

The Effect of Tax-Deferred Retirement Saving Accounts: A Dynamic General Equilibrium Analysis *

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Abstract

The present paper constructs a dynamic general-equilibrium OLG model with heterogeneous households and analyzes the effect of tax-deferred retirement saving accounts. When stylized 401(k)-type accounts are introduced, the government tax revenue decreases by 26% in the short run and by 15% in the long run. If the government finances these costs by a onetime income tax increase with government debt, it has to raise the marginal tax rates by 31% to make the policy change sustainable. National wealth and output will decline by 1.4% and 2.8%, respectively, in the long run, and households of all cohorts will be worse off.

1 Introduction

The importance of tax-deferred retirement saving accounts, such as 401(k) plans and individual retirement accounts (IRAs), has increased significantly since the introduction of the Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA). Through 2008 the annual contribution limits of 401(k) plans and IRAs are raised to \$15,500 and \$5,000, respectively. These tax-deferred accounts are expected to have significant effects on individual saving as well as aggregate economy. However, the short-run and long-run costs of these tax-favored accounts also have an impact on the government budget at the same time. Without taking the government's financing plan into account, we cannot fully evaluate the possible effects of introducing or extending tax-deferred retirement saving accounts.

*This is still very preliminary.

In this paper we construct a heterogeneous-agent overlapping-generations (OLG) model and analyze the effects of introducing stylized 401(k)-type tax-deferred accounts to the model economy by solving the general equilibrium model for equilibrium transition paths. The main questions of this paper are how much the introduction of tax-deferred accounts would increase individual savings and national wealth and whether those accounts would improve the social welfare. The emphasis is on the short-run and long-run costs of introducing these accounts and how the government would finance the costs. In other words, the question of this paper is whether introducing tax-favored accounts would be better than simply cutting marginal income tax rates.

A substantial literature has analyzed the possible effects of tax-deferred retirement saving accounts. However, most of those papers analyze these accounts either empirically or numerically only in a partial-equilibrium setting. To the best of our knowledge, Imrohoroglu, Imrohoroglu, and Joins (1998) are the first ones who analyze the tax-deferred accounts on household behaviors and aggregate economy in a dynamic general equilibrium setting. Following their paper, we also use a heterogeneous-agent OLG model with uninsurable income shocks and liquidity constraints.

Households in our model economy are heterogeneous with respect to age, working ability, and asset holdings in conventional taxable accounts and in 401(k)-type tax-deferred accounts. Households receive idiosyncratic working ability shocks in each period, and they choose optimal consumption, labor supply, and savings in these two accounts. The main advantage of tax-deferred accounts is the reduction of lifetime tax burden through the deferral of tax payments and the smoothing of taxable income, and the main disadvantage is the lower liquidity due to early-withdrawal penalties. Thus, a general-equilibrium OLG model with uninsurable working ability shocks and liquidity constraints is essential to analyze the effect of tax-deferred accounts.¹

We make three main extensions in this paper from that of Imrohoroglu et al (1998), however. First, the individual income tax in the model economy is progressive, thus, tax-deferred accounts have both income effect and substitution effect. Second, related to the first point, labor supply is endogenous and reflects the relative sizes of income effect and substitution effect of tax-deferred accounts. Third, we solve the model for equilibrium transition path to evaluate both the long-run (permanent) cost and the short-run (transition) cost of introducing tax-deferred accounts to the economy. We, however, made

¹A representative-agent stochastic OLG model does not work for this paper, since average households face liquidity constraints much less likely except for the early stage of their life. An OLG version of Krusell-Smith (1998) type growth model with both aggregate shocks and idiosyncratic shocks probably works better. However, adding three more state variables makes the computation of transition paths impractical, since we cannot use the linear-quadratic approximation in the economy with liquidity constraints and precautionary savings.

some simplifications of the model to see the model implications clearer. The model in this paper abstracts from lifetime uncertainty, bequests, social security pensions, and realistic age-wage profiles.²

In the present paper we first calibrate our model to the U.S. economy without tax-deferred retirement saving accounts (the baseline economy). The baseline economy is assumed to be in steady-state equilibrium, which means the economy is on a balanced growth path. Then, we introduce tax-deferred accounts to the economy, solve the model for an equilibrium transition path, and evaluate the individual and aggregate effects of tax-deferred accounts both in the short run and in the long run. The policy change reduces income tax revenue significantly. It does especially in the short run, since the initial old households do not withdraw their savings from tax-deferred accounts. We mainly assume that the government raises the marginal income tax rates proportionately and issues government bonds to make the policy change sustainable.

The main findings of this paper are as follows: Introducing tax-deferred retirement saving accounts would increase capital stock and total output if the government could finance the costs by reducing its wasteful expenditure without difficulty. Age-21 households would be better off by 1.5% of total resource in the long run and other households would be better off, too. However, more realistically, if the government financed the long-run cost of tax-deferred accounts by increasing marginal income tax rates, introducing these accounts would decrease total output slightly, even though it would still increase capital stock of the economy. The welfare of age-21 households would improve only by 0.1% in the long run. Finally, if the government financed both the long-run cost and the short-run cost by raising marginal tax rates and issuing government bonds, capital stock, labor supply, and total output would all decrease, and age-21 households would be worse off by 0.5% of total resource in the long run and even more in the short run.

The rest of the paper is laid out as follows: Section 2 describes the heterogeneous-agent OLG model with taxable and tax-deferred accounts, Section 3 shows the calibration of the model economy, Section 4 explains the effects of introducing tax-deferred accounts to the economy with a progressive income tax, Section 5 checks the robustness of the results by using altering model economies, and Section 6 concludes the present paper. Appendix explains the complementarity problem we use to solve the household optimization problem and the algorithm to solve the model for equilibrium transition paths.

²In a partial equilibrium setting, Love (2006) models lifetime uncertainty, unemployment shocks, wage shocks, the rate of return shocks, as well as permanent skill heterogeneity, and he analyzed the effect of unemployment insurance on the economy with 401(k) plans by simulations.

2 The Model Economy

The economy consists of a large number of households, a perfectly competitive firm with constant-returns-to-scale technology, and a government with commitment technology. The households are ex ante identical but are heterogeneous with respect to their age, working ability, and beginning-of-period asset holdings. In each period, a household receives an idiosyncratic working-ability shock and choose its optimal consumption, working hours, and end-of-period asset holdings. In the baseline economy (the initial steady-state economy), we assume no tax-deferred accounts, that is, there are only one type of taxable assets. Later we introduce tax-deferred accounts as policy experiments.

Let $a_1 \in A_1$ be the beginning-of-period level of conventional taxable assets, let $a_2 \in A_2$ be the beginning-of-period level of tax-deferred assets, and let $e \in E$ be the working ability shock received by the household. The individual state variables are (a_1, a_2, e) as well as the household's age i . Let Ω_t be a time series of vectors of factor prices and government policy variables that describe the path of the aggregate economy,

$$\Omega_t = \{r_s, w_s, C_{G,s}, \varphi_s, W_{G,s}, s_{2,s}^{\max}\}_{s=t}^{\infty},$$

where r_t is the interest rate, w_t is the average wage rate, $C_{G,t}$ is government consumption, $\varphi_t = (\varphi_{0,t}, \varphi_{1,t}, \varphi_{2,t})^\top$ is the vector of individual income tax parameters, $W_{G,t}$ is the government net wealth, and $s_{2,t}^{\max}$ is the annual contribution limit to tax-deferred accounts in period t . Let $v_i(a_1, a_2, e; \Omega_t)$ be the value function of a household of age i in period t .³ The household's optimization problem is

$$(1) \quad v_i(a_1, a_2, e; \Omega_t) = \max_{c, l, a'_1, a'_2} \{u(c, l) + \tilde{\beta} E[v_{i+1}(a'_1, a'_2, e'; \Omega_{t+1}) | e]\}$$

subject to

$$(2) \quad c + (1 + \mu)(a'_1 + a'_2) \leq (1 + r_t)(a_1 + a_2) + w_t e(1 - l) - \tau_{i,t}(r_t a_1, w_t e(1 - l), s_2),$$

$$s_2 = (1 + \mu)a'_2 - (1 + r_t)a_2 \leq s_{2,t}^{\max},$$

³Let $S_t = \{x_t(a_1, a_2, e, i), W_{G,t}\}$ be the state of the economy, where $x_t(a_1, a_2, e, i)$ is the distribution of households, and let $\Psi_t = \{C_{G,s}, \varphi_s, W_{G,s}, s_{2,s}^{\max}\}_{s=t}^{\infty}$ be the government policy schedule. Then, the value function of a household is shown as $v_i(a_1, a_2, e; S_t, \Psi_t)$, and the factor prices and government policy variables are $r_s(S_t, \Psi_t)$, $w_s(S_t, \Psi_t)$, $\varphi_{0,s}(S_t, \Psi_t)$, and so on, for $s \geq t$. However, we cannot solve the household problem in this form because the dimension of S_t is infinite. We avoid this dimensionality problem by replacing (S_t, Ψ_t) with Ω_t . Since there are no aggregate shocks in the model economy, the time series Ω_t is deterministic, and it will suffice to find the fixed point of Ω_t .

$$c \in (0, \infty), \quad l \in (0, 1], \quad a'_1 \in A_1 = [0, \infty), \quad a'_2 \in A_2 = [0, \infty);$$

where c is the household's consumption, l is leisure, $1 - l$ is working hours, a'_1 is the level of taxable assets at the beginning of next period, a'_2 is the level of tax-deferred assets at the beginning of next period, e' is the working ability in the next period, and $\tilde{\beta}$ is the growth-adjusted time discount factor. Individual variables except for l are growth-adjusted. In the budget constraint, μ is the labor-augmenting productivity growth rate, thus, $w_t e$ denotes the individual wage rate, and $\tau_{i,t}(\cdot)$ is the individual income tax function. In the economy with tax-deferred accounts, the individual income tax is dependent on both tax-deferred savings s_2 and age i . The last constraint assume the non-borrowing constraint (the artificial borrowing constraint at 0) in taxable assets, so that the early-withdrawal penalty of tax-deferred accounts affects the portfolio decision.

Solving the above problem for c , l , a'_1 , and a'_2 , we obtain the household's decision rules (policy functions). Let $c_i(a_1, a_2, e; \Omega_t)$, $l_i(a_1, a_2, e; \Omega_t)$, $a'_{1,i}(a_1, a_2, e; \Omega_t)$, and $a'_{2,i}(a_1, a_2, e; \Omega_t)$ be the corresponding decision rules, and let $h_i(a_1, a_2, e; \Omega_t)$ and $s_{2,i}(a_1, a_2, e; \Omega_t)$ be

$$h_i(a_1, a_2, e; \Omega_t) = 1 - l_i(a_1, a_2, e; \Omega_t),$$

$$s_{2,i}(a_1, a_2, e; \Omega_t) = (1 + \mu)a'_{2,i}(a_1, a_2, e; \Omega_t) - (1 + r_t)a_2.$$

Let $x_{i,t}(a_1, a_2, e)$ be the growth-adjusted population density of age i in period t , and let $X_{i,t}(a_1, a_2, e)$ be the corresponding cumulative distribution. We assume that households enter the economy with no assets, i.e., $a_1 = a_2 = 0$, and that the growth-adjusted population of age 1 households is normalized to unity,

$$\int_{A_1 \times A_2 \times E} dX_{1,t}(a_1, a_2, e) = \int_E dX_{1,t}(0, 0, e) = 1.$$

Let $\pi_i(e'|e)$ be the transition probability of working ability from e at age i to e' at age $i + 1$. Let ν be the population growth rate. Then, the law of motion of the growth-adjusted population distribution is

$$X_{i+1,t+1}(a'_1, a'_2, e') = (1 + \nu)^{-1} \int_{A_1 \times A_2 \times E} \mathbf{1}_{[a'_1 = a'_{1,i}(a_1, a_2, e; \Omega_t), a'_2 = a'_{2,i}(a_1, a_2, e; \Omega_t)]} \pi_i(e'|e) dX_{i,t}(a_1, a_2, e),$$

which is a function of the decision rules for a'_1 and a'_2 as well as $\pi_i(e'|e)$.

Let I be the highest age of households. The growth-adjusted private taxable wealth $W_{1,t}$ and tax-deferred wealth $W_{2,t}$ are

$$W_{1,t} = \sum_{i=1}^I \int_{A_1 \times A_2 \times E} a_1 dX_{i,t}(a_1, a_2, e),$$

$$W_{2,t} = \sum_{i=1}^I \int_{A_1 \times A_2 \times E} a_2 dX_{i,t}(a_1, a_2, e),$$

and capital stock (national wealth) K_t and labor supply in efficiency units L_t are

$$K_t = W_{1,t} + W_{2,t} + W_{G,t},$$

$$L_t = \sum_{i=1}^I \int_{A_1 \times A_2 \times E} e h_i(a_1, a_2, e; \Omega_t) dX_{i,t}(a_1, a_2, e).$$

The representative firm's problem is, for all t ,

$$(3) \quad \max_{\tilde{K}_t, \tilde{L}_t} F(\tilde{K}_t, \tilde{L}_t) - (r_t + \delta)\tilde{K}_t - w_t\tilde{L}_t,$$

where δ is the depreciation rate of capital. The profit maximizing conditions are

$$(4) \quad F_K(\tilde{K}_t, \tilde{L}_t) = r_t + \delta, \quad F_L(\tilde{K}_t, \tilde{L}_t) = w_t,$$

and the factor markets clear when $K_t = \tilde{K}_t$ and $L_t = \tilde{L}_t$.

The individual income tax revenue is

$$(5) \quad T_t = \sum_{i=1}^I \int_{A_1 \times A_2 \times E} \tau_{i,t}(r_t a, w_t e h_i(a_1, a_2, e; \Omega_t), s_{2,i}(a_1, a_2, e; \Omega_t)) dX_{i,t}(a_1, a_2, e),$$

and the government budget constraint is

$$(6) \quad C_{G,t} + (1 + \mu)(1 + v)W_{G,t+1} \leq (1 + r_t)W_{G,t} + T_t.$$

The recursive competitive equilibrium of this model economy is defined as follows.

DEFINITION Recursive Competitive Equilibrium: Let (a_1, a_2, e, i) the individual state of households. A time series of factor prices and the government policy variables,

$$\Omega_t = \{r_s, w_s, C_{G,s}, \varphi_s, W_{G,s}, s_{2,s}^{\max}\}_{s=t}^{\infty},$$

the value functions of households, $\{v_i(a_1, a_2, e; \Omega_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\{c_i(a_1, a_2, e; \Omega_s), l_i(a_1, a_2, e; \Omega_s), a'_{1,i}(a_1, a_2, e; \Omega_s), a'_{2,i}(a_1, a_2, e; \Omega_s)\}_{s=t}^{\infty},$$

and the disribution of households, $\{x_{i,s}(a_1, a_2, e)\}_{s=t}^{\infty}$ are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$, each household solves the optimization problem (1)-(2), taking Ω_s as given; the firm solves its profit maximization problem (3)-(4); the government policy schedule satisfies (5)-(6); and the goods market and factor markets clear.

The economy is in a steady-state equilibrium if, in addition, $x_{i,s+1}(a_1, a_2, e) = x_{i,s}(a_1, a_2, e)$ and $\Omega_{s+1} = \Omega_s$ for all $s = t, \dots, \infty$.

3 Calibration

Table 1 summarizes the main parameter values and the target values. The period utility function is one of Cobb-Douglas and constant relative risk aversion:

$$u(c, l) = \frac{(c^\alpha l^{1-\alpha})^{1-\gamma}}{1-\gamma} = \frac{(c^\alpha (1-h)^{1-\alpha})^{1-\gamma}}{1-\gamma}.$$

The share parameter of consumption, α , is 0.36, the coefficient of relative risk aversion, γ , is 2.0. The production function is also one of Cobb-Douglas with constant-returns-to-scale technology:

$$F(K, L) = AK^\theta L^{1-\theta}.$$

The share parameter of capital, θ , is 0.30, the depreciation rate of capital stock, δ , is 0.048, and the total factor productivity is calculated so that the average wage rate w is normalized to unity in the baseline economy.

We assume a household enters the economy at the beginning of age 21 and dies at the end of age 80. Thus, model age $i = 1$ is actual age 21, and model age $i = I = 60$ is actual age 80. The working

Table 1: Main Parameters and Factor Prices

α	γ	θ	δ	μ	ν	ρ	σ_ε	β	r	w	K/Y
0.36	2.0	0.30	0.048	0.018	0.010	0.90	0.10	0.9834	0.0520	1.0	3.0

ability of a household is cut proportionately by 60% at age 65 or $i = 45$, thus, most households stop working at this age. The labor-augmenting productivity growth rate, μ , is 0.018, and the population growth rate, ν , is 0.01. The population of the model economy is 45.40 when the population of age 21 households is normalized to unity.

The working ability (the individual wage rate) e follows the AR1 process:

$$\ln e' = \rho \ln e + \varepsilon, \quad \varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2),$$

where $E(e)$ is normalized to unity. In this calibration, the auto-correlation parameter, ρ , is 0.9, and the standard deviation, σ_ε , is 0.1. The log working ability, $\ln e$, is normally distributed with standard deviation $\sigma_{\ln e} = \sigma_\varepsilon / \sqrt{1 - \rho^2} = 0.2294$ and mean $-\sigma_{\ln e}^2/2 = -0.0263$. We discretize the distribution of $\ln e$ into five levels by the Gauss-Hermite quadrature. The discretized working ability levels are $e = (0.5057, 0.7137, 0.9740, 1.3293, 1.8760)^\top$ for $i \leq 64$ and $0.4e$ for $i \geq 65$, and the corresponding Hermite weights are $\pi = (0.0113, 0.2221, 0.5333, 0.2221, 0.0113)^\top$, where $\pi^\top e = 1$. We also calculate the Markov transition matrix by the bivariate normal distribution with correlation $\rho = 0.9$. The transition matrix is

$$\Gamma = \begin{pmatrix} 0.548137 & 0.451542 & 0.000321 & 0.000000 & 0.000000 \\ 0.022889 & 0.730252 & 0.246735 & 0.000124 & 0.000000 \\ 0.000007 & 0.102738 & 0.794510 & 0.102738 & 0.000007 \\ 0.000000 & 0.000124 & 0.246734 & 0.730252 & 0.022889 \\ 0.000000 & 0.000000 & 0.000318 & 0.451543 & 0.548129 \end{pmatrix}.$$

The progressive income tax function in this paper is one of Gouveia and Strauss (1994):

$$\tau_{i,t}(r_t a_1, w_t e(1-l), s_2) \equiv \tilde{\tau}_{i,t}(y) = \varphi_{0,t}(y - (y^{-\varphi_{1,t}} + \varphi_{2,t})^{-1/\varphi_{1,t}}) - \mathbf{1}_{[i < 40]} 0.1 \min(s_2, 0),$$

where $y = \max\{r_t a_1 + w_t e(1-l) - s_2, 0\}$. The taxable income y is of capital income in taxable

accounts and labor income less contribution to tax-deferred accounts. The parameters $\varphi_{1,t}$ and $\varphi_{2,t}$ in the progressive income tax function are 0.839 and 0.029, which are the simple averages of their estimated parameters in years 1979-89. The remaining parameter $\varphi_{0,t}$ explains the limit of the effective marginal income tax rate, and it is set at 0.30 or 30% in the baseline economy. The income tax revenue is equal to 2.6823 or 14.1 as a percentage of total output (GDP), which is roughly consistent with the income tax revenue (federal, state, and local) in the United States. The second term on the right-hand side is the 10% early withdrawal penalty on tax-deferred accounts. The penalty is applicable when the contribution is negative and the household is younger than age 60 (the model age $i < 40$).

The subjective time discount factor, β , is chosen to be 0.9834 (before the growth adjustment) so that the capital-output ratio in the baseline economy with a progressive income tax is equal to 3.0. When the population-average wage rate, w , is normalized to unity, the interest rate, r , is 0.0520 in the baseline economy. The growth-adjusted time discount factor is calculated as $\tilde{\beta} = \beta(1 + \mu)^{\alpha(1-\gamma)}$.

In the baseline economy, the government net wealth, W_G , is assumed to be 0 for simplicity. Thus,

$$C_{G,t} = (1 + r_t)W_{G,t} + T_t - (1 + \mu)(1 + v)W_{G,t+1} = T_t,$$

that is, the government consumption, which has no effect on the household utility, is equal to its income tax revenue.

4 Introducing Individual Retirement Accounts

In the baseline economy (the initial steady-state equilibrium), households are not allowed to contribute their money to tax-deferred retirement saving accounts. In the model, the parameter $s_2^{\max}$ for the maximum annual contribution to tax-deferred accounts is set at 0. In this section, we analyze the long-run effects of introducing tax-deferred accounts to the economies with a 30% progressive income tax and a 16% flat income tax. In the model, the parameter $s_{2,t}^{\max}$ is changed to 0.1. Because labor income per working-age household is 0.3732 in the baseline economy with a progressive income tax, the new level of $s_2^{\max}$ is roughly consistent with the contribution limit of the 401(k) plans, \$15,500 in 2008, in the United States. We also assume a 10% early withdrawal penalty for households younger than age 60.

Table 2: The Long-Run Effects of Tax-Deferred Accounts in an Economy with up to 30% Progressive Income Tax ($s_2^{\max} = 0.1$, % changes from the baseline)

Financing Assumption	(1) Government Consumption	(2) Income Tax Rates	(3) Income Tax Rates+Debt
Capital Stock	8.5	4.5	-1.4
Labor Supply	-0.4	-2.0	-3.4
Total Output (GDP)	2.2	-0.1	-2.8
Tax Revenue	-14.6	0.0	6.9
Consumption	3.7	-1.7	-3.9
Working Hours	-0.4	-1.9	-3.5
Welfare of Age-21 Households	1.5	0.1	-0.5
Interest Rate	-11.2	-8.5	-2.7
Wage Rate	2.6	1.9	0.6
Private Wealth	8.5	4.5	13.0
Taxable Wealth* ¹	16.9	13.3	10.6
Tax-Deferred Wealth* ¹	83.1	86.7	89.4
Government Consumption	-14.6	0.0	0.0
Income Tax Rates	0.0	21.1	31.0
Government Wealth* ²	0.0	0.0	-43.3
Private Wealth* ³	9.5	5.0	12.9

*¹ The shares in total private wealth. *² The change as a percentage of the baseline output. *³ The change as a percentage of the change in tax-deferred wealth. The other numbers are % changes from the the levels in the baseline economy.

4.1 Long-Run Effects

Run 1 of Table 2 shows the long-run effect of introducing tax-deferred accounts when it is financed with government consumption cuts. Government consumption is treated as wastes in this model economy. Since this experiment does not consider any budgetary feedback effect, it shows a sort of primary effect of the tax-deferred accounts. The after-tax rate of return to their savings goes up when households are allowed to invest their money in tax-deferred accounts, private saving is encouraged, and capital stock (= national wealth) increases by 8.5%. Labor supply, however, decreases by 0.4%, because the income effect of the effective tax cuts on average exceeds the substitution effect of the marginal tax rate reductions. National output (GDP) increases by 2.2%. The interest rate falls by 11.2% and the wage rate rises by 2.6%. As the result, the average value (welfare level) of age-21 households measured in terms of total resource increases by 1.5%. The welfare gains or losses in Table 2 are calculated by

$$\left(\frac{\sum_e v_1(0, 0, e; \Omega_\infty) \pi(e)}{\sum_e v_1(0, 0, e; \Omega_1) \pi(e)} \right)^{\frac{1}{1-\gamma}} - 1,$$

where $v_1(0, 0, e; \Omega_1)$ and $v_1(0, 0, e; \Omega_\infty)$ are the values of age-21 households in the baseline and the final steady state, respectively, and $\pi(e)$ is the unconditional distribution of working ability e .

Introducing tax-deferred accounts decreases income tax revenue by 14.6% in the long run. The size of revenue loss depends significantly on the steady-state population growth rate (or the population distribution by age). When the population growth rate is zero, tax-deferred accounts generate a slight tax revenue increase in the long run, because the tax revenue increase from elderly households exceeds the tax revenue loss from working-age households. When the population growth rate is 1%, however, the tax revenue increase from retired households does not fully compensate the revenue loss from working-age households, since the population of elderly households is smaller.

When tax-deferred accounts are available, 83.1% of private wealth are saved in tax-deferred accounts and 16.9% are saved in traditional taxable accounts. The difference in the shares between two accounts might be over-emphasized by the following reasons: First, the wealth in tax-deferred accounts is measured in the before-tax basis, whereas the wealth in taxable accounts is measured in the after-tax basis. Second, in this model economy, the tax benefits in a capital-gain tax are not considered in traditional taxable accounts, and all capital gains are treated as income gains. Third, the model doesn't consider uncertainty other than wage shocks, and early withdrawals occur only if the marginal income tax rate is relatively low.

The second row of Table 2 shows the long-run effect of tax-deferred accounts when the government increases the marginal income tax rates proportionately (shifts up the marginal tax parameter from 0.3 in the model). This shows the effect of tax-deferred accounts with the government's long-run budgetary feedback. The main difference is that, instead of cutting its consumption, the government has to raise the marginal tax rates by 21.1%. The maximum marginal tax rate goes up from 30% to 36.3%.

Because of the income tax increase, capital stock (= private wealth) increases by only 4.5%, labor supply decreases by 2.0%, and total output decreases slightly by 0.1%. The interest rate falls by 8.5% and the wage rate rises by 1.9%. The average value of age 21 households declines by 0.1%. The introduction of tax-deferred accounts is not welfare improving even if we ignore its transition cost. The share of wealth in tax-deferred accounts is even higher. Because of the higher marginal tax rates, 86.7% of private wealth are stored in tax deferred accounts and only 13.3% are stored in traditional accounts.

Run 3 of Table 2 shows the long-run effect of introducing tax-deferred accounts when the government increases the marginal income tax rates immediately but also issues government bonds to finance

the transition cost of the tax-deferred accounts. To calculate this final steady-state economy with government debt, we solve the model for an entire equilibrium transition path, which is explained in Section 5. It turns out that the marginal income tax rates have to increase by 31.0% immediately to 39.3% from 30%. With these new marginal tax rates, the growth-adjusted government wealth decreases gradually to its final steady-state level, which is -44.5 as a percentage of GDP. In this final steady-state economy, the increase in the marginal tax rates exactly finances the long-run cost of tax-deferred accounts and the debt service cost.

Because the increase in the marginal tax rates is even larger than Run 2, the share of private wealth in tax-deferred accounts is even higher: 89.5% of private wealth are invested in tax-deferred accounts and 10.5% are in traditional taxable accounts. Private wealth increases by 13.0% from the baseline level, because households have to pay higher taxes when they withdraw money from their retirement accounts. However, capital stock (= national wealth) declines by 1.4% because of the government debt. The higher marginal income tax rates also decrease labor supply by 3.4%, and total output (GDP) decreases by 2.8% in the long run.

The welfare level of age-21 households declines by 0.5% measured in terms of total resources of households explained above. The introduction of tax-deferred accounts hurt households in the long run if we take both the permanent cost and the transition cost into account and if these costs are financed by government bonds and one-time marginal income tax rate increase. We will see more about the transition analysis in Section 5.

Figure 1 shows the long-run effects of tax-deferred accounts by age. In each of six charts, the solid line shows the life-cycle profile in the baseline economy, and the other 3 lines show the profiles in the alternative final steady-state economies: one with government consumption cuts, one with marginal tax rate increases, and one with marginal tax rate increases and government debt.

The first chart shows the average tax payments of households by age. In the baseline economy with traditional taxable accounts only, households pay most of their taxes when they are working, and the tax payments increase gradually as their taxable wealth increases. When tax-deferred accounts are available, the tax payment during the working period decreases and the payment after the retirement increases. The reduction in the tax payment is especially large during the last 20 years of working period. The kink at age 59 is shown in the chart, because age 59 is the last year of the 10% early-withdrawal penalty. At age 60 some households start withdrawing money from tax-deferred accounts, thus, the tax payments increase until age 65. At age 65 most households retire in this economy. Since they first

withdraw their money from traditional taxable accounts, the tax payment declines significantly at age 65, then, it goes up as more money is withdrawn from tax-deferred accounts. The lines that represent three alternative experiments are roughly in parallel. The economy with government consumption cuts requires the lowest tax payments, then, the economy with the marginal income tax rate increases, and the economy with tax increases and government debt requires the highest tax payment to cover the debt service.

The second and the third charts show the consumption and working hours profiles of households, respectively. Because we assume the flat age-wage profile in these model economies, consumption and working hours are almost flat during the working period. When households are young, however, consumption is smaller (increasing) and working hours is larger (decreasing) due to the precautionary motive against the wage uncertainty. Once their wealth (especially more liquid taxable wealth) reach the optimal level of precautionary savings, both the consumption and labor supply profiles become flat.

The fourth and the fifth charts show the profiles of taxable wealth and tax-deferred wealth by age. In the economies with tax-deferred accounts, wealth in traditional taxable accounts is small and it is peaked at ages between 43 and 46, which is roughly the mid point of their working period (from age 21 through 64). As the remaining years of working period becomes shorter, the lifetime income risk gets smaller, which requires less precautionary saving. The wealth in tax-deferred accounts are due mostly to the life-cycle saving motive, thus, it is peaked at the beginning of age 65, the age at which most households stop working. The last chart show the profiles of tax-deferred saving by age. The tax-deferred saving increases until age 59 and decreases in ages between 60 and 64. At age 65 most households start dis-saving from tax-deferred accounts.

4.2 Transition Effects

To obtain the long-run effect of Run 3, which assumes one-time proportionate increase in the marginal income tax rates and period-by-period adjustment of government debt, we need to solve the model for an equilibrium transition path. In this section we further examine the transition paths when the government introduces tax-deferred individual retirement accounts. We assume that the economy is in the initial steady state without tax-deferred accounts in period 1 and that the government introduces tax-deferred accounts at the beginning of period 2. Regarding the government financing options, we first assume that the government cuts its consumption (expenditure) to finance the cost of tax-deferred accounts. The final steady state economy is same as Run 2 in the previous section. We next assume that

the government increases the marginal income tax rates proportionately and issues government bonds. The final steady state economy of this transition path is same as Run 3.

Figure 2 shows the transition effects of introducing tax-deferred accounts when the government cuts its consumption to finance the policy change. In this experiment, we assume that the government budget is balanced in every period and that the government consumption is changed period by period. Thus, the government budget constraint is

$$C_{G,t} = T_t + (1 + r_t)W_{G,t} - (1 + \mu)(1 + v)W_{G,t+1} = T_t,$$

since $W_{G,t} = 0$ for all t . The top-left chart of Figure 2 shows the transition paths of macroeconomic variables for the first 100 periods. Capital stock, which is equal to private wealth, increases monotonically. Labor supply declines at the beginning, because the income effect of the marginal tax rate cuts is larger than the substitution effect. Although labor supply recovers gradually, labor supply does not go beyond the level in the baseline economy. Total output (GDP) also declines first, and it goes up for the rest of the transition path. The top-right chart shows the paths of household variables. Both the disposable income and consumption of household jump up when tax-deferred accounts are introduced. The disposable income increases further in the short run then decrease when some households start withdrawing money from their tax-deferred accounts and paying higher income taxes. The household saving rate (shown on the right axis) is 11.8% in the baseline economy, and it shows the similar transition path to the disposal income of households.

The mid-left chart of Figure 2 shows the changes in private wealth as a percentage of total output and as a percentage of tax-deferred wealth. For the first 10 periods, slightly over 5% of wealth in tax-deferred accounts is the net increase in private wealth, and the remainder is wealth transferred from traditional taxable accounts. In the long run, 9.2% of tax-deferred wealth contributes to the increase in private wealth. The mid-right chart shows the shares of taxable wealth and tax-deferred wealth in private wealth. The changes in the shares are monotonic in the transition path. The bottom-left chart shows the paths of the interest rate and the average wage rate, and the bottom-right chart shows how much the government has to cut its consumption each period to balance the budget. The cost of tax-deferred accounts are about 25% of the baseline government consumption, which is equal to income tax revenue, in the short run. The cost decreases gradually and reaches its long-run level, which is 14.6% of the initial tax revenue.

Figure 3 shows the equilibrium transition path of introducing tax-deferred accounts when the gov-

ernment use a onetime proportional increase in the marginal income tax rates and period-by-period increases in government debt to finance the cost of tax-deferred accounts. The government inter-temporal budget constraint is

$$W_{G,t+1} = [(1 + \mu)(1 + v)]^{-1} \{T_t(\varphi_0) + (1 + r_t)W_{G,t} - C_{G,t}\},$$

for all t , with the sustainability condition

$$T_\infty(\varphi_0) = C_{G,\infty} - [(1 + r_\infty) - (1 + \mu)(1 + v)]W_{G,\infty},$$

where $t = \infty$ indicates the values in the final steady state and the tax parameter φ_0 shows the limit of marginal income tax rates. The bottom-right chart of Figure 3 shows the limit of the marginal income tax rates, φ_0 , and the government wealth, $W_{G,t}$, as a percentage of total output (shown on the right axis) that satisfy the above conditions. As we saw in the previous section, the marginal income tax rates have to increase proportionately by 31.0% for tax-deferred accounts to be sustainable to the infinite future. In the model, the parameter φ_0 goes up from 30% to 39.3% in period 2 and, with that change, the government wealth declines overtime and eventually reaches its steady-state level, which is -43.3 as a percentage of total output.

The top-left chart of Figure 3 shows the transition path of macroeconomic variables. Under this financing assumption, capital stock, labor supply, and total output (GDP) are all below the levels in the baseline economy throughout the transition path. Capital stock decreases by about 4% in the short run, then, it goes up to the level, which is 1.4% below the level in the baseline economy. The 31.0% increase in marginal tax rates discourage labor supply. It decreases by almost 5% in period 2 and recovers the level about 3.4% below the baseline economy. Total output decreases by about 3.5% in the short run and eventually reaches the long-run level, which is 2.8% below the baseline economy. The top-right chart shows the disposable income and consumption of households and the saving rate (shown on the right axis). The path of the disposable income shifts down from that in Figure 2 by about 6.5 percentage points due to the income tax increase. Consumption decreases by 4.7% in the short run, and it recovers to the level 3.9% below the baseline economy. The saving rate goes down to 10.9% from 11.8% at the beginning, then, it goes up to 15.4% in the short run and reaches the long-run level 13.6%. The saving rate has to be higher, because households expect much higher taxes when they withdraw money from tax-deferred accounts.

The mid-left chart shows the changes in private wealth as a percentage of total output and as a percentage of tax-deferred wealth. Private wealth decreases at the beginning, but it increases by 39.1 as a percentage of output in the long run. However, the increase is not large enough to compensate the decrease in the government wealth. The mid-right chart shows the shares of taxable wealth and tax-deferred wealth in total private wealth, and the bottom-left chart shows the transition paths of the interest rate and the average wage rate.

5 Alternative Model Economies

In this section we examine the effect of introducing tax-deferred individual retirement accounts to three alternative economies: an economy with a flat income tax, an economy with more risk averse households, and an economy with a larger income shocks. In each of these economies, we choose a subjective time discount factor β that generates the capital-output ratio of 3.0 in the initial steady state.

5.1 Economy with a 16% Flat Income Tax

Tax-deferred IRAs have two sources of effects that reduce the income tax payments of households. The first one is deferring tax payments, which is similar to the effect of capital gain taxes. Households can reduce the lifetime tax payments in present value by using tax-deferred IRAs. The second one is smoothing marginal income tax rates, which is applicable only when the income tax is progressive. Households can reduce the average tax payments by smoothing their taxable income over the lifetime. This effect is even larger when the economy is growing ($\mu > 0$) and the government adjusts the tax brackets to avoid real bracket creeps.

In the economy with a flat income tax, deferring taxable income from working ages to retirement ages will not reduce the marginal and average tax rates. Thus, the experiment in the economy with a flat income tax shows the effect of tax-deferred IRAs when we consider the first source of tax saving only and ignore the second source. We choose the flat tax rate to be 16% so that the income tax revenue is about the same level as before. In this new economy it is 2.7332 or 13.7 as a percentage of total output. The time discount factor, β , is chosen to be 0.9795 so that the capital-output ratio is 3.0 and the interest rate is 0.052 in the baseline economy without IRAs.

Table 3 shows the long-run effects of introducing tax-deferred IRAs to the economy with a 16% flat tax. Run 4 assumes that the government cuts its consumption to finance the permanent cost of tax-

Table 3: The Long-Run Effects of Tax-Deferred Accounts in an Economy with a 16% Flat Income Tax ($s_2^{\max} = 0.1$, % changes from the baseline)

Financing Assumption	(4) Government Consumption	(5) Income Tax Rates	(6) Income Tax Rates+Debt
Capital Stock	8.0	7.2	1.2
Labor Supply	-0.2	-0.2	-1.0
Output (GDP)	2.2	1.9	-0.3
Tax Revenue	-7.2	0.0	5.5
Consumption	2.2	0.5	-0.9
Working Hours	0.2	0.1	-0.9
Welfare of Age-21 Households	1.0	0.4	-0.2
Interest Rate	-10.4	-9.4	-2.9
Wage Rate	2.4	2.2	0.7
Private Wealth	8.0	7.2	12.5
Taxable Wealth* ¹	19.3	18.8	18.5
Tax-Deferred Wealth* ¹	80.7	81.2	81.5
Government Consumption	-7.2	0.0	0.0
Income Tax Rates	0.0	8.0	14.1
Government Wealth* ²	0.0	0.0	-34.0
Private Wealth* ³	9.2	8.2	13.7

*¹ The shares in total private wealth. *² The change as a percentage of the baseline output. *³ The change as a percentage of the change in tax-deferred wealth. The other numbers are % changes from the the levels in the baseline economy.

deferred IRAs, where government consumption does not affect the utility of any households. When tax-deferred IRAs are available, 80.7% of private wealth is invested in IRAs. The share of wealth is smaller than 83.1% in Run 1 in the economy with an up-to 30% progressive income tax, since the tax-saving effect of IRAs is smaller. The direct cost of introducing tax-deferred IRAs is 7.2% of baseline tax revenue, which is about the half of the cost in the economy with a progressive tax. Interestingly, however, the effect of tax-deferred IRAs on the macro economy is about the same. Capital stock increases by 8.0%, labor supply decreases by 0.2%, and total output increases by 2.2%. The welfare of age-21 households improves by 1.0%, which is smaller than that in Run 1.

Run 5 assumes that the government increases the income tax rate to finance the long-run cost of tax-deferred IRAs. The tax rate has to go up by 8.0% from 16% to 17.3%. The effect of tax rate increase on macroeconomic variables is relatively small. Capital stock still increases by 7.2%, labor supply decreases by -0.2%, and total output increases by 1.9%. The last mentioned number is only 0.3 percentage points lower than the increase in total output in Run 4. The average welfare of age-21 households increases by 0.4% in this experiment. Run 6 assumes that the government raises the tax

Table 4: The Long-Run Effects of Tax-Deferred Accounts in an Economy with up to 30% Progressive Income Tax and $\gamma = 3.0$ ($s_2^{\max} = 0.1$, % changes from the baseline)

Financing Assumption	(7) Government Consumption	(8) Income Tax Rates	(9) Income Tax Rates+Debt
Capital Stock	6.8	3.1	-3.6
Labor Supply	-0.4	-2.0	-3.3
Output (GDP)	1.7	-0.5	-3.4
Tax Revenue	-13.8	0.0	5.7
Consumption	3.3	-1.8	-4.1
Working Hours	-0.6	-2.1	-3.8
Welfare of Age-21 Households	2.8	0.1	-1.2
Interest Rate	-9.1	-6.6	0.4
Wage Rate	2.1	1.5	-0.1
Private Wealth	6.8	3.1	7.4
Taxable Wealth* ¹	23.2	17.6	16.2
Tax-Deferred Wealth* ¹	76.8	82.4	83.8
Government Consumption	-13.8	0.0	0.0
Income Tax Rates	0.0	20.4	29.0
Government Wealth* ²	0.0	0.0	-34.3
Private Wealth* ³	8.3	3.7	8.2

*¹ The shares in total private wealth. *² The change as a percentage of the baseline output. *³ The change as a percentage of the change in tax-deferred wealth. The other numbers are % changes from the the levels in the baseline economy.

rate at once and also issues government bonds. Similar to Run 3, the long-run steady-state economy is obtained by solving the model for an equilibrium transition path. It turns out that the income tax rate has to go up at the time of the policy change by 14.1% from 16% to 18.3% and that with this tax rate the government debt is 34.0 as a percentage of total output in the long run. Although private wealth increases by 12.5% from the baseline economy, combined with the government debt, capital stock increases by only 1.2%. Labor supply decreases by 1.0%, and total output also decreases by -0.3%. The average value of age-21 households declines by 0.2%. Again, when we take both the permanent cost and the transition cost into account, introducing tax-deferred IRAs makes the households worse off.

5.2 Economy with More Risk-Averse Households

The main difference between traditional taxable accounts and tax-deferred accounts is the reduced liquidity in tax-deferred IRAs due to the 10% early withdrawal penalty. The optimal size of wealth

in tax-deferred IRAs correlates negatively to the importance of precautionary savings against income uncertainty. In this experiment, we assume the coefficient of relative risk aversion, γ , to be 3.0 instead of 2.0. The time discount factor, β , is chosen to be 0.9928 before the growth adjustment so that the capital-output ratio in the baseline economy is 3.0. Tax revenue in the baseline economy is 2.6574 or 14.0% of total output, which is about the same level as before.

Table 4 shows the long-run effects of tax-deferred IRAs on the economy with an up-to 30% progressive income tax and with more risk-averse households. To finance the cost of introducing IRAs, Run 7 assumes government consumption cuts, Run 8 assumes marginal income tax rate increases, and Run9 assumes the increases in both marginal tax rates and government debt. The shares of wealth held in tax-deferred IRAs are smaller than those in Table 2. Accordingly, the changes in macroeconomic variables are also smaller. Capital stock increases by 6.8%, labor supply decreases by 0.4%, and total output increases by 1.7% in Run 7. Although the increases in the marginal tax rates required to finance tax-deferred IRAs are slightly smaller than before, the effect on capital stock and total output are worse in Runs 8 and 9 compared to those in Runs 2 and 3. The average welfare level of age-21 households improves by 2.8% when we assume government consumption cuts but decline by 1.2% when we assume the increases in marginal tax rates and government debt.

5.3 Economy with Higher Working-Ability Risk

Table 5 shows the long-run effects of tax-deferred IRAs, on the progressive tax economy with higher working ability risk. In this experiment we assume the standard deviation, σ_ε , of shocks on log wages to be 0.2 instead of 0.1. The corresponding working ability levels are $e = (0.2427, 0.4832, 0.9001, 1.6766, 3.3388)^\top$. The Hermite weights and the Markov transition matrix are the same as before. The time discount factor, β , is chosen to be 0.9741 to attain the capital-output ratio of 3.0 in the baseline economy, and tax revenue is 2.8696 or 14.8% of output, which is slightly higher than those in other economies we considered.

Similar to the previous tables, Run 8 of Table 5 assumes marginal income tax rate increases, and Run9 assumes the increases in both marginal tax rates and government debt. Since households that receive larger working ability shocks rely more on precautionary savings and less on tax-deferred IRAs with early-withdrawal penalties, the shares of tax-deferred wealth are uniformly lower than those in Table 2. The share is only 66.8% in Run 10 and 74.8% in Run 12. The direct cost of tax-deferred IRAs is 10.6 as a percentage of the baseline tax revenue. Interestingly, however, the increases in capital stock

Table 5: The Long-Run Effects of Tax-Deferred Accounts in an Economy with up to 30% Progressive Income Tax and $\sigma_\varepsilon = 0.2$ ($s_2^{\max} = 0.1$, % changes from the baseline)

Financing Assumption	(10) Government Consumption	(11) Income Tax Rates	(12) Income Tax Rates+Debt
Capital Stock	9.0	5.9	0.0
Labor Supply	-0.2	-1.3	-2.4
Output (GDP)	2.4	0.8	-1.7
Tax Revenue	-10.6	0.0	4.4
Consumption	3.2	-0.8	-2.7
Working Hours	0.1	-0.9	-2.4
Welfare of Age-21 Households	1.3	0.2	-0.3
Interest Rate	-11.5	-9.2	-3.3
Wage Rate	2.7	2.1	0.8
Private Wealth	9.0	5.9	9.8
Taxable Wealth* ¹	33.2	28.6	25.2
Tax-Deferred Wealth* ¹	66.8	71.4	74.8
Government Consumption	-10.6	0.0	0.0
Income Tax Rates	0.0	14.9	21.9
Government Wealth* ²	0.0	0.0	-29.9
Private Wealth* ³	12.3	7.8	11.9

*¹ The shares in total private wealth. *² The change as a percentage of the baseline output. *³ The change as a percentage of the change in tax-deferred wealth. The other numbers are % changes from the the levels in the baseline economy.

and total output are larger in Table 5. The marginal tax rates have to go up by 14.9% and 21.9% in Run 11 and Run 12, respectively. Since the tax increases are smaller, the effect of introducing IRAs on macroeconomic variables is more positive. Yet, capital stock is about the same level, labor supply decreases by 2.4%, total output decreases by 1.7%, and the average value of age-21 households also decline by 0.3% in Run 12.

6 Concluding Remarks

The primary contribution of the present paper is that tax-deferred retirement saving accounts, which are similar to the 401(k) plans, would make all generations worse off under the reasonable government financing assumption, i.e., when the government financed the short-run and long-run costs by raising income tax rates and issuing government bonds. The implication of the model experiment is robust. We get the qualitatively same result under several different assumptions of the model. Regarding the effect of tax-deferred accounts on wealth accumulation, it differs significantly depending on the government

financing assumption.

The supplemental contribution of the paper is that it presents a policy analysis tool for the economy with two assets. We can easily modify or extend the model to analyze the household decisions between housing assets and financial assets, risky assets and risk-free assets, social security individual accounts and other private assets, and cash and other less-liquid assets, etc. Since the solution algorithm presented in Appendix is efficient enough to solve the model for an equilibrium transition path of 150 years, with some refinements, the model will be useful for policy evaluations by the government institutions, too.

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Appendix

In the present paper, we solve the household problem by discretizing its individual state space. We discretize the asset space $A_1 = A_2 = [0, \infty)$ into 50 levels each, $\hat{A}_1 = \hat{A}_2 = \{a^1, a^2, \dots, a^{50}\}$, and the working ability space $E = [0, \infty)$ into 5 levels, $\hat{E} = \{e^1, e^2, \dots, e^5\}$. Thus, for households of age i in period t , we solve the above optimization problem on 12,500 individual states. We evaluate the marginal values $v_{1,i+1}(a'_1, a'_2, e'; \Omega_{t+1})$ and $v_{2,i+1}(a'_1, a'_2, e'; \Omega_{t+1})$ in the Kuhn-Tucker conditions with bilinear interpolation.

In this appendix, we first explain the algorithm to solve the household's optimization problem for each node in the individual state space. Then, we explain the algorithm to find an equilibrium transition path in which the government changes the marginal income tax rates and issues government bonds. For the general algorithm to compute a steady-state equilibrium and an equilibrium transition path, see Conesa and Krueger (1999).

Algorithm to Solve the Household Problem

Let λ be the Lagrange multiplier for the budget constraint. The Lagrangian of the household problem is

$$\begin{aligned} \mathcal{L}(c, l, a'_1, a'_2, \lambda) = & u(c, l) + \tilde{\beta} E[v_{i+1}(a'_1, a'_2, e'; \Omega_{t+1})|e] \\ & + \lambda \{(1 + r_t)(a_1 + a_2) + w_t e(1 - l) - \tau_{i,t}(r_t a_1, w_t e(1 - l), s_2) - c - (1 + \mu)(a'_1 + a'_2)\}. \end{aligned}$$

Let $v_{k,i}(a_1, a_2, e; \Omega_t)$ and $\tau_{k,i,t}(r_t a_1, w_t e(1 - l), s_2)$ denote the marginal value and marginal tax functions with respect to the k -th argument. The Kuhn-Tucker conditions for this problem are

$$\begin{aligned} c : & u_c(c, l) = \lambda, \\ l : & u_l(c, l) \geq \lambda w_t e (1 - \tau_{2,i,t}(r_t a_1, w_t e(1 - l), s_2)), \\ a'_1 : & \tilde{\beta} E[v_{1,i+1}(a'_1, a'_2, e'; \Omega_{t+1})|e] \leq \lambda(1 + \mu), \\ a'_2 : & \tilde{\beta} E[v_{2,i+1}(a'_1, a'_2, e'; \Omega_{t+1})|e] \leq \lambda(1 + \mu)[1 + \tau_{3,i,t}(r_t a_1, w_t e(1 - l), s_2)], \end{aligned}$$

and the budget constraint

$$c + (1 + \mu)(a'_1 + a'_2) \leq (1 + r_t)(a_1 + a_2) + w_t e (1 - l) - \tau_{i,t}(r_t a_1, w_t e (1 - l), s_2),$$

where weak inequalities in the bottom 4 conditions hold with equalities if $l < 1$, $a'_1 > 0$, $a'_2 > 0$, and $\lambda > 0$, respectively. For each household with state (a_1, a_2, e, i) and period t , we solve the above problem for $(c, l, a'_1, a'_2, \lambda)$ with the following simple conditions:

$$c \in (0, \infty), \quad l \in (0, 1], \quad a'_1 \in [0, \infty), \quad a'_2 \in [0, ((1 + r_t)a_2 + s_{2,t}^{\max})/(1 + \mu)], \quad \lambda \in [0, \infty).$$

Let $\mathbf{d} \equiv (c, l, a'_1, a'_2)^\top$, $u_{\mathbf{d}} \equiv (\infty, 1, \infty, \infty)^\top$, and $l_{\mathbf{d}} \equiv (\epsilon, \epsilon, 0, 0)^\top$, where $u_{\mathbf{d}}$ and $l_{\mathbf{d}}$ are the vector of upper bounds and lower bounds, respectively, and ϵ is a small positive number such as 10^{-3} . Let's also redefine the objective function and the budget constraint as

$$f(\mathbf{d}) \equiv u(c, l) + \tilde{\beta} E[v_{i+1}(a'_1, a'_2, e'; \Omega_{t+1}) | e],$$

$$g(\mathbf{d}) \equiv (1 + r_t)(a_1 + a_2) + w_t e (1 - l) - \tau_{i,t}(r_t a_1, w_t e (1 - l), s_2) - c - (1 + \mu)(a'_1 + a'_2).$$

Then, the Kuhn-Tucker conditions with simple conditions are expressed as the complementarity problem:

$$\begin{aligned} CP \left[\begin{pmatrix} \mathbf{d} \\ \lambda \end{pmatrix}, \begin{pmatrix} l_{\mathbf{d}} \\ 0 \end{pmatrix}, \begin{pmatrix} u_{\mathbf{d}} \\ \infty \end{pmatrix} \right] \\ \equiv \min \left\{ \max \left[\begin{pmatrix} \nabla f(\mathbf{d}) + \lambda \nabla g(\mathbf{d}) \\ -g(\mathbf{d}) \end{pmatrix}, \begin{pmatrix} l_{\mathbf{d}} - \mathbf{d} \\ 0 - \lambda \end{pmatrix} \right], \begin{pmatrix} u_{\mathbf{d}} - \mathbf{d} \\ \infty - \lambda \end{pmatrix} \right\} = \mathbf{0}, \end{aligned}$$

where $\nabla f(\mathbf{d})$ and $\nabla g(\mathbf{d})$ are the gradient vectors. For more details about the complementarity problem, see Miranda and Fackler (2002). We solve the complementarity problem with a nonlinear equation solver, NEQNF, in the Fortran IMSL library. Once we solve the problem for $(\mathbf{d}; \lambda)$, we also calculate the value and the marginal values with respect to two assets of household. The marginal values are

$$\begin{aligned} v_{1,i}(a_1, a_2, e; \Omega_t) &= u_c(c_i(a_1, a_2, e; \Omega_t), l_i(a_1, a_2, e; \Omega_t)) \times \\ &\quad [1 + r_t(1 - \tau_{1,i,t}(r_t a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_{2,i}(a_1, a_2, e; \Omega_t)))] \end{aligned}$$

$$v_{2,i}(a_1, a_2, e; \Omega_t) = u_c(c_i(a_1, a_2, e; \Omega_t), l_i(a_1, a_2, e; \Omega_t)) \times \\ (1 + r_t)(1 + \tau_{3,i,t}(r_t a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_{2,i}(a_1, a_2, e; \Omega_t))).$$

Algorithm to Find a Sustainable One-time Tax Increase

In this paper, we assume that the economy is in the initial steady-state equilibrium in period 1 and that tax-deferred IRAs are introduced at the beginning of period 2 and that the economy reaches its final steady-state equilibrium within $T = 150$ periods. We calculate an equilibrium transition path from the initial steady state to its final steady state as follows:

1. Set the initial values of aggregate variables and government policy variables, $\Omega_2^0 = \{K_s^0/L_s^0, \varphi_s^0, W_{G,s}^0, s_{2,s}^{\max}\}_{s=2}^T$, where $\{\varphi_s^0\}_{s=2}^T = \varphi^0$ is the time-invariant marginal income tax rate parameter, and $W_{G,2}^0 = W_{G,1}$.
2. Compute the final steady-state equilibrium in period T , assuming $W_{G,T}$ to be endogenous. Let $W_{G,T}^0 = W_{G,T}$.
3. For $t = T - 1, T - 2, \dots, 1$, compute backward the household decision rules, $\{\mathbf{d}_i(a_1, a_2, e; \Omega_t^0)\}_{i=1}^I$, and the marginal value functions, $\{v_{1,i}(a_1, a_2, e; \Omega_t^0)\}_{i=1}^I$ and $\{v_{2,i}(a_1, a_2, e; \Omega_t^0)\}_{i=1}^I$.
4. For $t = 1, 2, \dots, T - 1$, compute forward the aggregate variables, $\{K_t^1/L_t^1, W_{G,t+1}^1\}$, and the distributions of households, $\{x_{i,t+1}(a_1, a_2, e)\}_{i=1}^I$, using the decision rules, $\{\mathbf{d}_i(a_1, a_2, e; \Omega_t^0)\}_{i=1}^I$.
5. Let $\{\varphi_t^1\}_{t=2}^T = \{\varphi_t^0\}_{t=2}^T = \varphi^0$. If $W_{G,T}^1$ and $W_{G,T}^0$ are close enough, go to Step 6; otherwise, update $\{\varphi_t^1\}_{t=2}^T$ and return to Step 4.
6. If $\Omega_2^1 = \{K_s^1/L_s^1, \varphi_s^1, W_{G,s}^1, s_{2,s}^{\max}\}_{s=2}^T$ and Ω_2^0 are close enough, stop; otherwise, replace update Ω_2^0 by using Ω_2^1 and return to Step 2.

In Step 5, we update $\{\varphi_t^1\}_{t=2}^T = \varphi^1$ by using the ratio of the present value of government consumption and the increases in government wealth to the present value of tax revenue,

$$\varphi^1 \leftarrow \eta \frac{\sum_{t=2}^{T-1} R_t^{-1} C_{G,t} + R_{T-1}^{-1} (\tilde{r}_T^{-1} C_{G,T} + W_{G,T}^0) - W_{G,2}^0}{\sum_{t=2}^{T-1} R_t^{-1} T_t + R_{T-1}^{-1} \tilde{r}_T^{-1} T_T} \varphi^1 + (1 - \eta) \varphi^1,$$

where $R_t = \prod_{s=2}^t (1 + \tilde{r}_s)$, $\tilde{r}_t = (1 + r_t)/[(1 + \mu)(1 + \nu)] - 1$, and $\eta \in (0, 1]$.

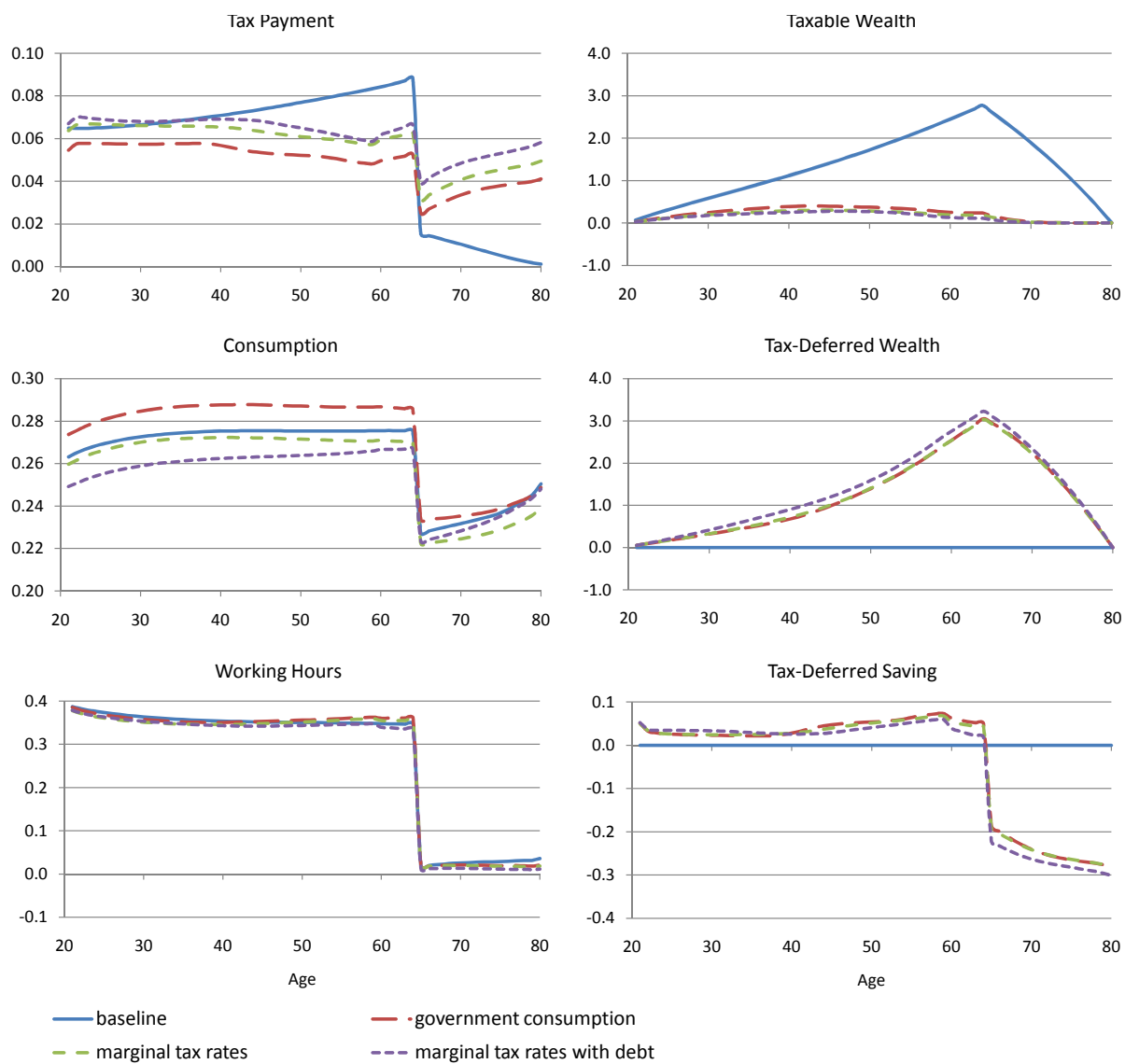


Figure 1: The Long-Run Effects of Tax-Deferred Accounts by Ages

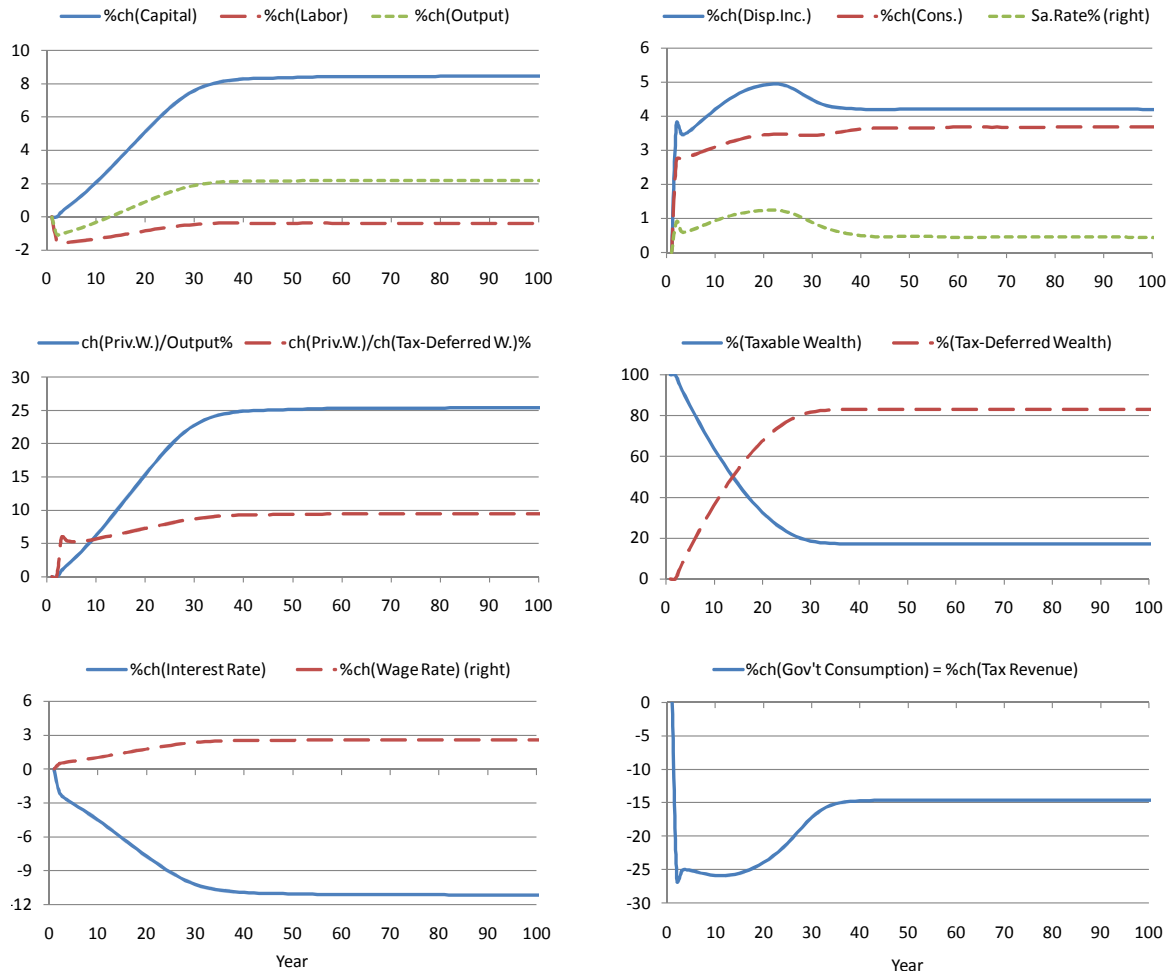


Figure 2: The Effects of Tax-Deferred Accounts Financed with Government Consumption Cuts

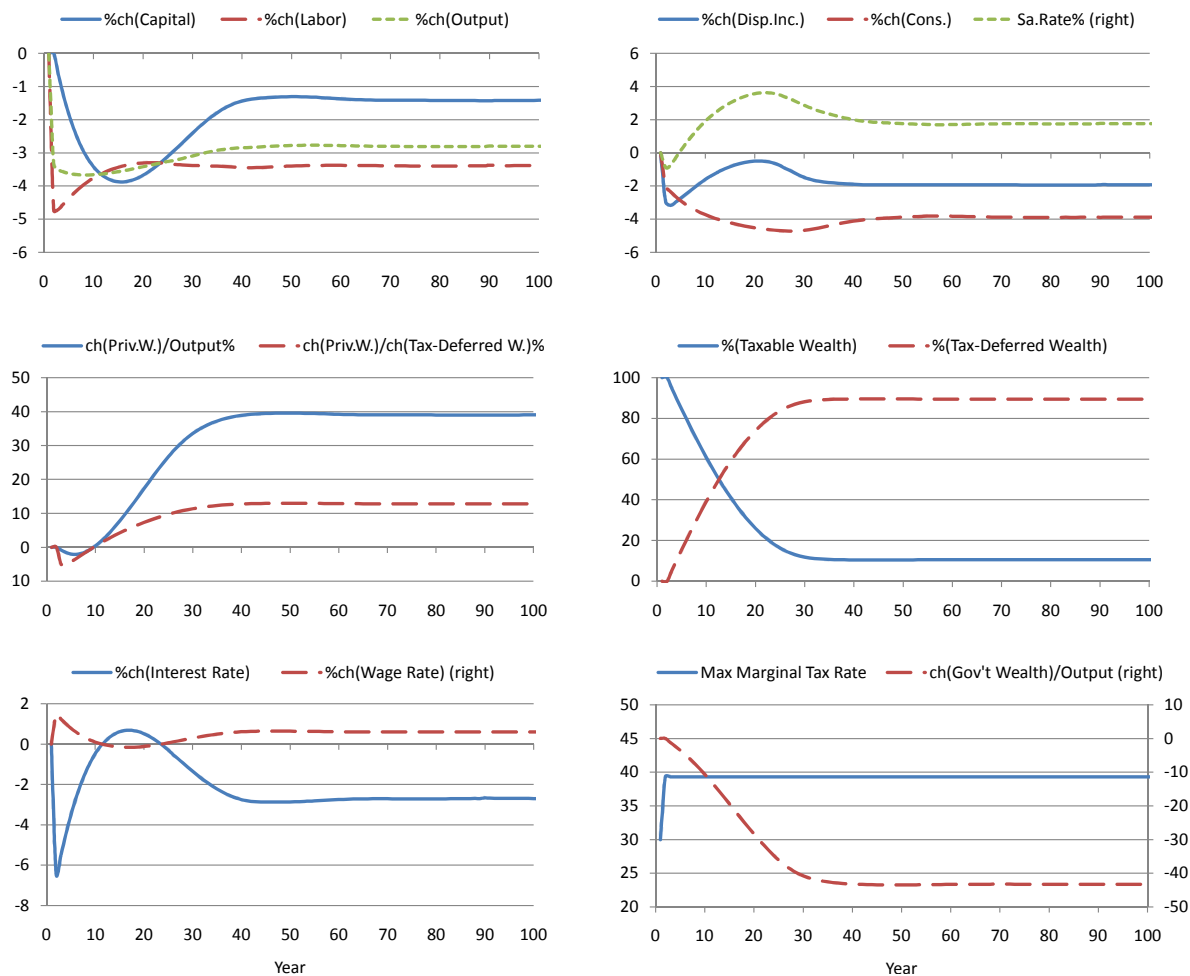


Figure 3: The Effects of Tax-Deferred Accounts Financed with Income Tax Increases and Government Bonds