

News and knowledge capital

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Abstract

We explore the ability of a model with knowledge capital to generate expectations-driven business cycles. Knowledge capital is an input in production which is endogenously produced through a learning-by-doing process. We show that a standard real business cycle model augmented with only a learning-by-doing technology can exhibit an expectations-driven business cycle in response to news about a future change in total factor productivity. News about future productivity immediately increases the value of knowledge. This induces agents to accumulate knowledge now by working harder. The ensuing expansion of output is sufficient that both current consumption and investment can increase above steady state levels despite the absence of any contemporaneous productivity shock. Moreover, if knowledge capital is accumulated by firms the boom in real variables is accompanied by an appreciation in the price of equity shares, a feature that has empirical support.

KEYWORDS: expectations-driven business cycle, Pigou cycle, news shock, learning-by-doing, asset pricing

JEL CLASSIFICATION: E3

1 Introduction

A number of recent studies have attempted to develop models capable of generating expectations-driven business cycles. A key aspect of these cycles is that a boom is created in anticipation of future increases in productivity as opposed to the typical real business cycle model where the boom is driven by a contemporaneous rise in productivity¹. Subsequent busts may follow if the anticipated increases in productivity are not fully realized. As discussed by Beaudry and Portier ([4]), Jaimovich and Rebelo ([17]) and others, the typical business cycle model is unable to deliver booms in which consumption, investment and hours all rise along with output in the periods after the news arrives but before the shock to productivity actually occurs².

In this paper we offer a simple variant of a standard business cycle model that generates the aforementioned co-movement through an intuitive mechanism. The modification is an environment in which agents' actions endogenously create productivity-increasing knowledge through a learning-by-doing (LBD) process. The idea is simple: news of impending total factor productivity (TFP) increases the value of knowledge capital which is an input into the production technology along with labour and physical capital. This creates an incentive for the agents to increase the use of labour and accumulate more knowledge capital. The increase in knowledge capital induces investment expenditure by raising the productivity of physical capital thus generating a boom which exhibits co-movement.

An important feature of these booms, often discussed in the popular press and highlighted in Beaudry and Portier ([5]), is that stock prices respond in advance of the increase in TFP. To capture this feature we model knowledge capital as being accumulated by firms as opposed to workers. This implies that when news about future increases in TFP raises the value of knowledge capital, it also raises the value of the firm, leading to an appreciation in its share price. Moreover, since the learning-by-doing mechanism in our model resides on the production side of the economy, the model is able to generate an expectational-boom over a range of preference specifications while still producing pro-cyclical asset prices. In addition, factor prices are pro-cyclical because a rise in knowledge capital raises the marginal productivity of both labour and capital.

Our work builds on recent business cycle models by Chang et al ([8]) and Cooper and Johri ([10]) that incorporate various forms of learning-by-doing

¹Evidence in favour of these boom-bust cycles is provided in Beaudry and Portier ([4]).

²This is closely related to the analysis of Barro and King's ([3]) which showed that consumption and hours-worked will negatively co-move for shocks other than contemporaneous productivity shocks.

into dynamic general equilibrium models and show that they can be an effective propagation mechanism for shocks. While details differ, the two models share the feature that knowledge capital accumulation is a by-product of production activity. Both papers offer aggregate evidence in favour of learning-by-doing and build on an extensive empirical literature which documents the existence of learning effects in all sectors of the economy. Recent studies include Bahk and Gort ([2]), Irwin and Klenow ([16]), Jarmin ([18]), Benkard ([7]), Thornton & Thompson ([27]). The specification used in this paper follows that of Chang et al ([8]) where learning occurs as a by-product of past hours-worked. While many other specifications are possible, this one has the advantage of simplicity while still delivering the result ³.

There is a small but growing literature on expectations driven business cycles. Beaudry and Portier ([4]) consider a model with a durable and non-durable good that are produced in two distinct sectors. A complementarity between the two allows both consumption and investment to rise in response to news about a productivity increase in the non-durable goods sector. Jaimovich and Rebelo ([17]) propose preferences that reduce or eliminate the strong wealth effect on leisure of an expected future increase in TFP or investment specific technical change. They demonstrate that when combined with capital utilization and a “flow specification” of investment adjustment costs based on Christiano et al ([11]), the model produces a strong expectational-boom. Christiano et al ([12]) show that the combination of the flow specification of investment adjustment costs with habit formation in consumption produces an expectational-boom, however, they find that the model requires an implausible rise in the real interest rate and produces a counterfactual countercyclical asset price. They as result present a monetary version of the model with nominal wage rigidities and an inflation-targeting monetary authority that creates an expectational boom without as large a rise in the real interest rate. Den Haan and Kaltenbrunner ([13]) present a matching model whereby matching frictions induce firms to post more vacancies in response to news, leading to an increase in employment which allows aggregate consumption and employment to co-move. Schmitt-Grohe and Uribe ([26]) investigate the role of news shocks in generating economic fluctuations by performing a structural Bayesian estimation on a model featuring habit-formation in consumption and leisure, a flow-specification of investment adjustment costs, and capacity utilization. By allowing for both anticipated (news) and unanticipated components for various shocks, they are able to perform a variance decomposition to determine the relative contribution of

³Our work is also related to the ideas of human capital and organizational capital which have been explored in several studies, too numerous to cite

anticipated versus unanticipated shocks, and find that anticipated shocks to the permanent and temporary components of TFP account for more than two-thirds of aggregate fluctuations in U.S. postwar quarterly data.

In the remainder of the paper we proceed as follows. In section 2 we discuss an example economy based on Chang et al ([8]) that clearly demonstrates the ability of learning-by-doing to generate expectations driven cycles. Like Chang et al, this section treats knowledge capital as being symmetric with human capital because it is accumulated by the worker. In the next section we present a model of firm-level learning which is capable of displaying procyclical stock prices. Calibrating the model to US data implies a relatively small contribution of knowledge capital to firm output. As a result we augment firm technology with variable capacity utilization which magnifies the expectational-boom. We also discuss the impact of changing preferences on the results. The final section concludes.

2 An example

We begin with a simple example economy based on Chang et al that makes clear how the learning-by-doing mechanism allows co-movement of hours, investment and consumption to occur in response to news about a future rise in exogenous total factor productivity. Since this economy is taken more or less directly from Chang et al, we offer very little discussion of the modeling assumptions.⁴

The economy is populated by a large number of identical infinitely-lived households. The representative household has preferences defined over sequences of consumption C_t and leisure L_t with expected lifetime utility defined as

$$U = E_0 \sum_{t=0}^{\infty} \ln C_t + \chi L_t \quad (1)$$

where β is the representative household's subjective discount factor and where χ parameterizes the household's relative preference for leisure over consumption.

The representative household operates a production technology that produces output Y_t according to the technology

$$Y_t = A_t \tilde{N}_t^{\alpha} K_t^{1-\alpha}, \quad (2)$$

⁴While Chang et al present a decentralized model, we focus on the associated planner's problem.

where A_t is the level of an exogenous stationary technology process, $\tilde{N}_t$ is effective labour, and K_t is physical capital which accumulates according to

$$K_{t+1} = (1 - \delta)K_t + I_t. \quad (3)$$

Effective labour is defined as

$$\tilde{N}_t = H_t N_t, \quad (4)$$

where N_t is hours-worked and H_t is the stock of knowledge capital which accumulates according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^\gamma N_t^{1-\gamma}. \quad (5)$$

The idea here is that the actual contribution of labour to production is a combination of raw labour and knowledge capital. The latter is acquired by households as a by-product of engaging in production and itself takes previous levels of knowledge capital as an input.

Combining (2) and (4) we then define

$$Y_t = A_t F(N_t, K_t, H_t) = A_t (H_t N_t)^\alpha K_t^{1-\alpha}. \quad (6)$$

The common exogenous total factor productivity process A_t evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \theta_{At}, \quad (7)$$

where $\rho_a < 1$ and θ_{At} is an exogenous period t innovation which we will define further below.

Each period, the representative household is endowed with one unit of time that can be allocated between leisure and hours-worked N_t according to

$$N_t + L_t = 1. \quad (8)$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \quad (9)$$

Letting λ_t and Υ_t be the period Lagrange multipliers on (9) and (5) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \quad (10)$$

$$u_L(C_t, L_t) = \lambda_t A_t F_{N_t} + \Upsilon_t \Psi_{N_t} \quad (11)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} [A_{t+1} F_{Kt+1} + 1 - \delta] \} \quad (12)$$

$$\Upsilon_t = \beta E_t \{ \lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1} \}. \quad (13)$$

where $F_{Nt} = A_t \frac{\partial F(N_t, K_t, H_t)}{\partial N_t}$, $\Psi_{Nt} = \frac{\partial \Psi(H_t, N_t)}{\partial N_t}$ etc. These first-order conditions differ from those of the standard RBC model only by the addition of a wedge in the hours first-order condition (11) and an Euler equation for knowledge capital (13).

To interpret these two equations, first define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out λ_t , we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1}] \right\} \quad (14)$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt+1} \quad (15)$$

Equation (14) shows that the value of the marginal unit of knowledge capital in terms of consumption is the stochastically-discounted future lifetime stream of additional output generated from the additional knowledge capital. Note that the two terms on the right hand side of the equation suggest that additional knowledge not only contributes to output but also raises the marginal effectiveness of each hour in the learning process. Recognizing the connection between hours worked and knowledge capital accumulation, the planner does not merely equate the household's marginal rate of substitution of consumption for leisure to the marginal product of labour as would occur in the standard model. Instead, in this model, we see from equation (15) that the planner equates the household's marginal rate of substitution to the sum the marginal product of labour and the marginal consumption value of the additional stock of knowledge generated by increasing hours-worked today. From the perspective of explaining how the model can generate an increase in hours in response to news about a future productivity shock, it is helpful to note that a change in the value of knowledge capital, q_{ht} , will act as a shift factor for the labour supply curve mapped out in hours-productivity space. This shift factor is missing in standard models.

Re-writing (15) as

$$\left[\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} - A_t F_{Nt} \right] \frac{1}{\Psi_{Nt+1}} = q_{ht} \quad (16)$$

shows that the households net cost of obtaining an additional unit of knowledge capital is the additional $\frac{1}{\Psi_N(H_t, N_t)}$ hours that it must work to obtain it multiplied by the excess of its marginal rate of substitution of consumption for leisure over the market compensation for that period. In this sense, the household “invests” in knowledge capital by working more than it would if it simply considered the additional current consumption that the extra work would bring in the present.

2.1 The impact of news shocks

In this section we explore how news of an impending rise in total factor productivity is received by the economy described above. We contrast this with the response of a similar economy without the learning technology. Our representation of news shocks is standard and follows Christiano et al ([12]). We provide for news about A_t by defining the innovation θ_{At} in equation (7) as

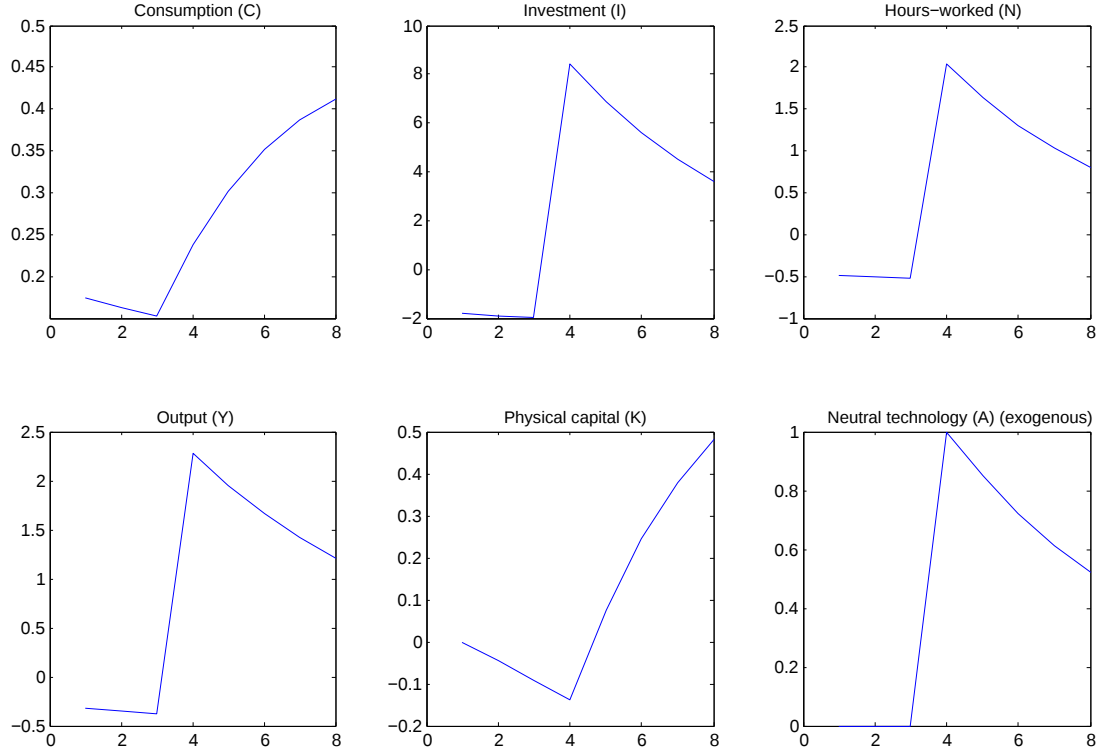
$$\theta_{At} = \epsilon_{A,t-p}^p + \varepsilon_{A,t}, \quad (17)$$

where ϵ_{At}^p is a news shock that agents receive in period t about the innovation θ_{At+p} , and $\varepsilon_{A,t}$ is an unanticipated contemporaneous shock to θ_{At} . The news shock ϵ_{At}^p has properties $E\epsilon_{At}^p = 0$ and standard deviation $\sigma_{\epsilon_A^p}$, and the contemporaneous shock $\varepsilon_{A,t}$ has properties $E\varepsilon_{A,t} = 0$ and standard deviation σ_{ε_x} . The shocks ϵ_{At}^p and $\varepsilon_{A,t}$ are uncorrelated over time and with each other.

Figure 1 shows the response of our benchmark standard RBC model to news in period 1 that a temporary but persistent increase in productivity will occur in period 4, represented by $\varepsilon_{A,t} = 0$ so that $\theta_{At} = \epsilon_{A,t-p}^p$. This and the next figure for the LBD model are based on the news turning out to be correct, ie., productivity actually does rise in period 4.⁵ The figure clearly illustrates the difficulties in generating co-movement in response to news shocks. In period 1 consumption rises but investment and hours-worked decrease below steady state. Thereafter, these three variables slope downwards slightly before reacting positively in the usual manner to the contemporaneous productivity shock in period 4. In response to the news, the wealth effect of the expected increase in future productivity causes households to increase consumption in the initial period, driving down the marginal utility of consumption, and producing a corresponding wealth effect on leisure through the hours first-order condition, causing households to reduce hours-worked.

⁵The parameterization behind these impulse responses is relatively standard: $\beta = 0.99$, $\alpha = 0.67$, $\gamma = 0.8$, $\delta = 0.025$, $\rho_A = 0.85$ and $p = 3$. A fully calibrated model will be presented in the next section.

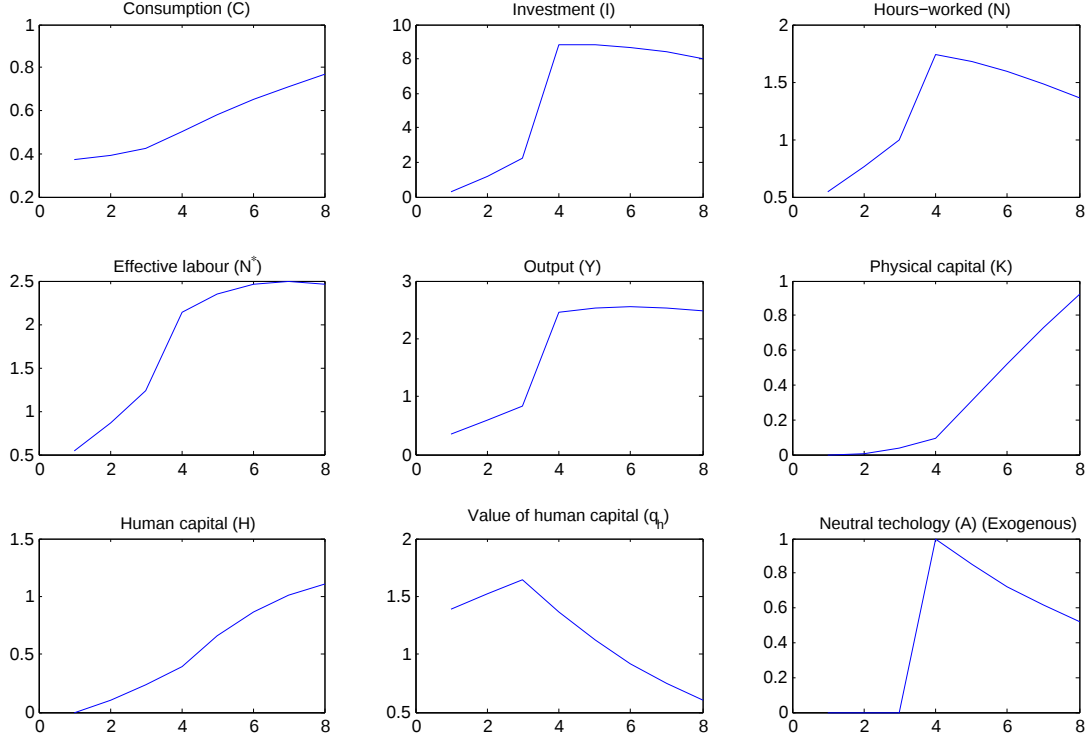
Figure 1: **Standard RBC** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4



With the capital stock fixed in the initial period, the reduction in hours-worked reduces production, and therefore households “fund” the increase in consumption through a decrease in investment.

Figure 2 shows the response of the LBD model to the same news shock. In sharp contrast to the previous figure, hours-worked, investment, consumption and output all increase and slope upwards in response to the news shock. Note in particular the sharp increase in the value of knowledge capital, q_{ht} , in response to the news about future productivity shocks. This increase in the value of knowledge capital, on its own, induces the agent to work harder. The aforementioned wealth effect on leisure is still in operation but is trumped by the desire to learn. The increase in hours worked leads to a rise in output in the current period as well as an increase in knowledge capital in the subsequent period. Anticipating this, agents realize that the productivity of capital will rise in the next period too and therefore are induced to increase investment. The increase in output allows agents to simultaneously satisfy

Figure 2: **Household knowledge** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4



their desire for more consumption and investment.

What causes q_{ht} to rise in period 1? To understand this, we have to work backwards from period 4. In that period TFP actually rises and raises the marginal productivity of labour services. While hours can respond immediately, knowledge capital cannot, being a state variable. As a result, to take advantage of the productivity shock, knowledge capital must be accumulated in advance by raising hours worked. The euler equation for knowledge capital shows that, *ceteris paribus*, an increase in A will raise the shadow value of h in the previous period. Tracing this back through the Euler equation for knowledge capital (13) then means that in period 1 Υ_t will increase. The plot of $q_h = \frac{\Upsilon}{\lambda}$ in Figure 2 demonstrates how the value of knowledge capital rises slowly and peaks in period three due to the curvature in the learning by doing function as well as the desire to smooth leisure.

While the results of this section illustrate the manner in which learning-by-doing is able to generate expectations driven business cycles, we think

important characteristics of expectational booms cannot be explained by it. The most important of these is the co-movement of firm equity share prices. Often discussion of booms in the media do not distinguish between increases in the value of financial assets and in real quantities that macroeconomists tend to focus on. To an extent this could be because both tend to rise together in these boom periods. Beaudry and Portier ([5]) show that for the US, “news”, as captured by innovations in their VAR results, lead to immediate increases in stock market values which are subsequently followed by increases in TFP. It would appear that stocks rise in anticipation of future increases in profits due to the increase in TFP. Our knowledge capital model has similar features. News about impending increases in TFP leads to an increase in the value of knowledge capital. If knowledge capital were accumulated by firms, this rise in value would also raise the value of the firms themselves. Share prices would rise in anticipation of the extra profits to be generated in the future. Interestingly, this suggests the learning mechanism can simultaneously explain not only the co-movement in real quantities like hours worked and investment, but also the increase in asset values.

This concept of firm-value as a function of firm knowledge is consistent with the idea of firm value in the organizational capital literature where organizational capital is typically viewed as an unobserved input into production. For example, Prescott and Visscher ([23]) refer to information accumulation within the firm as an explanation for the firm’s existence. This information affects its production possibilities set, and thus acts as an asset for the firm which gives it value. Our interpretation of this value is similar: knowledge capital is productivity-enhancing, allowing firms to produce additional output for given levels of labour and capital without having to pay out additional rents in the future, creating a stream of profits which provide value to the firm. It differs from many models of organizational capital such as Atkeson and Kehoe ([1]) where the evolution of organizational capital is exogenous and not controlled by the firm.

3 An economy with firm-specific capital

We now present our full model where knowledge is accumulated by firms as opposed to by workers as in the example economy. This will imply that firms will increase labour demand in response to the news, as opposed to workers increasing labour supply ⁶.

⁶It is entirely possible that both mechanisms are present in the data, but we explore only the former for clarity.

The economy consists of a large number of identical households and a single competitive firm. In addition to the standard market assumptions, we assume the existence of a stock market where households can buy and trade equity shares in the firm that represent claims to the firm's future profits.

3.1 Household

The household side of the model is relatively standard so we discuss it briefly. The representative household has preferences defined over sequences of consumption c_t and leisure l_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad (18)$$

where β is defined as in our example economy.

Each period, the representative household receives wage rate w_t for supplying hours-worked n_t , rental rate r_t for supplying capital services $\tilde{k}_t$, and dividend income d_t for each unit of its outstanding holdings of firm equity a_t . For convenience, we normalize the firm's outstanding number of shares to unity, and thus the household trades fractions of the firm's single equity share. The household allocates its earnings between consumption, investment in physical capital and equity shares. The period t household's budget constraint is given by

$$c_t + i_t + v_t a_{t+1} = w_t n_t + r_t \tilde{k}_t + [v_t + d_t] a_t, \quad (19)$$

where c_t is consumption, i_t investment in physical capital and v_t the price of equity. Capital services are defined as

$$\tilde{k}_t = u_t k_t, \quad (20)$$

where k_t is the household's stock of physical capital and u_t is the utilization rate of that capital. The household's physical capital evolves according to

$$k_{t+1} = [1 - \delta(u_t)] k_t + i_t \quad (21)$$

where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The household's problem is to choose sequences c_t , n_t , u_t , k_{t+1} and a_{t+1} to maximize (18) subject to (19), (20) and (21), yielding the standard first-order conditions

$$u_c(c_t, l_t) = \lambda_t \quad (22)$$

$$u_t(c_t, l_t) = \lambda_t w_t \quad (23)$$

$$\delta'(u_t) = r_t \quad (24)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} [r_{t+1} u_{t+1} + 1 - \delta(u_{t+1})] \} \quad (25)$$

$$v_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [v_{t+1} + d_{t+1}] \right\} = \sum_{s=1}^{\infty} E_t \left\{ \beta^s \frac{\lambda_{t+s}}{\lambda_t} d_{t+s} \right\}. \quad (26)$$

where it is clear from (26) that as usual the price of the firm's share v_t will equal the stochastically-discounted lifetime stream of the firm's profits beginning in period $t + 1$.

3.2 Firm

Since we will impose constant returns to scale in production to labour, capital services and knowledge capital, it will be convenient to assume that production occurs at a single firm that nonetheless behaves competitively and takes factor prices as given. This assumption has the advantage of suppressing notation associated with shares belonging to different firms.

The firm produces output according to

$$y_t = A_t F(n_t, \tilde{k}_t, h_t) = A_t n_t^\alpha \tilde{k}_t^\theta h_t^{1-\alpha-\theta}, \quad (27)$$

where A_t is aggregate exogenous neutral productivity defined as in (7) and h_t is the firm's stock of firm-specific knowledge capital.

The firm's knowledge capital evolves as in our example economy as

$$h_{t+1} = \Psi(h_t, n_t) = h_t^\gamma n_t^{1-\gamma}. \quad (28)$$

Each period, the firm pays out a dividend d_t to shareholders defined as

$$d_{it} = y_t - w_t n_t - r_t \tilde{k}_t. \quad (29)$$

Since the firm accumulates firm-specific knowledge capital through an internal learning-by-doing process, it faces the dynamic problem of choosing sequences of n_t , $\tilde{k}_t$ and h_{t+1} to maximize current and expected future lifetime dividends

$$V_t = d_t + E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \{d_{t+s}\} = d_t + \bar{V}_t \quad (30)$$

subject to (27), (28), and (29), where the term $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the household's stochastic discount rate for period $t + s$, and where we have defined $\bar{V}_t = E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \{d_{t+s}\}$ as the end-of-period discounted value of the firm's future lifetime stream of profits. Letting q_t be the Lagrange multiplier associated with (28), and making the appropriate substitutions, the firm's first-order conditions are then

$$w_t = A_t F_{nt} + q_t \Psi_{nt} \quad (31)$$

$$r_t = A_t F_{kt} \quad (32)$$

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [A_t F_{ht+1} + q_{t+1} \Psi_{ht+1}] \right\}, \quad (33)$$

where analogous to our example economy, the firm's knowledge capital first order condition (33) shows that the value in terms of profits of an additional unit of firm-specific knowledge is the stochastically discounted future lifetime stream of additional output created by that additional knowledge. As such, the firm's hours first-order condition now shows that in determining its optimal use of labour, the firm considers both direct marginal productivity of that labour in current production plus the value in terms of profits of the additional future lifetime output brought about by increasing its stock of firm-specific knowledge from hiring more labour hours today.

Note that once the firm has created the extra unit of knowledge capital, its contribution to additional output each period thereafter, as given by $A_t F_{ht+1}$ in (33), represents a stream of pure profits for the firm over the life of the knowledge capital. This occurs because once it is created, it is held costlessly by the firm without any rent payments. To see this, combine (29), (31), (32) and (33) along with the specific functional forms of $F(\cdot)$ and $\Psi(\cdot)$ to give

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{d_{t+1}}{h_{t+1}} + q_{t+1} \frac{h_{t+2}}{h_{t+1}} \right] \right\}, \quad (34)$$

which shows that the marginal value of an additional unit of knowledge capital is the additional profit created by the extra unit (which happens to also equal the average profit per unit).

The extent to which the second term in (31) influences the firm's labour decision will be thus affected by the current value of marginal firm-specific knowledge, q_t . When the value of knowledge is greater than zero, the firm will wish to use labour at a level in excess of that without internal learning.

From the lens of the standard firm's problem (where firms hire labour up to the point where the marginal product of labour equals the wage rate), it appears as if the firm is hiring "too much" labour because the marginal product is below the current wage rate. In fact, the firm is trading off lower current profit for higher future profit by "investing" in additional knowledge. This investment in knowledge capital responds to q_t , the value of knowledge. Like the example economy, if news about future TFP increases q_t the firm will respond by attempting to hire more labour. This in turn will raise the value of the firm and its shares. We show in the appendix that the end-of-period t value of the firm and therefore price of equity can be expressed as

$$\bar{V}_t = q_t h_{t+1}, \quad (35)$$

which shows that the value of the firm is determined by the total value of its existing stock of knowledge, obtained as a product of the marginal value of firm-specific knowledge and the stock of firm-specific knowledge.

3.3 Equilibrium

Equilibrium in this economy is defined by a contingent sequence of allocations C_t , N_t , H_{t+1} , I_t , K_{t+1} , and prices w_t , r_t and v_t that satisfy the following conditions: (i) the allocations solve the household's problem taking prices as given, (ii) the allocations solve the firm's problem taking prices as given, (iii) the equity market clears, $a_t = 1$ (iv) the labour market clears, (vi) the capital services market clears (vi) the consistency of individual and aggregate quantities, $c_t = C_t$, $i_t = I_t$, $y_t = Y_t$, $n_t = N_t$, $l_t = L_t$, $k_t = K_t$, $\tilde{k}_t = \tilde{K}_t$, $h_t = H_t$, (viii) the resource constraint $C_t + I_t = Y_t$ holds.

Finally, we note that (26) and (30) imply that $v_t = V_t$.

3.4 Solution Method and Calibration

In order to solve the model it is convenient to work with the associated central planner's version given in the appendix. The model is solved by linearizing the model equations around the steady-state and then using the singular linear difference system reduction method of King and Watson [21]. The solution method requires values to be assigned to the parameters of the model. We set N_{SS} (hours in steady state) = 0.2, $\beta = 0.99$ and $\delta = 0.025$.

For the learning-by-doing process, we set $\gamma = 0.8$, which is based on estimates obtained by Chang et al ([8]). The calibration of the parameters in the production technology requires some explanation. We start by imposing constant returns to N_t , $\tilde{K}_t$ and H_t in the production function. Next we

require the model deliver a steady-state labour share, $S_N = 0.67$, which in this model is given by

$$S_n = \alpha + \left(\frac{(1 - \gamma)}{\xi + (1 - \gamma)} \right) [1 - \alpha - \theta], \quad (36)$$

where $\xi = 1/\beta - 1$. Given ξ and γ , multiple combinations of α and θ can satisfy a given S_n , and so we must also tie down one of these. We choose $1 - \alpha - \theta = 0.15$, which is approximately the midpoint of the range of 0.08-0.26 estimated in Cooper and Johri ([10]). This value is equivalent to a learning rate of around ten percent which is half of that typically estimated in the learning literature. This is also the value of the contribution of organizational capital used in Atkeson and Kehoe. Using this value in (36) then yields $\alpha = 0.527$ and from our constant returns assumption, $\theta = 0.323$. We then round both of these to two decimal places which yields a labour share of $S_n = 0.672$.

The parameterization of the learning by doing technology has implications for steady state profit. We show in the appendix that with constant returns in both $F(\cdot)$ and $\Psi(\cdot)$ the share of profit is very small but positive and is given by

$$\frac{d}{y} = \left(\frac{\xi}{\xi + (1 - \gamma)} \right) [1 - \alpha - \theta], \quad (37)$$

where $\xi = 1/\beta - 1$ is the household's subjective discount rate, and γ is the parameter in the accumulation equation for knowledge capital.

For our parameterization, $\left(\frac{\xi}{\xi + (1 - \gamma)} \right) \approx 0.048$ yielding $\frac{d}{y} \approx 0.007$ which is indeed very small. It is conventional in DGE models to have a steady state profit share of zero. This could be achieved either by a small fixed cost or as we show in the appendix, by introducing a slight degree of increasing returns in either $F(\cdot)$ or $\Psi(\cdot)$, without materially altering the results.

For the exogenous technology shock process that includes news shocks, we set the persistence to $\rho_A = 0.85$, which is in the middle of the values of 0.83 and 0.89 estimated by Christiano et al ([12]) and Schmitt-Grohe and Uribe ([26]) respectively. Following the literature, we set $p = 3$, implying that in each period agents receive news about total factor productivity 3 periods in the future.

We set the elasticity of the marginal capital depreciation function $\epsilon_u = \frac{\delta''(u)}{\delta'(u)}u = 0.15$ which is at the lower end of the values considered by King and Rebelo ([20]) and the same as the value used by Jaimovich and Rebelo ([17]).

Since the learning-by-doing mechanism in this model is primarily a production-side mechanism, we explore the impact of three different forms of preferences on our results. These are:

1. Standard indivisible labour preferences separable in consumption and leisure with specification

$$u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{(L_t)^{1-\nu}}{1-\nu}, \quad (38)$$

with $\sigma = 1$ and $\nu = 0$, therefore implying log consumption and linear leisure per Hansen's ([15]) indivisible labour model.

2. Indivisible labour preferences not separable in consumption and leisure of the form used by King and Rebelo ([20]) in their application of Hansen and Rogerson's ([15]) generalization of indivisible labour to nonseparable preferences. These still fall within the general class of "KPR preferences" described in King, Plosser and Rebelo ([19]). With these preferences, the stand-in representative agent has the preference specification

$$u(C_t, L_t) = \frac{1}{1-\sigma} \left\{ C_t^{1-\sigma} v^*(L_t)^{1-\sigma} \right\} \quad (39)$$

where $v^*(L) = \left[\left(\frac{1-L_t}{H} \right) v_1^{\frac{1-\sigma}{\sigma}} + \left(1 - \frac{1-L_t}{H} \right) v_2^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$, and where H is the fixed shift length, and v_1 and v_2 are constants representing the leisure component of utility of the underlying employed group (who work H hours) and unemployed group (who work zero hours) respectively. We set $\sigma = 2$ in this case.

3. "JR preferences" of the form proposed by Jaimovich and Rebelo ([17]), with specification ⁷

$$u(C_t, L_t, X_t) = \frac{(C_t - \psi(1 - L_t)^\nu X_t)^{1-\sigma} - 1}{1-\sigma} \quad (40)$$

$$X_t = C_t^\zeta X_{t-1}^{1-\zeta} \quad (41)$$

and where we set $\sigma = 1$, $\nu = 1.16$ and $\zeta = 0.01$ based on Schmitt-Grohe and Uribe's ([26]) estimation of these preferences. As detailed in Jaimovich and Rebelo ([17]), the ζ parameter has the effect of parameterizing the wealth effect to leisure and nests both "GHH preferences" ($\zeta = 0$) proposed by Greenwood, Hercowitz and Huffman ([14]) and "KPR preferences", with lower ζ implying a lower wealth effect to leisure.

Table 1 summarizes our parameterization of the model.

⁷The inclusion of the term X_t introduces another state variable into our system (X_{t-1}) and another first-order condition for X_t to the household problem. See Jaimovich and Rebelo ([17]) for a complete discussion of these preferences.

Table 1: Firm knowledge capital interpretation - calibration

σ	ν	ζ	β	NSS	α	θ	ε	$\delta(u)$	ϵ_u	ρ_A	p	γ	η
<i>KPR separable indivisible labour preferences</i>													
1	0	n/a	0.99	0.2	0.53	0.32	0.15	0.025	0.15	0.85	3	0.8	0.2
<i>KPR nonseparable indivisible labour preferences</i>													
2	n/a	n/a	0.99	0.2	0.53	0.32	0.15	0.025	0.15	0.85	3	0.8	0.2
<i>JR preferences</i>													
1	1.16	0.01	0.99	0.2	0.53	0.32	0.15	0.025	0.15	0.85	3	0.8	0.2

4 Results

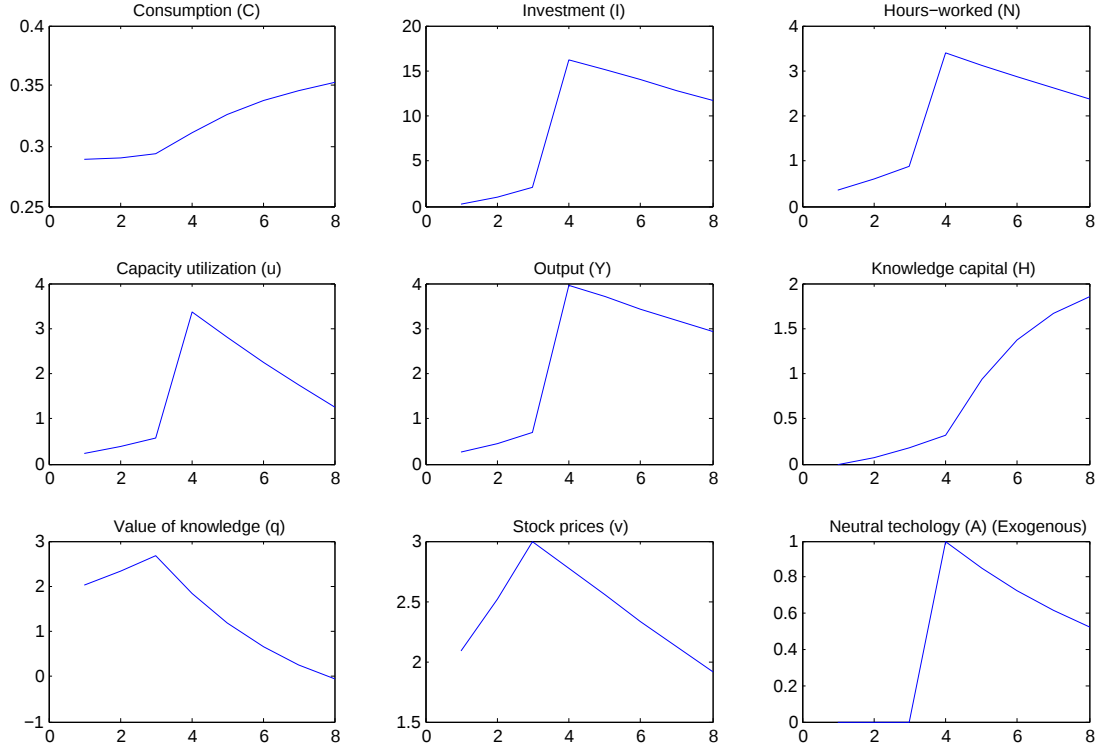
We begin this section with a discussion of how the economy described above would react to news of a one percent increase in TFP which would occur in period four. As discussed above, we present results for three different preferences. While all of these generate expectational booms with co-movement, the details differ, especially with regard to the relative increase of consumption versus investment.

Figure 3 shows the response of the economy using separable preferences. As expected from the results of section 1, consumption, investment, hours-worked and output all rise above steady state levels immediately. On receipt of the news, the firm realizes it needs to invest in accumulating more knowledge capital since its value has increased. This can be seen in the movement of q which rises two percent above steady state. A look back at the hours first order condition (31) shows that q acts as a shift factor for labour demand. The firm therefore hires more labour and increases production, leading an increase in H above steady state in period two. The rise in q and H together raise the value of shares as households anticipate an increased future flow of dividend payments. While H peaks in period ten, q peaks in period three, just before the shock is realized, as does the value of equity shares v .

4.1 The role of capacity utilization

Due to the assumption of constant returns to all factors in production, the contribution of knowledge capital to output is much smaller than was the case in the human capital example discussed earlier. As a result, without capacity utilization, the increase in hours-worked alone cannot raise output sufficiently to finance both an increase in consumption and investment in

Figure 3: **Firm-specific knowledge: separable indivisible pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *realized* in period 4

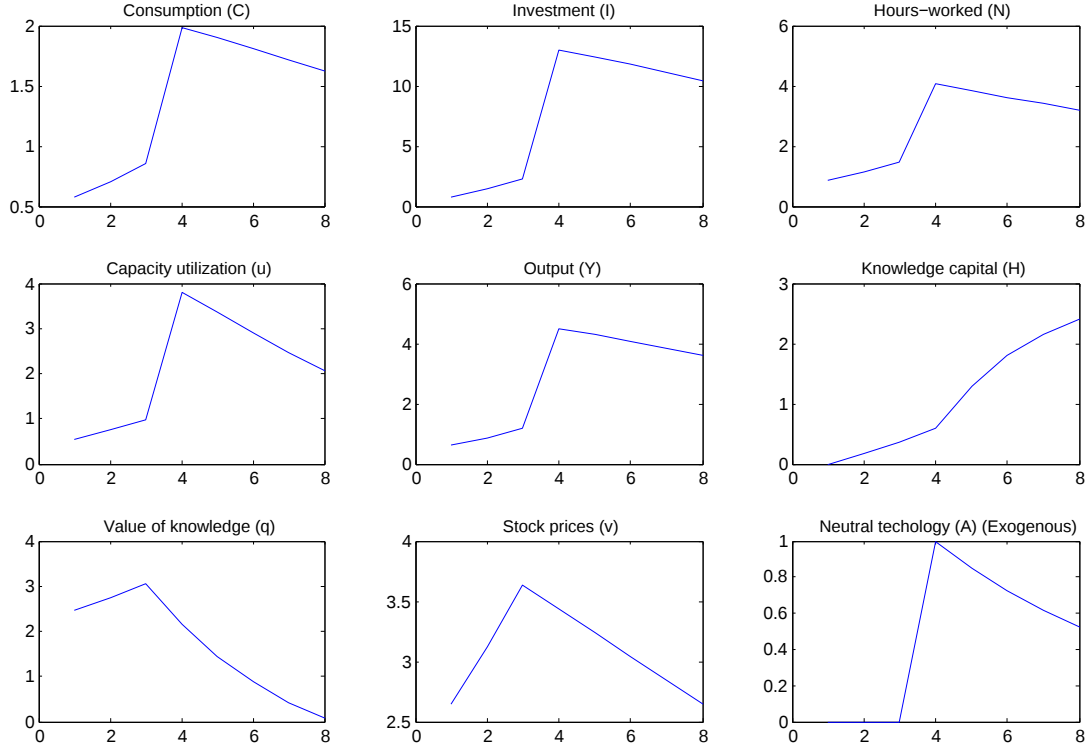


period 1. Adding capacity utilization to the model allows capital services to expand along with labour and therefore increase the responsiveness of output. The optimal determination of utilization can be seen by combining the household's first-order condition for capacity utilization (24) with the firm's first-order condition for capital services (32), and imposing equilibrium to give

$$\delta'(u_t)k_t = A_t F_{ut}. \quad (42)$$

Note that unlike models that include both capacity utilization and intertemporal adjustment costs to capital or investment, here there is no direct intertemporal link to the optimal level of utilization (through say changes in the relative price of investment or capital which would alter the cost of adjusting utilization), and thus utilization will simply respond to changes in its marginal product through the variation of the other factors of production, or

Figure 4: **Firm-specific knowledge: nonseparable indivisible pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *realized* in period 4

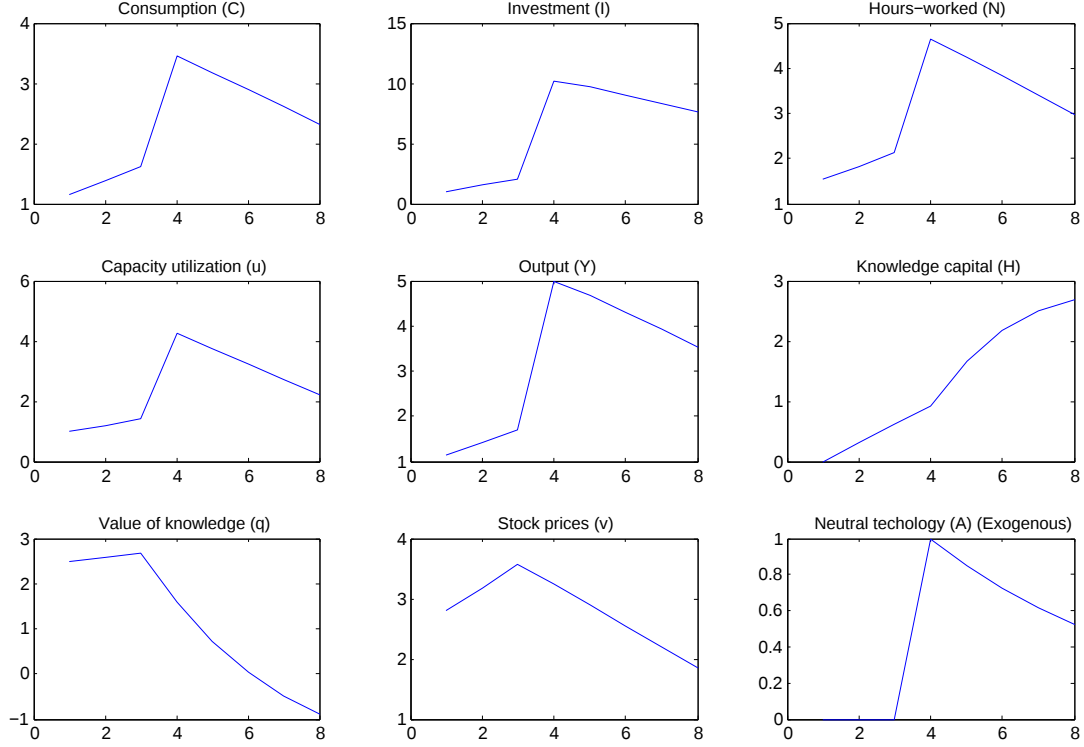


changes to the stock of capital. Thus the role of capacity utilization in the model is to simply amplify the boom.

However, while capacity utilization acts as a magnification device for the boom, it cannot deliver one in the absence of the learning-by-doing mechanism. As we showed with the standard RBC model, upon receiving news, hours-worked drops below steady state due to the wealth effect to leisure. Since the capacity utilization first-order condition (42) responds only to changes in its marginal product or the stock of capital, adding capacity utilization to the standard RBC actually worsens the model's response to news since the decrease in hours-worked induces a reduction in capacity utilization which further contracts output ⁸.

⁸Impulse response plots for this case are available from the authors

Figure 5: **Firm-specific knowledge: JR pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *realized* in period 4



4.2 The effect of varying preferences

We can increase the response of consumption and temper that of investment by altering preferences. Below we consider two possibilities. In figure 6 we show the case of indivisible labour with nonseparability in consumption and leisure. Figure (5) shows the response of preferences based on Jaimovich and Rebelo (2008). As both figures show, consumption now rises more both in response to the news and when the TFP shock hits, and investment responds less aggressively. Moreover, with both preferences specifications the model continues to deliver procyclical movement in the firm asset price v .

For the nonseparable indivisible labour preferences, as discussed by King and Rebelo([20]), when $\sigma > 1$, the combination of nonseparability of consumption and leisure and indivisibility in hours worked imply that the consumption of the employed group will exceed that of the unemployed group. An increase in total hours, which occurs along the extensive employment

margin, represents an increase in the number of individuals moving from unemployment to employment. Since the employed enjoy higher consumption levels, total consumption responds more than in the separable case.

The nonseparability in the Jaimovich and Rebelo preferences also boosts the response of consumption by making the marginal utility of consumption depend on labour. These preferences, however, also offer the benefit of being able to parameterize the wealth effect to leisure. With $\gamma = 1$ this wealth effect is very small, and as such labour-supply in our model will only respond to the contemporaneous wage. The firm's labour demand equation however remains unchanged by the addition of these preferences, and thus labour demand continues to respond intertemporally to the labour wedge driven by the value of knowledge. Unlike with our previous two sets of preferences, the wealth effect on leisure does not mitigate the shift in labour demand, and thus the response of hours is greater and the model is less dependent on the high-substitution elements discussed above to create an expectations-driven business cycle. Using their estimation results of a model with Jaimovich and Rebelo preferences along with capacity utilization and investment adjustment costs, Schmitt-Grohe and Uribe ([26]) note that while the model is able to improve upon features of their baseline model, it predicts a decline in the relative price of installed capital, and thus a decline in the value of the firm. Since the learning-by-doing mechanism in our model resides on the production side of the economy and is unaffected by preferences, we are able to avoid the issue of a counterfactual price of capital and continue to generate a procyclical asset value for the firm.

4.3 Stock prices and news

Thus far we have focussed on the ability of the model to generate stock prices that rise along with the boom in real variables as is observed in the data. However, as Beaudry and Portier ([5]) discuss, there is also a general notion that stock prices reflect changes in investors' beliefs about the future cash flow of firms. To the extent that future changes in TFP affect firms' future cash flows, then under this premise, stock prices should react immediately to news about these changes. To better understand how stock price movements are related to TFP, Beaudry and Portier ([5]) perform two distinct orthogonalization schemes on a bivariate VAR of TFP and stock prices, and using both schemes find strong support for the idea that permanent changes in productivity growth are preceded by stock market booms. We investigate this idea in the context of our model. We are particularly interested in exploring the possibility that a boom in stock prices can precede the rise in

TFP ⁹.

For this exercise we construct a TFP measure using a “Naive Solow Residual” approach that is ignorant of the presence of knowledge capital as an input into production. Beaudry and Portier ([5]) construct their baseline production function with constant returns to labour and capital and using average observed labour shares. Our Solow Residual similarly assumes constant returns to labour and capital (or capital services). As we show in the appendix, in steady state our production function with physical capital, labour and knowledge capital reduces down to $y = AF(n, k, h) = n^\alpha (\tilde{k})^\theta h^{1-\alpha-\theta} = An^{1-\theta} \tilde{k}^\theta = A\tilde{F}(n, \tilde{k})$, and thus we use a labour share of $1 - \theta$ in constructing our residual. Furthermore, we construct two versions of the residual - one that ignores capacity utilization and one that controls for it - to line up with Beaudry and Portier’s results. We define the former in log-linear terms as

$$\widehat{SR} = \hat{y}_t - (1 - \theta)\hat{n}_t - \theta\hat{k}_t \quad (43)$$

and the latter as

$$\widehat{SR} = \hat{y}_t - (1 - \theta)\hat{n}_t - \theta\hat{u}_t - \theta\hat{k}_t. \quad (44)$$

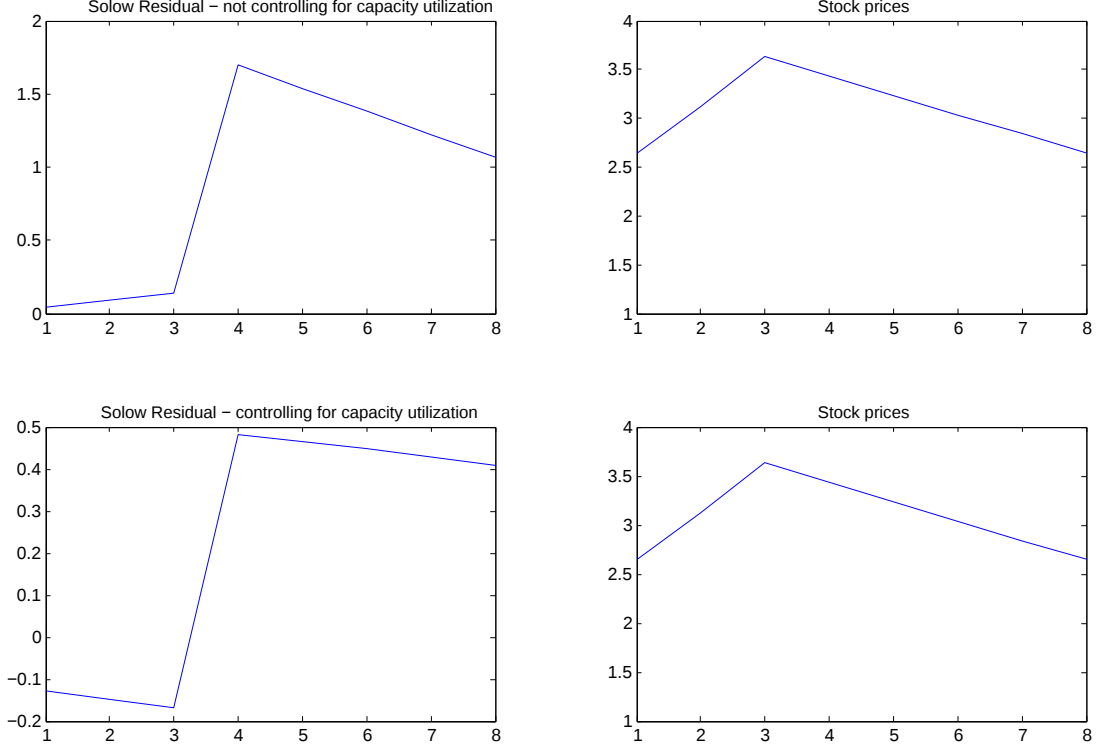
Figure 6 shows the impulse response of these measures, along with our stock price measure, to news about an increase in future productivity that is eventually realized. The figures show that our model predictions are generally consistent with the evidence of Beaudry and Portier. In line with their results, stock prices lead the rise in the Solow Residual and reach a local peak approximately around the time that both measures of the Solow Residual peak.

4.4 Robustness

In this section we explore the model’s sensitivity to key parameter values with regards to its ability to generate an expectations-driven business cycle. We maintain a strict definition of this type of cycle that in response to news, C , I and N must be at or above steady-state in period 1, and slope upwards beginning either in period 1 or thereafter. Table 2 shows the results of our robustness check for the three different preference specifications. Following the approach of Jaimovich and Rebelo ([17]), we vary only 1 parameter from each baseline parameterization and report the range of the parameterization over which the model can still exhibit an expectations-driven business cycle.

⁹Direct comparisons with Beaudry and Portier ([5]) are precluded by a number of crucial differences most prominently that they consider permanent changes to TFP while we model them as persistent but temporary.

Figure 6: **Firm-specific knowledge:** Response of Stock prices and “Naive” Solow Residual to news shock in period 1 about neutral tech. shock in period 4, tech. shock *realized* in period 4



It is clear that the parameter ranges are most limited for the KPR separable preferences, especially for the parameters ϵ_u and ν that provide for the high-substitution response. Allowing for nonseparability significantly expands the parameter range, especially by lowering the required $1 - \alpha - \theta$. We note that without adjustment costs to investment or capital, a typical parameterization for the first two KPR preference specifications that “fails” to exhibit an expectations-drive business cycle under this definition is often characterized by C and I that are above steady state and sloping upwards in accordance with our definition, but an I that begins below steady state - in some cases less than 0.01% below steady state - yet still sloping upwards and often above steady-state in periods 2 or 3, and giving the overall impression of a “boom”. While we could no doubt remedy this behaviour and expand our robustness set by adding a slight degree of traditional capital adjustment costs, for the purposes of illustrating our primary mechanisms we abstain from additional features and disqualify any parameterization that

Table 2: Robustness

<i>Preferences</i>	$(1 - \alpha - \theta)^a$	ϵ_u	γ^b	ν	ζ
KPP separable	0.14-0.52	0.01-0.16	0.63-0.85	0-0.26	n/a
KPR nonseparable	0.07-0.33	0.01-0.28	0.33-0.93	n/a	n/a
JR	0.01-0.25	0.01-0.70	0-0.98 ^c	1-1.58	0-0.15

^a θ is constant and therefore $\frac{K}{Y}$ is fixed throughout. Due to CRS in $F(\cdot)$, a change to $1 - \alpha - \epsilon$ implies a change to α . This alters the labour share but by no more than 0.02 over all parameter ranges in this column.

^bA change in *gamma* affects the labour share, but the change is less than 0.02 unless otherwise indicated.

^clabour share varies 0.679-0.630 over this range

causes either C , I or N to move even slightly below steady state.

Allowing for JR preferences expands the parameter range even further, especially for $1 - \alpha - \theta$, ϵ_u and ν because the absence of a strong wealth effect to leisure eliminates the need for as elastic a response of output. Even though JR preferences allow for very low learning rates as captured by $1 - \alpha - \theta < 0.05$, a $1 - \alpha - \theta < 0.05$ leads to small increases in variables above their steady state values implying a very low learning rate.

Across all the preference specifications it is clear that our model requires a high labour supply elasticity. Even with JR preferences with no wealth effect, our range of $\nu = 1 - 1.58$ is low compared to the range of robustness for the equivalent parameter found by Jaimovich and Rebelo ([17]) in their model. This is not surprising, however, given that the critical mechanism in our model requires a substantial increase in labour supply in response to the shift in labour demand induced by the news.

5 Conclusion

In this paper we highlight the role of knowledge capital in enabling the existence of expectations driven cycles. Knowledge capital refers to that knowledge which is accumulated by agents as a by-product of productive activities, a phenomenon that is often referred to as learning-by-doing. A key characteristic is that an increase in knowledge capital raises the productivity of other inputs like physical capital and labour.

We present a model in which firms accumulate knowledge capital as a function of hours worked at the firm. Since the learning process is internalized by firms, their demand for labour exceeds that implied by equating the wage rate to the marginal product of labour. This occurs because firms take into account not only the current increase in output but also the value of the additional knowledge capital generated by the marginal hour of work hired. This latter effect operates as a "shift" factor for a labour demand curve drawn in wage and hours space and is key to enabling an expectations driven cycle. When news of future increases in technology arrive, the value of the firm's knowledge capital rises. This induces the firm to hire more labour at any given wage rate and results in increased production. The subsequent increase in knowledge capital also induces the accumulation of physical capital in anticipation of higher productivity next period. Meanwhile households wish to consume more in anticipation of higher income when the new technology eventually arrives. The increase in hours allows output to rise enough for both consumption and investment to co-move. We note that unlike most models of expectations driven cycles, the rise in investment occurs in the absence of any investment adjustment costs. An interesting implication of the rise in the value of knowledge capital and its accumulation is that the value of the firm will rise leading to an appreciation in the price of equity shares. Evidence suggests that the boom in stock prices leads future increases in total factor productivity. We show our model is consistent with this lead lag relationship. Moreover the rise in asset values is an integral part of the very mechanism that generates the expectational boom.

Learning-by-doing has two important implications. First labour decisions become dynamic as agents control the optimal amount of learning to do as the value of knowledge capital varies. Second, knowledge capital accumulation leads to productivity increases. Both these features play a key role in enabling co-movement.

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A Appendix

A.1 Central Planner's problem

The representative household has preferences defined over sequences of consumption C_t and leisure L_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \quad (45)$$

where β is the representative household's subjective discount factor and the period utility $u(C_t, L_t)$ function falls within the standard general class of preferences detailed in King, Plosser and Rebelo ([19]).

The representative household operates a production technology that produces output Y_t according to the technology

$$Y_t = A_t F(N_t, K_t, H_t) = A_t N_t^\alpha \tilde{K}_t^\theta H_t^\epsilon, \quad (46)$$

where A_t is the level of an exogenous stationary technology process, N_t is hours-worked, $\tilde{K}_t$ is capital services and H_t is the stock of knowledge capital.

Capital services are defined as

$$\tilde{K}_t = u_t K_t, \quad (47)$$

where K_t is the stock of physical capital, and u_t is the utilization rate of that capital. Physical capital evolves according to

$$K_{t+1} = [1 - \delta(u_t)] K_t + I_t \quad (48)$$

where I_t is investment, and where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The common exogenous total factor productivity process A_t evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \theta_{At}, \quad (49)$$

where $\rho_a < 1$ and θ_{At} is an exogenous period t innovation which we will define further below.

The stock of knowledge capital H_t evolves according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^\gamma N_t^{1-\gamma}. \quad (50)$$

Each period, the representative household is endowed with one unit of time that can be allocated between leisure and hours-worked N_t according to

$$N_t + L_t = 1. \quad (51)$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \quad (52)$$

The central planner chooses contingent infinite sequences of C_t , N_t , u_t , K_{t+1} and H_{t+1} to maximize (1) subject to (??), (??) and (??).

Letting λ_t and Υ_t be the period Lagrange multipliers on (??) and (??) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \quad (53)$$

$$U_L(C_t, L_t) = \lambda_t A_t F_{Nt} + \Upsilon_t \Psi_{Nt} \quad (54)$$

$$\delta'(u_t)K_t = A_t F_{ut} \quad (55)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} [A_{t+1} F_{Kt+1} + 1 - \delta(u_{t+1})] \} \quad (56)$$

$$\Upsilon_t = \beta E_t \{ \lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1} \} \quad (57)$$

Define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out λ_t , we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1}] \right\} \quad (58)$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt+1} \quad (59)$$

A.2 The value of the firm

In this section we investigate steady-state firm profits and the time t value of the firm.

A.2.1 Steady-state firm profits

First, we re-write the firm's two technologies $F(\cdot)$ and $\Psi(\cdot)$ without imposing any particular returns to scale as

$$y_t = A_t F(n_t, \tilde{k}_t, h_t) = n_t^\alpha (\tilde{k}_t)^\theta h_t^\varepsilon \quad (60)$$

and

$$\Psi(h_t, n_t, \tilde{k}_t) = h_t^\gamma n_t^\eta. \quad (61)$$

Since $h_{t+1} = h_t^\gamma n_t^\eta$, in steady state $h = n^{\frac{\eta}{1-\gamma}}$ and thus

$$y = An^{\alpha + \frac{\eta\varepsilon}{1-\gamma}} \tilde{k}^\theta = A\tilde{F}(n, \tilde{k}), \quad (62)$$

so that in steady state the firm's production function can be expressed as a function of just labour and capital services. From (62) we can see that for a given α , θ and γ , we can impose constant returns to labour and capital services in steady-state such that $\alpha + \theta + \frac{\eta\varepsilon}{1-\gamma} = 1$ by either: (i) $\eta < 1 - \gamma$ (ie DRS in $\Psi(\cdot)$) and $\varepsilon > 1 - \alpha - \theta$ (IRS in $F(\cdot)$), or (ii) $\eta > 1 - \gamma$ (ie IRS in $\Psi(\cdot)$) and $\varepsilon < 1 - \alpha - \theta$ (DRS in $F(\cdot)$), or (iii) $\eta = 1 - \gamma$ (ie CRS in $\Psi(\cdot)$) and $\varepsilon = 1 - \alpha - \theta$ (CRS in $F(\cdot)$). In this paper we impose case (iii), and as such in steady-state,

$$y = An^{1-\theta} \tilde{k}^\theta. \quad (63)$$

We can see how this relates to firm profits in steady-state by expressing $d = y - wn - r\tilde{k}$ as a share of output as

$$\frac{d}{y} = 1 - \frac{wn}{y} - \frac{r\tilde{k}}{y} = 1 - S_n - S_{\tilde{k}}, \quad (64)$$

where S_n is the steady-state labour share and $S_{\tilde{k}}$ is the steady-state capital services share. Applying (60) and (61) to the firm's first-order conditions gives

$$S_n = \alpha + \left(\frac{1 - \gamma}{1/\beta - \gamma} \right) \varepsilon, \quad (65)$$

and

$$S_{\tilde{k}} = \theta, \quad (66)$$

so that

$$\frac{d}{y} = 1 - \alpha - \theta - \left(\frac{\eta}{1/\beta - \gamma} \right) \varepsilon = 1 - [\alpha + \left(\frac{\eta}{1/\beta - \gamma} \right)] - \theta, \quad (67)$$

or in our case with $\eta = 1 - \gamma$ and $\varepsilon = 1 - \alpha - \theta$

$$\frac{d}{y} = \left(\frac{1/\beta - 1}{1/\beta - \gamma} \right) \varepsilon = \left(\frac{\xi}{\xi + (1 - \gamma)} \right) [1 - \alpha - \theta], \quad (68)$$

where $\xi = 1/\beta - 1$ is the household-owner's subjective discount rate. From (67) and (68) it is evident that the steady-state profit share will be affected

not only by the steady-state returns to scale to n and $\tilde{k}$ as implied in (62), but also the household-owner's subjective rate of time discount β . With $\beta < 1$, the share of profits is slightly positive, even though $F(\cdot)$ exhibits constant returns to n and $\tilde{k}$. The reason for this is that since the firm evaluates the benefit of additional knowledge capital taking as given the household-owners subject discount factor, it will thus discount the effect of these future profits. This reduces q_h (relative to if $\beta = 1$), thereby reducing the magnitude of the wedge-term in the hours first-order condition and thus reducing the number of hours and share of output paid to labour.

A.2.2 Period t dynamic value of firm

Having established the firm's steady-state profits, we now obtain an expression for the dynamic period t value of the firm. In what follows we base our approach on that used by Jaimovich and Rebelo ([17]).

First, we re-write the firm's problem (30) recursively as

$$V(H, A) = \max_{n, \tilde{k}, H'} \left\{ \lambda [AF(n, \tilde{k}, H) - wn - r\tilde{k}] + \beta EV(H', A') \right\} \quad (69)$$

subject to

$$H' = \Psi(H, n) \quad (70)$$

and where the firm takes as given from the aggregate

$$K' = K(H, K, A) \quad (71)$$

$$\ln A' = \rho_A \ln A + \theta_A \quad (72)$$

$$\lambda' = \lambda(A, H, K) \quad (73)$$

$$w = w(H, K, A) \quad (74)$$

$$r = r(H, K, A) \quad (75)$$

Note that we have defined $V(A, H)$ in terms of the household-owners' utility as given by λ , so that in terms of the notation used in (30) in the main text, $V_t = \frac{V(A, H)}{\lambda}$. Letting Υ be the Lagrange multiplier on (70), we write (69) as

$$V(H, A) = \max_{n, \tilde{k}, H'} \left\{ \lambda [AF(n, \tilde{k}, H) - wn - r\tilde{k}] + \Upsilon [\Psi(H, n) - H'] + \beta EV(H', A') \right\}. \quad (76)$$

Solving the maximization on the right-hand side gives

$$w = AF_n(n, \tilde{k}, H) + \frac{\Upsilon}{\lambda} \Psi_n(H, n) \quad (77)$$

$$r = AF_{\tilde{k}}(n, \tilde{k}, H) \quad (78)$$

$$\Upsilon = \beta EV_1(H', A). \quad (79)$$

Now define $\bar{V}(A, H) = \beta EV(A', H')$ as the end-of-period value of the firm, which is related to (30) in the main text by $\bar{V}_t = \frac{\bar{V}(A, H)}{\lambda}$, and to the price v_t of a share of the firm's equity through the household's Euler equation for equity (26) as $v_t = \bar{V}_t = \frac{\bar{V}(A, H)}{\lambda}$.

Since it can be shown that $\bar{V}(A, H)$ is homogenous of degree 1 in H , we can write

$$\bar{V}(A, H) = \beta EV_1(H', A)H'. \quad (80)$$

Substituting the firm's h' first-order condition (79) into (80) then gives

$$\bar{V}(A, H) = \Upsilon H' \quad (81)$$

as the end-of-period value of the firm in terms of the household-owners' utility. Using the notation in the main text, this then gives

$$\bar{V}_t = v_t = \frac{\Upsilon_t}{\lambda_t} H_{t+1}. \quad (82)$$

Or, defining $q_t = \frac{\Upsilon_t}{\lambda_t}$,

$$\bar{V}_t = v_t = q_t H_{t+1}, \quad (83)$$

so that the period t price v_t of the firm's equity share is the product of the value of knowledge capital in terms of consumption today and next period's stock of knowledge capital.