Hot and Cold Seasons in the Housing Market*

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Abstract

Every year during the second and third quarters (the "hot season") housing markets in the U.K. and the U.S. experience systematic above-trend increases in both prices and transactions. During the fourth and first quarters (the "cold season"), housing prices and transactions fall below trend. A similar seasonal cycle is observed in other developed countries. We present a search-and-matching model that can quantitatively mimic the seasonal fluctuations in transactions and prices observed in the U.K. and the U.S. The model features a "thick-market" effect that can generate substantial differences in the volume of transactions and prices across seasons, with the extent of seasonality in prices depending positively on the bargaining power of sellers. As a by-product, the model sheds new light on the mechanisms governing fluctuations in housing markets and can be adapted to study lower-frequency movements in prices and transactions.

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1 Introduction

A rich empirical and theoretical literature has been motivated by dramatic boom-to-bust episodes in regional and national housing markets.[1] Booms are typically defined as times when prices rise and there is intense trading activity, whereas busts are times when prices and trading activity fall below trend.

While the boom-to-bust episodes motivating the extant work are relatively infrequent and of unpredictable timing, this paper shows that in several housing markets, booms and busts are just as frequent and predictable as the seasons. In particular, in all regions of the U.K. and the U.S., as well as in other developed economies, every year a housing boom of considerable magnitude takes place in the second and third quarters of the calendar year (the "hot season"), followed by a bust in the fourth and first quarters (the "cold season"). The predictable nature of house price fluctuations is furthermore confirmed by estate agents, who in conversations with the authors observed that during winter months there is less activity and "owners tend to sell at a discount." And, perhaps more compelling, publishers of house price indexes go to great lengths to produce seasonally adjusted versions of their indexes, usually the index that is published in the media. As stated by the publishers:

"House prices are higher at certain times of the year irrespective of the overall trend. This tends to be in spring and summer, when more buyers are in the market and hence sellers do not need to discount prices so heavily in order to achieve a sale." and *"...we seasonally adjust our prices because the time of year has some influence. Winter months tend to see weaker price rises and spring/summer see higher increases all other things being equal."* (From Nationwide House Price Index Methodology.)

"Houses prices are seasonal with prices varying during the course of the year irrespective of the underlying trend in price movements. For example, prices tend to be higher in the spring and summer months when more people are looking to buy." (From Halifax Price Index Methodology.)

The first contribution of this paper is to systematically document the existence, quantitative importance, and cross-regional variation of these seasonal booms and busts.[2]

[1] See for example Stein (1995), Muellbauer and Murphy (1997), Genesove and Mayer (2001), Krainer (2001), Ortalo-Magne and Rady (2005), Brunnermeier and Julliard (2008), and the contributions cited therein.

[2] Studies on housing markets have typically glossed over the issue of seasonality. There are a few exceptions, albeit they have been confined to only one aspect of seasonality (e.g., either quantities or prices) and/or to a relatively small geographical area. In particular, Goodman (1991) documents pronounced seasonality in *moving patterns* in

The surprising size and predictability of seasonal fluctuations in housing prices poses a challenge to standard models of durable-good markets. In those models, anticipated changes in prices cannot be large: If prices are expected to be much higher in May than in December, then buyers will shift their purchases to the end of the year, narrowing down the seasonal price differential. More formally, in the absence of risk, the asset-market equilibrium condition states that the one-period rental value of a house plus its appreciation should equal the one-period gross cost of housing services.[3] Calling p_t and d_t the real price of housing and rental services, respectively, and assuming that the gross real service cost is a (potentially changing) proportion c_t of the property price, the equilibrium asset-market condition is:

$$d_{t+1} + (p_{t+1} - p_t) = c_t \cdot p_t \tag{1}$$

where c_t is the sum of the (potentially time-varying) depreciation rate, maintenance and repair expenditure rate, property tax rate, and the tax-adjusted interest rate.[4] The arbitrage condition thus states that the seasonals in real prices must be accompanied by seasonals in the cost of housing services c_t or in the rental service flow d_t. Rents, however, display no seasonality, implying a substantial and, as we shall argue, unrealistic degree of seasonality in service costs c_t.[5,6,7]

It is important to remark that no-arbitrage conditions such as (1) may become irrelevant when

the U.S., Case and Shiller (1989) find seasonality in prices in Chicago and—to a lesser extent—in Dallas, and Hosios and Pesando (1991) find seasonality in prices in the City of Toronto; the latter conclude "that individuals who are willing to purchase against the seasonal will, on average, do considerably better."

[3] For an early asset-market approach to the housing market, see Poterba (1984).

[4] The effective interest rate is a weighted average between mortgage interest rate plus the opportunity cost of housing equity, where the weights are given by the loan-to-value ratio.

[5] The paucity of (good) data on rents (and particularly, new rents) may be one reason why we do not find seasonality in the data. Note, however, that one should observe extremely high levels of seasonality in rents (together with extremely high discount rates) for rents to be driving seasonality in prices; this is because prices should in principle reflect the present discounted value of a presumably long stream of rental services. Data on rentals, however, display no discernible pattern of seasonality.

[6] For example, the degree of price seasonality observed in the U.K. implies that service costs should be at least 300 percent higher in the cold season than in the hot season—see Appendix 7.1. This seems unlikely, particularly because interest rates and tax rates, two major components of c_t, display no seasonality.

[7] The seasonal in housing markets does not seem to be driven by seasonal differences in liquidity related to overall income. Income is typically high in the last quarter, a period in which housing prices and the volume of transactions tend to fall below trend. Beaulieu and Miron (1992) and Beaulieu, Miron, and MacKie-Mason (1992) show that in most countries, including the U.K., income peaks in the fourth quarter of the calendar year. There is also a seasonal peak in output in the second quarter, and seasonal recessions in the first and third quarters. Housing price

transaction costs are very high, as it is likely to be the case in housing markets.[8] Still, the question remains as to why do not (presumably informed) buyers try to buy in the low-price season. The seasonal behavior of housing prices and the failure of *a priori* appealing explanations, thus poses a challenge to models of the housing market based on standard asset-pricing conditions. This paper offers a novel explanation by resorting to a search-and-matching framework.

The model starts from the premise that every house is a little different and families have different housing needs. In that context, buyers are more likely to find a better-quality match (and thus their willingness to pay is more likely to increase) when there are more houses for sale.[9] Hence, in a thick market (or hot season), sellers can charge higher prices. Because prices are higher, more houses are put up for sale, better matches are formed, and so on. This self-reinforcing dynamics leads to higher number of transactions and prices in the hot season.

In the baseline model, we distinguish seasons by differences in the ex-ante propensity to move. These differences may arise, for example, from the school calendar: Families may prefer to move in the summer, before sending their children to new schools;[10] good weather may also make the search more convenient in the summer. We show that a higher *ex-ante* probability of moving in a given season can trigger thick-market externalities that make it appealing to a large number of agents to buy and sell during that season. This amplification mechanism can create substantial seasonality in transactions; the extent of seasonality in prices, in turn, increases with the bargaining power of sellers. The calibrated model can quantitatively account for most of the seasonal fluctuations in transactions and prices in the U.K. and the U.S.[11]

The contribution of the paper can be summarized as follows. First, it systematically documents seasonal booms and busts in housing markets; it argues that the predictability and high extent of seasonality in prices observed in some of them cannot be quantitatively reconciled with the standard asset-pricing equilibrium condition embedded in most models of housing markets (or

seasonality is not in line with income seasonality: prices are above trend in the second and third quarters. Finally, mortgage rates and tax rates do not display any seasonal pattern.

[8] See, for example, Englund and Quigley ().

[9] The labor literature distinguishes the thick-market effects due to faster arrival of offers and those due to the quality of the match. See for example Diamond (1981) and Petrongolo and Pissarides (2006). Our focus is entirely on the quality effects.

[10] See Goodman (1991) and Harding, Rosenthal, and Sirmans (2003).

[11] Our focus on these two countries is largely driven by the reliability and quality of the data.

consumer durables, more generally). Second, it develops a search-and-matching model that can quantitatively account for the seasonal patterns of prices and transactions observed in the U.K. and the U.S., shedding new light on the mechanisms governing fluctuations in housing markets. As a by-product, the model can be adapted to study lower-frequency movements in prices and transactions.

The paper is organized as follows. Section 2 presents the empirical evidence and discusses different potential (though ultimately unsuccessful) explanations; it discusses why, given the evidence, we need to deviate from the standard asset-pricing approach to housing markets. Section 3 presents the model. Section 4 presents a quantitative analysis of the model and confronts it with the empirical evidence. Section 5 presents extensions of the baseline model; in particular, it studies the role of seasonal moving and transaction costs as alternative triggers of seasonality in housing markets. Section 6 presents concluding remarks. Analytical derivations and proofs are collected in the Appendix.

2 Hot and Cold Seasons

This section documents the behavior of housing prices across what we call the two main seasonal terms: the summer term (second and third quarters of the calendar year) and the winter term (first and fourth quarters).

2.1 Data

We focus our analysis on housing markets in the U.K. and U.S., the two countries for which the available data are of highest quality. A supplementary Appendix provides a description of seasonal patterns in other countries (for which price indices are typically not quality-adjusted or are only published in seasonally-adjusted fashion).

U.K.

In the U.K. there are two main sources providing quality-adjusted non-seasonally adjusted prices: One is the Department of Communities and Local Government (DCLG) and the other is Halifax, one of the country's largest mortgage lenders.[12]

[12] Other price indices, like Nationwide Building Society, report quality adjusted data but they are already seasonally adjusted (the NSA data are not made publicly available). Nationwide Building Society, however, reports

Both sources report regional price indexes on a quarterly basis for the 12 standard planning regions of the U.K., as well as for the U.K. as a whole. The indexes calculated are 'standardized' and represent the price of a typically transacted house. The standardization is based on hedonic regressions that control for a number of characteristics, including location, type of property (house, sub-classified according to whether it is detached, semi-detached or terraced, bungalow, flat), age of the property, tenure (freehold, leasehold, feudal), number of rooms (habitable rooms, bedrooms, living-rooms, bathrooms), number of separate toilets, central heating (none, full, partial), number of garages and garage spaces, garden, land area, and road charge liability. Accounting for these characteristics allows to control for the possibility of seasonal changes in the composition of the set of properties (for example, shifts in the location or sizes of properties). The index reports transaction prices based on mortgages to finance house purchase at the time the mortgage is approved; re-mortgages and further advances are excluded.

The two sources differ in two respects. First, DCLG collects information from a sample of all mortgage lenders in the country, while the Halifax index uses all the data from Halifax mortgages only. Second, DCLG reports the price at the time of completion, while Halifax reports the price at the time of approval. Completion takes on average three to four weeks after the initial agreement but some agreed transactions do not reach completion. The DCLG index goes back to 1963 for certain regions, while Halifax starts in 1983.

To compute real price indexes, we later deflate the housing price indexes using the non-seasonally adjusted retail price index (RPI) provided by the U.K. Office for National Statistics.

As an indicator of the number of transactions, we use the number of mortgages advanced for home purchases; the data are collected by the Council of Mortgage Lenders and are also disaggregated by region.

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in its methodology description that June is generally the strongest month for house prices (with prices typically 1.3% above their SA level) and January is the weakest (with prices 1.9% below their SA level), differences that are comparable to the numbers we reported in Figure 1; this justifies the SA they perform in the published series. In a somewhat puzzling paper, Rosenthal (2006) argues that seasonality in Nationwide Building Society data is elusive; we could not, however, gain access to the NSA data to asses which of the two conflicting assessments (Nationwide Building Society's or Rosenthal's) was correct. We should perhaps also mention that Rosenthal (2006) finds different results from Muellbauer and Murphy (1997) with regards to lower-frequency movements.

The Land Registry data reports average prices, without adjusting for quality.

U.S.

The non-seasonally adjusted price index for the U.S. comes from the Office of Federal Housing Enterprise Oversight (OFHEO), which in turn builds its index from data provided by Fannie Mae and Freddie Mac; this is a repeat-sale, purchase-only index. The index is calculated for the whole of the U.S. and also disaggregated by Census region and state (the 50 states and the District of Columbia). We also study the Case-Shiller index carried out by Standard & Poor's for 20 big cities (and a composite of the 10 cities); this index is also a repeat-sale, purchase-only index.

To compute real price indexes, we use the non-seasonally adjusted consumer price index (CPI) provided by the U.S. Bureau of Labor Statistics.[13]

Data on the number of transactions come from the National Association of Realtors, and correspond to the number of sales of existing single-family homes. The data are disaggregated into the four major Census regions and are provided both in non-seasonally adjusted and seasonally adjusted format.

2.2 Aggregate Seasonality (as Reported by the Publishers)

In the U.K., the publishers of the Halifax index report both NSA and SA series (unlike DCLG, which publishes only NSA data or Nationwide, which publishes only SA data). From these two series we can obtain a first measure of seasonality, as gauged by the publisher. Figure 1 shows the (log) of the seasonal component of house prices, measured as the ratio of the non-seasonally adjusted (NSA) house price series, P_t, relative to the seasonally adjusted (SA) series, P_t^*, from 1983:01 to 2007:04, $\left\{\ln \frac{P_t}{P_t^*}\right\}$. (Both the NSA and the SA series come from Halifax and correspond to the U.K. as a whole.)

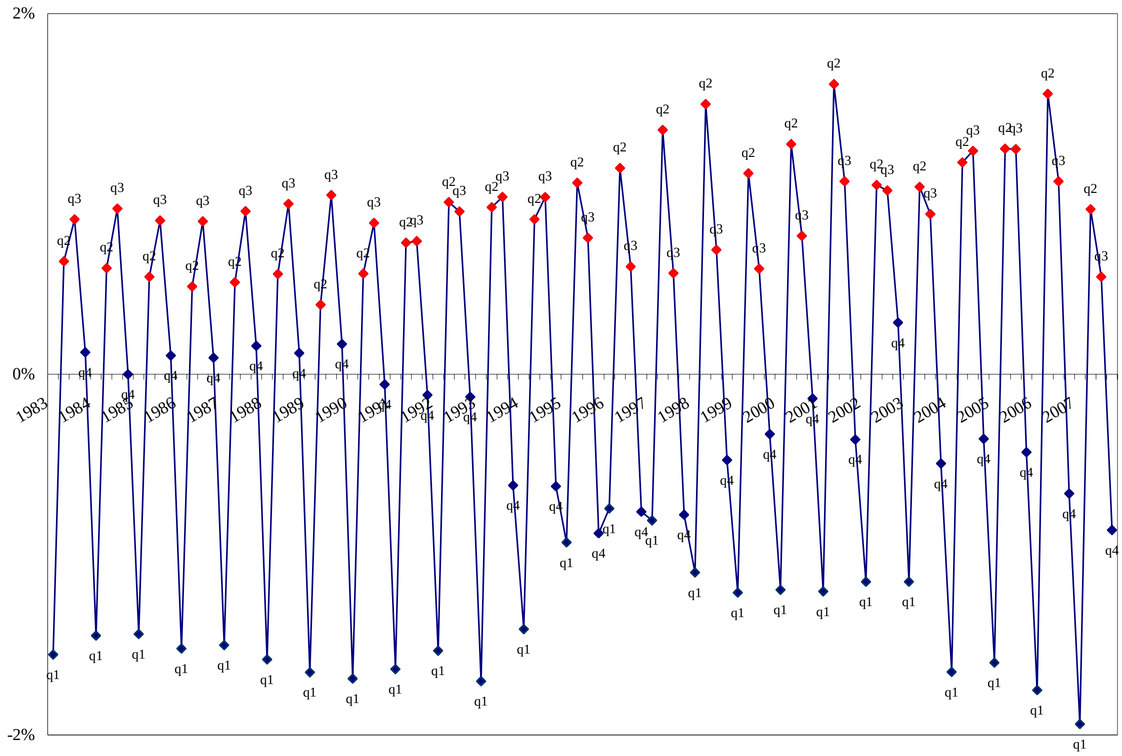
In the U.S., both OFHEO's purchase-only index and S&P's Case-Shiller index are published in NSA and SA form. Figure 2 depicts the seasonal component of the OFHEO series for the U.S. as a whole (measured as before as $\left\{\ln \frac{P_t}{P_t^*}\right\}$ from 1991:01 through to 2007:04. Figure 3 shows the corresponding plot for the Case-Shiller's composite of 10 cities, with the data running from 1989:01 through to 2007:04. (The start of the sample in all cases is dictated by data availability.)

All Figures seem to show a consistent pattern: house prices in the second and third quarters tend to be above trend (captured by the NSA series), and prices in the fourth, and particularly

[13]As it turns out, there is little seasonality in the U.S. CPI index, a finding first documented by Barsky and Miron (1989), and hence the seasonal patterns in nominal and real housing prices coincide.

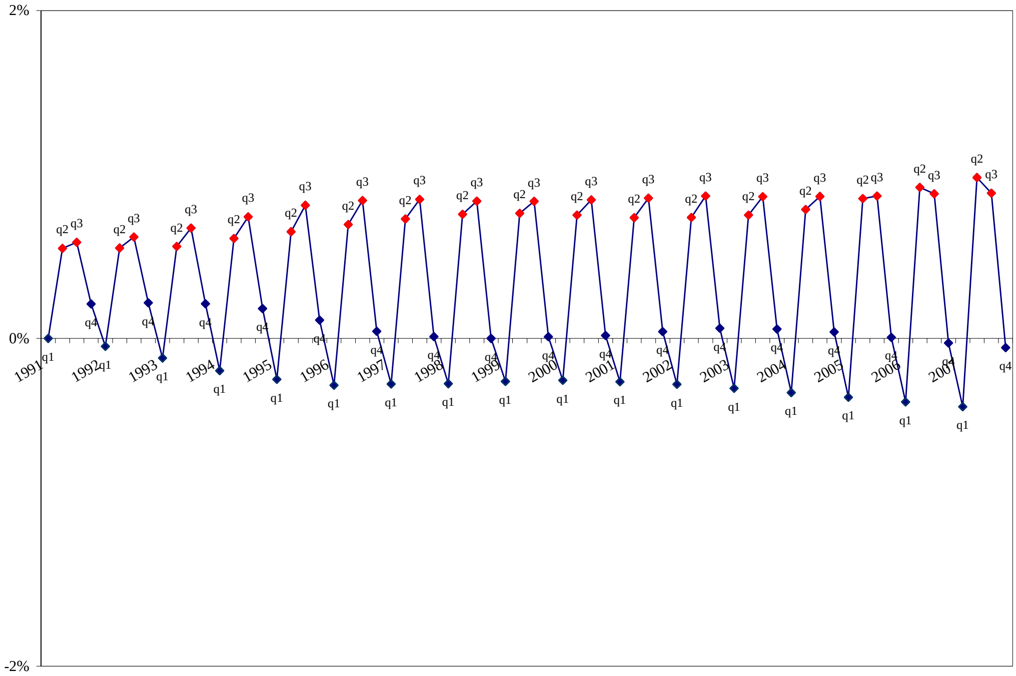
in the first quarter, tend to be in general at or below trend. Later on we show that this general pattern is also observed at finer levels of geographical aggregation for both countries. The Figures also make it evident that the extent of price seasonality is more pronounced in the U.K. than in the U.S. as a whole, though, certain cities in the U.S. seem to display seasonal patterns of the same magnitude as those observed for the U.K. as a whole. (Some readers might be puzzled by the lack of symmetry in Figure 2, as most expect the seasons to cancel out; this is exclusively due to the way OFHEO performs the seasonal adjustment; as will become evident later, we shall base our analysis only on the "raw", NSA series and hence the particular choice of adjustment by the publishers will be inconsequential.)

Figure 1: Seasonal Component of House Prices in the U.K. 1983-2008.



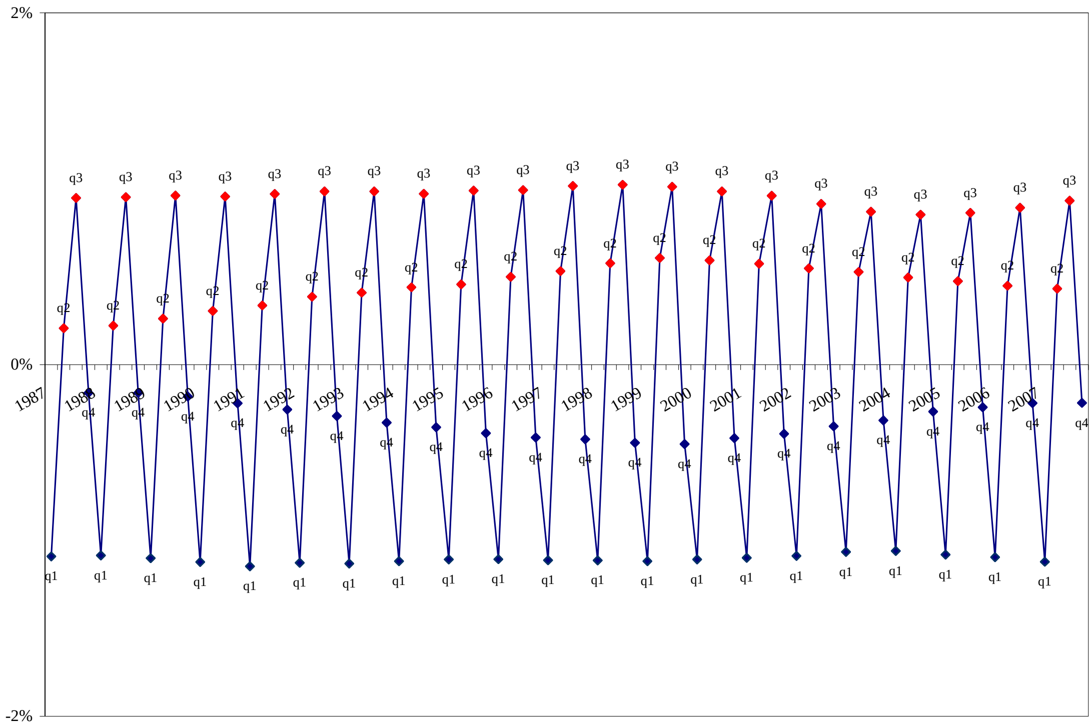
Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$, where P_t is the NSA and P_t^* is SA index. Source: Halifax.

Figure 2: Seasonal Component of House Prices in the U.S. 1991-2008.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: OFHEO-purchase only index.

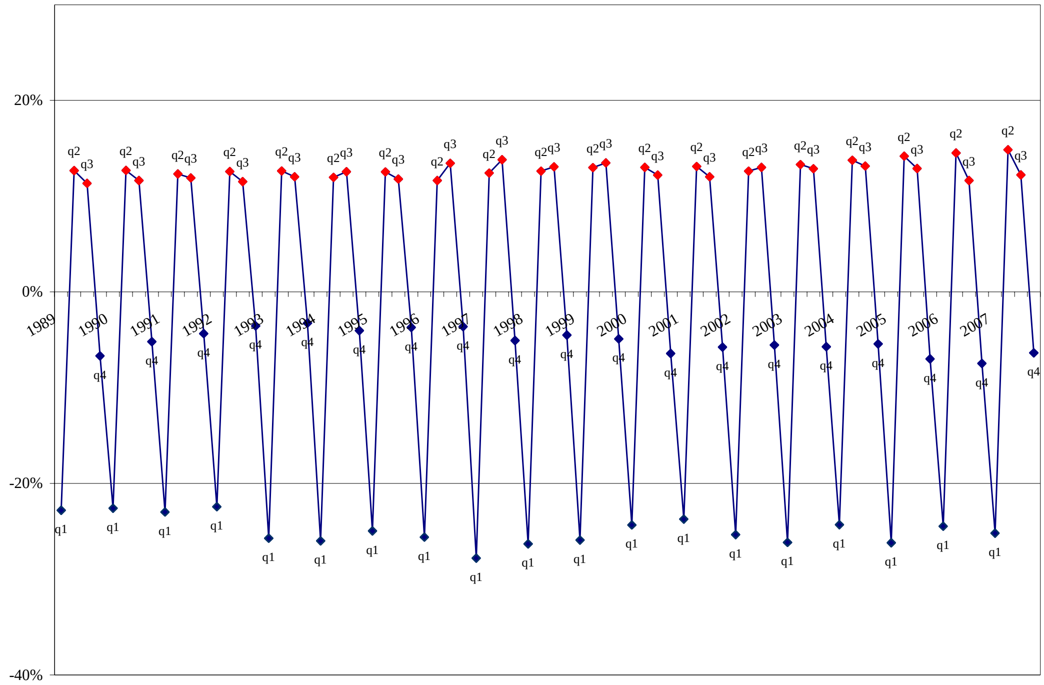
Figure 3: Seasonal Component of House Prices in U.S. cities 1987-2008.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: S&P's Case-Shiller 10-city comps

Last, but not least, the U.S. National Association of Realtors publishes its data on transactions both with and without seasonal adjustment. Figure 4 plots the seasonal component of house transactions, measured (as before) as the (log) of the (NSA) number of transactions Q_t divided by the SA number of transactions Q_t^*: $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$. The pattern is consistent with that for prices: transactions surge in the second and third quarters and stagnate or fall in the fourth and first quarters. (In the U.K., the Council of Mortgage Lenders publishes only NSA data.)

Figure 4: Seasonal Component of Housing Transactions in the U.S. 1989-2008.



Note: The plot shows $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$; Q_t is the NSA and Q_t^* the SA number of transactions. Source: NAR.

2.3 Within-Country Seasonality[14]

In this Section we study seasonal patterns at a more disaggregated geographical level. For ease of comparability across different datasets, we base the analysis entirely on the (raw) NSA series. We focus on deterministic seasonality, which is easier to understand (and to predict) for buyers and sellers (unlikely to be all econometricians), and hence most puzzling from a theoretical point of view.

Given the patterns described in the previous Section, we focus the analysis on the difference in

[14] We focus on the U.K. and the U.S., the countries with highest-quality data (in the supplementary Appendix we also describe seasonal patterns in different regions of Belgium and France).

relative prices across the two broadly defined seasons—second and third quarter, or "hot season", and fourth and first quarter, or "cold season". (We use interchangably the terms hot season and summer term to refer to the second-and-third quarters and cold season and winter term to refer to the first-and-fourth quarters.) The grouping into broad seasons will help simplify the exposition and allow for a clear mapping into the model we will develop later. For each geographical unit, we gauge (deterministic) seasonality as the average *annualized* difference in (logged) prices at the end of the hot season (that is, at the end of the third quarter (P_S)) relative to prices at the end of the (previous) winter, that is, at the end of the first quarter (P_W), $\frac{P_S}{P_W}$, vis-à-vis the price in winter relative to the price in summer, $\frac{P_{W'}}{P_S}$, where $P_{W'}$ indicates the end-of-winter price in the following year. In formula:

$$S = \left[\overline{\ln \left(\frac{P_S}{P_W} \right)^2} - \overline{\ln \left(\frac{P_{W'}}{P_S} \right)^2} \right] \tag{2}$$

This is a clean measure of deterministic seasonal variation, which nets out lower-frequency fluctuations affecting both seasons. In this case, (logged) relative prices are also the growth rates between the two broad seasons. So, put differently, S measures the difference in (annualized) inflation from winter to summer, $\ln\left(\frac{P_S}{P_W}\right)$, and inflation from summer to winter, $\ln\left(\frac{P_{W'}}{P_S}\right)$. We use an equivalent measure to quantify seasonality in transactions. In the model we will present later, we will focus on periodic steady states (without trend) for two broad seasons, and we will use a similar metric to gauge the extent of seasonality.

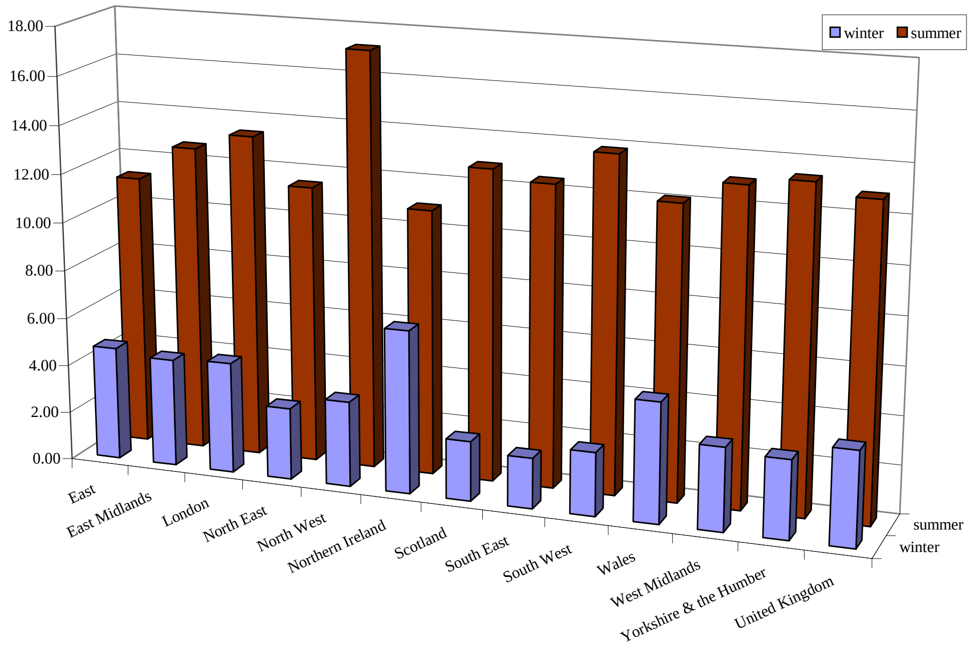
Housing Market Seasonality in the U.K.

Nominal House Price Figure 5 reports the average annualized percent price increases in the summer term (that is, the annualized price increase from the end of the first quarter to the end of the third quarter) and the winter term (the price growth from the end of the third quarter to the end of the first quarter in the following year) from 1983 through two 2007 using the regional price indices provided by DGLC.

During the period analyzed, the average nominal price increases in the winter term were below 5 percent in all regions except for Northern Ireland. In the summer term, the average growth rates were above 12 percent in all regions, except for Northern Ireland, East Anglia and North East. As shown in the graph, the differences in growth rates across the two broad seasons (our measure S) are generally very large and economically significant, with an average of 9 percent for all regions.

(The DGLC index goes back to 1968 for certain regions, and though the average growth rates are lower in the longer period, the average difference across seasons is still very high at above 8 percent.[15])

Figure 5: Average annualized housing price increases in summers and winters. DGLC, 1983-2007.

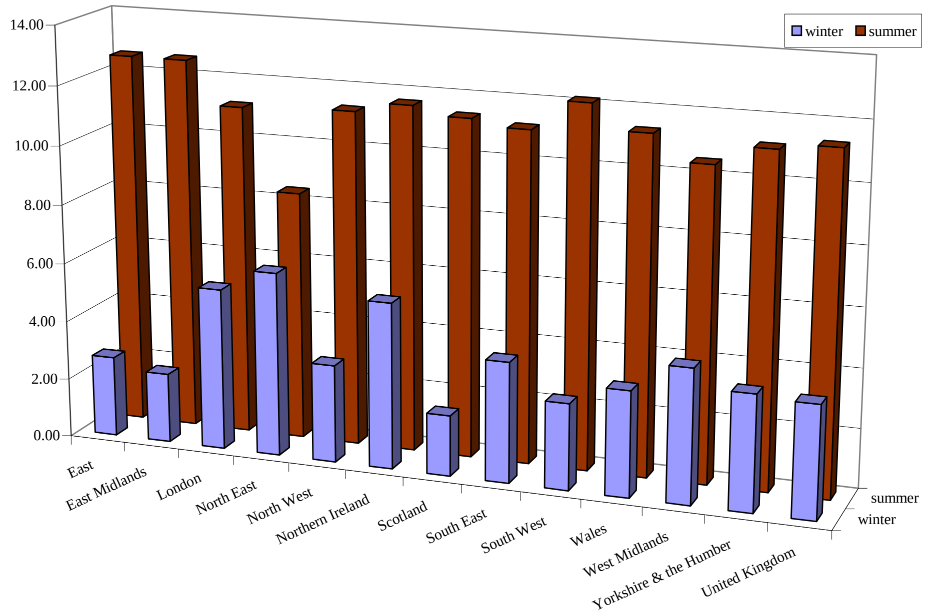


Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DGLC, 1983-2007.

Figure 6 shows a similar plot using the Halifax index. The patterns are qualitatively similar to those reported using DGLC. The annualized average price growth during the summer term is above 11 percent in all regions, with the exception of the North East and West Midlands, whereas the increase during the winter term is systematically below 5 percent, except for the the North East region and London, where the increase is just above 5 percent. The average difference in growth rates across seasons is 7.4 percent. There are some nonnegligible quantitative differences between the two sources, which might be partly explained by potential differences in coverage and by the lag between approval and completion, which, as mentioned, is one important difference between the two indices. The two sources, however, point to a similar pattern of prices surging in the summer and stagnating in the winter.

[15]The figures are available from the authors. We start in 1983 for comparability with the Halifax series.

Figure 6: Average annualized house price increases in summers and winters. Halifax 1983-2007.



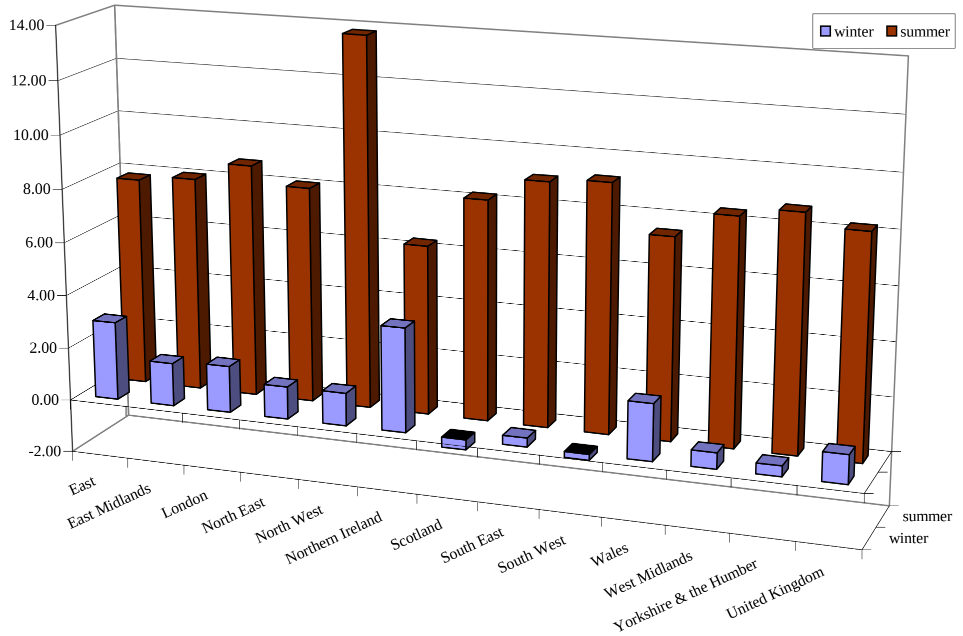
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DGLC, 1983-2007.

Real House Price

The previous Figures showed the seasonal pattern in nominal house price inflation. The seasonal pattern of real house prices (that is, house prices relative to the overall non-seasonally-adjusted price index) depends also on the seasonality of overall inflation. In the U.K. overall price inflation displays a slightly seasonal pattern. The difference across the two seasons, however, can hardly "undo" the differences in nominal house price inflation, implying a significant seasonal also in real house prices. This is illustrated in Figure 7 for the DGLC's index and in Figure 8 for the Halifax index. Netting out the effect of overall inflation reduces the differences in growth rates between winters and summers to a country-wide average of 7.3 percent in the DGLC series and 5.6 in the Halifax.[16]

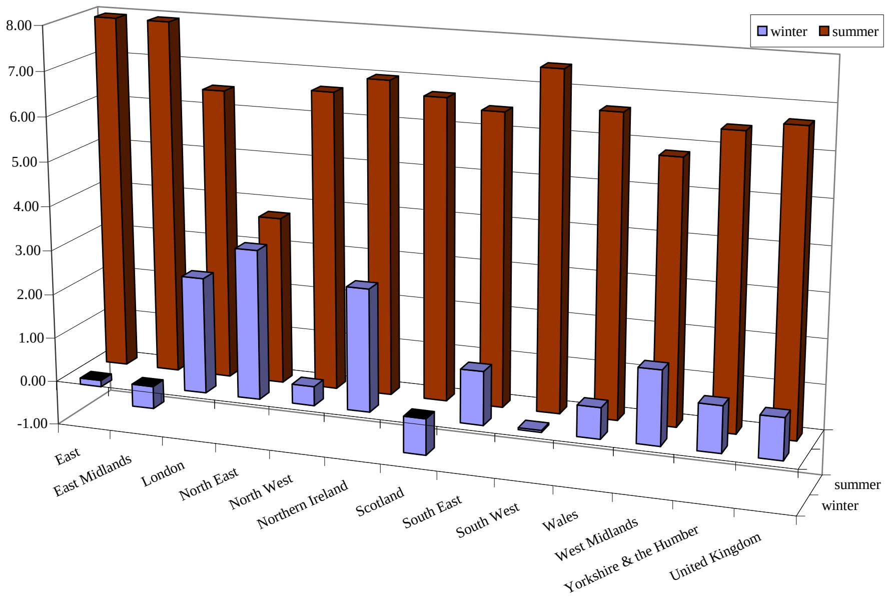
[16] We should note in addition that non-seasonally adjusted indexes of inflation are rarely used in practice (indeed it is even hard to find them for a long enough period), so they are unlikely to serve in contracts as financial means to "hedge" part of the seasonal nominal housing price fluctuations

Figure 7: Average annualized real house price increases in summers and winters. DGLC 1983-2007



Note: Annualized real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DGLC, 1983-2007.

Figure 8: Average annualized real house price increases in summers and winters. Halifax 1983-2007

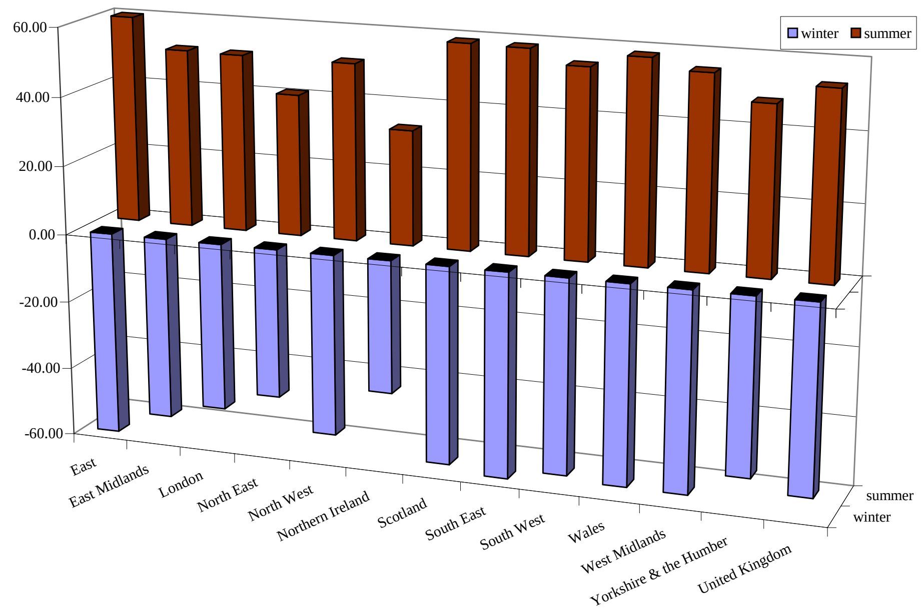


Note: Annualized real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DGLC, 1983-2007.

Number of Transactions

The seasonal differences in housing prices are mirrored by the patterns exhibited by the number of loans for housing purchases, which are a good proxy for the number of transactions. The data are collected by the Council of Mortgage Lenders. For comparability with the price sample, Figure 6 shows the growth rate in the number of loans for mortgage completions in the U.K. from 1983 to 2007. (The data for some regions go back to 1974 and the patterns are qualitatively in the earlier period.) As the Figure shows, the number of transactions increases sharply in the summer term and declines in the winter term.

Figure 9: Average annualized increases in the number of transactions in summers and winters. CML 1983-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DGLC, 1983-2007.

Statistical Significance of the Differences between Summers and Winters

We test the statistical significance of the differences in growth rates across seasons, as gauged in (2), in Tables 1 through 3, using a t-test on the equality of means.[17] The Tables report the

[17] The test on the equality of means is equivalent to the t-test obtained on the slope coefficient from a regression of annualized growth rates on a dummy variable that takes value 1 if the observation falls on the second and third quarter (or hot semester) and 0 otherwise. The coefficient on the dummy captures the annualized differences across the two seasons; note that this is the case regardless of the frequency of the data (provided the growth rates are annualized). To see this point, notice that the annualized growth rate in, say, the hot season, $\overline{\ln\left(\frac{P_S}{P_W}\right)^2}$, is equivalent to the average of annualized quarterly growth rates in the summer term: $\overline{\ln\left(\frac{P_S}{P_W}\right)^2} = 2\,\overline{\ln\left(\frac{P_3}{P_1}\right)} = \frac{1}{2}\left[4\ln\left(\frac{P_3}{P_2}\right) + 4\ln\left(\frac{P_3}{P_2}\right)\right]$ and, correspondingly, $2\,\overline{\ln\left(\frac{P_{1'}}{P_3}\right)} = \frac{1}{2}\left[4\ln\left(\frac{P_{1'}}{P_4}\right) + 4\ln\left(\frac{P_4}{P_3}\right)\right]$. Hence a regression with quarterly (or semester) data on a summer dummy will produce an unbiased estimate of the average difference in

average difference in growth rates across seasons and standard errors, together with the statistical significance. Table 1 reports the results for prices, both nominal and real, for all regions, using the data from DGLC (corresponding to Figures 5 and 7 before0 and and Table 2 shows the corresponding information using Halifax (corresponding to Figures 6 and 8). Table 3 shows the differences in transactions' growth rates from CML data (Figure 9).

The differences in price increases across seasons are not only sizeable but also statistically significant at standard levels for most regions, in the order of 7 to 9 percent on average in nominal terms and 5.7 to 7 percent on average in real terms. For transactions the differences reach 107 percent for the country as a whole.

Table 1: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. DCLG.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	6.536	(3.577)	4.87	(3.461)
East Midlands	8.231*	(3.148)	6.408*	(3.131)
Gr. London	8.788**	(3.273)	6.966*	(3.372)
North East	8.511*	(3.955)	6.845	(3.915)
North West	13.703***	(3.323)	12.583**	(3.245)
Northern Ireland	4.237	(3.431)	2.415	(3.467)
Scotland	10.393***	(2.793)	8.571**	(2.711)
South East	10.375**	(3.496)	8.709*	(3.301)
South West	11.244**	(3.419)	9.422**	(3.459)
Wales	7.180*	(3.504)	5.358	(3.442)
West Midlands	9.623**	(3.089)	7.801*	(3.070)
Yorkshire & the Humber	10.148**	(3.114)	8.325**	(3.056)
United Kingdom	9.008***	(2.304)	7.185**	(2.314)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%.
Source: Department of Communities and Local Government.

growth rates across seasons. In the estimation we use directly quarterly data to exploit all the information and gain on degrees of freedom.

Table 2: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. Halifax

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	9.885**	(3.604)	8.081*	(3.706)
East Midlands	10.247**	(3.393)	8.444*	(3.413)
Gr. London	5.696	(3.048)	3.892	(3.221)
North East	2.197	(2.945)	0.394	(2.864)
North West	8.019**	(2.653)	6.216*	(2.548)
Northern Ireland	6.053	(3.409)	4.25	(3.494)
Scotland	9.334***	(2.320)	7.530**	(2.272)
South East	7.104*	(3.019)	5.301	(3.149)
South West	9.258*	(3.474)	7.454*	(3.549)
Wales	7.786*	(3.329)	5.983	(3.288)
West Midlands	5.987	(3.540)	4.183	(3.505)
Yorkshire & the Humber	7.253*	(2.892)	5.45	(2.825)
United Kingdom	7.559**	(2.365)	5.756*	(2.400)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: Halifax.

Table 3: Difference in annualized percentage changes in the volume of transactions between summers and winters in the UK, by region. CML.

Region	Difference	Std. Error
East Anglia	119.420***	(11.787)
East Midlands	104.306***	(11.151)
Gr. London	99.758***	(11.577)
North East	84.069***	(9.822)
North West	103.525***	(8.963)
Northern Ireland	71.466***	(12.228)
Scotland	116.168***	(9.843)
South East	117.929***	(9.710)
South West	110.996***	(8.764)
Wales	115.900***	(13.850)
West Midlands	112.945***	(9.496)
Yorkshire & the Humber	98.904***	(8.192)
United Kingdom	107.745***	(8.432)

Note: The Table shows the average differences (and standard errors) by region for 1983-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: Council of Mortgage Lenders.

Put together, the data point to a strong seasonal cycle, with a large increase in transactions and prices during the summer relative to the winter term. Also, the seasonal patterns are qualitatively similar across regions, with some, such as the North East region or Northern Ireland (depending on the source), displaying less seasonality in prices.

Rents

Data on rents are not documented in as much detail as the data on prices. The series available corresponds to the aggregate of the U.K. and comes from ????. We run regressions using as dependent variables both the rent levels and the log of rents on the dummy variable S_t, which takes the value 1 in the second and third quarter and 0 otherwise. We also include, where indicated, a trend term. The results are summarized in Table 4, which shows that there is virtually no seasonality in rents for the U.K. as a whole. This is in line with anecdotal evidence suggesting that rents are fairly sticky. Given the paucity of data on rents, there is little we can say with high confidence. Still, note that for rents to be the driver of price seasonality, one would need an enormous degree of seasonality in rents (as well as a high discount rate), since prices should in principle, according to the standard asset-pricing approach, reflect the present values of all future rents (in other words, prices should be less seasonal than rents). The lack of even small discernible levels of seasonality in the data suggests that we need alternative explanations for the observed seasonality in prices.

Table 4: Summer Differentials in Rents in the U.K.

	Rents		log(Rent)	
Summer-dummy S_t	-47.90833	12.53771	-0.01406	0.00743
	(255.798)	(29.529)	(0.091)	(0.010)
Trend		61.67964**		0.02194**
		(1.276)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Mortgage Rates

Interest rates in the U.K. do not exhibit a seasonal pattern. The evidence is summarized in Table 5, which shows the coefficients on the summer dummy described before for different interest rate series provided by the Bank of England. The first column shows the results for the quarterly average of the repo (base) rate; the second column shows the corresponding results for the average interest rate charged by 4 U.K. major banks (Barclays Bank, Lloyds Bank, HSBC, and National Westminster Bank); and the third column shows the results for the weighted average standard variable mortgage rate from Banks and Building Societies.[18] As the Table shows, none of the interest rate measures appears to be different, on average, during the summer term.

[18]The first two series cover the period 1978 through 2005, whereas the third goes from 1994 through 2005.

Table 5: Summer Differentials in Interest Rates in the U.K.

	Repo rate	Bank-4 Rate	Mortgage Rate
Summer-dummy S_t	-0.163	-0.144	0.018
	(0.701)	(0.696)	(0.310)

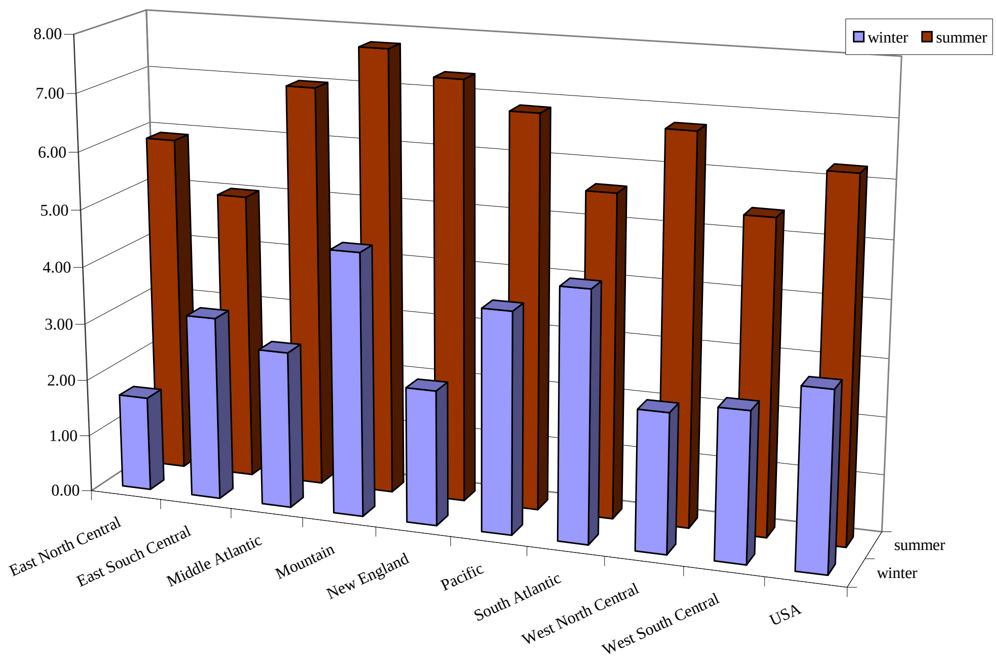
+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Housing Market Seasonality in the U.S.

House Price

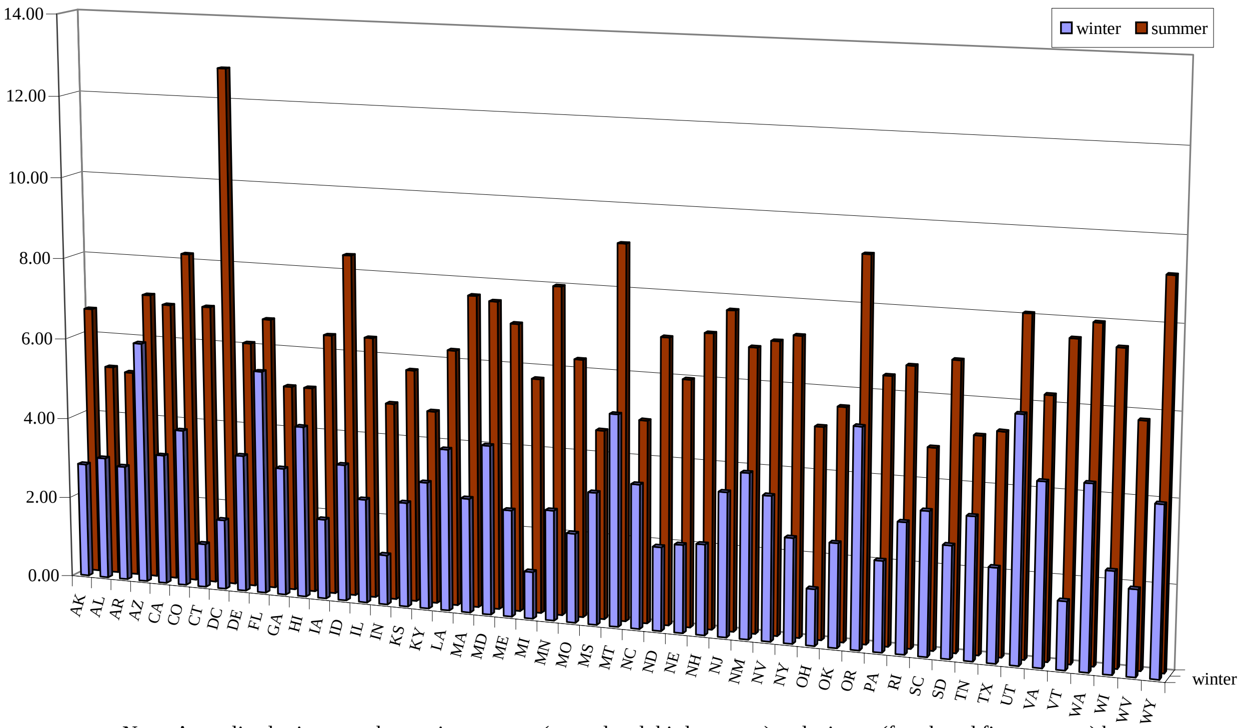
As noted before, the U.S. aggregate price index displays a consistent seasonal behavior, albeit the extent of seasonality is generally smaller than that in the U.K. Figures 10 illustrates the annualized price increases at the Census-Division level from OFHEO. Figure 11 shows the corresponding plot for different states, also from OFHEO, and Figure 12 shows the plot for the S&P's Case-Shiller index at the city level.

Figure 10: Average annualized house price increases in summers and winters, by region. OFHEO 1991-2007



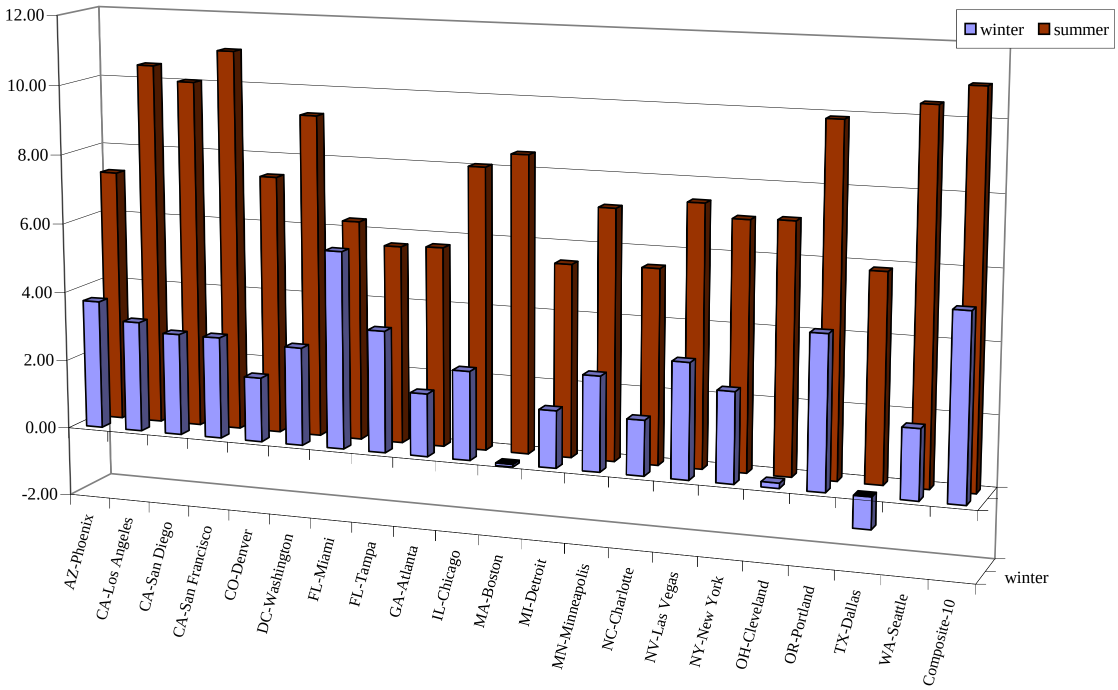
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. OFHEO, 1991-2007.

Figure 11: Average annualized house price increases in summers and winters by state. OFHEO 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. state. OFHEO, 1991-2007.

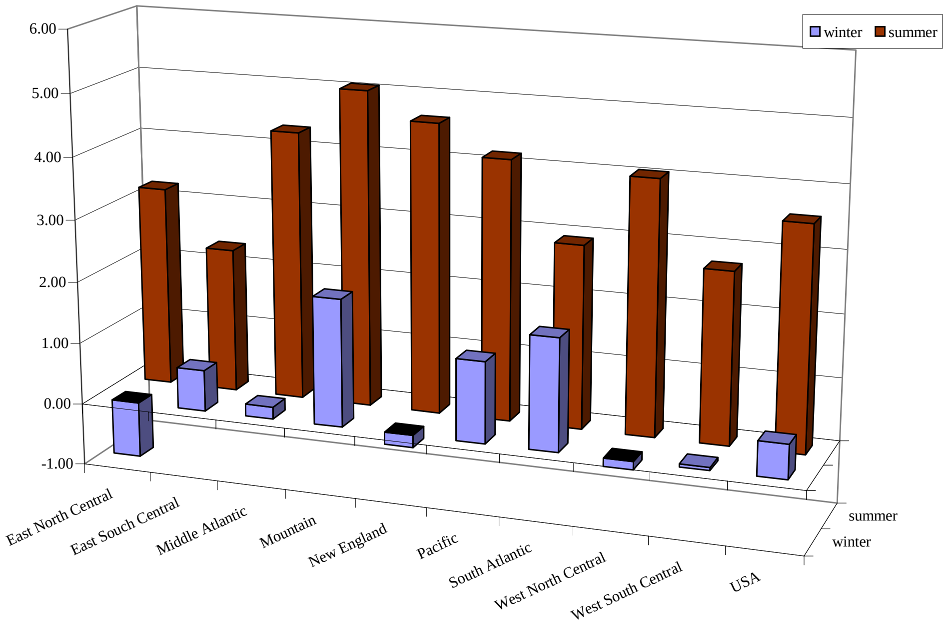
Figure 12: Average annualized house price increases in summers and winters by city. S&P Case-Shiller 1987-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2007.

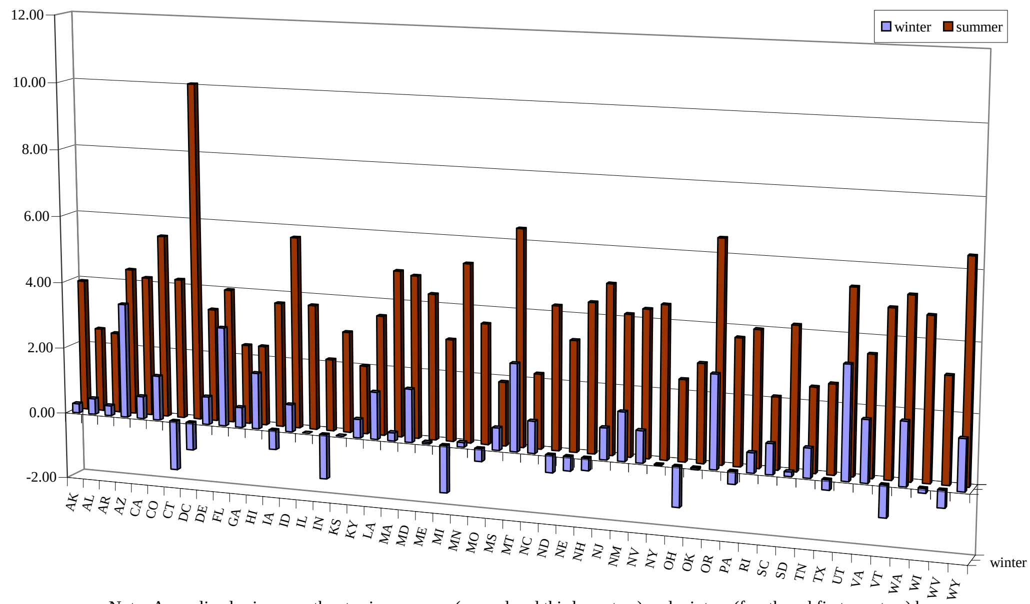
Real House Price Figures 13 through 15 show the average growth rates in real house price increases in the two seasons for the various indices: OFHE-Census-level division (Figure 13), OFHEO-state level (Figure 14) and S&P's Case-Shiller index. As mentioned, we use the NSA CPI index for the US to deflate the various nominal house price series; because the CPI displays a relatively low degree of seasonality and hence inflation rates hardly differ across seasons over the period analysed, the differences in real growth rates across seasons are very close to the differences in nominal growth rates.

Figure 13: Average annualized real house price increases in summers and winters, by region. OFHEO 1991-2007



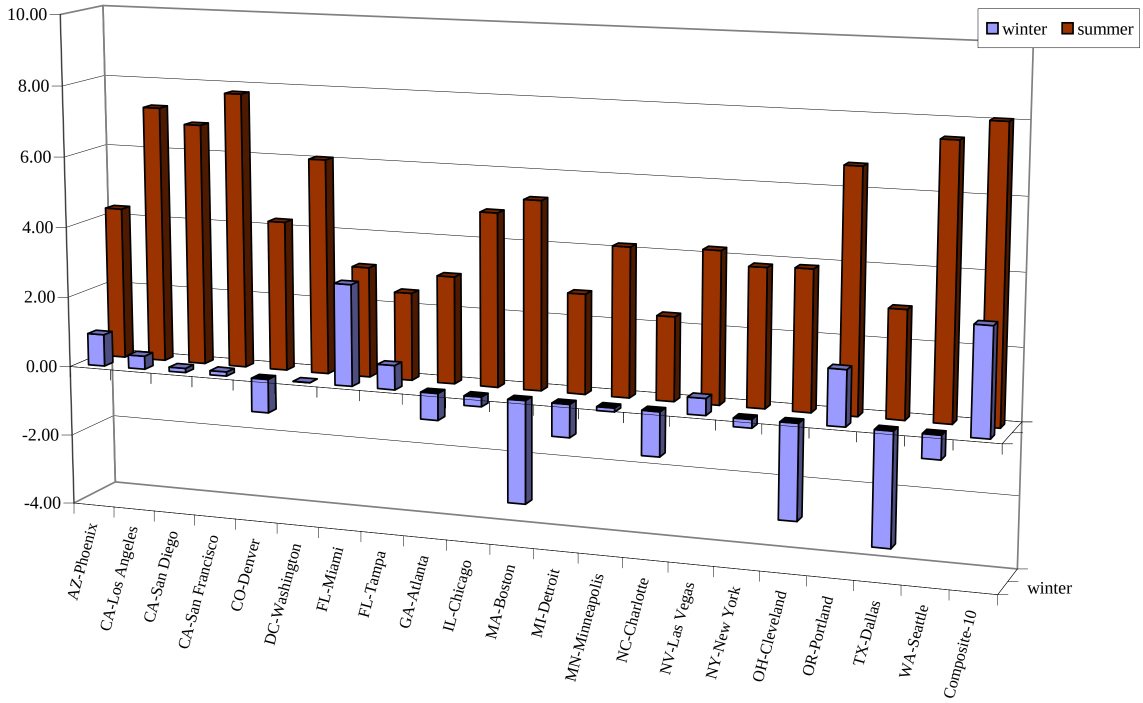
Note: Annualized real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. OFHEO, 1991-2007.

Figure 14: Average annualized real house price increases in summers and winters, by state. OFHEO 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. state. OFHEO, 1991-2007.

Figure 15: Average annualized real house price increases in summers and winters by city. S&P Case-Shiller 1987-2007

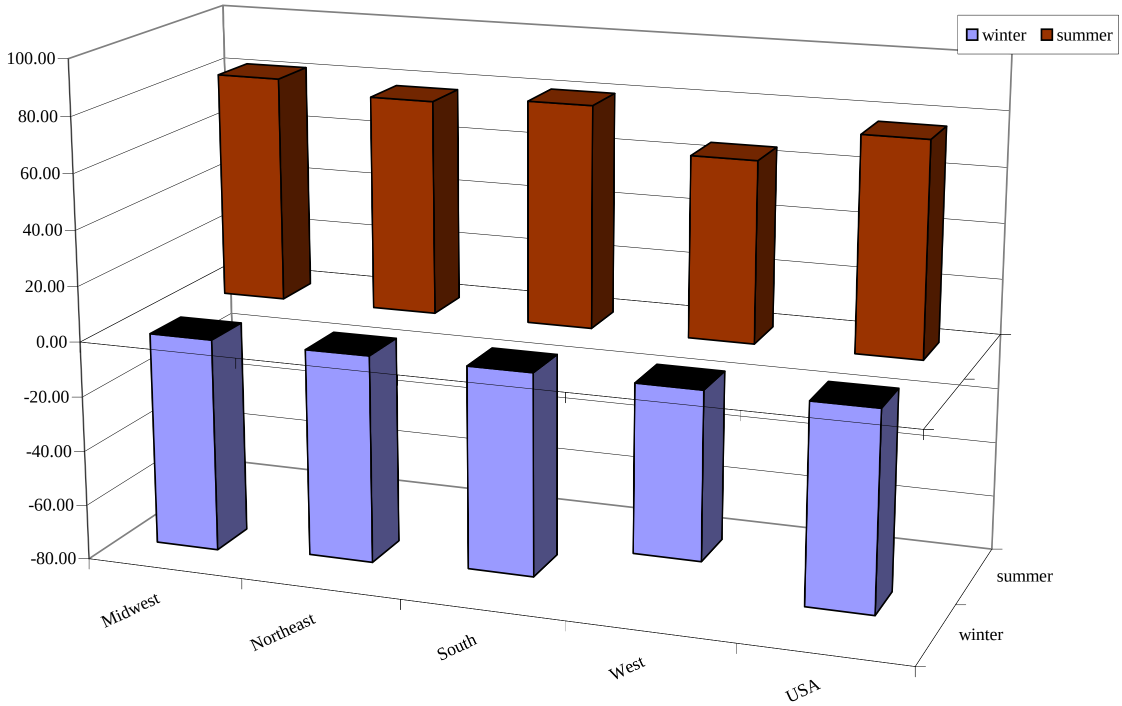


Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2007.

Transactions We show in Figure 16 the annualized growth rates in transactions during 1991 through to 2007 for main Census Regions; the data come from NAR.[19] Seasonality in transactions is overwhelming: the volume of transactions rises sharply in the summer and falls in the winter, by even bigger magnitudes than in the U.K.

[19]The series actually starts in 1989, but we use 1991 for comparability with the OFHEO-census-level division price series; adding these two years does not change the results

Figure 16: Average annualized increases in the number of transactions in summers and winters. NAR 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. NAR 1991-2007.

Statistical Significance of the Differences between Summers and Winters We summarize the differences in growth rates across seasons plotted in the previous Figures and report the results from a test on mean differences in Tables 6 through 9. Table 6 shows the results for prices using OFHEO's Census-division level (corresponding to Figures 10 and 13); Table 7 shows the results using OFHEO's state-level data (Figures 11 and 14); Table 8 shows the results using S&P's Case-Shiller city-level data; and Table 9 shows the results for transactions from NAR. Regarding prices, for the U.S. as a whole, the differences in annualized growth rates (nominal and real) are in the order of 3 percent. There is considerable variation across regions, with some displaying virtually no seasonality (South Atlantic) and others (East and West North Central, New England and Middle Attlantic) displaying significant levels of seasonality. This variability becomes more evident at the state level. Interestingly, the Case-Shiller index for cities displays higher levels of seasonality, comparable to the levels observed in U.K. regions. (This will be consistent with our model, which, ceteris paribus, generates more seasonality when the bargaining power of sellers is higher, as it is likely to be in cities where land is relatively scarcer.) Transactions are extremely seasonal in the U.S., as anticipated earlier, despite having, on average, lower seasonality in prices. (Our model will offer an explanation for this).

Table 6: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) in the US, by region

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East North Central	4.262***	(0.772)	4.106***	(0.924)
East South Central	1.811**	(0.535)	1.654*	(0.701)
Middle Atlantic	4.273*	(1.619)	4.116*	(1.660)
Mountain	3.166*	(1.205)	3.009*	(1.281)
New England	4.980*	(2.081)	4.823*	(2.181)
Pacific	3.01	(2.117)	2.853	(2.195)
South Atlantic	1.281	(1.277)	1.125	(1.370)
West North Central	4.333***	(0.743)	4.176***	(0.872)
West South Central	2.836***	(0.537)	2.679***	(0.650)
USA	3.169**	(0.967)	3.012**	(1.081)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 7: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US state.

	Nominal house price		Real house price	
State	Difference	Std. Error	Difference	Std. Error
Alabama	3.812*	(1.400)	3.655*	(1.378)
Alaska	2.189**	(0.692)	2.032*	(0.848)
Arizona	2.263*	(0.848)	2.106*	(0.950)
Arkansas	1.109	(2.586)	0.953	(2.583)
California	3.656	(3.398)	3.499	(3.479)
Colorado	4.285**	(1.323)	4.129**	(1.447)
Connecticut	5.819**	(2.055)	5.662*	(2.133)
District of Columbia	11.040*	(4.229)	10.883*	(4.150)
Delaware	2.687	(1.862)	2.530	(1.925)
Florida	1.185	(2.525)	1.028	(2.571)
Georgia	1.921*	(0.743)	1.764	(0.887)
Hawaii	0.850	(3.668)	0.693	(3.677)
Idaho	4.440***	(0.615)	4.283***	(0.711)
Illinois	5.035**	(1.659)	4.878**	(1.688)
Indiana	3.864***	(0.755)	3.707***	(0.859)
Iowa	3.621***	(0.768)	3.464***	(0.884)
Kansas	3.134***	(0.709)	2.977**	(0.925)
Kentucky	1.623**	(0.570)	1.466*	(0.707)
Louisiana	2.300**	(0.827)	2.143*	(0.921)
Maine	4.823*	(2.219)	4.666	(2.339)
Maryland	3.384	(2.341)	3.227	(2.396)
Massachusetts	4.407*	(2.146)	4.250	(2.231)
Michigan	4.573**	(1.568)	4.416*	(1.698)
Minnesota	5.290***	(1.376)	5.133**	(1.484)
Missouri	4.085***	(0.646)	3.929***	(0.758)
Mississippi	1.379	(1.028)	1.222	(1.108)
Montana	3.957*	(1.469)	3.800*	(1.510)
North Carolina	1.417*	(0.641)	1.260	(0.764)
North Dakota	4.908***	(1.353)	4.751**	(1.423)
Nebraska	3.842**	(1.082)	3.685**	(1.162)
New Hampshire	4.918*	(2.391)	4.761	(2.463)
New Jersey	4.197	(2.076)	4.041	(2.126)
New Mexico	2.857	(1.560)	2.700	(1.623)
Nevada	3.540	(2.946)	3.383	(3.026)
New York	4.662*	(1.815)	4.505*	(1.872)
Ohio	3.729***	(0.731)	3.572***	(0.911)
Oklahoma	3.095***	(0.477)	2.938***	(0.511)
Oregon	3.903**	(1.380)	3.746**	(1.310)
Pennsylvania	4.226**	(1.317)	4.069**	(1.329)
Rhode Island	3.544	(2.842)	3.388	(2.969)
South Carolina	1.360	(0.698)	1.203	(0.771)
South Dakota	4.201**	(1.171)	4.044**	(1.248)
Tennessee	1.759*	(0.685)	1.602	(0.834)
Texas	3.045***	(0.674)	2.888***	(0.763)
Utah	2.204	(1.820)	2.047	(1.803)
Virginia	1.873	(1.758)	1.716	(1.835)
Vermont	5.945*	(2.430)	5.788*	(2.373)
Washington	3.563*	(1.377)	3.406*	(1.377)
Wisconsin	5.007***	(0.738)	4.850***	(0.848)
West Virginia	3.753*	(1.702)	3.596*	(1.765)
Wyoming	5.091***	(1.365)	4.935**	(1.391)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 8: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US city.

	Nominal house price		Real house price	
City	Difference	Std. Error	Difference	Std. Error
AZ-Phoenix	3.571	(3.307)	3.405	(3.357)
CA-Los Angeles	7.273*	(3.478)	6.884	(3.535)
CA-San Diego	7.107*	(3.204)	6.717*	(3.275)
CA-San Francisco	8.051*	(3.009)	7.662*	(3.045)
CO-Denver	5.576**	(1.599)	5.186**	(1.805)
DC-Washington	6.439*	(2.604)	6.050*	(2.645)
FL-Miami	0.636	(2.744)	0.246	(2.838)
FL-Tampa	2.171	(2.384)	1.781	(2.484)
GA-Atlanta	3.920***	(0.903)	3.763**	(1.042)
IL-Chicago	5.530***	(1.342)	5.141**	(1.459)
MA-Boston	8.560***	(2.091)	8.170**	(2.325)
MI-Detroit	3.864	(1.909)	3.707	(2.060)
MN-Minneapolis	4.431**	(1.528)	4.265*	(1.741)
NC-Charlotte	3.968***	(0.721)	3.578***	(0.836)
NV-Las Vegas	4.149	(3.216)	3.76	(3.262)
NY-New York	4.477*	(2.161)	4.087	(2.342)
OH-Cleveland	6.942***	(0.973)	6.553***	(1.041)
OR-Portland	5.551***	(1.485)	5.161***	(1.388)
TX-Dallas	6.776***	(1.380)	6.138**	(1.823)
WA-Seattle	8.437***	(1.953)	8.175***	(1.942)
Composite-10 cities	6.051***	(1.717)	5.662**	(1.744)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: SP's Case&Shiller index.

Table 9: Difference in annualized percentage changes in house transactions between semesters (second-third quarters vis-à-vis fourth-first quarters) by US region.

Region	Coef.	Std. Error
Midwest	159.473***	(6.488)
Northeast	152.551***	(4.918)
South	153.009***	(4.702)
West	124.982***	(6.312)
United States	148.086***	(5.082)

Note: The Table shows the average differences (and standard errors) by region for 1991-2007. *Significant at the 10%; **significant at the 5%; ***significant at 1%. Source: National Association of Realtors.

Number of Transactions

As already observed, the U.S. as a whole displays a strong seasonality in the number of transactions. This remains true across all four major regions of the U.S. (state-level data are not available). The growth rates in the number of transactions in summers and winters are plotted in Figure 9. The average difference across seasons, together with the standard errors are summarized in Table 9.

Rents

Data on rents for the U.S. come from the Bureau of Labor Statistics (BLS); as a measure of rents we use the non-seasonally adjusted series of owner's equivalent rent and the non-seasonally adjusted rent of primary residence; both series are produced for the construction of the CPI and correspond to averages over all cities. For each series, we run regressions using as dependent variables both the rent levels and the log of rents on the summer-term dummy. We also include, where indicated, a trend term. The results are summarized in Tables 10 (owner's equivalent rent) and 11 (rent of primary residence). Both Tables show that there is no discernible pattern of seasonality in rents for the U.S. as a whole. To reiterate, if seasonality in rents were the driver of seasonality in prices, we should observed substantial seasonality in rents to justify the seasonality in prices according to the standard approach. In the model we present later, we will work under the constraint that rents are a-seasonal.

Table 10: Summer Differential in Rents in the U.S.: Owner's Equivalent Rent

	Rents		log(Rent)	
Summer-dummy S_t	-0.19638	-0.19638	-0.00102	-0.00102
	(8.133)	(0.269)	(0.051)	(0.006)
Trend		1.45183**		0.00905**
		(0.005)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Table 11: Summer Differential in Rents in the U.S.: Rent of Primary Residence

	Rents		log(Rent)	
Summer-dummy S_t	-0.16594	-0.16594	-0.00098	-0.00098
	(7.120)	(0.638)	(0.047)	(0.005)
Trend		1.26671**		0.00827**
		(0.012)		(0.000)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

Mortgage Rates

As in the U.K., interest rates in the U.S. do not exhibit a seasonal pattern (Barsky and Miron, 1989). Since housing service costs are of particular interest here, we summarize In Table 12 the summer effect (or lack thereof) in mortgage rates. The data come from the Board of Governors of the Federal Reserve and correspond to contract interest rates on commitments for fixed-rate first mortgages; the data are quarterly averages beginning in 1972; the original data are collected by Freddie Mac. As the Table shows, mortgage rates do not appear to be higher on average during the summer term, consistent with the findings in Barsky and Miron (1989).

Table 12: Summer Differential in Mortgage Rates in the U.S.

	Mortgage Rate
Summer-dummy S_t	0.104
	(0.477)

+ Significant at 10%; * Significant at the 5%; ** significant at 1%

2.4 Further Discussion

We have argued before that the predictability and size of the seasonal variation in house prices pose a puzzle to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal service cost. In Appendix 7.1 we carry out back-of-envelope calculations to assess to what extent seasonality in service costs might be driving the seasonality in prices.

The exercise makes clear that a standard asset-pricing approach that relies on perfect arbitrage leads to implausibly large levels of seasonality in service costs.[20] The findings suggests that there

[20] Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 328 and 209 percent of those in the summer. Depreciation and repair costs might be seasonal, being potentially lower during the summer. But income-tax-adjusted interest rates and property taxes, two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the U.K. is even larger. Assuming, quite conservatively, that the a-seasonal component accounts for only 50 percent of the service costs in the summer the implied ratio of depreciation and repair costs between summers and winters for rent-to-price ratios in the range of 2 through 5 percentbe between 557 and 318 percent. (If the a-seasonal componentaccounts for 80 percent of the service costs, the corresponding values are 1542 and 944 percent.

are important frictions in the market that impair the ability of investors to gain from any seasonal arbitrage and call for a deviation from the simple asset-pricing model.[21] A possible explanation for why the asset-pricing condition fails is of course that transaction costs are very high. Still, even if one takes that view, there still remain some puzzling observations: Why do not potential buyers try to buy in the low-price season? Could they be better-off waiting? Why do we observe a systematic seasonal pattern? (The lack of scope for seasonal arbitrage does not necessarily imply that most transactions should be carried out in one season, nor does it imply that prices and transactions should be correlated.) In the next Section, we develop a search and matching model for the housing market that seeks to offer answers to these questions.

3 A Search-and-Matching Model for the Housing Market

The model economy is populated by a unit measure of infinitely lived agents, who have linear preferences over a non-durable consumption good and housing services. Each period agents receive a fixed endowment of the consumption good which they can either consume or use to buy housing services. Agents can only enjoy housing services from living in one house a time, i.e. they can only be "matched" to one house at a time. A matched agent is a "homeowner" and an unmatched agent is a "buyer". There is a unit measure of housing stock. Each period houses can also be matched or unmatched. The matched house delivers a flow of housing services of quality ε to its homeowner, which we assume to be constant over time. The unmatched house is "for sale" and is owned by a "seller". The seller receives a flow of asset values u from an unmatched house he owns. Houses and agents are ex-ante identical. The asset flow value u of a house is common to all sellers. The quality of housing services ε, however, is match-specific, and it captures the quality of a match between a house and its homeowner. In other words, for any vacant house, the potential housing

[21] The need to deviate from the asset-market approach has been acknowledged, in a different context, among others, by Stein (1995). While static in nature, Stein's model is capable of generating unexpected booms and busts in prices (and transactions) in a rational-expectation setting. In a dynamic setting with forward-looking agents, however, predictably large changes in prices cannot be sustained: Expected price increases in the next season will actually be priced in the current season (or, in other words, sellers will refuse to sell at lower prices today given the perspective of higher prices in the next season); similarly, prospective buyers will benefit from waiting (at most a few months) and paying a significantly lower price. Even when agents are both sellers and buyers, if they are aware of the differences in prices, in a dynamic setting they will seek to sell in the summer and to buy in the winter; the excess supply in the summer will then push prices down, while the excess demand in the winter will push them up.

services are idiosyncratic to the match between the house and the buyer. Hence, ε is not the type of the house (or of the seller who owns a particular house); there is only one representative house in our model, but the utility derived from living in the house is idiosyncratic. This is consistent with our data, which are adjusted for houses' characteristics, such as size and location, but not for the (unobserved) quality of a match.[22] Since this is the key element of our model, we will first discuss in detail how we model it.

3.1 Match-specific Quality

The model embeds the intuitively appealing notion that in a market with many houses on sale a buyer can find a house closer to her ideal and hence her willingness to pay increases. We model this idea by assuming that a buyer draws the quality of a potential match, ε, from a distribution $F(\varepsilon, v)$ with positive support and finite mean, where v denotes the stock of vacant houses, and $f(\varepsilon, v)$ is the corresponding probability density function. Our notion of a "thick-market" is captured by the following assumptions:

Assumption 1 $F(., v)$ is decreasing in v.

Assumption 1 states that $F\left(., v^{'}\right)$ stochastically dominates $F(., v)$ if and only if $v' > v$. In words, when the stock of houses v is bigger, a random draw of match quality ε from $F(\varepsilon, v)$ is likely to be higher.[23] One useful implication of Assumption 1 is that higher v shifts up the expected surplus of quality above any threshold x :

$$h(x, v) \equiv [1 - F(x, v)]\, E(\varepsilon - x \mid \varepsilon > x) \text{ is increasing in } v. \tag{3}$$

To see this rewrite $h(x, v)$ using integration by part,

$$h(x, v) = \int_x^{\bar{\varepsilon}} (\varepsilon - x)\, dF(\varepsilon, v) = \bar{\varepsilon} - x - \int_x^{\bar{\varepsilon}} F(\varepsilon, v)\, d\varepsilon = \int_x^{\bar{\varepsilon}} [1 - F(\varepsilon, v)]\, d\varepsilon,$$

[22] Neither repeat-sale indices nor hedonic price indices can control for the quality of a match.

[23] One way to interpret our assumptions is to use order statistics. Let the potential match quality between a buyer and any house in the *entire housing stock* be randomly distributed according to a distribution $G(.)$, and let $g(.)$ be the corresponding probability density function. Suppose the buyer samples n units of vacant houses. Let $(\varepsilon_1, \varepsilon_2, ..\varepsilon_n)$ denote an *iid* random sample from the continuous distribution $G(.)$. Let ε be the maximum ε_i; then the distribution of ε is $F(., n) = [G(.)]^n$, which is decreasing in n. Intuitively, as the sample size increases, the maximum becomes "stochastically larger."

which is increasing in v from Assumption 1. Note that this expression also implies that the conditional surplus is:

$$E(\varepsilon - x \mid \varepsilon > x) = \frac{\int_x^{\bar{\bar{\varepsilon}}} [1 - F(\varepsilon, v)]\, d\varepsilon}{1 - F(., v)}.$$

Thus, the conditional surplus is also increasing in v under the following assumption:

Assumption 2 $\frac{\int_x^{\bar{\bar{\varepsilon}}} [1-F(\varepsilon,v)]d\varepsilon}{1-F(x,v)}$ is increasing in v.

Assumption 2 states that for any threshold x, as v increases, the increase in the integral of $[1 - F(., v)]$ for any ε above x is at least as large as the increase in $[1 - F(x, v)]$. This will be true if the increase in $[1 - F(x, v)]$, due to an increase in v, is not concentrated for any particular value of x. In words this rule out the case that a larger housing stock only makes a particular type of match-quality less likely.

Any buyer can sample the *entire stock of vacant houses* (e.g. by searching online or through newspapers).[24] After sampling the stock, the buyer chooses the house that ranks first and makes contact with the seller, i.e. we assume that each period a buyer visits only one house—her best house. Given the *iid* assumption, it follows that the best house is different for each buyer and, as a result, a house is visited by only one buyer. This assumption implies that the seller "negotiates" a price for each house he owns *independently* with each different buyer, that is, the price of a house is determined between one buyer and one seller.

3.2 Seasons and Timing

There are two seasons, $j = s, w$ (for summer and winter); each model period is a season, and seasons alternate. At the beginning of a period, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, and the house is for sale. The homeowner becomes a buyer and seller simultaneously. In our baseline model, the parameter ϕ^j is the only (*ex ante*) difference between the seasons (determined, for example, by the school calendar or summer marriages, among other factors). We focus on periodic steady states with constant v^s and v^w, where v^j is the stock of vacant (unmatched) houses in season $j = s, w$. We call b^j be the stock of buyers (unmatched

[24] This is different from the stock-flow literature (see e.g. Coles and Smith, 1998), where new buyers can only draw from the stock of old vacant houses, and the stock of old buyers can only draw from the stock of new vacant houses. We do not draw a distinction here between old and new buyers.

agents) in season $j = s, w$. Since a match is between one house and one agent, and there is a unit measure of agents and a unit measure of houses, it is always the case that $v^j = b^j$.

The sequence of events is as follows. At the beginning of season j, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, adding to the stock of vacant houses and buyers. The buyer observes ε (drawn from $F(., v^j)$) for her best house out of the available stock v^j and meets with the seller of this house. If the transaction goes through, the buyer pays a price (discussed later) to the seller, and starts enjoying the housing services from the same season j. If the transaction does not go through, the buyer looks for a house again next season, the seller receives the asset value flow in season j and puts the house up for sale again next period. An agent can hence be a homeowner, a buyer, a seller, both a seller and a homeowner, and both a buyer and a seller. Also, sellers may have multiple houses to sell.

3.3 The Homeowner

To study pricing and transaction decisions, we first derive the value of living in a house if a transaction goes through. The value function for a homeowner who lives in a house with quality ε in season s is given by:

$$H^s(\varepsilon) = \varepsilon + \beta\phi^w H^w(\varepsilon) + \beta(1-\phi^w)[V^w + B^w],$$

where $\beta \in (0,1)$ is the discount factor. With probability $(1-\phi^w)$ he receives a moving shock and becomes both a buyer and a seller (putting his house up for sale), with continuation value $(V^w + B^w)$, where V^j is the value of a vacant house to its seller and B^j is the value of being a buyer in season $j = s, w$, defined below. With probability ϕ^w he keeps receiving housing services of quality ε and stays in the house. (Notice that the formula for $H^w(\varepsilon)$ is perfectly isomorphic to $H^s(\varepsilon)$; in the interest of space we omit here and throughout the paper the corresponding expressions for season w.) The value of being a homeowner can be therefore re-written as:

$$H^s(\varepsilon) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon + \frac{\beta(1-\phi^w)(V^w+B^w) + \beta^2\phi^w(1-\phi^s)(V^s+B^s)}{1-\beta^2\phi^w\phi^s}, \tag{4}$$

which is straightly increasing in ε.

3.4 Market Equilibrium

In any season $j = s, w$, the buyer visits a house with match quality ε, drawn from the distribution $F^j(\varepsilon) \equiv F(\varepsilon, v^j)$. The buyer meets with the seller of this house to "negotiate" a price. We focus on the case in which the seller also observes ε and discuss the case in which he does not observe ε in the Appendix. If the transaction goes through, the buyer pays the price to the seller, and starts enjoying the housing services flow from the same season j. If the transaction does not go through, the buyer receives zero housing services and looks for a house again next season. This can be the case, for example, if buyers searching for a house pay a rent equal to the utility they derive from the rented property; what is key is that the rental property is not owned by the same potential seller with whom the buyer meets. On the seller's side, when the transaction does not go through, he receives the asset flow value u from season j and puts the house for sale again next season. The flow value u can be interpreted as a net rental income received by the seller. Again, what is key is that the tenant is not the same potential buyer who visits the house.

As in the search-and-matching literature, the realized transaction yields positive surplus. We assume that the surplus is shared according to Nash bargaining. Let $S_v^s(\varepsilon)$ and $S_b^s(\varepsilon)$ be the surpluses of a transaction to the seller and the buyer when the match quality is ε and the price is $p^s(\varepsilon)$:

$$\begin{aligned} S_v^s(\varepsilon) &\equiv p^s(\varepsilon) - (u + \beta V^w), &(5)\\ S_b^s(\varepsilon) &\equiv H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w. &(6) \end{aligned}$$

The price maximizes the Nash product:

$$\begin{aligned} &\max_{p^s(\varepsilon)} \left(S_v^s(\varepsilon)\right)^{\theta} \left(S_b^s(\varepsilon)\right)^{1-\theta} \\ &s.t. \quad S_v^s(\varepsilon), S_b^s(\varepsilon) \geqslant 0; \end{aligned}$$

where θ denotes the bargaining power of the seller. The solution implies

$$\frac{S_v^s(\varepsilon)}{S_b^s(\varepsilon)} = \frac{\theta}{1-\theta}, \tag{7}$$

and a transaction goes through as long as the total surplus $S^s(\varepsilon)$ is positive,

$$S(\varepsilon) \equiv S_v^s(\varepsilon) + S_b^s(\varepsilon) = H^s(\varepsilon) - [\beta(B^w + V^w) + u] \tag{8}$$

Given $H^s(\varepsilon)$ is increasing in ε, a transaction goes through if $\varepsilon \geqslant \varepsilon^s$, where the reservation ε^s is defined by

$$\varepsilon^s =: H^s(\varepsilon^s) = \beta(B^w + V^w) + u, \tag{9}$$

and $1 - F^s(\varepsilon^s)$ is thus the probability that a transaction is carried out.

The value functions for being a seller and a buyer in season s are, respectively:

$$V^s = \beta V^w + u + [1 - F^s(\varepsilon^s)] E^s[S_v^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{10}$$

$$B^s = \beta B^w + [1 - F^s(\varepsilon^s)] E^s[S_b^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{11}$$

where $E^s[.]$ indicates the expectation is taken with respect to the distribution $F^s(.)$. The Nash solution (7) implies (see Appendix):

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + (1-\theta)\frac{u}{1-\beta}, \tag{12}$$

which is a weighted average of the housing value and the present discounted value of the flow value u. So the price guarantees the seller his alternative usage of the house ($\frac{u}{1-\beta}$) and a θ fraction of the social surplus generated by the transaction $\left[H^s(\varepsilon) - \frac{u}{1-\beta}\right]$.

The average price of a transaction is:

$$P^s \equiv E[p^s(\varepsilon) \mid \varepsilon > \varepsilon^s] = (1-\theta)\frac{u}{1-\beta} + \theta E[H^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{13}$$

which is increasing in the conditional expected surplus of housing value for transactions exceeding the reservation ε^s.

We next derive the reservation quality ε^s. Observe from (8) and (9) that

$$S^s(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}(\varepsilon - \varepsilon^s), \tag{14}$$

Using the value functions (10) and (11), the sum of value for the buyer and the seller is:

$$B^s + V^s = H^s(\varepsilon^s) + [1 - F^s(\varepsilon^s)] E^s[S^s(\varepsilon) \mid \varepsilon > \varepsilon^s], \tag{15}$$

which is the sum of the housing value $H^s(\varepsilon^s)$ of the marginal transaction and the expected surplus from a transaction above the reservation ε^s. Using the definition of $S^s(\varepsilon)$ and ε^s in (8) and (9), and the expression in (14), the sum of values is:

$$B^s + V^s = \beta(B^w + V^w) + u + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}[1 - F^s(\varepsilon^s)] E^s[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s]. \tag{16}$$

Solving this explicitly, we derive:

$$B^s + V^s = \frac{u}{1-\beta} + \frac{(1+\beta\phi^w) h^s(\varepsilon^s) + \beta(1+\beta\phi^s) h^w(\varepsilon^w)}{(1-\beta^2)(1-\beta^2\phi^w\phi^s)}, \tag{17}$$

where $h^s(\varepsilon^s) \equiv h(\varepsilon^s, v^s) = [1 - F^s(\varepsilon^s)] E[\varepsilon - \varepsilon^s \mid \varepsilon > \varepsilon^s]$ is the expected surplus of quality above threshold ε^s as described in (3). Using the definition of ε^s in (9) and the expression (4), we derive the reservation quality:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s = u - \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(B^s+V^s) + \frac{1-\beta^2\phi^s}{1-\beta^2\phi^w\phi^s}\beta\phi^w(B^w+V^w), \tag{18}$$

Note that (17) and (18) together imply that the reservation quality ε^s is independent of θ. This is because a transaction will go through as long as the total surplus is positive which is independent of how the surplus is divided among the buyer and the seller.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $(P^j, P^j, \varepsilon^j, \varepsilon^j)$ in season $j = s, w$ through two channels, as shown in (13) and (18): the unconditional expected surplus of quality above reservation ε^j, $h^j(\varepsilon^j)$, which affects the outside option $B^s + V^s$ as shown in (17); and the condition surplus of quality above reservation ε^j, $E^j[\varepsilon - \varepsilon^j \mid \varepsilon > \varepsilon^j]$, which affect the expected total surplus of a transaction $E^j[S^j(\varepsilon) \mid \varepsilon > \varepsilon^j]$ as in (14).

3.5 Stock of vacant houses

In any season s, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$v^s = (1-\phi^s)[v^w(1-F^w(\varepsilon^w)) + 1 - v^w] + v^w F^w(\varepsilon^w)$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = 1 - \phi^s + v^w F^w(\varepsilon^w)\phi^s \tag{19}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality ε^w and on the distribution $F^w(.)$.

An equilibrium is a vector $(B^s + V^s, B^w + V^w, \varepsilon^s, \varepsilon^w, v^s, v^w, P^s, P^w)$ that jointly satisfies equations (13), (17),(18), and (19), with $H^s(\varepsilon)$ given by (9), (14), and (17).

4 Model-generated Seasonality of Prices and Transactions

4.1 Qualitative Results

We now derive the extent of seasonality in prices and transactions generated by the model. The driver for seasonality in the model is the probability of a moving shock, which we assume to be higher in the summer: $1-\phi^s > 1-\phi^w$. Using (19), the stock of vacant houses in season s is given by:

$$v^s = \frac{1-\phi^s+\phi^s F^w(\varepsilon^w)(1-\phi^w)}{1-F^s(\varepsilon^s)F^w(\varepsilon^w)\phi^s\phi^w}. \tag{20}$$

(The expression for v^w is correspondingly isomorphic). The *ex ante* higher probability of moving in the summer ($1-\phi^s > 1-\phi^w$) clearly has a direct positive effect on v^s.[25]

Given $v^s > v^w$, the thick-market effect implies $h^s(\varepsilon^s) > h^w(\varepsilon^w)$ as in (3). It then follows from (17) that $(B^s+V^s) > (B^w+V^w)$, and finally from (9) that $H^s(\varepsilon^s) < H^w(\varepsilon^w)$. In other words, the housing value of the marginal transaction is lower in the hot season.

4.1.1 Seasonality in Prices

From the price equation (13), since the flow value u is a-seasonal, housing prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. The following result follows:

Result 1 *When sellers have some bargaining power ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in θ.*

To see this, note that from (36) the equilibrium price P^s is the discounted sum of the flow value ($\frac{u}{1-\beta}$) plus a positive surplus from the sale. The surplus $E^s\left[\left(H^s(\varepsilon)-\frac{u}{1-\beta}\right)\mid \varepsilon>\varepsilon^s\right]$ is seasonal. The price is higher in the hot season if the average quality of transacted houses is higher, i.e. if $E^s[H^s(\varepsilon)\mid\varepsilon>\varepsilon^s] > E^s[H^w(\varepsilon)\mid\varepsilon>\varepsilon^w]$. In words, if the quality of matches goes up in the summer (and hence the total surplus of a transaction), then sellers can obtain a higher surplus in the summer.

Recall from (14) that

$$E^s[H^s(\varepsilon)\mid\varepsilon>\varepsilon^s] = H^s(\varepsilon^s)+\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}E[\varepsilon-\varepsilon^s\mid\varepsilon>\varepsilon^s]. \tag{21}$$

[25] More specifically, the numerator is a weighted average of 1 and $F^w(\varepsilon^w)(1-\phi^w)$ with $1-\phi^s$ being the weight assigned to 1. Therefore, as along as $F^w(\varepsilon^w)(1-\phi^w)$ is sufficiently smaller than one (which is true when we calibrate ϕ^j to match the average duration of staying in a house), we have $v^s > v^w$.

There are two opposite forces on the average quality of transacted houses: a negative one from the lower housing value of marginal transaction and a positive one from the conditional expected surplus of quality above the reservation. Since $\beta^2\phi^w\phi^s$ is close to 1, the positive effect dominates. Given that θ affects P^s only through the equilibrium vacancies (recall the reservation quality ε^s is independent of θ), it follows that the extent of seasonality in prices is increasing in θ. Since (13) holds independently from the steady state equation for v^s and v^w, Result 1 holds independently of what drives $v^s > v^w$. Finally, the effect of the flow-value u on the seasonality of prices is as follows:

Result 2 *The extent of seasonality in prices is decreasing in the flow value u.*

Result 2 follows from the fact that the extent of seasonality in prices decreases as the a-seasonal component—the outside option u—increases.

We next turn to the degree of seasonality in transactions.

4.1.2 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left(1 - F^s\left(\varepsilon^s\right)\right). \tag{22}$$

(An isomorphic expression holds for Q^w). Seasonality in transactions stems from three sources. First, a bigger stock of vacancies in the summer, $v^s > v^w$, tends to increase transactions in the summer. Second, the thick-market effect shifts up the probability of a transaction for any given ε^s. The final effect is due to fact that the seasonality in sellers' and buyers' outside options, tends to reduce the cutoff ε^s in the hot season. This is because the outside option in the hot season s is linked to the sum of values in the winter season: $B^w + V^w$. To see this negative effect more explicitly, rewrite (18) as

$$\begin{aligned} & \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s \\ = \quad & u + \frac{\beta\phi^w\left(1-\beta\right)\left(1+\beta\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^w + B^w\right) + \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^w + B^w - V^s - B^s\right), \end{aligned} \tag{23}$$

which makes clear that $(B^w + V^w) > (B^s + V^s)$ has a negative effect on $\varepsilon^s/\varepsilon^w$.

It is important to note that the amplification mechanism present in the model: for any given level of seasonality in vacancies, the thick-market effect through the first-order stochastic dominance of $F^s(.)$ over $F^w(.)$ can generate *higher* seasonality in transactions.

Given ε^s is independent of θ, we can summarize the results as follow:

Result 3 Transactions are more seasonal than vacancies. The extent of seasonality is independent of the seller's bargaining power θ.

To see this, note that the outside option for both the buyer and the seller in the hot season is to wait and transact in the cold season. This makes both buyers and sellers less demanding in the hot season, yielding a larger number of transactions. In other words, the "counter-seasonality" in outside options increases the seasonality in transactions. Finally, the effect of the flow value u on the seasonality of transactions is as follows:

Result 4 The extent of seasonality of transactions is decreasing in the rental flow u.

Result 4 follows from the fact that the extent of the seasonality of outside options for buyers and sellers is decreasing in u. Hence, as u increases, transactions become less seasonal.

In the Appendix we derive the case when the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer and $1-\theta$ as the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[26] The main difference between the observable and unobservable is the following. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon's$ are not sufficiently high enough (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that Results 1,2, and 4 continue to hold, while Result 3 is slightly modified so that the seasonality in transactions is decreasing in θ. This is because when ε is unobservable there is a second effect through the seller's surplus on the seasonality of reservation quality which is opposite to the effects from the seasonality of outside option described above. The intuition is the following. When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions. The second effect through the seller's surplus is increasing in θ (and disappears when $\theta = 0$).

[26] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

4.2 Calibration of the model

4.2.1 Parameter values

We now calibrate the model to study its quantitative implications. We set the discount factor β so that the implied annual real interest rate is 5 percent.

We set the average probability of staying in the house $\phi = (\phi^s + \phi^w)/2$ to match survey data on the average duration of stay in a given house, which in the model is given by $\frac{1}{1-\phi}$). The median duration in the U.S. from 1993 through 2005, according to the American Housing Survey, was 18 semesters; the median duration in the U.K. during this period, according to the Survey of English Housing was 26 semesters. The implied (average) moving probabilities ϕ per semester are 0.056 and 0.039 for the US and the UK, respectively. These two surveys also report the main reasons for moving. Around 30 percent of the respondents report that living closer to work or to their children's school and getting married are the main reasons for moving.[27] These factors are of course not entirely exogenous, but they can carry a considerably exogenous component; in particular, the school calendar is certainly exogenous to housing market movements (see Tucker, Long, and Marx (1995)'s study of seasonality in children's residential mobility[28]). In all, the survey evidence supports our working hypothesis that the *ex ante* probability to move is higher in the summer (or, equivalently the probability to stay is higher in the winter).

We calibrate the asset flow value, u, to match the implied average (de-seasonalized) rent-to-price ratio *received by the seller.* In the UK, the average *gross* rent-to-price ratio is roughly around 5 percent per year, according to *Global Property Guide.*[29] For the US, Davis et. al. (2008) argue that the ratio was around 5 percent prior to 2000 when it fell by 1.5 percent. The u/p ratio in our model corresponds to the *net* rental flow received by the seller after paying taxes and other relevant costs. It is accordingly lower than the *gross* rent-to-price ratio. As a benchmark, we choose u so

[27] Using monthly data on marriages from 1980 through 2003 for the U.K. and the U.S., we find that marriages are highly seasonal in both countries, with most marriages taking place between April and September. (The difference in annualized growth rates of marriages between the broadly defined "summer" and "winter" semesters are 200 percent in the U.S. and 400 percent in the U.K.). Results are available from the authors.

[28] See also Harding, Rosenthal, and Sirmans (2003), who find that families with school-age children are willing to accept less favorable terms of trade during July through September. In other words, everything equal, they are more likely to move during school vacation.

[29] Data for the U.K. and other European countries can be found in http://www.globalpropertyguide.com/Europe/United-Kingdom/price-rent-ratio

that u/p is equal to 3 percent per year (equivalent to paying a 40 percent income tax on rent).[30] To do so, we use the equilibrium equations in the model without seasonality, that is, the model in which $\phi^s = \phi^w = \phi$. From (13) and (18), the average price and the reservation quality ε^d in the absence of seasonality in moving probabilities are (see Appendix 7.2.2):

$$P = \frac{u}{1-\beta} + \theta \frac{\left(1 - \beta F\left(\varepsilon^d\right)\right) E\left[\varepsilon - \varepsilon^d \mid \varepsilon > \varepsilon^d\right]}{(1-\beta)(1-\beta\phi)}, \tag{24}$$

and

$$\frac{\varepsilon^d}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi} \int_{\varepsilon^d}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon)}{1 - \beta\phi F\left(\varepsilon^d\right)}. \tag{25}$$

To obtain a calibrated value for u, we substitute $u = 0.015 \cdot p$ for $\theta = 1/2$, (when sellers and buyers have the same bargainining power) and find the equilibrium value of p given the calibrated values for β and $F(.)$. We then use the implied value of $u = 0.015 \cdot p$. as a parameter.[31]

To be completed

5 Efficiency

This Section discusses the efficiency of equilibrium in the decentralized economy under both a seller's and a buyer's market scenarios. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy

To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by h_{t-1}, as well as the beginning of period's stock of vacant houses, v_t. Thus, taking as given the initial levels h_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0\ldots}$,

[30] In principle, other costs can trim down the 3-percent u/p ratio, including maintenance costs, and inefficiencies in the rental market that lead to a higher wedge between what the tenant pays and what the landlord receives; also, it might not be possible to rent the house immediately, leading to lower average flows u. Note, however, that lower values of u/p lead to even higher seasonality in prices and transactions for any given level of seasonality in moving shocks. In that sense, lower u/p-ratios make it "easier" for our model to generate seasonality in prices.

[31] We also calibrated the model using different values of u for different θ (instead of setting $\theta = 1/2$), keeping the ratio u/p constant. Results are not significantly different under this procedure, but the comparability of results for different values of θ becomes less clear, since u is not kept fixed.

which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, h_t, v_t\}_{t=0\ldots}\right) \equiv \sum_{t=0}^{\infty} \beta^t \left[h_t + uv_t F\left(\varepsilon_t; v_t\right)\right] \tag{26}$$

subject to the law of motion for h_t :

$$h_t = \phi_t h_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \tag{27}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1} = v_t \phi_{t+1} F\left(\varepsilon_t; v_t\right) + 1 - \phi_{t+1}, \tag{28}$$

and the inequality constraint:

$$0 \leqslant \varepsilon_t \leqslant \bar{\varepsilon}\left(v_t\right). \tag{29}$$

Intuitively, the planner faces two types of trade-offs when deciding the optimal reservation quality ε_t : a static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacant houses in period t in the objective function (26). The utility per person generated from vacant houses is the rental income per person, captured by $uv_t F\left(\varepsilon_t\right)$. The utility generated by occupied houses in period t is captured by h_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in period t. The utility h_t, which follows the law of motion (27), is the sum of the pre-existing expected housing utility h_{t-1} that survives the moving shocks and the expected housing utility from the new matches. By increasing ε_t, the expected housing value h_t decreases, while the utility generated by vacant houses increases (since $F\left(\varepsilon_t\right)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacant houses in (28). By increasing ε_t (which in turn decreases h_t), the number of transactions in the current period decreases; this leads to more vacant houses in the following period, v_{t+1}, and consequently to a thicker market in the next period.

Assuming the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons, the optimal reservation quality, ε^j, $j = s, w$, in the periordic steady state is (see Appendix 7.3):

$$\varepsilon^j \left(\frac{1 + \beta\phi^{j'}}{1 - \beta^2 \phi^j \phi^{j'}}\right) = \frac{\left(1 + \beta\phi^{j'} A^{j'}\right) u + \beta^2 \phi^j \phi^{j'} A^{j'} D^j + \beta\phi^{j'} D^{j'}}{1 - \beta^2 \phi^j \phi^{j'} A^j A^{j'}}, \tag{30}$$

where

$$A^j \equiv F^j\left(\varepsilon^j\right) - v^j T_1^j; \qquad D^j \equiv \frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j} \varepsilon dF^j(\varepsilon) + v^j T_2^j\right), \tag{31}$$

and the stock of vacant houses, v^j, $j = s, w$, satisfies (68) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j \equiv \frac{\partial}{\partial v^j}\left[1 - F^j\left(\varepsilon^j\right)\right] > 0$ and $T_2^j \equiv \frac{\partial}{\partial v^j}\int_{\varepsilon^j}^{\bar{\varepsilon}^j} \varepsilon dF^j(\varepsilon) > 0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1 - F^j(\varepsilon)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (30) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j < v^j$, $j = s, w$. However, when there are no seasons, $\phi^s = \phi^w$, it follows immediately from (68) that the solution is interior, $\varepsilon < v$. On the other hand, when the exogenous difference in moving propensities across seasons is large enough, the Planner might find it optimal to close down the market in the cold season. Before we turn to such situation, it is helpful to understand the sources of inefficiency in the decentralized economy when there are no seasons.

Abstracting from seasonality for the moment, the inefficiency in the decentralized economy is due to the fact that the optimal decision rules of buyers and sellers take the stock of houses in each period as given, thereby ignoring the effects of their decision rules on the stock of vacant houses in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high for any given stock of vacant houses. More specifically, setting $\phi^s = \phi^w = \phi$ in (30) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) + vT_2\right)}{1 - \beta\phi F\left(\varepsilon^p\right) + \beta\phi v T_1}. \tag{32}$$

Comparing (32) with (25), the thick-market effect, captured by $T_1$and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1 - \beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome.[32]

[32] This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

We now return to the planner's problem in the case in which it is optimal to close down the market during the cold season. In this case, the solution implies setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$ in the planner's problem. The optimal reservation quality, ε^s, in the periodic steady state is (see Appendix 7.3):

$$\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s} = \frac{u + \frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} \varepsilon dF^s(\varepsilon) + v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s) - v^s T_1^s\right]}, \tag{33}$$

which is similar to the Planner's solution with no seasons in (32) with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

6 Concluding Remarks

This paper documents seasonal booms and busts in housing markets and argues that the predictability and high extent of seasonality in prices observed in some of them cannot be quantitatively reconciled with standard asset-pricing equilibrium conditions.

To explain the empirical patterns, the paper presents a search-and-matching model that can quantitatively account for most of the empirical puzzle. As a by product, the model sheds new light on interesting mechanisms governing fluctuations in housing markets that can potentially be useful in a study of lower-frequency movements. In particular, the model highlights the roles of thick-market externalities as an important determinant of the extent of housing markets' fluctuations.

In future work, the authors plan to adapt the model presented in the paper to study lower frequency movements in the housing markets.

7 Appendix

7.1 A back-of-the-envelope calculation

We argued before that the predictability and size of the seasonal variation in housing prices in some countries pose a puzzle to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[33]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j} \tag{34}$$

[33] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real *gross* (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[34] As in Poterba (1984), we make the following simplifying assumptions so that service-cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): *i*) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, also constant for a given season. *ii*) The income-tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; they are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, they are actually constant across seasons; we come back to this point below).[35] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$, for $j = s, w$.

Subtracting (34) from the corresponding expression in the following season and using the condition that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}} \tag{35}$$

Considering the *real* differences in house price growth rates documented for the whole of the U.K., $\frac{p_s - p_w}{p_w} = 7.04\%$, $\frac{p_w - p_s}{p_s} = 0.75\%$, the left-hand side of (35) equals $6.3\% \approx 7.04\% - 0.75\% \cdot \frac{1}{1.0075}$. Therefore, $\frac{c_w}{c_s} = \frac{0.063}{c_s} + \frac{1}{1.0075}$. The value of c_s can be pinned-down from equation (34) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table 15, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.75\%$).

[34] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[35] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume that the opportunity cost of funds equals the cost of borrowing.

Table 15: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	459%
2.0%	328%
3.0%	267%
4.0%	232%
5.0%	209%
6.0%	193%

As the Table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 328 and 209 percent of those in the summer. Depreciation and repair costs $(\delta_j + \kappa_j)$ might be seasonal, being potentially lower during the summer.[36] But income-tax-adjusted interest rates and property taxes $(r_j + \tau_j)$, two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the U.K. is even larger. Assuming, quite conservatively, that the a-seasonal component $(r_j + \tau_j = r + \tau)$ accounts for only 50 percent of the service costs in the summer $(r + \tau = 0.5c_s)$, then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w+\kappa_w+0.5c_s}{\delta_s+\kappa_s+0.5c_s}$ implies that the ratio of depreciation and repair costs between summers and winters is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = 2\frac{c_w}{c_s} - 1$.[37] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 557 and 318 percent of those in the summer. (If the a-seasonal component $(r + \tau)$ accounts for 80 percent of the service costs $(r + \tau = 0.8c_s)$, the corresponding values are 1542 and 944 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for.

[36] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though it is key here that leisure is not too valuable for the owners.

[37] Call λ the asesonal component as a fraction of the summer service cost rate: $r + \tau = \lambda c_s$, $\lambda \in (0, 1)$ (and hence $\delta_s + \kappa_s = (1 - \lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{\delta_s+\kappa_s+\lambda c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w-\lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w+\kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

7.2 Derivation for the model with observable value

7.2.1 Model with seasons

Derive $p^s(\varepsilon)$ in (12)

Using the Nash solution,

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\theta$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u. \tag{36}$$

Using the value functions (10) and (11),

$$(1-\theta)V^s - \theta B^s = \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (36) to obtain (12).

7.2.2 The model without seasons

The value functions for the model without seasonality are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$.

Using (13),(14) and (17), we can express the average price explicitly as:

$$P^s = \frac{u}{1-\beta} + \theta\left[\frac{\beta(1+\beta\phi^s)h^w(\varepsilon^w) + \left(1-\beta^2 F^s(\varepsilon^s)\right)(1+\beta\phi^w)E[\varepsilon-\varepsilon^s \mid \varepsilon > \varepsilon^s]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)}\right], \tag{37}$$

Using (18),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}(1-\beta)(V+B)$$

and $B+V$ from (17),

$$B+V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2}\left\{\begin{array}{c} \frac{1-F}{1-\beta\phi}E[\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon} > \varepsilon] \\ +\beta\frac{1-F}{1-\beta\phi}E[\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon} > \varepsilon] \end{array}\right\}$$

which reduces to:

$$B+V = \frac{u}{1-\beta} + \frac{1-F(\varepsilon)}{(1-\beta)(1-\beta\phi)}E(\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon} > \varepsilon).$$

so

$$\varepsilon = u + \frac{\beta\phi}{1-\beta\phi}\left[1-F(\varepsilon)\right]E\left(\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon}>\varepsilon\right).$$

and the final equilibrium condition is:

$$v = \frac{1-\phi}{1-\phi F(\varepsilon)}.$$

7.3 Analytical derivations of the planner's solution

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,\ldots}$alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (h_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$\begin{aligned} W(h_{t-1}, v_t, \phi_t) &= \max_{\varepsilon_t}\left[h_t + uv_tF(\varepsilon_t; v_t) + \beta W\left(h_t, v_{t+1}, \phi_{t+1}\right)\right] \\ & \quad s.t. \\ h_t &= \phi_t h_{t-1} + v_t\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t), \\ v_{t+1} &= v_t\phi_{t+1}F(\varepsilon_t; v_t) + 1 - \phi_{t+1}. \end{aligned} \tag{38}$$

The first-order condition implies

$$\left(1+\beta\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial h_t}\right)v_t\left(-\varepsilon_t f(\varepsilon_t; v_t)\right) + \left(\beta\phi_{t+1}\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}} + u\right)v_t f(\varepsilon_t; v_t) = 0,$$

which simplifies to

$$\varepsilon_t\left(1+\beta\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial h_t}\right) = u + \beta\phi_{t+1}\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}}. \tag{39}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W(h_{t-1}, v_t, \phi_t)}{\partial h_{t-1}} = \phi_t\left(1+\beta\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial h_t}\right) \tag{40}$$

and

$$\begin{aligned} \frac{\partial W(h_{t-1}, v_t, \phi_t)}{\partial v_t} &= \left(u + \beta\phi_{t+1}\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F(\varepsilon_t; v_t) - v_tT_{1t}\right) \\ &\quad + \left(1+\beta\frac{\partial W\left(h_t, v_{t+1}, \phi_{t+1}\right)}{\partial h_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t) + v_tT_{2t}\right) \end{aligned} \tag{41}$$

where $T_{1t} \equiv \frac{\partial}{\partial v_t}\left[1 - F\left(\varepsilon_t; v_t\right)\right] > 0$ and $T_{2t} \equiv \frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right) > 0$.

In the periodic steady state, the first order condition (39) becomes

$$\varepsilon^j \left(1 + \beta \frac{\partial W^{j'}\left(h^j, v^{j'}\right)}{\partial h^j}\right) = u + \beta \phi^{j'} \frac{\partial W^{j'}\left(h^j, v^{j'}\right)}{\partial v^{j'}} \tag{42}$$

The envelope condition (40) implies

$$\frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial h^{j'}} = \phi^j \left[1 + \beta\left(\phi^{j'} + \beta \phi^{j'} \frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial h^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial h^{j'}} = \frac{\phi^j\left(1 + \beta \phi^{j'}\right)}{1 - \beta^2 \phi^j \phi^{j'}} \tag{43}$$

Substituting this last expression into (41), we obtain:

$$\frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial v^j} = \left(u + \beta \phi^{j'} \frac{\partial W^{j'}\left(h^j, v^{j'}\right)}{\partial v^{j'}}\right) A^j + D^j,$$

where

$$A^j \equiv F^j\left(\varepsilon^j\right) - v^j T_1^j; \qquad D^j \equiv \frac{1 + \beta \phi^{j'}}{1 - \beta^2 \phi^j \phi^{j'}} \left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j} \varepsilon dF^j\left(\varepsilon\right) + v^j T_2^j\right).$$

Hence, we have

$$\frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial v^j} = \left\{u + \beta \phi^{j'} \left[\left(u + \beta \phi^j \frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial v^j}\right) A^{j'} + D^{j'}\right]\right\} A^j + D^j,$$

which implies

$$\frac{\partial W^j\left(h^{j'}, v^j\right)}{\partial v^j} = \frac{u A^j\left(1 + \beta \phi^{j'} A^{j'}\right) + \beta \phi^{j'} D^{j'} A^j + D^j}{1 - \beta^2 \phi^j \phi^{j'} A^j A^{j'}}. \tag{44}$$

Substituting (43) and (44) into the first-order condition (42), we get:

$$\varepsilon^j \left(1 + \beta \frac{\phi^{j'}\left(1 + \beta \phi^j\right)}{1 - \beta^2 \phi^j \phi^{j'}}\right) = u + \beta \phi^{j'} \frac{u A^{j'}\left(1 + \beta \phi^j A^j\right) + \beta \phi^j D^j A^{j'} + D^{j'}}{1 - \beta^2 \phi^j \phi^{j'} A^j A^{j'}}$$

simplify to (30).

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$, the Bellman equation (38) can be rewritten as:

$$\begin{aligned} W^s\left(h_{t-1}^w, v_t^s\right) &= \max_{\varepsilon_t^s} \left[\begin{array}{c} \phi^s h_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon dF_t^s(\varepsilon) + u v_t^s F_t^s(\varepsilon_t^s) \\ +\beta\left(h_{t+1}^w + u\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right]\right) \\ +\beta^2 W^s\left(h_{t+1}^w, v_{t+2}^s\right) \end{array} \right] \\ & \quad s.t. \\ h_{t+1}^w &= \phi^w \left[\phi^s h_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon dF_t^s(\varepsilon) \right], \\ v_{t+2}^s &= \phi^s\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right] + 1 - \phi^s. \end{aligned} \tag{45}$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(h_{t-1}^w, v_t^s\right)$, the state variables for next period are $\left(h_{t+1}^w, v_{t+2}^s\right)$, and the control variable is ε_t^s.

The first order condition:

$$\begin{aligned} 0 &= v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s f_t^s(\varepsilon_t^s) \\ & \quad +\beta\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s \phi^w f_t^s(\varepsilon_t^s)\right) \\ & \quad +\beta^2\left[\frac{\partial W^s}{\partial h_{t+1}^w}\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right)\right) + \frac{\partial W^s}{\partial v_{t+2}^s}\left(\phi^s v_t^s \phi^w f_t^s(\varepsilon_t^s)\right)\right], \end{aligned}$$

which simplifies to:

$$\begin{aligned} 0 &= -\varepsilon_t^s + u + \beta\left(-\phi^w \varepsilon_t^s + u\phi^w\right) \\ & \quad +\beta^2\left[\frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial h_{t+1}^w}\left(-\phi^w \varepsilon_t^s\right) + \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s \phi^w\right] \end{aligned}$$

and can be written as:

$$\varepsilon_t^s\left[1 + \beta\phi^w + \beta^2\phi^w \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial h_{t+1}^w}\right] = \left(1 + \beta\phi^w\right)u + \beta^2\phi^w\phi^s \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{46}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s\left(h_{t-1}^w, v_t^s\right)}{\partial h_{t-1}^w} = \phi^s + \beta\phi^w\phi^s + \beta^2\phi^w\phi^s \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial h_{t+1}^w}, \tag{47}$$

and

$$
\begin{aligned}
& \frac{\partial W^s\left(h_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\
= & \left(1+\beta \phi^w\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon d F_t^s(\varepsilon)+v_t^s T_{2t}^s\right)+\left(1+\beta \phi^w\right) u\left[F_t^s\left(\varepsilon_t^s\right)-v_t^s T_{1t}^s\right] \\
& +\beta^2 \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial h_{t+1}^w} \phi^w\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon d F_t^s(\varepsilon)+v_t^s T_{2t}^s\right) \\
& +\beta^2 \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \phi^s \phi^w\left[F_t^s\left(\varepsilon_t^s\right)-v_t^s T_{1t}^s\right],
\end{aligned}
$$

where $T_{1t}^s \equiv \frac{\partial}{\partial v_t^s}\left[1-F_t^s\left(\varepsilon^s\right)\right]>0$ and $T_{2t}^s \equiv \frac{\partial}{\partial v_t^s} \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon d F_t^s(\varepsilon)>0$. This last expression can hence be written as:

$$
\begin{aligned}
& \frac{\partial W^s\left(h_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\
= & \left(1+\beta \phi^w+\beta^2 \phi^w \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial h_{t+1}^w}\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} \varepsilon d F_t^s(\varepsilon)+v_t^s T_{2t}^s\right) \\
& +\left(\left(1+\beta \phi^w\right) u+\beta^2 \phi^s \phi^w \frac{\partial W^s\left(h_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\right)\left[F_t^s\left(\varepsilon_t^s\right)-v_t^s T_{1t}^s\right]
\end{aligned} \tag{48}
$$

In steady state, (47) and (48) become

$$
\frac{\partial W^s\left(h^w, v^s\right)}{\partial h^w}=\frac{\phi^s\left(1+\beta \phi^w\right)}{1-\beta^2 \phi^w \phi^s}, \tag{49}
$$

and

$$
\begin{aligned}
& \frac{\partial W^s\left(h^w, v^s\right)}{\partial v^s}\left(1-\beta^2 \phi^s \phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]\right) \\
= & \left(1+\beta \phi^w+\beta^2 \phi^w \frac{\phi^s\left(1+\beta \phi^w\right)}{1-\beta^2 \phi^w \phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} \varepsilon d F^s(\varepsilon)+v^s T_2^s\right) \\
& +\left(1+\beta \phi^w\right) u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right].
\end{aligned} \tag{50}
$$

Substituting into the FOC (46),

$$
\begin{aligned}
\varepsilon^s \frac{1+\beta \phi^w}{1-\beta^2 \phi^w \phi^s} = & \left(1+\beta \phi^w\right) u \\
& +\beta^2 \phi^w \phi^s \frac{\left(1+\beta \phi^w\right) u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]+\frac{1+\beta \phi^w}{1-\beta^2 \phi^w \phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} \varepsilon d F^s(\varepsilon)+v^s T_2^s\right)}{1-\beta^2 \phi^s \phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]}
\end{aligned}
$$

which simplifies to (33).

7.4 Model with unobservable value

Assume that the seller does not observe ε. As shown by Samuelson (1984) in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: the seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta \in [0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. Broadly speaking, we can interpret θ as the "market power" of the seller. The setup of the model implies that the buyer accepts any offer p^{sv} if $H^s(\varepsilon) - p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i = b, v$:

$$
\begin{aligned}
S_v^{si} &\equiv p^{si} - (u + \beta V^w), && (51)\\
S_b^{si}(\varepsilon) &\equiv H^s(\varepsilon) - p^{si} - \beta B^w. && (52)
\end{aligned}
$$

Note that the definition of S_v^{si} implies that

$$
p^{sv} = S_v^{sv} + p^{sb} \tag{53}
$$

i.e. price is higher when the seller is making an offer.

Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i = b, v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i = b, v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$
H^s\left(\varepsilon^{si}\right) - p^{si} = \beta B^w. \tag{54}
$$

$1 - F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (4), the response of the reservation quality ε^{si} to a change in price is given by:

$$
\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1 - \beta^2 \phi^w \phi^s}{1 + \beta \phi^w}. \tag{55}
$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$
S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta\phi^w}{1 - \beta^2\phi^w\phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{56}
$$

7.4.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_p \left\{ \left[1 - F^s\left(\varepsilon^{sv}\right)\right]\left[p - \beta V^w - u\right] \right\}$$

The optimal price p^{sv} solves

$$\left[1 - F^s\left(\varepsilon^{sv}\right)\right] - \left[p - \beta V^w - u\right] f^s\left(\varepsilon^{sv}\right) \frac{\partial \varepsilon^{sv}}{\partial p^s} = 0. \tag{57}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv} - \beta V^w - u}{p^{sv}}}_{\text{mark-up}} = \underbrace{\left[\frac{p^{sv} f^s\left(\varepsilon^{sv}\right) \frac{\partial \varepsilon^s}{\partial p^s}}{1 - F^s\left(\varepsilon^{sv}\right)}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1 - F^s\left(\varepsilon^s\right)$). The optimal decisions of the buyer (55) and the seller (57) together imply:

$$S_v^{sv} = \frac{1 - F^s\left(\varepsilon^{sv}\right)}{f^s\left(\varepsilon^{sv}\right)} \frac{1 + \beta \phi^w}{1 - \beta^2 \phi^w \phi^s}. \tag{58}$$

Equation (58) says that the surplus to a seller generated by the transaction is higher when $\frac{1 - F^s(\varepsilon^{sv})}{f^s(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.[38]

Given the price-setting mechanism, in equilibrium, the value of a vacant house to its seller is:

$$V^s = u + \beta V^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}. \tag{59}$$

Solving out V^s explicitly,

$$V^s = \frac{u}{1 - \beta} + \theta \frac{\left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1 - \beta^2}, \tag{60}$$

[38] When f is normal, $(1 - F)/f$ is also called the Mills ratio, which is proportional to the area of the tail of a frequency curve.

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv} = \frac{u}{1-\beta} + \theta \frac{\left[1-\beta^2 F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta\left[1-F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{61}$$

7.4.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_v^{sb} = 0 \Leftrightarrow p^{sb} = u + \beta V^w$$

Using the value function V^w from (60), the price offered by the buyer is:

$$p^{sb} = \frac{u}{1-\beta} + \theta \frac{\beta^2\left[1-F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta\left[1-F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{62}$$

The buyer's value function is:

$$\begin{aligned} B^s &= \beta B^w + \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\ &\quad + (1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right], \end{aligned} \tag{63}$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$\begin{aligned} B^s &= \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\ &\quad + \beta\left\{\theta\left(1-F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S_b^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1-F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S_b^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}. \end{aligned} \tag{64}$$

7.4.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i = v, b$, satisfies

$$H^s\left(\varepsilon^{si}\right) = S_v^{si} + u + V^w + \beta B^w, \tag{65}$$

which equates the housing value of a marginal owner in season s, $H^s(\varepsilon^s)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller ($\beta V^w + u$). Using (4), ε^{si} solves:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} = S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w+V^w\right) - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s+B^s\right). \tag{66}$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (60) and (64):

$$\begin{aligned}
& B^s + V^s \\
= \; & \frac{u}{1-\beta} + \\
& \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]E^s\left[S^{sv}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{sv}\right] + (1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right]E^s\left[S^{sb}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{sb}\right] + \\
& \beta\left\{\theta\left(1-F^w\left(\varepsilon^{sv}\right)\right)E^w\left[S^{wv}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{wv}\right] + (1-\theta)\left[1-F^w\left(\varepsilon^{sb}\right)\right]E^w\left[S^{wb}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{wb}\right]\right\},
\end{aligned} \tag{67}$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon) + S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (66) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv}-\varepsilon^{sb}\right) = S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv}, p^{jb}, \varepsilon^{jv}, \varepsilon^{jb}\right)$ in season $j = s, w$ through two channels, as shown in (61), (62), and(66)): the conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j\left(\varepsilon^{jv}\right)}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1-F^j\left(\varepsilon^{ji}\right)\right)E^j\left[\varepsilon-\varepsilon^{ji}\mid\varepsilon>\varepsilon^{ji}\right]$, $i = b, v$. As shown in (58), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (61) and (62). Similarly as shown in (56), a higher expected surplus quality above ε^{jv} (follows from (3)) implies a higher expected surplus to the buyer $\left(1-F^j\left(\varepsilon^{ji}\right)\right)E^s\left[S_b^{si}(\varepsilon)\mid\varepsilon\geqslant\varepsilon^{si}\right]$, $i = b, v$. These two channels affect V^j and B^j in (60) and (64), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (18).

7.4.4 Stock of vacant houses

In any season s, the average probability that a transaction goes through is $\left\{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] + (1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$\begin{aligned} v^s &= (1-\phi^s)\left\{v^w\left[\theta\left(1-F^w\left(\varepsilon^{wv}\right)\right)+(1-\theta)\left(1-F^w\left(\varepsilon^{wb}\right)\right)\right]+1-v^w\right\} \\ &\quad +v^w\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = v^w\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s, \tag{68}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv},\varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv},p^{sb},p^{wv},p^{wb},B^s+V^s,B^w+V^w,\varepsilon^{sv},\varepsilon^{sb},\varepsilon^{wv},\varepsilon^{wb},v^s,v^w\right)$ that jointly satisfies equations (61),(64),(66), (67) and (68), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j=s,w$, derived as in (58), and (56).

Using (68), the stock of vacant houses in season s is given by:

$$v^s = \frac{(1-\phi^w)\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s}{1-\phi^w\phi^s\left\{\theta F^s\left(\varepsilon^{sv}\right)+(1-\theta)F^s\left(\varepsilon^{sb}\right)\right\}\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}}. \tag{69}$$

Given $1-\phi^s>1-\phi^w$, as in the observable case, the equilibrium $v^s>v^w$.

7.4.5 Seasonality in Prices

Let

$$p^s \equiv \frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]p^{sv}+(1-\theta)p^{sb}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

be the average price observed in season s. Given $p^{sv}=S_v^{sv}+p^{sb}$, we can rewrite it as

$$p^s = p^{sb}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

using (62)

$$\begin{aligned} p^s &= \frac{u}{1-\beta}+\theta\frac{\beta^2\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{1-\theta F^s\left(\varepsilon^{sv}\right)} \\ &= \frac{u}{1-\beta}+\theta\left(\frac{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\beta^2+1-\beta^2}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}\right)\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\frac{\theta\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2} \end{aligned}$$

so finally,

$$p^s = \frac{u}{1-\beta}+\theta\left\{\frac{\left[1-\theta\beta^2F^s\left(\varepsilon^{sv}\right)\right]\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}+\frac{\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}\right\}. \tag{70}$$

Since the flow value u is a-seasonal, housing prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. As in Result 1, *when sellers have some "market power" ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ.* To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (58), is seasonal. Given $v^s > v^w$ and Assumption 2, the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv} > S_v^{wv}$. Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ. Since (70) holds independently from the steady state equation for v^s and v^w, Result 1 holds independently of what drives $v^s > v^w$. Finally, the effect of the flow-value u on the seasonality of prices is the same as Result 2.

7.4.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta \left(1 - F^w \left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1 - F^w \left(\varepsilon^{wb}\right)\right)\right]. \tag{71}$$

(An isomorphic expression holds for Q^w). A bigger stock of vacancies in the summer, $v^s > v^w$, tends to increase transactions in the summer. On the other hand, a relatively higher reservation quality in the hot season, $\varepsilon^{si} > \varepsilon^{wi}$, $i = b, v$, tends to decrease the number of transactions in the summer. As shown in (66), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. We have already shown that $S_v^{sv} > S_v^{wv}$ because of the thick market effect (Assumption 2). Using (3) and (56), the thick market effect also implies that the expected surplus to the buyer is higher in the hot season, so the expected total surplus is also higher in the hot season. It follows from (67) that $(B^s + V^s) > (B^w + V^w)$. The seasonality of S_v^{sv} implies a higher reservation value ε^{sv} in the hot season s (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). The seasonality in sellers' and buyers' outside options, on the other hand, tends to reduce the cutoff ε^{si} in the hot season for $i = b, v$. This is because the outside option in the hot season s is linked to the sum of values in the winter season: $B^w + V^w$. To see this negative effect more explicitly, rewrite (66) as

$$\begin{aligned} & \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} \\ = \quad & S_v^{si} + u + \frac{\beta\phi^w(1-\beta)(1+\beta\phi^s)}{1-\beta^2\phi^w\phi^s}(V^w+B^w) + \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(V^w+B^w-V^s-B^s), \end{aligned} \tag{72}$$

which makes clear that $(B^s + V^s) > (B^w + V^w)$ has a negative effect on $\varepsilon^{si}/\varepsilon^{wi}$. As in Result 3, transactions are seasonal but the extent of seasonality is decreasing in the seller's market power θ. To see this, note that the outside option for both the buyer and the seller in the hot season is to wait and transact in the cold season. This makes both buyers and sellers less demanding in the hot season, yielding a larger number of transactions. In other words, the "counter-seasonality" in outside options increases the seasonality in transactions. On the other hand, when the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions. Hence, the seasonality of outside options and of the seller's surplus (S_v^{sv}) have opposite effects on the seasonality of reservation quality. The second effect (through S_v^{sv}) is increasing in θ (and disappears when $\theta = 0$). Finally, the effect of the flow value u on the seasonality of transactions is the same as Result 4.

7.4.7 The model without seasons

The value functions for the model without seasonality are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$. for $i = b, v$,

Using (66),

$$\frac{\varepsilon^v}{1-\beta\phi} = S_v^v + u + \frac{\beta\phi}{1-\beta\phi}(1-\beta)(V+B) \tag{73}$$

where S_v^v follows from (58),

$$S_v^v = \frac{1-F(\varepsilon^v)}{f(\varepsilon^v)(1-\beta\phi)}.$$

and $B+V$ from (67),

$$\begin{aligned} & (1-\beta)(B+V) \\ = \quad & u + \frac{\theta[1-F(\varepsilon^v)]}{1-\beta\phi}\left\{[E(\varepsilon-\varepsilon^v \mid \varepsilon > \varepsilon^v)] + \frac{1-F(\varepsilon^v)}{f(\varepsilon^v)}\right\} \\ & + \frac{(1-\theta)\left[1-F\left(\varepsilon^b\right)\right]}{1-\beta\phi}\left[E\left(\varepsilon-\varepsilon^b \mid \varepsilon > \varepsilon^b\right)\right] \end{aligned}$$

substitute into (73), we can jointly solve for ε^v and ε^b. More specifically, rewrite it as

$$\varepsilon^b = \varepsilon^v - \frac{1 - F(\varepsilon^v)}{f(\varepsilon^v)} \tag{74}$$

and

$$\begin{aligned} & \frac{\varepsilon^b}{1-\beta\phi} - \frac{\beta\phi}{1-\beta\phi}(1-\theta)\left[1 - F\left(\varepsilon^b\right)\right]\left[E\left(\varepsilon - \varepsilon^b \mid \varepsilon > \varepsilon^b\right)\right] \\ = \quad & u + \frac{\beta\phi}{1-\beta\phi}\theta\left[1 - F(\varepsilon^v)\right]\left[E(\varepsilon - \varepsilon^v \mid \varepsilon > \varepsilon^v) + \frac{1 - F(\varepsilon^v)}{f(\varepsilon^v)}\right]. \end{aligned} \tag{75}$$

Given f is log-concave, we know $E\left(\varepsilon - \varepsilon^i \mid \varepsilon > \varepsilon^i\right)$ is decreasing in ε^i, $i = b, v$ and $\frac{1-F(\varepsilon^v)}{f(\varepsilon^v)}$ is increasing in ε^v. Therefore, in the $\varepsilon^v - \varepsilon^b$ space, ε^b is increasing in ε^v for (74) and decreasing in ε^v for (75). Therefore the equilibrium is unique given v.

To compare to the Planner's problem, rewrite (75) as

$$\frac{\varepsilon^b}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\left[(1-\theta)\int_{\varepsilon^b} x dF + \theta \int_{\varepsilon^v} x dF\right]}{1 - \beta\phi\left[(1-\theta) F^b + \theta F^v\right]},$$

so there is an additional source of inefficiency when $\theta > 0$ due to the asymmetric information. The positive term S_v^{sv} increases the reservation quality in the decentralized equilibrium with $\theta > 0$ and hence lowers the number of transactions with respect to the efficient (Planner's) outcome. As in the observable case, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. Hence, the number of transactions can be too low or too high, depending ultimately on the shape of the distribution $F(.)$.

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To be Completed

8 Data Sources

For U.K. and U.S. data, see text.

Australia The housing price index comes from the Australia Bureau of Statistics (ABS); it is a weighted average for eight capital cities, available from 1986; the series is based on prices at settlement and are based on data provided to the land titles office; it is not quality adjusted. The CPI (non seasonally adjusted, NSA) also comes from the ABS and is a national index, not available at a disaggregated level; in what follows, for all countries, the price index considered in the analysis corresponds to the national index.

Belgium The housing price index comes from STADIM (*Studies & advies Immobiliën*) and covers Belgium and its three main regions from 1981; the series is based on the average selling prices of small and average single-family houses; apartments are not included; the data come from registered sales, and are not quality adjusted. The CPI (NSA) comes from the National Institute for Statistics.

Denmark The housing price index comes from the Association of Danish Mortgage Banks and corresponds to existing single-family homes (including flats and weekend cottages). The data come from the Land Registry, where all housing transactions are registered; they are not adjusted by quality and start in 1992. The CPI (NSA) comes from *Danmarks Statistik*.

France The housing price index comes from INSEE (National Institute for Statistics and Economic Studies) and corresponds to existing single-family homes. The data are not quality adjusted and start in 1994. The index covers all regions, and comes also disaggregated into 4 regions. The CPI (NSA) comes from the same source.

Ireland The housing price index comes from *Permanent TSB*, which accounts for about 20 percent of residential mortgage loans in the country, starting in 1996; the index is adjusted by the size of the property, dwelling type (detached, semi-detached, terrace, or apartment), and heating system. The number of transactions (loans) comes from the same source. The CPI (NSA) comes from the Central Statistical Office in Ireland.

Netherlands The housing price index comes from the Dutch Land Registry; it is a repeat-sale index, starting in 1993. The CPI (NSA) comes from the CBS (Statistics Netherlands).

New Zealand The housing price index comes from the Reserve Bank of New Zealand, starts in 1968, and is not adjusted by quality; the CPI (NSA) comes from the same source.

Norway The housing price index comes from Statistics Norway, starting in 1992; the data are not adjusted by quality as meticulously as in the U.K., however, the properties considered need to satisfy a set of broadly defined) characteristics to be included in the index; the CPI (NSA) comes from the same source.

South Africa The housing price index comes from ABSA, a commercial bank that covers around 53 percent of the mortgage market in South Africa. The data are recorded at the application stage of the mortgage lending process and the series starts in 1975. There is no quality adjustment, although the properties considered need to satisfy a set of (broadly defined) characteristics to be included in the index. The CPI (NSA) comes from Statistics South Africa.

Sweden The housing price index comes from *Statistika Centralbyrån*; the data correspond to one and two-dwelling properties and are not quality-adjusted; the series starts in 1986; data on transactions and CPI (NSA) come from the same source.