

# Aggregate Implications of Employer Search and Recruiting Selection\*

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November 2008

## Abstract

This paper develops a general equilibrium model of nonsequential employer search with recruiting selection and heterogeneous workers, and characterizes its equilibrium. I depart from the standard search model by allowing firms to simultaneously meet several applicants and choose the best candidate. Recruiting selection is important: firms interview a median of 5 applicants per vacancy and spend 2.5% of their total labor cost –about US\$4200 per recruit– in these activities.

The model provides an endogenous matching process with heterogeneous workers in which the hazard rate out of unemployment increases in productivity. The model also accounts for the empirical evidence of negative duration dependence of both hazard rates and re-employment wages. Under recruiting selection, lifetime inequality increases relative to the sequential search benchmark because low wage workers go through longer and more volatile unemployment spells, and have less valuable outside options to bargain with firms. I also show that stronger recruiting selection worsens the productivity of the unemployed and may not generate a more efficient job assignment at the aggregate level. Search frictions coupled with recruiting selection generate new kinds of externalities that affect not only transition probabilities, but also the expected productivity of recruited workers.

The calibrated model can replicate moments of the distribution of wages and unemployment durations in CPS data. Using this parametrization, I also show that an increase of screening costs reduces inequality and productive efficiency, and decreases negative externalities on other employers.

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\*I am very grateful to Mark Aguiar for his constant advice and encouragement; to William Hawkins for innumerable suggestions and comments; to Yongsung Chang for very useful discussions, and to the participants of seminars at University of Rochester, University of Chile, Central Bank of Chile and Catholic University of Chile. Last but not least, I thank my wife for being who she is.

**Keywords:** Employer search, Nonsequential search, Recruiting selection, Duration dependence, Hazard rate heterogeneity, Inequality, Search externalities.

# 1 Introduction

Firms devote considerable resources to recruiting new workers. After posting a job opening, employers typically evaluate resumes and conduct interviews to identify applicants' qualifications. In the National Employer Survey 1997 (NES97) firms report that they interview a median of 5 applicants per vacancy and spend 2.5% of their total labor cost in recruiting activities, with an average close to US\$4200 per recruited worker<sup>1</sup>. Even though such widespread recruiting activities affect outcomes of employers and workers in real labor markets, this feature is absent in most popular models of labor search. This paper embeds the recruiting selection process into an otherwise standard stationary general equilibrium search model of the labor market and characterizes its equilibrium.

I model the recruiting selection process as a *nonsequential* employer search strategy. Firms simultaneously meet various heterogeneous workers to fill vacancies and screen them to select the best. This approach nests the sequential search model as a particular case and solves several of its shortcomings. First, the sequential approach empirically runs into trouble to replicate the fact that several applicants are interviewed before filling a vacancy. Second, sequential employer search makes selection a random process, ignoring the role of workers' heterogeneity in the job assignment. In contrast, with a screening technology available, firms optimally evaluate several candidates at the same time; this allows them to increase the productivity of their hiring from among their applicant pool. Therefore, the model provides microfoundations for an endogenous matching process with heterogeneous workers. The approach is also justified empirically. Several papers<sup>2</sup> have documented that firms fill job openings by choosing an applicant from a pool that is formed shortly after the posting of the vacancy, and that almost no applications arrive afterwards. Vacancy durations are selection periods.

The recruiting selection model also generates important macroeconomic implications that benchmark models of sequential search (McCall 1970; Mortensen and Pissarides 1994) do not readily produce. My model qualitatively replicates and provides a simple explanation for cross-sectional features of CPS data. Specifically, the mean and variance of log re-employment wages decrease in unemployment duration, and the mean and variance of the unemployment duration are negatively correlated with log re-employment wages.

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<sup>1</sup>Reported statistics are controlled for nonresponse bias. A description of NES is in Cappelli (2001)

<sup>2</sup>Barron, Bishop, and Dunkelberg (1985), van Ours and Ridder (1992), Abbring and van Ours (1994), Weber (2000)

The economic environment of the model is deliberately simple to highlight the effect of recruiting selection. There is no aggregate uncertainty and jobs are homogeneous. Workers are risk-averse and heterogeneous only in time-invariant productivity. Their only decision is whether to submit a costly application per period. Since all vacant jobs are *ex ante* identical, in a symmetric equilibrium all firms receive the same expected number of applications.

To fill a vacancy, firms observe the number of applicants they receive and decide how many to interview considering screening costs. The employer perfectly observes the screened applicants' productivities and picks the best candidate. Rejected applicants remain in the unemployed pool. In equilibrium, the best workers get jobs faster on average and the unemployment pool worsens as the duration of unemployment increases. Therefore, the hazard rate out of unemployment –or job finding probability– decreases in productivity and the average productivity of workers is negatively correlated with the duration of unemployment.

Once a worker is chosen, the wage is determined by Nash-bargaining<sup>3</sup>. Because both the firm's profit and the value of worker's outside option increase in productivity, it is not readily clear that the most productive workers are also the most profitable. I show conditions for the existence of the symmetrical "Coincidence Ranking" equilibrium in which the productivity and profitability rankings are the same for all workers. In the steady-state of the model, the joint distribution of productivities and unemployment durations and the hazard rates are *equilibrium objects* that arise from the employers' recruiting selection technology, worker-selection strategies of firms, and the exogenous distribution of workers' productivity types.

The presented model is consistent with empirical evidence on hazard rates and wages. Several empirical studies<sup>4</sup> conclude that after controlling for observable variables and allowing for unobserved idiosyncratic heterogeneity, the negative duration dependence of the hazard rate disappears, i.e. elapsed duration does not negatively impact the job finding rate. Additionally, variables that are usually associated with higher earnings are also related to higher hazard rates. Both facts are consistent with the recruiting selection model because (i) high productivity workers find jobs more easily and (ii) the negative duration dependence disappears once productivity determinants are controlled for. Using data generated by my model, an econometrician who does not perfectly observe productivity will find some negative duration dependence. A closely related finding is that the unobserved heterogeneity accounts for a great deal of hazard rate variation

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<sup>3</sup>In Villena-Roldan (2008) using a similar framework, I study the implications of an alternative wage determination mechanism

<sup>4</sup>Machin and Manning (1999) (survey), Abbring et al. (2002) and Cockx and Dejemeppe (2005).

after controlling for elapsed duration and observables<sup>5</sup>. Instead of linking the unobserved heterogenous component to some idiosyncratic feature that affects the job finding rate, the recruiting selection model establishes a clear relationship between a worker's productivity and her hazard rate.

Related to this evidence, several empirical studies find that conditional on observables, re-employment wages exhibit significant negative duration dependence. Additionally, individuals who have experienced long unemployment durations tend to go through long unemployment spells frequently<sup>6</sup>. Stewart (2007) also finds that recurrence of unemployment spells is not only caused by past unemployment duration, but also by low past wages. The model has a simple explanation for all these phenomena: less productive workers have lower chances of being hired due to recruiting selection. In a labor market with search frictions, the recruiting selection model generates more lifetime inequality than a sequential search model. While in the latter case all workers are affected in the same way by the search frictions, in a model of recruiting selection low productivity workers experience longer unemployment spells and earn lower wages because their outside options are less valuable. Since low productivity workers face more volatile unemployment spells, under recruiting selection lifetime welfare inequality is even greater if workers are risk-averse.

In the recruiting selection model, some labor market outcomes have untraditional meaning. Indeed, positive unemployment rate plays a role for the efficient assignment of jobs that is absent in sequential search models. Since employers do not observe productivities before meeting applicants, only with some level of unemployment can firms meet several candidates to hire the best one. Another labor market outcome, the median unemployment duration, reflects the degree of firms' screening effort in this model. In the model, long unemployment durations are due to effective employer screening, which tends to reject low productivity applicants *given some distribution of unemployed workers*.

However, more screening is not always better. Private firms' screening and entry decisions generate externalities on other firms. As a side-effect of interviewing a large number of applicants and taking out the best, firms indirectly deteriorate the productivity of the unemployment pool and make it harder for other employers to find good workers. On the other hand, if more vacancies are posted, the number of expected applicants decreases so that firms cannot hire the best available workers very effectively. For this reason, the average productivity of the unemployment pool increases, with positive consequences for all employers. If other firms do not post too many vacancies, then a given firm may not find it worth posting either because the unemployment pool may consist mostly of low

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<sup>5</sup>See for instance, van den Berg and van Ours (1996), van den Berg and Ridder (1998), Machin and Manning (1999)

<sup>6</sup>See Omori (1997) and Gregg (2001).

productivity workers. If screening is very effective these externalities are strong enough to generate multiple equilibria.

In Section 4 I calibrate the model to target the unemployment rate, median number of interviewed applicants and the mean and variance of log wages in CPS data 1985-2006 for the whole labor market and for a segment of production workers. Using a reasonable parametrization, model-generated moments of the joint distribution of wages and unemployment durations qualitatively mimic their empirical counterparts in CPS data. However, because the simple model exaggerates the correlations observed in the data, I develop a slight variation of the model with stochastic screening costs that fits the data better. Using these calibrations, I also perform the counterfactual experiment of increasing the marginal cost of screening so that firms choose to search sequentially. This change substantially decreases both the average and the cross-sectional variation of welfare. However, because the higher screening cost shuts down selection, the losses due to lack of hiring selectivity are partially offset by the improvement of the unemployed pool. Considering that a more effective screening technology creates a worse unemployment pool composition, the increase in the marginal screening cost generates the largest welfare reduction in the economy with *less effective* screening technology.

The paper is organized as follows. Section 2 describes the model in detail and characterizes the most relevant equilibrium of the model. Section 3 discusses the model's implications. Section 4 contains a calibration and empirical evaluation of the model and runs some counterfactual experiments. The last section concludes.

## 2 The Model

In the model time is discrete and there is a continuum of homogenous risk-neutral firms or employers that post *ex ante* identical job vacancies. There is also a fixed mass of size 1 of workers who have a time-invariant productivity  $\theta$  from a exogenous distribution with density  $f_\theta(\theta)$ . Workers cannot borrow or save. Productivity  $\theta$  and unemployment duration  $\delta$  characterize each worker. I use the convention that  $\delta = 0$  represents an employed worker and  $\delta > 0$  a worker who became unemployed  $\delta$  periods ago.  $f(\theta, \delta)$  stands for the joint density of productivities and unemployment durations in the economy. Since all jobs are *ex ante* identical, the specific assignment of workers to jobs is not relevant. The general state of the economy is denoted by  $X = (A, V, f(\theta, \delta))$ , where  $A$  represents the mass of unemployed applicants in the economy,  $V$  is the mass of aggregate vacancies. This paper focuses on the symmetric steady-state equilibrium of this economy. Henceforth, to ease notation the aggregate state  $X$  is dropped.

## 2.1 Worker's Problem

At the beginning of the period, all unemployed workers receive an exogenous income  $b$  and decide whether to submit an application costing  $bz$  with  $0 < z < 1$ . By doing so, a worker is randomly allocated into some firm's applicant pool and faces an *equilibrium probability* (hazard rate)  $\pi(\theta)$  of being hired. In case of obtaining the job, the worker gets the value of being employed  $W(\cdot)$  earning a wage  $\tilde{w}(\theta)$  bargained with a prospective employer that will remain constant during the employment spell. No wage offers will be rejected in equilibrium. If the worker receives no offers, she increases her unemployment spell by one unit of time and faces the participation decision again. If the worker chooses not to send the application, she cannot get a job and consumes her whole unemployment income  $b$ .

Workers value consumption of every period according to an increasing, concave and  $C^2$  utility function  $u(\cdot)$  and have a constant discount factor  $\beta$ . Hence, an unemployed worker's lifetime utility is  $Q(\theta, \tilde{w}(\theta)) = \max\{Q_a(\theta, \tilde{w}(\theta)), Q_n\}$ , where  $Q_a(\cdot)$  stands for the value of submitting an application and is given by

$$Q_a(\theta, \tilde{w}(\theta)) = u(b(1 - z)) + \beta\pi(\theta)W(\tilde{w}(\theta)) + \beta(1 - \pi(\theta))Q(\theta, \tilde{w}(\theta))$$

and  $Q_n$  is the value of not submitting an application given by

$$Q_n(\theta, \tilde{w}(\theta)) = u(b) + \beta Q(\theta, \tilde{w}(\theta))$$

When hired, the worker produces her productivity  $\theta$  and receives a bargained wage  $w(\theta)$ . Before the period ends, the worker faces the exogenous chance of separation,  $\eta$ . Note that  $w(\theta)$  is the wage bargained with the employer who is making an offer, as opposed to  $\tilde{w}(\theta)$ , which is the wage to be bargained with some other prospective employer. The value of a type  $\theta$  worker employed at wage  $w(\theta)$  is

$$\begin{aligned} W(w(\theta)) &= u(w(\theta)) + \beta((1 - \eta)W(w(\theta)) + \eta Q(\theta, \tilde{w}(\theta))) \\ &= \frac{u(w(\theta)) + \beta\eta Q(\theta, \tilde{w}(\theta))}{1 - \beta(1 - \eta)} \end{aligned} \quad (1)$$

In the steady state, the hazard rate  $\pi(\theta)$  does not change because the aggregate state  $X$  is always the same. It follows that a worker of productivity type  $\theta$  will always make the same optimal decision. Hence, by substituting (1) and rearranging, the utility of applying can be written as

$$\begin{aligned} Q_a(\theta, \tilde{w}(\theta)) &= \frac{1}{1 - \beta} [S(\theta)u(b(1 - z)) + (1 - S(\theta))u(\tilde{w}(\theta))] \\ \text{with } S(\theta) &\equiv \frac{\beta^{-1} - 1 + \eta}{\beta^{-1} - 1 + \eta + \pi(\theta)} \end{aligned} \quad (2)$$

Using the same reasoning, the lifetime utility of not applying is expressed by

$$Q_n = u(b)/(1 - \beta) \quad (3)$$

Let  $a(\theta)$  be the optimal participation policy. There is a threshold productivity  $\underline{\theta}$  such that  $Q_a(\underline{\theta}, \tilde{w}(\underline{\theta})) = Q_n$ . For all  $\theta \geq \underline{\theta}$ ,  $a(\theta) = 1$ . Otherwise,  $a(\theta) = 0$ .

## 2.2 Firm's Problem

Every period, having observed the aggregate state  $X$ , potential employers optimally create vacancies by paying a fixed cost  $\kappa$ . Vacancies receive a random number of  $K$  applications drawn from the distribution of unemployed workers  $\tilde{F}(\theta) = F(\theta|\delta > 0, a(\theta) = 1)$ . That a vacancy can simultaneously meet multiple applicants is the key difference from sequential search models.

Once the number of  $k$  applicants is realized, the firm optimally decides how many applicants to screen, considering that each interview costs  $\xi \geq 0$ . If no applicants arrive, the firm posts a vacancy again in the next period. From the pool of screened candidates, the employer either chooses the most profitable worker or posts a vacancy again in the next period. To keep the focus on the consequences of recruiting selection, once the firm pays the cost of screening an applicant, her productivity is totally revealed to the firm. Hence, the employment history of the worker holds no informational value for the employer.

Equation (4) states the value of posting a vacancy  $P$  and equation (5) represents the lifetime profits for a firm that has received  $k$  applicants.  $J(\theta)$  stands for expected the present value of profit stream generated by a worker of productivity  $\theta$ .

$$P = -\kappa + \beta \sum_{k=0}^{\infty} Pr(K = k) \max \{P, G(k)\} \text{ with} \quad (4)$$

$$G(k) = \max_{i \leq k} \left\{ \mathbb{E} \left[ \max_j \{J(\theta_j)\}_{j=1}^i | \delta > 0, a = 1, K = i \right] - \xi i \right\} \quad (5)$$

The firm and the top applicant bargain on a contract with constant wage  $w(\theta)$ . No commitment issues arise because both sides perfectly observe productivity. Workers generate profits of  $\theta - w(\theta)$  every period unless an exogenous separation shock hits the match with probability  $\eta$ . If the match is destroyed, the worker becomes unemployed and the job becomes vacant. Then,  $J(\theta)$  can be written as

$$\begin{aligned} J(\theta) &= \theta - w(\theta) + \beta((1 - \eta)J(\theta)) + \eta P \\ &= \frac{\theta - w(\theta) + \beta \eta P}{1 - \beta(1 - \eta)} \end{aligned} \quad (6)$$

Employers post vacancies in a free-entry market, so that  $P = 0$  in equilibrium. Moreover, the firm will never choose to reject all applicants and repost a vacancy since no applicant whose value to the firm is lower than  $P$  will ever submit a costly application.

I characterize the problem of (4) conditional on a distribution of unemployed workers  $\tilde{F}(\theta)$ . I denote the expected value of hiring the best worker out of  $k$  arrived applicants as

$$\tilde{J}(k) = \mathbb{E}[\max\{J(\theta_j)\}_{j=1}^k | \delta > 0, a(\theta) = 1, K = k] = \int J(\theta) k \tilde{F}(\theta)^{k-1} \tilde{f}(\theta) d\theta$$

The concavity of  $\tilde{J}(k)$  is intuitive: as the number of applicants increases it becomes progressively harder for a marginal applicant to be better than all the others. This result is formally established in Lemma 1.

**Lemma 1** *For a given distribution of unemployed workers  $\tilde{F}(\theta)$ , the expected lifetime profit conditional on receiving  $k$  applicants,  $\tilde{J}(k)$ , is strictly increasing and strictly concave in  $k$ , that is,  $\tilde{J}(k) - \tilde{J}(k-1) < \tilde{J}(k-1) - \tilde{J}(k-2)$ .*

**Proof.** See Appendix A ■

Using Lemma 1 and some other mild conditions, the firm's optimal policy for problem (5) is to set  $g^*(k) = \arg\max G(k) = \min\{k, i^*\}$ . The employer interviews all the applicants if their number is not greater than  $i^*$ . If the actual number of applicants surpasses  $i^*$ , the firm randomly selects  $i^*$  of them to interview

**Lemma 2** *For a given  $\tilde{F}(\theta)$ , if the expected profit obtained by interviewing one applicant is positive and finite, i.e.  $0 < \tilde{J}(1) - C(1) \equiv M(1) < \infty$  and  $\xi > 0$ , there is a finite integer  $i^* > 0$  such that a firm receiving  $k \geq i^*$  applicants optimally screens  $i^*$  of them, while a firm receiving  $k < i^*$  applicants optimally screens all of them.*

**Proof.** See Appendix A ■

Intuitively, provided the function  $\tilde{J}(1)$  exceeds the marginal screening cost  $\xi$ , the total cost function  $C(k) = \xi k$  surpasses the expected profits function  $\tilde{J}(k)$  at some point. Since the difference between the two is a concave function, a maximum is reached at some finite number of applicants. If  $\xi = 0$  then all applicants are interviewed, so  $i^* = +\infty$ . If  $\xi > \tilde{J}(1)$  then no applicants are interviewed and  $i^* = 0$ .

Using the results from Lemma 2, and the free-entry condition in (4), the average number of workers per vacancy  $\lambda$  is determined by

$$0 = -\kappa + \beta \sum_{k=0}^{\infty} \frac{e^{-\lambda} \lambda^k}{k!} \max\{\tilde{J}(k) - \xi k, \tilde{J}(i^*) - \xi i^*\} \quad (7)$$

In Proposition 3, given a distribution  $\tilde{F}(\theta)$ , I show there is a unique  $\lambda$  (and a mass of vacancies  $V$ ) that is consistent with free-entry in (7) if the expected profits obtained



are high enough to pay the cost of posting a vacancy  $\kappa$ . The result follows because the vacancy posting value  $P$  is strictly increasing in  $\lambda$ .

**Proposition 3** *For a given  $\tilde{F}(\theta)$ , there is a unique mean number of applicants per vacancy  $\lambda$  that is consistent with the free-entry condition if  $M_{i^*} = \tilde{J}(i^*) - \xi > \kappa/\beta$ .*

**Proof.** See Appendix A ■

## 2.3 Wage determination

A natural framework for wage determination is Nash bargaining. Once the firm has chosen some worker, both sides negotiate about how to split the generated surplus. For the worker, the outside option is the value of being unemployed next period. For the firm, the outside option is to post vacancies again under aggregate conditions  $X$ . Since firms are identical, all of them offer the same wage schedule. As a consequence, bargained wages are never turned down in equilibrium for two reasons. First, a worker can send only one application per period. If a worker could contact two or more firms at the same time, then it would be possible for a worker to receive multiple offers, in which case she would necessarily have to reject all but one of them. Such a situation, though interesting, is not analyzed in this paper. Secondly, because the value of a match is completely determined by the productivity of the worker, the expected utility obtained from a subsequent match is the same obtained at the current match. The Nash axiomatic solution solves the following problem.

$$\begin{aligned} \max_{w(\theta)} \{ & (W(w(\theta)) - Q(\theta, \tilde{w}(\theta)))^\alpha ((J(\theta) - P))^{1-\alpha} \} \\ \text{subject to } & W(w(\theta)) - Q(\theta, \tilde{w}(\theta)) \geq 0 \text{ and } J(\theta) - P \geq 0 \end{aligned}$$

Substituting equations (1), (2), and (6), and using the free entry condition  $P = 0$  for an interior solution the first order condition is

$$\begin{aligned} \frac{\alpha}{1-\alpha}(\theta - w(\theta))u'(w(\theta)) &= u(w(\theta)) - (1-\beta)Q_a(\tilde{w}(\theta), \theta) \\ &= u(w(\theta)) - S(\theta)u(b(1-z)) - (1-S(\theta))u(\tilde{w}(\theta)) \end{aligned} \quad (8)$$

The worker wants to keep the match (i.e, the solution is interior) only if  $w(\theta) > b(1-z)$ . Because of the free-entry condition, employers will hire the worker as long as  $\theta > w(\theta)$ . Since a worker who obtains less than  $b$  would never choose to participate, the match is accepted for both sides if and only if  $\theta > w(\theta) > b(1-z)$ .

Being *ex ante* identical, all employers bargain in the same way and therefore  $w(\theta) = \tilde{w}(\theta)$  in equilibrium. Substituting this into (8) and doing some algebra yields

$$(\theta - w(\theta))B(\theta) = u(w(\theta)) - u(b(1 - z)) \quad (9)$$

$$\text{with } B(\theta) = \frac{\alpha}{1 - \alpha} u'(w(\theta)) S(\theta)^{-1} = \frac{\alpha}{1 - \alpha} u'(w(\theta)) \left( 1 + \frac{\pi(\theta)}{\beta^{-1} - 1 + \eta} \right)$$

The expression  $B(\theta)$  stands for the worker's overall bargaining power. The productivity type  $\theta$  has two opposing effects. On one hand, higher productivity increases the total surplus and decreases the marginal utility of consumption of workers. On the other hand, a high productivity implies a high hazard rate  $\pi(\theta)$  and a high value outside option. It is not clear what is the best firm hiring policy because a high productivity  $\theta$  may not imply higher profits. The profitability ranking of applicants determines the hazard rate in equilibrium, which in turn affects the worker's outside option value and wage determination. The approach I take to solve the model is to *conjecture* a optimal hiring policy for firms and compute the implied distribution of unemployed workers  $\tilde{F}(\theta)$ . Then, I verify that the conjecture is actually an optimal employer policy. A natural guess is a “coincidence ranking” equilibrium in which a firm's profits are increasing in the productivity  $\theta$  of the hired worker.

## 2.4 Matching and Equilibrium probabilities

To derive a microfounded matching mechanism with heterogenous workers, consider an economy with finite number of applicants  $N_a$ . Since all employers are identical and observe  $X$ , they optimally post  $N_v$  vacant jobs per period. For the same reason, workers randomly sort themselves into vacancies. Thus, the probability that the application of a given worker arrives to a given vacancy is  $1/N_v$ . Thus, the number of applications arrived to a vacancy,  $K$ , follows a binomial distribution.

$$Pr(K = k) = \binom{N_a}{k} (1/N_v)^k (1 - 1/N_v)^{N_a - k}$$

In the interesting case, as the number of workers increases, the number of applicants per vacancy  $N_a/N_v$  converges to  $A/V = \lambda$ . The mass of unemployed workers who send an application is computed as  $A = \sum_{i=1}^{\infty} \int a(\theta) f(\theta, \delta = i) d\theta$  and  $V$  is the mass of posted vacancies. Then, by standard arguments, the number of applicants received per vacancy  $K$  converges to a Poisson distribution with mean  $\lambda = A/V$ .

The matching process differs from the typical urn-ball approach in which coordination failure creates unemployment (see Petrongolo and Pissarides (2001)). That argument works well if firms do not care about which ball they accept. If balls are heterogenous,

firms have incentives to select which ball to pick. Although coordination failure of workers still plays a role for unemployment (bad luck of being sorted into a vacancy with strong competitors or vice versa), recruiting selection is the main determinant of job assignment.

From the worker's point of view, the chance of being in an application pile of size  $k$  equals the probability of that  $k - 1$  workers arrive to the same pile. Hence, conditional on a worker being there, the chance of being in an application pile of size  $k$  equals  $\frac{\lambda^{k-1}e^{-\lambda}}{(k-1)!}$ .

To derive the probabilities of being hired, we need to know the optimal firm hiring policy. The coincidence ranking equilibrium conjecture (or any other) implies a specific functional form of the hazard rate as a function of  $\theta$ . As a first step, consider the case when a vacancy receives only one applicant; she gets hired for sure.

Secondly, if the worker arrives to a pool with  $k < i^*$  competitors, she gets hired as long as her  $\theta$  is greater than the productivities of the other applicants in the vacancy. Since all workers are independently drawn from the distribution of unemployed workers, the chance of being hired in that case is  $\tilde{F}(\theta)^{k-1}$ .

If the number of applicants exceeds  $i^*$ , the chance of being randomly selected into the screened group is  $i^*/k$ . Thus, the chance of being recruited conditional on  $k$  applicants arriving is  $\tilde{F}(\theta)^{i^*} i^*/k$ . The equilibrium probability of receiving an offer is obtained by using Bayes' total probability law

$$\pi(\theta) = \sum_{k=1}^{i^*} \frac{\lambda^{k-1}e^{-\lambda}}{(k-1)!} \tilde{F}(\theta)^{k-1} + \sum_{k=i^*+1}^{\infty} \frac{\lambda^{k-1}e^{-\lambda}}{(k-1)!} \tilde{F}(\theta)^{i^*-1} \frac{i^*}{k} \quad (10)$$

In the special case in which all applicants are interviewed ( $i^* = \infty$ ) the hazard rate becomes  $\pi(\theta) = e^{-\lambda(1-\tilde{F}(\theta))}$ .

An important result is that the average hazard rate does not depend on  $i^*$  nor on the specific shape of the distribution of unemployed workers  $\tilde{F}(\theta)$ .

**Lemma 4** *The expected value of the hazard rate  $\mathbb{E}[\pi(\theta)]$  equals  $\frac{1-e^{-\lambda}}{\lambda}$  for all  $i^* > 0$  and for all well-defined distributions of unemployed workers  $\tilde{F}(\theta)$ .*

**Proof.** See Appendix A ■

The lower the value of  $i^*$ , the flatter becomes the shape of the hazard rate across productivities. With a low  $i^*$ , screening becomes less intensive and unemployment probabilities depend more on coordination failure –or plain luck– than on selection. Since the mean hazard rate only depends on the average number of people interviewed, the result in Lemma 4 allows the model to accommodate different shapes of the hazard rate and unemployment duration without affecting the unemployment rate  $\mathcal{U}$ . The hazard rate depends only on the *relative ranking* of unemployed workers throughout  $\tilde{F}(\theta)$ .

In steady state,  $\lambda = A/V$  is the mean number of applicants per vacancy that equalizes job creation and job destruction. Since every applicant can be hired in equilibrium, job creation is the measure of vacant jobs that have received at least one applicant. Job destruction is the mass of employed workers who had a separation shock. A double equality arises by using Lemma 4

$$\frac{\eta(1 - \mathcal{U}^*)}{\mathcal{U}^*} = \frac{1 - e^{-\lambda}}{\lambda} = \mathbb{E}[\pi(\theta)] \quad (11)$$

We can see from these equalities that if  $0 < \lambda^* < \infty$ , the unemployment rate is bounded away from 0 and from 1. There is positive relationship between the mean number of applicants and the equilibrium unemployment rate. When  $\lambda$  is large, the number of hired workers is small. As in sequential search models such as Shimer (2005), the aggregate unemployment rate is completely determined by the average job-finding rate, which in turn depends only on  $\lambda$ .

## 2.5 Endogenous distribution of workers

So far the analysis has been in partial equilibrium because the distribution of unemployed workers  $\tilde{F}(\theta)$  is given. However the recruiting selection process itself affects the distribution of unemployed workers. To close the model, I show in this section how this distribution is endogenously determined in equilibrium. I also show that for every well-defined hazard rate function  $\pi(\theta)$  and a marginal density of productivity types  $f_\theta(\theta)$  there exists a unique invariant density of productivities and durations  $f(\theta, \delta)$

In steady state, the mass of workers with unemployment duration  $\delta + 1$  is the mass of workers of duration  $\delta$  who do not find a job. Considering that workers who do not apply so that  $(a(\delta) = 0)$  have no chance of being hired,

$$f(\theta, \delta + 1) = (1 - \pi(\theta)a(\theta))f(\theta, \delta) \quad \forall \delta \geq 1 \quad (12)$$

Individuals of duration 1 are those who just had a separation shock. Employed workers are those who were not hit by the separation shock and those who find a job regardless of duration. The laws of motion for agents with  $\delta = 0$  or  $\delta = 1$  are given by

$$f(\theta, 1) = \eta f(\theta, 0) \quad (13)$$

$$f(\theta, 0) = (1 - \eta)f(\theta, 0) + \pi(\theta)a(\theta) \sum_{i=1}^{\infty} f(\theta, \delta = i) \quad (14)$$

Doing some algebra and recalling that  $f_\theta(\theta)$  is the marginal distribution of productivities and that  $f_\theta(\theta) - f(\theta, 0) = \sum_{i=1}^{\infty} f(\theta, \delta = i)$  I obtain that

$$f(\theta, \delta > 0) = f_\theta(\theta) - f(\theta, 0) = \frac{\eta f_\theta(\theta) a(\theta)}{\eta + \pi(\theta)}$$

Since the equilibrium unemployment rate  $\mathcal{U} = f(\delta > 0 | a(\theta) = 1)$  is well-defined for a positive finite  $\lambda$ , the cumulative distribution function of productivities of unemployed workers is

$$\tilde{F}(\theta) = \int_{-\infty}^{\theta} f(v | \delta > 0, a(\theta) = 1) dv = \int_{-\infty}^{\theta} \frac{\eta f_{\theta}(v)}{\mathcal{U}^*(\eta + \pi(v))} dv \quad (15)$$

This is a Volterra nonlinear integral equation whose solution is the endogenous cumulative distribution function of the unemployed workers. Given an optimal hiring policy consistent with a hazard function  $\pi(\theta)$ , the next Proposition establishes its existence and uniqueness by means of a fixed point argument under mild regularity conditions.

**Proposition 5** *There exists a unique steady-state joint distribution of productivities and durations  $F(\theta, \delta)$  with unique density  $f(\theta, \delta)$  provided that  $\sup_{\theta} f_{\theta}(\theta) < \infty$*

**Proof.** See Appendix A ■

Proposition 5 is a partial characterization of the equilibrium because the hazard rate depends on the average number of applicants per vacancy  $\lambda$ , which in turn depends on  $F(\theta)$  through the free entry condition in 7. To resolve this circularity, the coincidence ranking conjecture needs verification. I discuss this point in the following Section.

## 2.6 Stationary Symmetric Recursive Equilibrium

The Stationary Symmetric Recursive Equilibrium of this model equilibrium is defined as

- i. A set of value functions  $Q(\theta, \tilde{w}(\theta))$ ,  $Q_a(\theta, \tilde{w}(\theta))$ ,  $Q_n$ ,  $W(w(\theta))$ ,  $P$  and  $J(\theta)$  defined in equations (2), (3), (1),(4) and (6) that reflect optimal choices by workers and firms given an aggregate state  $X$ .
- ii. Policy functions  $a^*(\theta)$  and  $g^*(k)$  that solve the worker's application problem in  $Q(\theta, \tilde{w}(\theta))$  and the firm's number of screened applicants in  $G(k)$ , given an aggregate state  $X$ .
- iii. Equilibrium hazard rate functions  $\pi(\theta)$  as described in equation (10) conditional on the aggregate state  $X$ .
- iv. A wage schedule  $w(\theta)$  that solves condition (8) given  $X$ .
- v. An aggregate state  $X = (A, V, f(\theta, \delta))$  consistent with individuals' behavior and free entry  $P = 0$ , in which  $A$  is defined according to  $a^*(\theta)$  and  $f(\theta, \delta)$  is defined as in equation (15).

## 2.7 Characterization of the equilibrium

The equilibrium of the model generates several features of the cross-sectional distribution of wages and unemployment durations that are characterized in the following propositions<sup>7</sup>. In the first result, workers with long unemployment durations decrease their wages in expectation. This is a direct consequence of the recruiting selection process.

**Proposition 6** *Conditional on the duration of an unemployment spell, the mean and variance of the wage are decreasing in this duration.*

1. Given a distribution  $\tilde{F}(\theta)$ , for all  $\delta > 0$  and  $a(\theta) = 1$ ,  $\mathbb{E}[w(\theta)|\delta] > \mathbb{E}[w(\theta)|\delta + 1]$  and  $\mathbb{V}[w(\theta)|\delta] > \mathbb{V}[w(\theta)|\delta + 1]$
2. If  $\delta = 0$ ,  $\mathbb{E}[w(\theta)|\delta] = \mathbb{E}[w(\theta)|\delta + 1]$  and  $\mathbb{V}[w(\theta)|\delta] = \mathbb{V}[w(\theta)|\delta + 1]$

**Proof.** See Appendix A ■

The second result shows that expected durations are longer for individuals of low productivity.

**Proposition 7** *The expected duration conditional on the productivity type is*

$$\mathbb{E}[\delta|\theta] = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)} \quad (16)$$

*which is a differentiable and strictly decreasing function in  $\theta$ .*

**Proof.** See Appendix A ■

Since for a given worker the probability of leaving unemployment is  $\pi(\theta)$ , the duration of unemployment follows a geometric distribution conditional on  $\theta$ . For that reason, Proposition 7 shows that the unconditional duration of unemployment is the probability of being unemployed  $\frac{\eta}{\eta + \pi(\theta)}$  multiplied by the expected duration conditional on being unemployed  $\frac{1}{\pi(\theta)}$ . Using the same rationale, the variance of the duration conditional on being unemployed is  $\frac{1}{\pi(\theta)^2}$ . The next result concerns the unconditional variance of unemployment durations

**Proposition 8** *The variance of the unemployment duration conditional on the productivity type is*

$$\mathbb{V}[\delta|\theta] = \frac{\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta + \pi(\theta))^2\pi(\theta)^2} \quad (17)$$

*which is a differentiable and strictly decreasing function in  $\theta$ .*

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<sup>7</sup>The propositions obviously hold only for individuals in the labor force ( $a(\theta) = 1$ )

**Proof.** See Appendix A ■

A pending point is verifying that the conjectured Coincidence Ranking (CR) firms' hiring policy is truly optimal. The equilibrium is verified if  $J'(\theta) > 0$  for all  $\theta$  in a computed numerical solution. Unfortunately, sufficient conditions that only rely on primitives are not possible to obtain, because they depend on  $\tilde{F}(\theta)$ , which does not have an analytical solution. Nevertheless, it is possible to get some insights about the kind of primitives that generate such an equilibrium, which in turn, enhances our understanding of the model. In Appendix B, I obtain –for a given  $\tilde{F}(\theta)$  and  $\lambda$ – a “partial equilibrium” condition for  $J'(\theta)$  to be positive. A first conclusion is that the wage function  $w(\theta)$  is always increasing in productivity. Secondly, I show that for the specific case in which screening applicants is free, so that  $i^* = +\infty$ , the coincidence ranking equilibrium exists if the following inequality holds

$$\lambda \tilde{f}(\theta) < \frac{1 - \alpha}{\alpha \pi(\theta)(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right) \quad (18)$$

Condition (18) says that in order to have a CR equilibrium we need high enough dispersion of productivity for unemployed workers, i.e. low enough values for the density of the unemployed at every  $\theta$ . The shape of  $\tilde{f}(\theta)$  depends on the population's productivity dispersion and the screening technology. As discussed, the relative productivity ranking of workers is what determines the hazard rate. If the dispersion of productivities is small, a great difference in hazard rates can be due to a small increase in productivity. The outside option of a worker rises much more than her productivity does, making it unprofitable for the firm to hire high productivity workers.<sup>8</sup> On the other hand, workers' risk aversion plays a role for the existence of the CR equilibrium because the worker's share of the surplus decreases in wages. In order to have this equilibrium, the effect of low productivity dispersion may be overcome by the fact that the employer yields a share of the surplus that is decreasing in wages and productivity.

Additionally, a high  $\lambda = A/V$  –or equivalently a high unemployment rate– makes it difficult for a coincidence ranking equilibrium to exist. Intuitively, if the average number of applicants per vacancy is high, available jobs are scarce and it is extremely easy for very good workers to get hired. Since the top applicant's outside option is too high, firms have to yield almost all the surplus to the worker, which creates incentives to hire someone else. The greatest difficulty in having a coincidence ranking equilibrium is precisely achieved when  $i^* = +\infty$ , because top applicants have the highest outside option value. Hence,

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<sup>8</sup>There is a similar effect in Shimer (1999), but he shows that in his auction wage determination mechanism wages can be locally decreasing in productivity to achieve a coincidence ranking equilibrium in spite of the reduced local variability of productivities. Due to the Nash bargaining framework I assume here, locally decreasing wages are not possible.

although condition (18) is derived for a special case  $i^* = +\infty$ , if the equilibrium exists under that scenario, it exists for a finite  $i^*$ .

## 3 Discussion

### 3.1 Sequential vs. Nonsequential search

In this subsection, I present some arguments to show that a nonsequential search model circumvents some problematic theoretical and empirical consequences of the sequential approach.

The sequential model is a particular case of the recruiting selection model in which the marginal cost of interviewing is high enough to make firms optimally screen one applicant at most, i.e.  $i^* = 1$ . In this case, the firm's problem collapses to a simpler hiring problem. Employers receive at least one application with probability  $e^{-\lambda}$ , randomly select one worker and obtain the average lifetime profits associated to the productivity of the hired worker,

$$P = -\kappa + \beta [e^{-\lambda}P + (1 - e^{-\lambda})\mathbb{E}[J(\theta)|\delta > 0, a = 1]]$$

The profits from hiring a worker are just those of hiring a worker chosen at random from the unemployment pool. Firms' optimal hiring policy consists of making offers to applicants of productivity above some threshold  $\underline{\theta}$ . All workers with productivity greater than  $\underline{\theta}$  face the same hazard rate; for all productivities below  $\underline{\theta}$ , there is no chance of being hired. Consequently, in equilibrium no worker below that threshold submits a costly application. Thus, the partial equilibrium model suggested by Burdett and Cunningham (1998) –essentially the sequential model just presented– cannot explain why a candidate is not hired if the cost of application is arbitrarily low but positive. On the other hand, if applications are free ( $z = 0$ ), firms receive unprofitable applicants and interview multiple workers before filling the vacancy. However, the number of candidates interviewed in the data makes this implausible. According to NES97 data, roughly a median of 4 out of 5 applicants are not hired by any firm in the economy. Consequently, the steady-state unemployment rate would be enormous.

The sequential search model, when augmented with firm/match heterogeneity in the matching process, may replicate the number of applicants per vacancy observed in the data. In such a setup matches are rejected if the received shocks are not good enough, which explains that firms interview multiple candidates. However, estimated structural search models find that workers rarely turn down job offers and that the acceptance rate does not significantly vary with workers characteristics (van den Berg 1990; Devine and Kiefer 1991; Eckstein and van den Berg 2007). Barron et al. (1997) and Andrews et al.



(2008) advocate the importance of employer search behavior in light of this evidence. Indeed, a sequential model with firm/match heterogeneous productivity replicates the data if workers on average turn down no less than 80% of job offers so that firms interview roughly 5 applicants in expectation, which is a realistic number according to NES97.

An objection to the previous argument is that whether the firm or the worker rejects the match is generally irrelevant in matching models because both sides will always agree or disagree on the acceptability of the match. However, some direct measures of job offer rejection are available. Lollivier and Rioux (2005) report that 8% of unemployed workers have rejected one or more offers in a month in France. Even Blau (1992), the most unfavorable evidence for my arguments I find in the literature, reports a job acceptance probability around 50%, implying two interviews per vacancy in a sequential search environment. In addition, Lollivier and Rioux (2005) and Blau (1992) find that the number of self-reported job offers substantially increases in education, as predicted by the recruiting selection model.

Since in the sequential search model firm/match heterogeneity is the driving force generating a realistic number of interviews, this mechanism is exposed to the criticism of Hornstein, Krusell, and Violante (2007). These authors point out that, under a realistic match/firm heterogeneity dispersion, workers should turn down an implausible large number of job offers because the option value of waiting for a good draw is huge. This argument puts even more strain on the sequential search model because the amount of match/firm productivity dispersion is only compatible with very low job offer arrival rates. The latter implies a ridiculously large number of interviewed candidates.<sup>9</sup>

Another argument in favor of nonsequential search is the empirical evidence of van Ours and Ridder (1992), who show that applications' arrival concentrates shortly after the vacancy is posted. A sequential employer strategy is only consistent with evenly spaced applications on average. Abbring and van Ours (1994) propose a different empirical test to distinguish among sequential and nonsequential employer search strategies. If the sequential case is prevalent we should observe that the duration of the vacancy decreases in the number of applicants per vacancy. Under sequential search, the position is filled if a sufficiently high mutually acceptable match quality realizes. When the number of interviews is large, it is more likely that such realization occurs. In contrast, in the nonsequential case, we should observe that a greater number of applicants does not decrease the vacancy duration. In fact, it is even possible that the vacancy spell increases

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<sup>9</sup> Hornstein et al. (2007) conclude that an unemployment duration of 91 months is compatible with the empirical firm/match heterogeneity dispersion in the data. This exorbitant number suggests that more than 540 candidates are interviewed per vacancy in that employers make at least 1.5 interviews per week according to NES97.

because employers need more time to interview candidates. Using a similar argument, Abbring and van Ours (1994) find evidence for nonsequential search. In line with this, there is a significant positive correlation between vacancy durations and the number of applicants in NES97 data.<sup>10</sup> Thus, the recruiting selection model accounts for empirical evidence that is hard to reconcile with sequential employer search.

The recruiting selection model is related to the literature of nonsequential search models, whose tradition goes back to the seminal work of Stigler (1961), Wilde (1977) and Burdett and Judd (1983). However, empirical and theoretical contributions focus on the worker’s side: after submitting multiple applications, the workers choose the best offer received from employers<sup>11</sup>. To my knowledge, only Shimer (1999) share an employer-side viewpoint that is similar to this paper<sup>12</sup>. He also proposes a matching mechanism for *ex ante* identical employers and heterogenous workers. He suggests job auctions as a wage setting mechanism, while I assume a standard Nash-bargaining coupled with screening technology, which makes easier to compare to the rest of the literature. In Shimer’s paper there is no screening because workers self-reveal their types while bidding. In contrast, I assume firms devote resources to obtain information on prospective employees. In addition, I stress the consequences of the recruiting technology in aggregate labor market outcomes, and quantitatively compare the model’s predictions to the data.

### 3.2 Unobserved heterogeneity and negative duration dependence

Another shortcoming of the simplest employer sequential model is that hazard rates out of unemployment are the same for all workers’ productivities. Most studies on unemployment duration show that unobserved heterogeneity component accounts for a high proportion of the hazard variation (see for instance, Machin and Manning (1999), van den Berg (2001))<sup>13</sup>. Empirical studies try to disentangle true duration dependence of the probability of leaving unemployment from compositional effects. Allowing for unobserved hazard rate heterogeneity takes into account that individuals with intrinsic low hazard rates (or longer unemployment spells) tend to be over-represented in the stock of unem-

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<sup>10</sup>Non reported estimations of Mixed Proportional Hazard rate models for NES97 vacancy durations, show that (i) the vacancy filling hazard rate decreases in the number of applications, and (ii) the magnitude and significance of the effect holds for different choices of baseline hazard rates, unobserved heterogeneity distributions and covariates.

<sup>11</sup>See for instance Stern (1989), Acemoglu and Shimer (2000), Kandel and Simhon (2002) and Albrecht, Gautier, and Vroman (2006)

<sup>12</sup>I thank Daron Acemoglu for pointing this out.

<sup>13</sup>These findings typically derive from the estimation of Mixed Proportional Hazard models in which the hazard rate is decomposed into three multiplicative components: a baseline hazard common to all individuals, observable and unobservable heterogeneity.

ployed. In contrast, in this paper the hazard rate heterogeneity naturally arises from the interaction of workers' productivities and the recruiting technology of firms, rather than simply assuming an idiosyncratic exogenous worker characteristic.

In the model, negative duration dependence of the hazard rate in the data arises due to the imperfect observation of workers' productivities. My model suggests that as productivity heterogeneity is better captured by covariates, the negative duration dependence should fade away. Empirical evidence<sup>14</sup> indeed shows that after controlling for worker's observed and unobserved heterogeneity, the negative duration dependence pattern weakens or vanishes. In addition, consistent with the recruiting selection model, covariates that are associated to higher earnings are also related to higher hazard rates (with the exception of age).

Allowing for match/firm heterogeneity does not help the sequential model to explain the mentioned evidence. Long unemployment spells are due to high reservation wages, which in turn are positively related to high wages. To avoid this counterfactual implication, most sequential models allow for some non-stationarity such as decreasing unemployment benefits, and negative duration dependence of arrival rates and re-employment wages. Three leading examples are "stigma" effect of long unemployment spells, human capital depreciation and unemployment benefits exhaustion. In the first case, under imperfect observability of workers' productivity, a long duration becomes a bad signal encompassing information of multiple application rejections<sup>15</sup> (Lockwood 1991; Kollman 1994). In these models, firms' hiring selection creates informational externalities. In contrast, my paper isolates the importance of selection by shutting down informational issues. Since every model of stigma has employers selectively recruiting workers, if unemployment duration is important *per se* for firm's hiring decision, then it should generate observationally different testable implications. To my knowledge, there is no such a distinctive prediction. To illustrate the point, consider the claim that stigma models predict that the informational content of long unemployment spells is greater in a boom than in a recession. Indeed, empirical evidence shows that past unemployment duration is a better predictor of current duration when the joblessness occurred during a recession (Omori 1997). However, this finding is also consistent with a simpler model of selection with irrelevant informational content of the unemployment duration. In a recession, the number of applicants per

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<sup>14</sup>Although there is evidence of negative duration dependence in the literature (van den Berg and van Ours(1996, 1999), Addison and Portugal (2003), Guell and Hu (2006) among many others), the findings are not generally robust. Machin and Manning (1999) assert that after controlling for readily observable workers' characteristics, the negative duration dependence pattern disappears in most European studies. Examples of the latter are Abbring et al. (2002), Cockx and Dejemeppe (2005).

<sup>15</sup>The ranking model of Blanchard and Diamond (1994) is slightly different because there is an equilibrium with arbitrary discrimination against long-term unemployed workers.

vacancy is higher so that the expected productivity of new hirings increases. On average, individuals experiencing long unemployment durations in booms are less productive than individuals of long durations in recessions. The prevailing cyclical condition when the worker's unemployment spell took place reveals some information to an econometrician who imperfectly observes productivity. For employers, however, that information may not be relevant in the likely case that recruiters observe productivity better than the econometrician.

The second approach, human capital depreciation, explains that wages decay as unemployment spell increases because workers become less productive. For the same reason, firms are reluctant to hire individuals of long unemployment durations. Keane and Wolpin (1997), in order to match the data, estimate a striking 30.5% yearly depreciation rate of human capital when white collars are nonemployed. Machin and Manning (1999) (p. 3119) argue that there is lack of direct evidence about human capital depreciation and that employer's surveys assess long-term unemployed workers no worse than the average recruit.

Third, although unemployment benefits exhaustion may theoretically generate a decreasing sequence of reservation wages that fit the hazard rate profile across productivities, it is unlikely that this effect dominates, especially for high wage workers. Hogan (2004) finds that unemployment benefits do not significantly impact reservation wages after controlling for previous and re-employment wages.

### 3.3 Permanent Income and Welfare

The recruiting selection technology influences the lifetime inequality in terms of permanent income and welfare. As a first step to discuss the relation between recruiting selection and inequality, I derive expressions for these variables. After doing the same algebra used to derive equations (2), the permanent income conditional on unemployment  $y^U$  is

$$y^U(\theta) = S(\theta)b(1 - z) + (1 - S(\theta))w(\theta) \quad (19)$$

where  $S(\theta)$  is defined as in (2). It follows that the permanent income for an employed worker is

$$y^E(\theta) = \frac{(1 - \beta)w(\theta) + \beta\eta y^U(\theta)}{1 - \beta(1 - \eta)} \quad (20)$$

The unconditional long-run probability of being employed is  $p^E(\theta) = \frac{\pi(\theta)}{\pi(\theta) + \eta}$  while the one of being unemployed is  $p^U(\theta) = 1 - p^E(\theta)$ . Therefore, the unconditional permanent income is

$$y(\theta) = p^E(\theta)y^E(\theta) + (1 - p^E(\theta))y^U(\theta) \quad (21)$$

With analogous logic, the unconditional welfare –certain equivalent<sup>16</sup>– measured in terms of consumption is

$$we(\theta) = u^{-1} \left( (1 - \beta) (p^E(\theta)W(w(\theta)) + (1 - p^E(\theta))Q(\theta, w(\theta))) \right) \quad (22)$$

Given that employers are risk neutral and make zero profits in equilibrium, workers' certain equivalent is a correct measure of welfare for the whole economy.

In a sequential search model the *ex ante* inequality is simply determined by the workers' productivity because all the applicants have the same chance of being hired. In contrast, under recruiting selection individuals of low productivity forgo more labor earnings during their lifetimes than more productive workers do. Moreover, the joint effect of search frictions and Nash bargaining creates another channel to intensify inequality because high productivity workers obtain higher wages due to their more valuable outside options. The inequality amplification effect is even greater in terms of welfare because of workers' risk-aversion. Low productivity workers not only have longer unemployment durations, but also face greater uncertainty about the length of their unemployment spells according to Proposition 8.

The recruiting selection model predicts a negative relationship between permanent income and unemployment durations. There is evidence of negative duration dependence of re-employment wages<sup>17</sup> and substantial earnings losses due to job displacement.<sup>18</sup> Stewart (2007) finds that not only does past joblessness predict low wages and future unemployment but also low wages forecast future unemployment. In line with the recruiting selection model, low productivity workers appear to be trapped in a cycle of long unemployment spells and low wages.

An alternative explanation that may create similar effects on permanent income and welfare is the existence of endogenous separation in which low-quality workers are more likely to be fired. That would explain why the unemployed have lower average productivity and wages than the employed. However, if in such a hypothetical model employers search sequentially, all participating workers face the same chance of being hired, as shown above. The endogenous separation story fails to explain why, conditional on being unemployed, durations and wages are negatively related in cross-sectional data. If empirically relevant, endogenous separation manifests as reoccurrence of unemployment. Such a relation is not significant in the US data (Heckman and Borjas 1980; Corcoran and Hill 1985; Choi and Shin 2002) though it is for European countries (Arulampalam et al. 2000). Endogenous

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<sup>16</sup>This is the amount of fixed consumption per period that, if given forever to the worker, generates as much utility as his uncertain lifetime stream given his productivity  $\theta$ .

<sup>17</sup>Addison and Portugal (1989), Belzil (1995), Rao Sahib (1998), Christensen (2002)

<sup>18</sup>Ruhm (1991), Kletzer (1998), Farber (2003)

recruiting, on the other hand, should reveal persistency in the length of unemployment durations for individuals<sup>19</sup>. Omori (1997) and Gregg (2001) find this effect for the US (NLSY79) and the UK (NCDS), respectively.

### 3.4 Search Externalities

In a Walrasian world, firms observe all workers' productivities, so that the job assignment is efficient. Under search frictions, firms can only observe a limited number of workers' productivities by costly screening of applicants. Within the "local" labor market defined by one vacancy and its heterogenous applicants, the job assignment is efficient among the interviewed candidates (under CR equilibrium). This local competition for the job among workers can only be achieved with some positive unemployment rate. It follows that there is a crucial role of unemployment for the efficiency of job assignment process in heterogenous labor markets.

The recruiting selection model generates externalities that differ from those arising in search and matching models with homogenous workers. Previous literature –notably Hosios (1990)– has identified side-effects of individuals' actions that impact labor market transition probabilities. Allowing for workers' heterogeneity I show that firms' entry and screening margins not only affect the chances of job market transitions, but also the composition of the unemployment pool.

Three new kinds of externalities arise in the recruiting selection model. First, there is a *selection externality*. The privately optimal decision of screening a number of applicants and choosing the best one decreases the expected productivity of new hirings for other vacancies. In general equilibrium, if firms interview more applicants, good workers are more easily taken out of the unemployment pool and the composition of unemployed workers worsens. Secondly, there is a *partial equilibrium entry externality*. Posting new vacancies not only makes it harder for other employers to find a worker and easier for workers to be hired, as typically occurs in search and matching models. Whenever a new vacancy is posted, the expected number of candidates decreases. Since the other employers receive fewer applications in expectation, the average productivity of their top applicants declines. Third, a *general equilibrium entry externality* arises when a firm posts a vacancy and indirectly reduces the expected number of applicants per vacancy. In this way, firms screen fewer applicants regardless of the maximum number of interviewed candidates  $i^*$  implied by the marginal cost of screening  $\xi$  and the distribution of unemployed workers  $\tilde{F}(\theta)$ . But in general equilibrium, the diminished ability of firms to take out good applicants improves the productivity composition of the unemployed workers, which is a clear

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<sup>19</sup>Heckman and Borjas (1980) name this effect "lagged duration dependence".

positive side-effect for other firms.

The previous analysis shows that from a macroeconomic perspective, some of the screening effort of firms is a waste of resources. If all firms could coordinate to reduce the number of applicants interviewed and/or to post more vacancies, they would receive less applicants per vacancy but those workers would be drawn from a distribution of higher average productivity. On the other hand, the existence of unemployment (i.e. a mass of vacancies that generates some  $\lambda > 0$ ) and some degree of recruiting selection are important for improving the efficiency of the job assignment for heterogenous workers.

If the screening technology is powerful, that is, the marginal interview cost is not very high, these externalities can be so important that the economy has multiple equilibria. In a computed example in Section 4.6, three Pareto-ranked equilibria exist with low, medium and high unemployment rates. The low unemployment rate holds when there is high entry of vacancies and the general equilibrium entry externality prevails. For instance, in this equilibrium, firms face a high productivity distribution of unemployed workers. Spurred by the high expected profits they may make, employers post a large number of vacancies. Consequently, few applicants per vacancy arrive and firms cannot effectively remove the best workers from the pool. The fact that other firms have posted a lot of vacancies so that the pool of unemployed contains attractive candidates on average is *ex ante* good news for a prospective employer. However, after posting a vacancy, having fierce competition is *ex post* bad news because the expected number of applications is low. In this way, the high-quality/low-quantity of applicants equilibrium is preserved because the firms' screening cannot deteriorate too much the quality of the unemployed pool.

Multiple equilibria portray high or low “unemployment traps” that may be a partial explanation for large differences in unemployment rates across otherwise similar economies. In high unemployment countries, firms do not post many vacancies because the productivity of unemployed workers is low. Since the unemployment rate is high, employers screen a large number of applicants and easily take out of unemployment the high productivity workers. In this way, firms' recruiting selection perpetuates high-quantity/low-quality equilibrium.

## 4 Empirical Assessment

### 4.1 The data

Taking the model to the data requires matching theoretical and empirical measured variables. Therefore, because in the model jobs are *ex ante* identical, a reasonable step is to “clean” the data from any measurable influence of firm/match specific heterogeneity.

However, different ways of controlling for these factors in the data does not perceptibly change the empirical conclusions<sup>20</sup>. To make conclusions as transparent as possible, I use unadjusted detrended log weekly earnings in dollars of 2000.

A second approach to matching the theoretical variables of the model is to focus on a narrower labor market. Calibrating the model to target aggregate labor market moments portrays surgeons and janitors competing for the same jobs. To avoid such criticisms, I also calibrate the model to a segmented labor market of production workers. Because the number of interviews in NES97 refers to production jobs, there is another reason to focus on this restricted labor market. In the NES97, a 45.5% of nonmissing employers' answers refer to typical job titles such as machine operators, assemblers, welders, laborers, line workers, etc. Using the Earnings Study of the CPS, I obtain reasonable empirical counterparts of wages and unemployment durations for this specific labor market segment.

The reasonable data counterpart of the predicted relationship between (re-employment) wages and durations is the empirical joint distribution of completed unemployment spells and re-employment weekly earnings. Due to the cross-sectional nature of CPS data, I take the workers' current weekly earnings and the reported number of weeks of unemployment in one stretch to measure re-employment wages and the corresponding completed unemployment duration<sup>21</sup>. I do not include workers with more than one reported spell in the previous year because any particular way to impute duration is highly arbitrary. If separation is roughly exogenous as in the model, the sample is not biased. I also assume that the reported unemployment spell ended when the worker got the job she reports in the Earnings Study. Since earnings are reported in March, given the separation rates observed in the data, this assumption should not introduce a relevant bias.<sup>22</sup>

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<sup>20</sup>I run several specifications of Mincer augmented regressions including polynomials of age; gender, race, education category, state and year dummies; industry and employer size category dummies; and last year weekly earnings March CPS. I "clean" the data by subtracting the estimated contribution of industry, employer size, state and year from the log weekly earnings. Besides a slight reduction of the unconditional standard deviation, the cross sectional distribution of adjusted log earnings and unemployment durations is very similar to the distribution of unadjusted log earnings and durations.

<sup>21</sup>Although preferable, longitudinal data like NLSY79 or PSID do not have enough observations – specially for segmented labor market analysis – or are not representative enough of the whole US economy. CPS data provides a limited longitudinal structure that suffices for the empirical exercise in this paper.

<sup>22</sup>Individuals who had more than one unemployment spell in the previous year may be a non random sample of the population of individuals who report some unemployment. If separation rate depended on workers' productivity, the exclusion of these workers would generate sample-selection bias.



## 4.2 Calibration

The period is set to two weeks since this is the median time to fill a vacancy in NES97 data. The calibration matches data from CPS 1985-2006 such as the unemployment rate and mean and standard deviation of log weekly earnings of individuals who did not report being unemployed last year. I also target the median number of applicants interviewed from the NES97. Using parameters implied by this choice of targets, I compute the joint density of log wages and unemployment durations and compare the simulated data to their empirical CPS counterparts. I assume that the productivity distribution of the labor force  $f_\theta(\theta|a(\theta) = 1)$  is lognormal truncated at the lowest productivity worker who earns the minimum wage.

### Targeting Unemployment and Recruiting Costs:

The double equality in equation (11) shows the relations among the average hazard rate, separation rate, unemployment rate and average number of applications. All these pieces of information come from independent sources. To measure the average job finding (hazard) and separation rates, I follow the procedures described in Shimer (2005). In steady state, the empirical job finding rate  $\widehat{\mathbb{E}}[\pi(\theta)]$  equals the ratio between just-separated workers,  $A_{\delta=1}$ , and unemployed ones,  $A$ . Denoting by  $\mathcal{L}$  the number of employed workers, the separation rate is computed as  $\widehat{\eta} = A_{\delta=1} / \left( \mathcal{L}(1 - 0.5\widehat{\mathbb{E}}[\pi(\theta)]) \right)$  to correct for the fact that some workers find jobs within the period. I use monthly flows to compute these statistics due to the zigzag pattern of the empirical distribution of durations, usually attributed to recall bias. Because of the different time span used, the numbers obtained are lower than those reported by Shimer (2005). The empirical unemployment rate  $\widehat{\mathcal{U}}$  is traditionally measured.

Strictly speaking, NES97 reports *the number of interviewed candidates* instead of the number of applications. Taking the model literally, the average number of interviews is the mean of a Poisson random variable that is right-censored at the maximum number of applicants  $i^*$ . For calibration purposes  $i^*$  is treated as a parameter. The choice of  $i^*$  is crucial to mimicking the shape of the conditional distribution of unemployment durations  $f_\delta(\delta)$  according to the discussion in Section 2.4.

To choose  $i^*$  and  $\lambda$ , I use the computed separation and unemployment rates to calculate the implied number of applications per vacancy  $\lambda$  as the root of

$$\frac{1 - e^{-\widehat{\lambda}}}{\widehat{\lambda}} = \widehat{\eta} (1/\widehat{\mathcal{U}} - 1)$$

Using  $\widehat{\lambda}$ , the value of the maximum number of screened applicants  $i^*$  is the right-censoring

point that equalizes the median number of interviewed candidates in NES97 data and in the model.<sup>23</sup>

After solving the model using the computed  $\lambda$  and  $i^*$ , the marginal screening cost is calculated to be consistent with the choice of  $i^*$ , for instance<sup>24</sup>  $\xi = 0.5(\tilde{J}(i^* + 1) - \tilde{J}(i^* - 1))$ . The vacancy posting cost  $\kappa$  is calibrated so that the numerical solution of  $M(k) = \max\{\tilde{J}(k) - \xi k, \tilde{J}(i^*) - \xi i^*\}$  and  $\tilde{F}(\theta)$  is consistent with the free entry condition. Proposition 3 guarantees that the already computed  $\lambda$  is a unique solution for free entry condition.

### Targeting unemployment benefits and minimum wage:

Assuming CRRA preferences with parameter  $\gamma$ , the Nash condition becomes

$$\theta = w(\theta) + \frac{1 - \alpha}{\alpha(1 - \gamma)} S(\theta) (w(\theta) - (b(1 - z))^{1 - \gamma} w(\theta)^\gamma) \quad (23)$$

A worker of the lowest type  $\underline{\theta}$  earns the lowest wage  $w(\underline{\theta})$ . Moreover she has the lowest hazard rate  $\pi(\underline{\theta}) = e^{-\lambda}$  because she only gets hired if she does not face any other competitor. Moreover, the type  $\underline{\theta}$  must also be indifferent between being in the labor force or not, i.e.

$$Q_a(\underline{\theta}, w(\underline{\theta})) = Q_n \quad \Rightarrow \quad S(\underline{\theta})u(b(1 - z)) + (1 - S(\underline{\theta}))u(w(\underline{\theta})) = u(b)$$

Using these conditions, the unemployment income  $b$  is expressed in terms of other parameters.

$$b = w(\underline{\theta}) \left[ \frac{1 - S(\underline{\theta})}{1 - S(\underline{\theta})(1 - z)^{\gamma - 1}} \right]^{\frac{1}{1 - \gamma}} \quad (24)$$

Substituting (24) into (23) and doing some algebra,

$$\underline{\theta} = w(\underline{\theta}) \left( 1 + \frac{1 - \alpha}{\alpha(1 - \gamma)} S(\underline{\theta}) \left( \frac{1 - (1 - z)^{1 - \gamma}}{1 - S(\underline{\theta})(1 - z)^{1 - \gamma}} \right) \right) \quad (25)$$

Two additional restrictions are needed: (i) the observed minimum wage  $w(\underline{\theta})$  is greater than  $b(1 - z)$  and (ii) Unemployment income of an applicant  $b(1 - z)$  is positive. There are three cases to analyze. If  $\gamma > 1$ , by algebraic manipulation of (24), the conditions are met if  $z < \hat{z} \equiv 1 - S(\underline{\theta})^{\frac{1}{\gamma - 1}}$ . In case  $\gamma < 1$ , it is needed that  $z > \hat{z} \equiv 1 - S(\underline{\theta})^{\frac{1}{\gamma - 1}}$ . Finally, the restrictions are always satisfied if  $\gamma = 1$ .

<sup>23</sup>In this case, targeting the median number of interviews instead of the mean is preferable for two reasons. First, the NES97 contains several outliers that reduce the reliability of the sample average. Second, trying to correct the nonresponse bias (17% in this question) by different strategies yields an adjusted average ranging between 4.8-6.2. In contrast, regardless of the adjustments, the median remains invariable.

<sup>24</sup>There are a range of compatible values.

### Targeting wage distribution:

I choose the mean and variance of the log productivities to target the mean and variance of the log earnings of employed workers with no unemployment during the previous year. A Newton-Raphson procedure suffices to find a solution for the implied system of non-linear equations (see Appendix C for more details). Other parameters such as the relative risk aversion  $\gamma$ , the exogenous bargaining power  $\alpha$  and the ratio application cost to unemployment income  $z$  are set at conventional values.

### Solving the model:

Solving the model is to find a solution for the Volterra integral equation in (15), which is the cumulative distribution function of unemployed workers,  $\tilde{F}(\theta)$ . The computational algorithm is simple and fast. I describe it in Appendix C. Using the numerical solution, it is straightforward to compute hazard rates, wages and value functions.

The results obtained are presented in the section 4.4. Although the model qualitatively replicates the features of the joint distribution of re-employment log wages and unemployment durations, the magnitudes are exaggerated. The ability of firms to distinguish high and low productivity workers seems overstated. As a result, compared to the data, high productivity workers leave unemployment too soon and low productivity ones leave too late. This is a direct consequence of two assumptions: (i) Firms can perfectly observe productivity once workers have been screened, and (ii) Wages entirely depend on market productivity. While it is possible to extend the basic model to include these features in several ways (leisure value heterogeneity, noisy productivity signals, measurement error of wages, etc.), I present next a very simple modification that can generate more realistic moments of the joint distribution of wages and durations.

## 4.3 A simple extension of the model: Stochastic screening costs

If workers' applications are noisy signals of their productivities, firms will exert greater screening effort –high marginal costs  $\xi$ – to assess noisier candidates. One way to capture this idea is to allow for a stochastic marginal screening cost<sup>25</sup>. In the simplest case, firms face only two possible cost realizations<sup>26</sup>. With probability  $\chi$  all applicants are interviewed because  $\xi = 0$  (hence,  $i^* = \infty$ ). With probability  $1 - \chi$  firms face high enough  $\xi > 0$  that makes them optimally interview only one applicant ( $i^* = 1$ ). The *ex ante* hazard rate

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<sup>25</sup>Explicitly introducing noisy signals would generate comparable results at the cost of introducing complicated and inessential modifications to the structure of the model.

<sup>26</sup>It is straightforward to create cases with more complicated random screening costs.

becomes a mixture of two extreme cases: unrestricted nonsequential search and sequential search. It is expressed as

$$\pi(\theta) = \chi e^{-\lambda(1-\tilde{F}(\theta))} + (1-\chi) \frac{1-e^{-\lambda}}{\lambda} \quad (26)$$

Compared to the baseline case, this modification flattens out the hazard rate and the endogenous outside value functions, which implies a milder relation between wages and productivities. The model is solved exactly as before. The modified hazard rate is plugged into the Volterra integral equation (15). The average hazard rate of the economy does not change.<sup>27</sup> Facing this new uncertainty, the modified firms' free-entry condition becomes<sup>28</sup>

$$0 = -\kappa + \beta\chi \sum_{k=1}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!} \tilde{J}_k + \beta(1-\chi)(1-e^{-\lambda})[\tilde{J}_1 - \xi]$$

To calibrate the model, we can set  $\chi$  to target the observed (censored) expected unemployment duration of the lowest wage worker  $\bar{\delta}_{min}$ . Using Proposition 7, the following equality approximately holds

$$\bar{\delta}_{min} = \frac{\eta}{\left(\chi e^{-\lambda} + (1-\chi) \frac{1-e^{-\lambda}}{\lambda}\right) \left(\eta + \chi e^{-\lambda} + (1-\chi) \frac{1-e^{-\lambda}}{\lambda}\right)}$$

This is a quadratic equation in  $\chi$ . In the case of all occupations,  $\chi$  is targeted to match an unconditional expected duration of 0.9 fortnights. For production workers, I target an unconditional expected duration of 1.3 fortnights. The next section shows the results of the basic and the extended model. All parameters for Model 1 (deterministic screening costs) and Model 2 (stochastic screening costs) are summarized in Table 1. Parameters in Table 1 are set to exactly match the unemployment rate, mean and standard deviation of wages of the employed workers ( $\delta = 0$ ) for all occupations and for production workers<sup>29</sup>.

## 4.4 Results

Table 2 summarizes the main results of the model. For Model 1, the choice of  $i^*$  makes the model match the median number of interviewed applicants. For Model 2, the value of  $\chi$

<sup>27</sup>The proof is analogous to the one of Lemma 4.

<sup>28</sup>Since the right-hand size is strictly increasing in  $\lambda$ , it follows that there is a unique solution for this equation given some  $\tilde{F}(\theta)$ , using the argument of Proposition 3.

<sup>29</sup>Production workers category includes the following groups for 1985-2003: Supervisors, production (628-633); Precision metal working (634-655); other precision production (656-699); machine operators and tenders, not precision (703-779); fabricators, assemblers and hand working (783-795); production inspectors, testers, samplers and weighers (796-799). For 2004-2006, the category includes production occupations (7700-8960)

Table 1: Calibrated parameters

| Parameter                            | All occupations   |                   | Production workers |                   | Note                                         |
|--------------------------------------|-------------------|-------------------|--------------------|-------------------|----------------------------------------------|
|                                      | Model 1           | Model 2           | Model 1            | Model 2           |                                              |
| $\alpha$ : Worker barg. Power        | 0.5               | 0.5               | 0.5                | 0.5               | Symmetric Nash                               |
| $\beta$ : Discount rate              | $\sqrt[26]{0.95}$ | $\sqrt[26]{0.95}$ | $\sqrt[26]{0.95}$  | $\sqrt[26]{0.95}$ | Standard                                     |
| $\eta$ : Separation rate             | 1.212%            | 1.212%            | 1.451%             | 1.451%            | CPS data                                     |
| $\gamma$ : Relative risk aversion    | 2                 | 2                 | 2                  | 2                 | Standard                                     |
| $z$ : Search cost (1)                | 0.01              | 0.01              | 0.01               | 0.01              |                                              |
| $w$ : Min wage                       | 20                | 20                | 20                 | 20                | US\$ in 2000                                 |
| $b$ : Unem. income                   | 19.58             | 19.97             | 19.16              | 19.97             | From eq. (24)                                |
| $\mu_\theta$ : Mean of $\theta$ (2)  | 6.745             | 7.032             | 6.930              | 7.196             |                                              |
| $\sigma_\theta$ : SD of $\theta$ (2) | 0.828             | 0.943             | 0.539              | 0.610             |                                              |
| $\xi$ : Interview cost               | 3924              | 26729             | 2993               | 15828             | Match $i^*$ and $\lambda$                    |
| $\kappa$ : Vac. post cost            | 23302             | 67166             | 26129              | 55318             | FE condition                                 |
| $\chi$ : Prob of low $\xi$ (3)       |                   | 0.462             |                    | 0.463             | Target mean duration of $\underline{\theta}$ |

Notes: (1) Ratio application cost to unemployment income  $b$ ; (2) Target mean and standard deviation of  $\log w(\theta)$  according Algorithm 2 in Appendix C; (3) Marginal cost under high cost shock. Low cost shock is 0.

targets the minimum unconditional expected duration. While the number of interviewed candidates is below the range suggested by NES97 data in the case of all occupations, for production workers the average value matches the range of estimates.

The median and mean duration of ongoing unemployment spells is overestimated by Model 1, and underestimated by Model 2. A similar pattern is observed for completed durations although Model 1 is closer in this case. Figure 1 displays the marginal distribution of both ongoing and completed unemployment spells. The data show spikes at durations that are multiples of 4, which is usually attributed to recall or rounding bias. For ongoing durations (Panels A and C), both models replicate well the shape of the empirical distributions, except for the censoring point at 52 weeks (26 biweeks). While “all occupations” data show that about 8% of unemployed workers have durations longer than one year, Model 1 generates more than 25% and Model 2 only about 3%. The production workers distribution shows a similar pattern. The excessive number of long-term unemployed in Figure 1 generated by Model 1 can be reconciled with the evidence if unemployed workers include passive searchers, who want a job although they declare not

to be actively searching<sup>30</sup>. Although there is no exact recorded nonemployment duration for these passive searchers, I compute an approximated measure with the available information<sup>31</sup>, called duration A, that displays a spike at the censoring point that is similar to the one of Model 1.

The overall fit to the distributions of log wages of employed workers seems fine (see Figures 4 and 5) although only the mean and variance are explicitly targeted. With respect to Model 1, Model 2 substantially improves the fit to the distributions of unemployed workers with 12 or less weeks, 26 or less weeks and all the unemployed. Intuitively, the random screening cost decreases firms' ability to select high productivity workers. For this reason, wages and hazard rates are less positively correlated and the composition of long-term unemployed workers does not deteriorate too much due to the limited screening power.

Other predictions of the model are those coming from Propositions 6, 7 and 8. In order to compare those predictions to CPS data, I computed the relevant statistics in the model and in the data. Figures 2 and 3 show the moments generated by the models and by the data for all occupations and production workers. Panels A show that while Model 1 exaggerates the negative correlation between log wages and durations, Model 2 generates a good fit to the data. This is partially unsurprising because the parameter  $\chi$  was calibrated to approximately hit the unconditional expected duration of the lowest wage. Panels B in Figures 2 and 3 show the negative relation between the unconditional variance of duration and log wages. Again, Model 2 generates a sensible improvement with respect to Model 1.

Panels C and D show that both Models have limitations in mimicking the magnitude of the negative relation between wages and durations conditional on unemployment, although Model 2 performs somewhat better. Comparing results of Panels C and A suggests that other factors such as separation rate and leisure value heterogeneity may play some role in explaining the wage gap between recently hired (workers with  $\delta > 0$ ) and tenured workers ( $\delta = 0$ ). Models' performance is somewhat better for the variance of unemployment duration in Panel D in Figures 2 and 3.

Panels E and F depict the mean and variance of log wages decreasing in duration. For the expected wage, Model 2 performs relatively well, especially for production workers,

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<sup>30</sup>For instance, Yashiv (2006) considered this extended definition to assess the cyclical properties of the standard search and matching model

<sup>31</sup>This measure is constructed by using a CPS variable indicating how long ago the respondents out of the labor force worked for the last time. Individuals answering more than a year ago are included as top coded. For individuals out of the labor force, another variable registers how many weeks the respondent looked for a job last year. The nonemployment spell is the value of this variable plus ten weeks since the individual is interviewed in March.

although both models overstate the magnitude of the relation. For the variance of log wages, both models perform similarly. From these results, I conclude that the pure recruiting selection mechanism with perfect productivity observability generates a selection that is too strong. Since the simple modification in Model 2 makes the model perform better, it is possible the fit to the data possibly improves by introducing more realistic informational and stochastic structure. I see this mixed success in replicating data features as the price paid for having the simplest model with the recruiting selection mechanism.

Finally, I provide computations of the derivative of the wage function  $w'(\theta)$  for all the models and samples in Figure 6, Panel A. The derivative is below 1 in the entire domain, so the existence of a Coincidence Ranking equilibrium is verified for all the calibrations. In Panel B, I depict the equilibrium hazard rates  $\pi(\theta)$  for all cases. As expected, Model 2, which allows for stochastic screening costs, generates flatter profiles. Panels C, D, E and F show different distributions of types conditional on duration. The recruiting selection makes the conditional distributions progressively shift to the left as unemployment duration increases. For Model 2, the differences between distributions are less pronounced than for Model 1. Since firms can sometimes not screen workers in Model 2, the recruiting selection is less effective and the average productivity of workers decreases less markedly with duration.

## 4.5 An experiment: Efficiency vs. Equality

I use previously calibrated models to quantify the differences in efficiency and inequality between sequential and nonsequential employer search. I perform an experiment consisting of raising the marginal cost of screening  $\xi$  to make firms optimally search sequentially, i.e.  $i^* = 1$ . In the case of the model with stochastic screening costs (Model 2), the sequential search case arises when  $\chi = 0$ . Then I compare labor market outcomes of the initial and final steady states. To do so, I first compute the allocation cost of the shock by compensating the vacancy-posting cost  $\kappa$  so that the unemployment rate remains constant across steady states. The differences between the initial economy and the shocked one are explained by changes in the job assignment process and in the composition of the unemployment pool. Second, I compute the full adjustment effect using the initial post-vacancy cost  $\kappa$ , so that the vacancy entry margin endogenously adjusts to satisfy free entry condition.

In the initial steady state of Model 1 (all occupations and production workers), firms' screening technology generates hazard rates out of unemployment that increase in productivity. In the initial equilibrium, low productivity workers have systematically lower chances of being selected by a screening firm, which implies higher inequality and higher

Table 2: Data vs. Model generated moments

| Parameter                                  | All occupations |         |         | Production workers |         |         |
|--------------------------------------------|-----------------|---------|---------|--------------------|---------|---------|
| Statistic                                  | Data            | Model 1 | Model 2 | Data               | Model 1 | Model 2 |
| $\mathcal{U}$ Unem. rate                   | 5.74%           | 5.74%   | 5.74%   | 7.46%              | 7.46%   | 7.46%   |
| Median ongoing dur                         | 5               | 7       | 4       | 5                  | 8       | 4       |
| Mean ongoing dur(1)                        | 7.58            | 11.54   | 6.10    | 8.06               | 12.1372 | 6.80281 |
| Std.Dev ongoing dur(1)                     | 13.55           | 10.28   | 5.92    | 14.47              | 10.4988 | 6.48563 |
| Median completed dur                       | 6               | 7       | 4       | 7                  | 8       | 4       |
| Mean completed dur(2)                      | 8.06            | 6.25    | 5.64    | 8.45               | 6.11    | 6.12    |
| Std.Dev completed dur(2)                   | 6.10            | 8.53    | 5.15    | 6.21               | 6.18    | 5.47    |
| $i^*$ , max # interview                    |                 | 8       |         |                    | 5       |         |
| med( $K$ ) interview                       | 5               | 5       |         | 5                  | 5       |         |
| $\mathbb{E}[K]$ interview (3)              | 4.26-6.48       | 4.90    | 2.84    | 2.74-5.91          | 4.35    | 3.09    |
| Mean $\log(w)$ Emp.                        | 6.19            | 6.19    | 6.19    | 6.21               | 6.21    | 6.21    |
| Std.Dev $\log(w)$ Emp.                     | 0.77            | 0.77    | 0.77    | 0.57               | 0.57    | 0.57    |
| Perc 10 $\log(w)$ Emp.                     | 5.21            | 5.14    | 5.21    | 5.54               | 5.44    | 5.49    |
| Perc 25 $\log(w)$ Emp.                     | 5.76            | 5.70    | 5.62    | 5.86               | 5.87    | 5.76    |
| Perc 50 $\log(w)$ Emp.                     | 6.26            | 6.25    | 6.18    | 6.23               | 6.28    | 6.20    |
| Perc 75 $\log(w)$ Emp.                     | 6.71            | 6.74    | 6.76    | 6.60               | 6.62    | 6.65    |
| Perc 90 $\log(w)$ Emp.                     | 7.09            | 7.13    | 7.20    | 6.91               | 6.88    | 6.98    |
| Mean $\log(w)$ , $1 \leq \delta \leq 6$    | 5.90            | 5.75    | 5.96    | 6.05               | 5.93    | 6.04    |
| Std.Dev $\log(w)$ , $1 \leq \delta \leq 6$ | 0.72            | 0.79    | 0.72    | 0.50               | 0.61    | 0.52    |
| Perc 10 $\log(w)$ , $1 \leq \delta \leq 6$ | 4.94            | 4.07    | 5.00    | 5.47               | 4.44    | 5.37    |
| Perc 25 $\log(w)$ , $1 \leq \delta \leq 6$ | 5.50            | 4.40    | 5.34    | 5.73               | 4.71    | 5.57    |
| Perc 50 $\log(w)$ , $1 \leq \delta \leq 6$ | 5.94            | 5.01    | 5.74    | 6.01               | 5.29    | 5.83    |
| Perc 75 $\log(w)$ , $1 \leq \delta \leq 6$ | 6.38            | 5.79    | 6.26    | 6.34               | 5.97    | 6.23    |
| Perc 90 $\log(w)$ , $1 \leq \delta \leq 6$ | 6.78            | 6.46    | 6.80    | 6.73               | 6.46    | 6.65    |

Notes: (1) Model-generated statistics censored at 51 weeks to be compared to data statistics. (2) Model-generated statistics truncated at 51 weeks to be compared to data statistics. (3) Average number of interviews widely varies with different weighting and nonresponse bias correction procedures. Higher and lower estimates are reported.



median unemployment duration. In contrast, under sequential search, firms randomly select applicants, so the hiring process does not affect the composition of the unemployed.

Due to selection externalities discussed in Section 3.4, the average productivity of unemployed workers *increases* due to the diminished ability of firms to select good applicants. As shown in the left columns of Table 3, the economy adjusts by lowering the unemployment rate after a very large increase of screening costs. How is this explained? Since the initial recruiting technology is relatively powerful, firms' screening generates a strong negative externality on other vacancies. After the shock, selection is suppressed and the quality of the unemployment pool substantially increases. In the new equilibrium, firms randomly hire applicants who are drawn from a much better pool than the one they had before. Firms' inability to distinguish productivities is offset by the fact that available unemployed workers are better. In fact, firms choose to post more vacancies –which lowers the unemployment rate in equilibrium– to take full advantage of the improved quality of unemployed workers. In contrast, in the right columns of Table 3 that show the effect of having zero probability of free screening ( $\chi = 0$ ), after the economy fully adjusts the unemployment rate *increases*. Because the initial screening technology is not so effective at distinguishing applicants' productivities, the generated selection externalities are not very important in the initial equilibrium. The improved quality of the unemployed pool is not enough to compensate firms' inability to detect good workers. As a consequence, firms choose to post fewer vacancies and, indirectly, increase the unemployment rate.

Comparing the initial, compensated and final steady states, we can see that average wages, permanent income and welfare are higher under recruiting selection in all the cases. Under recruiting selection, more productive workers get jobs more frequently and have more valuable outside options to bargain with firms. On the other hand, inequality is substantially lower under sequential search by any measure. The recruiting selection economy assigns jobs to the most productive workers –in spite of the strong externalities– but severely increases inequality. After considering total effects, the average welfare reduction is higher for Model 2. The decomposition in the latter case also shows that most of this effect comes from the increased unemployment rate and not from the effects on job assignment. Perhaps surprisingly, the rise of unemployment also contributes to greater equality in this case, which suggests that its negative effect on the value of outside options is an important channel for wage inequality. In the case of Model 1, the increased screening cost generates less average welfare reduction because the economy avoids the strong screening externalities. In contrast, the reduction in permanent income and welfare inequality is much higher in initial steady state of Model 1.

Behind the average effects shown in Table 3, the increase in screening cost has heterogeneous effects across the population. Figure 7 shows the largely different effects of the

Table 3: Recruiting Selection vs. Sequential search

| Statistic                    | All occupations |                     |                     |         |                     |                     |
|------------------------------|-----------------|---------------------|---------------------|---------|---------------------|---------------------|
|                              | Model 1         | Model 1<br>seq comp | Model 1<br>seq full | Model 2 | Model 2<br>seq comp | Model 2<br>seq full |
| $\xi$ Cost screening         | 3924            | 29796               | 29796               | 26729   | 26729               | 26729               |
| $\kappa$ Cost vacancy        | 23302           | 24769               | 23302               | 67166   | 60729               | 67166               |
| $i^*$ , max # interview      | 8               | 1                   | 1                   | -       | -                   | -                   |
| $\chi$ Prob high $\xi$       | -               | -                   | -                   | 0.46    | 0.00                | 0.00                |
| $\mathcal{U}$ Unem. rate     | 5.74%           | 5.74%               | 5.09%               | 5.74%   | 5.74%               | 8.99%               |
| $\lambda$ # applic.          | 4.99            | 4.99                | 4.37                | 4.99    | 4.99                | 8.15                |
| Median ongoing dur           | 7               | 4                   | 3                   | 4       | 4                   | 6                   |
| med( $K$ ) / mean screen (1) | 5               | 1                   | 1                   | 2.84    | 1                   | 1                   |
| Mean log( $w$ )              | 6.13            | 5.92                | 5.96                | 6.17    | 6.10                | 5.94                |
| SD log( $w$ )                | 0.81            | 0.53                | 0.54                | 0.76    | 0.59                | 0.57                |
| Mean Perm inc $y(\theta)$    | 6.08            | 5.87                | 5.91                | 6.12    | 6.05                | 5.85                |
| SD Perm inc $y(\theta)$      | 0.87            | 0.53                | 0.54                | 0.79    | 0.59                | 0.56                |
| Mean Welf $we(\theta)$       | 5.58            | 5.17                | 5.25                | 5.39    | 5.26                | 4.94                |
| SD Welf $we(\theta)$         | 0.82            | 0.26                | 0.27                | 0.61    | 0.27                | 0.21                |
| Production workers           |                 |                     |                     |         |                     |                     |
| $\xi$ Cost screening         | 2993            | 17763               | 17763               | 15828   | 15828               | 15828               |
| $\kappa$ Cost vacancy        | 26129           | 29221               | 26129               | 55318   | 51745               | 55318               |
| $i^*$ , max # interview      | 5               | 1                   | 1                   | -       | -                   | -                   |
| $\chi$ Prob high $\xi$       | -               | -                   | -                   | 0.46    | 0.00                | 0.00                |
| $\mathcal{U}$ Unem. rate     | 7.46%           | 7.46%               | 5.62%               | 7.46%   | 7.46%               | 10.12%              |
| $\lambda$ # applic.          | 5.53            | 5.53                | 4.03                | 5.53    | 5.53                | 7.75                |
| Median ongoing dur           | 8               | 4                   | 3                   | 4       | 4                   | 6                   |
| med( $K$ ) / mean screen (1) | 5               | 1                   | 1                   | 3.09    | 1                   | 1                   |
| Mean log( $w$ )              | 6.15            | 5.95                | 6.06                | 6.19    | 6.13                | 6.01                |
| SD log( $w$ )                | 0.58            | 0.33                | 0.34                | 0.56    | 0.37                | 0.36                |
| Mean Perm inc $y(\theta)$    | 6.08            | 5.88                | 6.00                | 6.12    | 6.06                | 5.91                |
| SD Perm inc $y(\theta)$      | 0.72            | 0.33                | 0.34                | 0.60    | 0.36                | 0.35                |
| Mean Welf $we(\theta)$       | 5.43            | 5.05                | 5.25                | 5.26    | 5.14                | 4.90                |
| SD Welf $we(\theta)$         | 0.73            | 0.13                | 0.15                | 0.53    | 0.13                | 0.11                |

Notes: (1) For Model 1, this row reports the median number of applicants, while for Model 2, it reports the average number of applicants.

increase of the marginal screening cost for low and high productivity workers. Panel A depicts the permanent income and welfare (certain equivalent) functions for Model 1 (all occupations) according to equations (21) and (22) for both the initial and final steady states. Under sequential employer search, low productivity workers have greater permanent income and welfare than they do under recruiting selection. Both curves become flatter in the sequential search case, indicating that inequality is much lower. Two factors are important for these results. First, the expected unemployment durations are much lower for highly productive workers in the initial equilibrium, but all workers face the same expected duration in the sequential case. Secondly, due to the Nash bargaining wage determination, the outside option of high productivity workers is more valuable under recruiting selection, so they obtain higher wages in the latter steady state. From Panel C in Figure 7, which displays the production workers case, I derive conclusions and explanations that are similar to those for Panel A.

Panels B and D show a different picture, though. In these cases, the initial economies have stochastic screening costs (Model 2) for all occupations and productive workers, respectively. Although switching to a sequential search economy would make low productivity workers slightly better off in terms of permanent income and welfare inequality, the medium and high productivity workers are considerably worse off after the probability of a high screening cost raises to 1, i.e.  $\chi = 0$ . Since in the initial steady state low productivity workers already have a not-so-low hazard rate, their increases in wages, permanent income and welfare are not as high as under Model 1. Moreover, most of the gains that low productivity workers obtain by a random hiring process are lost when the economy adjusts to a higher unemployment rate.

## 4.6 Multiple Equilibria

If the recruiting selection technology is powerful enough, externalities become so strong that multiple equilibria can exist. Among the calibrated models, Model 1 for all occupations has the most effective recruiting selection technology in that  $i^* = 8$ . Using a computational procedure described in Appendix C, I find that this particular calibration of the model has three equilibria. However, the more realistic calibrations for Models 2 do not show this multiplicity. Thus, the main purpose of this subsection is to illustrate a theoretical point.

The main statistics of multiple equilibria are shown in Table 4. The baseline case analyzed in previous sections corresponds to a Medium unemployment rate equilibrium (5.74%). Even though firms are endowed with exactly the *same* recruiting technology and workers with the *same* distribution of productivities, the economy can also be in a

Table 4: Multiple Equilibria

| Statistic                         | All occupations |                   |                |
|-----------------------------------|-----------------|-------------------|----------------|
|                                   | Model 1<br>High | Model 1<br>Medium | Model 1<br>Low |
| $\xi$ Cost screening              | 3924            | 3924              | 3924           |
| $\kappa$ Cost vacancy             | 23302           | 23302             | 23302          |
| $i^*$ , max # interview           | 7               | 8                 | 15             |
| $\mathcal{U}$ Unem. rate          | 11.01%          | 5.74%             | 2.15%          |
| $\lambda$ # applic.               | 10.20           | 4.99              | 1.34           |
| Median ongoing dur                | 26              | 7                 | 1              |
| med( $K$ ) screen                 | 5               | 5                 | 5              |
| Mean $\log(w)$                    | 6.00            | 6.13              | 6.24           |
| SD $\log(w)$                      | 0.72            | 0.81              | 0.69           |
| Mean Permanent income $y(\theta)$ | 5.89            | 6.08              | 6.24           |
| SD Permanent income $y(\theta)$   | 1.01            | 0.87              | 0.69           |
| Mean Welfare $we(\theta)$         | 5.32            | 5.58              | 5.83           |
| SD Welfare $we(\theta)$           | 0.89            | 0.82              | 0.56           |

High unemployment equilibrium (11.01%) and a Low unemployment equilibrium (2.15%). The strong selection externalities can self-sustain different unemployment rates in the following way. Suppose a firm faces a very high productivity distribution of unemployed workers. Given a fixed vacancy-posting cost  $\kappa$ , it is very attractive to open positions to hire applicants. By doing so, the number of applicants per vacancy  $\lambda$  is small, so that firms cannot use their powerful screening technology for too many applicants on average. In this situation employers are not very effective at hiring the top workers in the pool of unemployed. The average quality of the unemployed does not deteriorate too much and the high-quality/low-quantity equilibrium perpetuates. Despite their limited opportunity to screen candidates due to the low number of applicants per vacancy, firms paradoxically recruit better workers on average in this equilibrium than in the others. They interview fewer candidates but these applicants are drawn from a “better” distribution.

Exactly the opposite situation can occur. If firms face a very bad unemployed pool, few vacancies are posted and each employer interviews a large number of workers. Since the screening technology is powerful, firms succeed in taking out the few good workers among the large number of applications received. Consequently, firms interview a large number of applicants coming from a very low productivity distribution and the low quality of the

unemployment pool persists.

Figure 8 displays different features of the three equilibria. Panel A shows that due to the differences in unemployment rates, even high productivity workers have greater chances of finding a job in a low unemployment equilibrium. Panel E depicts the ratio between log permanent incomes of Low and Medium unemployment equilibrium for each productivity type. Since the ratio is never below 1, in the Low unemployment equilibrium everyone is better off. The same analysis is done for Medium and High unemployment equilibrium in Panel F. For this particular calibration, I conclude that the three equilibria are Pareto-ranked. High-welfare equilibrium is characterized by low-quantity/high-quality hiring and vice versa. The presence of these externalities suggests that there may be room for active intervention in high unemployment labor markets.

## 5 Conclusions

The model presented in this paper embeds a recruiting selection process into a benchmark sequential search model with heterogeneous workers. The empirical relevance of nonsequential search by employers has been documented in several empirical papers. I also show that several augmentations of the sequential employer search model cannot satisfactorily explain why multiple applicants are interviewed to fill a vacancy. Recruiting selection, a highly realistic feature of the job search process, provides a simple explanation for documented empirical facts. In the data, high hazard rates and wages are positively correlated, there is substantial hazard heterogeneity after controlling for observables, and the negative duration of the hazard rate is generally found spurious once adjusted for workers' heterogeneity.

The model also explains the fact that workers with long unemployment spells have low permanent incomes. As shown, the more intense the recruiting selection is, the greater the welfare and permanent income inequality are. Although it is tempting to conclude that stronger screening leads to a more efficient job assignment, an improved selection technology does not guarantee that result at the aggregate level. Firms may devote too much effort to screen applicants and by doing so worsen the composition of the pool of unemployed workers. For this reason, the average productivity of recruited workers may improve if all the firms screen less intensively and/or post more vacancies.

The model suggests new ways to interpret labor markets outcomes. The existence of unemployment is crucial in this model for employers to assign jobs efficiently. From the perspective of a single firm, a low unemployment rate generates low chances of making good hirings. For the whole economy, low unemployment rate discourages overscreening of applicants, which would imply a great waste of resources.

Instead of being an indication of labor market rigidities or frictions –as some conventional wisdom may subscribe– long unemployment spells reflect the intensity of employers’ recruiting selection technologies. Prevalence of long term unemployment may be an indication of a significant deterioration of the productivity of unemployed workers. If the model describes real labor markets to some extent, policy makers should worry about long unemployment durations, but for reasons that are different from the traditional labor market flexibility considerations.

Even though the calibration of the model overstates the features of the joint distribution of wages and unemployment durations observed in CPS data, this result seems to be driven by simplifying assumptions that make the model more tractable. The simple extension that allows for random screening costs substantially improves the empirical performance of the model. The tension arising from confronting a highly stylized model to the data is natural: models cannot usually make strong points without a great deal of simplification.

## Appendices

### Appendix A Proofs

**Proof of Lemma 1.** Consider the lifetime firm’s profit  $J = J(\theta)$  generated by a worker of productivity  $\theta$ , and denote  $F_J(J)$  and  $f_J(J)$  the cumulative distribution function and density of  $J$ , respectively. Remember from Section 2.2 that  $\tilde{J}(k+1) = \mathbb{E}[J^k | \delta > 0, a = 1, K = k]$  where  $J^i$  denotes the  $i$ -th highest profitability in a group of  $k$  applicants. Given the discreteness of the number of applicants, consider the following difference

$$\tilde{J}(k+1) - \tilde{J}(k) = \mathbb{E}[J^{k+1} | \delta > 0, a = 1, K = k+1] - \mathbb{E}[J^k | \delta > 0, a = 1, K = k]$$

Writing the expectations in terms of integrals I find that

$$\begin{aligned} & \tilde{J}(k+1) - \tilde{J}(k) \\ &= \frac{1}{k+1} \left( \int J(k+1) f_J(v) F_J(v)^k dv - \int Jk(k+1) f_J(v) F_J(v)^{k-1} (1 - F_J(v)) dv \right) \\ &= \frac{1}{k+1} (\mathbb{E}[J^{k+1} | \delta > 0, a = 1, K = k+1] - \mathbb{E}[J^k | \delta > 0, a = 1, K = k+1]) > 0 \end{aligned}$$

Since the first term is the expected value of the best worker among  $k+1$  applicants and the second term is the expected value of the second-best among  $k+1$  applicants<sup>32</sup>, the

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<sup>32</sup>In general, the density function of the  $m$ -th highest value within  $k$  elements is

$$f^m(x^m) = \frac{k!}{(m-1)!(k-m)!} F(x^m)^{m-1} (1 - F(x^m))^{k-m} f(x^m)$$

with  $F(\cdot)$ ,  $f(\cdot)$  being the CDF and PDF of  $x$ .

difference is always strictly positive, which proves that  $\tilde{J}(k)$  is strictly increasing in  $k$ .

To prove concavity, the function is differentiated twice

$$\begin{aligned} & \left( \tilde{J}(k+1) - \tilde{J}(k) \right) - \left( \tilde{J}(k) - \tilde{J}(k-1) \right) \\ &= \int J f_J(v) \left[ F_J(v)^{k-1} (F_J(v) - k(1 - F_J(v))) - F_J(v)^{k-2} (F_J(v) - (k-1)(1 - F_J(v))) \right] dv \\ &= \int J(v) f_J(v) F_J(v)^{k-1} \left[ (k+2)F_J(v)^2 - 2(k+1)F_J(v) + k \right] dv \end{aligned}$$

Doing a bit of algebra, the last expression equals

$$\begin{aligned} & -\frac{1}{k+1} \left[ \int (k+2)(k+1)J(v)F_J(v)^{k-1}(1 - F_J(v))f_J(v)dv \right. \\ & \left. + \int k(k+1)J(v)f_J(v)F_J(v)^{k-1}(1 - F_J(v))dv \right] \\ &= -\frac{1}{k+1} \left[ \mathbb{E}[J(\theta^k)|\delta > 0, a = 1, K = k+1] + \mathbb{E}[J(\theta^{k-1})|\delta > 0, a = 1, K = k] \right] < 0 \end{aligned}$$

The last two terms are the expected value of the second-best worker when  $k+1$  and  $k$  applicants arrive. The expression is always negative, which proves concavity. ■

### Proof of Lemma 2.

The second assertion is almost trivial. The screening process being free, all applicants are going to be interviewed, so  $i^* = \infty$ . For the first part, consider a distribution of unemployed workers  $\tilde{F}(\theta)$  such that  $\tilde{J}(1) > \xi$ . Due to the concavity of  $\tilde{J}(k)$  proved in Lemma 1, it follows that  $\tilde{J}(2) - \tilde{J}(1) < \tilde{J}(1) - \tilde{J}(0) = \tilde{J}(1)$ . Denoting  $\Delta(k) = (\tilde{J}(k) - \xi k) - (\tilde{J}(k-1) - \xi(k-1))$ , the marginal profit of interviewing an additional applicant is strictly decreasing in the number of applicants

$$\Delta(1) > \Delta(2) - \Delta(1) > \Delta(3) - \Delta(2) > \dots$$

The maximum number of interviewed applicants  $i^*$  cannot be 0 because firms obtain positive profits by interviewing 1 applicant. The maximum number of applicants interviewed would be infinity ( $i^* = +\infty$ ) only if screening an additional applicant increased profits for any arbitrarily large  $k$ . Formally,

$$\Delta(k-1) - \Delta(k) > \underline{\zeta} > 0, \text{ for all } k$$

In the following, to assume the latter inequality holds will lead to a contradiction. Taking an arbitrarily large  $k$ , it is true that

$$\Delta(k-1) - \Delta(k) > \underline{\zeta} > 0 \Rightarrow \Delta(k-1) > \underline{\zeta} + \Delta(k) > \Delta(k)$$

It also holds that

$$\begin{aligned}\Delta(k-2) - \Delta(k-1) &> \Delta(k-1) - \Delta(k) > \underline{\zeta} > 0 \Rightarrow \\ \Delta(k-2) &> 2\Delta(k-1) - \Delta(k) > \underline{\zeta} + \Delta(k-1) > 2\underline{\zeta} + \Delta(k)\end{aligned}$$

Using similar reasoning,

$$\Delta(k-3) > \underline{\zeta} + \Delta(k-2) > 3\underline{\zeta} + \Delta(k)$$

Repeating the same argument  $k-1$  times obtains

$$\Delta(1) > (k-1)\underline{\zeta} + \Delta(k) \tag{27}$$

By making  $k \rightarrow \infty$  in the inequality (27) with  $0 < \tilde{J}(1) - \xi = \Delta(1) < \infty$ , the term  $\Delta(k)$  must necessarily be negative. Therefore, the optimal maximum number of interviewed applicants must be lower than  $k$  so that the marginal screened worker generates a nonnegative change in profits. ■

**Corollary 9** *The value of a vacancy with  $k$  applications,  $M(k)$  is a nondecreasing function in  $k$ .*

**Proof.**

Using the argument for the proof of Lemma 2, for any  $k < i^*$  it must be true that  $\Delta(k-1) - \Delta(k) > 0$ , which implies that  $M(k-1) - M(k-2) > M(k) - M(k-1)$ .

Assuming that  $M(k)$  is not increasing for  $k < i^*$ , I show that a contradiction arises. Suppose  $M(k-1) > M(k)$ . By the previous condition, it must be true that  $M(k-1) - M(k-2) > 0$ . Therefore, doing backward substitution we obtain the contradiction  $M(0) = 0 > M(1)$ , which proves that  $M(k)$  is increasing.

For any  $k > i^*$ , the employer randomly picks  $i^*$  applicants to interview, so that  $M(k) = \max\{\tilde{J}(k) - \xi k, \tilde{J}(i^*) - \xi i^*\}$  is always nondecreasing. ■

**Proof of Proposition 3.**

Denoting  $M(k) = \max\{\tilde{J}(k) - \xi k, \tilde{J}(i^*) - \xi i^*\}$ , a firms' value  $P$  is represented by the following equation as a function of  $\lambda$

$$P(\lambda) = \beta \sum_{k=1}^{\infty} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!} M(k) - \kappa$$

To prove the Proposition, the function  $P(\lambda)$  must have a unique zero. First notice that  $P(0) = -\kappa < 0$ . Second,  $P(\lambda)$  is continuous and increasing in  $\lambda$  because  $M(k)$  is increasing



in  $k$  as established in Corollary 9. To see this, simply take the derivative

$$P'(\lambda) = \beta e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} (M(k+1) - M(k)) \geq 0$$

Given the distribution  $\tilde{F}(\theta)$ , the economy is productive enough to not shut down by assumption, i.e  $M_{i^*} > \kappa/\beta$ . Then,

$$\lim_{\lambda \rightarrow \infty} P(\lambda) = \beta M_{i^*} - \kappa > 0$$

This condition ensures that there is some  $\lambda$  such that  $P(\lambda) > 0$ . Since  $P(\lambda)$  is strictly increasing and continuous in  $\lambda$ , there exists some  $\lambda^*$  such that  $P(\lambda^*) = 0$  by the Intermediate Value Theorem. ■

#### Proof of Lemma 4.

Taking the expectation of equation (10) yields

$$\mathbb{E}[\pi(\theta)] = \sum_{k=1}^{i^*} \frac{\lambda^{k-1} e^{-\lambda}}{k!} \int k \tilde{F}(v)^{k-1} \tilde{f}(v) dv + \sum_{k=i^*+1}^{\infty} \frac{\lambda^{k-1} e^{-\lambda}}{k!} \int i^* \tilde{F}(v)^{i^*-1} \tilde{f}(v) dv$$

Since  $k \tilde{F}(v)^{k-1} \tilde{f}(v)$  and  $i^* \tilde{F}(v)^{i^*-1} \tilde{f}(v)$  are the densities of the best applicant in a pool of size  $k$  and in a pool of size  $i^*$  respectively, the previous expression collapses to

$$\mathbb{E}[\pi(\theta)] = e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{k!} = \frac{1 - e^{-\lambda}}{\lambda}$$

which is the desired result. ■

#### Proof of Proposition 5.

By means of the Volterra nonlinear integral equation in (15) I show the existence, uniqueness and differentiability<sup>33</sup> of the distribution of unemployed workers  $\tilde{F}(\theta)$  for a given hazard conjectured hazard rate function  $\pi(\theta)$  and  $\lambda$ .

In equation (10) the hazard rate  $\pi(\theta)$  depends on  $\tilde{F}(\theta)$ . I define a function  $\tilde{\pi} : [0, 1] \rightarrow [0, 1]$  such that

$$\tilde{\pi}(z) = \pi(\tilde{F}(\theta)) = \sum_{k=1}^{i^*} \frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} z^{k-1} + \sum_{k=i^*+1}^{\infty} \frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} z^{i^*-1} \frac{i^*}{k}$$

Although I prove the existence of  $\tilde{F}(\theta)$  for the particular case of the Coincidence Ranking equilibrium, this Proposition holds for any conjectured hazard rate function satisfying the

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<sup>33</sup>This proof adapts a standard solution in Hackbusch (1995), chapter 2.

Lipschitz condition  $|\widetilde{\pi}(\widetilde{F}(\theta_0)) - \widetilde{\pi}(\widetilde{F}(\theta_1))| \leq \Xi |\widetilde{F}(\theta_0) - \widetilde{F}(\theta_1)|$  with  $\Xi < \infty$  for all  $\theta_0, \theta_1$ . Using the definition of  $\widetilde{\pi}(z)$ , the Volterra integral equation in (15) is expressed rewritten in terms of an operator  $O(\cdot)$ .

$$O(\widetilde{F})(\theta) = \int_{-\infty}^{\theta} \frac{\eta f_{\theta}(v)}{\mathcal{U}^*(\eta + \widetilde{\pi}(\widetilde{F}(v)))} dv \quad (28)$$

Establishing that (28) is a contraction mapping, there exists a unique function  $\widetilde{F}(\theta)$  that is a fixed point of  $O(\cdot)$ . A sufficient condition to show this is that the kernel function

$$K(v, \widetilde{F}(v)) = \frac{\eta f_{\theta}(v)}{\mathcal{U}^*(\eta + \widetilde{\pi}(\widetilde{F}(v)))}$$

integrated in the right-hand side of (28) satisfies a global Lipschitz condition so that there exists a finite constant  $\Lambda$  for which holds

$$|K(v, z_0) - K(v, z_1)| \leq \Lambda |z_0 - z_1| \quad \forall v \in \mathbb{R} \quad \text{and } z_0, z_1 \in [0, 1]$$

In what follows, I show that the Lipschitz condition is satisfied for the Coincidence Ranking Equilibrium. Since  $0 \leq z \leq 1$ , the following result holds

$$\begin{aligned} |\pi(z_0) - \pi(z_1)| &= \sum_{k=1}^{i^*} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!} |z_0^{k-1} - z_1^{k-1}| + \sum_{k=i^*+1}^{\infty} \frac{e^{-\lambda} \lambda^{k-1} i^*}{(k!)} |z_0^{i^*-1} - z_1^{i^*-1}| \\ &\leq \sum_{k=1}^{\infty} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!} |z_0^{k-1} - z_1^{k-1}| \leq \sum_{k=1}^{\infty} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!} k |z_0 - z_1| \\ &= |z_0 - z_1| \lambda \left[ \sum_{k=2}^{\infty} \frac{e^{-\lambda} \lambda^{k-2}}{(k-2)!} + 1 \right] \leq (\lambda + 1) |z_0 - z_1| \end{aligned}$$

The second line follows from use of the standard factorization result  $x^k - y^k = (x - y)(x^{k-1} + x^{k-2}y + \dots + y^{k-1})$ . Then, the absolute value of the difference of the two kernels is

$$|K(v, z_0) - K(v, z_1)| = \frac{\eta f_{\theta}(v)}{\mathcal{U}^*} \left| \frac{\pi(z_0) - \pi(z_1)}{(\eta + \widetilde{\pi}(z_0))(\eta + \widetilde{\pi}(z_1))} \right| \leq \frac{\lambda + 1}{\eta \mathcal{U}^*} \sup_{\theta} f_{\theta}(\theta) |z_0 - z_1|$$

Since  $\sup_{\theta} f_{\theta}(\theta) < \infty$ , the Lipschitz constant is  $\Lambda = \sup_{\theta} f_{\theta}(\theta) \frac{\lambda+1}{\eta \mathcal{U}^*}$ . Although the argument holds for the particular conjectured hazard rate based on CR equilibrium, in general it will hold for any conjecture that can satisfy the Lipschitz condition  $|\pi(z_0) - \pi(z_1)| \leq \Xi |z_0 - z_1|$  with  $\Xi < \infty$  as stated in the statement of the proposition. This condition is weaker than differentiability.

Let  $\mathcal{C}$  be the space of continuous functions with the special norm  $\|g\| = \max_v |\exp(-\phi\Lambda v)g(v)|$  and  $\phi > 1$ , which is equivalent to the sup-norm  $\|\cdot\|_\infty$ . As integration preserves continuity, the operator  $O(\tilde{F})$  maps from the space  $\mathcal{C}$  into the same space because  $f_\theta(\theta)$  and the kernel function  $K(v, z)$  are continuous.

Moreover, it follows that *for all*  $\theta$

$$\begin{aligned}
|(O(\tilde{F}_0) - O(\tilde{F}_1))(\theta)| &= \left| \int_{-\infty}^{\theta} (K(v, \tilde{F}_0(v)) - K(v, \tilde{F}_1(v))) dv \right| \\
&\leq \int_{-\infty}^{\theta} \Lambda |\tilde{F}_0(v) - \tilde{F}_1(v)| dv \\
&= \int_{-\infty}^{\theta} \Lambda \exp(\phi\Lambda v) \left| \exp(-\phi\Lambda v) (\tilde{F}_0(v) - \tilde{F}_1(v)) \right| dv \\
&\leq \left| \max_{\hat{v}} \left\{ \exp(-\phi\Lambda \hat{v}) (\tilde{F}_0(\hat{v}) - \tilde{F}_1(\hat{v})) \right\} \right| \int_{-\infty}^{\theta} \Lambda \exp(\phi\Lambda v) dv \\
&= \|\tilde{F}_0 - \tilde{F}_1\| \phi^{-1} \exp(\phi\Lambda\theta)
\end{aligned}$$

Since the former inequality holds for all possible  $\theta$ , it also holds for the productivity that maximizes the value of the operator on the left-hand side. Therefore, the conclusion is that

$$\|(O(\tilde{F}_0) - O(\tilde{F}_1))(\theta)\| \leq \phi^{-1} \|\tilde{F}_0 - \tilde{F}_1\|$$

Since  $\phi > 1$ , the integral equation is a Contraction Mapping. Due to the Banach Fixed Point Theorem, existence, uniqueness and continuity of  $\tilde{F}(\theta)$  are proven. Moreover, thanks to the Fundamental Theorem of Calculus, the density  $\tilde{f}(\theta)$  also exists and is unique. Because of its definition  $\pi(\theta) \equiv \tilde{\pi}(\tilde{F}(\theta))$  the hazard rate function also exists and it is unique. Then, using the fact that a positive  $\lambda$  implies an unemployment rate bounded away from 0 and 1, the existence and uniqueness of the density of unemployed workers  $\tilde{f}(\theta)$  is established. ■

**Proof of Proposition 6.** The second claim is obvious for expected values and variances since all separations are exogenous. For the case  $\delta > 1$ , we have that

$$\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta + 1] = \int \theta \left( \frac{f(\theta, \delta)}{f_\delta(\delta)} - \frac{f(\theta, \delta + 1)}{f_\delta(\delta + 1)} \right) d\theta$$

Substituting (12) into the latter equation, the right-hand side becomes

$$\int \theta \frac{f(\theta, \delta)}{f_\delta(\delta)} \left( 1 - (1 - \pi(\theta)a(\theta)) \frac{f_\delta(\delta)}{f_\delta(\delta + 1)} \right) d\theta \quad (29)$$

Integrating both sides of equation (12) and considering the case when  $a(\theta) = 1$  yields

$$\begin{aligned}\int f(\theta, \delta + 1) d\theta &= \int (1 - \pi(\theta)) f(\theta, \delta) d\theta \\ f_\delta(\delta + 1) &= f_\delta(\delta) - f_\delta(\delta) \int \pi(\theta) \frac{f(\theta, \delta)}{f_\delta(\delta)} d\theta\end{aligned}$$

Hence,

$$\frac{f_\delta(\delta)}{f_\delta(\delta + 1)} = \frac{1}{1 - \mathbb{E}[\pi(\theta)|\delta]}$$

Substituting this expression in equation (29) yields

$$\begin{aligned}\mathbb{E}[\theta|\delta] - \mathbb{E}[\theta|\delta + 1] &= \int \theta \frac{f(\theta, \delta)}{f_\delta(\delta)} \left( 1 - \frac{1 - \pi(\theta)}{1 - \mathbb{E}[\pi(\theta)|\delta]} \right) d\theta \\ &= \mathbb{E}[\theta|\delta] - \frac{\mathbb{E}[\theta(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\ &= \mathbb{E}[\theta|\delta] - \frac{\text{Cov}[\theta, (1 - \pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]\mathbb{E}[1 - \pi(\theta)|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\ &= \frac{\text{Cov}[\theta, \pi(\theta)|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} > 0\end{aligned}\tag{30}$$

Since  $\pi(\theta)$  is strictly increasing in  $\theta$ , the covariance is positive, which proves the first part of the proposition.

To establish the inequality for the variance, I proceed by using an expression that is proven analogously to (30). Hence,

$$\mathbb{E}[\theta^2|\delta] - \mathbb{E}[\theta^2|\delta + 1] = \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]}$$

Thus, the difference of variances conditional on different durations is

$$\mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta + 1] = \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} - \left( \mathbb{E}[\theta|\delta] - \frac{\mathbb{E}[\theta(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \right)^2$$

Doing some algebra and using the fact that  $0 \leq \mathbb{E}[\pi(\theta)|\delta] \leq 1$ , I obtain that

$$\begin{aligned}&\mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta + 1] \\ &\geq (\mathbb{E}[\theta^2|\delta] - (\mathbb{E}[\theta|\delta])^2) - \underbrace{\left( \frac{\mathbb{E}[\theta^2(1 - \pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]\mathbb{E}[\theta(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \right)}_{\text{Term 1}} \\ &\quad - \underbrace{\left( \frac{(\mathbb{E}[\theta(1 - \pi(\theta))|\delta])^2 - \mathbb{E}[\theta|\delta]\mathbb{E}[\theta(1 - \pi(\theta))|\delta]}{(1 - \mathbb{E}[\pi(\theta)|\delta])^2} \right)}_{\text{Term 2}}\end{aligned}$$

Term 1 of the previous expression is equivalent to

$$\begin{aligned}
& \frac{\mathbb{E}[(\theta^2 - \theta\mathbb{E}[\theta|\delta])(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[(\theta - \mathbb{E}[\theta|\delta])^2(1 - \pi(\theta))|\delta] - \mathbb{E}[\mathbb{E}[\theta|\delta]^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[(\theta - \mathbb{E}[\theta|\delta])^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} - \mathbb{E}[\theta|\delta]^2
\end{aligned}$$

Term 2 is always non-positive because  $0 \leq \pi(\theta) \leq 1$ . It can be written as

$$\frac{\mathbb{E}[\theta(1 - \pi(\theta))|\delta] [\mathbb{E}[\theta(1 - \pi(\theta))|\delta] - \mathbb{E}[\theta|\delta]]}{(1 - \mathbb{E}[\pi(\theta)|\delta])^2} \leq 0$$

Using the previous expressions, a lower bound of the difference of variances is

$$\begin{aligned}
& \mathbb{V}[\theta^2|\delta] - \mathbb{V}[\theta^2|\delta + 1] \\
&\geq \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[(\theta - \mathbb{E}[\theta|\delta])^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\
&\geq \mathbb{E}[\theta^2|\delta] - \frac{\mathbb{E}[\theta^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \\
&= \frac{\mathbb{E}[\theta^2|\delta]\mathbb{E}[1 - \pi(\theta)|\delta] - \mathbb{E}[\theta^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} = -\frac{\text{Cov}[\theta^2(1 - \pi(\theta))|\delta]}{1 - \mathbb{E}[\pi(\theta)|\delta]} \geq 0
\end{aligned}$$

where the last expression is positive because  $\pi(\theta)$  is increasing in  $\theta$ . If the hazard rate is strictly increasing in  $\theta$  as it is in a CR equilibrium, the inequality becomes strict, which is the desired result. ■

**Proof of Proposition 7.** Applying the definition of conditional expectation and equation (12)

$$\mathbb{E}[\delta|\theta] = \sum_{k=0}^{\infty} k \frac{f(\theta, \delta = k)}{f_{\theta}(\theta)} = f(\theta|\delta = 1) \sum_{k=0}^{\infty} k(1 - \pi(\theta)a(\theta))^{k-1}$$

Considering the case with  $a(\theta) = 1$  and differentiating the absolutely summable series  $\pi(\theta)^{-1} = \sum_{k=0}^{\infty} (1 - \pi(\theta))^k$  gives that

$$\sum_{k=0}^{\infty} k(1 - \pi(\theta))^{k-1} = \frac{1}{\pi(\theta)^2}$$

Therefore, using equations (14) and (13) yields

$$\mathbb{E}[\delta|\theta] = \frac{f(\theta|\delta = 1)}{\pi(\theta)^2} = \frac{\eta}{(\eta + \pi(\theta))\pi(\theta)}$$

Deriving the former expression, we obtain

$$\begin{aligned}\frac{d\mathbb{E}[\delta|\theta]}{d\theta} &= -\frac{\eta\pi'(\theta)(1+2\pi(\theta))}{(\eta+\pi(\theta))^2\pi(\theta)^2} \\ &= -\frac{\pi'(\theta)(1+2\pi(\theta))}{\eta}\mathbb{E}[\delta|\theta]^2\end{aligned}$$

which proves the claim. ■

**Proof of Proposition 8.** First consider that  $\mathbb{V}[\delta|\theta] = \mathbb{E}[\delta^2|\theta] - \mathbb{E}[\delta|\theta]^2$ . Applying the definition of conditional expectation and equation (12)

$$\mathbb{E}[\delta^2|\theta] = \sum_{k=0}^{\infty} k^2 \frac{f(\theta, \delta = k)}{f_{\theta}(\theta)} \quad (31)$$

$$= f(\theta|\delta = 1) \sum_{k=0}^{\infty} k^2 (1 - \pi(\theta)a(\theta))^{k-1} \quad (32)$$

Assuming that  $a(\theta) = 1$  and differentiating twice the absolutely summable series  $\pi(\theta)^{-1} = \sum_{k=0}^{\infty} (1 - \pi(\theta))^k$  gives that

$$\sum_{k=0}^{\infty} k^2 (1 - \pi(\theta))^{k-1} = \frac{2 - \pi(\theta)}{\pi(\theta)^3}$$

Replacing this expression into (33) yields

$$\mathbb{E}[\delta^2|\theta] = \frac{\eta(2 - \pi(\theta))}{(\eta + \pi(\theta))\pi(\theta)^2} \quad (33)$$

By substituting equation (16) into the variance identity stated at the beginning of the proof, the expression for (17) is obtained

$$\mathbb{V}[\delta|\theta] = \frac{\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2}{(\eta + \pi(\theta))^2\pi(\theta)^2}$$

To prove that  $\partial\mathbb{V}[\delta|\theta]/\partial\theta < 0$  involves tedious algebra. For this statement to be true it suffices to show that  $\partial\mathbb{V}[\delta|\theta]/\partial\pi(\theta) < 0$  since  $\pi'(\theta) > 0$ . Hence,

$$\begin{aligned}\frac{\partial\mathbb{V}[\delta|\theta]}{\partial\pi(\theta)} &= \frac{2\eta - \eta^2 - 2\eta\pi(\theta)}{(\eta + \pi(\theta))^2\pi(\theta)^2} \\ &\quad - \frac{(2(\eta + \pi(\theta))\pi(\theta)^2 + 2\pi(\theta)(\eta + \pi(\theta))^2)(\eta^2 + (2\eta - \eta^2)\pi(\theta) - \eta\pi(\theta)^2)}{(\eta + \pi(\theta))^4\pi(\theta)^4}\end{aligned}$$

Since the denominator of the former expression is always positive, I focus only on the numerator, which after some simplifications becomes

$$\begin{aligned}
& - (2\eta - \eta^2)(\eta + \pi(\theta))^2\pi(\theta)^2 - 2\eta^2(\eta + \pi(\theta))\pi(\theta)^2 - 2(2\eta - \eta^2)(\eta + \pi(\theta))\pi(\theta)^3 \\
& + 2\eta(\eta + \pi(\theta))\pi(\theta)^4 - 2\eta^2(\eta + \pi(\theta))^2\pi(\theta) \\
& = -\eta(2 - \eta)(\eta + \pi(\theta))\pi(\theta)^3 - 2\eta(\eta + \pi(\theta))\pi(\theta)^3 - 2\eta^2(\eta + \pi(\theta))\pi(\theta)^2(1 - \pi(\theta)) \\
& - 2\eta(\eta + \pi(\theta))\pi(\theta)^3(1 - \pi(\theta)) - 2\eta^2(\eta + \pi(\theta))\pi(\theta) < 0
\end{aligned}$$

Therefore, the numerator is always negative for  $\pi(\theta)$  between 0 and 1, which proves the proposition. ■

## Appendix B Deriving conditions for a Symmetric Coincidence Ranking Equilibrium

In this section I obtain the condition (18). Deriving expression (9) yields

$$\begin{aligned}
& (1 - w'(\theta))u'(w(\theta)) + (\theta - w(\theta))u''(w(\theta))w'(\theta) = \\
& \frac{1 - \alpha}{\alpha} [S'(\theta)(u(w(\theta)) - u(b(1 - z))) + S(\theta)u(w(\theta))w'(\theta)]
\end{aligned}$$

Since  $u''(w(\theta)) < 0$  and  $S'(\theta) = -\frac{(\beta^{-1}-1+\eta)\pi'(\theta)}{(\beta^{-1}-1+\eta+\pi(\theta))^2} < 0$  the wage function is strictly increasing in productivity  $w'(\theta) > 0$ . For  $J'(\theta)$  to be positive, it suffices that  $w'(\theta) < 1$ . By substituting (9) into the previous expression and rearranging, the former condition becomes

$$w'(\theta) = \frac{1 + \frac{\pi'(\theta)}{\beta^{-1}-1+\eta+\pi(\theta)} (\theta - w(\theta))}{1 + \frac{1-\alpha}{\alpha} \frac{\beta^{-1}-1+\eta}{\beta^{-1}-1+\eta+\pi(\theta)} + \gamma(w(\theta)) \frac{\theta-w(\theta)}{w(\theta)}} < 1$$

with  $\gamma$  as CRRA parameter. The previous condition becomes

$$\frac{\pi'(\theta)}{\pi(\theta)} < \frac{1 - \alpha}{\alpha\pi(\theta)(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right)$$

Focusing on the special case when  $i^* = \infty$  provides some intuition to analyze the result, because in this case  $\pi'(\theta)/\pi(\theta) = \lambda\tilde{f}(\theta)$ . For  $i^*$  finite, additional terms show up. Thus,

$$\lambda\tilde{f}(\theta) < \frac{1 - \alpha}{\alpha\pi(\theta)(\theta - w(\theta))} + \frac{\gamma}{w(\theta)} \left( \frac{\beta^{-1} - 1 + \eta}{\pi(\theta)} + 1 \right)$$

which is the condition discussed in Section 2.7.

## Appendix C Computational Algorithms

I use different algorithms to compute the model. The common building block for all of these is the solution for the Volterra nonlinear integral equation in (15). The computation is completely analogous to the solution of a Bellman equation via Value Function Iteration.

### Algorithm 1: Solve Volterra integral equation (15)

- Step 0 Generate a grid of log-productivities of  $\mathcal{N}$  points. Set  $j = 0$  and some tolerance  $\epsilon > 0$ .
- Step 1 Start iteration  $j$ . Have a guess  $\tilde{F}^j(\theta)$ .
- Step 2 Solve the right-hand side of the Volterra equation using a quadrature rule and linear interpolation of  $\tilde{F}^j(\theta)$ . In this way, obtain an updated guess  $\tilde{F}^{j+1}(\theta)$
- Step 3 If  $\|\tilde{F}^{j+1}(\theta) - \tilde{F}^j(\theta)\| < \epsilon$ , stop. Otherwise, start a new iteration at Step 1 using the updated distribution.

### Algorithm 2: Targeting mean and variance of log weekly earnings via Newton-Raphson

- Step 0 Have an initial guess for mean and variance of log productivities  $\mu_\theta^{(0)}$  and  $\sigma_\theta^{2(0)}$ . Set a  $j = 0$ , a tolerance  $\epsilon > 0$  and a marginal change to compute numerical derivatives  $\varrho > 0$ . Have the sample mean and variance of log wages  $\overline{\log w}$  and  $S^2(\log w)$ .
- Step 1 Using the guessed parameters, solve the distribution of unemployed workers  $\tilde{F}(\theta)$  according to Algorithm 1.
- Step 2 Using some quadrature rule, compute the mean and variance of the generated log wage distribution of employed workers as  $\mathbb{E}[\log(w(\theta))] = \int \log(w(\theta)) \frac{f_\theta(\theta)\pi(\theta)}{(1-\mathcal{U})(\eta+\pi(\theta))} d\theta$  and  $\mathbb{V}[\log(w(\theta))] = \mathbb{E}[\log(w(\theta)^2)] - \mathbb{E}[\log(w(\theta))]^2$ . If  $\|\mathbb{E}[\log(w(\theta))] - \overline{\log w}\| < \epsilon$  and  $\|\mathbb{V}[\log(w(\theta))] - S^2(\log w)\| < \epsilon$ , stop. Otherwise follow to Step 3.
- Step 3 Evaluate these moments again using small changes. First proceed<sup>34</sup> with changes for  $\mu_\theta + \varrho$  and then for  $\sigma_\theta^2 + \varrho$ .

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<sup>34</sup>Although it is generally recommended to compute two-side numerical derivatives, one-side numerical ones perform very well and require one-half the function evaluations.



Step 4 Compute the Jacobian matrix  $D$  with numerical derivatives and update the guess for  $\mu_\theta$  and  $\sigma_\theta^2$  using  $\begin{bmatrix} \mu_\theta^{(j+1)} \\ \sigma_\theta^{2(j+1)} \end{bmatrix} = \begin{bmatrix} \mu_\theta^{(j)} \\ \sigma_\theta^{2(j)} \end{bmatrix} - D^{-1} \begin{bmatrix} \mathbb{E}[\log(w)]^{(j)} - \overline{\log w} \\ \mathbb{V}[\log(w)]^{(j)} - S^2(\log w) \end{bmatrix}$ . With the updated guess, go back to Step 1.

Step 5 Having  $\tilde{F}(\theta)$  satisfying the targeted moments, compute the marginal cost of screening  $\xi = 0.5 \left( \tilde{J}(i^* + 1) - \tilde{J}(i^* - 1) \right)$ , which is consistent with the choice of  $i^*$  and the post-vacancy cost  $\kappa$  to satisfy the free entry condition (7) (or its stochastic screening cost version).

### Algorithm 3: Solving for counterfactuals.

This algorithm is used to solve for endogenous results for  $\lambda$  and  $i^*$  given posting-vacancy cost  $\kappa$ , marginal screening cost  $\xi$  and parameters for the exogenous distribution of productivities  $f_\theta(\theta)$ . I use this algorithm to compute counterfactual solutions analyzed in Sections 4.5 and 4.6. The possible existence of multiple equilibria makes it harder to compute solutions. The problem can be stated as finding zeroes for the free entry condition. Notice that this does not violate Proposition 3, because for any choice of  $\lambda$  we have a unique distribution of unemployed workers  $\tilde{F}(\theta)$ .

$$P(\lambda; i^*) = \beta \sum_{k=1}^{\infty} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!} M(k) - \kappa$$

If there is only one equilibrium, the unique zero is located in an increasing interval of the  $P(\lambda)$  function. To see why, consider the case when  $\lambda \rightarrow 0$ . Since no applicants are hired in the limit, the function converges to  $-\kappa$ . Thus, necessarily the unique zero is in an increasing interval. If there are more than one equilibria, at least one of the other zeroes takes place in a decreasing interval of  $P(\lambda)$ . With this in mind, I introduce the following algorithm to compute those equilibria.

Step 0 Define whether you are searching for an increasing or decreasing equilibrium. Define a level of tolerance  $\epsilon$ .

Step 1 Have an upper and a lower bound for the number of applicants per vacancy, such that  $\lambda_L^j < \lambda_H^j$ . Set the new guess for  $\lambda$  as  $\lambda^j = 0.5(\lambda_L^j + \lambda_H^j)$ . Set a guess for the maximum number of interviewed candidates,  $i^{*(j)}$ .

Step 2 Solve the Volterra integral equation using Algorithm 1 and the guessed values of  $\lambda^j$  and  $i^{*(j)}$  to calculate the hazard rate  $\pi^j(\theta)$  and obtain  $\tilde{F}^j(\theta)$ .

- Step 3 Solve the Nash bargaining problem and compute  $J(\theta)$ . Using a quadrature rule compute the expected value of profits conditional in the number of applicants interviewed  $\tilde{J}(k) = \int J(\theta)k\tilde{F}(\theta)^{k-1}\tilde{f}(\theta)d\theta$ .
- Step 4 Given  $\xi$  and  $\tilde{J}(k)$ , compute  $i^{*(j+1)}$ . Compute the free entry condition  $P(\lambda)$  given  $\kappa$  as in equation 7.
- Step 5 If  $|P(\lambda)| < \epsilon$ ,  $|\lambda^{j+1} - \lambda^j| < \epsilon$  and  $i^{*(j+1)} = i^{*(j)}$ , then stop. Otherwise, go to step 6.
- Step 6 If you are searching for an increasing equilibrium and  $P(\lambda) > 0$ , set  $\lambda_H^{j+1} = \lambda^j$ ; otherwise  $\lambda_L^{j+1} = \lambda^j$ . If you are searching for a decreasing equilibrium and  $P(\lambda) > 0$ , set  $\lambda_L^{j+1} = \lambda^j$ ; otherwise  $\lambda_H^{j+1} = \lambda^j$ . Follow analogous rules  $P(\lambda) < 0$ . Go back to Step 1.
- Step 7 If  $|\lambda^{j+1} - \lambda^j| < \epsilon$  but  $|P(\lambda)|$  is not low enough, stop and report failure. The kind of equilibrium searched does not exist in the original range analyzed.
- Step 8 If  $|\lambda^{j+1} - \lambda^j| < \epsilon$ ,  $|P(\lambda)| < \epsilon$  but  $i^{*(j+1)} \neq i^{*(j)}$  update  $i^{*(j+1)} = i^{*(j)} + \text{sgn}(i^{*(j+1)} - i^{*(j)})$ , set  $j = 0$  and new  $\lambda_H^0$  and  $\lambda_L^0$ . Go back to Step 1. Since the change is discrete, the update must be in gradual. Updating in every step usually leads to instability. It is more innocuous to make a larger update when  $\lambda$  is low.

The solutions provided are computed using  $\mathcal{N} = 1000$  evenly spaced points of log-productivity in the range  $[\underline{\theta}, \bar{\theta}]$ , with  $\underline{\theta}$  satisfying equation (25) and  $\bar{\theta} = \overline{\log(w)} + 6\sqrt{S^2(\log w)} + \log 2$ . All the integration steps are computed numerically using Gauss-Chebyshev quadrature with 80 nodes, due to its simplicity. The code was written in C++ and the first and second algorithm usually converge very fast. Algorithm 3 may take longer due to the discreteness of  $i^*$ . The results do not perceptively change using a larger grid.

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Figure 1: Marginal distribution of ongoing and completed unemployment durations

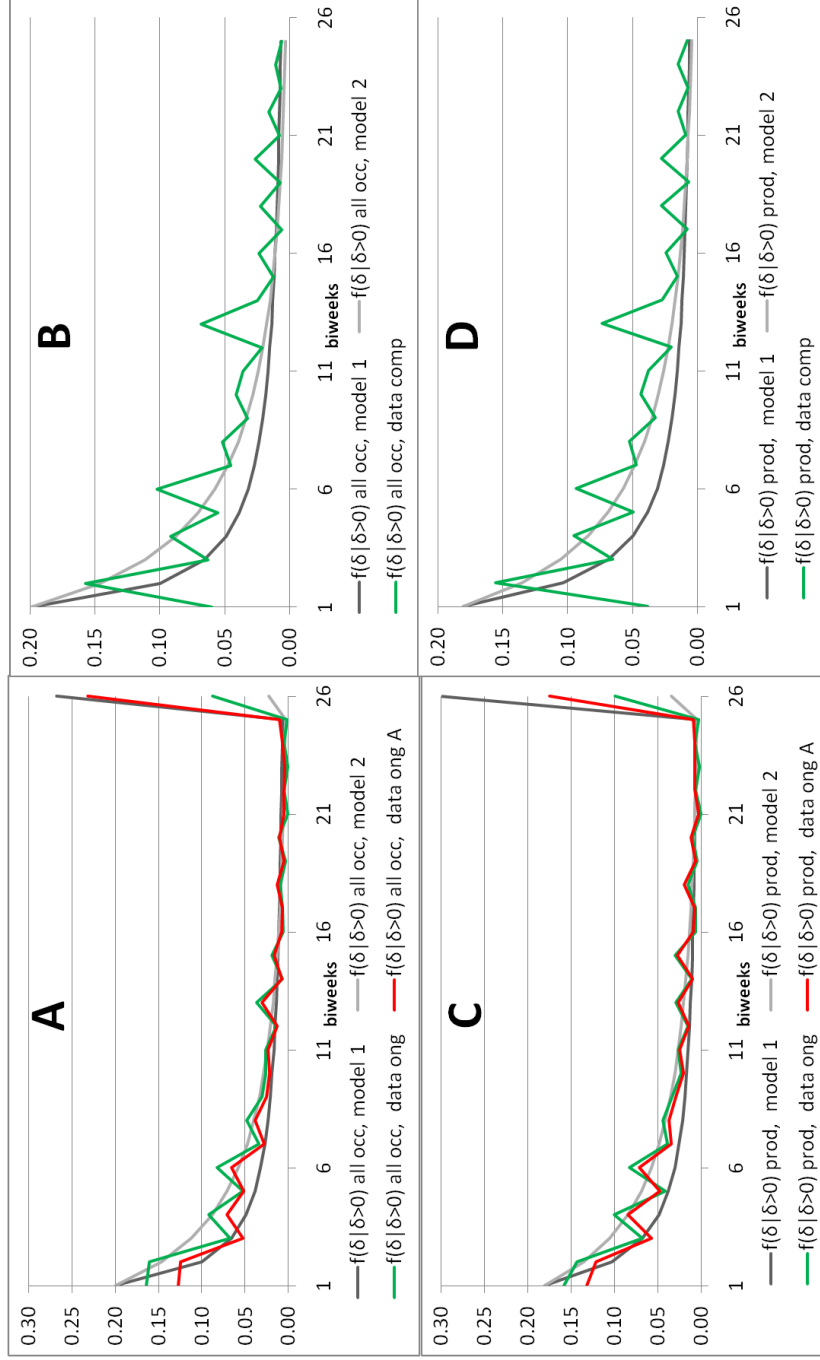
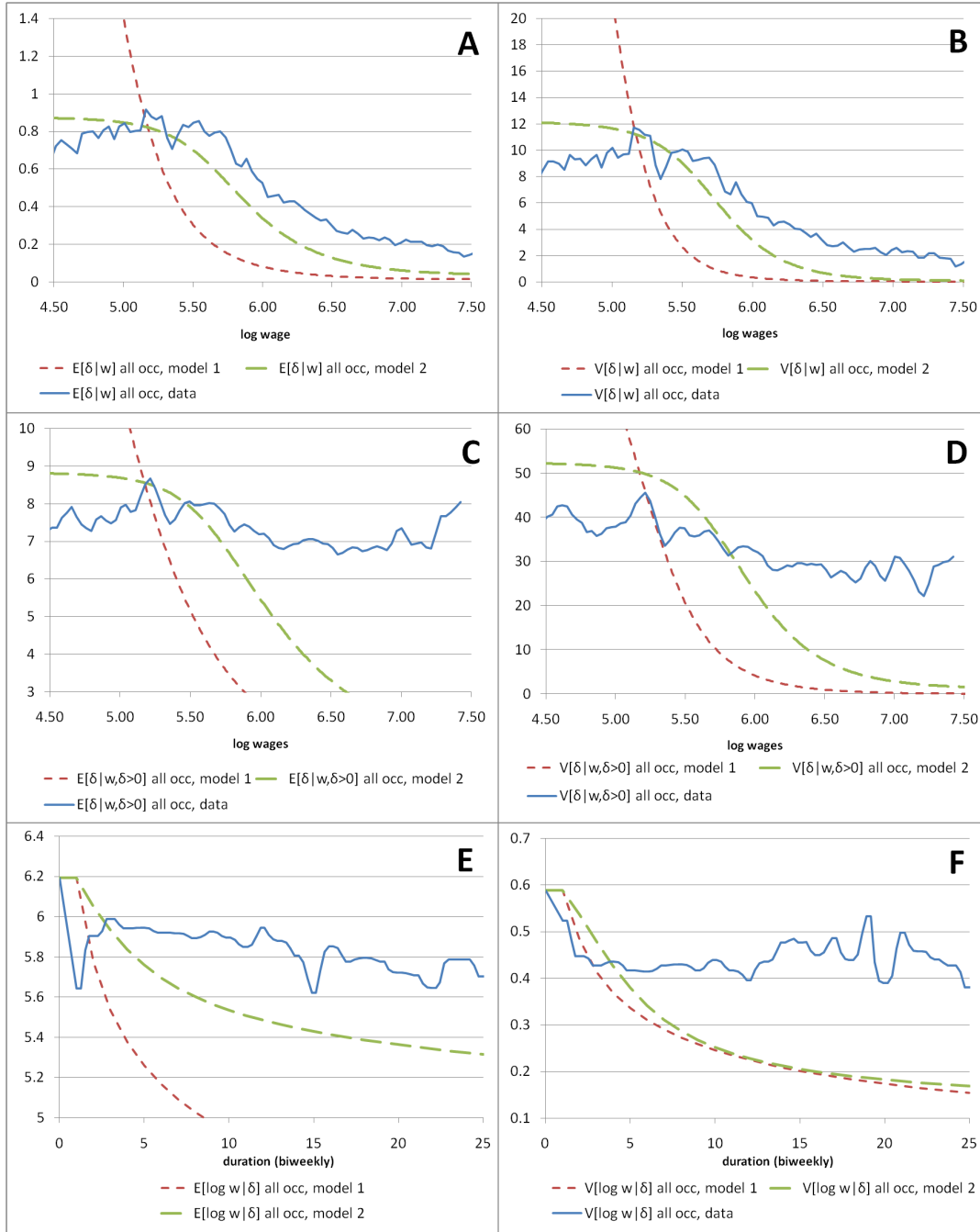


Figure A: Ongoing unemployment durations, all occupations; Figure B: Completed unemployment durations, all occupations; Figure C: Ongoing unemployment durations, production workers; Figure D: Completed unemployment durations, production workers. Duration A includes individuals who want a job but are counted as nonparticipants in the CPS.

Figure 2: Conditional Moments for All Occupations

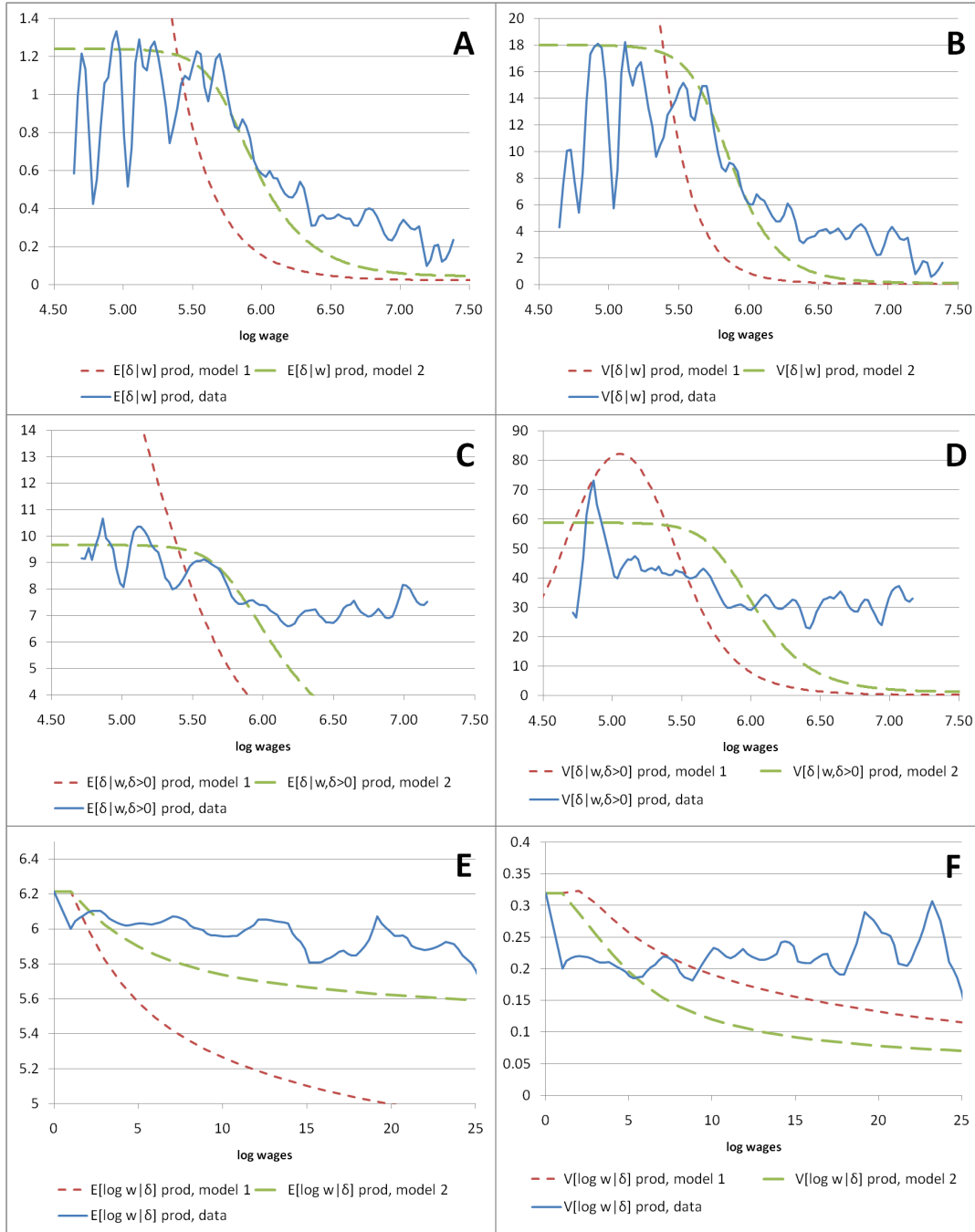


Notes: Panel A: Expected duration conditional on log wages; Panel B: Variance of duration conditional on log wages; Panel C: Expected duration conditional on log wages and unemployment; Panel D: Expected variance conditional on log wages and unemployment; Panel E: Expected log wage conditional on duration; Panel F: Variance of log wages conditional on duration.

All moments are computed using Nadaraya-Watson nonparametric regression with Silverman (1986) rule-of-thumb bandwidth.



Figure 3: Conditional Moments for Production workers



Notes: Panel A: Expected duration conditional on log wages; Panel B: Variance of duration conditional on log wages; Panel C: Expected duration conditional on log wages and unemployment; Panel D: Expected variance conditional on log wages and unemployment; Panel E: Expected log wage conditional on duration; Panel F: Variance of log wages conditional on duration.

All moments are computed using Nadaraya-Watson nonparametric regression with Silverman (1986) rule-of-thumb bandwidth.

Figure 4: Marginal density of log wages, All occupations

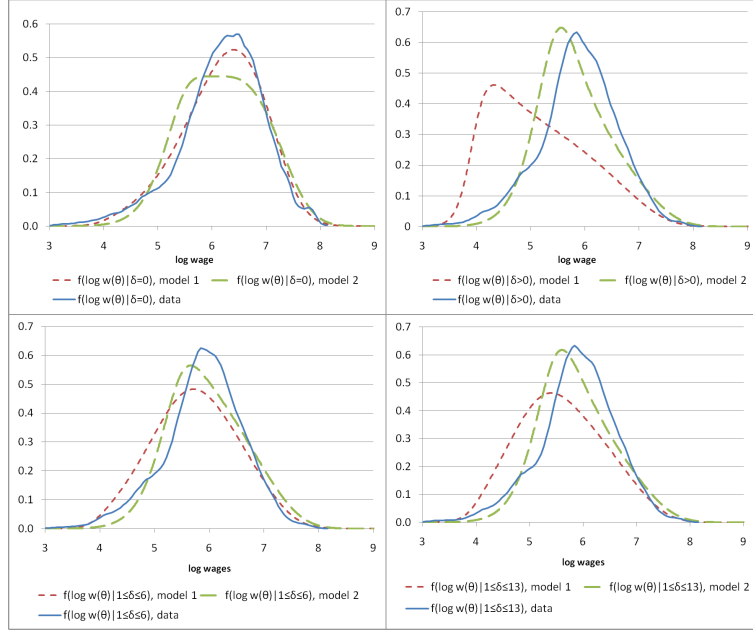


Figure 5: Marginal density of log wages, Production workers

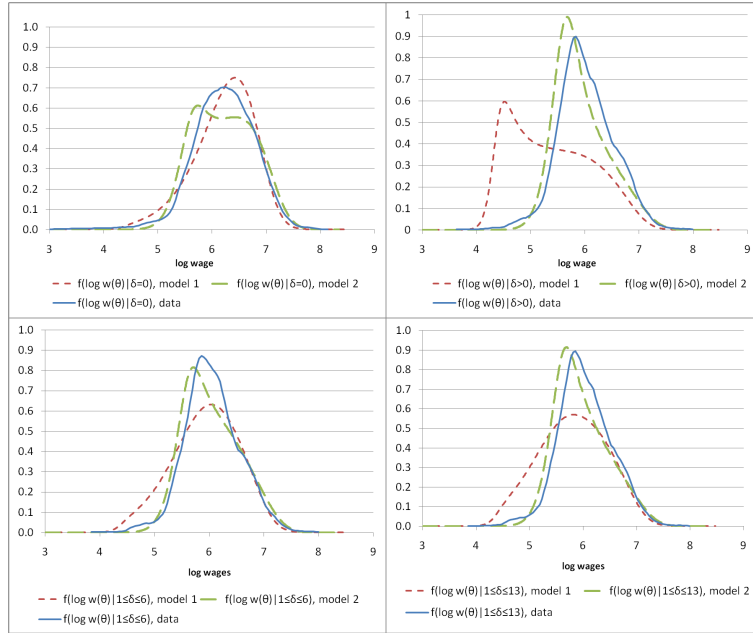
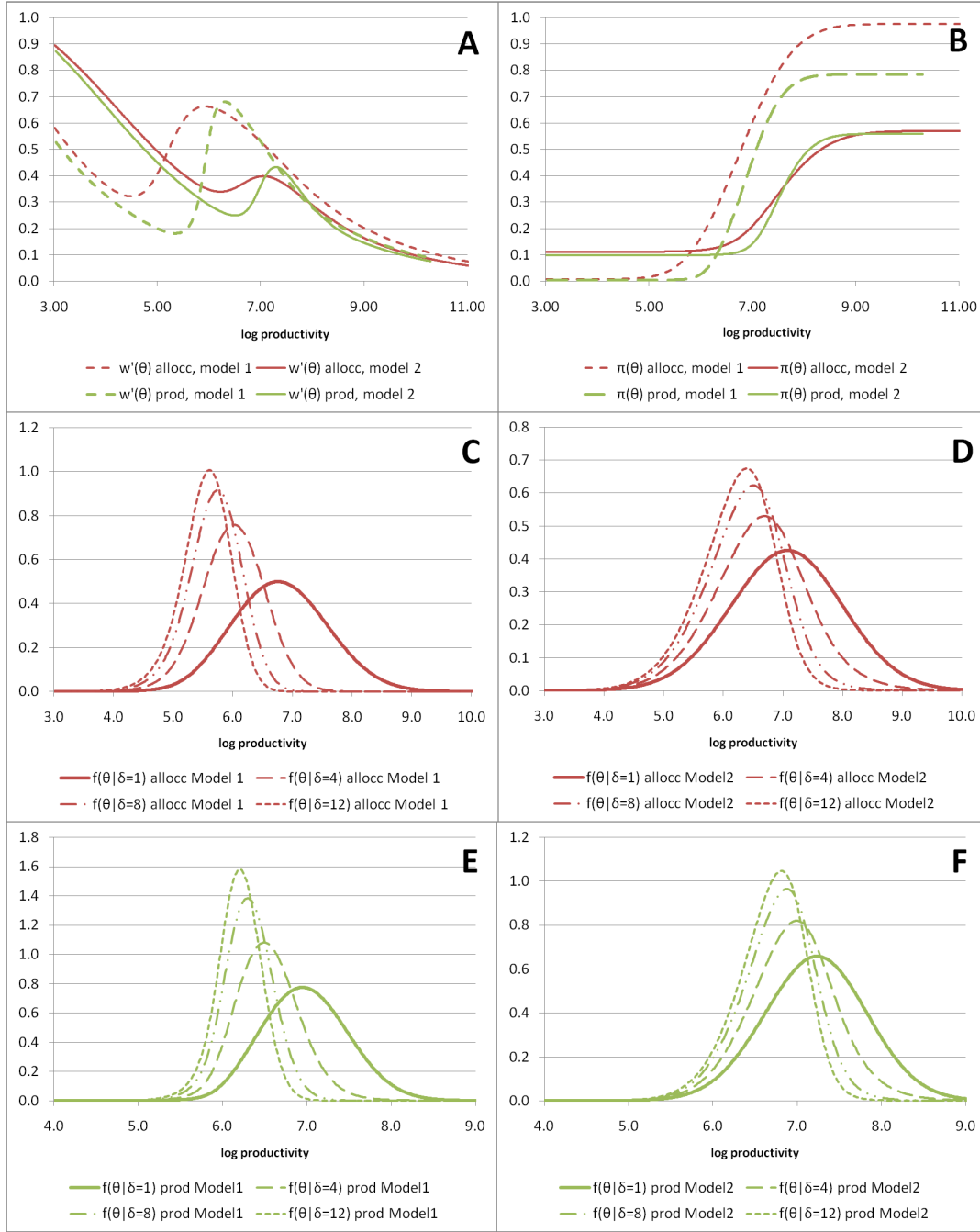
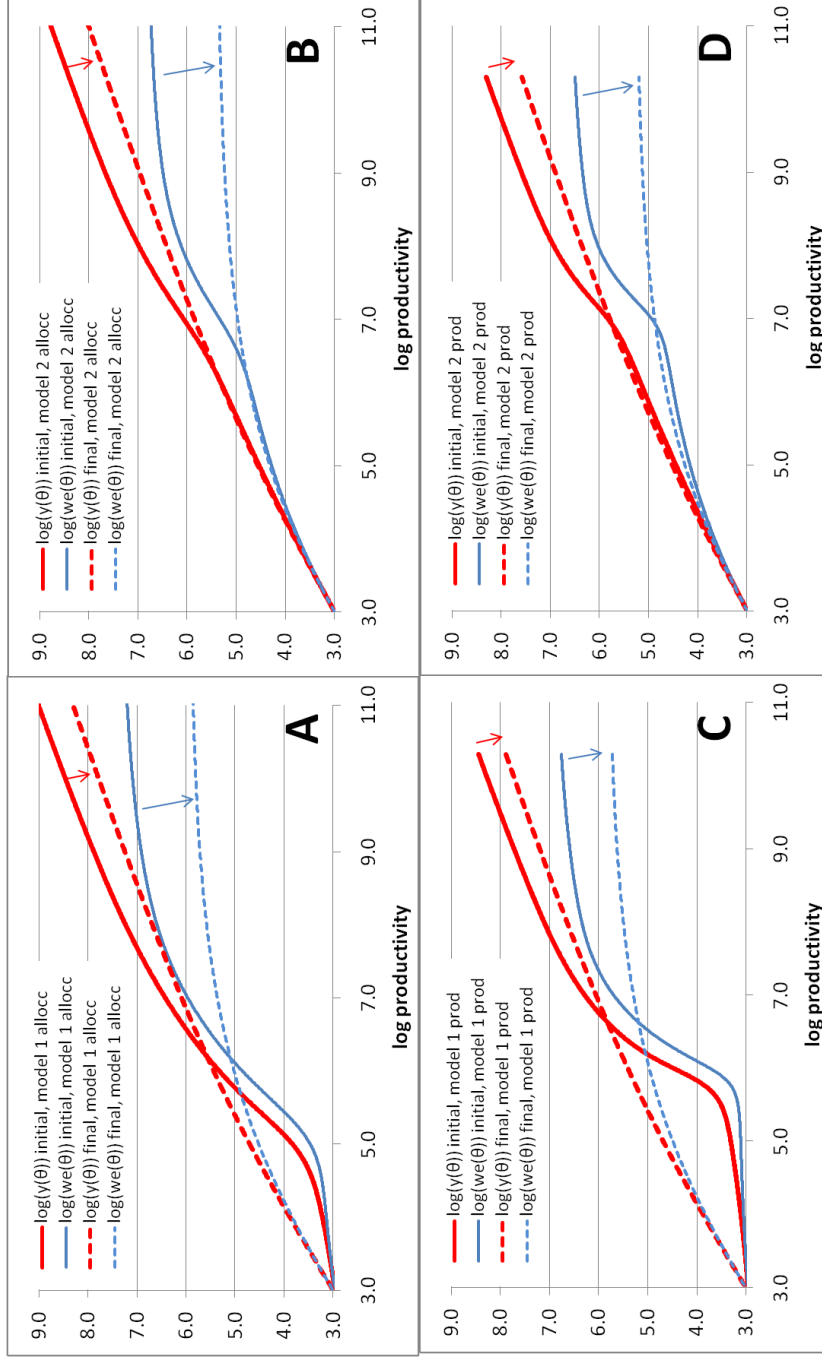


Figure 6: Wage derivative, hazard rates and conditional distributions.



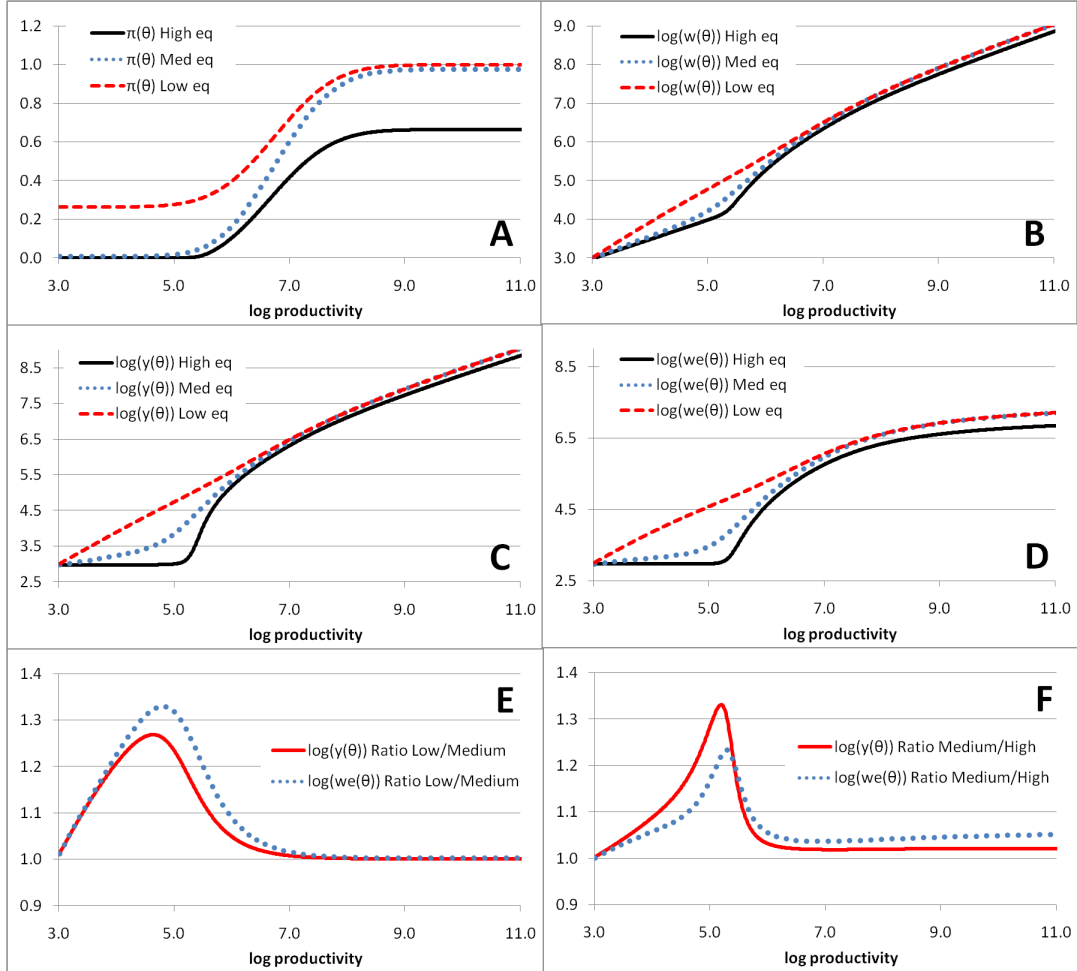
Notes: Panel A: Derivatives of wages  $w'(\theta)$  for each productivity  $\theta$ ; Panel B: Hazard rates out of unemployment  $\pi(\theta)$ ; Panel C: Distributions of productivity conditional on duration, Model 1, all occupations; Panel D: Distributions of productivity conditional on duration, Model 2, all occupations; Panel E: Distributions of productivity conditional on duration, Model 1, production workers; Panel F: Distributions of productivity conditional on duration, Model 2, production workers.

Figure 7: Permanent Income and Welfare for Sequential and Nonsequential models



Notes: Panels depict initial nonsequential equilibrium and final sequential equilibrium (not compensated). Panel A: Permanent income  $y(\theta)$  and welfare  $we(\theta)$  changes for Model 1, all occupations; Panel B: Permanent income  $y(\theta)$  and welfare  $we(\theta)$  changes for Model 2, all occupations; Panel C: Permanent income  $y(\theta)$  and welfare  $we(\theta)$  changes for Model 1, production workers; Panel D: Permanent income  $y(\theta)$  and welfare  $we(\theta)$  changes for Model 2, production workers.

Figure 8: Comparison among multiple equilibria, Model 1, all occupations



Notes: Panel A compares hazard rates out of unemployment  $\pi(\theta)$ ; Panel B compares wages  $w(\theta)$ ; Panel C compares permanent income  $y(\theta)$ ; Panel D compares welfare (certain equivalent)  $we(\theta)$ ; Panel E depicts the ratios of permanent income and welfare between Low unemployment and Medium unemployment equilibria; Panel F depicts the ratios of permanent income and welfare between Medium unemployment and High unemployment equilibria.