

House Price Dynamics with Heterogeneous Expectations*

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Abstract

This paper presents a dynamic equilibrium model of the housing market in which agents consume housing services and speculate on future price changes. The model features a fixed supply of housing and a random variation in demand, originating from the fact that agents hold heterogeneous expectations about the future course of prices. The important feature of the model is that heterogeneous expectations generate a non linear demand for housing: agents expecting higher future prices buy in anticipation of capital gains; agents holding pessimistic expectations prefer to rent to avoid capital losses. Because pessimistic agents rent their expectations are not incorporated in the price for owned houses. As a consequence, the equilibrium price reflects only the expectations of optimistic agents and is thus biased upward. We test the predictions of the model with US city data, using the dispersion in city income as a proxy for information dispersion. The empirical evidence supports the prediction that house prices are higher in cities with more dispersed beliefs about future economic conditions.

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1 Introduction

The US housing market has experienced substantial price variations in the last two decades. Figure 1 gives an example of such variations for the aggregate US economy and a representative sample of US cities.¹ As shown, in some cities, such as Los Angeles, the housing price has moved in tandem with the overall national index, yet in other cities prices movements have been quite heterogenous. In Miami, for example, the house price index has been steady for almost two decades before increasing exponentially in 2000. In San Antonio, the same index has declined since the 1980s and has not recovered since then. In Rochester, the real index of house prices has displayed an inverse “U shaped” history, while in Memphis it has gone through periodic cycles.

In the opinion of many housing-market observers (see e.g. Glaeser and Gyourko, 2007), these high frequency price variations are difficult to explain through the lens of a standard user cost approach (e.g., Poterba, 1984, 1991) in which house prices are determined by an indifference condition between owning and renting given current and expected future fundamentals, such as the user cost, income, and construction costs. For US cities, for example, Case and Shiller (2003) find that house prices movements cannot be explained by income variations alone; Himmelberg, Mayer and Sinai (2005) argue that the price-income ratio and the price-rent ratio cannot account for the bulk of house price changes; Glaeser, Gyourko and Saks (2005b) find a weak relationship between house prices and construction costs.

The purpose of this paper is to present a simple variant of the user cost approach to rationalize some of the price movements displayed in Figure 1. Specifically, we build a model featuring a fixed supply of housing and a demand that fluctuates stochastically because households hold heterogeneous beliefs about the future course of house prices. Heterogeneous beliefs arise because households are imperfectly informed about the state of the economy and use their own income, together with other signals, in the estimation of the underlying fundamentals. Thus, idiosyncratic income shocks translate into heterogeneous expectations about future aggregate demand and housing demand, and *a fortiori* — given the fixed supply — into heterogeneous expectations of house prices. Guided by the logic of this model, we create an index of difference in expectations based on the dispersion of local income shocks and find that this index influences significantly the dynamics of house prices across and within US cities.

Our analysis rests on three building blocks — household income, heterogeneous expectations, and fixed housing supply — which are motivated by several aspects of the US housing market. First, the available evidence suggests that income remains

¹We use the OFHEO constant quality house price index for single-family one-unit properties financed with a mortgage below the conforming loan limit.

the main determinant of housing demand, either because richer households tend to demand more (Poterba, 1991, Englund and Ioannides, 1997) or because higher income relaxes credit constraints (Ortalo-Magne' and Rady, 2006, Almeida et al., 2006, Benito, 2005). Second, surveys of households expectations (Case and Shiller, 1988, 2003) reveals a strong investment motive of home-buyers: agents' desire to buy is largely influenced by their expectations of reselling houses at higher prices. These surveys document also that home buyers' expectations tend to be extrapolative and largely influenced by past and current economic conditions (see Case, Quigley and Shiller, 2003). Third, the supply of houses is inelastic in the short-run, and adjust only slowly to local demand shocks because of locals regulations, zoning laws or technological constraints (Glaeser and Gyourko, 2003, Glaeser, Gyourko and Saks, 2005, 2007, Gyourko, Mayer and Sinai, 2006).

Taken together, these observations suggest a specific mechanism through which variations in income generate large swings in house prices: if household income is an important determinant of housing demand *and* shapes expectations of future house prices, a small income shock may initiate a dynamic process that, through heterogeneous expectations and the fixed supply of housing, runs from expected prices to house demand and back to house prices.

To formalize this mechanism we consider a model in which two groups of households, with different expectations about future prices, participate in the housing market to consume housing services (by either owning or renting) and to speculate on future price changes. The equilibrium price is pinned down by a non arbitrage condition equating the cost of renting to the cost of owning. However, since in our model expectations are heterogenous and anticipated capital gains reduces the cost of owning, this non arbitrage condition holds only for optimistic households, who are willing to buy in anticipation of future capital gains. For pessimistic households, instead, who expects future capital losses, the user cost is perceived higher then the cost of renting. Consequently, they strictly prefer to move out of the market of "homes for sale" and to use the rental market to consume housing services, where rental units are supplied by the optimists who buy units in excess of their demand for housing services. The final result is that the rental price reflects the average opinion in the market, while the equilibrium price of owned occupied houses is biased upward since it reflects only the views of the optimists.

Our model delivers two main predictions. First, house prices and their volatility are higher the larger the difference in expectations between pessimistic and optimistic households. Second, informational shocks have an asymmetric effect on prices: positive shocks bias the equilibrium house price upwards while negative shocks are moot. Both predictions stem from the fact that the market for owner occupied houses is segmented, because households use their private information to make inference about the unobservable aggregate income and housing demand.

In contrast, if households had homogenous expectations, the demand for housing would be linear, the individual private signals would get “washed out” in aggregate and the equilibrium price would depend only on average income. It turns out that our results survive even if agents use the equilibrium rental and house prices — which are summary statistics of the dispersed information in the economy — to update their beliefs about the economy-wide income, provided these prices are not perfectly revealing due to unobservable preference shocks for housing services.

To test the predictions of our model we run panel regressions using US city data. Lacking a direct measure of imperfect information, we use the city-level dispersion of industry income shocks to proxy for difference in beliefs. This choice is motivated by the fact that ours can be interpreted as a closed city model where moving costs prevent households to move in and out of cities, and the speculative demand for housing depends only on expected local economic conditions. If city residents are employed in different industries and they are imperfectly informed about the city income, then industry income shocks become a source of confusion about the current and future local economic conditions. In line with the model’s predictions, we find that house prices are higher and more volatile in cities with more heterogenous expectations. We also find an asymmetric response of house prices to positive and negative informational shocks: positive shocks explain significantly house price increases, while negative shocks lack statistical predictive power.

In the rest of the paper we proceed as follows. In Section 2 we relate our model to the relevant literature. In Section 3 we introduce the baseline model and discuss the determinants of the equilibrium rental and house prices. In Section 4 we study the benchmark case in which agents hold imperfect but common information about local economic conditions, while in Section 5 we derive the main predictions of the model when the information is not only imperfect but also dispersed. Section 6 discusses the robustness of our model’s predictions when the inference problem depends also on the equilibrium house and rental prices. In Section 7 we introduce our proxy of information dispersion and present our empirical analysis and results. We conclude in Section 8. All proofs are in the Appendix.

2 Related Literature

Methodologically, our paper follows the user-cost approach of Poterba (1984) and Henderson and Ioannides (1982) in which a prospective buyer is indifferent between renting and owning, and the cost of owning depends, among other variables, on property taxes, the opportunity cost of capital and the expected capital gains on the housing unit. While some papers have studied the house price implications of changes in taxes (Poterba, 1991) and interest rates (Himmelberg et al., 2006,

McCarthy and Peach, 2004) the role played by differences in the expected rate of price changes has so far remained unexplored. In part this is because buyers' expectations are difficult to measure, but also because difference in expectations cannot arise in the standard user cost framework, given the simplifying assumption that the housing market is populated by a representative agent. We complement this literature by showing that difference in expectations can lead house prices to deviate from their fundamental values and that a given degree of variation in expected prices across markets *and* within markets over time can account for some of the house price changes documented in Figure 1, more so than changes in property taxes — which are fairly constant over time — or interest rates — which are constant across markets.

In the housing literature, Stein (1995) and Ortalo-Magne' and Rady (2006) explain large swings in house prices by using borrowing constraints and household leverage. In their models buyers finance the purchase of houses by borrowing and the ability to borrow is directly tied to the value of the houses they own. Therefore, a positive income shock that increases the demand for houses and hence their prices relaxes the borrowing constraint, further increasing the demand for houses and so on. Our paper is related to both studies because changes in households' income may have a more than proportional effects on house prices. There are, however, three important differences. First, in our story there are no borrowing constraints. Instead, the amplification mechanism operates from expected price, via household income, back to current prices, via changes in speculative demand. Second, households do not need to own houses to consume housing services; they can also use the rental market. Finally, in our set-up not only the level of income but also its dispersion matters for explaining the dynamic of house prices.

In this regard, our paper relates to the recent works of Gyourko, Mayer and Sinai (2006) and Van Nieuwerburgh and Weil (2007). The first paper argues that the interaction between an inelastic supply of houses and the skewing of the income distribution generates a significant price appreciation in superstar cities — cities with unique characteristics preferred by the majority of the population. Wealthy households are willing to pay a significant financial premium to live in these areas, bidding up prices in the face of a relatively inelastic supply of houses. Van Nieuwerburgh and Weil (2006) use a similar mechanism (though in their model households move across cities for productive rather than preference reasons) to explain why the dispersion and the level of house prices increases with the cross-sectional wage dispersion in US cities. Our paper differs from these contributions because it highlights a different channel through which income dispersion matters. In our setup, income shocks affect households's perception of local economic conditions, leading to the formation of heterogeneous beliefs about future economic fundamentals. As a consequence, heterogeneous expectations are more pronounced

when, *ceteris paribus*, income is more dispersed. Moreover, in our model positive and negative income shocks have an asymmetric effect on prices, a prediction absent in Gyourko et al., and van Nieuwerburgh and Weil. In our framework the speculative motive for buying housing units is enhanced when households expect better economic conditions to prevail in the future; on the contrary, the speculative motive is moot following negative income shocks, since in this case households use the rental market. Another important difference is methodological. In our model prices are determined by a non arbitrage condition between buying and renting, while Gyourko et al., and van Nieuwerburgh and Weil follow the urban economics tradition in which house prices are determined by a spatial no arbitrage condition, with owners indifferent between different locations, given local wages and amenities. The spatial equilibrium approach is, however, more suitable for studying the long run distribution of housing prices as opposed to high frequency price variations, which are the main focus of our analysis.

Our analysis is also related to a large literature in macroeconomics and finance that studies the role of imperfect information among decision makers. In fact, our story can be seen as an adaptation of the Phelps-Lucas hypothesis to the housing market, in the sense that imperfect information about the nature of disturbances to the economy makes different economic agents react differently to changes in market conditions. Part of our work shares also many features with the literature on the pricing of financial assets in the presence of heterogeneous expectations and short-sale constraints (i.e., Miller, 1977, Harrison and Kreps, 1979, Hong, Scheinkman, and Xiong, 2004 and Sheinkman and Xiong, 2003). In this literature, if agents have heterogeneous beliefs about asset fundamentals and face short sales constraint, the equilibrium asset price reflects the opinion of the more optimistic investors, and it is thus biased upward. We adapt the same idea to the housing market where the short sale constraint arises as a natural constraint when households have the option to consume housing services by either renting or owning. In fact, in our model renting act as if relatively pessimistic investors were facing a binding short-sale constraint. Expecting lower prices in the future, this group of households would like to short their houses but cannot do so since they have to consume housing services. Consequently, they move out of the market of “home for sale” and the price of owned houses ends up reflecting only the more optimistic view in the market, rather than the average opinion.

3 The Model

3.1 Information

The economy is populated by an infinite sequence of overlapping generations of households with constant population. Each generation has unit mass and lives for two periods. In the first period, households supply labor and make saving and housing decisions; in the second period, they consume the return on savings. The wage W_t^j , at which labor is supplied inelastically, is equal to

$$W_t^j = \exp(\theta_t + \varepsilon_t^j), \quad (1)$$

where θ_t is the economy income and ε_t^j is an individual-specific wage shock. We make the assumption that θ_t follows an AR(1) process,

$$\theta_t = \rho\theta_{t-1} + \eta_t, \quad \text{with } \rho \in (0, 1] \quad (2)$$

with η_t independent and normally distributed innovations with zero mean and variance σ_η^2 . The individual-specific shocks, ε_t^j , which are the only source of income heterogeneity, are serially independent and have normal distribution with zero mean and variance σ_ε^2 . When households cannot observe the realization of θ_t at time t , ε_t^j is also a source of information heterogeneity. In other words, the wage W_t^j is the household j 's private signal about the unobservable aggregate shock, θ_t . As usual in this context, we make the assumption that idiosyncratic shocks cancel in aggregate, or equivalently the average private signal is an unbiased estimate of the underlying fundamental:

Assumption 1: $\int \varepsilon_t^j dj = 0$

3.2 Preferences

Households have logarithmic preferences over housing services, V_t^j , and second period consumption, C_{t+1}^j ,²

$$\mathcal{U}_t^j = A_t^j \log V_t^j + E_t^j \log C_{t+1}^j, \quad (3)$$

where E_t^j denotes the expectation operator based on household j 's information set at time t (to be specified later) and the parameter A_t^j is a preference shock,

$$A_t^j = \exp(2(a_t + \nu_t^j)),$$

²Underlying this utility function is the assumption that the demand for second-period housing services is constant which, for simplicity, we normalize to zero.

which consists of an aggregate taste shock, a_t , and an idiosyncratic noise ν_t^j . We assume that a_t and ν_t^j are independent and normally distributed with zero mean and variance σ_a^2 and σ_ν^2 . We also consider the limiting case where the variance of ν_t^j is arbitrarily large, so that knowing one's own individual taste provides no information about the aggregate taste.³

3.3 Budget constraint

In the first period, after the realization of the idiosyncratic income, households decide how many housing units to buy, $H_t^j \geq 0$, at unit price, P_t . They also choose the quantity of housing services to consume, V_t^j , and by implication the units of housing to rent out, $H_t^j - V_t^j$, at the rental price Q_t .

At the end of the first period, the residual income is saved at the gross interest rate, R , and at the beginning of the second period the stock of owned houses is sold to the young of the new generation, at the price P_{t+1} . For a the typical household j the flow of funds constraint is thus:

$$C_{t+1}^j = R(W_t^j - P_t H_t^j + Q_t(H_t^j - V_t^j)) + P_{t+1} H_t^j, \quad (4)$$

with

$$H_t^j \geq 0. \quad (5)$$

3.4 Optimal house demand

Households inter-temporal decisions consist of choosing H_t^j and V_t^j to maximize (3) subject to (4) and (5). It is immediate to establish that the optimal demand for housing service, V_t^j , and housing units, H_t^j , satisfy the following first-order conditions,

$$\frac{A_t^j}{V_t^j} = E_t^j \left[\frac{RQ_t}{C_{t+1}^j} \right], \quad (6)$$

$$E_t^j \left[\frac{R(U_t - Q_t)}{C_{t+1}^j} \right] \geq 0, \quad (7)$$

where

$$U_t = P_t - \frac{P_{t+1}}{R}, \quad (8)$$

³While this assumption implies that a_t is unobservable, the law of motion of a_t is known by all agents.

denotes the (per unit) user cost of housing, which increases with the current house price, p_t , and is inversely related to the next period house price, P_{t+1}/R .⁴

Equation (6) establishes that households consume housing services until the current period marginal utility of housing services, A_t^j/V_t^j , equal the expected marginal cost in terms of next period consumption utility, $E_t^j [RQ_t/C_{t+1}^j]$. The optimal quantity of housing units to buy is implicit in equation (7), which relates the user cost, U_t , to the cost of renting housing services, Q_t . Whether this condition holds with equality depends on households' expectations about future prices. For households holding pessimistic expectations the user cost is perceived to be higher than the cost of renting. Thus, constraint (5) binds and (7) is satisfied with inequality. The opposite holds for households with relatively optimistic expectations. They demand a strictly positive amount of housing units, $H_t^j > 0$, and (7) holds with equality.

3.5 The linearized optimality conditions

To deliver explicit solutions, we find it convenient to work with a linear approximation of equations (6) and (7) around the “certainty” equilibrium, i.e., the equilibrium prevailing when both aggregate and idiosyncratic shocks are zero. Denoting with lower case letters variables in percentage deviations from the equilibrium with certainty, Appendix I shows that a linear approximation to (7) leads to

$$E_t^j u_t \geq q_t, \quad (9)$$

where

$$u_t = \frac{(1+r)p_t - p_{t+1}}{r}, \quad (10)$$

and r is the net interest rate $r \equiv R - 1 > 0$. Similarly, a linear approximation of (6) leads to

$$v_t^j = w_t^j + a_t^j - q_t, \quad (11)$$

indicating that the consumption of housing services is positively related to individual income and preferences, and negatively related to the rental price.

⁴Our specification of the user costs is deliberately simple. Alternatively, we could have assumed that for each unit owned, households also incur a cost equal to a fraction M_t of the nominal value of housing, $P_t H_t^j$. M_t can be thought of as including maintenance and depreciation costs, property taxes, interest payments on mortgages, etc. Under this alternative specification, the user cost of housing would be

$$U_t = P_t(1 + M_t) - \frac{P_{t+1}}{R}.$$

As long as house market participants are homogeneously informed about M_t , none of the results presented below are affected, though the algebra would be much more cumbersome.

From now on, in order to make the analysis tractable, we consider only two groups of households, $j = 1$ and $j = 0$, each with equal mass, and adopt the convention that households in the first group receive a relatively more optimistic signal about the current fundamental, i.e., $\varepsilon_t^1 > \varepsilon_t^0$. As a consequence $E_t^1 p_{t+1} > E_t^0 p_{t+1}$, and using (10) $E_t^0 u_t > E_t^1 u_t$. Equation (9) can then be written as

$$E_t^0 u_t > q_t \quad \text{and} \quad h_t^0 = 0 \quad (12)$$

$$E_t^1 u_t = q_t \quad \text{and} \quad h_t^1 > 0. \quad (13)$$

In other words, pessimistic households choose to own no housing units, $h_t^0 = 0$, as they hold lower expectations about next period prices and thus perceive the cost of ownership to be higher than the cost of renting. On the contrary, optimistic households perceive the user cost to be equal to the cost of renting and are thus indifferent between owning and renting. The implication is that optimists choose the units of housing services to consume, v_t^1 , out of those owned, h_t^1 , and rent out the difference, $h_t^1 - v_t^1 = v_t^0$ to the relatively more pessimists households, at the equilibrium rent, q_t .

3.6 The equilibrium house price and rental price

Using (10), the indifference condition (13) can be written as

$$p_t = \frac{r}{1+r} q_t + \frac{1}{1+r} E_t^1 p_{t+1}, \quad (14)$$

suggesting that for a given rental price, the equilibrium house price reflects only the expectations of optimistic households. With the maintained assumption of fixed housing supply, s , the equilibrium rental price, q_t , is pinned down by the market clearing condition,

$$s = \frac{v_t^1 + v_t^0}{2},$$

which using (11) yields

$$q_t = \theta_t + a_t - s, \quad (15)$$

where

$$\theta_t = \frac{w_t^1 + w_t^0}{2} \quad \text{and} \quad a_t = \frac{a_t^1 + a_t^2}{2},$$

are, respectively, the average income and the average preference for housing services. Plugging (15) back into (14), the equilibrium price can be written as

$$p_t = \frac{r}{1+r} f_t + \frac{1}{1+\kappa} \bar{E}_t p_{t+1} + \frac{1}{1+r} \tilde{E}_t p_{t+1}, \quad (16)$$

where

$$f_t \equiv (\theta_t + a_t - s), \quad (17)$$

summarizes fundamental variables, and

$$\bar{E}_t p_{t+1} \equiv \frac{E_t^1 p_{t+1} + E_t^0 p_{t+1}}{2}, \quad \tilde{E}_t p_{t+1} \equiv \frac{E_t^1 p_{t+1} - E_t^0 p_{t+1}}{2},$$

denotes, respectively, the average expectation and the difference in expectations about tomorrow's price.

In equation (16), as in a standard asset pricing equation, the equilibrium price, p_t , depends, on fundamentals, f_t , and on the expected average capital gain from house price appreciation. The extra term, $\tilde{E}_t p_{t+1}$, is non-standard and arises because in our setup households hold heterogenous expectations. In the next two sections, we make different assumptions about households' information sets in order to evaluate how $\bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1}$ influence the relationship between house prices and fundamental variables.

4 Homogenous Information

In order to have a benchmark against which to compare the results, we start with the case where households are imperfectly but homogeneously informed about the state of the economy, θ_t . In other words, households receive identical information about the underlying unobservable aggregate fundamental: $\varepsilon_t^1 = \varepsilon_t^0$. In this benchmark case individual expectations coincide with average expectations, $E_t^j p_{t+1} = \bar{E}_t p_{t+1}$, and differences in expectation are zero, $\tilde{E}_t p_{t+1} = 0$.

As Appendix II shows, iterating equation (16) forward and imposing a transversality condition on housing prices, the average expectation of tomorrow's price can be written as ,

$$\bar{E}_t p_{t+1} = \bar{E}_t f_t = \phi \rho \theta_{t-1} - s, \quad (18)$$

where

$$\phi \equiv \frac{r\rho}{1+r-\rho}.$$

The second equality in the equation above arises because a_t , has mean zero and θ_t , which is not observable, follows an AR(1) process. Thus, households' forecast of θ_t depends on its past realization. Inserting (18) into (16), and recalling that $\tilde{E}_t p_{t+1} = 0$, the equilibrium price under imperfect but homogenous information, p^* , can be written as

$$p_t^* = f_t + \Lambda_t, \quad (19)$$

where f_t is given in (17) and

$$\Lambda_t \equiv \frac{\phi \rho \theta_{t-1} - \theta_t - a_t}{1 + r},$$

is an expectation error.

In what follows, we interpret p_t^* as the “fundamental” house price, because it reflects the average opinion in the market which, by Assumption 1, is an unbiased estimate of the unknown fundamental. As we will see, when information is not only imperfect but also heterogeneous among house market participants, the market becomes segmented, in the sense that not all households participates in the market of homes for sale. The upshot is that idiosyncratic shocks are not “washed out” in aggregate and the equilibrium price ends up reflecting only the most optimistic view in the market.

5 Heterogeneous Information

We now consider a setting where households use the current realization of their income, w_t^j , and the exogenous public signal, θ_{t-1} , to make an optimal inference about θ_t . Household j ’s information set at t is therefore⁵

$$\Omega_t^j = \{w_t^j, \theta_{t-1}\} \quad j = 0, 1.$$

Under the assumption that households do not share their private information with each other, and because w_t^j is buffeted by idiosyncratic shocks, households end up holding heterogeneous information about θ_t .

Before proceeding, it is important to notice that the equilibrium prices (both the housing and rental prices) are not included in Ω_t^j . This assumption is made only to simplify the characterization of the channels through which information dispersion affect the equilibrium price. As we will discuss in the following section this assumption is inessential for our results. A way to think about this assumption is to consider the special case where the variance of the aggregate unobservable preference shock, σ_a^2 , is arbitrarily large. In such a case the house price (16) and the rental price (15) become uninformative about θ_t and house market participants do not learn much upon observing p_t or q_t .⁶

⁵To know the entire history of aggregate shocks is superfluous since θ_t follows an $AR(1)$ process. Similarly, knowing the past realization of household private signals is irrelevant, given the *iid* assumption for ε_t^j .

⁶In excluding the equilibrium prices from the household information set, we make our analysis akin to a “difference of opinion model”, widely used in the finance literature. In such a model investors agree to disagree about the distribution of payoff and signals and therefore do not use

With signals w_t^j and θ_{t-1} , the ability of household j to estimate θ_t from available data depends on the relative magnitude of σ_ε^2 and σ_η^2 . Because of our assumption of independent and normally distributed errors, the projection theorem implies

$$E_t^j \theta_{t+1} = \rho E_t^j \theta_t = \rho \left((1 - \lambda) \rho \theta_{t-1} + \lambda w_t^j \right), \quad (20)$$

where the weight $\lambda \equiv \sigma_\eta^2 / (\sigma_\eta^2 + \sigma_\varepsilon^2)$ reflects the relative precision of the two signals. Thus, with $\lambda > 0$, expectations among households are heterogeneous and both average expectations and expectations differences become important determinants of the equilibrium house prices. Iterating equations (16) and (20) forward and excluding explosive price paths, Appendix III shows that the difference in expectations, and the average expectation of future prices are, respectively,

$$\tilde{E}_t p_{t+1} = \phi \lambda i_t, \quad (21)$$

$$\bar{E}_t p_{t+1} = (\phi \rho \theta_{t-1} - s) + \frac{\phi \lambda}{r} I + \phi \lambda (\theta_t - \rho \theta_{t-1}), \quad (22)$$

where

$$i_t \equiv \varepsilon_t^1 - \varepsilon_t^0,$$

denotes the informational difference between the two groups of households and

$$I \equiv \int_0^\infty x d\Gamma(x),$$

measures the average degree of information heterogeneity in the economy, with Γ denoting the distribution of i_t .

Equation (21), stems from the fact that households are disparately informed and in estimating θ_t they assign a positive weight to their private signal, w_t^j . Differences in expectations, $\tilde{E}_t p_{t+1}$, are therefore proportional to the difference in private signals, $i_t \equiv \varepsilon_t^1 - \varepsilon_t^0$. Equation (22), is the equivalent of (18), that is the average expectation under imperfect but homogeneous information. It differs, however, from (18) because heterogenous information introduces two additional terms, each proportional to the weight that households give to their private information. The first term, $\phi \lambda I / r$ arises because prices are forward looking: it is not only the current degree of information heterogeneity that matters but also the average difference in expectation that is expected to prevail in the future, given that the future dispersion in information will also affect the course of house prices. The second term,

the equilibrium prices to infer other investors' beliefs. An alternative reason households may not condition on the equilibrium prices is because they do not know how to use prices correctly (e.g. they display bounded rationality, as in Hong and Stein, 1999) or because they exhibit behavioral biases (e.g., they are overconfident, as in Scheinkman and Xiong, 2003).

$\phi\lambda(\theta_t - \rho\theta_{t-1})$, reflects instead the average degree of misperception in the economy and arises because households use only partially the public signal, θ_{t-1} , to infer the current state of the world. The larger the weight λ assigned to the private signal and the larger the degree of misperception in the economy, $(\theta_t - \rho\theta_{t-1})$, the more the expected price deviates from the one prevailing under homogenous information. This partial reaction of households to shifts in the fundamentals has the effect of introducing inertia in the way expectations are formed, which accords well with the idea that housing market expectations are formed in an extrapolative manner (see Case and Shiller, 1988, 2003).

Plugging these expressions into (16), the equilibrium house price can be written as

$$p_t = p_t^* + \lambda\Upsilon_t. \quad (23)$$

where, p_t^* , is the fundamental price given in (19), and

$$\Upsilon_t \equiv \phi \frac{(\theta_t - \rho\theta_{t-1})}{1+r} + \phi \frac{I}{r(1+r)} + \phi \frac{i_t}{1+r} \quad (24)$$

summarizes the role of information heterogeneity among households.

Thus, in the presence of heterogenous information, i.e., $\lambda > 0$, p_t differs from p_t^* , due to shifts in Υ_t . In turn, shifts in Υ_t are more pronounced the larger is the current and expected degree of information heterogeneity: i_t , and I . The reason is quite intuitive. Households that receive a positive signal expect higher income in the future and thus higher housing demand and prices. Conversely, households with a negative signal expect lower future prices. However, while households with optimistic expectations demand more houses for speculative reasons the higher the expected price, pessimistic households prefer to consume housing services through the rental market. In other words, renting act as if relatively pessimistic investors were facing a binding short-sale constraint: holding pessimistic expectations about tomorrow's prices these households would like to short their houses. Since they cannot do so, pessimists drop out of the market of homes for sale and their beliefs are not fully reflected in the equilibrium price. The final result is that the equilibrium price reflects only the expectations of relatively optimistic households, and is thus biased upward. Equation (23) suggests that the larger the divergence in beliefs among households, the more pronounced is this effect. Moreover, in comparing (23) with (19) it is straightforward to see that heterogenous information increases also the volatility of house prices by a factor proportional to the noise in the difference in beliefs:

$$V(p_t) - V(p_t^*) = \left(\frac{\lambda\phi}{1+r} \right)^2 (\sigma_\eta^2 + \sigma_i^2).$$

6 Learning from the equilibrium price

The result just derived — that differences in beliefs impart an upward shift in the level and volatility of house prices — hold under the assumption that the variance of the unobservable aggregate preference for housing services, a_t , is arbitrarily large, so that to infer θ_t households rely only on the exogenous public and private signals (θ_{t-1} and w_t^j), but not on the endogenous public signals, i.e. the house price, p_t , and the rental price, q_t .

We now relax this assumption and allow households to condition on prices. This extension is desirable because house and rental prices, like any other financial prices, are a useful summary statistics of the dispersed private information in the economy. If households use these endogenous public signals to update their beliefs on the underlying fundamental, dispersion in beliefs may vanish, and in the limit the upward bias in the equilibrium house price may disappear.

As shown in Appendix IV, of the two available endogenous public signals, q_t and p_t , households use only the information contained in the equilibrium rental price. The reason p_t is redundant is because q_t conveys the same information on θ_t as in p_t , but it is not influenced by the extra source of noise, i_t , originating from the fact that house market participants hold different expectations about the future course of house prices (compare equations (15) and (23). However, since q_t depends also on a_t , the underlying fundamental, θ_t , is not fully revealed. Specifically, households cannot tell whether the equilibrium rent is high because aggregate economic conditions improve or because unobservable taste shocks drive the demand for housing services. For a given noise in the exogenous public and private signals, the informative content of the rental price will be decreasing in the preference noise, σ_a^2 , as typical in a noisy rational expectation model *à la* Grossman and Stiglitz (1976) and Hellwig (1980).

Using a standard linear solution method, Appendix IV shows that the equilibrium price with learning can be written as,

$$p_t = p_t^* + \pi_2 \Upsilon_t + \pi_3 \Phi_t, \quad (25)$$

where π_2 and π_3 are weights on the private and the endogenous public signals (the equilibrium rent), Υ_t is given in (24) and

$$\Phi_t \equiv \frac{\phi}{1+r} ((\theta_t - \rho\theta_{t-1}) + a_t)$$

is a term that summarizes the degree of magnification of shocks induced by the process of learning. Intuitively, in the presence of unobservable shocks, households who observe a change in the rental price do not understand whether this change is driven by a change in the aggregate income or by changes in preferences. Thus,

when $\pi_3 > 0$, each of these shocks will have an amplified effect on the house price, since households respond to whatever is the source of movement in the rental price.

A key observation to make in comparing equation (25) with (23) is that i_t – our measure of dispersion in beliefs – continues to shift the equilibrium price away from its fundamental value, p_t^* , for the same reasons discussed in the previous section. The relative importance of Υ_t and Φ_t depends, however, on π_2 and π_3 which, as shown in Appendix IV, are related to λ as follows,

$$\lambda > \pi_2 \gtrless \pi_3,$$

with the property that $\pi_2 \rightarrow \lambda$, and $\pi_3 \rightarrow 0$, as $\sigma_a^2 \rightarrow \infty$. In words, as the noise in the preference for housing services becomes larger — that is the rental price gets less and less revealing — the equilibrium price (25) becomes identical to the one prevailing in absence of learning (23). This is illustrated in Figure 2, where we plot the percentage deviation of the equilibrium price from its fundamental level, $p_t - p_t^*$, for different values of the preference noise, σ_a^2 .⁷ As the figure shows, regardless of whether households are able to learn, the price misalignments remain of comparable size, for large values of σ_a^2 . When the preference shock is smaller, q_t is informative about θ_t and households rely less on their private source of information, i.e. $\pi_2 < \lambda$.

Nonetheless, provided preference shocks prevent prices from being fully revealing, i.e., σ_a^2 is not very small, the private signals continue to provide some information in estimating the underlying fundamental. This is illustrated in Figure 3, which plots the price differential, $p_t - p_t^*$, for different values of the private signal's noise σ_ε^2 . As it can be seen, the misalignment of the equilibrium price from its fundamental level is hump-shaped in the signal noise, regardless of whether households learn from the equilibrium price. The bias is increasing for intermediate values of the private signals' noise, and it is falling for smaller values of σ_ε^2 , since in this case households receive similar information. The bias will also be decreasing for larger values of σ_ε^2 because households tend to assign less and less weight to their private signals.

Finally, when the equilibrium rent is not perfectly revealing, the volatility of the equilibrium house price continues to be higher than in the benchmark scenario of imperfect but homogenous information. By comparing (25) with (19), it is easy to see that the difference of variances

$$V(p_t) - V(p_t^*) = \left(\frac{\pi_2 \phi}{1+r} \right)^2 (\sigma_\eta^2 + \sigma_i^2) + \left(\frac{\pi_3 \phi}{1+r} \right)^2 (\sigma_\eta^2 + \sigma_a^2)$$

⁷Figure 2 plots the average price differential, $p_t - p_t^*$, of 1000 simulated series using the following parameter values: $\theta_{t-1} = a_{t-1} = s = 0$ and $r = 0.01$. These parameter values are chosen without loss of generality given that we consider the deviation of the equilibrium price from its fundamental value.

is proportional to the noise in difference in beliefs — as in the case without learning — and to the noise in preference shocks — because of the amplification of shocks induced by the process of learning from the equilibrium rent.

7 Testing the implications of the model

Our model delivers three main predictions: 1) the deviation of house prices from their fundamental value is higher the larger the dispersion of beliefs; 2) the dispersion of beliefs is positively related to the volatility of house prices; and 3) optimistic expectations bias the equilibrium price upward more than pessimistic expectations.

The most difficult part of testing these predictions is to have data on heterogeneous information among house market participants. These data, in fact, are conspicuous by their absence. To deal with this problem we adopt the following strategy. We take US cities as units of observation and use the dispersion (within cities) of shocks to industry earnings as a proxy for dispersion of beliefs about local housing market conditions.

While debatable, this proxy is motivated by empirical and theoretical considerations. From an empirical viewpoint, there is widespread consensus that high frequency variations in house prices are mostly local, not national (Glaeser and Gyourko, 2006). In addition, the evidence suggests that the bulk of short run movements in local house prices is due to changes in demand, driven by economic conditions within a region as opposed to changes in preferences for local amenities.⁸

From a theoretical perspective, our proxy of information dispersion is motivated by the logic of our model, which can be thought as describing the time series dynamics of house prices in a typical city, where the speculative demand for housing ultimately depends on the realization of income shocks. If residents in each cities are employed in different industries, and they are imperfectly informed about the city income, then industry specific income shocks become a source of confusion about the city average income, as in the signal extraction problem discussed in our theoretical framework. With this interpretation, equation (1) and (2) in the model, which governs the dynamics of households' income, can be reinterpreted and rewritten as follows,

$$w_{k,t}^l = \theta_{k,t} + \varepsilon_{k,t}^l \quad \text{and} \quad \theta_{k,t} = \rho\theta_{k,t-1} + \eta_{k,t} \quad (26)$$

where $w_{k,t}^l$ is the time t earnings of a representative household resident in city k and employed in industry l , $\theta_{k,t}$ is the average city income at time t , and $\varepsilon_{k,t}^l$ is

⁸Endogenous supply-side changes may also affect variations in house prices (Glaeser, Gyourko and Saiz, 2008). However, in the short run, due to regulations and technological constraints, quantity changes tend to respond slowly to shifts in demand.

a time- t industry- l specific shock. At any point in time, a proxy for the degree of information dispersion in city k can then be computed using a measure of the dispersion of earnings shocks across industries, as we now explain.

7.1 Data description and summary statistics

We collect annual data for a sample of approximately 350 metropolitan areas (MSA) during the period 1980 to 2000. To infer the time series properties of local income shocks we use (per employed) earnings data for 10 one-digit industries, based on the SIC classification code.⁹ With these data, the dispersion of earnings shocks across industries is computed in two steps. First, based on equation (26), we run 10 regressions, one for each industry, in which we pool the growth rate of industry earnings for the full sample of MSAs,

$$\Delta w_{k,t}^l = \alpha_0 + \alpha_1 \Delta \theta_{k,t} + \alpha_2 \Delta \theta_{k,t-1} + \gamma_t + \varepsilon_{k,t}^l \quad \text{for } l = 1, 2, \dots, 10, \quad (27)$$

where Δ is the first difference operator, and γ_t is a time fixed effect. In this specification, the residuals $\varepsilon_{k,t}^l$ measure shocks to earnings growth in each city-industry, controlling for city-specific income dynamics and nationwide effects.¹⁰ Second, we measure the dispersion of earnings shocks across industries and within each MSA as the weighted average of the absolute value of industry-MSA residuals,

$$i_{k,t} = \sum_{l=1}^{10} \omega_l |\varepsilon_{k,t}^l|, \quad (28)$$

with weights ω_l equal to the share of MSA workers employed in industry l , to control for the size of each industry.¹¹

⁹Specifically, we use earnings data for the following industries: 1) Farm, 2) Mining, 3) Construction, 4) Manufacturing, 5) Transportation and public utilities, 6) Wholesale trade, 7) Retail trade, 8) Finance, insurance, and real estate, 9) Services, and 10) Government and government enterprises. These data are available at <http://www.bea.gov/regional/reis/>. Our sample period stops in 2000 because in that year the Standard Industrial Classification (SIC) system has been replaced by the North American Industry Classification System (NAICS). This different system for classifying economic activity makes it impossible to extend our data beyond 2000. Available data based on the NICS system cover only the period 2001 to 2006. To be able to better exploit the time series variation in the data we have preferred to use data based on the SIC classification codes for the period 1980-2000.

¹⁰We have also experimented with specifications that include lags of $\Delta w_{k,t}^l$ to control for industry-city specific dynamics. All the results reported below are robust to such changes.

¹¹None of the results presented below (in terms of economic and statistical significance) are affected by using squared deviations rather than absolute deviations. We prefer to model absolute deviations to be able to maintain the same unit as the change in industry earnings, so the coefficients in the house price regressions reported below are easily interpreted.

We focus our analysis on MSAs because these are the smallest units of observation for which income and quality-adjusted housing price data is available. For each MSA we take the nominal house price index for single-family houses from the Office of Federal Housing Enterprise Oversight, and per capita income data from the Bureau of Economic Analysis. Nominal variables are then converted in real dollars using the national CPI index less shelters from the Bureau of Labor Statistics. We use annual observations because income data is only available annually.

Table 1 reports basic summary statistics. As shown, our data display considerable variation across MSA. Over the full period 1980-2000, our proxy of information dispersion is less than 1.5% in Minneapolis, Cleveland, Kansas City and Tampa but greater than 4% in Chicago, Dallas, Los Angeles, and New York, among other cities. Real house price changes also exhibits considerable variation. For instance, Boston, San Francisco, and San Jose all experienced growth rates in house prices over 3% per annum over the 20 year period studied, while Houston, Oklahoma City, and San Antonio experienced house price changes of less than minus 1.5%.

We now exploit the rich cross-city variation of these data to test the implications of our model.

7.2 The baseline regression

We start the analysis by examining the empirical relevance of the price equation prevailing under common information. To empirically use equation (19), we take first differences of each variable and estimate the following regression¹²:

$$\Delta p_{k,t} = \beta_0 + \beta_1 \Delta \theta_{k,t} + \beta_2 \Delta \theta_{k,t-1} + \gamma_t + \gamma_k + \epsilon_{k,t}. \quad (29)$$

Here, $\Delta p_{k,t}$ is the log change of the real house price index in MSA k in year t , $\Delta \theta_{k,t}$ the log change in real per capita income and $\epsilon_{k,t}$ is a standard error term. In this regression, and those that follow, we include also year and MSA dummies, γ_t and γ_k , to account for unobservable aggregate and city-specific determinants of house price changes. Thus, each variable in any year is measured in deviation from both national average and long run city average.

Table 2 reports OLS estimates of this baseline regression, with standard errors clustered at the MSA level to allow for within-city autocorrelation in the errors. According to the model, β_1 and β_2 are expected positive and as shown in the first column of Table 2 these predictions are strongly supported by the data: higher current and lagged income changes are significantly associated with higher housing prices changes.

¹²We use each variable in first difference because the OFHEO house price index is not standardized to the same representative house across markets. Thus, price levels cannot be compared across cities, but they can be used to calculate growth rates.

The role of heterogeneous information is examined in columns 2 where we report estimates of the empirical counterpart of equation (23). More specifically, we add to the baseline regression (29) our proxy of difference in beliefs, $i_{k,t}$. In line with the prediction of the model, the results show a statistically significant relationship between difference in beliefs and house prices. Our indicator of information dispersion enters the regression positively and with a sizeable estimated effect: a 1% per cent increase in $i_{k,t}$ results in a 0.2% increase in the growth rate of house prices. To better gauge the economic effect of these results consider an exogenous increase of $i_{k,t}$, from the 10th percentile value (which is approximately 1.2%) to the 90th percentile value (which is approximately 4%). This increase would lead to an acceleration of the growth rate of house price by 0.6% per year, which is large considering that the mean change in real house prices over the 1980-2000 period is 0.4%.

7.3 Alternative empirical specifications

The results in Table 2, although based on the equilibrium price equations implied by the theoretical model, do not control for some patterns of the house price dynamics that prior works have documented to be important. For example, starting with Case and Shiller (1989), it is well known that house prices exhibit momentum and mean reversion over time. To control for these effects, we add three lags of the dependent variable to our baseline regression. The results shown in column 1 of Table 3 indicate that house prices indeed exhibits positive correlation at short lags and negative correlation at longer lags. However, as reported in column 2, our proxy of information dispersion continue to play a large and significant role in explaining house price changes over time.

Column 3 to 4 explore the robustness of our findings to an alternative empirical specification, suggested by the work of Lamont and Stein (1999). In their study of the house price dynamics in US cities, Lamont and Stein show that house prices (a) exhibit short run movements, (b) respond to contemporaneous income shocks, and (c) exhibit a long run tendency to fundamental reversion. They thus propose to estimate the following regression,

$$\Delta p_{k,t} = \gamma_0 + \gamma_1 \Delta p_{k,t-1} + \gamma_2 \Delta \theta_{k,t} + \gamma_3 (p/\theta)_{k,t-1} + \gamma_k + \gamma_t + \epsilon_{k,t} \quad (30)$$

where $(p_k/\theta_k)_{t-1}$ is the lagged ratio of house prices to per-capita income.¹³ Column 3 shows that these variables have all the expected sign and explain significantly a large fraction of house price variations. To this three-variable specification we add

¹³The inclusion of this variable implies that the long run elasticity of prices to city income is one.

our variable of interest, $i_{k,t}$, in column 4. In line with the results in Table 2, we find that our proxy of information dispersion continues to be related significantly to house price variations. House prices are higher in cities where information about local income shocks is more dispersed.

A common objection to the correlations reported so far is that they disappear when we control for demographic factors. In fact, high frequency changes in the demand for housing may be driven not only by changing economic conditions within a region but also by population movements, a shifter of housing demand that has been omitted in our theoretical analysis. In the attempt to control for these effects, columns 3 and 4 report estimates with the rate of population growth as an additional regressor. Population growth is expected to enter the regression with a positive sign since new buyers willing to move into a city push up housing demand and prices.¹⁴ The results in columns 3 and 4 show that population growth has indeed a positive and significant coefficient. Our core results, however, do not seem to depend on the inclusion of this additional control. The estimated effects for our proxy of information dispersion continue to be large and significant, supporting the model’s prediction that dispersion in information exerts an upward impact on house prices.

7.4 The volatility of house prices

We now turn to the second prediction of the model that the volatility of house prices increases with the variance in the dispersion of beliefs. To examine the strength of this prediction we compute the volatility of house prices by running a pooled regression for the change in house prices, controlling for year effects, and we then take the standard deviation of the residuals in each MSA. This gives us a measure of the volatility of house prices, within a metropolitan area, controlling for aggregate effects. With one observation for MSA, we exploit the cross sectional variation of house price volatility and regress our measure of volatility of house prices on the standard deviation of dispersion of beliefs across MSA.

Figure 2 graphs the volatility of house price against the fitted values from this regression, for a sample of 321 MSAs:

$$\text{volatility} = .0127 (11.37) + 1.3034 (6.00) \times \text{s.d of dispersion of beliefs},$$

where robust t-statistics are in parenthesis, and the $R^2 = 0.12$. The results highlight the empirical validity of our model’s prediction: MSAs with higher dispersion

¹⁴Given that the population of a city is almost perfectly correlated with the size of its housing stock (Glaeser, Gyourko and Saks, 2006) the inclusion of population growth in our regressions serves also to control for changes in housing supply. In this case, the estimated coefficient is expected positive provided construction firms are forward looking and new homes are built in response to rising prices.

of beliefs have also more volatile house prices. Moreover, this result holds even if we control for the standard deviation of the aggregate MSA income, which turned out to be not statistically significant.¹⁵

7.5 Positive and negative shocks

The final implication of our model is that house prices respond more to positive than to negative “news” shocks. In our model, the speculative motive for buying housing units is enhanced when, controlling for the average city income, households expect better economic conditions to prevail in the future. On the contrary, the speculative motive is moot following negative income shocks.

It is worthwhile to check the empirical relevance of this prediction since its validity distinguishes the theoretical implications of our model from those emanating in a setup where income dispersion affects house prices because households move in and out of cities for productive or preference reasons, as discussed in Gyourko, Mayer and Sinai (2006) or Van Nieuwerburgh and Weil (2007).

Using the residuals in equation (27), we define positive and negative shocks as follows:

$$\begin{aligned} POS_{k,t} &= \sum_{l=1}^{10} \omega_l \varepsilon_{k,t}^l \quad \text{if } \varepsilon_{k,t}^l > 0 \\ NEG_{k,t} &= \sum_{l=1}^{10} \omega_l \varepsilon_{k,t}^l \quad \text{if } \varepsilon_{k,t}^l < 0, \end{aligned}$$

where again l is an industry index and ω_l an industry weight. We then enter these variables in piecewise linear form into the empirical specification (29) and (30) to allow for differential effects between positive and negative shocks to industry earnings. The results are displayed in Table 4 using the same specifications as in Table 3. As shown, the estimates conform strongly with our model’s predictions that positive and negative shocks have asymmetric effects on housing prices. In fact positive shocks have a large and significant impact on house prices. Instead, the importance of negative shocks is negligible and not statistically significant.

¹⁵For this latter specification the regression output is:

$$\begin{aligned} \text{house price volatility} &= .0200 (6.30) + 1.1229 (4.93) \times \text{s.d. dispersion of beliefs} + \\ &\quad .100 (0.67) \times \text{s.d. average income}, \quad R^2 = .12 \end{aligned}$$

where robust standard errors are in parenthesis.

8 Conclusion

In this paper we have examined the implications of heterogeneous expectations within a standard user cost model of the housing market, and observed that the degree of information dispersion among market participants affect the equilibrium house prices. The emphasis on household expectations was motivated by the desire to formalize the popular idea that large house price swings occur when current prices depend upon expectations of future price increases.

Our theoretical model — in which agents have the option to rent and buy houses to consume housing services and/or to speculate on future price change — suggests that the equilibrium housing price is higher, the larger the difference in expectations among house market participants. The intuition is that all households face *de facto* a short position in housing. Therefore, those who hold pessimistic expectations about future prices decide to rent to avoid future capital losses, while those who have optimistic expectations decide to buy to speculate on future price increases. The upshot is that the equilibrium price incorporates only the expectations of the optimists and is thus biased upward.

This theoretical prediction holds empirically in a panel data of US cities, if dispersion in industry earnings is used as a proxy for dispersion in beliefs. This proxy was motivated by our model’s assumption that households’ “window on the world” is given by their individual income, which is perturbed by idiosyncratic shocks leading households to form different views of the economy. Although in line with the logic of our model, this proxy is undoubtedly a crude one. To the best of our knowledge, however, no direct measures of dispersion of beliefs in the housing markets are available to perform a direct test of our theory.

In keeping our model simple, we have abstracted from a number of issues that might play an important role in the development of a more complete model. For example, we have abstracted from the general equilibrium effects of the interest rate. Changes in R , however, may affect our analysis since the return on the safe asset influences households’ desire to rent rather than owning, for a given level of pessimistic expectations. We have also prevented households from re-trading. An extension of the model that allows for re-trading, as in Stein (1995) or Ortalo-Magne’ and Rady (2006), may shed new light on whether heterogeneous expectations induce a positive correlation between house prices and housing transactions. These extensions are left for future research.

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Appendix I: Linearization

We linearize equations (6) and (7) around the equilibrium with "certainty", i.e., when $\varepsilon_t^j = 0$, $\eta_t = 0$, $a_t = 0$ and $\nu_t^j = 0 \forall t$. In this equilibrium $V = H$, because there is no uncertainty and no heterogeneity among households. Denoting with X any variable X_t in the "certainty" equilibrium, the first order conditions (6) and (7), with interior solutions, can be written as

$$V^j = V > 0 \implies V = \frac{C}{R} \frac{1}{Q}, \quad (31)$$

$$H^j = H > 0 \implies Q = U. \quad (32)$$

Moreover, using equations (4), (8) and the fact that $V = H$,

$$\begin{aligned} \frac{C}{R} &= W - HP \left(1 - \frac{1}{R}\right) \\ &= W - VQ. \end{aligned} \quad (33)$$

Thus combining (33) and (31) one obtains

$$V = \frac{W}{2Q}.$$

Under the assumption of fixed housing supply, S , the market clearing condition is,

$$V = S,$$

which implies that the following relationships must hold in a certainty equilibrium,

$$U = Q, \quad Q = \frac{W}{2S}, \quad C = \frac{RW}{2}.$$

Denoting with lower-case letters variables in percent deviation from the equilibrium with certainty, and recalling our definition of user cost,

$$U_t = P_t - \frac{P_{t+1}}{R} \quad (34)$$

a linearization of (7) around the certainty equilibrium yields,

$$E_t^j \left[\frac{RP}{C} (1 + p_t - c_{t+1}^j) - \frac{RQ}{C} (1 + q_t - c_{t+1}^j) - \frac{P}{C} (1 + p_{t+1} - c_{t+1}^j) \right] \geq 0.$$

Rearranging,

$$\begin{aligned} E_t^j \left[\frac{RP}{C} p_t - \frac{RQ}{C} q_t - \frac{P}{C} p_{t+1} - c_{t+1}^j \left(\frac{RP}{C} - \frac{RQ}{C} - \frac{P}{C} \right) \right] &\geq 0 \Rightarrow \\ E_t^j [RPp_t - RQq_t - Pp_{t+1}] &\geq 0 \Rightarrow \\ E_t^j \left[p_t - \frac{Q}{P} q_t - \frac{1}{R} p_{t+1} \right] &\geq 0, \end{aligned}$$

we obtain

$$p_t \geq \frac{r}{1+r} q_t + \frac{1}{1+r} E_t^j p_{t+1}, \quad (35)$$

where

$$r = R - 1.$$

Notice also that a linearization of (34) gives

$$\begin{aligned} u_t &= \frac{P}{U} p_t - \frac{P}{RU} p_{t+1} \\ &= \left(\frac{1+r}{r} \right) p_t - \frac{1}{r} p_{t+1}. \end{aligned}$$

Therefore, (35) can be rewritten as

$$E_t^j u_t \geq q_t. \quad (36)$$

Since, by assumption, $E_t^1 p_{t+1} > E_t^0 p_{t+1}$, it follows that $E_t^0 u_t > E_t^1 u_t$. Thus, in equilibrium, equation (36) can be written as,

$$E_t^1 u_t = q_t \quad \text{and} \quad h_t^1 > 0, \quad (37)$$

$$E_t^0 u_t > q_t \quad \text{and} \quad h_t^0 = 0. \quad (38)$$

Proceeding as above, a linearization of equation (6), around the certainty equilibrium, gives

$$\begin{aligned} E_t^j \frac{RQ}{C} (q_t - c_{t+1}^j) &= \frac{A}{V} (2a_t^j - v_t^j) \\ E_t^j \frac{1}{S} (q_t - c_{t+1}^j) &= \frac{1}{V} (2a_t^j - v_t^j) \end{aligned}$$

which defines the optimal demand of housing services

$$v_t^j = 2a_t^j - q_t + E_t^j c_{t+1}^j. \quad (39)$$

The term $E_t^j c_{t+1}^j$ in (39) is obtained by linearizing the flow of budget constraint (4), that for the two groups of households reads as follows,

$$\begin{aligned} C_{t+1}^1 &= R(W_t^1 - P_t H_t^1 + Q_t(H_t^1 - V_t^1)) + P_{t+1} H_t^1, \\ C_{t+1}^0 &= R(W_t^0 - Q_t V_t^0). \end{aligned} \quad (40)$$

A bit of algebra establishes¹⁶

$$E_t^1 c_{t+1}^1 = 2w_t^1 - v_t^1 - \left(\frac{r+1}{r}\right) p_t + \frac{1}{r} E_t^1 p_{t+1} \quad (41)$$

$$\begin{aligned} &= 2w_t^1 - v_t^1 - E_t^1 u_t, \\ E_t^0 c_{t+1}^0 &= 2w_t^0 - v_t^0 - q_t. \end{aligned} \quad (42)$$

Plugging these expressions in (39) and using equation (37) it follows that

$$\begin{aligned} v_t^1 &= w_t^1 + a_t^1 - \frac{1}{2}(q_t + E_t^1 u_t) \\ &= w_t^1 + a_t^1 - q_t, \\ v_t^0 &= w_t^0 + a_t^0 - q_t. \end{aligned}$$

Using the market clearing condition,

$$s = \frac{1}{2} h_t^1 \quad (43)$$

and the fact that

$$\frac{1}{2}(h_t^1 - v_t^1) = \frac{1}{2} v_t^0$$

equation (43) can be written as

$$\frac{1}{2} v_t^0 + \frac{1}{2} v_t^1 = s,$$

¹⁶Linearizing (40) yields

$$\begin{aligned} E_t^1 c_{t+1}^1 &= \frac{RW}{C} w_t^1 - \frac{RPH}{C} (p_t + h_t^1) + \frac{RQH}{C} (q_t + h_t^1) - \frac{RQV}{C} (q_t + v_t^1) \\ &\quad + \frac{PH}{C} (E_t^1 p_{t+1} + h_t^1) \\ &= 2w_t^1 - \frac{P}{U} (p_t + h_t^1) + (q_t + h_t^1) - (q_t + v_t^1) + \frac{P}{RU} (E_t^1 p_{t+1} + h_t^1) \end{aligned}$$

Rearranging this equation gives (41). Proceeding in a similar way one obtains (42).

from which it is immediate to pin down the equilibrium rental price,

$$q_t = \theta_t + a_t - s, \quad (44)$$

where

$$\theta_t = \frac{w_t^1 + w_t^0}{2} \quad \text{and} \quad a_t = \frac{a_t^1 + a_t^0}{2}.$$

Finally, inserting (44) into (37) gives,

$$\begin{aligned} p_t &= \frac{r}{1+r} (\theta_t + a_t - s) + \frac{1}{1+r} E_t^1 p_{t+1} \\ &= \frac{r}{1+r} (\theta_t + a_t - s) + \frac{1}{1+r} \bar{E}_t p_{t+1} + \frac{1}{1+r} \tilde{E}_t p_{t+1}, \end{aligned} \quad (45)$$

where

$$\bar{E}_t p_{t+1} = \frac{E_t^1 p_{t+1} + E_t^0 p_{t+1}}{2} \quad \text{and} \quad \tilde{E}_t p_{t+1} = \frac{E_t^1 p_{t+1} - E_t^0 p_{t+1}}{2}.$$

Appendix II: Common Information

When information is imperfect but homogeneous, $E_t^j p_{t+1} = \bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1} = 0$. Therefore, equation (45), shifted one period forward, gives

$$p_{t+1} = \frac{r}{1+r} (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r} \bar{E}_{t+1} p_{t+2}.$$

Taking expectations on both sides conditional on time t information, and excluding explosive price paths, a forward iteration of the expression above gives

$$\bar{E}_t p_{t+1} = \frac{r}{1+r} \sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau} \bar{E}_t (\theta_{t+1+\tau} + a_{t+1+\tau} - s)$$

Since θ_t and a_t are unobservable at time t and

$$\theta_t = \rho \theta_{t-1} + \eta_t, \quad \text{with } \rho \in (0, 1]$$

we have

$$\bar{E}_t [\theta_{t+1} + a_{t+1} - s] = \rho^2 \theta_{t-1} - s.$$

It is therefore immediate to obtain

$$\bar{E}_t p_{t+1} = \bar{E}_t f_t = \phi \rho \theta_{t-1} - s, \quad (46)$$

where $\phi \equiv \frac{r\rho}{1+r-\rho}$. Plugging (46) back into (45) and recalling that $\tilde{E}_t p_{t+1} = 0$, the equilibrium price under common information can then be written as

$$p_t^* = (\theta_t + a_t - s) + \frac{1}{1+r} ((\phi \rho \theta_{t-1} - \theta_t) - a_t).$$

Appendix III: Heterogeneous Information

In the presence of heterogeneous expectations, $E_t^j p_{t+1} \neq \bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1} \neq 0$. Shifting equation (45) one period forward

$$p_{t+1} = \frac{r}{1+r}(\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r}\bar{E}_{t+1}p_{t+2} + \frac{1}{1+r}\tilde{E}_{t+1}p_{t+2}$$

denoting,

$$i_t = \left| \varepsilon_t^j - \varepsilon_t^i \right| \quad \text{for } i \neq j.$$

and guessing that $\tilde{E}_t[p_{t+1}] = \phi \lambda i_t$, we have

$$\begin{aligned} E_t^j p_{t+1} &= \frac{r}{1+r} E_t^j (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r} E_t^j \bar{E}_{t+1} p_{t+2} + \frac{\phi \lambda}{1+r} I \\ \bar{E}_t p_{t+1} &= \frac{r}{1+r} \bar{E}_t (\theta_{t+1} + a_{t+1} - s) + \frac{1}{1+r} \bar{E}_t \bar{E}_{t+1} p_{t+2} + \frac{\phi \lambda}{1+r} I, \\ \tilde{E}_t p_{t+1} &= \frac{r}{1+r} \tilde{E}_t \theta_{t+1} + \frac{1}{1+r} \tilde{E}_t \bar{E}_{t+1} p_{t+2}, \end{aligned}$$

where the last equality holds because households hold heterogeneous expectations with respect to θ_{t+1} but not with respect to a_{t+1} . In the expressions above,

$$I \equiv \int_0^\infty x d\Gamma(x),$$

is the average degree of information heterogeneity where Γ is the density of i_t .

Iterating these expressions forward and excluding explosive price paths, we obtain:

$$\begin{aligned} E_t^j p_{t+1} &= \frac{r}{1+r-\rho} E_t^j \theta_{t+1} - s + \frac{\phi \lambda}{r} I, \\ \bar{E}_t p_{t+1} &= \frac{r}{1+r-\rho} \bar{E}_t \theta_{t+1} - s + \frac{\phi \lambda}{r} I, \\ \tilde{E}_t p_{t+1} &= \frac{r}{1+r-\rho} \tilde{E}_t \theta_{t+1}. \end{aligned}$$

Moreover, using equation (20), it is easy to see that:

$$E_t^j \theta_{t+1} = \rho E_t^j \theta_t = \rho \left[(1-\lambda) \rho \theta_{t-1} + \lambda w_t^j \right],$$

and thus,

$$\begin{aligned} \bar{E}_t p_{t+1} &= \phi (\rho(1-\lambda) \theta_{t-1} + \lambda \theta_t) - s + \frac{\phi \lambda}{r} I, \\ &= (\phi \rho \theta_{t-1} - s) + \phi \lambda (\theta_t - \rho \theta_{t-1}) + \frac{\phi \lambda}{r} I, \\ \tilde{E}_t p_{t+1} &= \phi \lambda i_t. \end{aligned}$$

so that $\tilde{E}_t p_{t+1} = \phi \lambda i_t$ as claimed. Plugging $\tilde{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1}$ in (45), the equilibrium house prices can be written as

$$\begin{aligned} p_t &= (\theta_t + a_t - s) + \frac{1}{1+r} ((\phi \rho \theta_{t-1} - \theta_t) - a_t) \\ &\quad + \frac{\phi \lambda}{1+r} (\theta_t - \rho \theta_{t-1}) + \frac{\phi \lambda}{r(1+r)} I + \frac{\phi \lambda}{1+r} i_t. \\ &= p_t^* + \lambda \Upsilon_t \end{aligned}$$

where

$$\Upsilon_t \equiv \frac{\phi (\theta_t - \rho \theta_{t-1})}{1+r} + \frac{\phi I}{r(1+r)} + \frac{\phi i_t}{1+r}.$$

Appendix IV: Learning from the equilibrium rental price

We now provide a solution to the signal extraction problem when households condition also on the rental and equilibrium prices to learn about θ_t . We proceed in two steps. First, we show that households use only the equilibrium rental price in the inference problem. Second, we apply a standard method to solve the inference problem for θ_t and the resulting equilibrium price.

Omitting constant variables, we can write the equilibrium house price as follows,

$$p_t = P(\theta_{t-1}, \theta_t, a_t, i_t). \quad (47)$$

Notice that it is superfluous to account for q_t as this variable can be substituted for θ_t and a_t using (44). We guess that P is a quasi-linear function:

$$p_t = \psi \theta_t + \hat{P}(\theta_{t-1}, a_t, i_t). \quad (48)$$

and using (44), we rewrite it as

$$p_t = \psi (q_t + s) + \underbrace{-\psi a_t + \hat{P}(\theta_{t-1}, a_t, i_t)}_{\text{noisy information}}. \quad (49)$$

Since $-\psi a_t + \hat{P}(\theta_{t-1}, a_t, i_t)$ in the RHS of (49) is just noisy information, and contain no valuable information for inferring θ_t , this equation shows that households only learn from the rental price. Our guess that p_t is indeed linear in θ_t is confirmed below (see equation (54)).

We now turn to the inference problem. Household j estimates the unknown fundamental θ_t by solving a standard filtering problem, based on the normally distributed

(a) private signal, w_t^j , (b) exogenous public signal, θ_{t-1} , and (c) endogenous rental price $q_t + s$. Recalling that

$$\begin{aligned}\theta_t &= \rho\theta_{t-1} + \eta_t, \\ w_t^j &= \theta_t + \varepsilon_t^j, \\ q_t + s &= \theta_t + a_t.\end{aligned}$$

Assuming that a_t is normally distributed with zero mean and independent of ε_t^j and η_t , we can write down the log-likelihood function as

$$L = -\frac{1}{2\sigma_\eta^2} \left(\rho\theta_{t-1} - E_t^j \theta_t \right)^2 - \frac{1}{2\sigma_\varepsilon^2} \left(w_t^j - E_t^j \theta_t \right)^2 - \frac{1}{2\sigma_a^2} \left(q_t + s - E_t^j \theta_t \right)^2.$$

Thus, the optimal filtering solves the following first order condition,

$$\sigma_\varepsilon^2 \sigma_a^2 \left(\rho\theta_{t-1} - E_t^j \theta_t \right) + \sigma_\eta^2 \sigma_a^2 \left(w_t^j - E_t^j \theta_t \right) + \sigma_\eta^2 \sigma_\varepsilon^2 \left(q_t + s - E_t^j \theta_t \right) = 0,$$

or,

$$E_t^j \theta_t = \frac{\sigma_\varepsilon^2 \sigma_a^2 \rho \theta_{t-1} + \sigma_\eta^2 \sigma_a^2 w_t^j + \sigma_\eta^2 \sigma_\varepsilon^2 (q_t + s)}{\sigma_\varepsilon^2 \sigma_a^2 + \sigma_\eta^2 \sigma_a^2 + \sigma_\eta^2 \sigma_\varepsilon^2}.$$

The best linear estimate of θ_t is therefore,

$$E_t^j \theta_t = \pi_1 \rho \theta_{t-1} + \pi_2 w_t^j + \pi_3 (q_t + s), \quad (50)$$

where

$$\pi_1 = \frac{\sigma_\varepsilon^2 \sigma_a^2}{\sigma_\varepsilon^2 \sigma_a^2 + \sigma_\eta^2 \sigma_a^2 + \sigma_\eta^2 \sigma_\varepsilon^2} \quad (51)$$

$$\pi_2 = \frac{\sigma_\eta^2 \sigma_a^2}{\sigma_\varepsilon^2 \sigma_a^2 + \sigma_\eta^2 \sigma_a^2 + \sigma_\eta^2 \sigma_\varepsilon^2}, \quad (52)$$

$$\pi_3 = \frac{\sigma_\eta^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 \sigma_a^2 + \sigma_\eta^2 \sigma_a^2 + \sigma_\eta^2 \sigma_\varepsilon^2}. \quad (53)$$

Notice that if $\sigma_a^2 \rightarrow \infty$ (i.e., the preference shock has a very large variance), then

$$\pi_1 \rightarrow \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} = 1 - \lambda, \quad \pi_2 \rightarrow \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} = \lambda \quad \text{and} \quad \pi_3 \rightarrow 0.$$

In other words, households have nothing to learn from the equilibrium price and the weights used for inferring the unobservable aggregate fundamental are the same as in Section 5. Notice also that away from this limiting case the following is always true

$$\lambda > \pi_2.$$

To solve for the equilibrium price we follow the same steps as in Appendix III. By guessing that $\tilde{E}_t p_{t+1} = \phi \pi_2 i_t$, we have

$$\begin{aligned} E_t^j p_{t+1} &= \frac{r}{1+r-\rho} E_t^j \theta_{t+1} - s + \frac{\phi \pi_2}{r} I, \\ \bar{E}_t p_{t+1} &= \frac{r}{1+r-\rho} \bar{E}_t \theta_{t+1} - s + \frac{\phi \pi_2}{r} I, \\ \tilde{E}_t p_{t+1} &= \frac{r}{1+r-\rho} \tilde{E}_t \theta_{t+1}. \end{aligned}$$

Using (50), the last two equations can be written as:

$$\begin{aligned} \bar{E}_t p_{t+1} &= -s + \phi \pi_1 \rho \theta_{t-1} + \phi \pi_2 \theta_t + \phi \pi_3 (q_t + s) + \frac{\phi \pi_2}{r} I, \\ \tilde{E}_t p_{t+1} &= \phi \pi_2 i_t, \end{aligned}$$

confirming the claim that $\tilde{E}_t p_{t+1} = \phi \pi_2 i_t$. Inserting $\bar{E}_t p_{t+1}$ and $\tilde{E}_t p_{t+1}$ in (16), the equilibrium price becomes

$$p_t = \frac{r}{1+r} (\theta_t + a_t - s) + \frac{1}{1+r} \left(-s + \phi \pi_1 \rho \theta_{t-1} + \phi \pi_2 \theta_t + \phi \pi_3 (q_t + s) + \frac{\phi \pi_2}{r} I \right) + \frac{\phi \pi_2}{1+r} i_t, \quad (54)$$

which using (15) gives,

$$\begin{aligned} p_t &= (\theta_t + a_t - s) + \frac{1}{1+r} ((\phi \rho \theta_{t-1} - \theta_t) - a_t) + \frac{\phi \pi_2}{1+r} (\theta_t - \rho \theta_{t-1}) + \frac{\phi \pi_2}{r(1+r)} I \\ &\quad + \frac{\phi \pi_2}{1+r} i_t + \frac{\phi \pi_3}{1+r} ((\theta_t - \rho \theta_{t-1}) + a_t) \end{aligned}$$

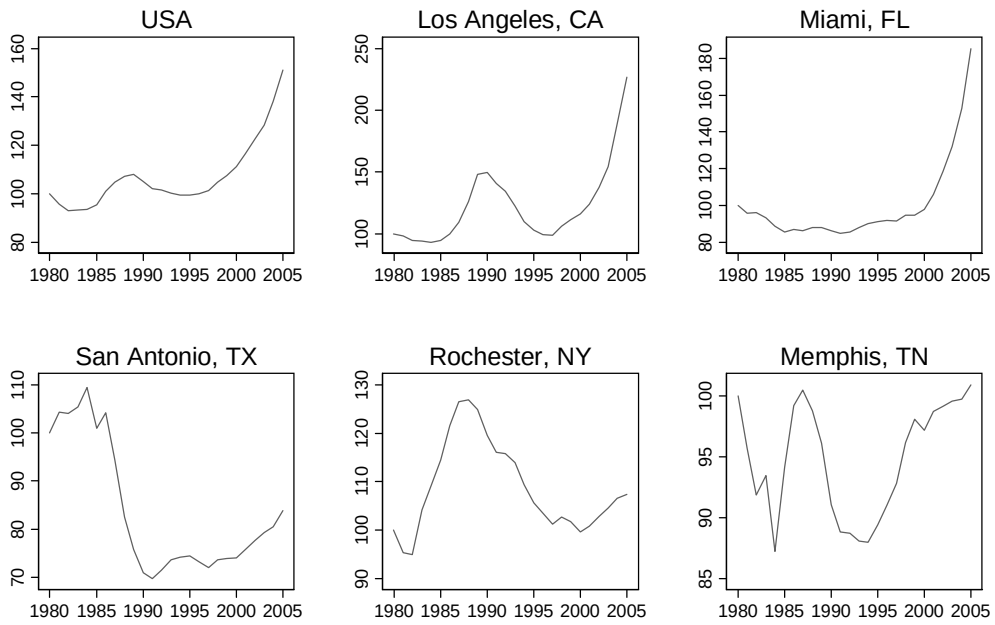
or

$$p_t = p_t^* + \pi_2 \Upsilon_t + \pi_3 \Phi_t$$

where

$$\Phi_t \equiv \frac{\phi}{1+r} ((\theta_t - \rho \theta_{t-1}) + a_t).$$

Figure 1: Real House Price Index in the US



Source: OFHEO and BLS

Figure 2: House price volatility and volatility of dispersion in beliefs in 321 MSAs

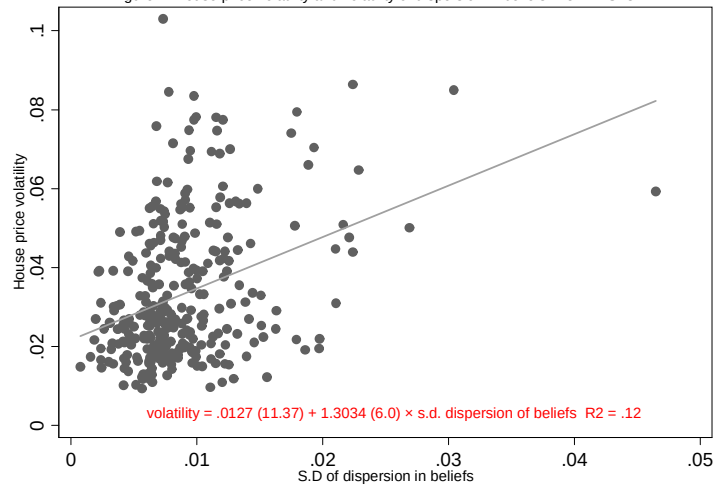


Figure 2: Equilibrium price and the preference noise σ_μ

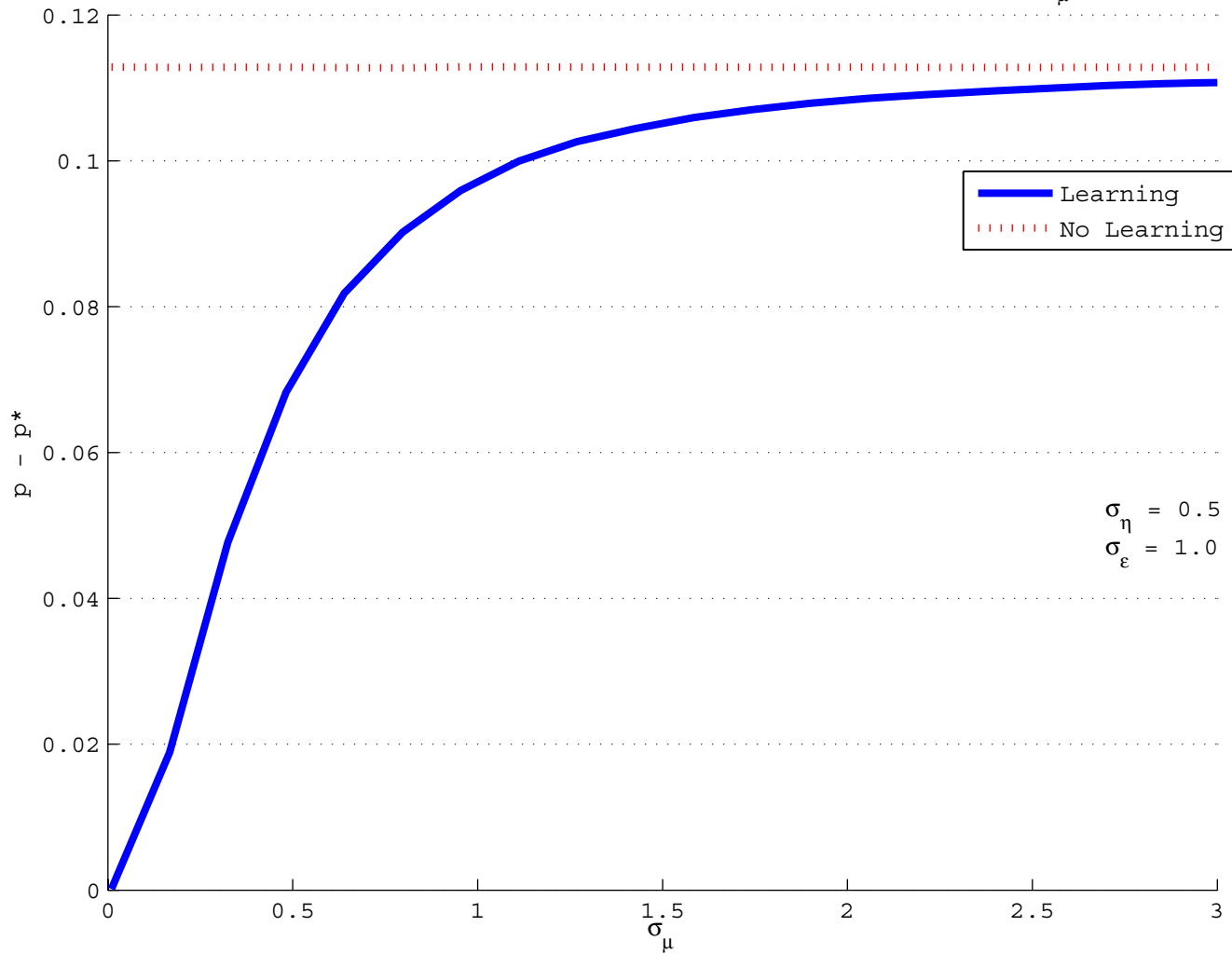


Figure 3: Equilibrium price and the idiosyncratic shock σ_ε

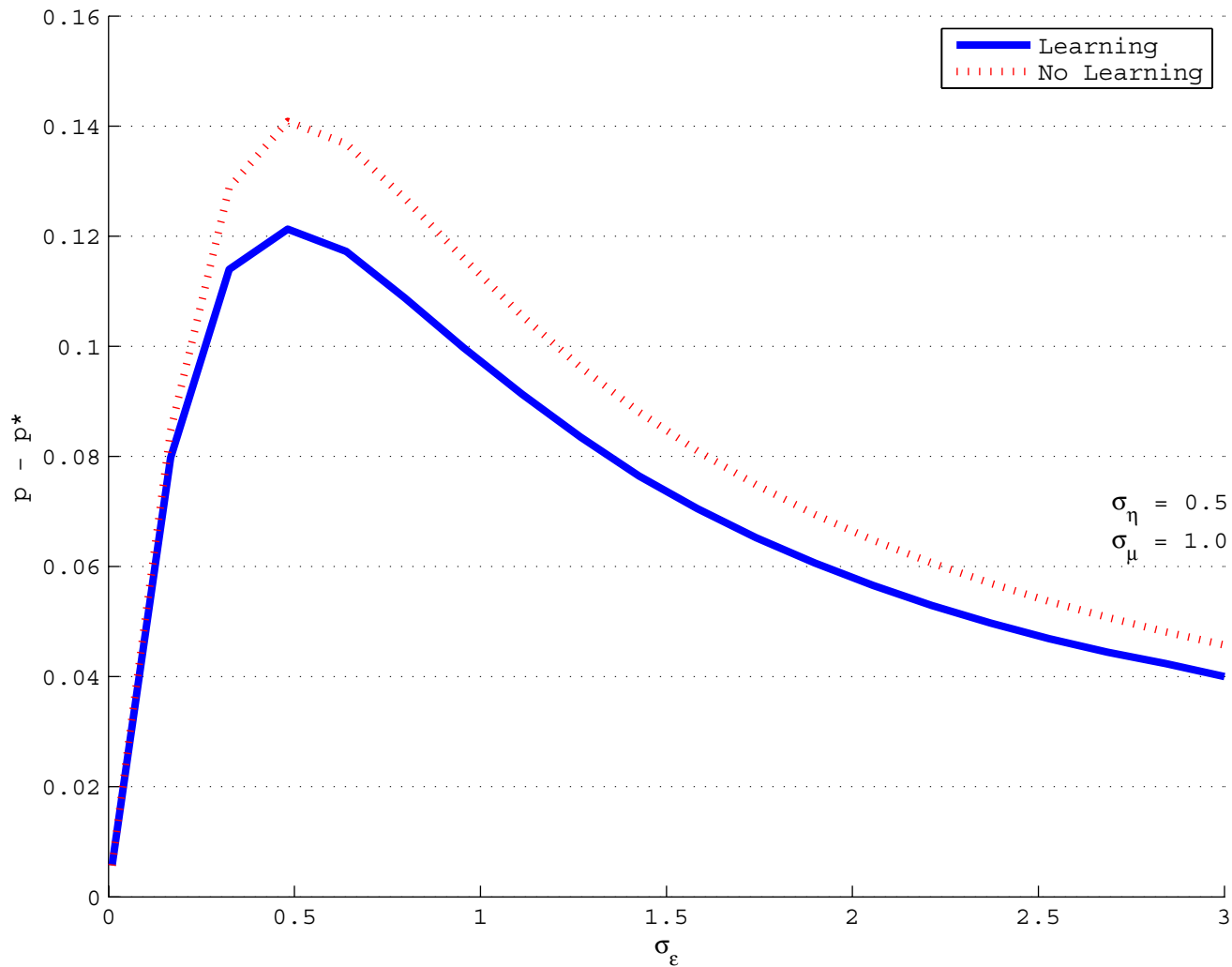


Table 1

Summary Statistics, 381 MSAs, 1980-2000

	Description	Mean	Standard Deviation	Minimum	Maximum
ΔP_t	Change in log of real house price	.004021	.044600	-.297416	.233374
$\Delta \theta_t$	Change in log of real per capita personal income	.014956	.025590	-.168891	.179146
ΔPop_t	Change in log of population	.011968	.015082	-.110857	.133832
Disp_t	Proxy for dispersion of beliefs	.025007	.012729	.003807	.196372
Pos_t	Positive shocks to Disp_t	.012200	.010211	.000029	.192546
Neg_t	Negative shocks to Disp_t	-.011717	.008969	-.091725	-.000014

Statistics shown are for annual observations pooled across MSAs. The house price data comes from the Office of Federal Housing Enterprise Oversight. Income and Population data are from the Regional Economic Accounts of the Bureau of Economic Analysis. Variables Disp_t , Pos_t and Neg_t are computed as described in Section?? using (per employed) earnings data for one-digit industries from the Regional Economic Accounts of the Bureau of Economic Analysis. The deflator for nominal variables is the aggregate CPI index from the Bureau of Labor Statistics.

Table 2

Effect of dispersion of beliefs on house prices growth

Dependent variable: ΔP_t --- Sample: 1980-2000		
Independent Variable		
$\Delta \theta_t$	0.426*** (0.041)	0.492*** (0.057)
$\Delta \theta_{t-1}$	0.639*** (0.044)	0.701*** (0.055)
Disp_t		0.199** (0.092)
Year effects	Yes	Yes
MSA effect	Yes	Yes
Obs	5393	3285
N. Units	350	306
R2 within	0.240	0.251
R2 overall	0.242	0.268

This table presents OLS estimates of the log change of the real house prices, ΔP_t on the independent variables shown. Standard errors (which appear below coefficients in parenthesis) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level. $\Delta \theta_t$ is the log change in real per capita personal income. $\Delta \theta_{t-1}$ is the lagged value of $\Delta \theta_t$. Disp_t is our proxy of dispersion of beliefs which is calculated as explained in Section ??.

Table 3

Effect of dispersion of beliefs on house price growth

Independent Variable	Dependent Variable: ΔP_t --- Sample: 1980-2000					
	(1)	(2)	(3)	(4)	(5)	(6)
ΔP_{t-1}	0.511*** (0.014)	0.494*** (0.017)	0.540*** (0.024)	0.565*** (0.023)	0.373*** (0.025)	0.442*** (0.030)
ΔP_{t-2}	0.226*** (0.034)	0.280*** (0.037)			0.279*** (0.031)	
ΔP_{t-3}	-0.101*** (0.013)	-0.110*** (0.016)			-0.111*** (0.014)	
$\Delta \theta_t$	0.304*** (0.028)	0.338*** (0.038)	0.273*** (0.029)	0.285*** (0.038)	0.375*** (0.035)	0.347*** (0.037)
$\Delta \theta_{t-1}$	0.336*** (0.035)	0.366*** (0.047)			0.212*** (0.037)	
$(P/\theta)_{t-1}$			-0.158*** (0.006)	-0.165*** (0.008)		-0.141*** (0.007)
ΔPop_t					1.353*** (0.123)	1.197*** (0.103)
Disp_t		0.140** (0.068)		0.209*** (0.060)	0.133** (0.062)	0.154*** (0.057)
Year effects	Yes	Yes	Yes	Yes	Yes	Yes
MSA effect	Yes	Yes	Yes	Yes	Yes	Yes
Obs	4466	2616	5166	3127	2616	3127
N. Units	350	289	350	302	289	302
R2 within	0.553	0.565	0.521	0.541	0.618	0.586
R2 overall	0.583	0.603	0.386	0.419	0.533	0.452

This table presents OLS estimates of the log change of the real house prices, ΔP_t , on the independent variables shown. Standard errors (which appear below coefficients in parenthesis) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level. ΔP_{t-1} is the lagged value of the log change in house price. $\Delta \theta_t$ is the log change in real per capita personal income. $(P/\theta)_{t-1}$ is the one period lagged log-ratio of the house price to per-capita income. ΔPop_t is the log difference of MSA population. Disp_t is our proxy of dispersion of beliefs which is calculated as explained in Section ??.

Table 4

Effect of positive and negative news shocks on house price growth

Independent variables	Dependent variable: ΔP_t --- Sample: 1980-2000			
	(1)	(2)	(3)	(4)
ΔP_{t-1}	0.505*** (0.015)	0.404*** (0.020)	0.532*** (0.024)	0.431*** (0.029)
ΔP_{t-2}	0.232*** (0.035)	0.232*** (0.029)		
ΔP_{t-3}	-0.105*** (0.013)	-0.101*** (0.011)		
$\Delta \theta_t$	0.297*** (0.028)	0.332*** (0.027)	0.271*** (0.029)	0.317*** (0.029)
$\Delta \theta_{t-1}$	0.342*** (0.034)	0.218*** (0.029)		
$(P/\theta)_{t-1}$			-0.161*** (0.006)	-0.142*** (0.006)
ΔPop_t		1.194*** (0.100)		1.085*** (0.090)
Pos_t	0.229*** (0.071)	0.147** (0.064)	0.310*** (0.053)	0.207*** (0.055)
Neg_t	0.058 (0.061)	-0.045 (0.057)	0.086 (0.058)	0.004 (0.055)
Year effects	Yes	Yes	Yes	Yes
MSA effect	Yes	Yes	Yes	Yes
Obs	4391	4391	5076	5076
N. Units	350	350	350	350
R2 within	0.55	0.60	0.53	0.56
R2 overall	0.58	0.53	0.38	0.41

This table presents OLS estimates of the log change of the real house prices, ΔP_t , on the independent variables shown. Standard errors (which appear below coefficients in parenthesis) are clustered at the MSA level. *** denotes significance at 1% level, ** denotes significance at 5% level. ΔP_{t-1} is the lagged value of the log change in house price. $\Delta \theta_t$ is the log change in real per capita personal income. $(P/\theta)_{t-1}$ is the one period lagged log-ratio of the house price to per-capita income. ΔPop_t is the log difference of MSA population. Pos_t and Neg_t refer to positive and negative “news” shocks, which are calculated as explained in Section (??).