

Optimal Regulation in the Presence of Reputation Concerns*

Andrew Atkeson[†]

Guillermo Ordoñez[‡]

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Abstract

In many markets, buyers cannot observe the quality of the seller, but can learn about it over time. Using a learning model with entry and exit of firms, we evaluate the effects of entry restrictions on welfare. In the presence of sellers' reputation concerns, more regulation can be pervasive. When the average quality of entrants is low, regulation improves welfare but, after a certain point, more regulation reduces welfare by shrinking the size of the market. This result is particularly relevant in designing optimal regulation in financial markets, which are characterized by long-term relations and strong reputation concerns.

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[†]UCLA, Federal Reserve Bank of Minneapolis, and NBER.

[‡]Federal Reserve Bank of Minneapolis and Yale University.

1 Introduction

Licensing, minimum standards, and certification requirements to firms entering an industry or to individuals entering a profession are common government practices. Professionals as diverse as doctors, lawyers, accountants, hairdressers, and contractors must pass state examinations that supposedly ensure a minimum quality level of the service they provide¹. Drugs and other potentially hazardous products have to pass federal safety standards. Financial intermediaries's capital and portfolios are initially evaluated and regularly monitored to guarantee their solvency.

The main argument for these regulatory interventions rests on the notion of imperfect information in these markets and the need for consumer protection. Even when professionals know ex-ante the quality of the service they provide, consumers typically can only ex-post observe it, but not verify it. Similarly, in many financial markets, buyers of securities may have highly imperfect information about the quality of the securities they may purchase.

The theoretical justification for this argument was first proposed by Leland (1979), who rely on the Akerlof's lemons model to consider licensing as equivalent to truncating the quality distribution at its bottom tail. However, in markets where long-term relations are common, if regulation is justified by imperfect information about the quality of the sellers, we should evaluate it in a context in which consumers try to learn about that quality by using sellers' previous results.

For example, it has been widely accepted that financial intermediaries have strong reputation concerns because their operations are based on long-term relationships. Furthermore, it is believed these concerns impose self-discipline in their operations. However, the recent financial crisis developed in the United States has reopened the discussion about whether self-regulation in financial markets is enough or additional government regulation is needed. In this paper we show the answer depends on the average quality of firms entering the market. If this quality is low, more regulation is preferred because reputation concerns complement it. However, if this quality is high enough, more regulation is harmful because reputation concerns counteract it.

In our model there are two types of firms, good and bad. Good firms provide a high quality service with a higher probability than bad firms. Consumers do not observe

¹Shapiro (1896) reports that, on average, forty occupations are licensed in each state in the U.S.

the type of a firm but learn about it from the quality of previous services it provided. We define reputation as the probability the firm is good. Firms are concerned about their reputation and can enter and exit the market when they so decide.

We find that an increase in regulatory entry restrictions always raise the average quality of entrants (and hence the reputation assigned to them). However, more regulation only increases market size and welfare if the reputation of entrants is low enough. Contrarily, more regulation reduces welfare if the reputation of entrants is already high. In other words, when considering reputation concerns, there is always an optimal level of regulation that maximizes welfare.

The intuition of this result relies on three properties of a firm expected profits. First, firms' expected profits increase with the level of reputation. This is because consumers are willing to pay a higher price for services provided by a firm that is good with a higher probability. Second, good firms' expected profits are higher than those of bad firms at any reputation level. This is because good firms are more likely to provide high quality services and hence more likely for their reputation to increase. Finally, firms' expected profits increase with the aggregate price level in the market, or, which is the same, decrease with market size.

When regulatory entry costs increase, both good and bad firms require more expected profits to enter in order to compensate those costs. There are two channels to raise expected profits: an increase in the reputation assigned to entrants and an increase in aggregate prices in the market. If the reputation of entrants is low, any increase induced by regulation raises expected profits more for good firms than for bad ones. This is because learning becomes easier and good firms can take more advantage in terms of a future reputation improvement. The large affluence of good firms increases the market size, decreasing the aggregate price and improving welfare. Contrarily, if the reputation of entrants is already high, any further increase induced by regulation raises expected profits less for good firms than for bad ones. This is because learning becomes more difficult and bad firms are insured against a reputation loss. The large affluence of bad firms decreases the market size, increasing the aggregate price and reducing welfare.

These results also allow us to compare the welfare implications of different forms of regulation. For example, we can compare between frontloading costs (such as licensing, entry requirements, and minimum standards) and backloading costs (such

as regular supervision, taxes or subsidies to age, and periodic monitoring). Our results suggest the best way to improve quality in the market without reducing welfare is to impose frontload costs that affect good and bad firms initially and to provide backload subsidies increasing in age or reputation, that compensate good firms more, since in expectation they live longer and achieve better reputations.

This paper is related to two strands of literature that have not been systematically connected: regulation and reputation. With respect to the regulation strand, this paper contributes to Leland (1979), extended later by Shaked and Sutton (1981) and Shapiro (1896) who introduced moral hazard and investment decisions. It also complements von Weizsacker (1980), who discusses how barriers to entry may reduce welfare once we consider economies of scale and differentiated products. More recently, Albano and Lizzeri (2001) analyze the efficiency effects of certification intermediaries, but without making reference to reputation concerns.

With respect to the reputation strand, Mailath and Samuelson (2001) discusses a reputation model where firms enter to replace those that exogenously die. Similarly, the work of Tadelis (1999 and 2002) studies the market for names and the endogenous value of reputation as a tradeable asset. Among models with exit of firms, our paper is related to Horner (2002) and Bar-Isaac (2003). However, none of these papers consider a reputation model with both entry and exit decision by firms nor use it to discuss regulation implications.

In the next Section we describe the model. In Section 3 we solve for the exit and entry decisions in equilibrium and discuss the effects of regulation on welfare. In Section 4 we show a numerical example. In Section 5 we make some final remarks.

2 The Model

2.1 Environment

There is a market for an experience good composed by a continuum of firms. Firms that enter in a given calendar period survive for at most T periods, where t denotes firms' age. Each firm produces and sells one unit of the good each period that it operates. These firms can be either good (G) or bad (B). Reputation is defined as

$Pr(G) = \phi$, for each firm in the market. Each unit of the good sold generates one of two possible results, good (g) or bad (b) outcomes, with probabilities depending on the firm's type. $Pr(g|G) = \alpha > Pr(g|B) = \gamma$. Bad outcomes can be interpreted as a defective product.

Firms decide whether to enter (age 0) and whether to exit before they reach age T . Good firms spend an unobservable amount C to enter. This can be interpreted as an initial investment to be a good type (for example, the differential cost of operating a more efficient machine or the opportunity cost for not operating more efficiently in an alternative activity). Bad firms enter for free. There are no marginal costs of production. At age T , good firms recover a scrap value $c > 0$ from the initial investment C . Regulators may additionally impose a fixed observable entry cost F .²

There is a unit mass of consumers in the market. When buying one unit of the experience good, they cannot observe firms' types, hence we have a pure adverse selection problem³. Consumers get an effective consumption of 1 from good products and $-\kappa$ from bad products, where $\kappa > \frac{\gamma}{1-\gamma}$ so that the expected effective consumption from purchasing a unit of the experience good from a firm with expected quality $\phi = 1$ is positive and is negative if purchased from a firm with expected quality $\phi = 0$. They also consume a numeraire good E .

To rule out firms using their price as a signal of quality, we assume consumers compete to purchase each unit of the experience good produced. Consumers in the market choose how many experience goods of a given expected quality ϕ and how many of the numeraire good to buy, having an income that is normalized to 1. Hence, consumers maximize period utility:

$$U(m(\phi)) = \max_{m(\phi) \geq 0} \frac{1}{1-\eta} \left[\sum_{\phi} \theta(\phi) m(\phi) \right]^{1-\eta} + 1 - \sum_{\phi} p(\phi) m(\phi) \quad (1)$$

²Allowing both types to burn money publicly at the initial date would complicate the exposition and lead to multiple equilibria. As we show in the Appendix however, the equilibrium we analyze in which firms do not burn money is in fact the unique equilibrium of the more general model under an intuitive stability criterion.

³Preliminary results from extending the model by introducing also moral hazard considerations (good firms also can decide whether or not to exert high efforts in using their more efficient technology) suggest to reinforce our conclusions in this paper.

where $0 < \eta < 1$, we define the total size of the experience good market as,

$$M = \sum_{\phi} \theta(\phi) m(\phi) \quad (2)$$

and $\theta(\phi)$ is the expected effective consumption from buying one good from a firm ϕ , given by $\theta(\phi) = [\alpha\phi + \gamma(1 - \phi)] + [(1 - \alpha)\phi + (1 - \gamma)(1 - \phi)](-\kappa)$. Hence $\theta(\phi)$ is linear in ϕ .

$$\theta(\phi) = \phi[(\alpha - \gamma)(1 + \kappa)] + [\gamma(1 + \kappa) - \kappa] \quad (3)$$

2.2 Preliminaries

From the consumers' first order conditions, $p(\phi) = \theta(\phi)M^{-\eta}$. This implies profits of a firm with reputation ϕ is given by $\pi(\phi, M) = \theta(\phi)M^{-\eta}$, since we assume no marginal costs. Hence, profits are increasing on reputation ϕ and decreasing on market size M .

From replacing first order conditions into the definition of consumers' utility (equation 1) and the definition of market size (equation 2), we can express welfare directly as a function of market size.

$$U(M) = 1 + \frac{\eta}{1 - \eta} M^{1 - \eta} \quad (4)$$

Existing firms of age $t < T$ and reputation ϕ decide the probability continuing operating in the market, after each possible result. This continuation or exit decision is made subject to a tremble that implies that the firm must exit with probability at least ϵ and continue with probability at least ϵ . For example, $Pr(\text{continuation} | G, b, \phi, t) = \sigma_G^b(\phi, t) \in [\epsilon, 1 - \epsilon]$. For simplicity in notation, let $\sigma_i(t) = \{\sigma_i^g(t); \sigma_i^b(t)\}$ for $i = \{G, B\}$ and $\sigma^k = \{\sigma_G^k(t); \sigma_B^k(t)\}$ for $k = \{g, b\}$, where we suppress reference to ϕ .

Given strategies $\sigma^g(t)$, firms' reputation after a good outcome is updated according to Bayes Rule

$$\phi^g(\phi, \sigma^g(t)) = \frac{\alpha \sigma_G^g(t) \phi}{\alpha \sigma_G^g(t) \phi + \gamma \sigma_B^g(t) (1 - \phi)} \quad (5)$$

and likewise, given strategies $\sigma^b(t)$, after a bad outcome

$$\phi^b(\phi, \sigma^b(t)) = \frac{(1 - \alpha) \sigma_G^b(t) \phi}{(1 - \alpha) \sigma_G^b(t) \phi + (1 - \gamma) \sigma_B^b(t) (1 - \phi)} \quad (6)$$

We compute the expected profits of a cohort of firms in a steady-state, with market size $M = 1$, by backward induction as follows. For firms of age T , continuation profits are given by

$$V_G(\phi, T) = \theta(\phi) + c \quad \text{and} \quad V_B(\phi, T) = \theta(\phi) \quad (7)$$

For firms of age $t < T$, the continuation value function for firms of type G is given by

$$V_G(\phi, t) = \max_{\sigma_G} \left\{ \theta(\phi) + \beta \left[\alpha \sigma_G^g V_G(\phi^g(\phi, \sigma^g), t+1) + (1 - \alpha) \sigma_G^b V_G(\phi^b(\phi, \sigma^b), t+1) \right] \right\} \quad (8)$$

The continuation value function for firms of type B is given by

$$V_B(\phi, t) = \max_{\sigma_B} \left\{ \theta(\phi) + \beta \left[\gamma \sigma_B^g V_B(\phi^g(\phi, \sigma^g), t+1) + (1 - \gamma) \sigma_B^b V_B(\phi^b(\phi, \sigma^b), t+1) \right] \right\} \quad (9)$$

It is clear that the continuation values for both good and bad firms in steady-state scale with the market size $M^{-\eta}$. Hence, free entry conditions for a new cohort of firms (of age $t = 0$) for an imposed regulatory entry cost F , are:

$$\begin{aligned} V_G(\phi_0, 0) M^{-\eta} &= C + F \\ V_B(\phi_0, 0) M^{-\eta} &= F \end{aligned} \quad (10)$$

These two equations determine two unknowns, the reputation of entrants ϕ_0 and the size of the market M in steady-state.

3 Equilibrium decisions

3.1 Continuation decisions

It is straightforward to show by backward induction that there is a unique steady-state equilibrium. We will show that the continuation value functions $V_i(\phi, t)$ are all continuous, strictly increasing, strictly positive at $\phi = 1$, strictly negative at $\phi = 0$, that $V_G(\phi, t) > V_B(\phi, t)$ for $\phi \in (0, 1)$, and that the firms' continuation decisions $\sigma_i^k(\phi, t) \in [\epsilon, 1 - \epsilon]$ are weakly increasing and have a form defined by six cutoff levels, four for bad firms defining regions of pure and mixed strategies following good and

bad outcomes respectively and two for good firms following good and bad outcomes respectively.

These cutoff levels are defined relative to reputation thresholds $\phi_B^*(t+1)$ and $\phi_G^*(t+1)$ defined by

$$V_i(\phi_i^*(t+1), t+1) = 0 \quad (11)$$

for $i = G, B$. That this threshold is well defined follows from the result that the continuation value functions are continuous, strictly increasing, and cross zero. This result will also imply that it is optimal for good firms with an updated reputation $\phi \geq \phi_G^*(t+1)$ to continue with the maximum probability $1 - \epsilon$ and likewise for bad firms with an updated reputation $\phi \geq \phi_B^*(t+1)$. That firms' continuation decisions are weakly increasing implies that for sufficiently high values of ϕ , both good and bad firms choose to continue with the maximum probability $1 - \epsilon$ following either a good or a bad outcome. For bad firms of age t , there are cutoff reputation levels $\bar{\phi}_B^k(t)$ such that

$$\phi_B^*(t+1) = \frac{\alpha \bar{\phi}_B^g(t)}{\alpha \bar{\phi}_B^g(t) + \gamma(1 - \bar{\phi}_B^g(t))} = \frac{(1 - \alpha) \bar{\phi}_B^b(t)}{(1 - \alpha) \bar{\phi}_B^b(t) + (1 - \gamma)(1 - \bar{\phi}_B^b(t))}. \quad (12)$$

These thresholds are defined such that, if both good and bad firms of age t and with reputation ϕ greater than these cutoffs continue with the maximum probability $1 - \epsilon$, then these firms will have updated reputations at age $t+1$ following good and bad outcomes respectively sufficiently high to justify continuing with the maximum probability $1 - \epsilon$.

Bad firms of age t and reputation ϕ below these cutoffs will not have updated reputations at age $t+1$ to justify a pure strategy of continuation. We will show that bad firms of age t with reputations immediately below these thresholds choose to mix ($\sigma_B^k(\phi, t) \in (\epsilon, 1 - \epsilon)$) while good firms continue with the maximum probability $1 - \epsilon$. The mixing probability for bad firms is chosen so that the updated reputation for continuing bad firms is $\phi_B^*(t+1)$, delivering a continuation value of 0, as required to justify mixing. That good firms with reputations ϕ immediately below this threshold continue with the maximum probability $1 - \epsilon$ follows from the result that $V_G(\phi, t+1) > V_B(\phi, t+1)$, which implies that $V_G(\phi_B^*(t+1), t+1) > 0$.

The mixing probabilities chosen by bad firms of age t with reputation $\phi < \bar{\phi}_B^k(t)$

following good and bad outcomes respectively are given by

$$s_B^g(\phi, t) = \frac{\alpha(1-\epsilon)(1-\phi_B^*(t+1))}{\gamma\phi_B^*(t+1)} \frac{\phi}{1-\phi} \quad (13)$$

and

$$s_B^b(\phi, t) = \frac{(1-\alpha)(1-\epsilon)(1-\phi_B^*(t+1))}{(1-\gamma)\phi_B^*(t+1)} \frac{\phi}{1-\phi} \quad (14)$$

Since we have imposed a lower bound of ϵ on these mixing probabilities, there are two additional threshold reputation levels for bad firms of age t defined by the reputations $\underline{\phi}_B^k(t)$ such that this lower bound is binding on the strategies above, i.e.

$$\epsilon = \frac{\alpha(1-\epsilon)(1-\phi_B^*(t+1))}{\gamma\phi_B^*(t+1)} \frac{\underline{\phi}_B^g(t)}{1-\underline{\phi}_B^g(t)} \quad (15)$$

and

$$\epsilon = \frac{(1-\alpha)(1-\epsilon)(1-\phi_B^*(t+1))}{(1-\gamma)\phi_B^*(t+1)} \frac{\underline{\phi}_B^b(t)}{1-\underline{\phi}_B^b(t)} \quad (16)$$

We will show that it is a dominant strategy for the bad firms of age t and reputation ϕ below these cutoffs to continue with the minimum probability ϵ after a good and bad result respectively.

Finally, there are two thresholds for $\underline{\phi}_G^k(t) < \phi_B^k(t)$ for $k = g, b$ such that good firms of age t and reputation ϕ above these thresholds continue with the maximum probability $1-\epsilon$ after good and bad results respectively and good firms of age t and reputation ϕ below these thresholds continue with the minimum probability ϵ after good and bad results respectively. These thresholds $\underline{\phi}_G^k(t)$ are defined by

$$\phi_G^*(t+1) = \frac{\alpha(1-\epsilon)\underline{\phi}_G^g(t)}{\alpha(1-\epsilon)\underline{\phi}_G^g(t) + \gamma\epsilon(1-\underline{\phi}_G^g(t))} = \frac{(1-\alpha)(1-\epsilon)\underline{\phi}_G^b(t)}{(1-\alpha)(1-\epsilon)\underline{\phi}_G^b(t) + (1-\gamma)\epsilon(1-\underline{\phi}_G^b(t))} \quad (17)$$

Proposition 1 *The unique equilibrium continuation strategies have the following form*

$$\sigma_G^k(\phi, t) = \begin{cases} 1-\epsilon & \text{if } \phi \geq \underline{\phi}_G^k(t) \\ \epsilon & \text{if } \phi < \underline{\phi}_G^k(t) \end{cases} \quad (18)$$

$$\sigma_B^k(\phi, t) = \begin{cases} 1 - \epsilon & \text{if } \phi \geq \bar{\phi}_B^k(t) \\ s_B^k(\phi, t) & \text{if } \phi \in (\underline{\phi}_B^k(t), \bar{\phi}_B^k(t)) \\ \epsilon & \text{if } \phi \leq \underline{\phi}_B^k(t) \end{cases} \quad (19)$$

where the cutoffs are defined in equations (11), (12), (15), (16), and (17), while the mixing probabilities $s_B^k(\phi, t)$ are given by (13) and (14).

Proof The proof is by backward induction.

From (7), we have that value functions $V_i(\phi, T)$ are continuous and strictly increasing, such that $V_i(0, T) < 0 < V_i(1, T)$. Hence, thresholds $\phi_i^*(T)$ from equation (11) are well defined. Also, at period T , $V_G(\phi, T) > V_B(\phi, T)$ for all ϕ , and hence $\phi_G^*(T) < \phi_B^*(T)$.

From Bayes Rule given in (5) and (6), this implies that firms' updated reputations at age T following either a good or bad result are bounded below by the updated reputation that follows from Bayes Rule when both good and bad firms choose to continue with maximum probability $1 - \epsilon$. This implies that

$$\sigma_i^k(\phi, T - 1) = 1 - \epsilon \quad \text{for } \phi \leq \bar{\phi}_i^k(T - 1).$$

Likewise, from Bayes Rule given in (5) and (6), this implies that firms' updated reputations at age T following either a good or bad result are bounded above by the updated reputation that follows from Bayes Rule when good firms continue with maximum probability $1 - \epsilon$ and bad firms continue with minimum probability ϵ . This implies that

$$\sigma_i^k(\phi, T - 1) = \epsilon \quad \text{for } \phi \leq \underline{\phi}_i^k(T - 1).$$

Now we show bad firms cannot play a pure strategy in $\phi \in (\underline{\phi}_B^k(T - 1), \bar{\phi}_B^k(T - 1))$. This result follows from the bounds of reputation given above. That is, if a bad firm of age $T - 1$ and reputation ϕ in this region continues with maximum probability $1 - \epsilon$, it will have updated reputation less than $\phi_B^*(T)$ leading to a negative continuation value while if it continues with minimum probability ϵ , it will have updated reputation greater than $\phi_B^*(T)$ leading to a positive continuation value. Neither is possible in equilibrium. Hence, this bad firm must mix, and it must choose a mixing probability such that its updated reputation is exactly equal to $\phi_B^*(T)$ where it is indifferent between continuing and exiting.

Since $\phi_B^*(T) > \phi_G^*(T)$, good firms choose to continue with a maximum probability $1 - \epsilon$ when the updated reputation is $\phi_B^*(T)$. This means we have unique equilibrium strategies for each reputation in the range $(\phi_B^k(T-1), \bar{\phi}_B^k(T-1))$, given by the pair of continuation strategies $\{\sigma_B^k(\phi, T-1); \sigma_G^k(\phi, T-1) = \{s_B^k(\phi, T-1); 1 - \epsilon\}$ that update reputation to $\phi_B^*(T)$, where $s_B^k(\phi, T-1)$ are given by equations (13) and (14).

Under these strategies, continuation values at $T-1$ satisfy three properties. First, $V_i(\phi, T-1)$ are continuous and strictly increasing. Second, $V_B(0, T-1) < 0 < V_B(1, T-1)$, hence, the threshold $\phi_B^*(T-1)$ from equation (11) is well defined. Finally, $V_G(\phi, T-1) > V_B(\phi, T-1)$ for all $\phi \in (0, 1)$, hence $\phi_G^*(T-1) < \phi_B^*(T-1)$.

Iterating the argument above we can solve for the value functions and unique continuation strategies for firms of all ages $t < T-1$. Q.E.D.

Remark 1 *In our model we have assumed $T < \infty$, $\epsilon > 0$, and $c > 0$ to ensure a unique pair of continuation strategies for good and bad firms at all ages $t = \{0, 1, \dots, T\}$. The assumption that $T < \infty$ ensures that continuation values for both good and bad firms are strictly increasing in their reputation. The assumption that $\epsilon > 0$ ensures there is no out of the equilibrium outcomes and hence beliefs can be always constructed using Bayes' rule. The assumption that $c > 0$ ensures there are unique continuation strategies for firms of age $T-1$. Relaxing this third assumption by setting $c = 0$, delivers multiple equilibrium strategies for firms of age $T-1$, but a unique pair of continuation value functions $V_i(\phi, T-1)$ that satisfy the requirements of our Proposition, still guaranteeing unique continuation strategies for firms of age $t < T-1$.*

In what follows we will focus on equilibria with small ϵ and c . Given the equilibrium continuation strategies for good and bad firms are a Bellman equation, it is a contraction mapping, hence as T grows large our value functions $V_i(\phi, 0)$ converges to a fixed point of the Bellman equation, $V_i(\phi)$, that defines fixed cutoffs ϕ_i^ and $\bar{\phi}_i^k$.*

These results imply that, effectively, at any period there will not be firms of age t with a reputation lower than $\phi_B^*(t)$, since bad firms' continuation decisions guarantee that the update after a bad result never drops below that threshold.

3.2 Entry decisions

Now we solved continuation decisions we can analyze entry decisions, which depend on regulation. Then we can evaluate the impact of regulation on the reputation assigned to entrants ϕ_0 and the size of the market M , which determines consumers' utility.

The next Proposition shows three important entry properties. First, there is a unique reputation to entrants ϕ_0 that is determined by regulatory entry costs F . Second, higher entry costs imply higher reputation assigned to entrants. The reason is that firms need a higher reputation when entering to compensate higher entry cost. Finally, higher entry costs imply the reputation assigned to entrants ϕ_0 is higher than the lowest reputation operating in the market $\phi_B^*(0)$. This means that, when $F > 0$, firms can run down their reputation after entrance and before exiting.

Proposition 2 *A regulatory entry cost F uniquely determines the reputation assigned to entrants, ϕ_0 . Furthermore, an increase in F monotonically increases ϕ_0 .*

Proof

From entry conditions 10,

$$\frac{V_B(\phi_0, 0)}{V_G(\phi_0, 0)} = \frac{F}{C + F}, \quad (20)$$

which is independent of M . Then, to prove the Proposition it is enough to prove the ratio $\frac{V_B(\phi_0, 0)}{V_G(\phi_0, 0)}$ is an increasing function of ϕ_0 that maps from $\phi_0 = [\phi_B^*(0), 1]$ to $[0, 1]$.

First, we define the domain and image of the function. From Proposition 1, the lower reputation in the market at period 0, as $\epsilon \rightarrow 0$, is $\phi_B^*(0)$, where $V_B(\phi_B^*(0), 0) = 0$ and $V_G(\phi_B^*(0), 0) > 0$. Also from Proposition 1, we know that $V_B(1, 0) = V_G(1, 0)$ when $c = 0$.⁴ Finally, $V_B(\phi_0, 0) < V_G(\phi_0, 0)$ for all other $\phi_0 \in [\phi_B^*(0), 1)$. This implies $\frac{V_B(\phi_0, 0)}{V_G(\phi_0, 0)}$ is a mapping from $\phi_0 = [\phi_B^*(0), 1]$ to $[0, 1]$.

Now, working by backward induction, we prove $\frac{V_B(\phi_0, 0)}{V_G(\phi_0, 0)}$ is an increasing function.

Step 1) Increasing $\frac{V_B(\phi, T-1)}{V_G(\phi, T-1)}$.

⁴When $c > 0$, this is still true but instead of C in equation (20), we have $C' = C - \sum_{t=0}^T \beta^t (1 - \epsilon)^t c$.

From continuation decisions at $T - 1$,

$$V_G(\phi, T) = \begin{cases} \theta(\phi) + c & \text{if } \phi \geq \phi_B^*(T) \\ c & \text{if } \phi < \phi_B^*(T) \end{cases} ; \quad V_B(\phi, T) = \begin{cases} \theta(\phi) & \text{if } \phi \geq \phi_B^*(T) \\ 0 & \text{if } \phi < \phi_B^*(T) \end{cases}$$

This is a continuous and differentiable function almost everywhere (except at the kink $\phi_B^*(T)$). Hence the derivative of the function also has a kink at $\phi_B^*(T)$,

$$V'_i(\phi, T) = \begin{cases} (\alpha - \gamma)(1 + \kappa) & \text{if } \phi \geq \phi_B^*(T) \\ 0 & \text{if } \phi < \phi_B^*(T) \end{cases}$$

Now, the value function for bad firms at $T - 1$ (equation 9) considering equilibrium continuation strategies and value functions at T can be rewritten as,

$$V_B(\phi, T - 1) = \begin{cases} \theta(\phi) + \beta(1 - \epsilon) [\gamma\theta(\phi^g(\phi)) + (1 - \gamma)\theta(\phi^b(\phi))] & \text{if } \phi \geq \bar{\phi}_B^b(T - 1) \\ \theta(\phi) + \beta(1 - \epsilon)\gamma\theta(\phi^g(\phi)) & \text{if } \phi \in [\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)] \\ 0 & \text{if } \phi \leq \phi_B^*(T - 1) \end{cases}$$

and for good firms

$$V_G(\phi, T - 1) = \begin{cases} \theta(\phi) + \beta(1 - \epsilon) [\alpha\theta(\phi^g(\phi)) + (1 - \alpha)\theta(\phi^b(\phi)) + c] & \text{if } \phi \geq \bar{\phi}_B^b(T - 1) \\ \theta(\phi) + \beta(1 - \epsilon) [\alpha\theta(\phi^g(\phi)) + c] & \text{if } \phi \in [\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)] \\ \beta(1 - \epsilon) [(\alpha - \gamma)\theta(\phi_B^*(T - 1)) + c] & \text{if } \phi \leq \phi_B^*(T - 1) \end{cases}$$

These value functions are continuous with kinks at $\phi_B^*(T - 1)$ and $\bar{\phi}_B^b(T - 1)$. The two kinks are basically generated by the exit options at period $T - 1$ (given by $\phi_B^*(T - 1)$) and at period T (given by $\bar{\phi}_B^b(T - 1)$, since for all reputations below it, the firm that experiences a bad result gets $\phi_B^*(T)$).

The strategy is to prove the ratio of value functions $\frac{V_B(\phi, T-1)}{V_B(\phi, T-1)}$ is increasing in the two ranges $[\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)]$ and $[\bar{\phi}_B^b(T - 1), 1]$, where the functions are differentiable. To do this, we need to work with both first and second derivatives of the ratio. In

general, at any age t , first derivatives are,

$$\frac{\partial \frac{V_B(\phi, t)}{V_G(\phi, t)}}{\partial \phi} = \frac{V'_B V_G - V'_G V_B}{V_G^2}, \quad (21)$$

and second derivatives are,

$$\frac{\partial^2 \frac{V_B(\phi, t)}{V_G(\phi, t)}}{\partial \phi^2} = \frac{V_G[V''_B V_G - V''_G V_B] - 2V'_G[V'_B V_G - V'_G V_B]}{V_G^3}. \quad (22)$$

Now, taking derivatives of the value function for bad firms at $T - 1$,

$$V'_B(\phi, T - 1) = \begin{cases} (\alpha - \gamma)(1 + \kappa)[1 + \beta(1 - \epsilon)(\gamma \frac{\partial \phi^g}{\partial \phi} + (1 - \gamma) \frac{\partial \phi^b}{\partial \phi})] & \text{if } \phi \geq \bar{\phi}_B^b(T - 1) \\ (\alpha - \gamma)(1 + \kappa)[1 + \beta(1 - \epsilon)\gamma \frac{\partial \phi^g}{\partial \phi}] & \text{if } \phi \in [\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)] \\ 0 & \text{if } \phi \leq \phi_B^*(T - 1) \end{cases}$$

The first derivative of value functions for good firms is the same, but replacing γ by α , since it is more likely for good firms to obtain good results. The main difference between the derivative in the lower reputation range $[\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)]$ and the upper reputation range $[\bar{\phi}_B^b(T - 1), 1]$ is that, in the upper range, reputation decreases after a bad result. Contrarily, in the lower range, regardless of the initial ϕ , the updated reputation is constant $\phi_B^*(T)$ after a bad result. Hence changes in reputation modify values in expectation more in the upper range than in the lower range. This intuition is key in proving kinks are convex.

At each range, the function is also twice differentiable,

$$V''_B(\phi, T - 1) = \begin{cases} (\alpha - \gamma)(1 + \kappa)\beta(1 - \epsilon)[\gamma \frac{\partial^2 \phi^g}{\partial \phi^2} + (1 - \gamma) \frac{\partial^2 \phi^b}{\partial \phi^2}] & \text{if } \phi \geq \bar{\phi}_B^b(T - 1) \\ (\alpha - \gamma)(1 + \kappa)\beta(1 - \epsilon)\gamma \frac{\partial^2 \phi^g}{\partial \phi^2} & \text{if } \phi \in [\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)] \\ 0 & \text{if } \phi \leq \phi_B^*(T - 1) \end{cases}$$

Given profits are linear on reputation, the first and second derivatives of value functions only depend on the first and second derivatives of reputation updating, respectively. Deriving equations (5) and (6) for the cases of continuation with maximum

probability $(1 - \epsilon)$, which are the only terms that enter into the derivatives of value functions, we get

$$\frac{\partial \phi^g}{\partial \phi} = \frac{\alpha\gamma}{[\alpha\phi + \gamma(1 - \phi)]^2} > 0, \quad (23)$$

$$\frac{\partial \phi^b}{\partial \phi} = \frac{(1 - \alpha)(1 - \gamma)}{[(1 - \alpha)\phi + (1 - \gamma)(1 - \phi)]^2} > 0, \quad (24)$$

and taking second derivatives

$$\frac{\partial^2 \phi^g}{\partial \phi^2} = -\frac{\alpha\gamma(\alpha - \gamma)}{[\alpha\phi + \gamma(1 - \phi)]^3} < 0, \quad (25)$$

$$\frac{\partial^2 \phi^b}{\partial \phi^2} = -\frac{(1 - \alpha)(1 - \gamma)(\gamma - \alpha)}{[(1 - \alpha)\phi + (1 - \gamma)(1 - \phi)]^3} > 0. \quad (26)$$

First we show the ratio $\frac{V_B(\phi, T-1)}{V_G(\phi, T-1)}$ cannot achieve a local maximum for reputations strictly inside each range. Second, we show the derivative of the ratio evaluated both at the beginning and at the end of each range, is non-negative. These two properties guarantee the ratio of value functions at $T - 1$ is weakly increasing for all relevant reputations $\phi \in [\phi_B^*(T - 1), 1]$.

First, $V_B''(\phi, T - 1) > V_G''(\phi, T - 1)$ for all ϕ in both ranges. This is straightforward from computing $V_B'' - V_G''$ by using equations (25) and (26). By inspecting equations (21) and (22). This means that, in these ranges, if the function $\frac{V_B(\phi, T-1)}{V_G(\phi, T-1)}$ does not change (i.e., has a zero slope, or $V_B'V_G = V_B V_G'$), then the second derivative is always positive (this is clearly true when $V_B'' > 0$, but it is easy to check this is also the case when $V_B'' < 0$). Hence, the ratio never has a local maximum inside each range.

Second, we need to evaluate the derivative of the ratio at the two extremes of the two ranges and check these are non-negative. In the lower range $\phi \in [\phi_B^*(T - 1), \bar{\phi}_B^b(T - 1)]$:

- $\frac{\partial \frac{V_B(\phi_B^*(T-1))}{V_G(\phi_B^*(T-1))}}{\partial \phi} > 0$. This is because $V_B(\phi_B^*(T - 1)) = 0$ and $V_B'(\phi_B^*(T - 1)) > 0$.
- $\frac{\partial \frac{V_B(\bar{\phi}_B^b(T-1))}{V_G(\bar{\phi}_B^b(T-1))}}{\partial \phi} \geq 0$. This is obtained combining the previous bullet point and the fact that the ratio has not local maximum in the range.

In the upper range $\phi \in [\bar{\phi}_B^b(T - 1), 1]$:

- $\frac{\partial V_B(\bar{\phi}_B^b(T-1))}{\partial \phi} > 0$. First, this derivative is non-negative when evaluated from the left (at the end point of the lower range). Second, $V'_i(\bar{\phi}_B^b(T-1))_{\mathbf{L}} < V'_i(\bar{\phi}_B^b(T-1))_{\mathbf{R}}$, where the subscript $\mathbf{L}$ represents the derivative at $\bar{\phi}_B^b(T-1)$ from the left (at the end point of the lower range) and $\mathbf{R}$ from the right (at the initial point of the upper range). Furthermore, $V'_B(\bar{\phi}_B^b(T-1))_{\mathbf{R}} - V'_B(\bar{\phi}_B^b(T-1))_{\mathbf{L}} = (\alpha - \gamma)(1 + \kappa)(1 - \gamma)\beta(1 - \epsilon)\frac{\partial \phi^b}{\partial \phi}$ which is greater than $V'_G(\bar{\phi}_B^b(T-1))_{\mathbf{R}} - V'_G(\bar{\phi}_B^b(T-1))_{\mathbf{L}} = (\alpha - \gamma)(1 + \kappa)(1 - \alpha)\beta(1 - \epsilon)\frac{\partial \phi^b}{\partial \phi}$, by using equations (23) and (24). This means the derivative of the ratio from the right is strictly greater than the derivative of the ratio from the left.
- $\frac{\partial V_B(1, T-1)}{\partial \phi} > 0$. This is because $V_B(1, T-1) = V_G(1, T-1)$ and $V'_B(1, T-1) > V'_G(1, T-1)$ because $\frac{\partial \phi^g}{\partial \phi} = \frac{\gamma}{\alpha}$ and $\frac{\partial \phi^b}{\partial \phi} = \frac{(1-\gamma)}{(1-\alpha)}$ when evaluating equations (23) and (24) at $\phi = 1$.

Hence the function $\frac{V_B(\phi, T-1)}{V_G(\phi, T-1)}$ is continuously increasing in $\phi \in [\phi_B^*(T-1), 1]$ and not differentiable at the kink $\bar{\phi}_B^b(T-1)$.

Step 2) Increasing $\frac{V_B(\phi, 0)}{V_G(\phi, 0)}$.

Now we generalize the proof for any $t < T-1$, in particular, for $t = 0$.

At any age t , the value functions for good and bad firms in the range $\phi \in [\phi_B^*(t), 1]$ will have at most as many kinks as future possible histories that end in bad results and produce an update equal to indifference points $\phi_B^*(t+\tau)$ for all $\tau \in \{1, 2, \dots, T-t\}$. More specifically, define s^τ the possible histories after τ periods. If $\tau = 1$, there are only two possible histories in the next period (g and b), if $\tau = 2$, there are four possible histories two periods from the current one (gg, gb, bg and bb) and so on. Define $\rho_i(s^\tau)$ the probability history s^τ occurs, which depends on the firm type $i = \{G, B\}$. Finally, define $\phi(s^\tau | \phi)$ the updated reputation after τ periods, given history s^τ and prior ϕ .

Now, denote the kinks by $\hat{\phi}_{s^{\tau-1}b}$ where $s^0 = \emptyset$ and $\rho(s^0) = 1$, where the kinks at age t are given by the reputation priors that solve the following equation, for all t and $1 < \tau < T-t$,

$$\phi_B^*(t+\tau) = \frac{\rho_G(s^{\tau-1})(1-\alpha)\hat{\phi}_{s^{\tau-1}b}}{\rho_G(s^{\tau-1})(1-\alpha)\hat{\phi}_{s^{\tau-1}b} + \rho_B(s^{\tau-1})(1-\gamma)(1-\hat{\phi}_{s^{\tau-1}b})}. \quad (27)$$

Hence, at any age t there are at most $N(t) = 2^{T-t} - 1$ kinks in the region $[\phi_B^*(t), 1]$.

For example, at $T - 2$ there are at most three kinks in the relevant range $[\phi_B^*(T - 2), 1]$, determined by the reputation levels whose update may reach an indifference point during the rest of the firm's life. An obvious kink is $\hat{\phi}_b = \bar{\phi}_B^b(T - 2)$, since after a bad result reputation in $T - 1$ is $\phi_B^*(T - 1)$. However there are two more kinks. One is given by a reputation level $\hat{\phi}_{gb}$ such that, after a history gb , reputation will be updated to $\phi_B^*(T)$. The other is given by a reputation level $\hat{\phi}_{bb}$ such that, after a history bb , reputation will be updated to $\phi_B^*(T)$ as well. There are three kinks because, starting from $T - 2$, there are 3 possible future histories that end in bad results.

These kinks $\hat{\phi}_{s^{\tau-1}, b}$ can be ranked from lowest $\hat{\phi}_1$ to highest $\hat{\phi}_{N(t)}$. For each reputation ϕ , define $\hat{\Phi}(\phi, t)$ the subset of histories s^τ for all $\tau \in \{1, 2, \dots, T - t\}$ that generate kinks at the reputation level $\phi \in [\phi_B^*(t), 1]$ or higher, where the number of elements on $\hat{\Phi}(\phi, t)$ increase with ϕ . For example, at $T - 2$, if $\phi_B^*(T) = \phi_B^*(T - 1)$, then $\hat{\phi}_{gb} < \hat{\phi}_b < \hat{\phi}_{bb}$ and we can construct $\hat{\Phi}(1, t) = \emptyset$, $\hat{\Phi}(\hat{\phi}_{bb}, t) = \{bb\}$, $\hat{\Phi}(\hat{\phi}_b, t) = \{b, bb\}$ and $\hat{\Phi}(\hat{\phi}_{gb}, t) = \{gb, b, bb\}$.

Considering this notation, the value function for bad firms in the top reputation range $\phi \in [\hat{\phi}_{N(t)}, 1]$ has a derivative,

$$V_i'(\phi, t) = (\alpha - \gamma)(1 + \kappa) \left[1 + \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau} \rho_i(s^\tau) \frac{\partial \phi(s^\tau | \phi)}{\partial \phi} \right] \quad (28)$$

in the range just below it (i.e., $\phi \in [\hat{\phi}_{N(t)-1}, \hat{\phi}_{N(t)})$) the derivative is

$$V_i'(\phi, t) = (\alpha - \gamma)(1 + \kappa) \left[1 + \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau \setminus \hat{\Phi}(\hat{\phi}_{N(t)}, t)} \rho_i(s^\tau) \frac{\partial \phi(s^\tau | \phi)}{\partial \phi} \right] \quad (29)$$

and so on.

Similarly, the second derivatives of value functions for bad firms in these two upper ranges are

$$V_i''(\phi, t) = (\alpha - \gamma)(1 + \kappa) \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau} \rho_i(s^\tau) \frac{\partial^2 \phi(s^\tau | \phi)}{\partial \phi^2} \quad (30)$$

and

$$V_i''(\phi, t) = (\alpha - \gamma)(1 + \kappa) \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau \setminus \hat{\Phi}(\hat{\phi}_{N(t)}, t)} \rho_i(s^\tau) \frac{\partial^2 \phi(s^\tau | \phi)}{\partial \phi^2} \quad (31)$$

These derivatives, as in $T - 1$, just depend on derivatives of reputation updating with respect to the prior ϕ . Generalizing equations (23) and (24), first derivatives for any possible history s^τ are,

$$\frac{\partial \phi(s^\tau | \phi)}{\partial \phi} = \frac{\rho_G(s^\tau) \rho_B(s^\tau)}{[\rho_G(s^\tau) \phi + \rho_B(s^\tau) (1 - \phi)]^2} > 0. \quad (32)$$

Generalizing equations (25) and (26), second derivatives for any possible history s^τ are,

$$\frac{\partial^2 \phi(s^\tau | \phi)}{\partial \phi^2} = - \frac{\rho_G(s^\tau) \rho_B(s^\tau) (\rho_G(s^\tau) - \rho_B(s^\tau))}{[\rho_G(s^\tau) \phi + \rho_B(s^\tau) (1 - \phi)]^3}. \quad (33)$$

Now we can proceed in the proof following the same steps as in period $T - 1$. First, at any range between kinks at t , $V_B''(\phi, t) \geq V_G''(\phi, t)$ at any point where $V_B'(\phi, t) \geq V_G'(\phi, t)$. This comes from subtracting $V_B''(\phi, t) \geq V_G''(\phi, t)$ (from equations like 30) using equation 33. This guarantees that, inside each range, the ratio of value functions never achieve a local maximum.

Second, the derivative of the ratio at $\phi_B^*(t)$ and at $\phi = 1$ are positive, following the same argument than in the analysis for $T - 1$. Finally, since moving to upper ranges we add bad events, which are more likely for bad firms, it is still the case that $V_B'(\hat{\phi})_{\mathbf{R}} - V_B'(\hat{\phi})_{\mathbf{L}} > V_G'(\hat{\phi})_{\mathbf{R}} - V_G'(\hat{\phi})_{\mathbf{L}}$, at all kinks $\hat{\phi}$. This shows that the ratio of value function increase at kinks as well, or kinks are convex.

The combination of these three properties is enough to show that $\frac{V_B(\phi, t)}{V_G(\phi, t)}$ is increasing in the range $[\phi_B^*(t), 1]$ for all t , including the initial age $t = 0$. Hence, the ratio $\frac{V_B(\phi, 0)}{V_G(\phi, 0)}$ increases with initial reputation ϕ_0 .

Q.E.D.

The next proposition shows that, without regulation (this is, $F = 0$), the unobservable entry cost for good firms C , uniquely determines the market size M . It also shows that, without regulation, firms enter at the lowest possible reputation level operating in the market. Hence reputation does not go down during the firm's life.

Proposition 3 *Without regulation (i.e., $F = 0$), entry costs for good firms C uniquely determines the size of the market in equilibrium M .*

Proof Parameters uniquely determine the stopping point for bad firms $\phi_B^*(0)$, and value functions for bad and good firms $V_B(\phi_B^*(0), 0) = 0$ and $V_G(\phi_B^*(0), 0) > 0$. From entry conditions 10, when $F = 0$, $\phi_0 = \phi_B^*(0)$, since $V_B(\phi_0, 0) = 0$. Then, equation $V_G(\phi_0, 0)M^{-\eta} = C$ uniquely pins down market size M without regulation. Q.E.D.

Now we can analyze the impact of regulation on welfare, through the impact of regulatory entry costs F on the market size M .

Proposition 4 *The increase of regulatory entry costs F increase the size of the market M when the prior assigned to entrants ϕ_0 is low and decrease the size of the market M when the prior assigned to entrants ϕ_0 is high.*

Proof Take the difference between the entry conditions (10), such that,

$$(V_G(\phi_0, 0) - V_B(\phi_0, 0)) M^{-\eta} = C.$$

From Proposition 2 we know that a given $F \in [0, \infty)$ uniquely determines the entry reputation $\phi_0 \in [\phi_B^*(0), 1)$ for a given C . If the previous equation were differentiable, the condition for the market size to increase would be given by $\frac{\partial V_G(\phi_0, 0)}{\partial \phi_0} > \frac{\partial V_B(\phi_0, 0)}{\partial \phi_0}$. This comes from taking derivative with respect to ϕ_0 ,

$$\frac{\partial M}{\partial \phi_0} = \frac{\left[\frac{\partial V_G(\phi_0, 0)}{\partial \phi_0} - \frac{\partial V_B(\phi_0, 0)}{\partial \phi_0} \right] M}{\eta [V_G(\phi_0, 0) - V_B(\phi_0, 0)]}$$

However, we also know from Proposition 2 that value functions are not differentiable everywhere but almost everywhere, except at the kinks. We will show the impact of F on M in the ranges that are piece-wise differentiable for high and low reputation of entrants ϕ_0 . When ϕ_0 is already high, a further increase in F reduces the market size M . To see this, consider a ϕ_0 very close to 1, in the range of value functions that are differentiable above the last kink. In this range we can write the value function at the initial period sequentially as in equation (28). Evaluated at $\phi_0 = 1$, we can rewrite the

difference of derivatives between good and bad firms as,

$$V'_G(1, 0) - V'_B(1, 0) = (\alpha - \gamma)(1 + \kappa) \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau} [\rho_G(s^\tau) - \rho_B(s^\tau)] \frac{\rho_B(s^\tau)}{\rho_G(s^\tau)} < 0, \quad (34)$$

from evaluating equation (32) at $\phi = 1$.

The expression is negative because $\sum_{s^\tau} \frac{\rho_B^2(s^\tau)}{\rho_G(s^\tau)} > 1$.

Contrarily, when ϕ_0 is low enough, a further increase in F increase the market size M . To show this, take the difference of derivatives, evaluating equation (32) at $\phi = 0$,

$$V'_G(0, 0) - V'_B(0, 0) = (\alpha - \gamma)(1 + \kappa) \sum_{\tau=1}^{T-t} \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau} [\rho_G(s^\tau) - \rho_B(s^\tau)] \frac{\rho_G(s^\tau)}{\rho_B(s^\tau)} > 0, \quad (35)$$

which is positive since $\sum_{s^\tau} \frac{\rho_G^2(s^\tau)}{\rho_B(s^\tau)} > 1$.

From the proof of Proposition 2, we know that, close to the indifference point $\phi_B^*(0)$, derivatives are given by

$$V'_i(\phi, 0) = (\alpha - \gamma)(1 + \kappa) \left[1 + \sum_{\tau=1}^T \beta^\tau (1 - \epsilon)^\tau \sum_{s^\tau \setminus \hat{\Phi}(\phi_B^*(0), 0)} \rho_i(s^\tau) \frac{\partial \phi(s^\tau | \phi_0)}{\partial \phi_0} \right], \quad (36)$$

where the derivatives of reputation updating in some states are erased from the computation of the value function derivatives. As discussed, this effect reduced $V'_B(\phi)$ more than $V'_G(\phi)$. This means that equation (35) is a sufficient condition for $V'_G(\phi_B^*(0), 0) > V'_B(\phi_B^*(0), 0)$ using equation (36) when evaluating $\phi_B^*(0) = 0$. Q.E.D.

The intuition for this non monotonic relation between regulation and reputation relies on learning properties. When regulatory entry costs increase, both good and bad firms require more expected profits to enter and compensate those costs. There are two channels to raise expected profits: an increase in the reputation assigned to entrants and an increase in aggregate prices in the market. If the entry reputation without regulation is low enough, any increase in the initial reputation raises expected profits more for good firms than for bad ones. This is because learning becomes easier and good firms can take more advantage in terms of a future reputation improvement. The large affluence of good firms increases market size, decreases aggregate

prices and improves welfare. Contrarily, if entry reputation without regulation is already high, any increase in the initial reputation raises expected profits more for bad firms than for good ones. This is because learning becomes more difficult and bad firms are insured against a reputation loss. The large affluence of bad firms decreases market size, increases aggregate prices and reduces welfare.

4 A Numerical Example

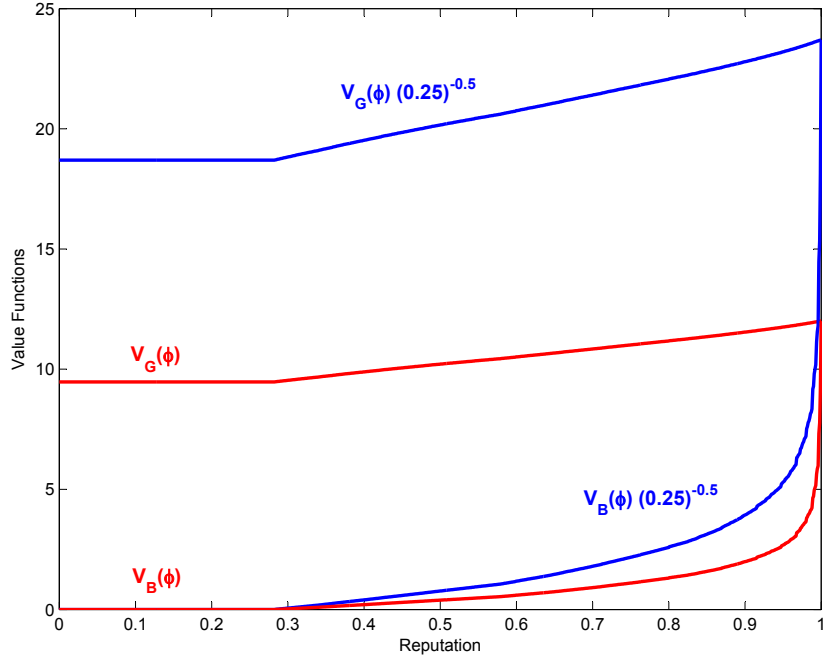
For illustrative purposes, we simulate the model for a large grid of ϕ values, using parameters, $\alpha = 0.8$, $\gamma = 0.3$, $\beta = 0.95$, $\eta = 0.5$, $C = 10$, $c = \epsilon = 0$ and $\kappa = -1$.

First, we compute the fixed points of value functions for good and bad firms $V_G(\phi)$ and $V_B(\phi)$ as $T \rightarrow \infty$. Figure 1 shows these values considering $M = 1$ and $M = 0.25$. We can confirm the main properties of value functions. First, good firms have higher continuation values than bad firms for all reputation levels $\phi \in (0, 1)$, since good firms construct reputation faster than bad firms. Second, value functions are the same for a reputation $\phi = 1$. Third, given that $M = 1$ represents a market size four times as large as $M = 0.25$ and since M scales value functions by a power of a half, continuation values with $M = 0.25$ double those with $M = 1$ at each reputation level ϕ .

Figure ?? shows market size M and entrants' reputation ϕ_0 corresponding to different regulatory entry costs F . As these regulation costs increase, ϕ_0 increases monotonically but M achieves a maximum and then starts to decrease.

The intuition for this result is the following. First assume $F = 0$. From Proposition 3, $\phi_0 = \phi_B^*(0) = 0.3$, which is determined by the level of reputation at which values for bad firms become zero. We also know from Proposition 3 that, at ϕ_0 , $V_G(\phi_0)M^{-\eta} = C$. In this example, $V_G(0.3) = 9.49$ and $C = 10$, which determines an equilibrium $M = 0.9$. This is why, in the first panel of Figure 2, $\phi_0 = 0.3$ and $M = 0.9$ cross the vertical axis, when $F = 0$. Even when difficult to note at first sight in Figure 1, for the range $\phi \in [0.3, 0.62]$, $V_G(\phi)$ grows faster than $V_B(\phi)$. Then, in the range $\phi \in [0.62, 1]$, $V_G(\phi)$ grows slower than $V_B(\phi)$. Hence, when imposing $F > 0$, but close to 0, both ϕ_0 and M increase. When $F = 0.85$, such that $\phi_0 = 0.62$, $M = 0.98$ and reaches the maximum. As F grows above 0.85, it generates further improvements in ϕ_0 but reductions in M .

Figure 1: Value Functions for Good and Bad Firms - Two Possible Market Sizes



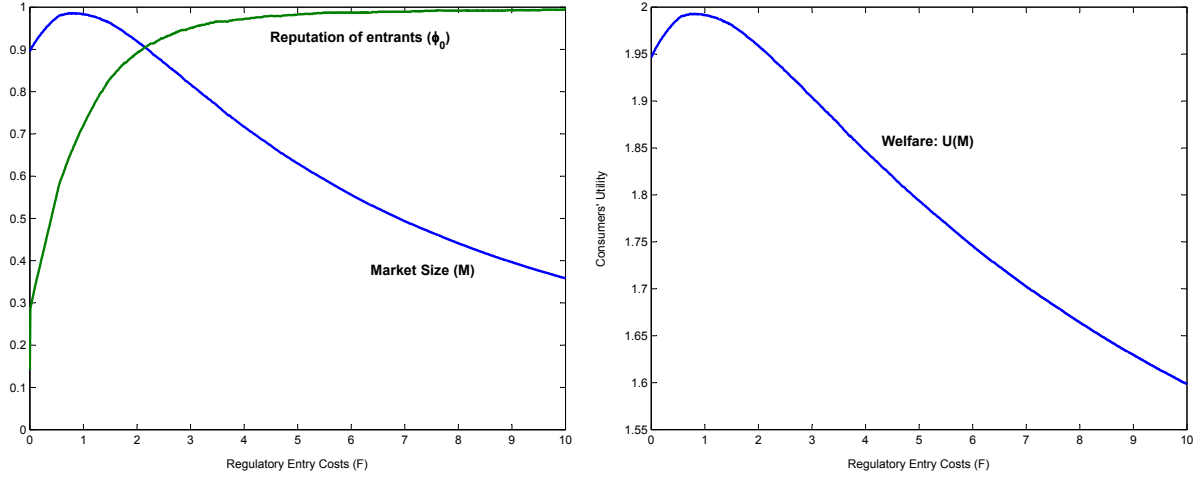
The second panel of Figure 2 shows the effects of regulation on consumers' utility. From equation (4), the maximum level of welfare is achieved when regulation maximizes M . This happens when $F = 0.85$ such that $M = 0.98$ and entrants have a reputation $\phi_0 = 0.62$.

5 Conclusions

This paper evaluates the welfare effect of regulatory entry costs (such as certification and minimum standard requirements) when firms are concerned about the consumers' perception of their quality, this is, their reputation. We find that there is an optimal level for the quality of new firms in the market, which can be achieved as an equilibrium without regulation only by chance. Even when this justifies regulation to improve welfare, excessive entry costs may indeed reduce welfare below levels achievable without regulation. This is because reputation concerns complement regulatory efforts when the quality of entrants is low, but counteract those efforts when the quality of entrants is high.

The model is flexible enough to discuss other forms of regulation, such as periodic

Figure 2: Reputation of Entrants, Market Size and Welfare Effects of Regulation



monitoring and regular supervision. Results suggest optimal ways of combining regulation to entrants with regulation to incumbents in order to maximize welfare. For example, it would be optimal to impose regulatory costs up front (taxes to entrants) followed by subsidies positively related to age and reputation (subsidies to old incumbents with good records).

These results are relevant in discussing optimal regulation in financial markets, since they are characterized by free entry and exit and by strong reputation concerns. In particular, this paper sheds light on whether reputation and regulation are complements or substitutes, a discussion reopened after the recent financial crisis because of the seemingly loss of market self-discipline. Our results suggest that reputation and regulation are complements or substitutes depending on the level of quality of new firms in the market. Hence, it is possible reputation concerns where insufficient to impose self-discipline in financial markets, leading to a collapse. However these reputation concerns may turn excessive regulation perilous to welfare as well.

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A Appendix

A.1 Allowing for Money Burning

In the main body of the paper, we have assumed good firms cannot burn money publicly when entering as a way to signal their type. Here we relax this assumption and allow entrants to choose how much money to burn, as an observable sunk cost $S \geq 0$, so as to maximize future profits. In this case, there are multiple equilibria equivalent to all possible equilibria discussed in Section 3.2 for $F \geq 0$ plus all their possible random combinations. Some of these equilibria are characterized by pooling strategies in which bad firms follow exactly the same strategies as good ones, which are the equilibria in Proposition 2 for different F . There are also equilibria based on good and bad firms having different random strategies. In these cases the reputation assigned to entrants is conditional on the amount of money burnt (i.e., $\phi_0(S)$). Finally, there are also multiple equilibria that depend on out of the equilibrium beliefs.

All these equilibria, in which firms burn money with positive probability, should still fulfill entry conditions with equality, such that good and bad entrants are ex-ante indifferent to follow the burning money strategies, given consumers' beliefs about these strategies for the two types. Defining $\sigma_i(S)$ the probability that firm i spend $S > 0$, $E_i(V_i(\phi_0(S))) = \int_{S=0}^{\infty} \sigma_i(S) V_i(\phi_0(S)) dS$ (with $\int_{S=0}^{\infty} \sigma_i(S) dS = 1$) the ex-ante expected value function for firm i and $E_i(S) = \int_{S=0}^{\infty} \sigma_i(S) S dS$ its ex-ante expected money burnt.

$$\begin{aligned}\Pi_G(\sigma_i(S)) &= E_G(V_G(\phi_0(S)))M^{-\eta} - C - E_G(S) = 0 \\ \Pi_B(\sigma_i(S)) &= E_G(V_B(\phi_0(S)))M^{-\eta} - E_B(S) = 0\end{aligned}\tag{37}$$

We show here that the equilibrium without money burning ($S = 0$) is the unique equilibrium with respect to an intuitive stability criterion or, which is the same, all other equilibria that burns some money with positive probability are unstable in an intuitive sense.

Without loss of generality, consider an equilibrium in which firms just randomize between burning an amount $\bar{S} > 0$ in public and no burning any money. A small perturbation to firms' behavior would lead this proposed equilibrium to unravel. This is because a small reduction in the probability of firms burning $\bar{S}$ will increase the incentives for them to do so. This in turn will lead firms to burn money less frequently, and so on until the process reaches a boundary. By contrast, the equilibrium in which good firms never burn money is stable in the sense that, if bad firms become more likely to burn money, that reduces the incentives to bad firms to do so, and the

behavior has a tendency to return to the equilibrium point in which no one burns money at all.

To define stability formally, we follow Gentzkow and Shapiro (2006). We say an equilibrium is stable if $\Pi_i(\sigma_i(S)) = 0$ and $\frac{\partial \Pi_i(\sigma_i(S))}{\partial \sigma_i(S)} \Delta \sigma_i(S) < 0$ for all possible $\Delta \sigma_i(S)$ and all firms' types. This means that, an increase of the probability of spending S strictly reduces the incentives to do it. This definition captures the idea that, when a type's behavior is perturbed (maintaining consumers' beliefs), it ought to have an incentive to move back to the equilibrium point. Formally,

Proposition 5 *There exists a unique stable equilibrium in which firms do not burn money.*

Proof Since $\frac{\partial \Pi_i(\sigma_i(S))}{\partial \sigma_i(S)} = -S < 0$ when maintaining consumers' beliefs, the only stable equilibrium is given by $\sigma_i(S) = 0$ for all S and all firms. These are the only strategies for which $\Delta \sigma_i(S)$ cannot be negative and then, the only situation in which $\frac{\partial \Pi_i(\sigma_i(S))}{\partial \sigma_i(S)} \Delta \sigma_i(S) < 0$ for all possible $\Delta \sigma_i(S)$.

Q.E.D.

This concept of stability is reminiscent of evolutionary stable strategies, that require basically for an stable equilibrium to "repel invaders". As discussed in Fudenberg and Levine (1999), a stable strategy should be a best reply to itself and a better reply to all other best replies than these are to themselves.