Household Leverage*

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Abstract

I propose a life-cycle model where a finitely lived risk averse agent finances his housing investment by a fixed rate mortgage contract. The agent is exposed to stochastic house price and stochastic labor income. After signing the mortgage contract, while increasing his equity share in the house through amortization, the agent optimally determines the time at which defaulting and moving into the rental market provides a greater expected continuation utility than continuing the mortgage payments. Lenders market efficiently price the mortgage, charging a default premium to compensate themselves for expected losses due to default on the mortgage. Three main results arise. First, the model predicts that the default premium is increasing in the volatility of house prices and decreasing in the down payment. Second, in order to justify a highly leveraged position in the house purchase whose price is highly volatile, the rent premium has to be substantial to make the agent *ex ante* indifferent between owning with a mortgage and renting the same house. Third, in the presence of borrowing constraints, a negative labor income shock induces the agent to early exercise his default option with a higher level of net wealth.

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1 Introduction

From a life-cycle point of view, an agent is willing to smooth housing consumption over time taking on mortgage when his current income and his available liquid assets are insufficient to afford the desired house; at the same time, he has income flow and expects to be able to pay the house over time. The long term durability of housing makes its purchase price relatively high, with the result being that most purchases imply that a relevant mortgage coupon has to be paid over the course of several years. Also, this mortgage contract necessarily has a risk of default which must be priced in the coupon. As a result, the cost of mortgage has a major impact on the owning decision and hence the household's net worth.

I argue that understanding the consequences of taking costly leverage is key to understanding the choice of owning a house. I propose a life-cycle model where a finitely lived risk averse agent finances his housing investment by a fixed rate mortgage contract. The agent is exposed to two sources of uncertainty: stochastic house price and labor income. After signing the mortgage contract, while increasing his equity share in the house through amortization, the agent optimally determines the time at which defaulting and moving into the rental market provides a greater expected continuation utility than continuing the mortgage payments. Lenders market efficiently price the mortgage, charging a default premium to compensate themselves for expected losses due to default on the mortgage. The default premium mainly depends on the down payment the agent provides, the agent's propensity to default and the dead weight losses lenders face in case of default.

Campbell and Cocco (2003) is the first paper that explores how owning a house is affected by a mortgage contract.[1] They assume that buying a house with a mortgage is strictly

[1]Campbell and Cocco (2003) is the first paper of the current literature on household risk management that extends the previous literature of life-cycle consumption and saving in the presence of liquidity constraints, Zeldes (1989), Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002), introducing housing consumption and mortgage contracts. Cocco (2005), Yao and Zhang (2005) and Van Hemert (2008) also contributed to the household risk management literature. Yao and Zhang (2005) and Van Hemert (2008) study the agent's choice of owning with a mortgage versus renting. Under their conditions, if the agent can provide the downpayment and his labor income is sufficiently high to pay the future mortgage payments, then owning is preferred. However, these results depend on two assumptions: (i) renting includes an arbitrary premium that makes it more expensive than owning per unit of housing services; (ii) the mortgage interest

preferred to renting the same house and that the agent can choose between a fixed rate mortgage (FRM) or an adjustable rate mortgage (ARM). Both mortgage contracts, however, have default premia that are not fairly priced. Furthermore, they assume that the agent must move into the rental market if he chooses to default on the mortgage and there he faces an exogenously determined rent premium. The default timing is endogenously determined and is affected by the default premium and the rent premium. A lower (higher) rent premium increases (decreases) the probability of default, but the exogenous default premia on the FRM and ARM do not adjust accordingly.[2]

Following Campbell and Cocco (2003), I assume that the agent changes his status from owning to renting when he defaults. The agent rents the same house paying continuously a constant fraction of the house price. This is the cost of ownership that incorporates a rent premium. Because the agents propensity to default is higher when the rent premium is lower, the lender *ex ante* charges a higher default premium. Instead of imposing an exogenous rent premium as in Campbell and Cocco (2003), I assume that the cost of ownership is determined such that the agent is indifferent *ex ante* between owning with a mortgage and renting the same house. As a result, the rent premium depends on the main parameters of the model. In reality, the rent premium is determined in equilibrium by local supply and demand conditions and it might not be specific to the agent's characteristics. Despite this critique, the indifference rent premium provides an important indication of when the agent is willing to own with a mortgage, given the initial loan to house value ratio. Moreover, arbitrarily setting the rent premium, as shown in this paper, crucially affects the model's predictions with respect to the default decision and the owning versus renting decision.

Three main contributions and results arise from this paper. First, the down payment and the default premium are endogenously related, while in Campbell and Cocco (2003) there is an apparent disconnect between the down payment and the default premium, because

rate contains no default premium. The effect is that the model's results may be significantly biased against renting.

[2]This limitation is acknowledged by the two authors.

they are given. In my model, the down payment and the default premium are inversely related: the higher the down payment, the lower the default premium the lender *ex ante* charges. One main implication is that the agent has incentive to provide a significant down payment to decrease the default premium. However, the agent has no incentive to provide all his initial savings as down payment. Because the agent is willing to consume numeraire consumption and has to pay the mortgage coupon, holding a risk free asset position relaxes the "cash-on-hand" constraint if labor income switches from 'Good' state to 'Bad' state.

Second, in order to justify a highly leveraged position in the house purchase whose price is highly volatile, the rent premium has to be substantial to make the agent *ex ante* indifferent between owning with a mortgage and renting the same house. The model sheds light on the choice between owning and renting. In the literature, Sinai and Souleles (2005) address the question of under which circumstances an agent is better off owning than renting. They identify a hedging benefit to owning if the agent has a long horizon and is exposed to volatile rents. Their claim that owning is a natural hedge against rent fluctuations is intuitive, but is built on the assumption that the agent does not need to take a mortgage, because he is endowed with enough wealth to pay the full purchase price. They abstract from how the agent finances the house purchase.[3] Thus, they rule out the effect of taking costly leverage on owning a house, significantly biasing their results in favor of owning over renting. However, the choice between owning or renting is a complicated one and I abstract from considering some factors that might affect the choice between owning and renting, such as tax law and individuals' mobility. However, while tax laws may offer incentives to owning in the United States, in many countries the tax treatment of renting and owning is not financially relevant, or may favor renting. Therefore, I concentrate on the noninstitutional economic aspects of

[3]Their argument is built on two other main assumptions as well. The first assumption is that they rule out fixed-price, long-term, lease contracts (whereby the tenant could lock in fixed rental payments for the duration of his tenancy). The second assumption is that the agent does not earn any risky labor income which might be positively correlated with rent fluctuations. In a partial equilibrium framework, Ortalo-Magné and Rady (2002) analyze an agent's tenure choice under uncertainty of income and house prices. They show that renting becomes relatively more attractive than homeownership as the covariance between labor income and house price increases. Davidoff (2006) shows that an agent with mean-variance preferences optimally purchases less housing as the covariance between labor income and housing prices rises.

the problem. The choice between owning and renting might be influenced by timing and frequency of the agent's move. Furthermore, transactions costs associated with selling and buying the house would make renting more attractive than owning.

A third major contribution of my model is that the parameters of the labor income process directly affect the default premium, which is endogenously determined. Instead, in the current mortgage pricing literature, the occurrence of default is mainly driven by the house price process, and is triggered when the owner has a negative equity position in the house. Mortgage termination is treated as the optimal response of a rational risk neutral owner who chooses when to default.[4] However, empirical analysis suggests that trigger events, such as divorce, loss of a job, or accidents affect default behavior (see Vandell (1995)). As highlighted by Campbell and Cocco (2003), the limit of the current mortgage pricing literature is to not recognize the role played by labor income and borrowing constrains in determining default. In fact, the implicit assumption is that the agent could borrow immediately and costlessly from other sources if labor income falls and is not sufficient to make the required mortgage payment. In my model, I account for the agent's ability to pay the mortgage coupon in pricing the mortgage. Despite the fact that my model is too stylized to capture all of the features that many mortgages offer in the market place,[5] it captures some important aspects. Default may be unlikely if the agent has substantial equity in a home and loses his job, because the agent would simply sell the house or borrow on a home equity line and transform the equity into income. However, if the agent has a slightly negative equity position and a negative shock on income occurs, it is optimal for the agent to early exercise the default option. In the presence of borrowing constraints, a negative labor income shock induces the agent to default with a higher level of net wealth and then to move into the rental market with a higher *buffer-stock* of savings.

The paper is organized as follows. Section 2 describes the agent's optimization problem.

[4] The agent is exposed to two sources of uncertainty: interest rates and house prices. Under this approach, standard contingent claims techniques are used to calculate the agent's optimal exercise policy, (see Kau and Keenan (1995) and Downing et al. (2005))

[5] Moreover, I do not include stochastic interest rates in my analysis.

Subsection 2.1 derives the renter's problem. Subsection 2.2 derives the owner's problem in the presence of a mortgage, describing how default is endogenously determined. Subsection 2.2.2 provides the mortgage valuation. Section 3 provides a brief overview of the numerical schemes implemented to solve the agent's problem and the mortgage valuation. Section 4 describes the calibration of the model parameters, while Section 5 presents the numerical results of the model. Finally, Section 6 concludes.

2 Statement of the problem

Consider an agent whose preferences are represented by a constant relative risk aversion utility function (CRRA), and depend on housing services, H, and other goods, C_t:

$$u(C_t, H) = \frac{(C_t^{\beta} H^{1-\beta})^{1-\gamma}}{1-\gamma}, \tag{1}$$

where β measures the relative importance of numeraire consumption versus housing services, $\beta \in (0, 1)$, and γ is the curvature parameter and the coefficient of relative risk aversion, $\gamma > 0$.

The agent is unconstrained with respect to the owning versus renting choice. He chooses the status that maximizes his expected utility at time zero. A housing stock H provides a constant flow of housing services to the agent. I make the assumption that (i) the housing stock H does not depreciate; (ii) the flow of housing services is δH_t where $\delta = 1$ for convenience. As in Campbell and Cocco (2003), the house size H at time zero is given and the agent does not change the house size regardless of the path of his income and wealth. Thus, I ignore the possibility that the agent can adjust the level of housing services and move to a larger or smaller house.[6] The agent chooses consumption of all goods other than housing,

[6] I make this assumption to make the model numerically tractable. If the agent is allowed to adjust the housing services, the amount he borrows to buy the bigger or smaller house, might vary. In my model, the default premium directly depends on the house size and the mortgage amount. Therefore, I would need to identify the default premium for different combinations of housing and mortgage amounts and to verify whether the agent could get a higher indirect utility with a different combination of housing and mortgage amount. However, I could identify the optimal house size H^* at time 0. Identifying H^* requires running the numerical algorithm for different levels of housing stock H and verifying at which level of the housing stock,

C_t, at any time t.

The agent has a finite horizon and discounts the utility function at the constant rate of time preference ρ:

$$\int_0^T e^{-\rho t} \frac{(C_t^\beta H^{1-\beta})^{1-\gamma}}{1-\gamma} \mathrm{d}t, \tag{2}$$

where T is the terminal date. The agent also derives utility from terminal wealth W_T, which can be interpreted as bequest motive:

$$V(W,T) = \frac{W_T^{1-\gamma}}{1-\gamma}. \tag{3}$$

The agent is exposed to two sources of uncertainty: (i) the stochastic flow of labor income, L_t; and (ii) the stochastic house price per square foot, P_t.

To examine the impact of labor uncertainty in the simplest possible environment, I assume that the labor income $L_t \geq 0$ can only take two values: L_G and L_B with $L_G > L_B > 0$ (G - 'Good',B - 'Bad'). L_G represents the normal level of income, while L_B is the level of income after a negative shock (i.e. unemployment, divorce, etc..). In addition, I presume that L_t is observable and that its transition probability follows a Poisson law, such that L_t is a two-state Markov chain. Let θ denote the rate of leaving state i and l_i denote the time to leave state i. Within the present model, the exponential law holds:

$$P(l_i > t) = \mathrm{e}^{-\theta_i t}, \quad \text{where} \quad i = B, G, \tag{4}$$

and there is a probability $\theta_i \Delta t$ that the value of the labor income L_t changes from L_i to L_j during an innitesimal time interval Δt. In addition, the expected duration of regime B is $1/\theta_B$ and the average fraction of time spent in that regime is $\theta_G/(\theta_B + \theta_G)$.

The house price is given and is governed by a geometric Brownian motion:

$$\mathrm{d}P_t = \mu_P P_t \mathrm{d}t + \sigma_P P_t \mathrm{d}Z_t, \tag{5}$$

H, the agent gets the highest indirect utility. This extension is left as further development of the model.

where μ_P is the rate of real home price appreciation, σ_P is the volatility parameter and Z_t is a Wiener process.

The wealth of the agent at time zero, W_0, corresponds to the dollar amount in the risk free asset, A_0, and does not take into account the present value of the future uncertain income stream. The risk free asset pays a constant interest rate r and its dynamic is governed by

$$\mathrm{d}A_t = rA_t\mathrm{d}t. \tag{6}$$

If the agent chooses to rent, he continuously pays a rent that is a constant fraction α, the rental cost, of the current house value, HP_t, exposing himself to the rent fluctuations. The rental cost α is defined as the sum of the user cost, the cost of consuming the house, and the constant rent premium λ. I set the rent premium such that the agent is indifferent *ex ante* as to owning with a mortgage or renting at time zero.

If the agent chooses to own, he can finance the house purchase at time zero, taking a mortgage F_0. By assumption, the agent signs a mortgage contract with a level of income of L_G at date 0. The face value of the mortgage at date 0 is $F_0 = (1-\omega)HP_0$, where ω is the agent's equity share in the house, $\omega \in (0, 1]$. The mortgage is a fully amortizing fixed rate contract

$$F_0 = \int_0^T Ne^{-is}\mathrm{d}s, \tag{7}$$

where T is the term to maturity, N is the mortgage coupon and i is the mortgage coupon rate. As a result, the total payout is

$$N = \frac{iF_0}{1-e^{-iT}} \tag{8}$$

and the mortgage balance at time t is

$$F_t = \int_t^T Ne^{-i(s-t)}\mathrm{d}s = \frac{N}{i}(1-e^{-i(T-t)}) = F_0\left(\frac{1-e^{-i(T-t)}}{1-e^{-iT}}\right). \tag{9}$$

Let re-express the total payout, N, as

$$N = \frac{iF_t}{(1 - e^{-i(T-t)})} = \psi(i,t,T)F_t,$$

where $\psi(i,t,T) = \frac{i}{1-e^{-i(T-t)}}$. The dynamics of the mortgage balance, F_t, is

$$\mathrm{d}F_t = -Ne^{-i(T-t)}\mathrm{d}t = -\frac{iF_t}{(1 - e^{-i(T-t)})}e^{-i(T-t)}\mathrm{d}t = -\phi(i,t,T)F_t\mathrm{d}t, \tag{10}$$

where $\phi(i,t,T) = ie^{-i(T-t)}/(1 - e^{-i(T-t)})$. Because the mortgage is a fully amortizing contract, the agent gradually increases his equity share in the house due to amortization. Therefore, the mortgage balance at time T is null. By assumption, the interest component of the coupon is not tax deductible.[7]

To model the occurrence of default, I assume that default is triggered endogenously at the stopping time τ. The agent optimally determines at which point to default based on whether defaulting and moving into the rental market or continuing to pay mortgage coupons provides him the greater expected continuation utility. The agent defaults with a negative equity position in the house. However, he optimally defaults with positive wealth, because his preferences are represented by a constant relative risk aversion utility function (CRRA).

At default, the agent and the lenders face several consequences. The agent loses the house and incurs the additional cost, ϵHP_τ, due to transaction and legal costs.[8] However, he disregards the mortgage balance F_τ. Thus, the agent moves into the rental market with the wealth $W_{i,\tau} - (1+\epsilon)HP_\tau + F_\tau$, where $i = B, G$.

On the other side, the lender incurs a loss proportional to the house value at default

[7] I disregard tax considerations as they are too country-specific. In the United States, the homeownership is substantially subsidized. A owner can deduct the mortgage interest payment from the income tax decreasing the after-tax cost of owning with a mortgage. Then, the owner can deduct the property taxes and the points charged when the mortgage contract is signed. Moreover, the most fundamental subsidy is that homeowners are not taxed on the implicit rent they receive from their housing investment. Jaffee and Quigley (2006) provide a review and taxonomy of U.S. federal housing programs, including direct public expenditures on housing and indirect expenditures through the tax system.

[8] The dead weight loss associated with default is also a *stigma*: the agent is forced into the rental market and he loses access to the mortgage market.

ψHP_τ, due to missing coupon payments, delay and legal costs associated with the possession of the collateral. The default premium κ compensates the lenders for the potential consequences of default. In my model, lenders have rational expectations and take the optimal homeowner default policy as given. Hence, the default premium κ is set such that the lenders are indifferent as to the return r on the risk free asset A_t or the return i on the risky mortgage F_t.

In the next sections, I separately analyze the renter's and the owner's problem (Subsection 2.1 and 2.2 respectively). While the notation $V_i^r(W_{i,t}, P_t, t)$ is used to indicate the indirect utility when the agent rents, where $i = B, G$, $V_i^{om}(W_{i,t}, P_t, t)$ is used to indicate the indirect utility when the agent owns with a mortgage. The mortgage valuation is discussed in the Subsection 2.2.2. The notation $M_i(W_{i,t}, P_t, t)$ is used to indicate the mortgage fair value.

The goal is to solve the owning with a mortgage versus renting problem to identify the default premium κ and the rent premium λ such that (i) the agent is indifferent *ex ante* as to owning with a mortgage or renting the house HP_0 in the 'Good' state; (ii) the mortgage fair value equals the mortgage face value. The two premia depend on the level of wealth W_0, the labor income L_G and the house price P_0. Specifically, the following conditions have to be satisfied:

$$V_G^r(W_0, P_0, 0) = V_G^{om}(W_0, P_0, 0), \tag{11}$$

$$M_G(W_0, P_0, 0) = F_0, \tag{12}$$

in order to identify the default premium κ and the rent premium λ.

In order to make the model numerically tractable, I make two simplifying assumptions. The first is that, as in Campbell and Cocco (2003), the owner can change his status, from owning to renting, by defaulting on the mortgage, while the renter can never change his status. The second major assumption is that while the default and rent premia are determined endogenously at the mortgage's origination and rent's signature, then they remain constant throughout the agent's horizon.[9] The second assumption precludes the possibility that, after signing the mortgage contract, the agent or the lender might renegotiate the mortgage coupon varying the default premium. It also precludes the possibility that, after signing the rental contract, a different rental cost might be charged varying the rent premium. While the assumption is restrictive, it may be realistic if the fixed costs of negotiating contracts limit the potential to renegotiate.

2.1 Renting

If the agent rents a house of size H, I assume that he continuously pays a fraction α, rental cost, of the current house value HP_t. The agent's wealth $W_{i,t}$ corresponds to the risk free asset A_t and the associated dynamic is:

$$\mathrm{d}W_{i,t} = \mathrm{d}A_t + (L_i - C_{i,t} - \alpha H P_t)\mathrm{d}t, \quad \text{where} \quad i = B, G. \tag{13}$$

Given $W_0 \geq 0$, the renter's problem is to choose the numeraire consumption, $C_{i,t}$, to solve:

$$V_i^r(W_0, P_0, 0) = \sup_{C_{i,t}} \mathrm{E}\left[\int_0^T e^{-\rho t}\frac{(C_{i,t}^{\beta}H^{1-\beta})^{1-\gamma}}{1-\gamma}\mathrm{d}t + e^{-\rho T}\frac{W_T^{1-\gamma}}{1-\gamma}\right], \quad \text{where} \quad i = B, G. \tag{14}$$

The associated system Hamilton-Jacobi-Bellman equations is the following:

$$\rho V_B^r = \sup_{C_{B,t}}\left\{\frac{(C_{B,t}^{\beta}H^{1-\beta})^{1-\gamma}}{1-\gamma} + \mathcal{D}V_B^r + \theta_B(V_G^r - V_B^r)\right\}, \tag{15}$$

$$\rho V_G^r = \sup_{C_{G,t}}\left\{\frac{(C_{G,t}^{\beta}H^{1-\beta})^{1-\gamma}}{1-\gamma} + \mathcal{D}V_G^r + \theta_G(V_B^r - V_G^r)\right\}, \tag{16}$$

[9] Campbell and Cocco (2003) assume an exogenous constant default and rent premia. Yao and Zhang (2005) and Van Hemert (2008) assume an exogenous constant rent premium and a zero valued default premium.

where

$$\mathcal{D}V_i^r = (rW_{i,t}+L_i-C_{i,t}-\alpha HP_t)V_{i,W}^r+\mu_P P_t V_{i,P}^r+V_{i,t}^r+\frac{\sigma_P^2}{2}P_t^2 V_{i,PP}^r, \quad \text{where} \quad i=B,G, \tag{17}$$

and $\theta_i(V_j^r - V_i^r)$ reflects the impact of the labor income on the value functions. This term is the product of the instantaneous probability of a regime shift and the change in the value function occurring after a regime shift.

Solving for the first order condition, I obtain the optimal numeraire consumption:

$$C_{i,t}^r = \left(\frac{H^{-(1-\beta)(1-\gamma)}V_{i,W}^r}{\beta}\right)^{1/(\beta(1-\gamma)-1)}, \quad \text{where} \quad i=B,G. \tag{18}$$

The optimal numeraire consumption, $C_{i,t}^r$, is subject to the constraint that the wealth position must be non negative at any time t: $W_{i,t} \geq 0$. If the agent meets the constraint with equality at some date t, he is forced to leave the house at that date, and is left with zero housing and numeraire consumption from then on.

The nature and properties of the solution in the presence of the liquidity constraint need to be numerically investigated. This is also the case for the saving-consumption problem studied in the macroeconomics literature of *buffer-stock* models (see Zeldes (1989), Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002)) and in the HRM literature.

I assume that the rental cost has two components: $\alpha = (r-\mu_P)+\lambda$. The first component, $r-\mu_P$, is the user cost of the house.[10] The second component, λ, is the rent premium.

[10] I derive the user cost, $r-\mu_P$, assuming that the landlord (i) is risk neutral, (ii) is endowed with enough wealth to afford the house purchase, (iii) is only exposed to the house price fluctuations. Hence, equilibrium in the housing market occurs when the expected cost of owning a house equals that of renting. Equating the *ex ante* utilities of renting and owning:

$$\underbrace{W_0 - \mathrm{E}\left[\int_0^T e^{-r(T-s)}\tilde{\alpha}HP_s \mathrm{d}s\right]}_{\text{Renting}} = \underbrace{W_0 - HP_0 + \mathrm{E}\left[e^{-rT}HP_T\right]}_{\text{Owning}}, \tag{19}$$

I obtain that $\tilde{\alpha} = (r-\mu_P)$, where r is the cost of foregone interest that the landlord could have earned on the risk free asset, A_t. It is adjusted to include the offsetting benefit given by the expected capital gain, μ_P, on the house value, HP_t. Alternatively, the user cost can be derived assuming that the current house value, HP_t, is equal to the expected present value of the future rents. The user cost is determined imposing that

Following Sinai and Souleles (2005), I set the rent premium, λ, such that the agent is indifferent *ex ante* to owning with a mortgage or renting.[11]

In order to derive it, I could explicitly model the landlord problem, assuming that the landlord is a risk averse agent who receives the rent as cash flow, who is borrowing from the lender in order to be the owner of the house, who receives a stochastic labor income flow, who is consuming the housing services of another house and so on. This approach would add unnecessary complications to the model, requiring the solution of the landlord's optimal control problem, while not adding to the generality of the problem.

Moreover, setting arbitrarily the rent premium, λ, and the default premium, κ, crucially affects the model's prediction with respect to the owning versus renting decision and the default decision. To illustrate this point, I provide evidence representing the indirect utility of owning with a mortgage, $V_G^{om}(W_{G,t}, P_t, t)$ and renting, $V_G^{r}(W_{G,t}, P_t, t)$ at time zero in 'God' state. Figure 1 plots the indirect utilities as function of the wealth state variable, $W_{G,t}$.

Insert Figure 1 here

The parameters I assume are reported in Table 1 and will be discussed in Section 4. For convenience, I assume that the agent can own or rent a house which has a price, P_0, of 1, and a size, H, of 10 square feet. I start by following Yao and Zhang (2005) and Van Hemert (2008) and I simply set the default premium, κ, to zero implying that the agent pays a risk free mortgage coupon rate. To illustrate the implications of the rent premium, λ, Figure 1

the expected value of capital gain on house value and rent is equal to the risk free rate over the interval $\mathrm{d}t$:

$$\mathrm{E}\left[\frac{\mathrm{d}(HP_t)}{HP_t} + \frac{\alpha HP_t \mathrm{d}t}{HP_t}\right] = r\mathrm{d}t.$$

[11] Alternatively, Campbell and Cocco (2003) identify the rent premium as the remuneration that covers the average realized maintenance costs: tenants have no incentive to look after the property so home maintenance is expensive. Henderson and Ioannides (1983) derive a full equilibrium model of the choice between owning and renting where the tenant may not properly care for the property. Because of limited information, landlords cannot distinguish *ex ante* good tenants from bad tenants. Under such circumstances, landlords charge rents which reflect average maintenance costs across potential tenants. It follows that tenants who have a predisposition to maintain their home pay rents which exceed the marginal costs they impose on landlords. These tenants have the incentive to own the house avoiding externalities in the rental market.

plots the renter's indirect utility for a null rent premium, $\lambda = 0$ percent, and for a positive one, $\lambda = 4$ percent. As the plot highlights, the λ parameter is capable of generating large differences in terms of indirect utility. Renting dominates owning when the rent premium is zero and the agent has to borrow in order to buy the house. However, if the rent premium is equal to 4 per cent, the agent prefers owning with a mortgage to renting. It is not surprising that a risk averse agent prefers renting to owning when the rent is based only on the user cost of housing services: $\lambda = 0$ percent. In this case, the agent can keep the savings as a *buffer-stock* instead of investing it in the house purchase. As Yao and Zhang (2005) and Van Hemert (2008), I need to increase the rent premium, λ, in order to induce the agent to become an owner. Therefore, exogenously setting the rent premium, λ, and the default premium, κ, can crucially affect the model's predictions.

2.2 Owning

2.2.1 Owner's problem

The dollar wealth amount is:

$$W_t = HP_t + A_t - F_t \tag{20}$$

and the associated dynamic of the wealth is:

$$\begin{aligned} \mathrm{d}W_{i,t} &= H\mathrm{d}P_t + \mathrm{d}A_t - \mathrm{d}F_t + (L_i - C_{i,t} - N)\mathrm{d}t \\ &= [\mu_P HP_t + rA_t + \phi(i,t,T)F_t + L_i - C_{i,t} - N]\mathrm{d}t + HP_t\mathrm{d}Z_t, \quad \text{where} \quad i = B, G, \end{aligned} \tag{21}$$

and $A_t = W_{i,t} - HP_t + F_t$. The wealth dynamic takes two different forms, depending on whether or not the level of labor income is L_B or L_G. The other terms represent respectively the return of housing investment, the return on the risk free asset, the mortgage balance amortization, the labor income flow, the numeraire consumption and the mortgage coupon. The diffusion component represents the house price uncertainty in the homeowner's wealth dynamic; the price uncertainty depends on the current house value, HP_t, and the house price volatility, σ_P.

The owner's problem is:

$$
\begin{aligned}
V_i^{om}(W_0, P_0, 0) = \sup_{C_{i,t},\tau} \mathrm{E}\Bigg[& \int_0^T e^{-\rho t} \frac{(C_{i,t}^{\beta} H^{1-\beta})^{1-\gamma}}{1-\gamma} \mathrm{d}t \\
& + \underbrace{\mathbf{1}_{\{\tau \le T\}} e^{-\rho\tau} V_i^r(W_{i,\tau} - (1+\epsilon)HP_\tau + F_\tau, P_\tau, \tau)}_{\text{Default}} + \mathbf{1}_{\{\tau > T\}} e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma} \Bigg], \\
& \text{where} \quad i = B, G, \qquad (22)
\end{aligned}
$$

and $V_i^r(W_{i,\tau} - (1+\epsilon)HP_\tau + F_\tau, P_\tau, \tau)$ is the indirect utility at default. Two locus of points, $(W_{i,\tau}, P_\tau)$, representing when the agent is *ex ante* indifferent between owning with a mortgage and defaulting, are identified and they are associated with the 'Good' state and 'Bad' state. Because the agent is forced into the rental market, the indirect utility at default is denoted by V_i^r.

The associated system of Hamilton-Jacobi-Bellman equations is:

$$
\rho V_B^{om} = \sup_{C_{B,t}} \left\{ \frac{(C_{B,t}^{\beta} H^{1-\beta})^{1-\gamma}}{1-\gamma} + \mathcal{D}V_B^{om} + \theta_B(V_G^{om} - V_B^{om}) \right\}, \qquad (23)
$$

$$
\rho V_G^{om} = \sup_{C_{G,t}} \left\{ \frac{(C_{G,t}^{\beta} H^{1-\beta})^{1-\gamma}}{1-\gamma} + \mathcal{D}V_G^{om} + \theta_G(V_B^{om} - V_G^{om}) \right\}, \qquad (24)
$$

where

$$
\begin{aligned}
\mathcal{D}V_i^{om} = {} & (\mu_P HP_t + rA_t + \phi(i,t,T)F_t + L_i - C_{i,t} - N)V_{i,W}^{om} + \mu_P P_t V_{i,P}^{om} + V_{i,t}^{om} \\
& + \frac{\sigma_P^2}{2} H^2 P_t^2 V_{i,WW}^{om} + \frac{\sigma_P^2}{2} P_t^2 V_{i,PP}^{om} + \sigma_P^2 H P_t^2 V_{i,WP}^{om} \quad \text{where} \quad i = B, G, \qquad (25)
\end{aligned}
$$

over a region where it is not optimal to default.

Solving for the first order condition, I obtain the optimal numeraire consumption $C_{i,t}$:

$$C_{i,t}^{om} = \left(\frac{H^{-(1-\beta)(1-\gamma)} V_{i,W}^{om}}{\beta} \right)^{1/(\beta(1-\gamma)-1)}, \quad \text{where} \quad i = B, G. \tag{26}$$

2.2.2 Contingent claims evaluation of the mortgage

In this section, I derive the mortgage fair value $M_i(W_{i,t}^*, P_t, t)$, where $i = B, G$. The contract is a fixed rate fully amortizing contract. Paying the mortgage coupon N, the agent gradually increases his equity share in the house value due to amortization. After signing the contract, I assume that the agent or the lender cannot re-negotiate the mortgage. Specifically, the default premium κ is determined at the contract's origination and it is constant.

I assume that the lenders' market has rational expectations and takes the optimal default strategy of the owner as given. The market is also competitive, implying that the mortgage coupon N is set equal to the full mortgage coupon so that the mortgage is issued at par. Moreover, the lender adopts a contingent claims approach to fairly price the mortgage contract. The solution approach is that of option pricing analysis. Traditionally, in the option pricing literature, prices of contingent claims are based on arbitrage arguments. However, such an approach requires assumptions about the liquidity of underlying assets. In the case of mortgage contracts, such arbitrage arguments are questionable. One underlying asset is the house that is subject to substantial transaction costs and the inability to be sold short. The second one is labor income for which contingent claims contracts are not observed in reality for reasons of moral hazard and adverse selection. Alternatively, an equilibrium approach relaxes the tradability assumptions needed for arbitrate pricing, although an appropriate model must be chosen. For simplicity, I assume the risk neutrality, so that all assets are priced to yield an expected rate of return equal to the risk free rate, r.[12] This restrictive assumption can be relaxed by adjusting the drift rates to account for a risk premium in the manner of Cox and Ross (1976).

[12] The assumption is the same made by Grenadier (1996) who proposes a unified framework for determining the equilibrium credit spread on commercial leases subject to default risk.

Consider the instantaneous return on $M_i(W^*_{i,t}, P_t, t)$, where $i = B, G$, over a region in which the mortgage has not yet defaulted. The value of the mortgage depends on $W^*_{i,t}$, which is the owner's wealth accounting for the optimal numeraire consumption $C^{om}_{i,t}$, the owner's labor income, L_i, and the house price, P_t. The lender receives a cash inflow due to the coupon payment of N over the interval dt. The total expected return on M_t per unit of time μ_M is defined as

$$\mathrm{E}\left[\frac{\mathrm{d}M_t}{M_t} + \frac{N\mathrm{d}t}{M_t}\right] = \mu_M \mathrm{d}t. \tag{27}$$

Setting the expected return μ_M to the risk free rate r and simplifying yields the following equilibrium partial differential equation:

$$rM_B = N + \mathcal{D}M_B + \theta_B(M_G - M_B), \tag{28}$$

$$rM_G = N + \mathcal{D}M_G + \theta_G(M_B - M_G), \tag{29}$$

where

$$\begin{aligned}\mathcal{D}M_i = {} & (\mu_P HP_t + rA_t + \phi(i,t,T)F_t + L_i - C_{i,t} - N)M_{i,W} + \mu_P P_t M_{i,P} + M_{i,t} \\ & + \frac{\sigma_P^2}{2}H^2P_t^2 M_{i,WW} + \frac{\sigma_P^2}{2}P_t^2 M_{i,PP} + \sigma_P^2 HP_t^2 M_{i,WP}, \quad \text{where} \quad i = B, G.\end{aligned} \tag{30}$$

The boundary conditions are:

$$M_i(W^*_{i,T}, P_T, T) = 0, \tag{31}$$

$$\lim_{P\to\infty} M_i(W^*_{i,t}, P_t, t) = \tilde{M}(t), \tag{32}$$

where $i = B, G$. The boundary condition (31) is the terminal condition, reflecting the residual mortgage balance at the terminal date T. The boundary condition (32) says that when the house price gets large, default no longer occurs: $\tilde{M}(t)$ is the value of a bond with the same promised cash ows as the mortgage, but with no house price dependence. At default, the boundary condition is:

$$M_i(W^*_{i,\tau}, P_\tau, \tau) = (1-\psi)HP_\tau, \quad \text{where} \quad i = B, G. \tag{33}$$

The lender seizes the house and recovers HP_τ at net of the loss ψHP_τ.

In the next section, I briefly present the numerical scheme adopted to solve the problems of both the owner and the renter as well as the mortgage valuation.

3 Numerical approach

The problems of both the owner and the renter as well as the mortgage valuation cannot be solved analytically. I adopt a finite difference scheme following Kushner and Dupuis (2002).[13]

The owning with a mortgage versus renting problem is the solution of a system of six partial differential equations: Eq.(23)-Eq.(24) (owning), Eq.(15)-Eq.(16) (renting), and Eq.(28)-Eq.(29) (mortgage valuation). For the equations Eq.(23), Eq.(24), Eq.(15) and Eq.(16), I obtain the optimal numeraire consumption of the owner and the renter respectively. To obtain the mortgage fair value M, I need to consider the default strategy of the owner and the value of the collateral at default. At the terminal date T the mortgage value is $M_T = F_T = 0$. Prior to maturity, the mortgage value satisfies the system of partial differential equations Eq.(28)-Eq.(29). The default strategy is derived solving simultaneously the system of partial differential equations Eq.(23)-Eq.(24) (owning) and Eq.(15)-Eq.(16) (renting) respectively, at any time step Δt, and verifying whether the agent does or does not exercise the default option. Conditional on the optimal exercise policy, the lender receives the mortgage coupon N or seizes the house, at net of the dead weight losses: $(1-\psi)HP_\tau$. Hence, the partial differential equations Eq.(23)-Eq.(24) (owning) and Eq.(28)-Eq.(29) (mortgage valuation) have to satisfy the associated default boundary conditions. I identify two default bound-

[13]The approach has been adopted in portfolio choice problems by Fitzpatrick and Fleming (1991), Hindy et al. (1997), Munk (2000), Øksendal and Sulem (2007) and Budhiraja and Ross (2007); in capital structure problems by Titman et al. (2004).

aries $(W_{i,\tau}, P_\tau)$ associated with the 'Good' state and 'Bad' state. Finally, I solve the model pinning down the default premium, κ, and the rent premium, λ, given the levels of state variables W_0 and P_0. Appendix A provides further computational details.

4 Model calibration

The following numerical analysis is based on the parameters listed in Table 1. Following Campbell and Cocco (2003), I assume a relative risk aversion, γ, of 3 and a rate of time preference, ρ, of 2 percent. The parameter β measures how much the agent values housing consumption relative to other goods. Housing preference, β, is set at 0.3 consistent with the average proportion of household housing expenditure in the United States.[14] I assume that the risk free rate r is equal to 3 percent. As Campbell and Cocco (2003), I set the house price appreciation parameter, μ_P, at 1.65 percent. I assume a standard deviation of house price, σ_P, of 10 per cent.[15] I set the owner's and the lender's default costs, ϵ and ψ, at 10 and 20 percent respectively for the baseline analysis. Finally, labor income can take the following these two values: $L_G = 1.5$ and $L_B = 0.75$ with $\theta_G = 0.02$ and $\theta_B = 0.2$ respectively.

5 Results

The goal is to solve the owning with a mortgage versus renting problem to identify the default premium κ and the rent premium λ such that (i) the agent is indifferent *ex ante* as to owning with a mortgage or renting the house HP_0 in the 'Good' state, given the equity share, ω; (ii) the mortgage fair value equals the mortgage face value. The two premia depend on the level of the labor income L_G and the wealth W_0 and the house price P_0 at time 0. Specifically, the agent finances the purchase of a house whose value is HP_0 is 10, where $H = 10$ and $P_0 = 1$.

[14] While Cocco (2005) sets β at 0.1, Yao and Zhang (2005) assume that β equals 0.2.

[15] Campbell and Cocco (2003) estimate a standard deviation parameter of the house price of 11.5 per cent; Cocco (2005) estimates a standard deviation of 6.2 per cent. Yao and Zhang (2005) set the standard deviation at 10 per cent.

Insert Figure 2 here

Figure 2 graphically represents the numerical solution of the model. The space (W_0, P_0) span all the possible combinations of values of W_t and P_t at time zero. The space is divided into three areas: (I) renting, (II) owning with a mortgage and (III) defaulting and renting. The existence and location of all these three areas depend on my parameters assumptions.

The owning with a mortgage area (II) borders the renting area (I) in which owning with a mortgage dominates renting, $V_G^{om} > V_G^r$. Area (II) is delimited by the renting frontier. The frontier is increasing in W_0, making an agent endowed with a relevant wealth, W_0, and high house price, P_0, indifferent to owning with a mortgage or renting. Given a housing stock H, pairs of points associated with low wealth and high house prices imply a negative risk free asset position, $A_0 < 0$: the agent is borrowing at the risk free rate r while paying the mortgage coupon N that includes a default premium. By assumption, these positions are ruled out making the renting status the only possible one.

In Figure 2, X corresponds to the solution of the problem: $\kappa = 1.311$ per cent and $\lambda = 3.352$ per cent. The agent is borrowing 90 per cent of the house value $HP_0 = 10$, $F_0 = 9$, and will pay a mortgage coupon, N, of 0.534. As a result, the agent holds a position in the risk free asset, $A_0 = W_0 - HP_0 + F_0 = 3.83 - 10 + 9 = 2.83$. Intuitively, the agent has incentive to provide a bigger down payment lowering the coupon payment N and the default premium κ. However, it is optimal to hold a risk free asset position. Because the agent is willing to consume numeraire consumption $C_{i,t}$ and has to pay the mortgage coupon N, holding a risk free asset position relaxes the "cash-on-hand" constraint if labor income switches from 'Good' state to 'Bad' state. In this case, the agent could take another debt contract, for example a home equity line contract, to relax the "cash-on-hand" constraint if his equity position in the house is positive. Pricing this contract complicates the numerical analysis. I would need to identify the amount the agent would borrow and the associated default premium lenders would charge accounting for the current mortgage. This interesting case is not modeled in this current version of the document, but it is left for future research.

Then, the agent faces a rent premium λ of 3.352 per cent. A novel implication is that the rent premium has to be substantial to make the agent *ex ante* indifferent between owning with a mortgage and renting the same house HP_0, in order to justify a highly leveraged position to finance the house purchase.

In Table 2, I present the parameters that mainly influence the default premium: the down payment and the house price volatility. For each combination of the down payment ω and the house price volatility σ_P, I report the initial wealth W_0, the agent has before signing the mortgage contract, the mortgage coupon N, the default premium κ and the rent premium λ. The extreme sensitivity of the default premium to house price volatilities may be a bit surprising. However, the magnitude the default premium at lower down payment for a house price volatility of 15 percent is by no means outside the range of the values suggested by the literature. In addition, the rent premium sharply rises implying that (i) as expected, the higher the house price volatility the higher the rent premium; (ii) the lower the down payment, the higher the rent premium to make the agent *ex ante* indifferent between owning with a mortgage and renting.

Insert Figure 3 here

Figure 3 graphically represents the numerical solution to the optimal default problem. The agent optimally determines at which point to default based on whether default or continuing to pay mortgage coupons provides him the greater expected continuation utility. Specifically, in the Figure 3, two default boundaries are represented: $(W_{G,\tau}, P_\tau)$ is associated with the 'Good' state, while $(W_{B,\tau}, P_\tau)$ with the 'Bad' state. As a result, the agent defaults with a negative equity position in the house. However, he optimally defaults with positive wealth, because his preferences are represented by a constant relative risk aversion utility function (CRRA). The default boundary is decreasing in $W_{i,\tau}$. Intuitively, the benefit of defaulting in terms of wealth, $W_{i,\tau}$, is higher for lower house prices, P_τ: the agent loses the house value at default HP_τ but he is not longer responsible for the mortgage balance F_τ. After defaulting, he moves into the rental market with the residual wealth $W_{i,\tau} - (1 +$

$\epsilon)HP_\tau + F_\tau$. The probability of default increases as the house value lowers, until reaching certainty at sufficiently low house values. As a result, the agent does not default as soon as equity is negative because the default option includes not only the right to terminate now but also the right to terminate in the future. The agent exercises the default option when it is "in the money" only when the level of wealth is substantially low.

Moreover, an implication of the model is that labor income and borrowing constraints affect the agent's default behavior. If labor income switches from a 'Good' state to a 'Bad' state, the agent early exercises his default option. As consequence, the area delimited by $(W_{B,\tau}, P_\tau)$ is wider that the one delimited by $(W_{G,\tau}, P_\tau)$. The prediction is consistent to the empirical analysis that suggests that trigger events, such as divorce, loss of a job, or accidents affect default behavior (see Vandell (1995)). If labor income falls and is not sufficient to make the required mortgage payment, the agent cannot borrow immediately and costlessly from other sources and has to finance his numeraire consumption and mortgage coupon withdrawing from his saving account. Therefore, in the presence of borrowing constraints, a negative labor income shock may induce the agent to early exercise his default option with a higher level of net wealth and then to move into the rental market with a higher *buffer-stock* of savings.

6 Conclusions

A main contribution of this paper is to derive jointly the default and the rent premia in a dynamic setting where the agent is exposed to both stochastic labor income and stochastic house prices. The agent finances the house purchase through a fixed rate mortgage and he has the option to default on the mortgage itself, moving into the rental market. Lenders market efficiently price the mortgage charging a default premium to compensate themselves for expected losses due to default on the mortgage. An alternative for the agent is to rent the same house, paying a rent fully adjustable to house prices. The rent premium is set such that the agent is indifferent *ex ante* between owning with a mortgage and renting.

An important implication of the model is that the default premium is increasing with respect to volatility of house prices and decreasing with respect to the down payment the agent provides. In addition, the rent premium has to be substantially high to justify a owning with mortgage status where the down payment is low and the house price is highly volatile.

A novel implication is that labor income and borrowing constraints crucially affect the leverage position the agent is willing to take. The agent has incentive to provide a significant down payment to decrease the default premium. However, he has no incentive to provide all his initial savings as down payment. Because the agent is willing to consume numeraire consumption and has to pay the mortgage coupon, holding a risk free asset position relaxes the "cash-on-hand" constraint if labor income switches from 'Good' state to 'Bad' state. Moreover, labor income and borrowing constraints directly affect the agent's default risk strategy. In the presence of borrowing constraints, a negative labor income shock induces the agent to early exercise his default option with a higher level of net wealth before moving into the rental market.

A Computational Details

The problem can be formulated as a standard finite-horizon dynamic program and solved by backward induction. At the terminal date T, the solution is trivial: consume everything. I solve for the indirect utility V at age $T - \Delta t$ conditional on wealth, income, house price and the (degenerate) policy rule at T. Next, I need to solve for numeraire consumption C. Usually, the consumption policy C is derived (i) using the first order condition $C = (H^{-(1-\beta)(1-\gamma)} V_W/\beta)^{1/(\beta(1-\gamma)-1)}$ and approximating V_W with its finite difference representation; (ii) discretizing the space of available policies and using simple grid search methods to find the optimal policy at each iteration. However, the first one lacks sufficient reliability in the presence of borrowing constraints as in my case, while the second one has the disadvantage that is very slow and not feasible if accurate solutions are necessary. Instead, I exploit the fact that the first order condition $C = (H^{-(1-\beta)(1-\gamma)} V_W/\beta)^{1/(\beta(1-\gamma)-1)}$ provides a simple mapping between C and V_W. Therefore, I take advantage of this relation by deriving the partial differential equation associated with the optimal numeraire consumption C. I provide the derivation for the rental case in 'Good state'. Differentiating the Hamilton-Jacobi-Bellman equation:

$$\rho V_G^r = \sup_{C_{G,t}} \left\{ \frac{(C_{G,t}^{\beta} H^{1-\beta})^{1-\gamma}}{1-\gamma} + \mathcal{D} V_G^r + \theta_G (V_B^r - V_G^r) \right\},$$

where

$$\mathcal{D} V_G^r = (rW_{G,t} + L_G - C_{G,t} - \alpha H P_t) V_{G,W}^r + \mu_P P_t V_{G,P}^r + V_{G,t}^r + \frac{\sigma_P^2}{2} P_t^2 V_{G,PP}^r,$$

with respect to W_t, I obtain

$$\begin{aligned} 0 &= (r-\rho) V_{G,W}^r + \theta_G (V_{B,W}^r - V_{G,W}^r) + V_{G,tW}^r \\ &\quad + (rW_{G,t} + L_G - C_{G,t} - \alpha H P_t) V_{G,WW}^r + \mu_P P_t V_{G,PW}^r + \frac{\sigma_P^2}{2} P_t^2 V_{G,PPW}^r. \end{aligned} \tag{A-1}$$

Using the first order condition $V_{G,W}^r = \beta H^{(1-\beta)(1-\gamma)} C_{G,t}^{-1+\beta-\beta\gamma}$, tedious algebra produces the following dynamics

$$\begin{aligned} 0 &= (-C_{G,P}^2((2+\beta(3+\beta(\gamma-1))(\gamma-1))\sigma_P^2 P_t^2)/(2C_G) - (r-\rho)C_G \\ &\quad + \theta_G (C_G - C_G^{2-\beta+\beta\gamma} C_B^{-1+\beta-\beta\gamma}) + ((1-\beta+\beta\gamma)(-C_G + L_G + rW_{G,t} - \alpha H P_t)) C_{G,W} \\ &\quad + \mu_P P_t (1-\beta+\beta\gamma) C_{G,P} + (1-\beta+\beta\gamma) C_{G,t} + \frac{\sigma_P^2}{2}(1-\beta+\beta\gamma) P_t^2 C_{G,PP}, \end{aligned} \tag{A-2}$$

where $C_G = C_G(W_{G,t}, P_t, t) = C_{G,t}$. The terminal boundary condition is $C_G(W_{G,T}, P_T, T) = W_{G,T}$. Following Candler (1999) I treat the non-linear terms 'explicitly', thus resolving the problem according to a finite difference approach. I impose the condition that the wealth position must be non negative at any time t: $C_G(W_{G,t}, P_t, t) \leq W_{G,t}$.

The system of the Hamilton-Jacobi-Bellman equations are evaluated on a discrete state space grid. For W, I set the lower bound of the domain at $W_{\min} = 0.001$ and the upper bound at $W_{\max} = 50$. I then construct the W grid with a $\Delta W = 0.25$ mesh. For P, I use $P_{\min} = 0$

and $P_{\max} = 3$ and construct the corresponding P grid using a $\Delta P = 0.015$ mesh. The boundary conditions are treated as follows. Economic intuition does not offer exact boundary conditions at $P_{\min}$, $P_{\max}$ and $W_{\max}$. Then, I impose that $V_W(W_{\max}, P, t) = V_P(W, P_{\max}, t) = V_P(W, P_{\min}, t) = 0$ and $C_W(W_{\max}, P, t) = C_P(W, P_{\max}, t) = C_P(W, P_{\min}, t) = 0$.

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Variable	Symbol	Value
Relative risk aversion	γ	3
House flow services	β	0.30
House stock	H	10
Time preference	ρ	0.03
Risk free rate	r	0.03
House price drift	μ_P	0.0165
House price standard deviation	σ_P	0.10
Level of labor income	L_B, L_G	$0.75, 1.5$
Persistence of labor shocks	θ_B, θ_G	$0.2, 0.02$
Owner's deadweight loss	ϵ	0.10
Lender's deadweight loss	ψ	0.20
Time horizon (years)	T	30

Table 1: Parameter values.

House Price Standard Deviation σ_P	Down Payment ω	Initial Wealth W_0	Mortgage Coupon N	Default Premium κ	Rent Premium λ
5%	10%	3.69	0.464	0.172%	1.323%
	20%	4.69	0.411	0.145%	1.126%
	30%	5.69	0.357	0.128%	1.083%
10%	10%	3.69	0.534	1.311%	3.352%
	20%	4.69	0.436	0.602%	2.812%
	30%	5.69	0.375	0.465%	2.573%
15%	10%	3.69	0.595	2.251%	8.332%
	20%	4.69	0.497	1.702%	7.523%
	30%	5.69	0.417	1.335%	6.933%

Table 2: This table reports the initial wealth W_0, the agent has before signing the mortgage contract, the mortgage coupon N, the default premium κ and the rent premium λ for each combination of the house price volatility σ_P and the down payment ω. The agent finances the purchase of a house whose value is HP_0 is 10, where $H = 10$ and $P_0 = 1$.

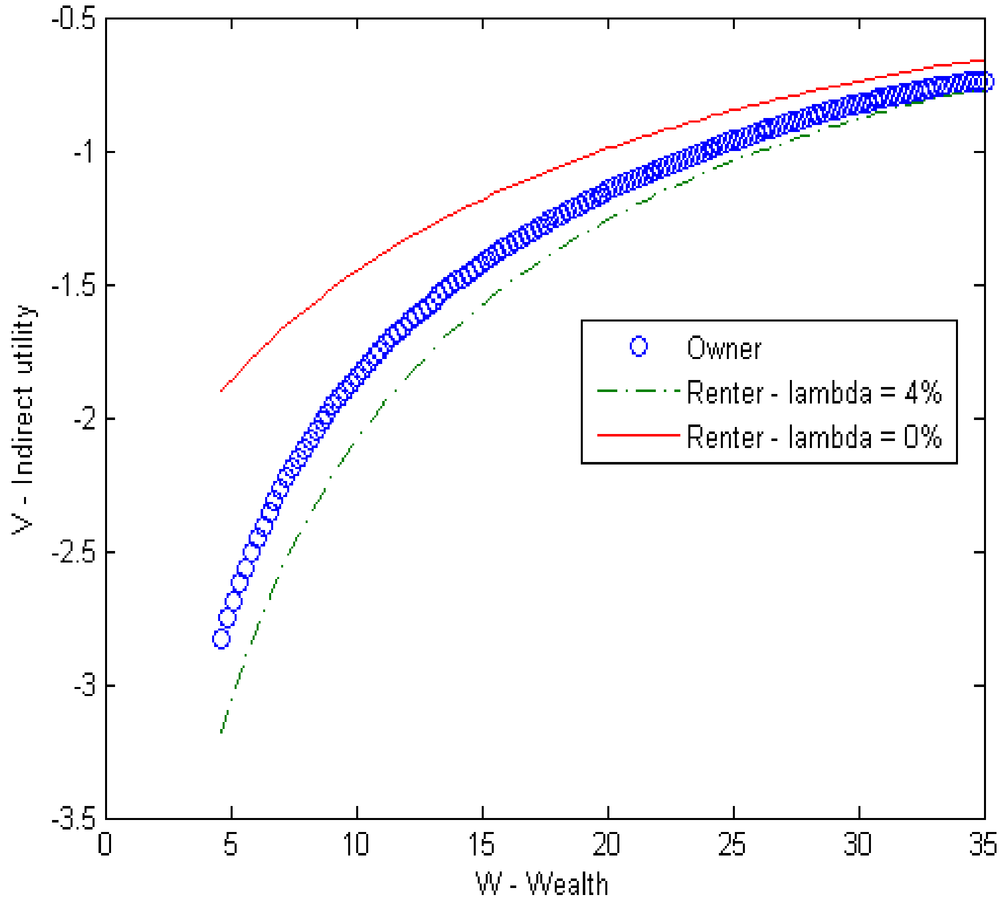

Figure 1: The figure displays the indirect utility of owning with a mortgage, V_G^{om}, and renting, V_G^r, as function of the wealth state variable W, at time 0. The parameters are reported in Table 1. The agent can own or rent a house whose price P_0, is 1 and size H is 10. The renting indirect utility, V_G^r, is represented with two different rent premia: $\lambda = 0$ percent (continuous line) and $\lambda = 4$ percent (dotted line).

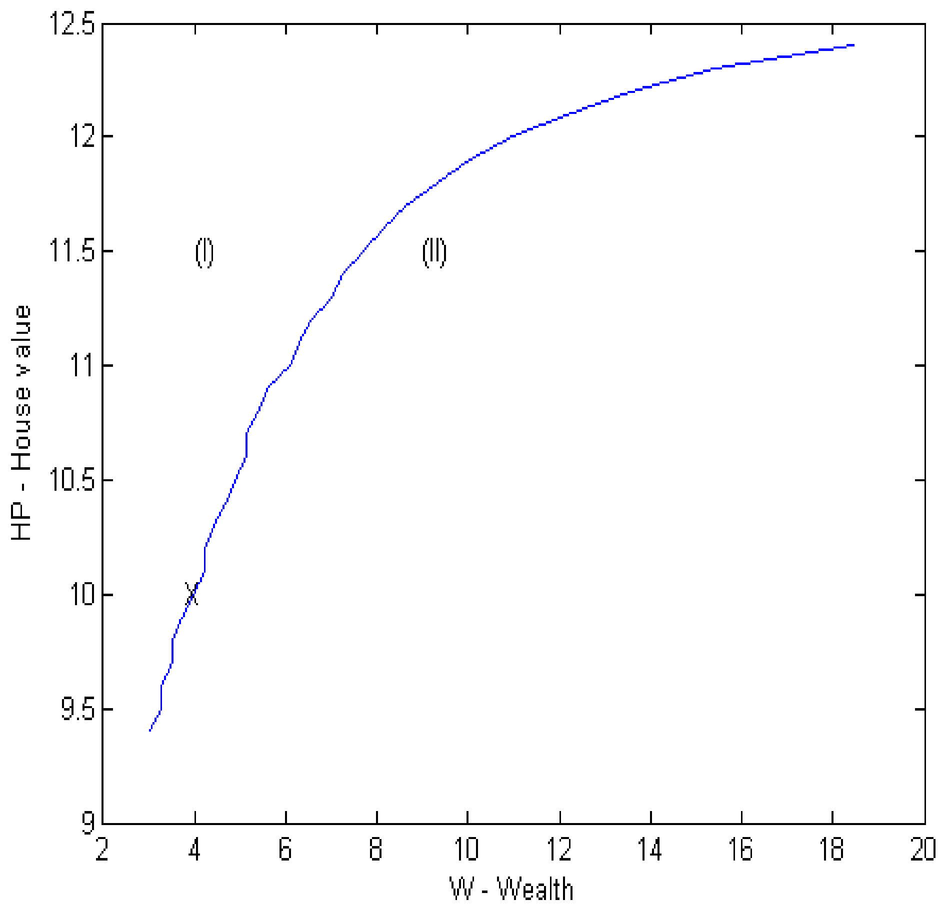

Figure 2: The figure represents the numerical solution of the model. The space (W_0, P_0) span all the possible combinations of values of W_t and P_t at time zero. The space is divided into two areas: (I) renting and (II) owning with a mortgage. The existence and location of all these three areas depend on my parameters assumptions. The owning with a mortgage area (II) borders the renting area (I) in which owning with a mortgage dominates renting, $V_G^{om} > V_G^r$. The pair (W_0^*, P_0^*) identifies where the agent is indifferent between owning with a mortgage and renting the same house HP_0. The points W_0^* and P_0^* are identified by the X point. Specifically, the optimal solution is $\kappa = 1.311$ per cent and $\lambda = 3.352$ per cent.

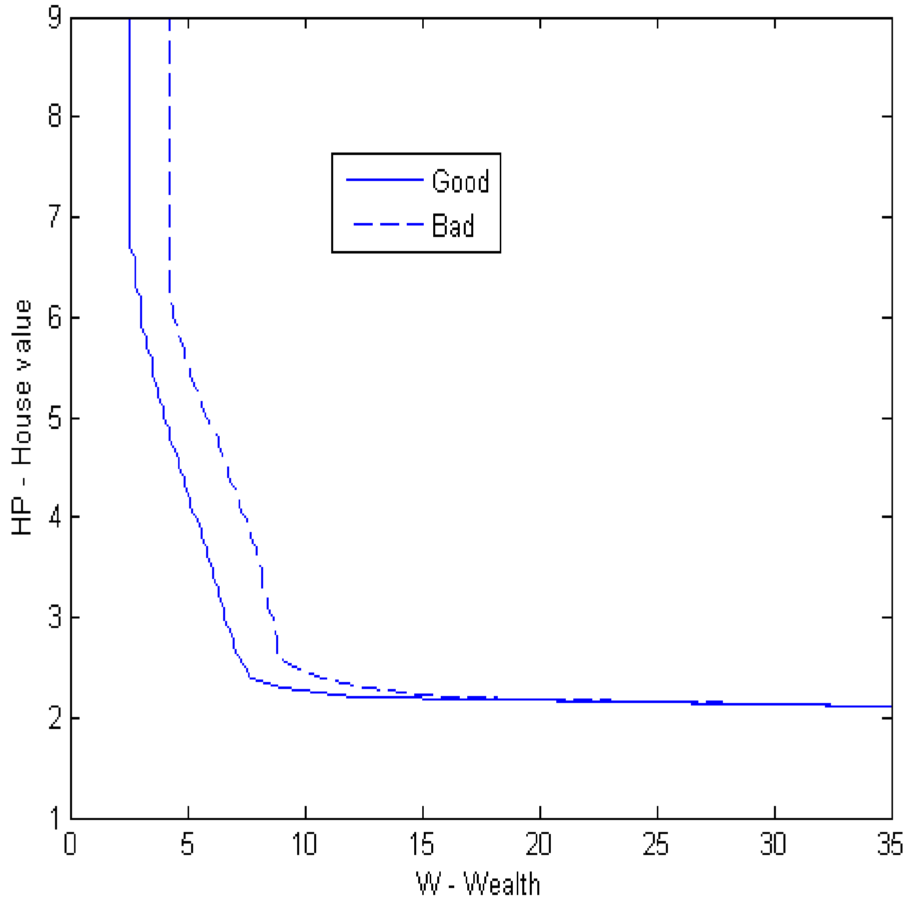

Figure 3: The figure represents two default boundaries are represented: $(W_{G,\tau}, P_\tau)$ is associated with the 'Good' state, while $(W_{B,\tau}, P_\tau)$ with the 'Bad' state.