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Who's afraid of population aging?*

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ABSTRACT

Most industrialized countries confront at the same time an increasing life-span, low current birth rates and the retirement of the relatively large post World War II generation. This paper considers different policies as potential solutions to the implied funding problem of any Pay-As-You-Go retirement system. The policies are: increasing social security taxes, decreasing retirement-benefits, increasing eligibility age for retirement benefits, and a privatization of the retirement system using debt to finance the transition. It asks the question: What are the welfare and macroeconomic consequences of different retirement policies in the presence of population aging? The question is answered in a general equilibrium lifecycle model with endogenous savings, labor supply and human capital accumulation, calibrated to match important characteristics of the U.S. economy. The paper finds that privatization is the only Pareto improving policy in the considered set. Furthermore, the presence of human capital accumulation has important policy implications, making the transition to a privatized system longer and more difficult to finance.

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1 Introduction

Today's advanced industrialized countries are confronted with large demographic changes leading to population aging. The main sources behind population aging are: First, each current generation is getting older than any generation before and this trend is projected to persist. Second, fertility rates have been low for a long time and are expected to remain low. Third, the relatively large post WWII generation is starting to retire. While the first two aspects represent long term problems, the last one is temporary in nature, though its consequences will persist for decades to come. As a consequence both the overall government budget and the Pay-As-You-Go (PAYG) funded retirement systems will be under severe stress. For the case of the United States, which has the least aging problems among the G7 countries, either retirement benefits to the old will decrease considerably or the taxes needed to fund these benefits have to increase substantially.[1] This paper focuses on the problems of the retirement system and asks the question: What are the welfare and macroeconomic consequences of different retirement policies in the presence of population aging and endogenous human capital accumulation? The policies I consider are: an increase in the tax that funds retirement, a decrease in retirement benefits, an increase in the retirement age combined with either tax or benefit changes, and a move to a fully funded system (sometimes also referred to as privatization). The key finding is that a move to a fully funded system is the only Pareto improving policy in the considered set of policies[2]. The paper also finds that the presence of human capital in the model implies a longer transition and considerable long run benefits in terms of human capital gains.

The paper uses an overlapping generations (OLG) framework with two forms of savings: physical capital and human capital. The model is calibrated to match key features of the U.S.

[1] Alternatively, the social security eligiblity age can be increased. This help with the funding but is welfare reducing for some cohorts.

[2] While the set of considered policies might appear narrow, it is quite broad in the sense that any convex combination of the two extreme cases, tax increase and benefit decrease, is part of the set.

economy for the period 1950 to 2005. For future demographic developments, I make use of the population projections for the United States as provided by the Bureau of the Census. The equilibrium on the transition path is found using a Gauss-Seidel-Quasi-Newton approach as outlined in Ludwig (2007). Equipped with historical average marginal tax rates for capital and labor income, Total Factor Productivity (TFP) and survival probabilities the model is successful in replicating long run trends and movements in the main macroeconomic variables (output, capital stock, and total hours worked) for the period 1950 to 2005. Considering the different policies, I find that among the four policies that preserve the PAYG system, none is welfare improving for all generations. The policies that increase the retirement funding tax (either by themselves or combined with an increase in the retirement age) hurt the future generations while making the currently alive generations happy. It is noteworthy, that the tax rates remain high (at about 18 per cent) even after the baby boomers have left the economy. This suggest that the problem is not just about financing the period of the baby boomer's retirement. The policies that decrease the benefits to the retired (either by themselves or combined with an increase in the retirement age) have negative consequences for the welfare of the currently alive generations, while stabilizing the welfare of future generations. The benefit adjusting policies have the advantage of stabilizing output, the capital stock, and total hours worked. This is not the case for tax increasing policies. All of them have negative implications for output. The analysis suggests that independent of whether taxes or benefits are changed in response to the demographic shifts, the government is served best (both in welfare and economic aggregates responses) by increasing the retirement age. A serious alternative to preserving the PAYG system is a move to a fully funded system, in which the persons that contributed to the old PAYG–system get generous transfers during their retirement period that reflect their contribution to the old system. The key to funding these transfers is an increase in economic activity, especially labor supply, after the distortionary retirement tax is abolished. This implies increased revenues from taxing labor income, capital income and consumption, which are used to pay for the debt that accumulates during the transition period while persons that paid into the old system are still alive. I find that this approach to privatizing the retirement system is Pareto improving relative to the considered alternatives in the sense that it makes every generation better off, without making anybody worse off. The drawback of this transition is that it takes a long period of time (about 195 years). The length of the transition is mainly due to the presence of human capital in the model because human capital acts as a substitute for physical capital and slows down the interest rate decrease relative to a world where the only savings vehicle is capital.[3] An advantage of the debt funded transition to a fully funded system is that the debt functions as a intermediary supply of extra equity and allows the supply of equity to adjust slowly. The long term consequences of a privatization are very positive. All the main aggregates rise and due to an increase in human capital accumulation during the transition, TFP rises by 5 per cent and labor productivity even by 18 per cent.

Regarding the policy aspect of this paper, there is a large number of recent and past papers to consider. The most comprehensive in scope is a 2007 paper by Kotlikoff, Schmetters, and Walliser. They consider various policy alternatives within the PAYG framework, using a large scale OLG model with heterogenous agents that mainly differ in their earnings profiles. Their focus is on the idea of pre-funding the transition by using debt/public savings. Their findings are similar to those in this paper for the common reforms.[4] Their picture is more heterogenous, since there are different income groups with different implications of various reforms for different income groups. There are two recent papers that mainly focus on the privatization of the social security

[3]The duration is shortened to the extend that international capital markets are able to supply the needed extra resources during the transition period without changes in the real interest rate. This is easy to show in a s small open economy version of the model at hand. To the extent that the extra demand for resources is a multiple of U.S. GDP, it is unlikely that this small open economy is realistic.

[4]The average type of a given cohort in their model is the representative cohort in my model and the prefunding is some convex combination of my extremes of tax increase and benefit decrease.

One reform they consider, that I ignore is that of using consumption taxes to finance the PAYG system.

system in the United States: Kotlikoff, Smetters, and Walliser (1999) and Prescott (2004). The first paper is in spirit close to the previously described paper by the same authors, just focusing on privatization. It differs from this paper, focusing on the transition financing through specific taxes on consumption or labor income instead of debt. For the closed economy case it shows similar results for macroeconomic variables as the study at hand. Regarding the long run, the two studies find similar beneficial effects of privatization for macroeconomic aggregates and welfare. Prescott's approach to moving to a fully funded system is very similar to the one in this paper in that it uses debt to finance the transition and treats the past contributions as a debt of the government to past contributors. Prescott's focus, though, is on a small open economy in which the capital return is constant and wages grow at a constant rate over time. In that sense the paper ignores the potential price implications of the large amount of debt needed to finance the transition. The main difference from the previous papers on evaluating policies that mitigate the problems of demographic change is the introduction of endogenous human capital accumulation. This appears to be an important margin to adjust in response to labor shortages, to changes in the price of the capital stock after a move to a fully funded system, and to a change in the retirement age. This is not the first paper that has introduced human capital into the debate on demographic change. Ludwig and Schelkle (2007) focused on the importance of introducing human capital into the overlapping generations model proposed by Auerbach and Kotlikoff (1987). Ludwig and Schelkle show that endogenous human capital adjustment is indeed an important factor. They also show that endogenous growth based on human capital adds little to the model.[5] A paper that focuses on the move to a fully funded retirement system incorporating human capital into an OLG structure is Dubovyk (2007). This paper uses a steady state analysis and shows that human capital and the way the fully funded

[5] This last finding is to some extent in contrast to the finding by Fougère and Mérette (1999). Their analysis suggests that endogenous growth due to human capital has helpful impulses along the transition path. Given that their analysis of the human capital accumulation process is only to a lesser extent data based, I am going to follow Ludwig and Schelkle and focus on exogenous growth.

system is implemented, either voluntary savings accounts or mandatory savings accounts, make a difference for the steady state outcome.

The paper is organized as follows: In section 2, facts regarding the aging population are presented. The main focus is the severity of the situation. This is followed by a presentation of the model and a description of the policies in Section 3. Afterwards in the Sections 4 and 5 the calibration method is explained and the parameters presented, respectively, the main experiments are outlined and the key results discussed. Conclusions are drawn in Section 6.

2 Aspects of population aging

Demographic variables are changing very slowly. Given this fact, it is very surprising, that it took so long for social security reforms to become a political issue. Potentially, the reasoning is exactly the reverse: given that demographic change is so slow, there always appears to be time left to react and thus waiting seems to be a good option, at least till it is too late. This section documents the main demographic changes that will put pressure on government budget constraints in the next decades: people are getting older, birth rates have been low, and the baby boomers are reaching retirement age. These facts are documented using data provided by the United Nations demographics group. I focus on the G7 countries. The data from 1950 to 2003 are actual data and the data from then onward are projections.[6] I choose the medium projection among the three available ones. The main reason is that this projection is well balanced when it comes to taking risks (birth rates, life expectancy) and opportunities (immigration) into account. The projections take immigration into account and thus already consider the help that can be gotten from external sources to alleviate the demographic pressure.

Fact 1: The industrialized world is getting older.

[6] Projections regarding population developments are fairly reliable for industrialized countries, given the highly peristent nature of the underlying processes.

This is illustrated in Figure 1, which shows the median age for the period 1950 to 2050. The figure shows that the median age for the majority of the G7 countries (except Italy and Japan) was fairly stable till the 1990s but since then has started to rise at a fast pace and will continue to do so till the 2030s. By then it will have risen from about 30 to about 45.

There are three main reasons for this overall aging. The first one is that people are living longer.

Fact 2: People are getting older than ever before.

The fastest increasing population group in the industrialized countries today is that of the 80 year old and older. Figure 2 shows this group as a fraction of the total population for various years and the G7 countries. The size of this group will increase from about 3 per cent in 2000 to about 10 per cent in 2050. While this is overall welcome, it will be a problem for any PAYG retirement system, since it means a steadily increasing fraction of the population will receive retirement benefits for a long time.

Another reason for the aging is that there have been fewer children over the past decades.

Fact 3: The total fertility rates have been low and will remain so for quite some time.

The total fertility rate is a measure of the number of children a woman will on average have during her life. For industrialized countries the total fertility rate necessary to keep the population at a constant level (abstracting from immigration) is 2.1 children per woman. For all of the G7 countries the fertility rate has been below this replacement rate (Figure 3). And not only has the rate been far below, it is projected to remain so. The United States, while being below the replacement rate, is still fairly close at this point in time. Other countries like Canada are far below with around 1.5 children per woman while Japan has the lowest rate among the G7 countries at 1.3 children per woman.

The implication of the last two facts is that the increasingly older population will not be balanced out by an increase in the young population. Quite the opposite is true: More and more old persons are confronting less and less young persons, making the retirement funding problem a more pressing matter.

The final aspect of population aging is the peculiar situation created by the very high birth rates in the 1945-1965 period and the subsequent drop in the birth rates. This is visible in Figure 3. The highest birth rates were observed in Canada and the United States where the generation born in this period is known as the Baby Boomers. But a similar drop is visible in the other G7 countries as well. While this might not seem immediately linked to the retirement system, it turns out to be quire relevant.

Fact 4: The 1945 to 1965 generation is relatively big and starts retiring.

The first wave of this baby boom generation will start to reach the retirement age (65 years and older) soon and this will imply, if nothing else changes, a severe increase in the retirement-age population relative to the working-age population (age 15 to 64), see Figure 4. We can see here that between 2010 and 2030 the fraction of retired over working age persons is expected to increase considerably. This will put particular stress on public funding for retirement. Thus even through most information regarding the population development and aging has been readily available for quite some time, little has been done. Most measures were aimed at temporarily stabilizing the system.

Summary: *This section presented dimensions of population aging. It established that the population is aging rapidly and that the problem is particularly pressing at this point in time, just before the first baby boomers reach the retirement age. The U.S. is the least severe case of population aging among the G7 countries. Therefore the results below can be considered a lower bound for the other G7 countries.*

3 Model

This section outlines an Overlapping Generations model that allows for human capital accumulation over the lifecycle and various retirement policies.[7]

3.1 Household problem

Starting point is the problem of a household representing the decision making process of a generation. For notational convenience, I drop the time and the generation superscripts. They will be introduced, when I reach the resource constraints. The focus is on the lifecycle aspect. A household consists of two individuals: male and female. They are solving a joint decision problem reading as follows:

$$\max \sum_{a=1}^{M} \beta^{a-1} \eta_a \left(\omega u\left(c_{f,a}, l_{f,a}\right) + (1-\omega)\, u\left(c_{m,a}, l_{m,a}\right)\right)$$

s.t.

$$\begin{aligned}
(1+\tau_c)\left(c_{f,a}+c_{m,a}\right)+k_{a+1} &= \\
R_a k_a & \quad + \left[(1-\tau_l-\tau_r)\, w_a \left[z_{m,a}+(1-\tau_d)\, z_{f,a}\right] 1_{a<M_r}\right] + T_a \\
l_{s,a} &= n_{s,a}+q_{s,a};\, s=f,m \\
z_{s,a} &= h_{s,a} n_{s,a};\, s=f,m \\
h_{s,a+1} &= (1-\delta_h)\, h_{s,a} + g\left(q_{s,a}, h_{s,a}, s\right);\, s=f,m \\
& \quad h_{s,1} \text{ given } \forall s=f,m;\ k_1=0
\end{aligned}$$

where $1_{a<M_r} = \left\{ \begin{array}{c} 1 \text{ for } a<M_r \\ 0 \text{ otherwise} \end{array} \right\}$ and η_a is the unconditional probability of reaching age a. As part of the problem the household decides for each of its members ($s = f, m$) on the

[7] The focus is on a closed economy, but the qualitative findings would be valid in a small open economy.

consumption, $c_{s,a}$, and the amount of non-leisure activity, $l_{s,a}$. This non-leisure activity can take the form of time spend working in the market, $n_{s,a}$, or time spend enhancing the individual specific human capital stock, $h_{s,a}$. Besides human capital there is another form to save and that is in physical capital, k_a.[8] These savings are pooled by all household members. The physical capital savings provide a next period gross-return, R_a. Given that there are two individuals that face a common problem, I need to introduce a way to solve the internal allocation problem. This is done by introducing a weight ω that captures the relative importance of the individual's utility in the household problem. Besides the weight ω, another main difference between males and females is that females only receive a fraction $(1-\tau_d)$ of the wage of a male. I assume that there are two sources of this wage difference: one part is due to actual productivity differences, $\tau_d \nu$, and another part is due to discrimination $(1-\nu)\tau_d$. The productivity differences are mostly considered to be due to educational differences but other things might be important too. Discrimination in this context refers to wage differences that can not be explained by any observable characteristic. The difference factor τ_d will be generational specific and determined using wage data. It captures the fact that hourly wages of males and females have converged over the last decades and that this induced an increase in female and a decrease in male labor supply. Here, I pick up different ideas put forth by other authors. Goldin (1990) emphasized that cohorts are the right way of thinking about the recent convergence in female wages. Beyond this, Goldin stresses that a sizable part of this wage difference is due to gender discrimination.[9] Jones, Manuelli and McGrattan (2003) provide an abstraction that allows to introduce the gender-wage gap into a general equilibrium model in a simple fashion. I make use of their abstraction, the gender factor, since it is able to

[8] As part of this project, I considered a version of the model with household borrowing constraints. While the household problems computationally solve easier in this case, the predictions regarding the labor supply decisions of households deviate strongly from what can be found in household data. Furthermore the borrowing constraint makes labor supply at the aggregate level fairly inelastic leading to distrepancies between model and data in the aggregate.

[9] Here discrimination is used in the sense that there exists a wage difference between males and females even if controlling for a large number of observables. Empirically discrimination is not exhausting the gender-wage-gap. A sizable fraction can be explained by observables like occupation and education differences.

capture various theories in a convenient way.

The final gender difference is captured by the human capital accumulation technology. The root for this assumption is that the life-cycle patterns of wages between males and females are very different over time. This is also consistent with work done by Erosa, Fuster and Restuccio (2005). The authors show that the wage difference can be accounted for by the fertility decision. On average women are decreasing their labor supply after the birth of a child and loose human capital relative to a person without a child. Since I abstract from the fertility decision, this will be captured in my model by τ_d and the male–female human capital parameter differences.

The last aspect of the household's problem is retirement which is modeled as mandatory and common among males and females starting with age M_r. This means that starting at age M_r, nobody is allowed to work. There are a couple of ways the government influences individuals. On the one hand the government taxes consumption, τ_c, labor income, $\tau_l + \tau_r$, and capital income, τ_k. On the other hand, it provides age specific transfers. These transfers, as will become clearer later-on, have various components: lump sum rebates of unspent government taxes, the distribution of accidental bequests (evenly over all agents alive), and starting with retirement there are retirement benefits for the male and the female members of the household.[10]

For the analysis in this paper, I restrict my attention the following functional forms:

$$
\begin{aligned}
u(c,l) &= \begin{cases} \dfrac{\left[c^{\alpha}(1-l)^{1-\alpha}\right]^{1-\sigma}}{1-\sigma} \end{cases} {}^{11} \\
g(q,h;s) &= B_s \left(q_s h_s\right)^{\rho_s}
\end{aligned}
$$

The chosen preferences allow a consistency between data and model along various dimensions. First, the Frisch elasticity of labor supply is close to what macroeconomic data suggest,

[10] I am aware of the problem, that the transfers are not completely exogenous, but rather functions of your life time earnings profiles. This is common practice in the literature.

given that the household is representative of a large group of person and thus captures both the extensive and intensive margin. Second, given the specification, the model's consumption pattern if more realistic in capturing the hump shape and the drop after retirement as seen in the data. Finally, these preferences allow a split of consumption between the female and the male member of the household that is asymmetric ($c_f < c_m$) and changing over time ($c_{f,t}/c_{m,t}$ increases over time), qualitatively in with the data. The functional form for the human capital production function is standard, having the Ben-Porath form.

3.2 Firm's problem

There is a market sector that employs effective labor and rents capital from the households to maximize profit. The sector's decision problem is given by:

$$\max A_t \left(K_t\right)^{\theta} \left(L_t\right)^{1-\theta} - r_t K_t - w_t L_t.$$

There is a representative form that rents capital and pays for labor services.

The firm's First Order Conditions are restrictions on prices:

$$\begin{aligned} r_t &= \theta A_t \left(\frac{L_t}{K_t}\right)^{1-\theta}, \\ w_t &= \left(1-\theta\right) A_t \left(\frac{K_t}{L_t}\right)^{\theta}. \end{aligned}$$

In the model the capital returns net of depreciation are taxed before they are given to the households. Thus the gross return that reaches the households is given by:

$$R_t = \left(1 + \left(1 - \tau_{k,t}\right)\left(r_t - \delta_k\right)\right).$$

3.3 Resource constraints

There are three goods traded in the market place: capital, labor services, and the final output good. Until now, I was able to work with the notation of time and generations separately, now I need to introduce some notation. For the individual variables, I follow the convention that superscripts denote the generation and subscript denote the period. Thus for example $c_{f,t+j}^{t-j}$ is the consumption of a female, f, born in period $t-j$ that is alive in period $t+j$ and is j years old. Note that according to this convention all individual variables are infinite sequences that have at most M non-zero components. Furthermore I follow the following conventions: The population size for a given generation is denoted by N_{t+j}^{t-j} and $\eta_{t+j+1}^{t-j-1}/\eta_{t+j}^{t-j}$ is the survival probability till age $t+j+1$ conditional on being alive at age $t+j$. It captures the fact that some persons of a generation die from one period to the next. The persons that die, leave some accidental bequests behind. I assume that the capital stock they owned through their savings is scraped and distributed, through lump sum transfers to the survivors, respectively, the newly born generation.

The resource constraints in the economy are given by:

$$\sum_j N_{t+j}^{t-j}\left(c_{f,t+j}^{t-j}+c_{m,t+j}^{t-j}+\frac{\eta_{t+j+1}^{t-j-1}}{\eta_{t+j}^{t-j}}\left(\gamma_A k_{t+j+1}^{t-j+1}\right)-(1-\delta_k)\,k_{t+j}^{t-j}\right)+\left(D_{t+1}-(1-\delta_k)\,D_t\right)+G_t=F\left(K_t,L_t\right)$$

$$\sum_j N_{t+j}^{t-j}\left[\left(1-\nu\tau_d^{t-j}\right)z_{f,t+j}^{t-j}+z_{m,t+j}^{t-j}\right]=L_t$$

$$\sum_j \frac{\eta_{t+j+1}^{t-j-1}}{\eta_{t+j}^{t-j}}N_{t+j}^{t-j}k_{t+j}^{t-j}+D_t=K_t$$

As already pointed out when the household problem was described, I assume that a fraction $\left(1-\nu\tau_d^{t-j}\right)$ of effective female labor supply is productive and thus enters the production process.

3.4 Government

There is a government present in this economy that collects taxes and uses them for transfers and to pay for government expenditures. The budget satisfies two main redistributive functions, one is using resources for general government spending and transfers to everybody, the other one is using funds to transfer them to retired persons. Government policies are at the heart of this paper. I consider five major government policies that I outline in detail below. To understand them fully, I start by outlining the government budget constraint and then explain the ways to save for retirement with and without a PAYG-retirement system.

Government budget and retirement policies

To think about government policies it is essential to understand the government budget constraint. All considered policies are fiscal policies that either go through taxes, transfers, bonds, retirement age changes, or any combination of the four. As a first step, I outline the equations that determine the variables that make up the policies.

The policies consist of sequences that affect the government budget constraint.

$$\overbrace{\begin{matrix}\sum_{i=0}^{M-1} N_{t+i}^{t-i}\left[\tau_c\left(c_{f,t+i}^{t-i}+c_{m,t+i}^{t-i}\right)+\tau_{k,t}\left(r_t-\delta_k\right)\frac{\eta_{t+j+1}^{t-j-1}}{\eta_{t+j}^{t-j}}k_{t+i}^{t-i}+\tau_{l,t}w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}+\right.\\ \left.\left(1+\tau_{l,t}+\tau_{r,t}\right)\left(1-\nu\tau_d^{t-i}\right)w_t z_{f,t+i}^{t-i}1_{i<M_r}+\{(1-s_{t+i+1}^{t-i-1})k_{t+i}^{t-i}\}\right]-G_t-\sum_{i=0}^{M-1} N_{t+i}^{t-i}T_{g,t+i}^{t-i}\end{matrix}}^{\text{General government account}}+ \tag{1}$$

$$\underbrace{+\sum_{i=0}^{M-1} N_{t+i}^{t-i}\left(\tau_{r,t}w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}-T_{r,t+i}^{t-i}\right)+\gamma_A D_{t+1}-R_t D_t=0}_{\text{Retirement account}}$$

$$\text{where } T^{t-i}_{r,t+i} = \left\{ \begin{array}{ccc} \chi_t b^{t-i} & & i > M_r \\ & \text{for} & \\ 0 & & otw. \end{array} \right\} \tag{2}$$

$$\text{and } b^{t-i} = \left[\sum_{s=1}^{M_r} \varepsilon_s w_{t-i+s} z^{t-i}_{t-i+s} \right]. \tag{3}$$

under the current system in the U.S., benefits are determined as the product between the replacement rate χ and the benefit base, b. The benefit base for an individual i is defined as the price indexed weighted average over all of individual i's earnings, where the weights are such that the maximum 35 year earnings are averaged over. The replacement rate is the percentage of the pre-retirement earnings that the social security system is supposed to replace. It is roughly 42%.

With the exception of the move to a fully funded system, I operate under the assumption that the government balances the retirement account independently from the general account. The exception is explained later in the paper. Given the balancing assumption the two accounts are separated, and the problem breaks down into two pieces, a general government account and a retirement account. The general account takes consumption tax revenue, capital income tax revenue and labor income tax revenue and uses them to finance government spending and equal transfers to all individuals in the economy.[12] There are two extra source of government revenue in this model. They mainly serve to close the model in a consistent way: there are accidental bequests and the income of females that is not given to them due to discrimination. Regarding the accidental bequests, I assume that the government liquidates the capital stock that is left behind by persons that die surprisingly and redistributed in an even way across the currently alive persons.

The retirement account takes social security tax income from males and females and poten-

[12] I am aware of the possibility of rising governement expenditures due to an aging population, but will abstract from it in the analysis below. This is done since government expenditures are hard to project and an increase of these expenditures would affect all persons alive equally and thus not change the preference for the policies under consideration.

tially issues debt and funds the transfers according to benefit eligibility. If there is prior debt, then gross interest is paid on the debt.

The main emphasis of this paper is on the retirement account. There are different ways of ensuring that this account is always balanced. One is to adjust social security taxes, τ_r, holding everything else constant. Another one is to adjust benefits as needed, T_r. This can happen in two ways: by changing the replacement rate, χ, or by changing the benefit base, b. I will focus in this paper on adjustments of the replacement rate. The third approach to balance the budget is to adjust the age at which a person can receive retirement benefits, M_r. The final one, also considered, is to pay with debt, D. I use a constraint on the behavior of debt to ensure a well behaved problem. I assume that that there exists some period $\tilde{T}$ after which all the government issued bonds have to be paid off and no new ones can be issued. Namely, I assume that $D_t = 0, \forall t \geq \tilde{T}$ with $\tilde{T} < \infty$. So, the policies I consider are such that I will fix three out of four objects and use the remaining one as the policy instrument, so that the government budget is balanced.

There exists a possibility of using some convex combination of the different 'pure' policies I am using, but those are mainly watered down versions of the extremes and their implications can be derived from the policies I do consider. I now describe the policies in more detail, starting with the Pay-As-You-Go preserving ones:

Retirement tax increase The government fixes the amount of benefits, debt and the retirement age and varies the social security tax rate, τ_r, to pay for the increased retirement transfers due to population aging.

Retirement benefit decrease The government fixes the retirement contribution tax rate, the debt, and the retirement age and varies the benefits.

The idea has been put forward (see for example Kotlifkoff et al., 2007) that the government charges higher taxes early on and then accumulates assets to finance the baby boom retirement period. I will not consider these policies. This step is justified by the fact that most of the problems confronting the financing of a PAYG social security system stem from more permanent problems and not from temporary ones. The increase in longevity is here to stay and the low fertility rates in many industrialized countries will have effects outlasting the baby boom retirement. So, pre-financing is not a long lasting solution. Thus I focus on policies that are sustainable in the long-run. The remaining policies I consider are:

Retirement age increase The government changes the age after which persons are eligible for social security payments. As a result of this policy it has to do something with the residual gap between social security tax revenue and outlays. I consider two alternatives: either taxes or benefits are used to balance the budget. In this case, debt and either taxes or benefits are held constant.

Sofar, I considered policies that attempt to preserve the PAYG nature of the retirement system. Now, I present a suggestion of how to transition to privatize the social security system.

Switch to fully funded system The government abolishes the Pay-As-You-Go system and starts a private-savings-based retirement system.[13] The policy is implemented as follows: The social security taxes are set to zero. No further payments into the old system are made and the benefit base can't be increased any further for anybody. The government incurs debt to pay the persons that *up to that point-in-time* have paid into the old system (even for a short period of time). The time-varying benefit factor is fixed at its target level and individual benefits are fixed to the level accumulated up to the switch. As a result of the tax increase, economic activity will increase and consumption, capital and labor will increase. The debt

[13] I am aware of the time inconsistency problem faced by a government arising from an undersaving of households in the hope that the government eventually will use transfers to help them out, when they are old. Following Prescott (200?), I assume that the government institutes mandatory private savings accounts against whose proceeds the household's can borrow when very young but are required to pay back in a fixed number of periods.

is paid off by excess tax revenue from these three sources.

For those who are interested in a mathematical description of the policies, I refer to Appendix A.5. There all policies are described in detail.

4 Calibration and model evaluation

This section has two parts. The first part describes the calibration process and the moments matched. The second part focus on the performance of the model for the past four decades. The focus will be on macroeconomic aggregates.

4.1 Calibration

The parameters and exogenous processes are listed in Tables 1 and 2. As a first step, I start by describing how I obtain the exogenous processes. Whenever possible they are taken straight from standard sources such as BEA or the Census bureau.

The population development is taken directly from the U.S. Census bureau, see Figure 5 for an overall development of the population. Historical data are used up to the year 2000, from then onwards I use of the Census Bureau's medium projection.[14] For the periods beyond the projection period, I start by fixing the long term population growth rate. γ_N such that it is consistent with the growth rate of population that prevailed from 2040 to 2055. Then I assume that this growth rate continues and that the population-age profile remains as it was in 2055. The key underlying assumptions regarding the medium projection for 2050 are: fertility rate $2,219$, life expectancy at birth males 79.8 years, females 81.2 years, net immigration as fraction of population 0.26 per cent per year.

For the survival probabilities, I made use of the life tables from 1890 to 2004 as provided by the National Center for Health Statistics. To extend the lifetables beyond the historical period,

[14] See <http://www.census.gov/population/www/projections/index.html>.

I used the assumption that the relative population increase at age 65 in the Census Bureau's projection reflected an increase in the survival probability rather than immigration and thus shift the life table curve for 2004 accordingly. The result is approximately in line with the projections for the population-age groups 65 and older.

Government spending and Total Factor Productivity were determined using National Income and Product Accounts (NIPA). I first determined government spending net of government capital depreciation and net exports. Then I took the ratio of this over the corresponding output measure. Then I average the resulting quotient for the period 1950 to 2007 to obtain g. As a result, government spending is defined as $G_t = g \times Y_t$, which states that government spending is a constant fraction of output and thus grows in line with output. Taking the capital income share as measured from the NIPA, I calculate TFP as: $A_t = Y_t / \left(K_t^{\theta} L_t^{1-\theta} \right)$. Where K_t is the net fixed assets in the economy and L_t are the total hours worked. The resulting TFP for the period 1950 to 2004 is plotted in Figure 5.

Consumption taxes are based on own calculations using the NIPA. The method and results are in line with Mendoza et al. (1992). The income taxes, both capital and labor, are taken from Joines (1981) for the period 1929 to 1975. I extend the period of coverage forward following the methodology of Mendoza et al. (1992). For the social security tax rate, I make use of the historical schedule as provided on the webpage of the U.S. Social Security Administration. I include both the employer and the employee rate. The resulting time series are shown in Figure 5.

One of the more difficult issues is the measurement of the female wage factor τ_d. Here, I make use of the the movement of the wage rate of females relative to males. The direct measurement from the CPS is presented in Figure 6. I follow the convention that generation g refers to a member of a cohort that turned 20 at the date g. The figure shows the wage of males relative to females for different artificial cohorts, averaging their earnings over the lifetime. Given this convention, the generations up to 1935 faced a $\tau_{f,d} = 0.393$. Generations born from 1985 onwards faced, $\tau_{f,d} = 0.087$. In between the factor followed a linear trend. For all time periods, I normalized the male factor to zero, $\tau_{d,m} = 0$.

Next, I turn to the parameters. A list can be found in Table 2. There I also list the moments that are matched. Since most of the parameters are fairly standard, I won't dwell on them, the table supplies sufficient information. There are two parameters that are hard to determine, the welfare weight in the household problem, ω, and the relative importance of productivity differences relative to discrimination ν. I make the assumption that both parameters are one half. I conducted a sensitivity analysis varying the parameters by $\pm 10\%$ and compared the results with those of the benchmark calibration. It turns out that the value of ν is fairly unimportant either for the relative policy ranking or the overall economic outcome. For the bargaining weight one half seems a natural first choice giving all household members the same weight. The sensitivity analysis shows that the choice of ω has important macroeconomic consequences. A lower weight on females for example implies a higher labor supply and thus a higher output. This is a simple consequence of the fact that if the household cares less about females, it makes them work harder in the market even if their time is not as productive. Any quantitative change is wiped out when the main labor parameters are re-calibrated.

The only other parameters that were difficult to calibrate were those governing the market labor supply and the human capital accumulation. To estimate these parameters $\left(\{B_s, \rho_s, h_{s,1}\}_{s=f,m}, \delta_h, \alpha\right)$, I use consumer data. In particular, I employ the Current Population Survey for the United States from 1961 to 2006, as provided by the IPUMS project[15]. From that dataset, I determine the wage

[15] Miriam King, Steven Ruggles, Trent Alexander, Donna Leicach, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Version 2.0. [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004. URL: cps.ipums.org/cps.

per hour and the average hours worked per capita.

Name	Symbol	Derivation
Wage per hour	wph	$\frac{\text{Total wage and salary income}}{\text{Number of employed persons*Hours worked per employed person}}$
Average hours	n	$\frac{\text{Number of employed persons*Hours worked per employed person}}{\text{Number of persons}}$

These objects are calculated for different age groups, starting at age 20 and proceeding in one year steps until age 64 inclusive. Next, for the cohort that reaches age 20 in 1971, I determine the wage per hour and the average hours in the data and in the model, where I assume that the model is on a balanced growth path and from 2010 onwards the policy is one with a PAYG retirement system with the social security tax increasing. Then I solve the problem:

$$\min_{P=\left(B,\rho_1,\rho_2,h_{0f},h_{1m},\alpha\right)} \frac{1}{10}\sum_{a=1}^{10}\sum_{g=f,m}\frac{5}{100}\left(\frac{w_g\left(P\right)z_g\left(P\right)}{n_g\left(p\right)}-wph_{g,d}\right)^2+\frac{95}{100}\left(n_g\left(P\right)-n_{g,d}\right)^2$$

$s.t.$

$\left(w_g\left(P\right),z_g\left(P\right),n_g\left(P\right)\right)$ are part of the balanced growth path equilibrium given policy P.

The fit of the resulting model is presented in Figure 7. The left side presented the result for males and the right side the results for females. In the first row are the wages by age group and in the second row are the hours worked by age group. Overall the fit is fairly good in the sense that both the wage and the labor supply results of the model coincide well with those of the data. The model also captures the relative labor supply and the relative wage rates of males and females well. There is a slight discrepancy between model and data regarding the age group 50 and above.

4.2 Aggregate implications of the model

Since I use the model to conduct policy analysis for the future, I wish to confirm that the model is a good abstraction regarding past macroeconomic developments. To do this, I consider the period 1950 to 2005, for which I have historical data, to look at aggregate output, capital stock, and labor supply, as well as Total Factor Productivity.

I start by evaluating the model performance for the four major aggregates: output, capital stock, total hours worked and TFP during the historical period 1950 to 2005. Notice that TFP in the model is determined in the same way that TFP was determined in the data, namely as $TFP_t = Y_t / \left(K_t^{\theta} \left(\sum_{i=1}^{M} N_{t+i}^{t-i} n_{t+i}^{t-i} \right)^{1-\theta} \right)$. I present only the results under one policy, since for the historical period these results are representative for all policies and thus focusing on one policy just helps to have a clearer comparison. The comparison is presented in Figure 8. The policy used here is that of a tax increase holding benefits, retirement age and debt constant. The key insight is that the model does very well regarding output till 1990 and for aggregate labor supply up to 1985. Furthermore, the movement of both TFP and the capital output ratio are well captured by the model. The main discrepancy is regarding labor supply starting in 1990. Here a gap opens that then shows up in output data one period delay. It appears that the model misses some aspect of increased aggregate labor supply.

To better understand labor supply and the importance of female labor supply, I conducted a counterfactual experiment. The experiment compares the benchmark model with a world in which males and females are identical in the sense that $\tau_{f,d} = \tau_{m,d} = 0$ for all generations. The implications of this experiment sheds light on the importance of having females incorporated in the model. I present output and labor supply under the two scenarios in Figure 9. The benchmark calibration (solid line) is contrasted data the data (circles) and the counterfactual (dashed line). It is clear for the labor supply plot, that the presence of the gender wage gap as taken from the data is capturing a significant part of the overall labor supply increase. Without it the discrepancy between the output in the model and in the data would be much larger, roughly by a factor of two. Similarly, aggregate labor supply would be considerably lower, than what is actually observed in the data. Note that while the inclusion of females is important for the aggregate development, it does not matter for the policy ranking in the model.

I conclude my comparison of model and data with a look at the average male and the average female labor supply decision. This is done in Figure 10. The first plot shows the data average hours of males (circles) and the model's (line with crosses) for the period 1965 to 2005.[16] The data are taken from the Current Population Survey. The model captures both the direction and the change correctly. The model misses out on the timing of the movement. It appears, that the decline in male hours took mostly place in the 1970s, while the model stretches it out till the 1990s. Regarding the increase in average hours worked by females, the model is able to capture a significant part of the increase, about 60 percent. It actually get the timing right for the largest part of the increase, 1980 to 2000. The main problem is that there appears to have been an increase in female labor supply from 1965 to 1980, that the model completely misses. This is most likely also the main reason, for the aggregate labor supply result.

This section has presented the calibration and a model–data comparison along multiple dimensions. The comparison has revealed, that the model captures past developments of labor supply, output, the capital–output ratio and of male and female labor supply fairly well. A key to this is the introduction of female labor supply and the movement of the labor factor, $\tau_{f,d}$.

[16] I only had access to the CPS micro data for the period 1961 to 2007.

5 Policy experiments

Now the model almost ready to conduct policy analysis. Before I proceed, I still have to specify a metric that allows for a comparison between different policies. I use as my welfare criterion the lifetime consumption equivalents (LTCE).

5.1 Welfare metric

Nearly all advanced industrialized countries have as part of their retirement funding a pay-as-you-go system in place, for example Canada, France, Germany, Italy, Japan, and the United States. Population aging makes the funding of the PAYG system problematic for policy makers as they are forced to either hurt the older population by decreasing benefits or the younger generations by increasing their tax burden. Different policies are feasible and to determine who gets hurt and by how much, I make use of the life-time consumption equivalent (LTCE) as my welfare measure. It provides me with a measure for each generation that tells me by how much I would have to change their lifetime consumption to make them indifferent between two different policies. Formally the definition of the LTCE for a generation j under a policy p', $\lambda^{j,p'}$, is defined as follows.

Definition 1. *Consider a reference household i that lives under policy p and let $\left\{c_{s,a}^{i,p}, l_{s,a}^{i,p}\right\}_{s=m,f;\forall a}$ be his optimal decision for consumption and leisure. Then the life time consumption equivalent (LTCE), $\lambda^{j,p'}$ for a household j that lives under the policy p' and chooses the optimal allocation $\left\{c_{s,a}^{j,p'}, l_{s,a}^{j,p'}\right\}_{s=m,f;\forall a}$ is defined as:*

$$\begin{aligned} & \sum_{t=1}^{M} \beta^{a-1} \eta_a \left\{ \omega u \left(\boldsymbol{\lambda}^{j,p'} c_{f,a}^{i,p}, l_{f,a}^{i,p} \right) + (1-\omega) \, u \left(\boldsymbol{\lambda}^{j,p'} c_{m,a}^{i,p}, l_{m,a}^{i,p} \right) \right\} \\ = \; & \sum_{t=1}^{M} \beta^{a-1} \eta_a \left\{ \omega u \left(c_{f,a}^{j,p'}, l_{f,a}^{j,p'} \right) + (1-\omega) \, u \left(c_{m,a}^{j,p'}, l_{m,a}^{j,p'} \right) \right\}. \end{aligned}$$

In words, the LTCE is the percentage increase in consumption that the reference person (i,p)

needs, to make him indifferent with the person (j, p').

For the particular utility function I consider, the measure is given by:

$$\lambda^{j,p'} = \left(\frac{\sum_{t=1}^{M} \beta^{a-1} \eta_a \left\{ \omega u \left(c_{f,a}^{j,p'}, l_{f,a}^{j,p'} \right) + (1-\omega) u \left(c_{m,a}^{j,p'}, l_{m,a}^{j,p'} \right) \right\}}{\sum_{t=1}^{M} \beta^{a-1} \eta_a \left\{ \omega u \left(c_{f,a}^{i,p}, l_{f,a}^{i,p} \right) + (1-\omega) u \left(c_{m,a}^{i,p}, l_{m,a}^{i,p} \right) \right\}} \right)^{1/(\alpha(1-\sigma))}$$

Note that the policy evaluation of a given generation is invariant to exogenous growth.

5.2 Policy experiments

Before I go to the implications of the policies under consideration, let me remind you of the policy options. The government has four key policy variables that it can change: retirement taxes, benefits, the retirement benefit eligibility age, and debt to fund benefits for a period of time. In the following experiments I normally fix three of these policy choices and change the fourth one as needed to make the retirement budget constraint hold with equality. To reduce redundant language, only the mentioned changes are made and the other options are constant at their 2005 level.

The basic three policies

The first three policies I consider are a tax increase, a benefit decrease and a move to a fully funded system. Figure 13 presents the main aspects of the three policies. Here benefits are measured as a fraction of the average pre tax earnings. Taxes are the tax rate. Debt is measured as debt over output at the annual rate and retirement age is fixed for the basic policies.

Under the first policy taxes are the main financing instrument while benefits are held constant at the replacement rate of 42%[17]. As a result, taxes have to increase to 18.5% by 2050, till they slightly decrease to 18.1%. The reduction is not nearly as much as one might expect and suggests

[17] This is the declared long term goal of the Social Security Agency. See for example the Final Report on the Social Security "Notch" Issue at <http://www.ssa.gov/history/notchfile1.html>.

that while the baby boomer's retirement might contribute to the fast rise, it just speeds up the inevitable. The next policy, Benefits move, fixes the taxes at their 2005 level and then adjusts benefits to accommodate the demographic change. I find that a benefit decrease to 28% is needed. Finally, the privatization touches nearly all policy instruments. It fixes benefits for the persons that have paid into the old system at 42%, and does not allow any further benefit eligibility increase afterwards. The social security tax is set to zero from 2005 onwards. To finance the transition, it makes use of debt. This debt peaks in 2075 at roughly 4 times the total output. The debt will be paid of by 2205. Two things about privatization at this stage appear to be unappealing: the huge amount of debt and the long time that the government needs to be committed to repay the debt. While the second concern is a valid one, the first concern is superficial given the approach of privatizing in this paper. The amount of debt that the transition generates is quite useful in mitigating the transition. Financial economist have suggested that the equity market is incapable in a short period of time to absorb all the savings that will accumulate as a result of the privatization. Thus the debt is a good place to absorb a large fraction of the higher savings and thus buys the equity market time to adjust to the increased demand.

Before I turn to the welfare results, I wish to clarify the nature of the different options. The tax on labor income is distortionary and thus leads to a welfare loss. The retirement benefits on the other hand are a lump sum transfer that does not distort any marginal condition and thus is under normal circumstances welfare neutral. In a sense, it represents a partial move to a fully funded system, since it requires a significant increase in private savings to finance the retirement period. Finally, a fully funded retirement system in itself is welfare improving by removing the distortionary income taxes of the current PAYG system, but might hurt the persons that have to pay for it during the transition. Keeping this in mind, we can see in Figure 12 the welfare consequences of the three policies. The x-axis represents the generations that turn 20 in the given year and thus start working in the model. The y-axis represent the Lifetime-Consumption-Equivalent of each generation relative the benchmark generation born in 2000 under the tax increase policy. A value of higher than one indicates that the considered generation gains relative to the benchmark generation and a value smaller than one means that it looses relative to the benchmark generation. What matters for the policy choice is the relative position of the lines. A generation x prefers a policy p over the alternative policy p', if its welfare is higher under p, than under p'. This implies for the considered policies, that all generations before 1995 (i.e. born before 1975) prefer a tax increase policy over a benefit decrease policy and all generations after 1995 (i.e. born after 1975) prefer the benefit decrease option over the tax increase option. The fact that the welfare benefits for the young from the benefit decrease option is much higher than the ones of the old from the tax increase option suggests that there might be the possibility of a welfare improving transfer scheme. The problem of such a scheme is to determine, how much the future generations should pay to the current old and how to facilitate the payment without creating any distortions. Last but not least, the move to a fully funded system as outlined above. The welfare consequences are such that the older persons are virtually bribed into the new system by the promise to get the same benefits as before. There are further gains that stem from an increase in the value of their private savings after the move, due to a relative shortage of capital. The middle generations, generations 1975 to 2005, gain particularly because they get some benefits from the old system and less taxes for most of their working life. The main reason for the dip in welfare around 2120 is the huge amount of debt that has to be paid off by the working age generations at that stage. I shall refer to a policy as being Pareto superior relative to the alternatives, if it makes all generations better off and none worse off. I conclude that the proposed move to a fully funded system is a Pareto superior policy. It is clearly preferred by all over any other considered alternative, including convex combinations.

To understand the reason for the result a bit better, I present the main aggregates under the different policies, see Figure 13. As already stated privatization is the least distortionary policy, followed by the benefit decrease option. The latter one essentially leaves the main aggregates unchanged as the demographic change unfolds. Note the slight increase in measured TFP. This is solely due to a change in the age structure of employed persons. The median age increases and as a result the human capital content increases. This is due to the fact that the median age increases on the upward sloping side of the human capital lifecycle profile. A different scenario unfolds in the case of the tax increase option. It has a depressing effect on labor supply and capital accumulation. The first effect stems directly from the distortionary nature, the latter one from the relatively smaller need for private savings. The decrease in capital also means that less human capital is needed leading to a decrease in TFP. Output goes down relative to its 2000 level. Finally, after the move to a fully funded system the labor supply will go up due to the decreased tax burden. This has only a slight effect on output, since the fast increase in debt crowds out private investment. As savings pick up this effect evaporates and the capital intensity goes up. The increase in capital intensity implies a decrease in the capital rental rate making human capital accumulation more profitable and thus leading to a substitution away from labor toward more human capital. In the long–run labor will decrease nearly back to its pre-reform level, while TFP, output and capital stock will be much higher.

Retirement age increase as a cushion

A natural alternative to moving taxes or benefits is to adjust the retirement benefit eligibility age, short retirement age. Unlike the previous two approaches to maintaining the PAYG system, retirement age increases work on both sides of the budget constraint at the same time. They lower the expenditures by decreasing the number of retirees and they increase the tax revenue by increasing the tax base. I consider the retirement age increase in conjunction with either a benefit decrease or a tax increase policy. The particular age increase I use is 5 years from 64 to 69. The main reason for the five year increase is that it is rounded to the closest whole year needed to fully cover the demographic transition period.

Figure 14 shows the policy variables and how they adjust. I contrast the results with those of a privatization. The first thing to notice is that regarding benefits the three policies are very similar, all three are ending up at about the same level. Based on the figure, the two retirement age increase policies are very similar reflecting the fact that a retirement age increase by 5 years is nearly offsetting the effect from population aging. This makes sense when considering the effect of population aging, which roughly increases the median age in the population by five years between 2000 and 2050, see Figure 1. So, the main difference between the two policies is a slight differences in the benefits and the taxes. To contrast the policies with the pure benefit decrease or tax increase policies, we see that both the benefits and the taxes are kept stable. Thus the policies neither create large substitution or income effects relative to the initial situation.

Next, I consider the implications of these policies for the economic aggregates as presented in Figure 15. The shift in the retirement age has an immediate implication in driving up labor supply and thus output. The increase is comparable to what the decrease in social security taxes under a privatization does. At the same time the retirement age policies have little impact on the capital–output ratio or on TFP. Thus under either of the two alternatives little to no change is initiated beyond the initial impact.

Now we are ready to assess the welfare consequences in Figure 16. The key insight is that both of the added policies are inferior to the privatization policy. The main reason is that they still leave the leisure–consumption distortion in place, while fixing the funding problem of the PAYG system. Both policies are superior to the pure policies, but each one has its own supporters. Retirement age increases with benefit decreases are desired by the future generations. This is due to the fact that distortions are slightly decreasing and benefits are still fairly high. Apparently the extra benefits are more than compensating for the longer working life. The tax increase version is preferred by the generations between 1960 and 2020. Those generations are not working that much under the increased taxes and prefer the relatively high benefits. Both policies are nearly equally liked by the pre 1960 generations and privatization is just able to make those generations indifferent between preserving the PAYG system and the fully funded system. Overall I conclude that privatization even in the set of five policies is Pareto improving. The next best solution appears to be a retirement age increase in some combination with a tax increase or a benefit decrease policy. This is likely one of the reasons, why many countries have gone the path of preserving the PAYG system by increasing the retirement age.

It is time to better understand what exactly the sources of the welfare gains are. I make this clear using Figure 17. The figure shows the transfers per person and the aggregate consumption. In each case the base is the 2005 level of a person living under a taxes increase policy. Focusing on the long run transfers it becomes clear that the fully funded system implies huge gains of 45 per cent in terms of transfers relative to any alternative scenario. These larger transfers are the result of increased economic activities due to the removal of the labor market distorting tax rate. Without these large gains, a move to a fully funded system based on debt funding would be infeasible since the debt servicing requires considerable resources. The other point that this transfer plot makes is that the privatization is balancing the bribing of the old and the debt payment. It is crucial for the success of the transition (regarding debt repayment and welfare gains) that this balancing act is feasible. I already mentioned that the decrease in social security taxes stimulates the economy. This is also visible in the plot of aggregate consumption. The initial impact is an increase by 10 per cent that subsequently increases to 28 per cent relative to the pre reform level. This gain in consumption is the main reason for the welfare ranking.

Something that has not been emphasized before, is that the long-term welfare differences between the different alternatives at ± 3 per cent in LTCE are not that large. It is mostly the look at the overall gains over multiple generations that makes the move to the fully funded system appealing.

The fact that the long term benefits on a per person level aren't that large might be one of the reasons, why the move to the fully funded system is not very popular. The analysis above makes it clear, that there are alternatives that work fine in welfare terms for the current generations and still allow the preservation of the PAYG system. The other important problem of a privatization is the long duration of the transition period. Nearly all PAYG policies imply a very fast transition, but privatization does not. Indeed in the version considered here, it requires a clear commitment of the policy makers to a strict repayment schedule.

5.3 How does human capital matter?

So far, I have paid little attention to the role of human capital. The only policy for which it was important was privatization. As a step toward understanding the role of human capital, I conducted a simple experiment. The heart of the experiment is a change of the model to allow for exogenous human capital development. So, individuals will not save any longer in the form of human capital instead they will get an exogenous time-dependent stream of human capital $\{\bar{h}_a\}$. I refer to these exogenous factors as efficiency units. They are chosen such that they coincide with the stream used for the benchmark calibration. Notice that the definition of effective labor supply changes to $z_a = \bar{h}_a n_a$. Aside from this the model remains unaffected. The comparison between a privatization with exogenous human capital and endogenous human capital is presented in Figure 18. One of the main differences between the two models is visible in the plot of the debt–output ratio. The model with efficiency units ends the transition period much faster and is debt-free 75 years earlier than the model with endogenous human capital. This is mainly due to the fact that during the transition with endogenous human capital a decrease in the return to capital implies a reallocation of labor toward more human capital investment which leads to a much slower long run decrease in the capital return and thus makes the debt repayment much more costly. The slower adjustment of capital and human capital is also visible in the figure. An implication of the more costly transition is that the transition in the presence of human capital might not be viable, because the debt goes out of hand. This happened for example, when I focused on increased labor income tax revenue as the basis of the retirement funding. Prescott (2004) shows that this form of funding is a viable option in a world where the capital rental rate and the wage rate are constant during the transition and no human capital present. I find that this is still a viable option, when there is no human capital adjustment but price movement. This funding option fails in the presence of human capital. A broader tax base is needed to fund the transition period or the debt goes to infinity.

6 Conclusion

In this paper, I analyze the welfare and macroeconomic consequences of different retirement policies in the presence of population aging. The policies that I consider are: an increase in the tax that funds retirement, a decrease in retirement benefits, an increase retirement age combined with either tax or benefit changes, and a debt funded move to a fully funded system (also referred to as privatization). The key finding is that while a move to a fully funded system is the only Pareto improving policy in the considered set of policies, it leads to large movements in macroeconomic aggregates that might make it undesirable from a policy makers perspective. Beyond that the paper establishes that the presence of human capital makes it harder to finance the transition and prolongs the transition period.

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Appendix

A.1 Tables

Table 1: Calibration of processes used.

Process	Approach
$\{N_t^g\}$	Population (national estimates for the U.S.)
$\{\eta_t^g\}$	Survival probabilities (life tables for the U.S.; National Center for Health Statistics)
$\{G_t\}$	Government spending (estimated government expenditures as fraction of output; BEA) $\frac{1}{55}\sum_{t=1950}^{2005} \hat{G}_t/\hat{Y}_t = g$
$\{A_t\}$	TFP from market production (Using NIPA data for capital, output, and CPS data for labor supply)
$\{\tau_k, \tau_l, \tau_c, \tau_r\}$	Taxes (taken from Mendoza et al. and the Social Security web page)

Table 2: Calibration values used.

Parameter	Value	Fact
M	17	Maximum life span in data 100+, starting at 20
M_r	11	Expected age of retirement 65
β	$(1.026)^5$	Real annual interest 4.2 per cent (ave. long run)
δ_k	$1-(1-0.07)^5$	Annual capital depreciation 7 per cent
σ	4	Intertemporal elasticity of substitution 2
θ	0.356	Capital income share
γ_A	$(1.018)^5$	Mean gross growth rate of output (1950-2007)
γ_N	$(1.007)^5$	Mean projected population growth beyond 2040
$\left[\begin{array}{c} \left\{\begin{array}{c} B_g \\ \rho_g \\ h_{0,g} \end{array}\right\}_{g=f,m} \\ \alpha \quad \delta_h \end{array}\right]$	$\left[\begin{array}{c} \left\{\begin{array}{ccc} 0.64 & & 0.74 \\ 0.28 & ; & 0.19 \\ 0.69 & & 0.75 \end{array}\right\} \\ 0.32 \quad 1-(1-0.05)^5 \end{array}\right]$	Calibrated to match lifecycle profile of labor and earnings (using a minimum distance estimator)
ω	0.5	*Sensitivity analysis*
ν	0.5	*Sensitivity analysis*

utility function: $\{\alpha, \sigma, \beta, \omega\}$; production technologies: $\{\theta, \gamma_A, \delta_k\}$; labor supply: $\left\{\{B_s, \rho_s, h_{s,1}\}_{s=f,m}, \delta_h\right\}$ endowments: $\{k_1\}$

Table 3: Key policy variables.

Sequence name	Sequence symbol
social security taxes	τ_r
labor income taxes	τ_l
bonds	D
transfers	T
benefit rate	χ

Table 4: Changes of main aggregates and welfare under different policies from 2000 to 2200.

	Output	Capital	Total hours	Labor Productivity	TFP	Welfare of final generation†
Fixed retirement age						
- Decreased benefits	104	105	103	101	100	100
- Increased taxes	97	92	101	96	99	99
Increased retirement age						
- Decreased benefits	106	101	107	98	101	100
- Increased taxes	106	100	109	98	100	100
Fully funded system	129	157	107	121	105	102

†The reference point is the welfare of a person born in 2000 under the fixed benefit, fixed retirement age policy.

A.2 Figures

Figure 1: Median age of population in G7 countries from 1950 to 2050.

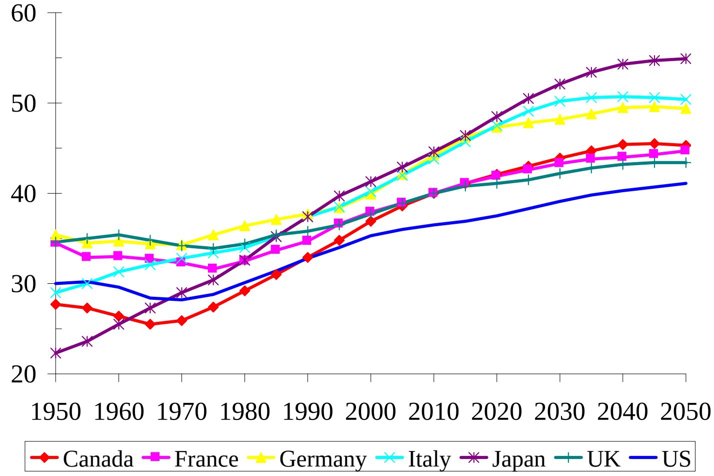


Figure 2: Fraction of 80 years and older in population for G7 countries from 1950 to 2050.

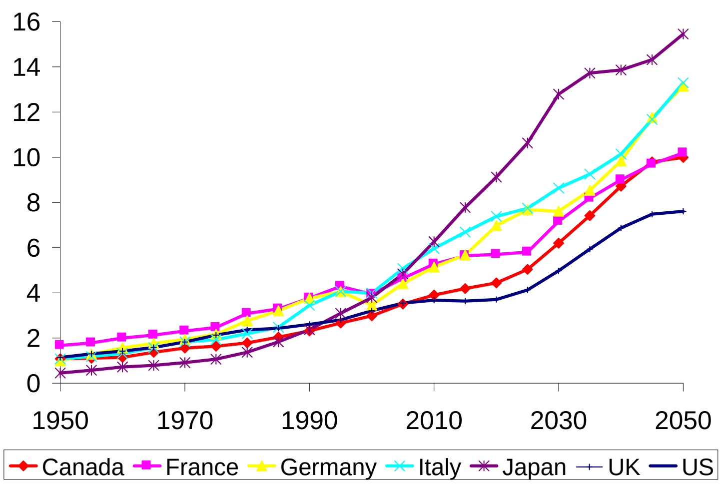


Figure 3: Total fertility rate in G7 countries, from 1950 to 2050.

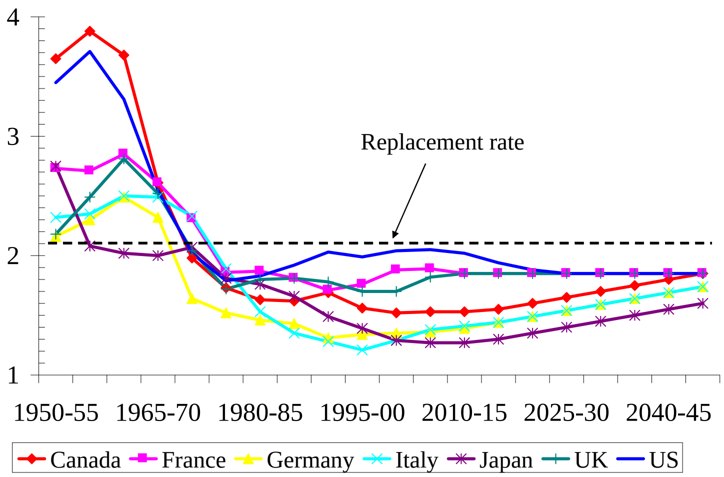


Figure 4: Fraction of 65 and older relative to population age 15 to 64 for G7 from 1950 to 2050.

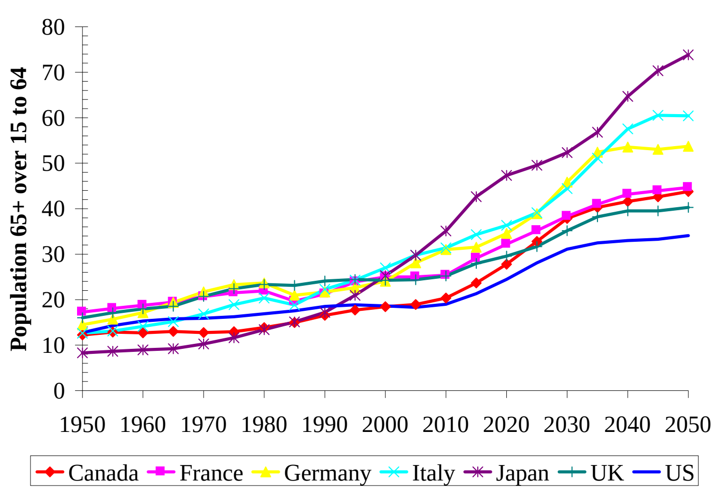


Figure 5: Exogenous processes.

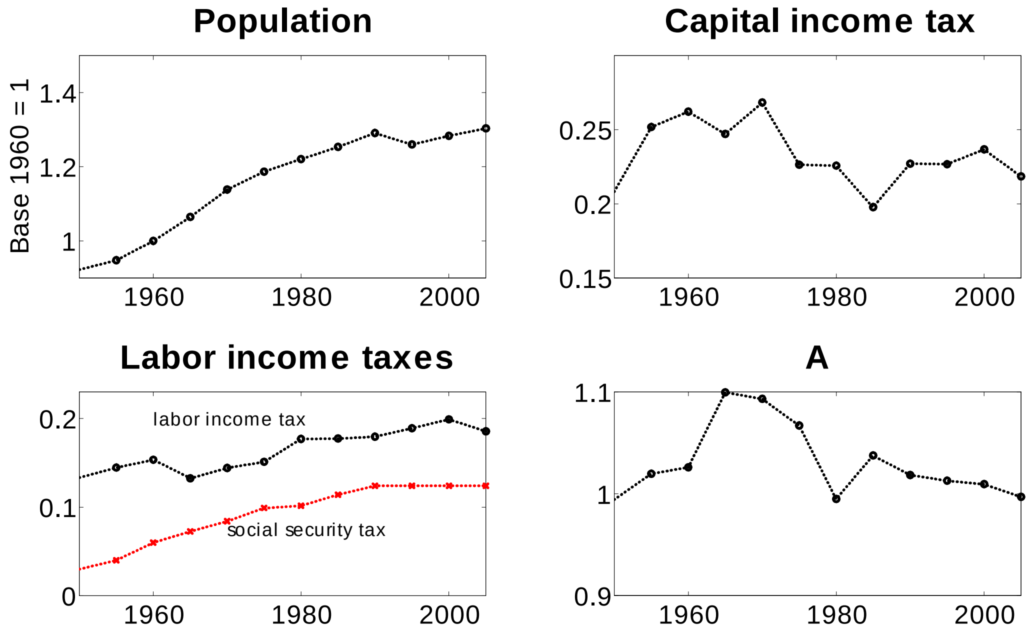


Figure 6: Female-male wage difference used as a measure for τ_d.

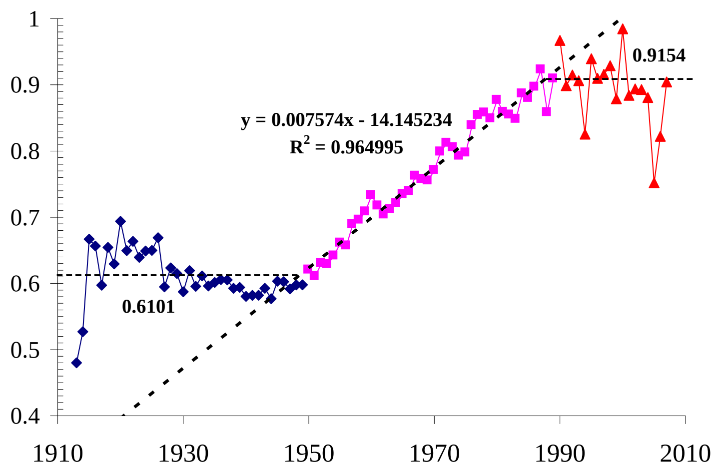


Figure 7: Calibration implications based on minimum distance estimation.

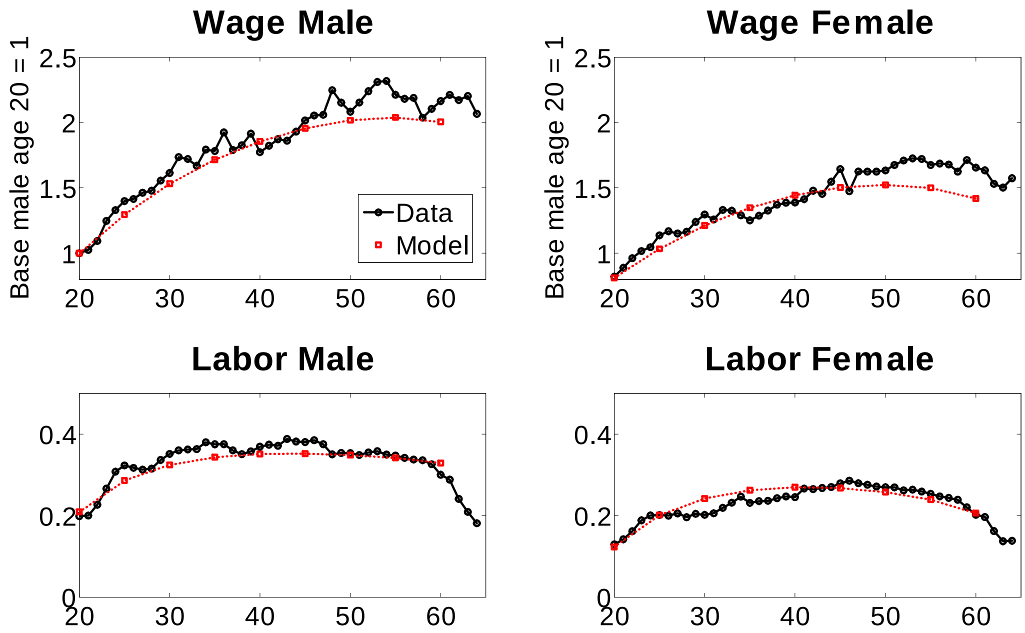


Figure 8: Comparison of Model (benefits adjust) and Data for the historical period 1950 to 2005.

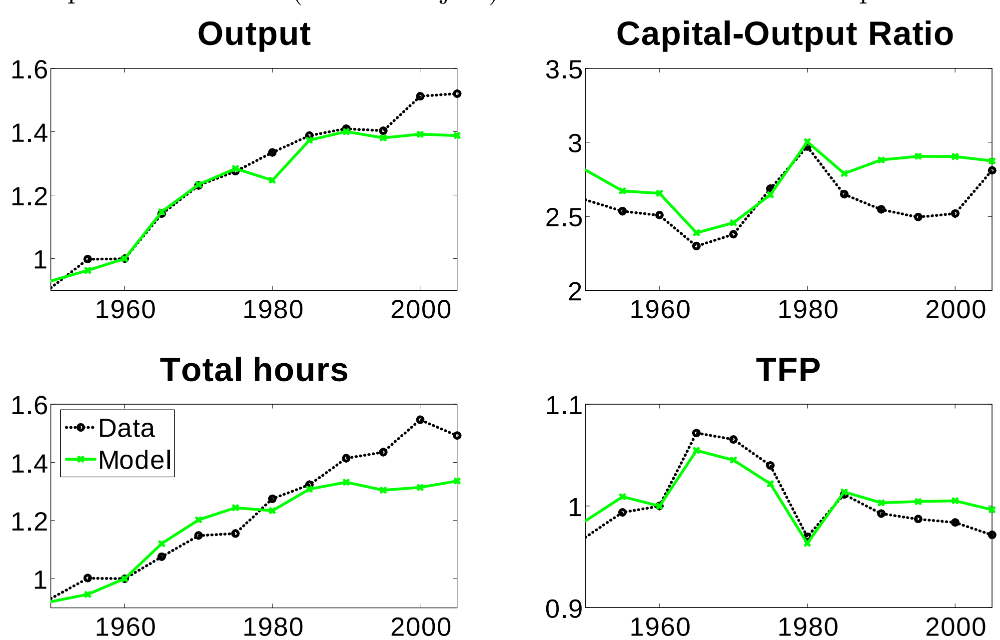


Figure 9: Importance of including females in the model.

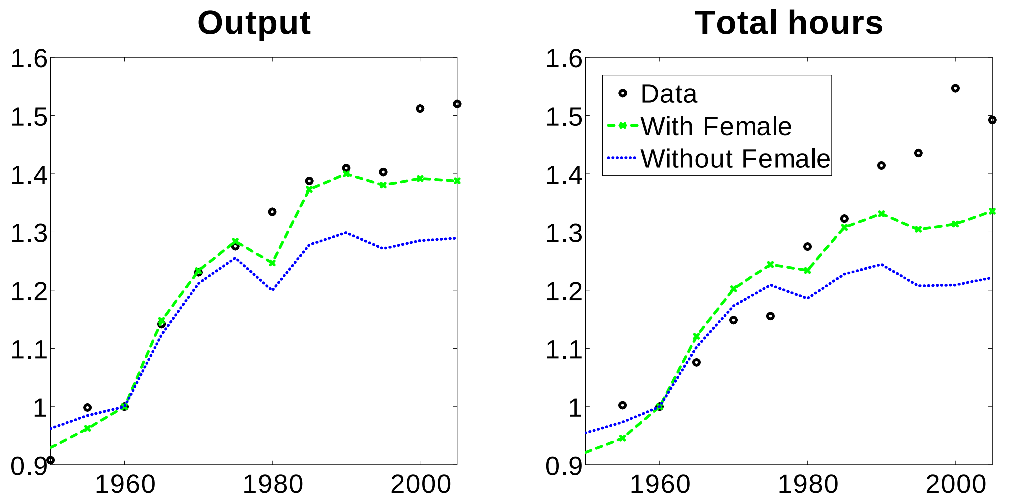


Figure 10: Model implications for average hours of males and females compared to data.

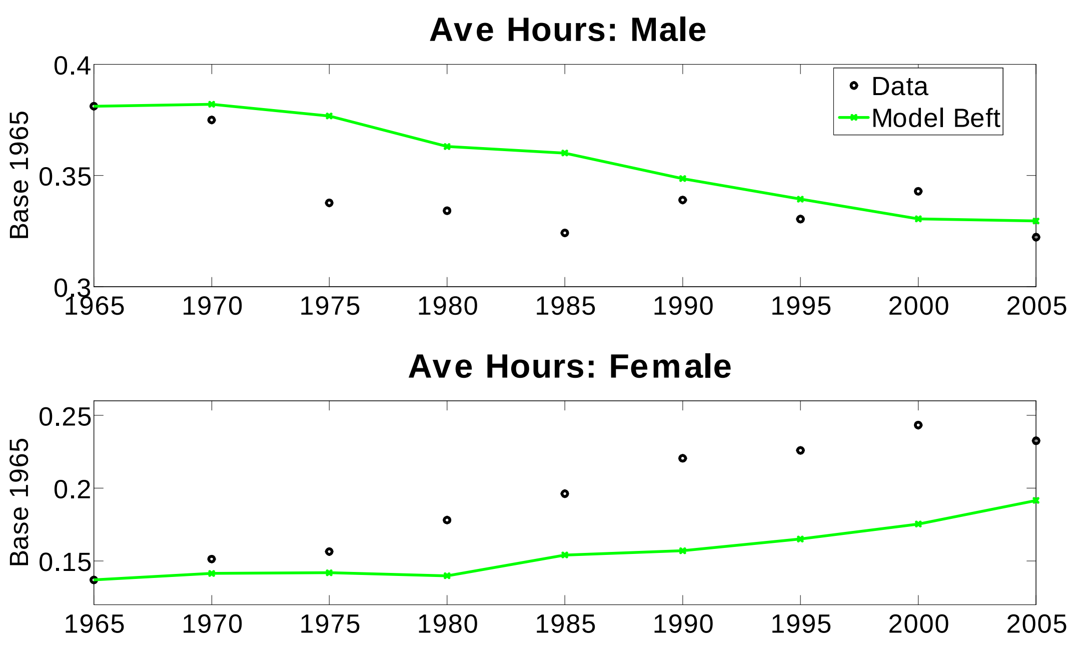


Figure 11: Key policy variables for Taxes move, Benefits move and Privatization.

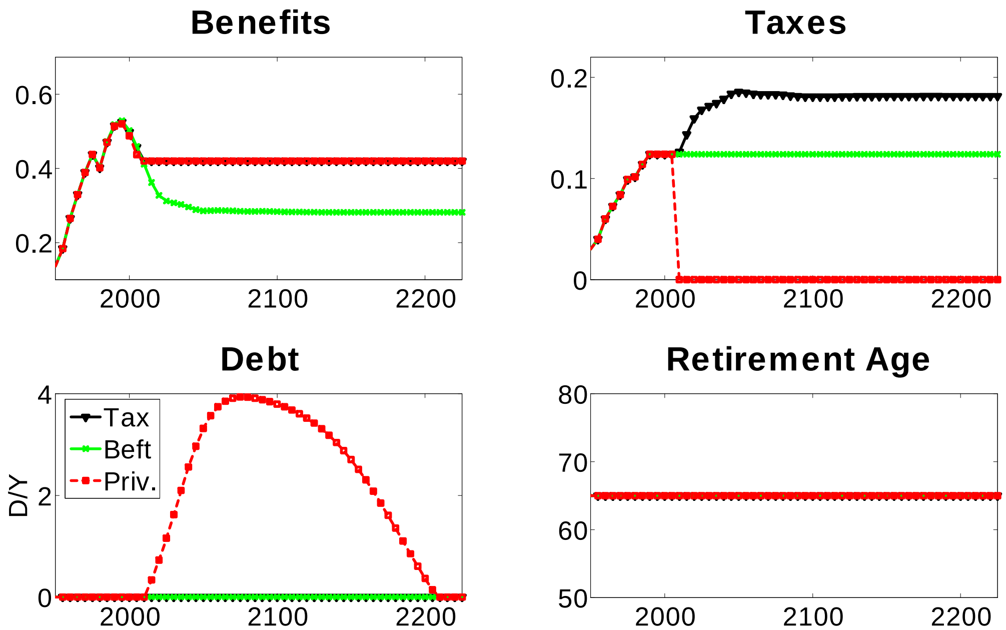


Figure 12: Welfare comparison of taxes move, benefits move, and privatization.

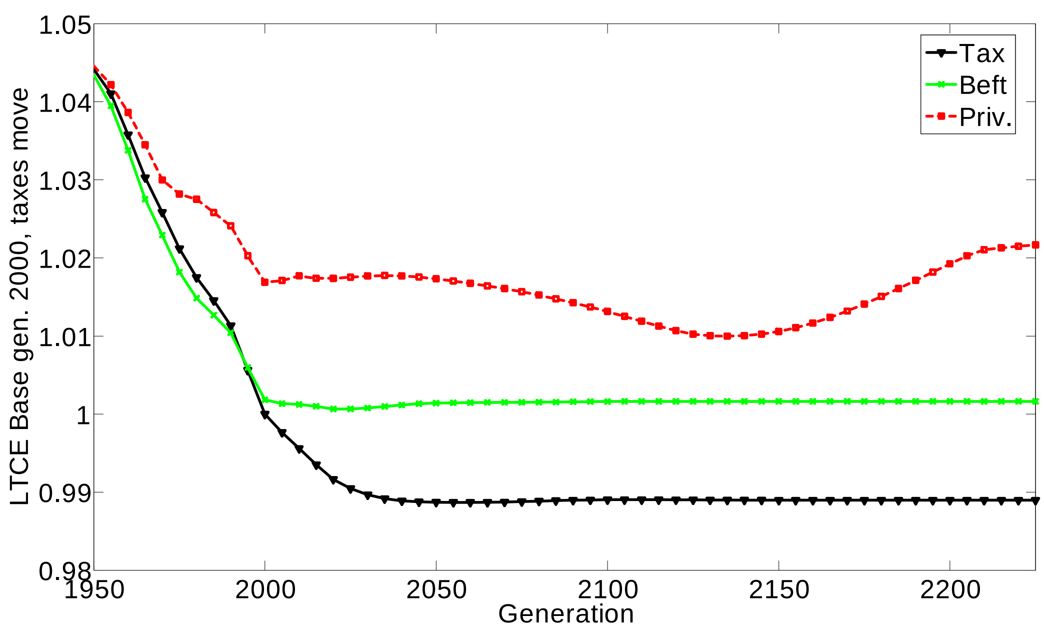


Figure 13: Aggregate implications of three fundamental policies.

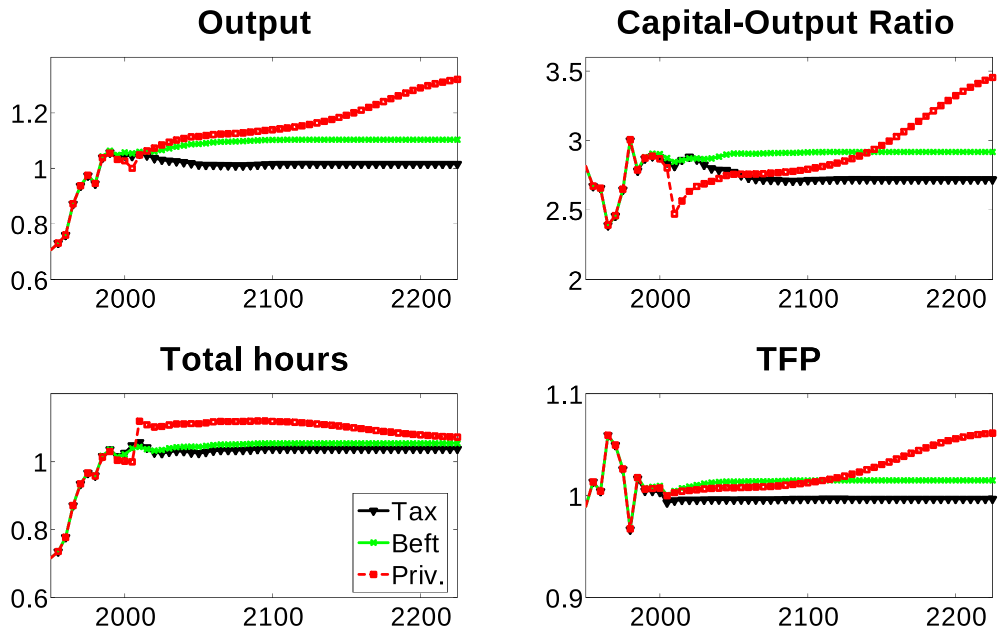


Figure 14: Policy description of retirement increase policies with taxes or benefits changing.

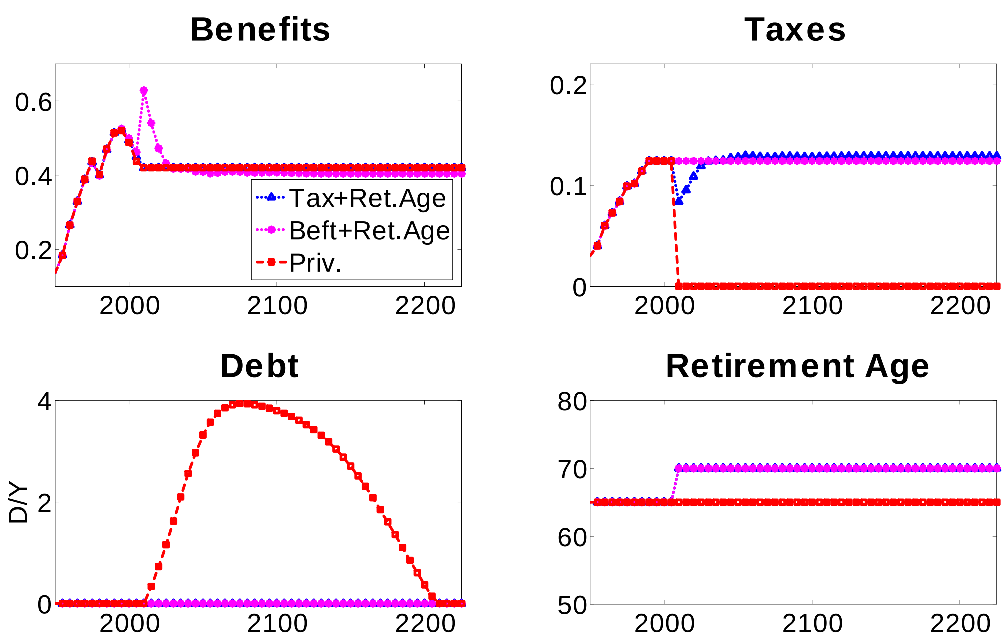


Figure 15: Aggregates for retirement age increase policies and privatization.

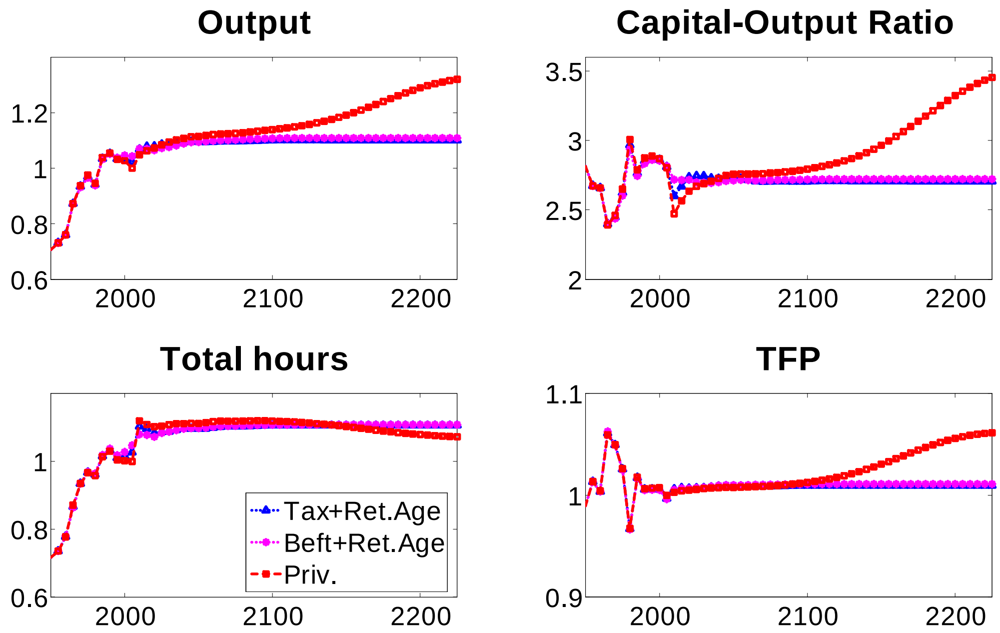


Figure 16: Welfare implications of all policies.

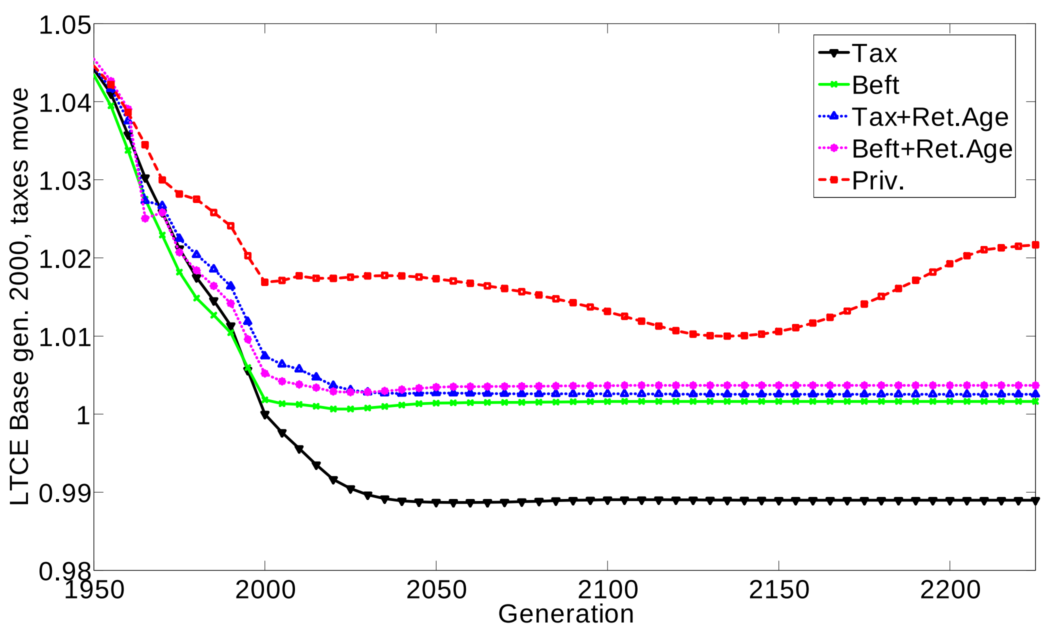


Figure 17: Transfers and consumption under different policies.

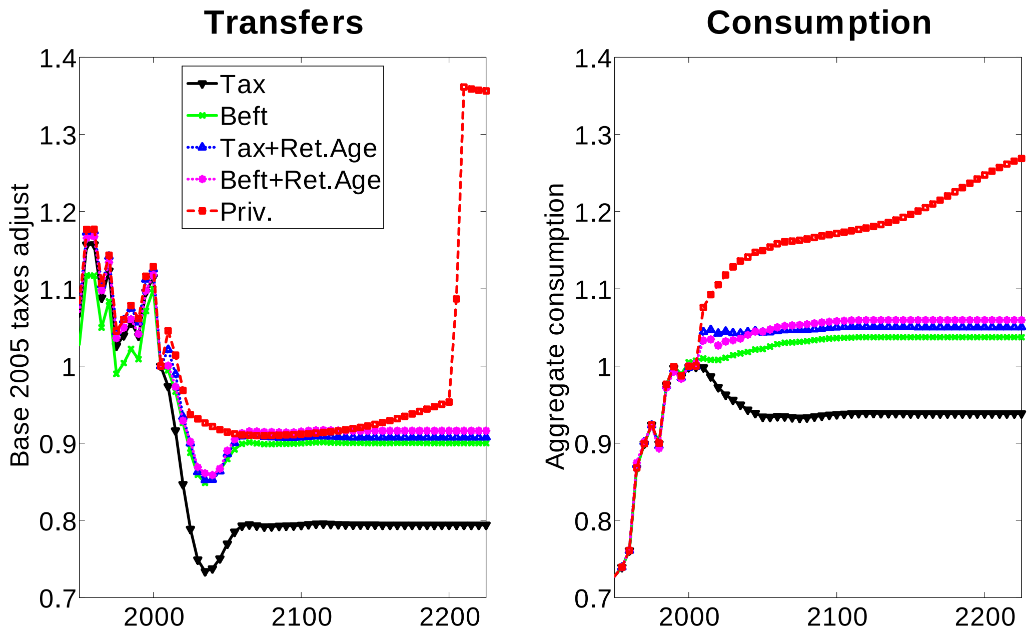
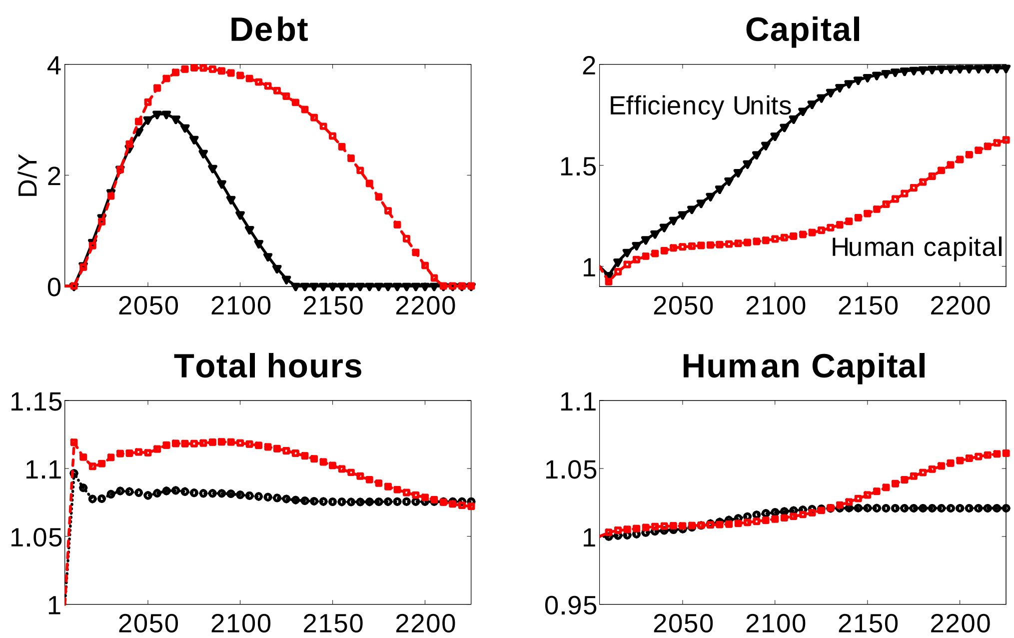


Figure 18: Importance of human capital accumulation.



A.3 Solving the model

Our approach to solving the model is as follows:

1. Find an initial guess for the houshold decisions under the different fiscal arrangments. This is done by solving the steady state, assuming that over the lifetime of a household, it faces the averages over the taxes and TFP.
2. Using this initial guess, a loop is started over transfers and prices:
   (a) The first part of the loop adjusts the policy variables, namely transfers (retirement and general), the benefit factors, the social security taxes, and the debt. Here the particular policies are taken into account.
   (b) The second part of the loop consists of a Quasi-Newton Gauss Seidel algorithm to solve for the path of capital rental rates and wage rates such that the labor and the capital market clear and the decisions of households are consistent with the aggregates given the transfers. As part of this loop all the household problems are solved given the prices, taxes, and transfers as well as the wage difference factors and TFP.

A.3.1 Solving the household's problem

At the heart of the model is the household's problem. I characterize the problem using the first order conditions derived from the household problem.

$$\max \sum_{a=1}^{M} \beta^{a-1} \eta_a \left(\omega u \left(c_{f,a}, l_{f,a} \right) + (1-\omega) u \left(c_{m,a}, l_{m,a} \right) \right)$$

s.t.

$$
\begin{aligned}
\lambda &: (1+\tau_c)(c_{f,a}+c_{m,a})+k_{a+1} = \\
&\quad R_a k_a + [(1-\tau_l-\tau_r) w_a [(n_{m,a}h_{m,a})+(1-\tau_d)(n_{f,a}h_{f,a})] 1_{a<M_r}] + T_a \\
\mu_s &: l_{s,a} = n_{s,a}+q_{s,a}; s=f,m \\
\xi_s &: h_{s,a+1} = (1-\delta_h) h_{s,a} + g(q_{s,a}, h_{s,a}, s); s=f,m
\end{aligned}
$$

The first order conditions for the married couple's problem are:

$$
\begin{aligned}
c_f &: \beta^{a-1}\eta_a \omega u_1(c_{f,a}, l_{f,a}) = (1+\tau_c)\lambda_a \\
c_m &: \beta^{a-1}\eta_a (1-\omega) u_1(c_{m,a}, l_{m,a}) = (1+\tau_c)\lambda_a \\
l_f* &: \beta^{a-1}\eta_a \omega u_2(c_{f,a}, l_{f,a}) = \mu_{f,a} \\
l_m* &: \beta^{a-1}\eta_a (1-\omega) u_2(c_{m,a}, l_{m,a}) = \mu_{m,a} \\
n_f* &: \mu_{f,a} = (1-\tau_l-\tau_r) w_a h_{f,a} 1_{a<M_r} \lambda_a \\
n_m* &: \mu_{m,a} = (1-\tau_l-\tau_r)(1-\tau_d) w_a h_{m,a} 1_{a<M_r} \lambda_a \\
q_f* &: \mu_{f,a} = \xi_{f,a} g_1(q_{f,a}, h_{f,a}, f) \\
q_m* &: \mu_{m,a} = \xi_{m,a} g_1(q_{m,a}, h_{m,a}, m) \\
h_f* &: \xi_{f,a} = \xi_{f,a+1}(1-\delta_h + g_2(q_{f,a}, h_{f,a}, f)) + \lambda_{a+1}(1-\tau_l-\tau_r)(1-\tau_d) w_{a+1} n_{f,a+1} 1_{a+1} \\
h_m* &: \xi_{m,a} = \xi_{m,a+1}(1-\delta_h + g_2(q_{m,a}, h_{m,a}, m)) + \lambda_{a+1}(1-\tau_l-\tau_r) w_{a+1} n_{m,a+1} 1_{a+1} \\
k &: \lambda_a = \lambda_{a+1} R_{a+1}
\end{aligned}
$$

$$\lambda \quad : \quad (1+\tau_c)(c_{f,a}+c_{m,a})+k_{a+1}=$$
$$R_a k_a + [(1-\tau_l-\tau_r)\, w_a\, [(n_{m,a}h_{m,a})+(1-\tau_d)(n_{f,a}h_{f,a})]\, 1_{a<M_r}] + T_a$$
$$\mu_s \quad : \quad l_{s,a} = n_{s,a}+q_{s,a}; s=f,m$$
$$\xi_s \quad : \quad h_{s,a+1} = (1-\delta_h)\, h_{s,a} + g(q_{s,a}, h_{s,a}, s); s=f,m$$

I denote equations that don't hold for all ages by a star (∗). These conditions have to be treated differently.

The model as it is contains 11 deep variables $\left\{ \{c_s, n_s, l_s, q_s, h_s\}_{s=f,m}, k \right\}$, with the last 3 as state variables. It also has 5 multipliers, which I eliminate next.

$$k \quad : \quad \frac{\eta_a u_1(c_{m,a+1}, l_{m,a+1})}{\beta \eta_{a+1} u_1(c_{m,a+1}, l_{m,a+1})} = R_{a+1}$$
$$c_{m,t} \quad : \quad \omega u_1(c_{f,a}, l_{f,a}) = (1-\omega)\, u_1(c_{m,a}, l_{m,a})$$
$$c_{f,t} \quad : \quad c_{f,a}+c_{m,a} = \frac{R_a k_a + [(1-\tau_l-\tau_r)\, w_a\, [(n_{m,a}, h_{m,a})+(1-\tau_d)(n_{f,a}, h_{f,a})]\, 1_{a<M_r}] + T_a - k_{a+1}}{(1+\tau_c)}$$
$$l_s \quad : \quad l_{s,a} = n_{s,a}+q_{s,a}; s=f,m$$
$$q_s \quad : \quad g(q_{s,a}, h_{s,a}, s) = h_{s,a+1} - (1-\delta_h)\, h_{s,a}; s=f,m$$
$$n_s \quad : \quad \frac{u_2(c_{s,a}, l_{s,a})}{u_1(c_{s,a}, l_{s,a})} = \frac{(1-\tau_l-\tau_r)(1-\tau_{s,d})}{(1+\tau_c)} w_a h_{s,a} 1_{a<M_r}; s=f,m$$

A special treatment is needed for human capital accumulation. Up to two periods prior to retirement the Euler equations take the form:

$$\begin{aligned} R_{a+1} \quad = \quad & \frac{(1-\tau_l-\tau_r)(1-\tau_d)\, w_{a+1} h_{s,a+1} 1_{a+1<M_r}}{(1-\tau_l-\tau_r)(1-\tau_d)\, w_a h_{s,a} 1_{a<M_r}} \frac{g_1(q_{s,a}, h_{s,a}, s)}{g_1(q_{s,a+1}, h_{s,a+1}, s)} \times \\ & \left[(1-\delta_h + g_2(q_{s,a+1}, h_{s,a+1}, s)) + g_1(q_{s,a+1}, h_{s,a+1}, s)\, \frac{n_{s,a+1}}{h_{s,a+1}} \right] \end{aligned}$$

For the last period before retirement the situation is changed. This is due to the fact that there are no indirect benefits any longer from human capital accumulation. All the benefits are direct from the increase in next periods earnings capability. The resulting Euler equations are:

$$R_{a+1} = \frac{(1-\tau_l-\tau_r)(1-\tau_d)\, w_{a+1} h_{s,a+1} 1_{a+1<M_r}}{(1-\tau_l-\tau_r)(1-\tau_d)\, w_a h_{s,a} 1_{a<M_r}} g_1(q_{s,a}, h_{s,a}, s)$$

Note that $\eta_{a+1}/\eta_a = (N_{a+1}/N_1)/(N_a/N_1) = N_{a+1}/N_a$, which is the probability of survival till age $a+1$ conditional on being alive at age a. I denote this conditional probability by S_{a+1}. Furthermore, I remove the long run trend growth in Total Factor Productivity, γ_A.

This results for the household problem under utility specification 1 is:

$$\begin{aligned}
k &: \left(\frac{c_{m,a}}{c_{m,a+1}}\right)^{\alpha(1-\sigma)-1}\left(\frac{1-l_{m,a}}{1-l_{m,a+1}}\right)^{(1-\alpha)(1-\sigma)} = \gamma_A^{\alpha(1-\sigma)}\left(\frac{\eta_{a+1}}{\eta_a}\beta\right)R_{a+1} \\
c_f &: c_{f,a}+c_{m,a} = \frac{R_a k_a + [(1-\tau_l-\tau_r)\, w_a [(n_{m,a}h_{m,a}) + (1-\tau_d)(n_{f,a}h_{f,a})]\, 1_{a<M_r}] + T_a - k_{a+1}}{(1+\tau_c)} \\
c_m &: \left(\frac{c_{m,a}}{c_{f,a}}\right)^{1-\alpha(1-\sigma)} = \frac{1-\omega}{\omega}\left(\frac{1-l_{m,a}}{1-l_{f,a}}\right)^{(1-\alpha)(1-\sigma)} \\
l_s &: l_{s,a} = n_{s,a} + q_{s,a};\, s = f,m \\
n_s &: \frac{(1-\alpha)\, c_{a,s}}{\alpha(1-l_{a,s})} = \frac{(1-\tau_l-\tau_r)(1-\tau_{s,d})}{(1+\tau_c)} w_a h_{s,a} 1_{a<M_r};\, s=f,m \\
q_s &: q_{s,a} = \left(\frac{h_{s,a+1}-(1-\delta_h)\, h_{s,a}}{B\,(h_{s,a})^\rho}\right)^{1/\rho};\, s=f,m
\end{aligned}$$

and for human capital:

$$\begin{aligned}
R_{a+1} &= \frac{(1-\tau_l-\tau_r)(1-\tau_d)\, w_{a+1} h_{s,a+1} 1_{a+1<M_r}}{(1-\tau_l-\tau_r)(1-\tau_d)\, w_a h_{s,a} 1_{a<M_r}}\left(\frac{q_{s,a}}{q_{s,a+1}}\right)^{\rho-1}\left(\frac{h_{s,a}}{h_{s,a+1}}\right)^{\rho}\times \\
&\quad \left[1-\delta_h+\rho B_s\,(q_{s,a+1})^{\rho-1}(h_{s,a+1})^{\rho-1}(q_{s,a+1}+n_{s,a+1})\right] \\
\text{for } s &= f,m \text{ and } a < M_r - 2
\end{aligned}$$

$$
\begin{aligned}
R_{a+1} &= \frac{\left(1-\tau_l-\tau_r\right)\left(1-\tau_d\right) w_{a+1} h_{s,a+1} 1_{a+1<M_r}}{\left(1-\tau_l-\tau_r\right)\left(1-\tau_d\right) w_a h_{s,a} 1_{a<M_r}} \rho B_s \left(q_{s,a}\right)^{\rho-1} \left(h_{s,a}\right)^{\rho} \\
\text{for } s &= f, m \text{ and } a = M_r - 2
\end{aligned}
$$

The household problem thus is reduced to solving for: $\left\{ \{h_s, n_s\}_{s=f,m}, k, c_m \right\}$.

A.3.2 Special issues

There are a couple of computational problems that came up during this project. One set is related to situations where the labor supply goes to zero. The other set is related to the best way of updating the Quasi-Jacobian. I will briefly outline the approach that I followed to deal with those issues.

Complementarity problems

In some cases the problem at hand leads to boundary problems. In particular for labor supply this is sometimes the case as the very young want to spend all their non-leisure time on human capital accumulation and non on working. This leads to the following complementarity problem:

$$
\begin{aligned}
x_i &> a_i \Rightarrow F(x_i) \geq 0, \forall i \\
x_i &< b_i \Rightarrow F(x_i) \leq 0, \forall i
\end{aligned}
$$

The main approach is to transform the problem into a simpler root finding problem of the standard formate:

$$
\begin{aligned}
f(x) &= \phi^{-}\left(\phi^{+}\left(F(x), a-x\right), b-x\right) \\
\phi_i^{\pm}(u, v) &= (u_i + v_i) \pm \sqrt{u_i^2 + v_i^2}
\end{aligned}
$$

or

$$
\begin{aligned}
f(x_i) = {} & \left[(F(x_i) + (a_i - x_i)) + \sqrt{(F(x_i))^2 + (a_i - x_i)^2}\right] + (b_i - x_i) - \\
& \sqrt{\left[(F(x_i) + (a_i - x_i)) + \sqrt{(F(x_i))^2 + (a_i - x_i)^2}\right]^2 + (b_i - x_i)}; \forall i
\end{aligned}
$$

where the lower bound is a vector of zeros and the upper bound is a vector of ones. There is an alternative approach that uses minimum and maximum operators, but that does not work since the problem at hand requires differentiability.

Updating of quasi-Jacobian

For the problem at hand the derivation of an analytic Jacobian is extremely difficult and the derivation of a numerical one is extremely costly in terms of computing time. As a result, the Jacobian has to be approximated. What I actually do is to update the inverse of the Jacobian in order to save time on the inversion. There are different updating rules that are popular and can be used. For the purpose of my paper, I tried five rules and compared the success and time the program took to converge. The five methods are: Davidson-Fletcher-Powell (DFP), Broyden-Fletcher-Goldfarb-Shanno (BFGS), Symmetric Rank 1 (SR1), and simple secant method (SSM) proposed by Miranda and Fackler (2002) on page 38. Given that there is very little guidance as to which method is best, I ran all policy experiments for all methods to find out which performed best. The result was the overall SSM is the most reliable methods in term of achieving convergence of the overall Quasi-Newton-Gauss-Seidel problem. It converges for all policies. BFGS was doing better for the taxes adjust and the privatization problem, but otherwise is very slow to converge. The other methods did not always converge.

A.4 Understanding the wage difference factor

This part analyzes the effect of the wage difference factor as well as the way one might think about it. As a first step a simple model is used to understand how it affects labor supply.

A.4.1 Labor supply effects of the wage difference factor

Consider the simple intratemporal problem of a married couple abstracting from savings and human capital accumulation and making all the wage differences real productivity differences.[18]

$$\max \omega \left[\log\left(c_{f,a}\right) + \alpha \log(l_{f,a})\right] + (1-\omega)\left[\log\left(c_{m,a}\right) + \alpha \log(l_{m,a})\right]$$

s.t.

$$\begin{aligned} \lambda &: c_{f,a} + c_{m,a} = w\left[\left(n_{m,a}\right) + \left(1-\tau_d\right)\left(n_{f,a}\right)\right] \\ \mu_s &: l_s = 1 - n_s; s = f, m \end{aligned}$$

The labor supply decision for this problem is given by:

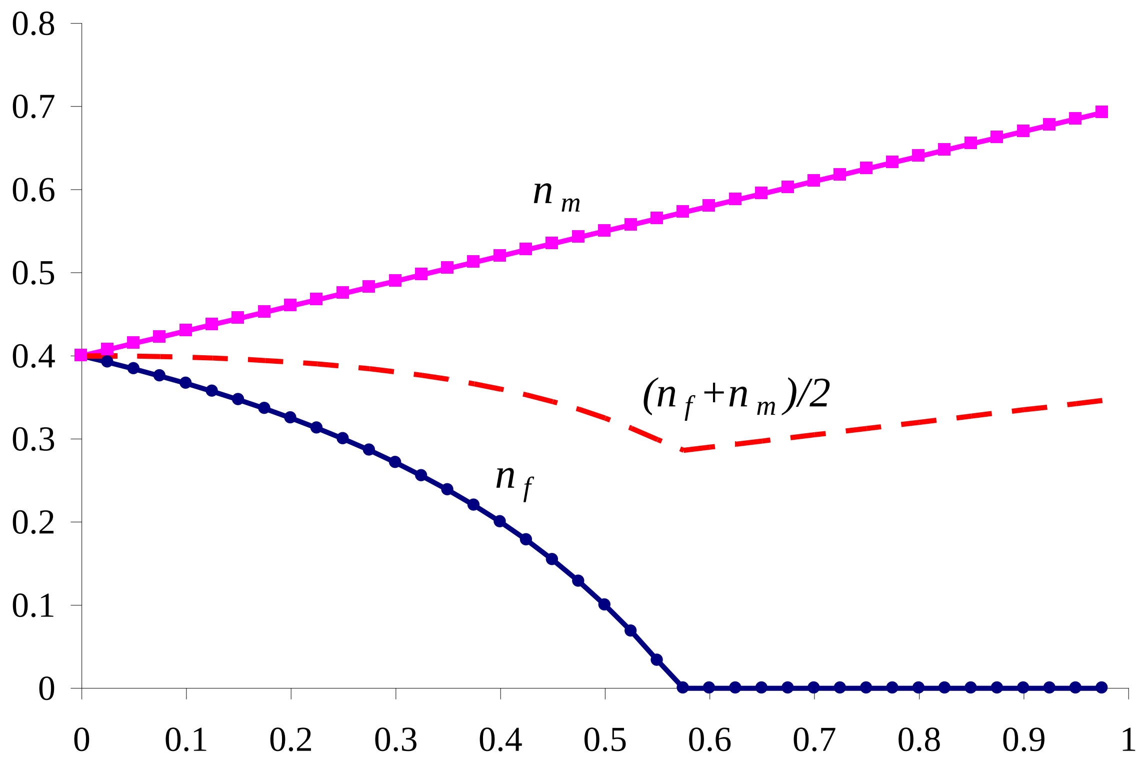
$$\begin{aligned} n_f &= 1 - \frac{\alpha\omega}{(1-\omega)/\omega + \alpha} \frac{2-\tau_d}{1-\tau_d} \\ n_m &= 1 - \frac{\alpha\omega}{1+\alpha\omega}\left(1 - \frac{\alpha\omega}{(1-\omega)/\omega + \alpha}\right)(2-\tau_d) \end{aligned}$$

I present the resulting individual and aggregate labor supply implications in Figure 19. It

[18] Neither of these assumptions is important for the derived conclusions. They are mainly for convenience to make an analytic solution feasible respectively easy to reach.

is clearly visible that there exists a very negative impact of wage differences on labor supply up to the point where all persons with lower wages have exited the market.

Figure 19: Labor supply decisions in a world with wage differences.



I conclude that in a world with married households pooling their incomes wage differences between the two household members have a significant effect on individual as well as aggregate labor supply.

A.4.2 Components of the wage difference

A key question regarding the gender wage gap is the question: What exactly does τ_d represent? There are two answers that are different in terms of their implication. The first answer is that it represent some real productivity difference between males and females. The second answer is that it represents some discrimination against females. I will consider the alternatives in turn.

Before I do so, I introduce a parameter of discrimination, ν, that can take values between zero and one which encompasse two extremes and anything in between. At the one extreme for $\nu = 0$ the wage difference is due to full discrimation in the sense that women will be equally productive as men but the fraction $(1 - \tau_d)$ is taken from them and redistributed lump sum to everybody. The lump sum distribution makes sure that the discrimination is only distorting the margin and does not create a wealth effect. At the other extreme for $\nu = 1$ there is no discrimination and all the wage gap between men and women is fully justified by a lower effective labor supply.

Object	Formula	What is done with it
Effective labor	$(1 - \tau_d \nu) z_f$	Supplied as labor
Female labor income	$(1 - \tau_d) w z_f$	Received income
Transfer component	$(1 - \nu)(1 + \tau_l + \tau_r) \tau_d w z_f$	Lump sum redistributed

Next, I consider different sources of wage difference.

Productivity difference

It might be that females are less productive than males. Examples for this are that the job requires a certain property that on average females compared to males have less off, for example physical strength. It might also be due to different inclinations of women compared to males. Women might prefer a teaching job compared to an engineering job, even if that means lower productivity measured by the wage difference (market valuation). Human capital differences due to a loss in job experience as a result of having a child might also be a source of the wage difference between males and females. All of these things are reflected in $\tau_{f,d}$.

If productivity differences are at the heart of the wage difference, then we should change the definition of effective labor of women to reflect this aspect. This also means that part of income that is lossed does not show up in any other part of the model. In pacticular we have:

Type	Formula	What happens with it
Labor income tax	$(1-\tau_d)\,\tau_l w z_f$	Go to general transfers
Retirement tax	$(1-\tau_d)\,\tau_r w z_f$	Go to retirement transfers
Labor income	$(1-\tau_d)\,w z_f$	Goes to households

Discrimination

Another source of the wage difference between males and females might be discrimination. This can take the form of pure wage discrimination, or other forms like discrimination regarding advancement (the so called 'glass ceiling'). In this case females are in reality as productive as comparable males, but part of their marginal product is not paid to them. We have to make adjustments to the transfers. In particular we have:

Type	Formula	What happens with it
Labor income tax	$(1-\tau_d)\,\tau_l w z_f$	Go to general transfers
Retirement tax	$(1-\tau_d)\,\tau_r w z_f$	Go to retirement transfers
Labor income	$(1-\tau_d)\,w z_f$	Goes to households
Correction term	$\tau_d\,(1-\tau_l-\tau_r)\,w z_f$	Goes to general transfers

A.5 Detailed policy description

Recall the fundamental government equation.

$$
\begin{aligned}
&\left\{\sum_{i=0}^{M-1} N_{t+i}^{t-i}\left[\tau_c\left(c_{f,t+i}^{t-i}+c_{m,t+i}^{t-i}\right)+\tau_k\left(r_t-\delta_k\right)\frac{\eta_{t+i+1}^{t-i-1}}{\eta_{t+i}^{t-i}}k_{t+i}^{t-i}+\left(\tau_{l,t}+\tau_{r,t}\right)w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}+\right.\right.\\
&\qquad\left.\left.\left(1+\tau_{l,t}+\tau_{r,t}\right)\left(1-\nu\tau_d^{t-i}\right)w_t z_{f,t+i}^{t-i}1_{i<M_r}+\left(1-\frac{\eta_{t+i+1}^{t-i-1}}{\eta_{t+i}^{t-i}}\right)k_{t+i}^{t-i}\right]-\left[G_t+\sum_{i=0}^{M-1}N_{t+i}^{t-i}T_{g,t+i}^{t-i}\right]\right\}\\
&+\left\{\left[\sum_{i=0}^{M-1}N_{t+i}^{t-i}\left[\tau_{r,t}w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}\right]+\gamma D_{t+1}\right]-\left[\sum_{i=0}^{M-1}N_{t+i}^{t-i}T_{r,t+i}^{t-i}1_{i\geq M_r}+R_tD_t\right]\right\}=0
\end{aligned}
$$

For notational convenience, I introduce some notational conventions:

$$
\begin{aligned}
I_{g,t} &= \sum_{i=0}^{M-1} N_{t+i}^{t-i}\left[\tau_c\left(c_{f,t+i}^{t-i}+c_{m,t+i}^{t-i}\right)+\left(\tau_{l,t}+\tau_{r,t}\right)w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}+\right.\\
&\quad\left.\tau_k\left(r_t-\delta_k\right)\frac{\eta_{t+i+1}^{t-i-1}}{\eta_{t+i}^{t-i}}k_{t+i}^{t-i}+\left(1+\tau_{l,t}+\tau_{r,t}\right)\left(1-\nu\tau_d^{t-i}\right)w_t z_{f,t+i}^{t-i}1_{i<M_r}+\left(1-\frac{\eta_{t+i+1}^{t-i-1}}{\eta_{t+i}^{t-i}}\right)k_{t+i}^{t-i}\right],\\
I_{r,t} &= \sum_{i=0}^{M-1} N_{t+i}^{t-i}\left[\tau_{r,t}w_t\left(\left(1-\tau_d^{t-i}\right)z_{f,t+i}^{t-i}+z_{m,t+i}^{t-i}\right)1_{i<M_r}\right],
\end{aligned}
$$

where $I_{g,t}$ stands for the total revenue received by the government in period t that is distributed evenly over all households and $I_{r,t}$ is the revenue received in period t that goes to fund retirement benefits.

The key to understanding retirement funding lies in understanding the policy instruments that can be used: (τ_r, T_r, M_r, D), i.e. the retirement taxes, the retirement benefits, the retirement age, and debt. There is one element in this list, that is not a primitive, but rather a function of two other parts, T_r. There are different ways on how to vary retirement benefits, T_r: either one adjust a time varying benefit factor χ_t, or one adjusts the way the individual entitlement to benefits are determined, b^i. The entitlement to benefits is described in equation 3 and the time dependent benefit factor is presented in equation 2. For most cases below, I adjust the benefit factor and don't touch the individual benefit formula. The exception is the switch to a fully funded sytem.

Next, I present the different retirement policies considered in this paper.

Tax variation

I start by outlining the way the government policy is determined in the case in which benefits are fixed (equal for all persons) and taxes for the purpose of financing the retirement benefits are the policy instrument. In that case the problem reduces to:

$$T_{g,t} = (I_{g,t} - G_t) \,/ \left(\sum_i N_{t+i}^{t-i} \right)$$

$$\tau_{r,t} = \left[\sum_{i=0}^{M-1} \left[N_{t+i}^{t-i} T_{r,t+i}^{t-i} 1_{i \geq M_r} \right] + R_t D_t - \gamma D_{t+1} \right] / \left[I_{r,t} \right]$$

Following the system as it is currently in place the personal benefits are defined as above in equation 3 and I assume that the benefit factor is fixed to the level it had in the last historical period $\chi_t = \bar{\chi}_{2005}; t \geq 2005$.

In this setup, I assume that the government reacts to changes in the demographic structure by increasing the social security taxes. In principle, one might also generate a deficit to cover temporary deficits. This deficit is paid off later-on by future generations. For my tax variation experiments, I assume that $D_t = 0; \forall t$.

Benefit variation

Next, I fix the tax that determine the the social security contributions and let the benefits vary. The problem thus becomes:

$$T_{g,t} = \left[I_{g,t} - G_t \right] / \left[\sum_i N_{t+i}^{t-i} \right]$$

$$\sum_i N_{t+i}^{t-i} G \left[\tau_{r,t} w_t \left(\left(1 - \tau_d^{t-i} \right) z_{f,t+i}^{t-i} + z_{m,t+i}^{t-i} \right) 1_{i < M_r} \right] + \gamma D_{t+1} = \sum_i N_{t+i}^{t-i} T_{r,t+i}^{t-i} 1_{i \geq M_r} + R_t D_t$$

Given the in place benefit structure, the retirement budget constraint solves then for a fixed benefit factor that makes benefit dependent on labor income, $T_{r,t+i}^{t-i} = \phi \sum_{a=1}^{M} w_a z_a^{t-i} 1_a$, the resulting problem is:

$$\chi_t = \left[\left[\sum_i N_{t+i}^{t-i} \tau_{r,t} w_t \left(\left(1 - \tau_d^{t-i}\right) z_{f,t+i}^{t-i} + z_{m,t+i}^{t-i} \right) 1_{i<M_r} \right] + R_t D_t - \gamma D_{t+1} \right] / \left[\sum_i N_{t+i}^{t-i} b^{t-i} 1_{t+i \geq M_r} \right];$$

$$\text{where } 1_{t+i \geq M_r} = \left\{ \begin{array}{c} 0 \text{ for } a < M_r \\ 1 \text{ otherwise} \end{array} \right\}.$$

As before there is the choice of using debt as a substitute for benefit adjustments. I focus on the pure case with only the benefit rate adjusting and assume that $D_t = 0; \forall t$.

Retirement age

The retirement age change is able to resolve any funding problem on its own by making retirement age a real number and moving to a continuous definition of time. This is not very plausible considering a retirement age of 68 years 212 days 4 hours 12 minutes 34 seconds. So, for more realistic szenarios I consider the retirement age to be a round annual number and then combine it with benefit or tax rate adjustments to ensure that the retirement budget still matches. In this setting tt is a a straight forward extension of the previous policies.

Furthermore, it is easy to see, what a change in the retirement age does. It immediately increases the funds available for distribution and decreases the number of persons eligible for benefits. Both of these effects help to take some weight off the other possible responses. It is a quantiative question, whether other measures are still necessary to augment the retirement age change. In the paper I use a retirement age increase of five years.

Privatization

The policy is the smooth transition to a privately funded retirement and complicated in its details. In essence, privatization leads to a decrease in the social security tax to zero $\tau_{r,t} = 0, \forall t \geq \bar{T}$ and the same holts for all further benefit accumulation under the old system. So, after date $\bar{T}$ all persons in the economy are responsible for their own retirement funding. The persons that entered the labor market before data $\bar{T}$ are still eligible for benefits according to the old system in proportion to their payments. The way it can be implemented is to do a benefit variation based on the szenario that taxes for retirement funding are zero. The smooth transition approach tries to apeace the persons that have paid into the old system. It will abandon to pay benefits to all the persons entering the labor market after period $\bar{T}$ and stop collecting taxes from that period onwards. I will focus on the transition period, because before the reform the old system was in place and that was described above and afterwards the privately optimal savings plan is implemented.

The transition starts by freezing the time varying benefit factor $\chi_t = 0.42, \forall t > \bar{T}$. Furthermore, we have to adjust the personal benefit formula to reflect the fact that nobody can accumulate any further benefits but the old benefits are honored.

$$b_s^i = \left\{ \begin{array}{ccc} \left[\sum_{j=1}^{M} \varepsilon_j w_j \left(1 - \tau_{d,s}^i\right) z_j^i\right] & & i \leq \bar{T} - M_r + 1 \\ \frac{1}{M_r - (\bar{T} - i)} \left[\sum_{j=1}^{M} \varepsilon_j w_j \left(1 - \tau_{d,s}^i\right) z_j^i\right] & \text{for} & \bar{T} - M_r + 1 < i < \bar{T} \\ 0 & & i > \bar{T} \end{array} \right\}.$$

The first row is capturing all the workers that have worked all their lives under the old system. The second row in the formula considers the case of those that have worked for a shorter than full period of their lives under the old system. The get a proportional factor of the total benefits. If you worked for only one year you get 1/40th of the normal benefits. The last row is covering all the persons born into the new system.

Now, given that there are no new tax collections the benefits have to be paid in a different way. Here the smooth transition idea relies on an increase in labor supply, consumption and savings due to the decrease in distortionary labor income taxes. So, there will be an accumulation of debt for a while that stops after the last person that paid into the old system dies. From then on there are only downpayments, till no debt is left. Once that situation is reached the retirement account will be closed. In order to make sure that the older persons in the economy can be bribed into the new system, I need to make sure that the general transfers are high enough. I start by fixing the general transfers during the transition to achive this:

$$T_{g,t} = \left\{ \begin{array}{lr} T_{g,\bar{T}} & \text{if} \quad D_t > 0 \\ \left[I_{g,t} - G_t\right] / \left[\sum_i N_{t+i}^{t-i}\right] & \text{if} \quad D_t \leq 0 \end{array} \right\}.$$

The following condition will hold during the transition period, i.e. $D_t > 0$ or $\left\{\exists i : T_{r,t+i}^{t-i} > 0\right\}$:

$$\gamma D_{t+1} + I_{g,t} + I_{r,t} = \sum_i N_{t+i}^{t-i} \left[T_{g,t} + T_{r,t+i}^{t-i} 1_{i \geq M_r}\right] + G_t + R_t D_t$$

$$\text{with } T_{r,t+i}^{t-i} = \chi_{t+i} b_{t+i}^{t-i} \text{ and } D_{\bar{T}-1} = 0.$$

The periods after the last person died that payed into the old system, the constraint simplifies to:

$$\gamma D_{t+1} + I_{g,t} + I_{r,t} = \sum_i N_{t+i}^{t-i} \left[T_{g,t}\right] + G_t + R_t D_t$$

$$\text{with } T_{r,t+i}^{t-i} = \chi_{t+i} b_{t+i}^{t-i} \text{ and } D_{\bar{T}-1} = 0.$$