

Vertical Specialization and International Business Cycle Synchronization*

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Abstract

We explore the impact of vertical specialization – trade in goods across multiple stages of production – on the relationship between trade and international business cycle synchronization. We develop a model in which the degree of vertical specialization is endogenously determined by comparative advantage across heterogeneous goods and varies with trade barriers between countries. We show analytically that fluctuations in measured productivity in our model are not linked across countries through trade, despite the greater transmission of technology shocks implied by higher degrees of vertical specialization. In numerical simulations, we find this transmission is insufficient in generating substantial dependence of business cycle synchronization on trade intensity.

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1 Introduction

In recent empirical work, several authors have documented a link between international trade and cross-country business cycle synchronization. Frankel and Rose (1998) established that country pairs that trade more exhibit on average higher correlations in their business cycles, as measured by fluctuations in GDP.¹ However, Kose and Yi (2001) and (2006) have illustrated what they have called a trade-comovement puzzle: standard international real business cycle models along the lines of Backus, Kehoe, and Kydland (1994) cannot quantitatively account for the relationship between trade and business cycle comovement.

In this paper, we develop and quantitatively assess the ability of a model with vertical specialization - which we define as the production of goods in multiple stages spread across countries - to generate stronger business cycle synchronization between countries that trade more. In addressing the empirical facts behind the trade-comovement puzzle, several authors have suggested that it is not only the volume of trade, but particular features of specialization patterns and industrial structure associated with increased trade that lead to business cycle synchronization. Frankel and Rose (1998) conjectured that intra-industry trade tends to make countries more correlated, while Kose and Yi (2001) have suggested that vertical specialization may be the key linkage that synchronizes business cycles of countries with close trade relationships.² The intuition for this is that if closer trade relationships are characterized by tighter links in the chain of production, fluctuations in one economy should be transmitted more to the other. Indeed, Ng (2007) finds that direct measures of bilateral vertical specialization are related to increased business cycle correlation, and that intra-industry trade plays no significant role once vertical specialization is taken into account.³ In addition, Di Giovanni and Levchenko (2008), using cross-country industry-level data, find that the correlation of output in industries with vertical production linkages is more sensitive to trade. In this paper, we ask whether an international business cycle model augmented with vertical specialization provides a mechanism that accounts for these empirical findings

¹Recent papers that have confirmed this finding are Clark and Van Wincoop (2001) and Baxter and Kouparitsas (2005).

²Regarding the magnitude and growth of vertical specialization, Feenstra and Hanson (1996) find that the share of imported intermediate inputs in total purchases of materials in the US increased from 5.3% to 10.7% from 1972 to 1990. Hummels, Rapoport, and Yi (1998) also document a considerable increase in value of imported intermediate inputs in exports across many industrialized and developing countries.

³Fidrmuc (2004) and Calderon, Chong, and Stein (2007) find that business cycle comovement is linked to intra-industry trade and to cross-country similarity in industrial structure.

linking vertical specialization and business cycle comovement.

Our model builds on a two-country model of international business cycles driven by productivity shocks, as in Backus, Kehoe, and Kydland (1992) and (1994). The main innovation of our work is to introduce a Ricardian framework for production and trade with two stages of production into an international business cycle setting. In each stage, the degree of specialization by each country is endogenously determined. In this respect, our model is similar to Yi (2003), extended to an environment with aggregate uncertainty. Additionally, our modeling of comparative advantage and specialization draws on the methods of Eaton and Kortum (2002) in formulating heterogeneity across producers probabilistically.

In our model, production of final tradeable goods occurs in two stages. In the first stage, a continuum of intermediate goods can be produced in each country using only domestic factors. The second stage of production uses all the intermediate inputs as well as value added from domestic factors to produce final goods. Countries possess technologies to produce every good in each stage, but the efficiency with which each good can be produced is randomly distributed across goods in each country. The differences in efficiency across countries for each good lead each country to specialize in a range of goods in each stage of production. The specialization that occurs in each stage means that each country requires inputs from the other to produce final output. Since this link is stronger when countries trade a wider range of goods, this vertical specialization provides a potential mechanism for the model to generate increased business cycle correlation with higher trade.

When we simulate our model under different assumptions on the costs of trade and the determinants of comparative advantage, we find that the model is able to generate significant increases in the degree of vertical specialization. However, the model is unable to quantitatively account for the increase in GDP correlation that is associated with higher trade in the data. There are two sides to this result: one analytical, and one numerical. On the analytical side, we derive expressions for changes in real GDP and measured Total Factor Productivity (TFP) in our model and show that, when standard national accounting methods are used to construct these aggregates, fluctuations in TFP in each country depend only on domestic shocks. Hence, trade links do not contribute to transmit technology shocks across countries directly into measure productivity, regardless of trade intensity. Changes in real GDP, though, are accounted for by changes in TFP as well as changes in factor inputs; the numerical side of our result is that the correlation across countries of changes in inputs

is not sufficient to generate substantial correlation in real GDP, nor a significant dependence of business cycle correlation on trade intensity.

This paper is most closely related to a series of papers that attempt to account for the trade-comovement relationship. While Kose and Yi (2006) show that the standard business cycle framework cannot account for the relationship between trade and comovement, Burstein, Kurz, and Tesar (2008) show that production structures that allow for production sharing among countries can deliver tighter business cycle synchronization.⁴ Their modeling of production sharing imposes high complementarity in inputs and exogenous patterns of specialization, while we develop an approach in which the degree of specialization is endogenous. Our results suggest that for looking at real GDP, relaxing key assumptions such as constant elasticity of substitution preferences and perfect competition could be crucial for changing the implications of trade intensity for business cycle behavior. In recent work Bergin, Feenstra, and Hanson (2007) combine a model of outsourcing of production with non-CES preferences to account for the variance of output in outsourcing industries in Mexico compared to the US. Additionally, Drozd and Nosal (2008) deviate from the standard neoclassical framework and attempt to explain the link between trade and comovement in a model featuring a low short-run price elasticity of trade coexisting with a high long-run price elasticity.

Our results on the inability for the mechanism of vertical specialization to generate changes in the behavior of macroeconomic aggregates resemble other results in the literature, particularly regarding models of international trade. Alessandria and Choi (2007) find that augmenting a standard model with producer heterogeneity and sunk costs of exporting do not affect business cycle dynamics. Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) find that a wide variety of models, including those of Eaton and Kortum (2002) and Armington (1969), have identical predictions for trade aggregates.

The rest of the paper is organized as follows: Section 2 lays out our model. Section 3 discusses the potential for vertical specialization in our model to affect the dynamics of real GDP and measured productivity. Section 4 briefly reviews some of the data on vertical specialization and links various measures to our model. Section 5 describes our numerical experiments and their results, and Section 6 concludes.

⁴Following a different approach, Huang and Liu (2007) argue that multiple stages of production increase business cycle comovement in the presence of nominal rigidities.

2 Setup of the Model

The model is a two-country real business cycle model in the tradition of Backus, Kehoe, and Kydland (1992) and (1994), modified to include vertical specialization in traded goods in a sense similar to Yi (2003). Goods are produced in two stages with the second stage of production (production of “final goods”) using goods produced in the first stage (“intermediate goods”).⁵ Producers of all goods are perfectly competitive, and the presence of Ricardian comparative advantage, which we model as in Eaton and Kortum (2002), leads countries to endogenously specialize across a continuum of goods in each stage. Production is vertically specialized to the extent that one country uses imported intermediate goods to produce output that is exported.

The time horizon is infinite and discrete, and periods are indexed by $t = 0, 1, \dots$. In each country, there are two sectors of production: a tradeable sector and a nontradeable sector. Producing final goods in either sector requires capital, labor, and intermediate inputs. There is a continuum of measure one of goods in the first stage of production, and in the second stage of tradeable production. To economize on notation, except where noted below, we index both intermediate and final goods in the tradeable sector by ω , although an intermediate good labelled ω and a final good labelled ω are distinct commodities. We use subscripts to refer to stages of production, $s = 1, 2$, and time periods, and we use superscripts to refer to countries, $i, j = 1, 2$. When a variable has a double superscript, the first index refers to the source country and the second refers to the destination.

We describe the technologies available for producing tradeable goods in each country, then discuss the pattern of specialization that arises.

2.1 Production of Tradeable Goods

Each first-stage intermediate input ω can be produced in each country i using a constant returns to scale technology combining physical capital and labor inputs with efficiency denoted by $A_{1t}^i z_1^i(\omega)$. A_{1t}^i is a country-specific time-varying productivity shock common to all intermediate goods producers in country i , and $z_1^i(\omega)$ is a good-specific efficiency that is

⁵Our specification of intermediate input use in production differs from that in Krugman and Venables (1995) and Eaton and Kortum (2002), who assume that all goods play the role of both intermediate and final goods. We do this to explore the possibility of a country specializing relatively more in intermediate or final goods. Our structure is more similar to Yi (2003), who uses two stages of production.

constant over time. Output of each intermediate good ω produced by country i is given by:

$$(1) \quad y_{1t}^i(\omega) = A_{1t}^i z_1^i(\omega) k_{1t}^i(\omega)^\alpha \ell_{1t}^i(\omega)^{1-\alpha} ,$$

where $k_{1t}^i(\omega)$ and $\ell_{1t}^i(\omega)$ denote capital and labor, respectively, used in the production of good ω , and $\alpha \in (0, 1)$.

The minimum unit cost of producing intermediate good ω in country i can be expressed as:

$$(2) \quad q_{1t}^i(\omega) = \frac{q_{1t}^i}{z_1^i(\omega)} ,$$

where q_{1t}^i is the cost of the input bundle scaled by aggregate productivity in first stage production:

$$q_{1t}^i = \frac{(r_t^i)^\alpha (w_t^i)^{1-\alpha}}{A_{1t}^i \alpha^\alpha (1-\alpha)^{1-\alpha}} .$$

Here, w_t^i denotes the wage and r_t^i is the rental rate of capital in country i .

Purchasers of intermediate goods in each country buy each good from the source country that offers the lowest price after accounting for trade costs. We adopt the standard “iceberg” cost formulation, so that delivering one unit of any stage- s good from country i to country j requires shipping τ_s^{ij} units, with $\tau_s^{ii} = 1$ and $\tau_s^{ij} \geq 1$. Therefore, the price at which country i purchases intermediate good ω is given by:

$$(3) \quad p_{1t}^j(\omega) = \min \{ q_{1t}^i(\omega) \tau_s^{ij} : i = 1, 2 \} .$$

The technology for producing output of final good ω is:⁶

$$(4) \quad y_{2t}^i(\omega) = A_{2t}^i z_2^i(\omega) (k_{2t}^i(\omega)^\alpha \ell_{2t}^i(\omega)^{1-\alpha})^\eta \left(\int m_t^i(\omega, \omega')^{\frac{\sigma-1}{\sigma}} d\omega' \right)^{\frac{(1-\eta)\sigma}{\sigma-1}} ,$$

where $k_{2t}^i(\omega)$, and $\ell_{2t}^i(\omega)$ denote capital and labor used in the production of final good ω , A_{2t}^i and $z_2^i(\omega)$ are time-varying aggregate and constant good-specific efficiency for final goods, respectively, and $m_t^i(\omega, \omega')$ is the use of intermediate good ω' in the production of final good

⁶Unless otherwise noted, integration is over the entire set of goods in the relevant stage of production.

ω . The parameter σ is the elasticity of substitution between different intermediate inputs. We define the aggregate composite of intermediate goods used in the production of final tradeable goods as

$$(5) \quad M_{Tt}^i = \int \left(\int m_t^i(\omega, \omega')^{\frac{\sigma-1}{\sigma}} d\omega' \right)^{\frac{\sigma}{\sigma-1}} d\omega ,$$

where the subscript T distinguishes variables for the tradeable sector from corresponding variables for the nontradeable sector, discussed below.

The unit cost of production for final good ω is given by

$$q_{2t}^i(\omega) = \frac{q_{2t}^i}{z_2^i(\omega)} ,$$

where

$$q_{2t}^i = \left(\frac{(r_t^i)^\alpha (w_t^i)^{1-\alpha}}{\eta A_{1t}^i \alpha^\alpha (1-\alpha)^{1-\alpha}} \right)^\eta \left(\frac{P_{1t}^i}{1-\eta} \right)^{1-\eta} .$$

Here, the term P_{1t}^i denotes the price index associated with purchases of all intermediate goods:

$$(6) \quad P_{1t}^i = \left(\int p_{1t}^i(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)} .$$

Similar to first-stage goods, final goods are also purchased in each country from the source offering the lowest price adjusted for trade costs. Thus, $p_{2t}^i(\omega)$ is given by:

$$p_{2t}^j(\omega) = \min \{ q_{2t}^i(\omega) \tau_s^{ij} : i = 1, 2 \} .$$

Final goods are purchased by households to form composite consumption and investment of tradeable goods, which we denote

$$(7) \quad X_{Tt}^j = \left(\int x_{Tt}^j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} ,$$

$$(8) \quad C_{Tt}^j = \left(\int c_{Tt}^j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} .$$

Therefore, the price index for final tradeable goods has the same form as (6), and is given by:

$$P_{2t}^i = \left(\int p_{2t}^i(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}.$$

In the next section, we impose further structure on the distribution of good-specific efficiencies, and derive how these assumptions shape the pattern of trade and the prices paid for goods in each country.

2.2 Technology structure for tradeable goods

We follow the probabilistic representation of Eaton and Kortum (2002) for good-specific efficiencies. For each country i and stage s , z_s^i in (1) and (4) is drawn from a Fréchet distribution characterized by the cumulative distribution function:

$$F_s^i(z) = e^{-T_s^i z^{-\theta}},$$

for $s = 1, 2$ and $i = 1, 2$, where $T_s^i > 0$ and $\theta > 1$. Efficiency draws are independent across goods, stages, and countries. The probability that a particular stage- s good ω can be produced in country i with efficiency less than or equal to z_s^i is given by $F_s^i(z_s^i)$. Since draws are independent across the continuum of goods, $F_s^i(z_s^i)$ also denotes the fraction of stage- s goods that country i is able to produce with efficiency at most z_s^i .

As in Eaton and Kortum (2002), the cross country differences in T_s^i reflect absolute advantage in the production of goods in each stage: a country with a higher T_s^i draws efficiencies for all goods in a given stage from a better distribution. The parameter θ determines the dispersion of efficiency draws, and hence governs heterogeneity across goods and leads to comparative advantage within each stage of production. In addition, since the terms T_s^i may differ for $s = 1, 2$, our technological structure allows for comparative advantage across stages, determined by the ratio T_1^i/T_2^i across countries. A country with a higher T_1^i/T_2^i , for example, is relatively more productive in intermediate inputs.

Following Eaton and Kortum (2002), it is straightforward to show that the distribution

of prices of stage-1 goods that country i offers to country j is equal to:

$$G_{st}^{ij}(p) = 1 - e^{-T_s^i (q_{st}^{ij})^{-\theta} p^\theta} ,$$

where

$$(9) \quad q_{st}^{ij} = q_{st}^i \tau_s^{ij} .$$

The probability that country j is able to purchase a certain good at price below p is the probability that either source country offers country j a price below p , that is, the probability that $\min \{q_{st}^{ij}, q_{st}^{jj}\} \leq p$. This means that the overall distribution of prices of stage- s goods available in country j is

$$(10) \quad G_{st}^j(p) = 1 - e^{-\Phi_{st}^j p^\theta} ,$$

where

$$(11) \quad \Phi_{st}^j \equiv T_s^i (q_{st}^{ij})^{-\theta} + T_s^j (q_{st}^{jj})^{-\theta} .$$

Since the only dimension of heterogeneity across goods in a given stage is efficiency, we can aggregate across goods by aggregating across efficiency levels or across prices. With this transformation, the price index for stage- s goods can be written as $P_{st}^j = \left(\int_0^\infty p^{1-\sigma} dG_{st}^j(p) \right)^{1/(1-\sigma)}$, which, using the distributions (10) is:

$$(12) \quad P_{st}^j = \left(\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right)^{1/(1-\sigma)} (\Phi_{st}^j)^{-1/\theta} ,$$

where Γ is the Gamma function, $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$.

The probability that country j buys a certain good from country i , or alternatively the fraction of goods that country j buys from country i , is given by the probability that $q_{st}^{ij} \leq \min \{q_{st}^{ij}, q_{st}^{jj}\}$. As Eaton and Kortum (2002) show, this is equal to:

$$(13) \quad \lambda_{st}^{ij} = \frac{T_s^i (q_{st}^{ij})^{-\theta}}{\Phi_{st}^j} .$$

As in Eaton and Kortum (2002), it is also true that, because the distribution of stage- s goods actually purchased by country j from country i is equal to the overall price distribution G_{st}^j , the fraction λ_{st}^{ij} of goods purchased from country i is also equal to the fraction of country j 's total expenditures on stage- s goods that it spends on goods from country i .

2.3 Production of nontradeable goods

Each country also produces a nontradeable good according to the following technology:

$$Y_{Nt}^i = A_{Nt}^i \left((K_{Nt}^i)^\alpha (L_{Nt}^i)^{1-\alpha} \right)^\eta \left(\int m_{Nt}^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{(1-\eta)\sigma}{\sigma-1}},$$

where $m_{Nt}^i(\omega)$ is the quantity of intermediate good ω purchased for use in the production of the nontradeable good. The composite intermediate inputs used in nontradeable production is

$$M_{Nt}^i = \left(\int m_{Nt}^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

The unit cost of producing nontradeable goods is

$$P_{Nt}^i = \left(\frac{(r_t^i)^\alpha (w_t^i)^{1-\alpha}}{\eta A_{Nt}^i \alpha^\alpha (1-\alpha)^{1-\alpha}} \right)^\eta \left(\frac{P_{1t}^i}{1-\eta} \right)^{1-\eta}.$$

2.4 Households

Each country is populated by an infinitely-lived representative household which values sequences of consumption of tradeable final goods, consumption of nontradeable goods, and leisure, according to the following preferences:

$$(14) \quad E \sum_{t=0}^{\infty} \beta^t \left(\left((C_{Tt}^i)^\gamma (C_{Nt}^i)^{1-\gamma} \right)^\mu (1-L_t^i)^{1-\mu} \right)^{1-\varepsilon} / (1-\varepsilon),$$

where L denotes the fraction of time devoted to labor services supplied to domestic industries, C_T denotes the tradeable composite consumption defined in (8) and C_N denotes consumption of nontradeable goods. E denotes the expectation over the entire time horizon, and $\beta \in (0, 1)$ is the household's discount factor. The parameter γ determines the fraction of aggregate

expenditure on tradeable goods.

The household also purchases tradeable and nontradeable investment goods, X_{Tt}^i and X_{Nt}^i , that are bundled to augment the aggregate capital stock K_t^i :

$$(15) \quad K_{t+1}^i = (X_{Tt}^i)^\gamma (X_{Nt}^i)^{1-\gamma} + (1 - \delta)K_t^i ,$$

where δ is the depreciation rate of capital, and again the fraction γ of investment expenditures are spent on tradeable goods. The household receives income from selling labor services and renting capital to firms in each period. We assume that countries do not trade financial assets - that is, trade in goods is balanced in each period - so the total expenditure of each country is constrained by its income:⁷

$$(16) \quad P_{2t}^i [C_{Tt}^i + X_{Tt}^i] + P_{Nt}^i [C_{Nt}^i + X_{Nt}^i] \leq w_t^i L_t^i + r_t^i K_t^i .$$

2.5 Market Clearing and Equilibrium

Let Ω_{st}^i denote the set of stage- s goods which country i produces. We define total capital stocks and labor in each country $i = 1, 2$ and in each tradeable stage $s = 1, 2$ as

$$(17) \quad K_{st}^i = \int_{\Omega_{st}^i} k_{st}^i(\omega) d\omega ,$$

$$(18) \quad L_{st}^i = \int_{\Omega_{st}^i} \ell_{st}^i(\omega) d\omega .$$

At each date t , the total supply of capital and labor by households equals the demand from producers,

$$(19) \quad K_t^i = K_{1t}^i + K_{2t}^i + K_{Nt}^i , \quad i = 1, 2 ,$$

$$(20) \quad L_t^i = L_{1t}^i + L_{2t}^i + L_{Nt}^i , \quad i = 1, 2 .$$

We use the expenditure shares defined in (13) as well as the form of the production

⁷The findings in Heathcote and Perri (2002) suggest that models with financial autarky generate international comovement that is closer to the data than models with complete financial markets. Models with complete markets have a strong risk-sharing channel from which we wish to abstract to isolate the mechanisms in our model.

technologies to write aggregated market clearing conditions for goods in each stage and sector. Since producers are perfectly competitive, the value of first-stage production of tradeable goods in country i equals the income paid to factors employed in the first stage, $w_t^i L_{1t}^i + r_t^i K_{1t}^i$. The first-stage output of country i is purchased by second-stage producers in country i and country j , totalling fractions λ_{1t}^{ii} and λ_{1t}^{ij} , respectively, of each destination's total expenditures on intermediate goods. That is,

$$w_t^i L_{1t}^i + r_t^i K_{1t}^i = \lambda_{1t}^{ii} P_{1t}^i (M_{Tt}^i + M_{Nt}^i) + \lambda_{1t}^{ij} P_{1t}^j (M_{Tt}^j + M_{Nt}^j) .$$

For final goods, payments to factors equal a fraction η of the value of final goods output, which can be constructed from foreign and domestic expenditures on final goods, so that

$$w_t^i L_{2t}^i + r_t^i K_{2t}^i = \eta (\lambda_{2t}^{ii} P_{2t}^i (C_{Tt}^i + X_{Tt}^i) + \lambda_{2t}^{ij} P_{2t}^{ij} (C_{Tt}^j + X_{Tt}^j)) .$$

Similarly, for nontradeable goods, market clearing implies that factor payments for the factors employed in the nontradeable sector in each country equal a fraction η of output:

$$(21) \quad w_t^i L_{Nt}^i + r_t^i K_{Nt}^i = \eta P_{Nt}^i (C_{Nt}^i + X_{Nt}^i) , \quad i = 1, 2 .$$

An equilibrium consists of stochastic processes for prices and quantities such that households' utility is maximized subject to (15) and (16), producers' costs are minimized, and the market clearing conditions for goods and factors inputs are satisfied each period. We solve for an equilibrium in terms of consumption, investment, and intermediate input expenditures, capital and labor supplies, factor prices, and composite price indices, with quantities aggregated across the continuum of goods in the case of the two tradeable stages. We use a standard linear approximation method to solve for recursive decision rules in the neighborhood of the model's deterministic steady state.

3 International Comovement of Real GDP and Productivity in the Model

Compared to standard international business cycle models, our model contains additional potential channels of international transmission of fluctuations through trade. The presence

of vertical specialization links countries' production processes. Also, the degree of specialization can potentially increase with trade intensity, so that countries that trade more can have production processes that are more closely linked.

In standard international business cycles in which all value-added occurs in a single stage of production, technology shocks are transmitted across countries through the imperfect substitutability of goods: a country that receives a favorable shock demands more imports as well as domestic goods for consumption and investment, so that its trading partner devotes more resources to production. In our model, there is an additional effect from having two stages of production: a country with a favorable productivity shock offers its trading partner lower prices for intermediate inputs, which makes production of final goods more efficient in the sense that the same amount of output can be produced with lower input expenditures. If the degree of vertical specialization increases with trade intensity, then there is more potential for a country to benefit from foreign technology improvements in this way.

To assess the potential impact of these mechanisms on business cycle comovement, we need to construct a measure of real value added, or GDP. In order to compare the model's predictions to data, we construct a measure of real GDP from the model's output comparable with standard national accounting methods. In our model, since there are a continuum of goods and changing trade patterns, we have some choice regarding how to compute aggregate quantities. Since our results depend heavily on the method of GDP measurement, we dwell on this point a bit here: in the first subsection below, we explain the definitions of aggregate statistics we use, and in the second subsection, we derive some of the implications of our choices for business cycle fluctuations in real GDP and aggregate measured Total Factor Productivity (TFP).

3.1 National Accounts Statistics in the Model

We construct an analogue of real GDP, that is, GDP measured in base period prices, as reported in actual data by national statistical agencies. In order to do this in a way that is as close as possible to the methods used by these agencies, we follow the recommendations in the UN's *System of National Accounts 1993* (SNA 93).

We define nominal GDP at current prices as aggregate value-added, or the difference between the total value of gross output less total expenditures on intermediate inputs. Gross

output at current producer prices is given by:

$$(22) \quad Z_t^i = \int_{\Omega_{1t}^i} q_{1t}^i(\omega) y_{1t}^i(\omega) d\omega + \int_{\Omega_{2t}^i} q_{2t}^i(\omega) y_{2t}^i(\omega) d\omega + P_{Nt}^i Y_{Nt}^i ,$$

where Ω_{st}^i is the set of stage- s goods which country i produces in period t .

Expenditure on intermediate inputs, valued at purchaser prices, is given by:

$$(23) \quad I_t^i = \int_{\Omega_{2t}^i} \left(\int p_{1t}^i(\omega') m_t^i(\omega, \omega') d\omega' \right) d\omega + \int p_{1t}^i(\omega) m_{Nt}^i(\omega) d\omega .$$

So GDP at current prices is then:

$$Y_t^i = Z_t^i - I_t^i .$$

To construct real GDP as measured in the data, we reconstruct the above formulas using constant, base-period prices for each good. Real gross output is:

$$(24) \quad \mathcal{Z}_t = \int_{\Omega_{1t}^i} q_{10}^i(\omega) y_{1t}^i(\omega) d\omega + \int_{\Omega_{2t}^i} q_{20}^i(\omega) y_{2t}^i(\omega) d\omega + P_{N0}^i Y_{Nt}^i ,$$

and real expenditures on intermediate inputs are:

$$(25) \quad \mathcal{I}_t = \int_{\Omega_{2t}^i} \left(\int p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' \right) d\omega + \int p_{10}^i(\omega) m_{Nt}^i(\omega) d\omega .$$

Real GDP is then defined as

$$(26) \quad \mathcal{Y}_t^i = \mathcal{Z}_t^i - \mathcal{I}_t^i .$$

Effectively, we are using what national statistical agencies refer to as a “double-deflation” method, deflating the current price values of gross output and intermediate consumption each by their own deflators.

A practical problem that this method raises is that country-specific period-0 producer prices are not well defined for all goods, due to the fact that specialization patterns change in the model. For example, it may be the case that a good ω is produced in country 1 in period t , but was not produced by country 1 in period 0. In this case, it is unclear at what

price we should value country 1's output of good ω in period t when calculating real gross output. In our model, country 1 would have imported the good in period 0, but the price at which the good is imported does not correspond to a producer price.

This is surely a problem in actual national accounting as well, as products are newly invented or disappear through time, and must be assessed using base period prices in order to construct a measure of real output. For these situations, the *SNA 93* recommends (paragraph 16.53) using average price changes of similar products as a proxy for the change in price of a new good between the base period and the current period. In the context of our model, following this recommendation results in using what *would have been* the base period price for a good produced in period t by a country *if the country had produced* that good in the base period. So, for example, we assign a stage-2 good ω produced by country 1 in period t but not in period 0 the base-period producer price $q_{20}^1(\omega)$, the price at which country 1 would have produced good ω in period 0.

In the appendix, we derive expressions for these constant-price accounting aggregates in terms of the composite variables defined in solving the model. This requires significant manipulation since, for example, while I_t as defined in (23) can simply be written as $P_{1t}(M_{Tt} + M_{Nt})$, no corresponding relation exists between the total value of intermediates at base period prices and the model's composite quantities.

3.2 Measured TFP correlation

In Section 5, we perform several numerical experiments to evaluate the extent to which increased trade intensity affects business cycle synchronization in the presence of vertical specialization. In this section, however, we show that, in one important respect, trade in our model does not make countries more correlated: changes in measured TFP, constructed in our model using real GDP as defined above, are not linked across countries through international trade. That is, although measured TFP is an endogenous object in our model, there is no endogenous link between TFP across countries, and hence no dependence on trade intensity of the correlation in TFP. This result is important because of the extent to which TFP synchronization affects GDP correlation. Without comovement in TFP, our model can produce endogenous comovement in GDP, but to a small degree, as we show in the next section. This is consistent with the finding in Kose and Yi (2006), that the standard model can account for the trade-comovement puzzle if it is exogenously imposed that the correlation

of TFP increases with trade intensity. While in our model, TFP is endogenous, we can prove that when measuring aggregate accounting statistics as in the data, the dynamics of TFP are pinned down by exogenous factors alone. In addition, the intuition behind the result suggests that it holds for a wide class of dynamic models with endogenous specialization and trade in intermediate goods.

Our result can be stated as follows:

Proposition 1 *Let variables with $\hat{\cdot}$ denote log deviations from the steady state. Measured TFP, defined by:*

$$\mathcal{A}_t^i = \frac{\mathcal{Y}_t^i}{(K_t^i)^\alpha (L_t^i)^{1-\alpha}} ,$$

follows, to a first order approximation:

$$(27) \quad \hat{\mathcal{A}}_t^i = \varphi_1^i \hat{A}_{1t}^i + \varphi_2^i \hat{A}_{2t}^i + \varphi_N^i \hat{A}_{Nt}^i ,$$

where the φ_1^i, φ_2^i , and φ_N^i are steady state gross output in each sector relative to GDP:

$$\begin{aligned} \varphi_1^i &= \frac{(w^i L_1^i + r^i K_1^i)}{Y^i} , \\ \varphi_2^i &= \frac{(w^i L_2^i + r^i K_2^i + P_1^i M_T^i)}{Y^i} , \\ \varphi_N^i &= \frac{(w^i L_N^i + r^i K_N^i + P_1^i M_N^i)}{Y^i} . \end{aligned}$$

Proof. See appendix ■

The proof simply makes use of expressions for first-order approximations to the equilibrium conditions and to the definitions of the national accounts aggregates in this section. The algebra is rather tedious, so we leave it to the appendix. However, the intuition for this stark result is straightforward, given the assumptions used in aggregating value-added across goods to construct real GDP. First, because we use good-specific prices in valuing the output of each producer, efficiency differences across goods do not show up in the value of output: in the presence of perfect competition, a producer with higher efficiency simply charges a proportionally lower price, so the measured value of output per unit of input does not vary across producers. Bernard, Eaton, Jensen, and Kortum (2003) and Gibson (2006) raise the same point regarding the effect of reallocation that occurs in response to trade liberalization

in models with heterogeneous producers and monopolistic competition. Second, by using base period prices, the fluctuations in the value of income reflected in movements in factor costs and prices do not directly show up in the computation of real GDP, a point made by Kehoe and Ruhl (2008). Finally, valuing intermediate inputs at base period prices removes the gains in efficiency that result from purchasing imported inputs at cheaper prices when the foreign country receives a favorable productivity shock.

The expression for changes in TFP in equation (27) suggests that one possible channel by which changes in trade patterns could make countries more correlated is as follows. If shocks to certain sectors (say, stage 1 tradeable goods) are more correlated across countries than shocks to other sectors (say, stage 2 tradeable goods), and if a change in trade patterns shifts countries' steady state output shares toward the sectors that are more correlated, then the correlation in TFP would increase. Since we are interested in examining whether linkages across countries through trade can by themselves increase comovement, we do not pursue this alternative approach here.

4 Vertical Specialization and Trade

Before proceeding to our numerical experiments, we review some measures of vertical specialization in the data, and show how to construct corresponding measures in the model. A commonly cited measure of vertical specialization is the one in Hummels, Ishii, and Yi (2001). The authors define an index of vertical specialization for a given sector as the ratio of imported inputs to gross output. To construct an economywide measure of vertical specialization, they aggregate this index across sectors, weighted by each sector's share of exports, to capture the degree to which imported inputs are important in exporting sectors. Their measure for country i is then given by:

$$VS^i = \frac{\sum_v \frac{II_v^i}{Z_v^i} \text{Exports}_v^i}{\sum_v \text{Exports}_v^i},$$

where v denotes a sector, II_v^i denotes imported intermediates in sector v , and Z_v^i denotes gross output in sector v . When the exports of one sector and/or its imported intermediate inputs are 0 then that sector does not influence the composition of the index. Countries are more vertically specialized if they export more in sectors that use imported intermediates

intensively. Using Input-Output tables Hummels, Ishii, and Yi (2001) report some examples of the increasingly vertically specialized nature of international trade evaluated using their index. Some numbers taken from their paper are illustrated in Table 1. Using the metric that Hummels, Ishii, and Yi (2001) propose most of the countries become increasingly vertically specialized over time.

In our model we do not explicitly map the continuum of goods into a sectoral categorization. For the purposes of constructing a measure of vertical specialization comparable to the above index, we can think of all the goods as being assigned to one sector in the data.⁸ The share of intermediates in gross output for all the second stage goods is η while the share of the value of imported intermediates out of the total value of intermediates is λ_{1t}^{ij} . This means that in our model, the analogue of the index in Hummels, Ishii, and Yi (2001) is given by:

$$\begin{aligned} VS^i &= \frac{II^i}{Z^i} \\ &= \eta \lambda_{1t}^{ij} \frac{Z_2^i}{Z_2^i + Z_1^i}, \end{aligned}$$

where Z_s^i , $s = 1, 2$ denote gross output in each stage.

Additionally, we can use input-output tables to directly construct measures in the data analogous to the shares of intermediate and final expenditures spent by a country on domestic goods, the λ_1^{ii} and λ_2^{ii} . Table 2 shows these statistics for the same countries as in Table 1. As trade has increased significantly over time in all these countries (a decrease in the fraction spent on domestic goods), both the shares of imported intermediate and final goods have increased. However, the imported share of intermediate inputs increased more than the imported share of final goods.

In the next section, we evaluate the ability of versions of our model with and without vertical specialization to generate higher synchronization of business cycles between countries that have higher trade intensity.

⁸Alternatively, a random assignment of subsets of goods along the continuum from each stage into sectors would give the same result.

5 Numerical Experiments

In the numerical experiments of this section we parameterize the model in order to have reasonable predictions with regard to the business cycles of aggregate statistics as well as to deliver reasonable magnitudes of vertical specialization. Using the measures of vertical specialization developed in the previous section we will evaluate the ability of the parameterized model to increase business cycles synchronization.

5.1 Parameter Values

We set several parameters to standard values in the international business cycle literature. We interpret one model period as one quarter, and set the discount factor $\beta = 0.99$ so that the steady state real interest rate is 4% per year. We assign α a value of 0.3, implying that 30% of income from value added is paid to capital services. The depreciation rate δ is set to 2.5% per period. The utility parameter μ is set to 0.34, implying that about 1/3 of the total time endowment is supplied as labor in the steady state. We set the share of tradeable output in GDP to match the share of nontradeable expenditures in the model to the share of final expenditures on services in US input-output tables, giving $\gamma = .3$.

We assume that the technology shocks are equal across sectors within a country, $A_{1t}^i = A_{2t}^i = A_{Nt}^i \equiv A_t^i$, and that each country's aggregate technology follows an AR(1) process in logs,

$$\log A_{t+1}^i = \rho \log A_t^i + \varepsilon_{t+1}^i ,$$

for $i = 1, 2$, where ε_t^i is a mean-zero normally distributed i.i.d. innovation with standard deviation σ_ε . We set $\rho = 0.9$ and $\sigma_\varepsilon = 0.01$. Notably, we do not build in any correlation into the shocks to technology across countries or across sectors, so that we isolate the degree to which our model endogenously generates cross-country correlation in measured real GDP. Assuming some positive correlation in the ε_{st}^i terms across countries would essentially additively increase our cross-country correlations.

The model contains two parameters relating to the elasticity of substitution between purchases of domestic and foreign goods. As in Eaton and Kortum (2002), the role of the parameter σ in determining the elasticity of trade flows with respect to technology

shocks, input costs, and transport costs, is entirely concealed by the role of θ . While σ governs substitutability in the intensive margin - within goods that are continuously traded - θ governs the heterogeneity across goods, and hence determines the extent to which the extensive margin of trade in new goods in a given time periods respond to variations in technology or trade costs. At the aggregate level, the elasticity of substitution between imported and domestic goods in our model is θ . As Ruhl (2008) notes, measures of this aggregate elasticity in the data differ depending on the source of price variation: measures from time series data find small elasticities of the magnitude typically used in international business cycle models. On the other hand, estimates from cross-section data relating trade patterns to tariff and non-tariff barriers find elasticities that are much higher. Since the parameter θ governs the elasticity in response to both types of price variation in our model, we balance between the two measures by choosing a value of $\theta = 3.6$ because it is the lowest of the three estimates from Eaton and Kortum (2002). For the elasticity of substitution σ that determines the substitution between different goods, we use the benchmark value of Backus, Kehoe, and Kydland (1994) of $\sigma = 1.5$, which is consistent with estimates of the elasticity of substitution between foreign and domestic goods by SITC commodity groups.^{9, 10}

The parameter η determines the share of gross output in final goods paid to intermediate inputs. We consider two versions of our model, a one-stage benchmark with $\eta = 1$, and a version with vertical specialization, with $\eta = 0.5$, which is approximately the ratio of intermediate inputs to gross output in US input-output tables.

In the various experiments below, we choose the technology parameters T_s^i and the trade costs τ_s^{ij} to generate different specialization and international trade patterns, and we look at the degree of business cycle comovement across these different patterns.

5.2 Results

Tables 3-5 show various cross-country correlations of H-P filtered variables in the different versions of the model we consider. Beyond the real GDP and TFP aggregates defined earlier, we also look at the correlations between countries of aggregate labor supply; real consumption

⁹Recent estimates by Broda and Weinstein (2006) place the median value of this elasticity across all sectors to be 2.5 and the average value much higher than that. The model constraints us to set $\theta > \sigma - 1$ but raising elasticity does not substantially alter our results.

¹⁰In Eaton and Kortum (2002), σ can literally be ignored. In our model, however, due to our methods of measuring real aggregate statistics, σ does appear to play some role, but changes in this parameter have little effect on our results overall.

expenditures at base period prices ($\mathcal{C}^i$); real investment expenditures at base period prices ($\mathcal{X}^i$); value added deflated by the model's CES price indices ($\frac{Y^i}{P^i}$), and real value added in different stages of production ($\mathcal{Y}_s^i$). We define these variables as follows:

$$\mathcal{C}_t^i = \int p_{20}^i(\omega) c_{Tt}^i(\omega) d\omega + P_{N0}^i C_{Nt}^i ,$$

$$\mathcal{X}_t^i = \int p_{20}^i(\omega) x_{Tt}^i(\omega) d\omega + P_{N0}^i X_{Nt}^i ,$$

$$\begin{aligned} \left(\frac{Y^i}{P^i}\right)_t &= \frac{w_t^i L_{1t}^i + r_t^i K_{1t}^i}{P_{1t}^i} + \frac{w_t^i L_{2t}^i + r_t^i K_{2t}^i + P_{1t}^i M_{Tt}^i}{P_{2t}^i} - \frac{P_{1t}^i M_{Tt}^i}{P_{1t}^i} \\ &\quad + \frac{w_t^i L_{Nt}^i + r_t^i K_{Nt}^i + P_{1t}^i M_{Nt}^i}{P_{Nt}^i} - \frac{P_{1t}^i M_{Nt}^i}{P_{1t}^i} , \end{aligned}$$

and

$$\begin{aligned} \mathcal{Y}_{1t}^i &= \int_{\Omega_{1t}^i} q_{10}^i(\omega) y_{1t}^i(\omega) d\omega \\ \mathcal{Y}_{2t}^i &= \int_{\Omega_{2t}^i} q_{20}^i(\omega) y_{2t}^i(\omega) d\omega - \int_{\Omega_{2t}^i} \int p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' d\omega \end{aligned}$$

We show the alternative aggregate value added measure and the sectoral value added measures to try to get a sense of where, if not in traditional national accounting statistics, the comovement due to increases in trade and vertical specialization show up. The main conceptual difference between $\frac{Y^i}{P^i}$, value added in which components are deflated by the theoretical price indices, and $\mathcal{Y}^i$, real GDP, is that the former is a welfare-based measure of income generated in each sector of production, since each is deflated by the price index at which purchasers of goods buy output. This has the effect that international transmission of efficiency gains as discussed in Section 3 above are reflected in the former measure, but not in the latter, which uses base period producer prices to value output.

We first look at a version of the model with no vertical specialization which we denote as the benchmark. The goal is to consequently introduce different experiments where vertical specialization arises for different reasons and compare the properties of these versions of the model with the no vertical specialization benchmark. The left half of Table 3 shows statistics for the one-stage version of our model, with $\eta = 1$, and with symmetric countries, so that $T_s^1 = T_s^2$. When we vary trade costs to generate steady state trade to GDP ratios between

3% and 15%, we see that the correlation of real GDP across countries increases very slightly, from .01 to .04. Confirming Proposition 1, the correlation of TFP across countries is zero, and essentially does not rise with trade intensity. The correlations of labor, investment, and consumption rise across trade intensities, from .02 to .10 for labor and investment, and from .01 to .12 for consumption. In addition, the correlation of the alternative measure of value-added, $\frac{Y}{P}$, increases with trade intensity much more than that of real GDP.

The right half of Table 3 shows our model with vertical specialization, in three cases in which the trade costs τ_s^{ij} and technology parameters T_s^i are the same across countries and across stages of production. As trade intensity increases in this model from 3% to 15%, the steady state vertical specialization measure increases as well, from 0.7% to 3.5%.¹¹ The patterns of cross country correlations are broadly similar to the one-stage model, except the increases are slightly smaller as trade intensity rises. The exception is the $\frac{Y}{P}$ measure of value-added, which increases to a larger degree with two stages of production than with only one. This suggests that, when countries are symmetric in our model, the main effect of having two stages of production is that it increases the international transmission of technology into purchasers' prices, since there are two stages at which this occurs.

In Table 4 we show statistics for two variations of our model with 15% trade intensity, in which trade costs differ across stages of production. The column labelled "Low VS" has relatively high trade costs for first stage goods, so that trade intensity in these goods is dampened, while the column labelled "High VS" has relatively low trade costs for first stage goods. The results indicate that a higher degree of vertical specialization does not significantly affect the business cycle correlations we consider if countries are symmetric.

Finally, in Table 5 we consider a case in which countries are asymmetric: we choose the T_s^i terms so that country 1 has a comparative advantage in the production of stage 1 goods, while country 2 has a comparative advantage in stage 2 goods, and both countries have the same steady state GDP. In the left column, labeled "low specialization", the degree of comparative advantage is smaller than in the right column, labeled "high specialization", so that the vertical specialization measure for country 2 is larger, and for country 1 is smaller, in the right column. We see again that most of the cross-country correlations change little

¹¹While the λ_1^{ij} fractions in our model results come close to the data for, for example, the US (compare to $1 - \lambda_1^{ii}$ in Table 1), the *VS* measures our model predicts are much smaller than in the data. This suggests that sectoral differences that are emphasized by the *VS* measure of Hummels, Ishii, and Yi (2001) may be important.

across these cases. The exception is the correlation of real value added across countries in different stages: while the correlation of real value added in the sector in which each country specializes is only .02 in the low specialization case, this correlation increases to .12 in the high specialization case. The extent to which countries are negatively correlated in the sectors in which they do not have comparative advantage increases as well. The negative change slightly outweighs the positive change here, so that even though it applies to sectors that are small in each country, the overall effect is that the correlation of aggregate real GDP is similar under low specialization or high specialization.

In these results, we have shown that several versions of our model can generate moderate increases in business cycle comovement with increases in trade intensity. However, for the cases we have considered here, these increases are small, and the addition of vertical specialization does not contribute significantly to magnifying them. While asymmetric countries in our model clearly do display the tight links across sectors that vertical specialization implies, additional mechanisms would be needed to translate these links to aggregate real GDP and measured TFP.

6 Conclusion

In this paper we investigated the ability of a business cycle model that features vertical specialization to explain the trade-comovement puzzle reported by Kose and Yi (2001, 2006). We parsimoniously model vertical specialization in a setting with aggregate fluctuations by generalizing the framework of Yi (2003) while making use of techniques developed by Eaton and Kortum (2002). We find that the mechanism of vertical specialization modeled in this way cannot account for higher business cycle synchronization due to more trade.

While the framework we develop does not resolve the puzzle, our work helps to take important steps in understanding the reasons behind its persistence under different modeling frameworks: we prove that in our model the measured TFP of countries does not depend on trade or vertical specialization intensity. Given the structure of our model, this result is likely to hold in a large variety of the models with similar assumptions about preferences, production and competition. Current and future research that deviates from these assumptions in modeling features of trade such as vertical specialization will show whether it is important in understanding international business cycle synchronization. While vertical

specialization as we have specified it provides an intuitive reason for countries that trade more to be more correlated, additional transmission mechanisms are needed to account for the extent to which this channel affects the behavior of measured business cycle statistics.

	Year	VS index	Year.	VS index
Canada	1971	.20	1990	.27
France	1972	.18	1990	.24
Germany	1978	.18	1990	.20
Japan	1970	.18	1990	.11
United Kingdom	1979	.25	1990	.26
United States	1972	.06	1990	.11
Korea	1970	.33	1990	.37

Table 1: Vertical Specialization Measures from Hummels et al. (2001)

	Year	λ_1^{ii}	λ_2^{ii}	Year	λ_1^{ii}	λ_2^{ii}
Canada	1971	.77	.71	1990	.70	.57
France	1972	.81	.86	1990	.70	.72
Germany	1978	.79	.81	1990	.75	.73
Japan	1970	.90	.95	1990	.89	.91
United Kingdom	1979	.74	.75	1990	.64	.62
United States	1972	.94	.93	1990	.88	.81
Korea	.	.	.	1995	.73	.78

Table 2: Domestic expenditure shares for intermediate and final goods from OECD Input-Output Tables

	Benchmark ($\eta = 1$)			Vert. Spec. ($\eta = .5$)		
variable	trade/GDP			trade/GDP		
	3%	9%	15%	3%	9%	15%
$corr(\mathcal{Y}^1, \mathcal{Y}^2)$	.01	.02	.04	.01	.01	.02
$corr(\mathcal{A}^1, \mathcal{A}^2)$	.00	.00	.01	.00	.00	-.01
$corr(L^1, L^2)$	.02	.05	.10	.02	.04	.07
$corr(\mathcal{X}^1, \mathcal{X}^2)$	.02	.05	.10	.02	.04	.06
$corr(\mathcal{C}^1, \mathcal{C}^2)$	.01	.03	.12	.01	.02	.04
$corr\left(\frac{Y^1}{P^1}, \frac{Y^2}{P^2}\right)$	.04	.06	.13	.04	.10	.17
$(\lambda_1^{ij}, \lambda_2^{ij})$	(-, .10)	(-, .30)	(-, .50)	(.04, .04)	(.11, .11)	(.19, .19)
<i>VS</i> index	-	-	-	.007	.021	.035

Table 3: Model business cycle correlations (see text for variable definitions).

	$\eta = .5$, Low VS	$\eta = .5$, High VS
variable	trade/GDP	trade/GDP
	15%	15%
$corr(\mathcal{Y}^1, \mathcal{Y}^2)$	.02	.02
$corr(\mathcal{A}^1, \mathcal{A}^2)$	.00	.00
$corr(L^1, L^2)$	.08	.06
$corr(\mathcal{X}^1, \mathcal{X}^2)$	.08	.06
$corr(\mathcal{C}^1, \mathcal{C}^2)$	.06	.05
$corr\left(\frac{Y^1}{P^1}, \frac{Y^2}{P^2}\right)$	.15	.14
$corr(\mathcal{Y}_1^1, \mathcal{Y}_2^2)$	.00	.00
$(\lambda_1^{ij}, \lambda_2^{ij})$	(.09, .35)	(.25, .09)
<i>VS</i> index	.017	.046

Table 4: Model correlations for high and low vertical specialization cases (See text for variable definitions)

	$\eta = .5$, Low specialization	$\eta = .5$, High specialization
variable	trade/GDP	trade/GDP
	15%	15%
$corr(\mathcal{Y}^1, \mathcal{Y}^2)$	.02	.03
$corr(\mathcal{A}^1, \mathcal{A}^2)$	.00	.01
$corr(L^1, L^2)$	.07	.07
$corr(\mathcal{X}^1, \mathcal{X}^2)$	.06	.07
$corr(\mathcal{C}^1, \mathcal{C}^2)$	.05	.09
$corr\left(\frac{Y^1}{P^1}, \frac{Y^2}{P^2}\right)$	.17	.16
$corr(\mathcal{Y}_1^1, \mathcal{Y}_2^2)$	.02	.12
$corr(\mathcal{Y}_2^1, \mathcal{Y}_1^2)$	-.05	-.19
$(\lambda_1^{21}, \lambda_2^{21})$	(.17, .23)	(.08, .38)
$(\lambda_1^{12}, \lambda_2^{12})$	(.21, .15)	(.25, .05)
VS^1 index	.029	.011
VS^2 index	.041	.059

Table 5: Model correlations with asymmetric countries. Country 1 has comparative advantage in first stage (See text for variable definitions)

7 Appendix

7.1 Model expressions for real GDP

7.1.1 Aggregate Gross Output

We use equation (24) and the fact that the value of gross output of each good evaluated at current prices is equal to the value of payments to inputs to write:

$$\begin{aligned}
\mathcal{Z}_t^i &= \int_{\Omega_{1t}^i} \frac{q_{10}^i(\omega)}{q_{1t}^i(\omega)} (w_t^i \ell_{1t}^i(\omega) + r_t^i k_{1t}^i(\omega)) d\omega \\
&+ \int_{\Omega_{2t}^i} \frac{q_{20}^i(\omega)}{q_{2t}^i(\omega)} \left(w_t^i \ell_{2t}^i(\omega) + r_t^i k_{2t}^i(\omega) + P_{1t}^i \left(\int m_t^i(\omega, \omega')^{\frac{\sigma-1}{\sigma}} d\omega' \right) \right) y_{2t}^i(\omega) d\omega \\
&+ \frac{P_{N0}^i}{P_{Nt}^i} (w_t^i L_{Nt}^i + r_t^i K_{Nt}^i + P_{1t}^i M_{Nt}^i) .
\end{aligned}$$

Because $q_{st}^i(\omega) = q_{st}^i/z_s^i(\omega)$ for all t , the $z_s^i(\omega)$ terms cancel out, and the input demands can be aggregated across sectors using the relationships in equations (5), (17) and (18):

$$(28) \quad \mathcal{Z}_t^i = \frac{q_{10}^i}{q_{1t}^i} (w_t^i L_{1t}^i + r_t^i K_{1t}^i) + \frac{q_{20}^i}{q_{2t}^i} (w_t^i L_{2t}^i + r_t^i K_{2t}^i + P_{1t}^i M_{1t}^i) + \frac{P_{N0}^i}{P_{Nt}^i} (w_t^i L_{Nt}^i + r_t^i K_{Nt}^i + P_{1t}^i M_{Nt}^i) .$$

7.1.2 Aggregate Expenditures on Intermediate Inputs

In equation (25), we need to split up the range of goods purchased by country i depending on the pattern of specialization both in the current period, and in the base period. We need to know from which country each good was bought in each period, to determine the base period price and the current period quantity purchased. Let Ω_{1t}^{ji} denote the set of goods that country i purchases from country j in period t . Then, for example, the set of goods which country i buys from home in period t but imported in period 0 is $\Omega_{1t}^{ii} \cap \Omega_{10}^{ji}$, so (25) can be written:

$$\begin{aligned} \mathcal{I}_t^i = & \int_{\Omega_{2t}^i} \left(\int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ii}} p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' + \int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ji}} p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' \right. \\ & + \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ii}} p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' + \left. \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ji}} p_{10}^i(\omega') m_t^i(\omega, \omega') d\omega' \right) d\omega \\ & + \int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ii}} p_{10}^i(\omega) m_{Nt}^i(\omega) d\omega + \int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ji}} p_{10}^i(\omega) m_{Nt}^i(\omega) d\omega \\ & + \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ii}} p_{10}^i(\omega) m_{Nt}^i(\omega) d\omega + \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ji}} p_{10}^i(\omega) m_{Nt}^i(\omega) d\omega . \end{aligned}$$

We use the definitions of prices from (2) and (3) and the CES demand function for each good to write:

$$\begin{aligned} (29) \quad \mathcal{I}_t^i = & \left(\frac{q_{1t}^{ii}}{P_{1t}^i} \right)^{-\sigma} (M_{Tt}^i + M_{Nt}^i) \left(q_{10}^{ii} \int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ii}} z_1^i(\omega)^{\sigma-1} d\omega + q_{10}^{ji} \int_{\Omega_{1t}^{ii} \cap \Omega_{10}^{ji}} \frac{z_1^i(\omega)^\sigma}{z_1^j(\omega)} d\omega \right) \\ & + \left(\frac{q_{1t}^{ji}}{P_{1t}^i} \right)^{-\sigma} (M_{Tt}^i + M_{Nt}^i) \left(q_{10}^{ji} \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ii}} \frac{z_1^j(\omega)^\sigma}{z_1^i(\omega)} d\omega + q_{10}^{ji} \int_{\Omega_{1t}^{ji} \cap \Omega_{10}^{ji}} z_1^j(\omega)^{\sigma-1} d\omega \right) . \end{aligned}$$

Next, we use the fact that, for example, goods ω in $\Omega_{1t}^{ii} \cap \Omega_{10}^{ii}$, are goods for which country i has comparative advantage in both periods 0 and t :

$$\Omega_{1t}^{ii} \cap \Omega_{10}^{ii} = \left\{ \omega : \frac{z_1^i(\omega)}{z_1^j(\omega)} \geq \max \left\{ \frac{q_{1t}^{ii}}{q_{1t}^{ji}}, \frac{q_{10}^{ii}}{q_{10}^{ji}} \right\} \right\} .$$

The other sets are similarly defined by

$$\Omega_{1t}^{ii} \cap \Omega_{10}^{ji} = \left\{ \omega : \frac{q_{1t}^{ii}}{q_{1t}^{ji}} \leq \frac{z_1^i(\omega)}{z_1^j(\omega)} \leq \frac{q_{10}^{ii}}{q_{10}^{ji}} \right\} .$$

$$\Omega_{1t}^{ji} \cap \Omega_{10}^{ii} = \left\{ \omega : \frac{q_{1t}^{ii}}{q_{1t}^{ji}} \geq \frac{z_1^i(\omega)}{z_1^j(\omega)} \geq \frac{q_{10}^{ii}}{q_{10}^{ji}} \right\} .$$

$$\Omega_{1t}^{ji} \cap \Omega_{10}^{ji} = \left\{ \omega : \frac{z_1^i(\omega)}{z_1^j(\omega)} \leq \min \left\{ \frac{q_{1t}^{ii}}{q_{1t}^{ji}}, \frac{q_{10}^{ii}}{q_{10}^{ji}} \right\} \right\} .$$

By changing the variable of integration from ω to pairs of z_1^i, z_1^j , (29) can be written:

$$(30) \quad \mathcal{I}_t^i = \left(\frac{q_{1t}^{ii}}{P_{1t}^i} \right)^{-\sigma} (M_{Tt}^i + M_{Nt}^i) \left(q_{10}^{ii} \int_0^\infty \int_{z_1^j \max\{q_{1t}^{ii}/q_{1t}^{ji}, q_{10}^{ii}/q_{10}^{ji}\}}^\infty (z_1^i)^{\sigma-1} dF_1^i(z_1^i) dF_1^j(z_1^j) \right. \\ \left. + q_{10}^{ji} \int_0^\infty \int_{z_1^j q_{10}^{ii}/q_{10}^{ji}}^{z_1^j q_{1t}^{ii}/q_{1t}^{ji}} (z_1^j)^{-1} (z_1^i)^\sigma dF_1^i(z_1^i) dF_1^j(z_1^j) \right) \\ + \left(\frac{q_{1t}^{ji}}{P_{1t}^i} \right)^{-\sigma} (M_{Tt}^i + M_{Nt}^i) \left(q_{10}^{ii} \int_0^\infty \int_{z_1^j q_{10}^{ii}/q_{10}^{ji}}^{z_1^j q_{1t}^{ii}/q_{1t}^{ji}} (z_1^i)^{-1} (z_1^j)^\sigma dF_1^i(z_1^i) dF_1^j(z_1^j) \right. \\ \left. + q_{10}^{ji} \int_0^\infty \int_0^{z_1^j \min\{q_{1t}^{ii}/q_{1t}^{ji}, q_{10}^{ii}/q_{10}^{ji}\}} (z_1^j)^{\sigma-1} dF_1^i(z_1^i) dF_1^j(z_1^j) \right) ,$$

which we can compute, given equilibrium values for prices and quantities, by numerically solving the integrals.

7.2 Proof of Proposition 1

Throughout, we make use of the approximation $g(x_t) \approx g'(x) \hat{x}_t$, where $\hat{x}_t = \log(x_t) - \log(x)$. Any variable without a time subscript refers to the steady state value, and $\hat{x}_t$ is the log deviation of variable x_t from its steady state value. Using the fact that base period quantities are equal to steady state quantities, the approximation for gross output, from equation (28), can be written as:

$$\mathcal{Z}^i \hat{\mathcal{Z}}_t^i = (w^i L_1^i + r^i K_1^i) \hat{A}_{1t}^i + (w^i L_2^i + r^i K_2^i + P_1^i M_T^i) \hat{A}_{2t}^i + (w^i L_N^i + r^i K_N^i + P_1^i M_N^i) \hat{A}_{Nt}^i \\ - (1 - \alpha) (w^i L_1^i + r^i K_1^i + \eta (w^i L_2^i + r^i K_2^i + P_1^i M_T^i + w^i L_N^i + r^i K_N^i + P_1^i M_N^i)) \hat{w}_t^i \\ - \alpha (w^i L_1^i + r^i K_1^i + \eta (w^i L_2^i + r^i K_2^i + P_1^i M_T^i + w^i L_N^i + r^i K_N^i + P_1^i M_N^i)) \hat{r}_t^i \\ - (1 - \eta) (w^i L_2^i + r^i K_2^i + P_1^i M_T^i + w^i L_N^i + r^i K_N^i + P_1^i M_N^i) \hat{P}_{1t}^i \\ + w^i L_1^i \hat{L}_{1t}^i + w^i L_2^i \hat{L}_{2t}^i + w^i L_N^i \hat{L}_{Nt}^i \\ + r^i K_1^i \hat{K}_{1t}^i + r^i K_2^i \hat{K}_{2t}^i + r^i K_N^i \hat{K}_{Nt}^i \\ + P_1^i M_T^i \hat{M}_{Tt}^i + P_1^i M_N^i \hat{M}_{Nt}^i ,$$

which can further be reduced to

$$\begin{aligned}\mathcal{Z}^i \hat{\mathcal{Z}}_t^i &= (w^i L_1^i + r^i K_1^i) \hat{A}_{1t}^i + (w^i L_2^i + r^i K_2^i + P_1^i M_T^i) \hat{A}_{2t}^i + (w^i L_N^i + r^i K_N^i + P_1^i M_N^i) \hat{A}_{Nt}^i \\ &\quad + w^i L^i \hat{L}_t^i + r^i K^i \hat{K}_t^i + P_1^i M_T^i \hat{M}_{Tt}^i + P_1^i M_N^i \hat{M}_{Nt}^i .\end{aligned}$$

The real value of intermediate inputs, in (30), can be written:

$$\mathcal{I}_t^i = (M_{Tt}^i + M_{Nt}^i) \left(\left(\frac{q_{1t}^{ii}}{P_1^i} \right)^{-\sigma} (q_{10}^{ii} I_{ddt}^i + q_{10}^{ji} I_{mdt}^i) + \left(\frac{q_{1t}^{ji}}{P_1^i} \right)^{-\sigma} (q_{10}^{ii} I_{mdt}^i + q_{10}^{ji} I_{mmt}^i) \right) ,$$

where the term I_{dd1t}^i refers to the integral term in (30) over the set of goods that country i purchases domestically in both periods 0 and t . (Hence the mnemonic dd in the subscript).

The approximation of $\mathcal{I}_t^i$ is then given by:

$$\begin{aligned}(31) \mathcal{I}^i \hat{\mathcal{I}}_t^i &= \left(\left(\frac{q_1^{ii}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{dd}^i + q_1^{ji} I_{md}^i) + \left(\frac{q_1^{ji}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{dm}^i + q_1^{ji} I_{mm}^i) \right) (M_T^i \hat{M}_{Tt}^i + M_N^i \hat{M}_{Nt}^i) \\ &\quad - (M_T^i + M_N^i) \sigma \left(\frac{q_1^{ii}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{dd}^i + q_1^{ji} I_{md}^i) \hat{q}_{1t}^{ii} \\ &\quad - (M_T^i + M_N^i) \sigma \left(\frac{q_1^{ji}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{dm}^i + q_1^{ji} I_{mm}^i) \hat{q}_{1t}^{ji} \\ &\quad + (M_T^i + M_N^i) \sigma \left(\left(\frac{q_1^{ii}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{dd}^i + q_1^{ji} I_{md}^i) + \left(\frac{q_1^{ji}}{P_1^i} \right)^{-\sigma} (q_1^{ii} I_{md}^i + q_1^{ji} I_{mm}^i) \right) \hat{P}_{1t}^i \\ &\quad + (M_T^i + M_N^i) \left(\frac{q_1^{ii}}{P_1^i} \right)^{-\sigma} q_1^{ii} I_{dd}^i \hat{I}_{ddt}^i + (M_T^i + M_N^i) \left(\frac{q_1^{ii}}{P_1^i} \right)^{-\sigma} q_1^{ji} I_{md}^i \hat{I}_{mdt}^i \\ &\quad + (M_T^i + M_N^i) \left(\frac{q_1^{ji}}{P_1^i} \right)^{-\sigma} q_1^{ii} I_{dm}^i \hat{I}_{dmt}^i + (M_T^i + M_N^i) \left(\frac{q_1^{ji}}{P_1^i} \right)^{-\sigma} q_1^{ji} I_{mm}^i \hat{I}_{mmt}^i .\end{aligned}$$

To proceed, we suppose that $\frac{q_{1t}^{ii}}{q_{1t}^{ji}} > \frac{q_{10}^{ii}}{q_{10}^{ji}}$ (the case $\frac{q_{1t}^{ii}}{q_{1t}^{ji}} \leq \frac{q_{10}^{ii}}{q_{10}^{ji}}$ is similar), so that the I terms

are equal to:

$$\begin{aligned}
I_{ddt}^i &= \int_0^\infty \int_{z_1^j q_t^{ii}/q_t^{ji}}^\infty (z_1^i)^{\sigma-1} dF_1^i(z_1^i) dF_1^j(z_1^j) , \\
I_{mdt}^i &= \int_0^\infty \int_{z_1^j q_t^{ii}/q_t^{ji}}^{z_1^j q_0^{ii}/q_0^{ji}} (z_1^f)^{-1} (z_1^i)^\sigma dF_1^i(z_1^i) dF_1^j(z_1^j) , \\
I_{dmt}^i &= \int_0^\infty \int_{z_1^j q_0^{ii}/q_0^{ji}}^{z_1^j q_t^{ii}/q_t^{ji}} (z_1^i)^{-1} (z_1^f)^\sigma dF_1^i(z_1^i) dF_1^j(z_1^j) , \\
I_{mmt}^i &= \int_0^\infty \int_0^{z_1^j q_0^{ii}/q_0^{ji}} (z_1^f)^{\sigma-1} dF_1^i(z_1^i) dF_1^j(z_1^j) .
\end{aligned}$$

These can be approximated as:

$$\begin{aligned}
(32) \quad I_{dd}^i \hat{I}_{ddt}^i &= -(\hat{q}_{1t}^{ii} - \hat{q}_{1t}^{ji}) \int_0^\infty (z_1^j q_1^{ii}/q_1^{ji})^\sigma f_1^i(z_1^j q_1^{ii}/q_1^{ji}) dF_1^j(z_1^j) , \\
I_{md}^i \hat{I}_{mdt}^i &= 0 , \\
I_{dm}^i \hat{I}_{dmt}^i &= (\hat{q}_{1t}^{ii} - \hat{q}_{1t}^{ji}) \int_0^\infty (f_1^i(z_1^j q_1^{ii}/q_1^{ji})) (z_1^j)^\sigma dF_1^j(z_1^j) , \\
I_{mm}^i \hat{I}_{mmt}^i &= 0 ,
\end{aligned}$$

where f_1^i is the pdf of the distribution F_1^i .

Now, the price index P_{1t}^i can be written as:

$$(P_{1t}^i)^{1-\sigma} = \int_0^\infty \left(\int_{z_1^j q_{1t}^{ii}/q_{1t}^{ji}}^\infty \left(\frac{q_{1t}^{ii}}{z_1^i} \right)^{1-\sigma} dF_1^i(z_1^i) + \int_0^{z_1^j q_{1t}^{ii}/q_{1t}^{ji}} \left(\frac{q_{1t}^{ji}}{z_1^j} \right)^{1-\sigma} dF_1^j(z_1^j) \right) dF_1^j(z_1^j) .$$

So that we can write:

$$(33) \quad (P_1^i)^{1-\sigma} \hat{P}_{1t}^i = (q_1^{ii})^{1-\sigma} I_{dd1}^i \hat{q}_{1t}^{ii} + (q_1^{ji})^{1-\sigma} I_{mm1}^i \hat{q}_{1t}^{ji} .$$

Substituting (32) and (33) into (31), we find:

$$\mathcal{I}_t^i \hat{\mathcal{I}}_t^i = P_1^i M_T^i \hat{M}_{Tt}^i + P_1^i M_N^i \hat{M}_{Nt}^i .$$

Using the definition of real GDP in (26),

$$\begin{aligned}
\mathcal{Y}^i \hat{\mathcal{Y}}_t^i &= \mathcal{Z}^i \hat{\mathcal{Z}}_t^i - \mathcal{I}^i \hat{\mathcal{I}}_t^i \\
&= (w^i L_1^i + r^i K_1^i) \hat{A}_{1t}^i + (w^i L_2^i + r^i K_2^i + P_1^i M_T^i) \hat{A}_{2t}^i + (w^i L_N^i + r^i K_N^i + P_1^i M_N^i) \hat{A}_{Nt}^i \\
&\quad + w^i L^i \hat{L}_t^i + r^i K^i \hat{K}_t^i .
\end{aligned}$$

The steady state value of Y^i is:

$$Y^i = w^i L_1^i + r^i K_1^i + w^i L_2^i + r^i K_2^i + w^i L_N^i + r^i K_N^i .$$

And using the fact that $w^i L^i = (1 - \alpha) Y^i$ and $r^i K^i = \alpha Y^i$, we can write:

$$\hat{\mathcal{Y}}_t^i = \varphi_1^i \hat{A}_{1t}^i + \varphi_2^i \hat{A}_{2t}^i + \varphi_N^i \hat{A}_{Nt}^i + (1 - \alpha) \hat{L}_t^i + \alpha \hat{K}_t^i ,$$

where the φ 's are shares of gross output in each sector relative to GDP:

$$\begin{aligned}
\varphi_1^i &= \frac{(w^i L_1^i + r^i K_1^i)}{Y^i} , \\
\varphi_2^i &= \frac{(w^i L_2^i + r^i K_2^i + P_1^i M_T^i)}{Y^i} , \\
\varphi_N^i &= \frac{(w^i L_N^i + r^i K_N^i + P_1^i M_N^i)}{Y^i} .
\end{aligned}$$

Therefore, measured TFP is

$$\begin{aligned}
\hat{\mathcal{A}}_t^i &= \hat{\mathcal{Y}}_t^i - \alpha \hat{K}_t^i - (1 - \alpha) \hat{L}_t^i \\
&= \varphi_1^i \hat{A}_{1t}^i + \varphi_2^i \hat{A}_{2t}^i + \varphi_N^i \hat{A}_{Nt}^i .
\end{aligned}$$

So, measured TFP is equal to a weighted sum of the productivity shocks in each sector, in which the weights are equal to each sector's steady state gross output as a fraction of steady state GDP.

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