlighted in this paper. In these models, the price of current bond issuances depend on past issuances through the effect of these past issuances on the amounts that have to be paid in the future. But past issuances do not affect current prices through the lenders' expectations. In contrast with the effect of past issuances through lenders' expectations, the effect through the amounts that have to be paid in the future is present both with and without commitment to exclusive contracts.

Kletzer (1984) studies an infinite-horizon model of sovereign default in which debt is not observable and, therefore, equilibrium bond prices depend on the issuance amount expected by lenders. However, assumptions in his model are such that the current debt stock does not influence the willingness to borrow. Consequently, current debt does not influence the issuance amount expected by lenders and equilibrium bond prices. Thus, Kletzer (1984) does not study the concerns about future borrowing conditions we highlight in this paper.

This paper is also related to a large literature on non-exclusive contracts in principal-agent models. Bizer and DeMarzo (1992), Bisin and Guaitoli (2004), Bisin and Rampini (2006), and other studies in this literature have analyzed unsecured-credit problems explicitly. However, these studies focus on two-period models with a single borrowing period and, therefore, do not consider the concerns about future borrowing opportunities highlighted in this paper.⁶

The rest of the article proceeds as follows. Section 2 introduces a benchmark model of credit risk with commitment to exclusive borrowing contracts. Section 3 proposes an extension of the benchmark model that allows us to study credit risk when borrowers cannot commit to exclusive contracts with lenders. Section 4 presents the results. Section 5 concludes and discusses extensions.

2 A benchmark with commitment to exclusive borrowing contracts

There is a single good in the economy, y . Borrowers receive a stochastic endowment stream of this good where

$$\log(y_t) = (1 - \rho) \mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

$|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Let F denote the cumulative distribution function for next-period endowment, y' .

⁶In contrast with other studies in this literature, Bisin and Rampini (2006) find that non-exclusivity of contracts reduces the amount borrowed in equilibrium. This is the case because non-exclusivity reduces the equilibrium level of insurance and, consequently, it increases equilibrium precautionary savings.

Borrowers have CRRA preferences over consumption:

$$u(c) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma},$$

where σ denotes the coefficient of relative risk aversion. Let β denote their discount factor.

Each period, borrowers first decide whether to default on previously issued debt and later borrow or save using one-period bonds. A defaulting borrower loses $\phi(y)$ of its output in the default period.

Let b denote the bond position at the beginning of a period. A negative value of b denotes that a borrower was an issuer of bonds in the previous period. Each bond delivers one unit of the good next period (if there is no default).

There is a continuum of risk neutral lenders. Each lender can borrow or lend at the risk free rate r . Lenders have perfect information regarding a borrower's endowment.

Borrowing occurs as follows. First, each lender offers to each borrower a borrowing contract that specifies a bond price q for each possible issuance level b' . Second, each borrower chooses one of the borrowing contracts offered to him and the amount he wants to borrow from this contract. This concludes the borrowing game. Thus, it is assumed that in each period, after a borrower trades with a lender, the borrower can commit to stop borrowing. This is the borrowing game assumed in quantitative studies that extend the framework proposed by Eaton and Gersovitz (1981).⁷

2.1 Recursive formulation

In this section we present a recursive formulation of the problems solved by a borrower when it decides whether to default and how much to borrow, and of the problem solved by a lender when he decides which borrowing contract he offers to a borrower. Let $V(b, y)$ denote a borrower's value function at the beginning of a period—before he decides whether to default. Let $\tilde{V}(d, b, y)$

⁷Most of these studies present directly a borrowing contract that is such that the bond price offered for each issuance level satisfies a lender's zero-profit condition. See, for example, Aguiar and Gopinath (2006), Arellano et al. (2007), Arellano (2008), Arellano and Ramanarayanan (2006), Bai and Zhang (2006), Bi (2006), Chatterjee et al. (2005), Chatterjee et al. (2007), Cuadra and Sapriza (2006a,b), Eyigungor (2006), Hatchondo et al. (2006a,b, 2007), Livshits et al. (2007), Lizarazo (2005, 2006), and Yue (2005). It is easy to show that offering this borrowing contract is an equilibrium strategy of the game described above.

denote the expected utility of a borrower after he made his default decision. The function $V(b, y)$ is computed as follows:

$$V(b, y) = \max_d \{d\tilde{V}(1, b, y) + (1 - d)\tilde{V}(0, b, y)\}. \quad (1)$$

Let $d(b, y)$ denote the optimal default decision that solves problem (1).

Let $s \equiv (d, b, y)$. Let $\tilde{g}(s; \hat{q}(b'))$ denote the optimal borrowing level for a borrower who accepted a borrowing contract $\hat{q}(b')$. That is, $\tilde{g}(s; \hat{q}(b'))$ solves

$$\max_{b'} \left\{ u(y - d\phi(y) + [1 - d(1 - \alpha)]b - \hat{q}(b')b') + \beta \int V(b', y')F(dy' | y) \right\}. \quad (2)$$

Let $\tilde{q}(b'; s)$ denote the equilibrium borrowing contract offered to a borrower with state s . In a competitive credit market, this contract is such that it maximizes the borrowers's expected utility provided that it satisfies the non-negative expected profit condition for lenders. That is, $\tilde{q}(b'; s)$ solves

$$\max_{\hat{q}(b')} \left\{ u(y - d\phi(y_t) + (1 - d)b - \hat{q}(\tilde{g}(s; \hat{q}(b'))))\tilde{g}(s; \hat{q}(b')) + \beta \int V(\tilde{g}(s; \hat{q}(b')), y')F(dy' | y) \right\} \quad (3)$$

subject to

$$\tilde{g}(s; \hat{q}(b')) \left[\hat{q}(\tilde{g}(s; \hat{q}(b'))) - \frac{1 - \int d(\tilde{g}(s; \hat{q}(b')), y')F(dy' | y)}{1 + r} \right] \geq 0. \quad (4)$$

Thus, the expected utility of a borrower after he made his default decision is given by

$$\tilde{V}(d, b, y) = \max_{b'} \left\{ u(y - d\phi(y) + (1 - d)b - \tilde{q}(b'; s)b') + \beta \int V(b', y')F(dy' | y) \right\} \quad (5)$$

Definition 1 *A recursive competitive equilibrium is characterized by*

1. a set of value functions $\tilde{V}(d, b, y)$ and $V(b, y)$,

2. a default decision rule $d(b, y)$, and a borrowing decision rule $\tilde{g}(s; \hat{q}(b'))$,
3. and a set of borrowing contracts $\tilde{q}(b'; s)$

such that

- (a) $V(b, y)$ and $\tilde{V}(d, b, y)$ satisfy functional equations (1) and (5), respectively;
- (b) the policies for default $d(b, y)$ and for borrowing $\tilde{g}(s; \hat{q}(b'))$ solve the dynamic programming problems (1) and (2), respectively;
- (c) and borrowing contracts $\tilde{q}(b'; s)$ solve problem (3) subject to constraint (4).

3 A model without commitment to exclusive borrowing contracts

In this section, we propose an extension of the benchmark presented in Section 2 that allows us to study credit risk when borrowers can deal with more than one lender in each period. In this preliminary version of the paper, we follow Bisin and Rampini (2006), and we build on two known properties of sequential borrowing games—in which borrowers can always increase their borrowing level by visiting a new lender.

The first of these properties is as follows. As explained by Bizer and DeMarzo (1992), in equilibrium “it is unnecessary for the borrower to visit more than one bank.” The intuition behind this result is straightforward. In equilibrium all lenders dealing with a borrower must have zero profits in expectation.⁸ Thus, since all lenders face the same default probability, they have to pay the same bond price. Since all lenders pay the same bond price, a borrower is

⁸This is the case in the environment studied in this paper. Bisin and Guaitoli (2004) and Bisin and Rampini (2006) discuss environments in which competitive risk-neutral lenders may have positive expected profits in equilibrium.

indifferent between selling all bonds to one lender and selling bonds to multiple lenders. This property implies that imposing that borrowers trade with only one lender (as we will do in this paper) does not rule out equilibrium allocations.

The second equilibrium property is the *no-further-borrowing* property. As explained by Bizer and DeMarzo (1992), the equilibrium allocation must be such that “any additional loan involving nonnegative profits for the banks makes the borrower worse off.” The intuition behind this result is straightforward. If exclusive contracts are not assumed and a borrower can always continue borrowing, the equilibrium borrowing level must be such that he does not want to continue borrowing. This constraint rules out all contracts that would imply negative expected profits if borrowers were not able to commit to exclusive contracts and were allowed to continue selling bonds (at fair prices).

Bisin and Rampini (2006) use the equilibrium properties described above to study equilibrium allocations without commitment to exclusive contracts (see also Detragiache (1994)). We follow their approach. The first property discussed above implies that one can restrict attention to equilibria in which a borrower deals with only one lender, as in the benchmark presented in Section 2. We do so.

In order to guarantee that the no-further-borrowing property is satisfied, we impose a no further-borrowing constraint to the borrowing contracts that can be offered by lenders (this constraint is equivalent to the incentive compatibility constraint used by Bisin and Rampini (2006)). That is, we assume that lenders can only offer to a borrower contracts that are such that after the borrower accepts the contract and chooses his preferred borrowing amount from this contract, he would not want to sell more bonds at prices that satisfy a new lender’s expected zero-profit condition.

Let $q(b', y)$ denote the bond price that gives zero profits in expectation to a lender when a borrower issued b' bonds and his current income is y . That is,

$$q(b', y) = \frac{1}{1+r} \left[1 - \int d(b', y') F(dy' | y) \right]. \quad (6)$$

The no-further-borrowing constraint stipulates that a borrowing contract $\hat{q}(b')$ can only be offered if, when $\tilde{g}(s; \hat{q}(b'))$ is negative, for all $b'_2 < 0$, the government prefers selling $\tilde{g}(s; \hat{q}(b'))$ bonds at

$\hat{q}(\tilde{g}(s; \hat{q}(b')))$ over selling $\tilde{g}(s; \hat{q}(b'))$ bonds at $\hat{q}(\tilde{g}(s; \hat{q}(b')))$ and also selling b'_2 bonds at $q(b'_2 + \tilde{g}(s; \hat{q}(b')), y)$. That is, lenders can only offer borrowing contracts $\hat{q}(b')$ such that if $\tilde{g}(s; \hat{q}(b')) < 0$, 0 maximizes

$$\max_{b'_2 \leq 0} \left\{ u(y - d\phi(y_t) + (1-d)b - \hat{q}(\tilde{g}(s; \hat{q}(b')))) \tilde{g}(s; \hat{q}(b')) - q(\tilde{g}(s; \hat{q}(b')) + b'_2, y)b'_2 + \beta \int V(\tilde{g}(s; \hat{q}(b')) + b'_2, y') F(dy' | y) \right\}. \quad (7)$$

We characterize the equilibrium that result from solving the model presented in Section 2 (in which a borrower only sells bonds to one lender), but imposing that contracts offered by lenders must not only maximize the borrower expected utility (problem (3)) subject to the expected non-negative profit constraint (equation (4)) but also subject to the *no-further-borrowing constraint* in equation (7). Adding the no-further-borrowing constraint captures the lack of commitment to exclusive contracts.⁹

We also impose that borrowing contracts must be such that the bond price is weakly decreasing with respect to the issuance volume. That is, we assume that a borrower is always able to sell fewer bonds at the price offered by lenders. It is easy to show that this constraint is not binding when commitment to exclusive contracts is assumed. Without exclusivity, this assumption rules out equilibria such that a borrower would prefer to borrow less at the price offered by the lenders (giving positive expected profits to the lenders), which is not allowed because borrowing less at the same price does not satisfies the no-further-borrowing constraint in equation (7). We find such equilibria unappealing.

4 Results (VERY PRELIMINARY AND INCOMPLETE)

In this section, we discuss how the exclusivity assumption influence results in studies of credit risk. We use the superscript E to denote allocations in the equilibrium with exclusive contracts. From now on, subscripts 1 are used to denote derivatives with respect to the first argument.

⁹Modeling the sequential borrowing problem allows Bizer and DeMarzo (1992) to study seniority, which we do not study in this paper.

For notational simplicity, we do not write the equilibrium consumption and borrowing levels as functions of the state.

4.1 Equilibrium with exclusive contracts

It is easy to show that an equilibrium borrowing contract of the model presented in Section 2 is given by the zero-profit price menu $q(b', y)$ presented in equation (6)—this is the borrowing contract used in previous studies. Plugging in this borrowing contract into a borrower's problem presented in (5), one can obtain the following condition that must be satisfied in equilibrium:

$$u_1(c^E)q(b'^E, y) = \beta \int u_1(c'^E) [1 - d(b'^E, y')] F(dy' | y) - q_1(b'^E, y) u_1(c^E) b'^E. \quad (8)$$

4.2 Equilibrium without exclusive contracts

We now characterize an equilibrium that satisfies the no-further-borrowing constraint in equation (7). First, we show that equilibrium borrowing contracts evaluated at the equilibrium borrowing level is such that a better bond price cannot be offered when the borrower issue less debt. Thus, borrowing contracts à la Chatterjee et al. (2007) (the zero-profit price menu $q(b', y)$) cannot be offered in equilibrium.

The first-order condition of a borrower's maximization problem in (2) when he is offered contract $\tilde{q}(b'; s)$ reads

$$u_1(c) \tilde{q}(b'; s) = \beta \int V_1(b', y') F(dy' | y) - \tilde{q}_1(b'; s) u_1(c) b'. \quad (9)$$

A necessary condition for the no-further-borrowing constraint in equation (7) to hold is

$$u_1(c) q(b', y) \leq \beta \int V_1(b', y') F(dy' | y). \quad (10)$$

In equilibrium the zero-expected-profit condition is satisfied. That is, evaluated at the equilibrium borrowing level $\tilde{q}(b'; s) = q(b', y)$. Therefore, $\tilde{q}_1(b'; s) \leq 0$ (i.e., the equilibrium borrowing contracts evaluated at the equilibrium borrowing level cannot offer a better bond price when the borrower issue less debt).

The previous discussion shows how removing the exclusivity assumption from a standard credit risk model results in borrowers behaving as borrowing from credit lines: without commitment to an exclusive contract, a borrower does not affect the interest rate in his current contract by changing the amount he borrows. It is easier to present the intuition for this result in the context of sequential borrowing. If sequential borrowing is possible, when a borrower wants a small loan from a lender, this lender anticipates that the borrower will ask for another loan after obtaining the small loan. Thus, the lender who is asked for a small loan does not offer a lower interest rate. The no-further-borrowing constraint in equation (7) allows us to capture this property of sequential borrowing.

Recall also that we assumed that borrowing contracts must be such that the bond price is weakly decreasing with respect to the issuance volume. Thus, the first-order condition of a borrower's maximization problem in (2) implies that

$$u_1(c) q(b', y) = \beta \int V_1(b', y') F(dy' | y). \quad (11)$$

Let $g(b, y)$ denote the equilibrium borrowing level when the initial debt level is b , income is y , the default decision is $d(b, y)$, and the borrowing contract is $\tilde{q}(b'; (d(b, y), b, y))$. Let $\bar{q}(b', b, y) \equiv \tilde{q}(b'; (d(b, y), b, y))$, and let $\bar{q}_2(b', b, y)$ denote the derivative of the equilibrium bond price with respect to the initial debt level (when it is well defined). The derivative of a borrower's expected utility with respect to his initial debt level (when it is well defined) is given by

$$V_1(b, y) = u_1(c) [1 - d(b, y)] - \bar{q}_2(b', b, y) u_1(c) g(b, y). \quad (12)$$

Plugging in equation (12) into equation (11) we obtain

$$u_1(c) q(b', y) = \beta \int u_1(c') [1 - d(b', y')] F(dy' | y) - \beta \int \bar{q}_2(g(b', y'), b', y') u_1(c') g(b', y') F(dy' | y). \quad (13)$$

The left-hand side of equation (13) can be interpreted as the marginal benefit of borrowing. This marginal benefit is given by the value of the increase in current consumption implied by an increase in borrowing. The right-hand side of equation (13) can be interpreted as the marginal

cost of borrowing. It represents the present value of the decrease in future consumption implied by a increase in current borrowing. The second term in the right-hand side of equation (13) represents the expected effect of current borrowing on future borrowing contracts.

Equations (8) and (13) are useful to compare equilibrium allocations with and without commitment to exclusive contracts. The main difference between these two equations is that the effect represented in the second term of the right-hand side of equation (8) is not present in equation (13), and the effect represented in the second term of the right-hand side of equation (13) is not present in equation (8). Which of these two effects is larger is crucial in determining whether there is overborrowing without commitment to exclusive contracts.

The second term in the right-hand side of equation (8) represents the value of an increase in current consumption implied by a decrease in current borrowing. If a borrower can commit to exclusive contracts, he can increase the bond price he is paid by selling fewer bonds to the lender he is dealing with (exclusively). As explained above, this is not the case if sequential borrowing is possible. With sequential borrowing, when a borrower wants a small loan from a lender, this lender anticipates that the borrower will ask for another loan after obtaining the small loan. Thus, the lender who is asked for a small loan does not offer a lower interest rate. Given that abandoning the exclusivity assumption implies eliminating a borrowing cost, the equilibrium without commitment could exhibit more borrowing. Since more borrowing implies a higher default probability, the equilibrium without commitment could exhibit lower bond prices—i.e., higher interest rates. This is the mechanism behind the result in previous studies.

The second term in the right-hand side of equation (13) represents the expected effect of current borrowing on future borrowing contracts. If a borrower cannot commit to exclusive contracts, the price at which lenders would buy his bonds depends on the issuance level expected by them. In order to infer the issuance level that will be chosen by a borrower, rational lenders use all relevant information, which includes the borrower's current debt level. As previous studies, we find that when initial debt is higher, the equilibrium issuance level is higher. Consequently, an increase in the issuance volume today increases future issuance volumes expected by the lenders and deteriorate future borrowing conditions. When lenders observe higher debt levels, they anticipate higher issuance levels, and therefore, they offer lower bond prices. This is not possible

in previous studies of debt dilution where borrowers can commit to exclusive contracts within each borrowing period (see, for example, Sachs and Cohen (1982) and Bi (2006)) and in studies with a unique borrowing period (see, for example, Bizer and DeMarzo (1992), and Detragiache (1994)). In Section 4.4 we show that concerns about future borrowing conditions may dominate and borrowing can be lower when commitment to exclusive contracts is not assumed.

Equation (13) also shows that without commitment, borrowing levels tend to be lower in environments where borrowers' discount factor is higher or debt needs to be rollover more often. In these environments, concerns about future borrowing conditions are more important. Equilibrium borrowing levels also tend to be lower when the default probability is lower. Recall that the bond price offered to a borrower depends on his debt level at the moment he decides the issuance volume, which is zero if he previously defaulted. Thus, if the default probability is higher, the expected impact of current borrowing on future borrowing conditions is less important. These findings indicate that borrowers who face a high default probability may benefit from issuing debt with shorter maturity because this may help reduce the overborrowing discussed in the debt dilution literature—for instance, this could help explain why emerging economies issue short term debt; see Broner et al. (2007) for a recent discussion of this issue. These findings also indicate that legislation that lower the frequency with witch credit card issuers can change borrowing conditions can reduce welfare because this would weaken the concerns about future borrowing conditions that moderate the overborrowing problem.

4.3 Computation of the equilibrium

The equilibrium can be found as follows. First, for any states s , the set of borrowing levels $b'(s)$ that satisfy the necessary condition in equation (11) can be found. Second, among these borrowing levels, we select the one that is preferred by the borrower. This borrowing level is the equilibrium borrowing level. An equilibrium borrowing contract is a credit line with a borrowing limit equal to the equilibrium borrowing level and a bond price that gives zero profits in expectation for the equilibrium borrowing level. This procedure for finding the equilibrium is not fundamentally more complex computationally that the one used when commitment is

Risk aversion	σ	2
Interest rate	r	1%
Endowment autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	3.4%
Mean (log) endowment	μ	$(-1/2)\sigma_\epsilon^2$
Endowment loss	λ	0.85%
Discount factor	β	0.95
Recovery rate	α	70%

Table 1: Numerical Example.

assumed.

The model is solved numerically using value function iteration and interpolation as in Hatchondo et al. (2006a).¹⁰ We compute the solution solving a finite-horizon version of the model backwards.

4.4 Numerical example

At this time, we only have a numerical example. The output loss function is parameterized as in Arellano (2006):

$$\phi(y) = \begin{cases} y - \lambda & \text{if } y > \lambda \\ 0 & \text{if } y \leq \lambda \end{cases}$$

Table 1 presents the parameter values for our example. In the example, we assume partial defaults. When a borrower defaults he pays back a proportion α of its debt.

Figure 1 shows that borrowing can be lower when commitment to exclusive contracts is not assumed. In fact, this is often the case in the example. Consequently, proposed policies that constraint the borrowers' ability to issue debt may actually be detrimental to welfare. The figure

¹⁰The value functions $\tilde{V}(0, b, y)$ and $\tilde{V}(1, b, y)$ are approximated using Chebychev polynomials. Fifteen polynomials on the asset space and ten on the endowment shock are used. Results are robust to using more polynomials.

also shows that borrowing is lower without commitment for high incomes. This is the case because the equilibrium default probability is lower for these incomes and, therefore, the effect of current borrowing on future borrowing conditions is more important for these incomes—when commitment to exclusive contracts is not assumed.

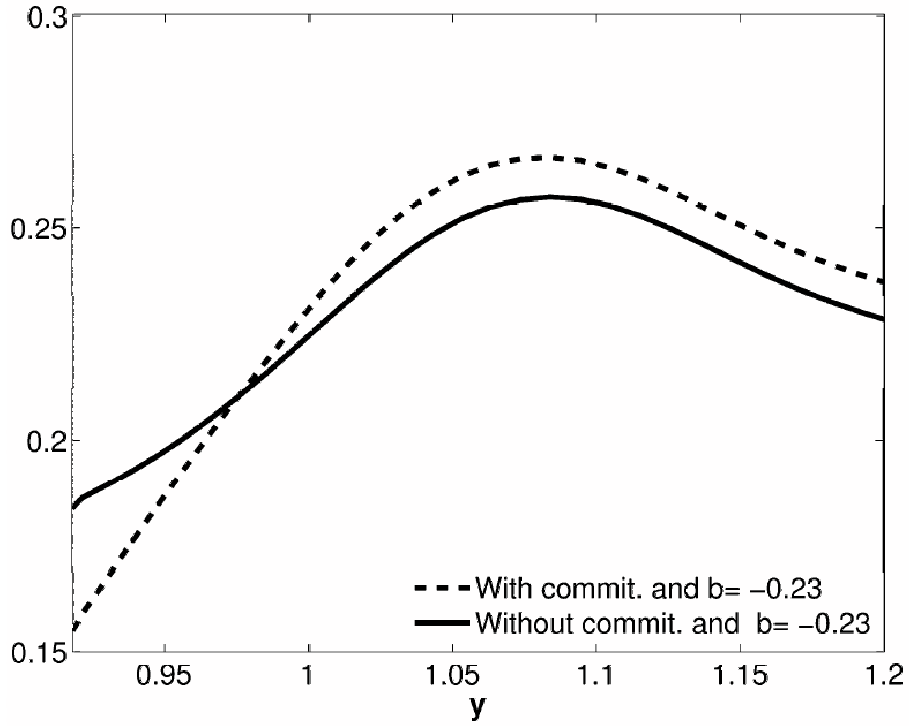


Figure 1: Equilibrium borrowing revenues ($g(b, y)q(g(b, y), y)$). The figure considers income levels for which the borrower chooses not to default (for $b=-0.23$).

5 Conclusions

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