

Optimal Monetary Policy

In a Model with Agency Costs

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Abstract: This paper integrates a fully explicit model of agency costs into an otherwise standard Dynamic New Keynesian (DNK) model. The model's utility-based welfare criterion is derived explicitly and includes a measure of credit market tightness which we interpret as a risk premium. A principle result is the characterization of agency costs as endogenous mark-up shocks in an output-gap version of the Phillips curve. This implies that the output gap is a bad metric for assessing monetary policy in a model with agency costs, and that including the gap in a policy rule is typically welfare-reducing. The paper also fully characterizes optimal monetary policy and provides conditions under which zero inflation is the optimal policy. Finally, the paper demonstrates that the degree of price stickiness and/or the nature of monetary policy alter the endogenous propagation of net worth across time.

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1. Introduction

The macroeconomic events in the latter half of 2008 have sparked renewed interest in the role of financial shocks in the business cycle and the appropriate response of monetary policy to these shocks. This paper adds to this discussion by formally integrating a model of agency costs into an otherwise standard Dynamic New Keynesian (DNK) model. We do so in such a way that the agency-cost mechanism is quite transparent so that interactions between sticky prices and agency cost distortions are clearly identified. In addition our framework enables us to derive analytical expressions for the model-consistent welfare function.

Bernanke and Gertler (1989) provided the first attempt to build agency cost effects into an otherwise standard model of the business cycle. Their analysis was entirely qualitative. In a series of papers, Carlstrom and Fuerst (1997, 1998, 2001) built on this earlier work to make the analysis quantitative. These papers were in the tradition of flexible prices. Bernanke, Gertler, and Gilchrist (1999) took the next logical step to integrate these agency cost effects into a DNK model. These models helped to quantify two distinct effects of agency costs: (1) a change in the economy's response to macroeconomic shocks because of the dynamics of borrower net worth, and (2) an additional source of shocks to the economy such as shocks to borrowers' balance sheets. But these previous analyses did not consider the question of optimal monetary policy.

In this paper, we study the interaction of agency costs and sticky prices in a simple extension of the standard DNK model. Agency costs are modeled as a constraint on the firm's hiring of labor as in the hold-up problem of Kiyotaki and Moore (1997). We assume that the entrepreneur's hiring of one productive factor (labor) is constrained by entrepreneurial net worth. More generally, the constraint proxies for the effect that asset prices have on the ability of firms

to finance operations. Net worth is accumulated over time via purchases of shares that are claims on the profit flow of sticky-price firms that produce the final good. This leads to a natural interplay between price stickiness and collateral constraints. In our setup, monetary policy affects dividends and thus share prices by altering the profit flow of these sticky price firms. Share prices in turn affect the hiring of labor via the collateral constraint.

How should monetary policy be conducted in such an environment? From a public finance perspective, prices stickiness implies that real marginal cost acts like a distortionary *subsidy* on both factor inputs, while agency costs act like a distortionary *tax* on only one input (the constrained input). This suggests that a tradeoff between stabilizing these two distortions may exist. We study this question formally by deriving the quadratic welfare function that is consistent with the underlying model and analyze optimal monetary policy in a linear-quadratic framework. Inflation and the output gap enter our loss function with the same coefficients as in the standard sticky price model. In addition, agency costs give rise to a new term that captures the variations in the tightness of the collateral constraints and that can be interpreted as a risk premium more generally. Thus, the recent concerns by central banks about credit market tightness and the volatility of risk premia have a counterpart in our welfare based loss function.

If credit constraints were absent from our model, we would obtain the standard result that central banks should fully stabilize inflation at all times in response to technology shocks. With agency costs and sticky prices, there is a special case where all distortions can again be closed in response to technology shocks by fully stabilizing inflation. This special case requires preferences that are logarithmic in consumption and an efficient initial value for the model's state variable. In this case net worth happens to move by exactly the amount required to hire the first best quantity of labor when inflation is fully stabilized. Outside of this special case, it is not

optimal to fully stabilize inflation in response to technology shocks. However, the optimal deviation from inflation stability is small, so inflation stabilization is nearly optimal in this environment. Hence, the preferred interest rate rule should feature a strong anti-inflationary response, i.e. an inflation coefficient that is well in excess of the value originally proposed by Taylor.

We model financial shocks as shocks to the net worth of entrepreneurs and show that they act like endogenous markup shocks. Net worth shocks imply that inflation stabilization comes at the cost of increased fluctuations in the output gap and in the tightness of the collateral constraint that enter our loss function. Consequently, a temporary deviation from full price stability is warranted under optimal policy when financial sector shocks hit the economy.

Most previous work on optimal monetary policy with agency costs and sticky prices has analyzed the performance of simple interest rate rules, e.g., Bernanke and Gertler (2001), Gilchrist and Leahy (2002), Faia and Monacelli (2007). Faia and Monacelli (2007) consider a model with sticky prices and credit frictions similar to Bernanke, Gertler and Gilchrist (1999). They employ numerical second-order approximations to evaluate welfare of a parametric family of interest rate rules. Faia and Monacelli (2007) find that strict inflation targeting is the welfare maximizing policy rule within the restricted set of rules. They suggest that there is no trade off between stabilizing the price stickiness distortion and the agency cost distortion. On the contrary, our linear-quadratic approach shows that a tradeoff generally exists unless special conditions on initial values and parameters are met. There are two likely reasons for these differing results. First our paper considers optimal policy versus restricting ourselves to a Taylor-type rule.¹

¹ It is unlikely that fully optimal monetary policy under commitment falls within a class of simple Taylor rules. As such the LQ approach can demonstrate policy tradeoffs that may be difficult to detect by analyzing Taylor rules.

Second we deal with consumption heterogeneity by assuming that entrepreneurial consumption is rebated to households when they die. Fai and Monacelli (2007) instead do not redistribute income and instead focus on just the utility of households.

Our work is most closely related to the papers by DeFiore and Tristani (2008) and Curdia and Woodford (2008). These authors also provide small scale sticky price models with credit frictions and characterize optimal policy within a linear-quadratic framework. Curdia and Woodford (2008) focus on interest rate spreads between bank's lending and deposit rates that arise because loans are costly to produce. The analysis of these credit frictions is limited to a reduced form relationship between credit spreads and macroeconomic conditions.

De Fiore and Tristani (2008) add a more complete underlying structure by including a costly state verification framework within the standard DNK model. But De Fiore and Tristani (2008) abstract from the endogenous evolution of net worth by assuming that entrepreneurs receive a fixed endowment in every period. This assumption simplifies matters but eliminates the endogenous state variable that is of fundamental importance to the agency cost mechanism. Further, both of these previous papers do not feature any feedback between asset prices and net worth of credit constrained agents. In contrast, the interplay between the share price, net worth and the agency cost distortion is central to the dynamics of our model.

Our paper differs from Curdia and Woodford (2008) and De Fiore and Tristani (2008) in another important respect. Agency costs manifest themselves as constraints on the *quantity* of factor inputs in our work, whereas they are effectively constraints on the *price* of inputs in the aforementioned papers. Some central bankers have voiced concern that looking exclusively at the price of credit misses the importance of quantity constraints. A-priori it is not clear how important this difference is for model dynamics and optimal policy. However, in the log-

linearized equilibrium conditions of our model, these quantity constraints look akin to a time-varying external finance premium, i.e. they are very similar to a framework where the price of credit fluctuates endogenously.

The paper proceeds as follows. In the next section we outline the model, culminating in an expression for the welfare criterion. Section three analyzes the model quantitatively. Section four concludes.

2. The Model.

The model consists of households, entrepreneurs that produce intermediate goods, and sticky price firms that produce the final good. The basic structure is as follows. Households supply two types of labor input to the entrepreneurs. These entrepreneurs use a constant-returns-to-scale production function to produce an intermediate good using these two labor inputs. The entrepreneurial choice of one of the two labor inputs is constrained by accumulated net worth. This is the manifestation of agency costs in the model. The unconstrained input is necessary to ensure equilibrium existence. The combination of demand-determined output (from sticky prices) and constrained supply (from the net worth constraint) can often result in an over-determined system. See the Appendix for details. The intermediate good is sold to monopolistically competitive final goods firms who use a linear production function to produce output. Pricing at this level is subject to Rotemberg (1982) adjustment costs. We now discuss each economic agent in turn.

Households.

The typical household consumes the final good (c_t) and sells two types of labor input (L_t and u_t) to the entrepreneurs at factor prices w_t and r_t . Preferences are given by

$$U(c_t, L_t, u_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - B_1 \frac{L_t^{1+\theta}}{1+\theta} - B_2 \frac{u_t^{1+\theta}}{1+\theta}.$$

This two-input framework can be interpreted in at least two ways. First, the aggregative household may be interpreted as a proxy for an economy with households that sell two distinct types of labor, but who insure each other in terms of final consumption. Alternatively, the second labor input can be interpreted as “capital utilization.” Suppose that each household has a fixed level of capital, and they sell capital services to the entrepreneur at factor price r_t . Utilization of the capital stock (u_t) causes a utility cost to the household. Both of these micro-level stories are consistent with the aggregative model analyzed here.

The household has access to two financial instruments. The first is a standard one-period bond. These are in zero net supply but are introduced to price the gross nominal interest rate denoted by R_t . Second, the household can purchase shares in the sticky price firm at price Q_t . These shares pay out dividends D_t . The aggregate supply of shares is normalized to unity. Ownership of these shares shifts back and forth endogenously between households and entrepreneurs. The input supplies, Fisher equation, and asset price equation are:

$$U_L(t)/U_c(t) = w_t(1 + w_{sub}) \tag{1}$$

$$U_u(t)/U_c(t) = r_t(1 + r_{sub}) \tag{2}$$

$$U_c(t) = E_t \beta U_c(t+1) R_t / \pi_{t+1} \tag{3}$$

$$Q_t U_c(t) = E_t \beta U_c(t+1) [Q_{t+1} + D_{t+1}] \tag{4}$$

where π_t denotes the gross inflation rate, w_{sub} and r_{sub} are wage subsidies, and E_t is the expectations operator.

Entrepreneurs.

We assume that there are a continuum of long-lived entrepreneurs with linear consumption preferences and inter-temporal discount rate β . Entrepreneurs hire two types of labor from households and use these inputs in a CRS production function to produce an intermediate good. Their profit function is given by:

$$profits_t = p_t L_t^\alpha u_t^{1-\alpha} - w_t L_t - r_t u_t \quad (5)$$

where p_t denotes the relative price of the intermediate good price (in terms of the final good), and $x_t \equiv L_t^\alpha u_t^{1-\alpha}$ will denote production of the intermediate good. The parameter, α , governs the importance of agency costs in the model, if $\alpha = 0$ the model collapses to the simple one-sector sticky price model without agency costs. As mentioned above, α cannot be too large or the system becomes over-determined.² Entrepreneurs face a collateral constraint on their hiring of the one labor input:

$$w_t L_t \leq e_{t-1}(Q_t + D_t) \equiv nw_t \quad (6)$$

where e_{t-1} denotes the entrepreneur's holding of equity shares at the beginning of time- t , and nw_t denotes their net worth. The collateral constraint is motivated by a hold-up problem as in Hart

² This modeling formulation is isomorphic to two sets of entrepreneurs, one using labor input L_t , and the other using labor input u_t . The former set of entrepreneurs is constrained by net worth, while the latter are not. These two inputs are then combined by a Cobb-Douglass production aggregator with coefficient α . In this case, the coefficient α has the interpretation as the share of intermediate-good firms that are credit constrained.

and Moore (1994).³ The model will be calibrated so that this constraint will always bind in equilibrium.⁴ The rigid constraint (6) implies that the loan-to-net-worth ratio is always unity. In contrast, Carlstrom and Fuerst (1997) assumed a costly-state-verification environment in which this leverage ratio varies endogenously with aggregate conditions, and the risk premium was linked to the leverage ratio. For typical calibrations this endogenous leverage ratio varies only modestly so that the rigid constraint (6) is imposed with little loss of generality but with a great increase in transparency.

Let ϕ_t denote the multiplier on the constraint (6). The optimization conditions include:

$$\alpha p_t x_t = w_t L_t (1 + \phi_t) \quad (7)$$

$$(1 - \alpha) p_t x_t = r_t u_t \quad (8)$$

There is an alternative interpretation of ϕ_t that now suggests itself. From (7), ϕ_t is isomorphic to a model in which the L -input wage bill must be paid in advance of production at real interest rate ϕ_t . Compared to the unfettered input choice (8), ϕ_t can thus be interpreted as a risk-premium on an intra-temporal loan, and fluctuations in ϕ_t are thus fluctuations in the risk premium.

Because of the collateral constraint firms will earn profits in equilibrium. These profits are given by

$$profits_t = \alpha p_t x_t - n w_t = \alpha p_t x_t \left(\frac{\phi_t}{1 + \phi_t} \right) \quad (9)$$

³ In the present context, the Hart-Moore (1994) hold-up problem would go something like this. After the L -input has been added to the production process, the entrepreneur must add his unique and inalienable human capital to the process for otherwise there would be no production. This means that ex post the entrepreneur can “hold up” the L -input supplier and force down her wages. To avoid this renegotiation problem, the L -input supplier requires the entrepreneur to back the promised wages with assets that can be seized if necessary. This creates the collateral constraint (6).

⁴ More precisely, we log-linearize the model around a steady state in which this constraint binds and assume that it is also binding for small fluctuations around the steady state. Iacoviello (2005) has examined the possibility that collateral constraints could be only occasionally binding and concluded that this is quite unlikely in his environment given a reasonable calibration of the volatility of the shocks.

In the neighborhood of the steady-state, profits are positive such that the rate of return on internal funds exceeds the time preference rate. Entrepreneur will thus desire to accumulate more net worth and postpone consumption. The entrepreneur's budget constraint is given by

$$c_t^e + e_t Q_t \leq e_{t-1}(Q_t + D_t) + profits_t \quad (10)$$

Using the binding collateral constraint and equilibrium profits we have:

$$c_t^e + e_t Q_t \leq \alpha p_t x_t \quad (11)$$

Linear preferences along with the profit potential from acquiring greater net worth to mitigate future agency costs implies that entrepreneurial consumption will always be zero in equilibrium. To ensure that entrepreneurs do not acquire so much collateral that the collateral constraint does not bind, we assume that fraction $(1-\gamma)$ of the entrepreneurs die each period, to be replaced by an equal number of new entrepreneurs each endowed with a trivial amount of net worth. These deaths are unexpected so the entrepreneurs do not consume their assets before dying. After dying, all assets of the entrepreneurs are redistributed to households.⁵ This simplifying assumption implies that the model's quadratic welfare function extends the standard loss function with only one additional term that accounts for agency costs.

⁵ See Carlstrom and Fuerst (1998) for a discussion of other methods of ensuring that the collateral constraint binds in equilibrium. For example, as in Carlstrom and Fuerst (1997), we could assume that entrepreneurs are risk-neutral and choose consumption optimally. To ensure that the collateral constraint binds, we could assume that entrepreneurs discount the future more heavily than do households. The Euler equation for entrepreneurial intertemporal consumption would be given by: $Q_t = \beta \gamma E_t(Q_{t+1} + D_{t+1})(1 + \phi_{t+1})$, where $\gamma < 1$ is the higher rate of discount. In log-deviations we would have: $\hat{q}_t = \beta E_t \hat{q}_{t+1} + (1 - \beta) E_t \hat{d}_{t+1} + E_t \hat{\phi}_{t+1}$. We do not follow this approach as we would then need to include entrepreneurial consumption in the model which would needlessly complicate the welfare analysis.

Sticky price firms.

Monopolistically competitive firms indexed by j produce final goods $y_{t,j}$ that are aggregated to an output bundle according to $y_t = \left[\int_0^1 y_{t,j}^{\varepsilon/(\varepsilon-1)} dj \right]^{(\varepsilon-1)/\varepsilon}$. The final goods firms purchase the intermediate good from entrepreneurs at relative price p_t . The production function is given by $y_{t,j} = a_t x_{t,j}$, where productivity a_t follows an exogenous AR(1) process. Since the production function is linear, real marginal cost is given by $z_t = p_t/a_t$. The final goods price is subject to a Rotemberg-style (1982) quadratic cost of price adjustment, which enter the profit function of firm j as $\frac{\varphi}{2} [(p_{t,j} - p_{t-1,j})/p_{t-1,j}]^2 y_t$. These costs disappear from the linearized version of the social resource constraint as long as the gross inflation rate is unity in the steady state. Proposition 1 demonstrates that it is indeed optimal for a central bank that adopts a present value welfare criterion to deliver such a steady state inflation rate.

We use hatted variables throughout to denote their percentage deviation from the deterministic steady state. In a symmetric equilibrium, the Rotemberg price setting problem gives rise to the standard DNK Phillips curve given by

$$\hat{\pi}_t = \lambda \hat{z}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \epsilon_t^\pi \quad (12)$$

where $\lambda \equiv (\varepsilon - 1)/\varphi$, and ϵ_t^π is a mark-up shock with standard deviation σ_π . Here $\varepsilon > 1$ is the elasticity in the CES aggregator and φ is the cost of price adjustment. Monopolistic competition implies that these firms earn profits in equilibrium. These profits are paid out as dividends to shareholders of the sticky-price firms. These dividends are given by $D_t = a_t x_t (1 - z_t)$.

A Tale of Two Distortions.

A helpful interpretation of the model is revealed by considering the two labor input choices.

$$\frac{U_L(t)}{U_c(t)} = \frac{z_t}{(1+\phi_t)} MPL_t(1 + w_{sub}) \quad (13)$$

$$\frac{U_u(t)}{U_c(t)} = z_t MPU_t(1 + r_{sub}) \quad (14)$$

Increases in real marginal cost (z_t) are isomorphic to a decrease in a distortionary tax on all labor income, while an increase in the agency cost distortion (ϕ_t) is isomorphic to an increase in a distortionary tax on labor income only from the choice of constrained labor, L_t . Hence, increases in z_t will increase output via *symmetric* movements in both types of labor, while increases in ϕ_t will decrease output via an *asymmetric* decline in L_t .

We introduce wage subsidies to render the steady-state of the model efficient. The appendix provides the details. The appendix also outlines the entire model in log deviations. For present purposes, we will collapse the model into a smaller system that has a clear economic interpretation in terms of these two distortions. Let us begin by defining the output gap, etc., to be the difference in log deviations between actual output and the level of output when there are no price or credit frictions, $\hat{y}_t^g \equiv (\hat{y}_t - \hat{y}_t^{eff})$, where

$$\hat{y}_t^{eff} = \frac{(1+\theta)}{\sigma+\theta} a_t.$$

In this case, (A1)-(A2), (A7)-(A8), and (A11) can be combined to yield:

$$\hat{u}_t^g = \frac{1}{\sigma+\theta} \hat{z}_t + \frac{\alpha(\sigma-1)}{(\sigma+\theta)(1+\theta)} \hat{\phi}_t \quad (15)$$

$$\hat{L}_t^g = \frac{1}{\sigma+\theta} \hat{z}_t - \left[\frac{\sigma(1-\alpha)+(\alpha+\theta)}{(\sigma+\theta)(1+\theta)} \right] \hat{\phi}_t \quad (16)$$

$$\hat{y}_t^g = \frac{1}{\sigma+\theta} \hat{z}_t - \frac{\alpha}{(\sigma+\theta)} \hat{\phi}_t \quad (17)$$

$$\hat{w}_t^g = \sigma \hat{y}_t^g + \theta \hat{L}_t^g \quad (18)$$

$$\hat{r}_t^g = \sigma \hat{y}_t^g + \theta \hat{u}_t^g \quad (19)$$

As noted above, output varies positively with marginal cost, and negatively with the agency cost distortion. Marginal cost affects both inputs symmetrically; while increases in the agency cost distortion necessarily lowers L_t while having an ambiguous effect on u_t (depending upon the size of σ). The Phillips curve (A9) and Fisher equation (A3) are given by:

$$\hat{\pi}_t = \lambda \hat{z}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \epsilon_t^\pi \quad (20)$$

$$\frac{\sigma}{\sigma+\theta} (E_t \hat{z}_{t+1} - \hat{z}_t) - \frac{\alpha\sigma}{\sigma+\theta} (E_t \hat{\phi}_{t+1} - \hat{\phi}_t) = \hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{\psi}_t \quad (21)$$

where $\hat{\psi}_t \equiv \sigma(E_t \hat{y}_{t+1}^{eff} - \hat{y}_t^{eff})$ is an exogenous fluctuation in the natural rate of interest. The agency cost movements $\hat{\phi}_t$ are endogenous to the model, so to close the model we need an expression for the dynamics of the agency cost distortion. As a first step, we use (A5) to solve for the asset price:

$$\hat{q}_t = \frac{1}{(\sigma+\theta)} [(1 + \sigma + \theta) \hat{z}_t - \alpha \hat{\phi}_t + (1 + \theta) \hat{a}_t] - \hat{e}_t \quad (22)$$

Equations (A6) and (A10) then provide a dynamic equation in $\hat{e}_t$:

$$\hat{e}_t = \frac{1}{\beta} [\hat{e}_{t-1} + \hat{n}_t - (1 - \beta)(1 + m) \hat{z}_t + \hat{\phi}_t] \quad (23)$$

where $m \equiv \left(\frac{z_{ss}}{1-z_{ss}} \right)$ and $\hat{n}_t$ is an exogenous shock to the net worth of entrepreneurs with variance σ_n^2 . Since $\hat{e}_{t-1}$ is the fraction of shares owned by entrepreneurs, $\hat{n}_t$ is a redistribution of wealth

from households to entrepreneurs. The last dynamic relationship comes from the asset-pricing formula (A4):

$$E_t \hat{e}_{t+1} = \frac{1}{\beta} \hat{e}_t + \frac{1}{\beta(\sigma+\theta)} \{ \alpha(\sigma-1)(E_t \hat{\phi}_{t+1} - \hat{\phi}_t) - [(1-\beta)(\sigma+\theta)m + (\sigma-1) - \beta(\sigma+\theta)] E_t \hat{z}_{t+1} - (1+\theta)\hat{z}_t - (\sigma-1)(1+\theta)(E_t \hat{a}_{t+1} - \hat{a}_t) \} \quad (24)$$

Alternatively we obtain a simpler relationship by scrolling (23) forward and subtracting it from (24).

$$\hat{\phi}_t = \frac{(1+\theta)}{\sigma+\theta} (E_t \hat{z}_{t+1} - \hat{z}_t) - \frac{\sigma(1-\alpha)+(\sigma+\theta)}{(\sigma+\theta)} (E_t \hat{\phi}_{t+1} - \hat{\phi}_t) - \frac{(\sigma-1)(1+\theta)}{\sigma+\theta} (E_t \hat{a}_{t+1} - \hat{a}_t) - n_{t+1} \quad (24b)$$

This completes the simplification of the model.

In summary, the model is given by the two familiar DNK equations (20)-(21), and equations (23) and either (24a) or (24b) which track the dynamic behavior of entrepreneurial wealth and the agency cost distortion. The model is then closed by articulating a rule for monetary policy. Several comments are in order.

First, if we set $\alpha = 0$ so that the constrained input is not part of the production process, the model collapses to the familiar DNK model without the agency cost distortion.⁶ Hence, it is quite easy to consider the model with and without this distortion.

Second, agency costs manifest themselves as a distortion, $\hat{\phi}_t$, in the Fisher equation (21). Fluctuations in $\hat{\phi}_t$ lead to an inefficient input mix (15)-(16), and thus fluctuations in the output gap (17). As noted earlier, one can interpret these movements as fluctuations in the risk premium on loans to finance the L -input. These effects are scaled by the production coefficient α .

⁶This statement is true assuming that the central bank interest rate rule does not include a response to endogenous variables in (22)-(24) such as share prices or the risk premium.

Finally, the addition of agency costs has no effect on the standard DNK pricing relationship (20). Inflation is still driven by fluctuations in marginal cost. However, if we write the pricing relationship in terms of the output gap (using (17)) we have:

$$\hat{\pi}_t = \lambda(\sigma + \theta)\hat{y}_t^g + \alpha\lambda\hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} + \lambda\epsilon_t^\pi \quad (25)$$

Fluctuations in agency costs affect inflation in a manner symmetric to mark-up shocks, but with the key difference that agency costs evolve endogenously and are not simply exogenous shocks tacked on to the pricing relationship. Hence, whether agency costs need to be considered when estimating a Phillips curve depends upon whether marginal cost or the output gap are part of the empirical estimation. Relatedly, even if there are no mark-up shocks (ϵ_t^π), a monetary policy that stabilizes the output gap does not stabilize inflation.

Endogenous Agency Cost Dynamics with Flexible Prices

In the standard DNK model there are no endogenous state variables but agency costs naturally introduce net worth as an endogenous state variable. Hence, to understand the agency cost mechanism one needs to examine the endogenous dynamics of entrepreneurial net worth—to what extent is current net worth propagated forward? The dynamic mechanism is particularly transparent for the case of flexible prices. In this DNK framework, these flexible-price dynamics are identical to a sticky price model in which the central bank stabilizes the inflation rate (assuming that there are no mark-up shocks). In this case, equations (23) and (24) become:

$$E_t \hat{\phi}_{t+1} = A\hat{\phi}_t + \frac{A(1+\theta)(\rho-1)}{\alpha}\hat{a}_t - \rho_n \hat{n}_t \quad (26)$$

$$\hat{\phi}_t = \beta\hat{e}_t - (\hat{e}_{t-1} + \hat{n}_t) \quad (27)$$

where $A \equiv \frac{\alpha(1-\sigma)}{\alpha+\theta+\sigma(1-\alpha)}$, and (ρ, ρ_n) are the autocorrelation in the productivity and net worth

shock, respectively. The characteristic equation of this system is given by:

$$h(x) \equiv \beta x^2 - (1 + \beta A)x + A.$$

The two eigenvalues are $1/\beta$ and A . Hence, there is determinacy if and only if A is in the unit circle. There are three cases to consider depending upon the size of σ . If $\sigma = 1$, $A = 0$ so that there are no endogenous agency cost dynamics. If $\sigma < 1$, then $0 < A < 1$ so that there are non-oscillatory endogenous dynamics. In contrast, if $\sigma > 1$, $A < 0$ so that the stable root is negative and the agency dynamics are oscillatory.⁷

The equilibrium behavior under flexible prices is thus given by:

$$\hat{e}_t = A\hat{e}_{t-1} + \left(\frac{A-\rho_n}{1-\beta\rho_n}\right)\hat{n}_t + \frac{A(1-\rho)(1+\theta)}{\alpha(1-\beta\rho)}\hat{a}_t \quad (28)$$

$$\hat{\phi}_t = (A\beta - 1)\hat{e}_{t-1} + \left(\frac{A\beta-1}{1-\beta\rho_n}\right)\hat{n}_t + \frac{A\beta(1-\rho)(1+\theta)}{\alpha(1-\beta\rho)}\hat{a}_t \quad (29)$$

Since $A\beta < 1$, the risk premium is decreasing in entrepreneurial net worth. However, the response of net worth and the risk premium to the productivity shock depends upon the sign of A . For example, if $A > 0$ ($\sigma < 1$), then the risk premium and net worth accumulation are positively correlated with productivity shocks (and thus output). The opposite is true for $A < 0$ ($\sigma > 1$). In either case, the output gap is negatively correlated with the risk premium (see (17)).

The size of σ matters because it determines the quantitative effect that entrepreneurial net worth has on share prices. The intuition goes something like this. When entrepreneurial net worth ($\hat{e}_{t-1}$) is high the risk premium $\hat{\phi}_t$ is low, and output is high. If $\sigma < 1$, this has a small effect on share prices, while if $\sigma > 1$ this has a large effect on share prices. From (A5), entrepreneurial savings are given by $\hat{e}_t + \hat{q}_t = \hat{y}_t$. When $\sigma < 1$, an increase in $\hat{e}_{t-1}$ increases

⁷ If α is too large and $\sigma > 1$, the system is over-determined. See the appendix.

output but has only a small effect on share prices implying that $\hat{e}_t$ must increase. The opposite is true for $\sigma > 1$.

These flexible price dynamics are independent of the interest rate policy conducted by the central bank. As we add sticky prices things become more complicated because monetary policy no longer separates out but impacts the endogenous dynamics of $\hat{e}_t$. Suppose we close the system (20)-(21) and (23)-(24) with the simple interest rate rule $\hat{R}_t = \tau \hat{\pi}_t$, for $\tau > 1$. We can then calculate the value of σ consistent with the single stable eigenvalue of the system being zero. This cut-off value then demarcates the oscillatory vs. non-oscillatory dynamics. In particular we have that the dynamics are oscillatory if $\sigma > \frac{\lambda\tau}{1+\lambda\tau}$ and non-oscillatory if $\sigma < \frac{\lambda\tau}{1+\lambda\tau}$.⁸ Notice that this bound collapses to the flexible price result as $\lambda \rightarrow \infty$. Similarly, if monetary policy acts to stabilize inflation, $\tau = \infty$, the dynamics of net worth are identical to the flexible price case because marginal cost does not respond to net worth. For smaller values of τ , increases in $\hat{e}_{t-1}$ lower inflation and marginal cost as well. The distortion on constrained labor also decreases by more. These decreases make it more likely for $\hat{e}_t$ to also decline. In summary, with flexible prices ($\lambda = \infty$), or a very aggressive monetary policy ($\tau = \infty$), there are oscillatory dynamics if and only if $\sigma > 1$. But with sticky prices, the region of oscillatory dynamics increases ($\sigma > \frac{\lambda\tau}{1+\lambda\tau}$) via the effect that entrepreneurial net worth has on marginal cost.

The Special Case of $\sigma = 1$.

Before proceeding to the welfare analysis, (28)-(29) demonstrate a key result. Recall that (absent mark-up shocks) these equations also describe the model under the case of sticky prices if the central bank stabilizes the inflation rate. Stabilizing the inflation rate also stabilizes

⁸ This bound can be easily calculated from (20)-(21) and (24) by setting all shocks and future variables to zero. See the appendix for details.

marginal cost (20). For the case of $\sigma = 1$, equation (29) implies that the risk premium does not respond to technology shocks. Hence, for the case of technology shocks and $\sigma = 1$, if the economy begins at the steady-state ($\hat{e}_{t-1} = 0$), then a policy of stabilizing inflation eliminates both the marginal cost and agency cost distortion.

Why does zero inflation eliminate all distortions arising from technology shocks when $\hat{e}_{t-1} = 0$ and $\sigma = 1$? The agency cost distortion arises if and only if the L -input does not move by the efficient amount. The L -input is constrained by the value of the entrepreneur's net worth and thus fluctuates with movements in share prices:

$$\hat{w}_t + \hat{L}_t = \hat{e}_{t-1} + \beta \hat{q}_t + (1 - \beta) \hat{d}_t \quad (\text{A6})$$

The efficient wage bill is given by:

$$\hat{w}_t^{eff} + \hat{L}_t^{eff} = \frac{(1+\theta)}{(\sigma+\theta)} \hat{a}_t.$$

The right-hand side of (A6) is given by:

$$\frac{(1+\theta)}{(\sigma+\theta)} \hat{a}_t + \frac{(1+\theta+\sigma)}{(\sigma+\theta)} \hat{z}_t - \frac{(\sigma+\theta+\alpha)}{(\sigma+\theta)} \hat{\phi}_t + \hat{e}_{t-1}$$

Note that if $\hat{e}_{t-1} = 0$, fluctuations in productivity lead to fluctuations in the right-hand side of (A6) of *precisely* the magnitude needed to finance the efficient wage bill. This logic suggests that zero inflation might eliminate distortions for all values of σ , assuming that $\hat{e}_{t-1} = 0$. The reason this is not the case is that entrepreneurial savings $\hat{e}_t$ in expression (24) is affected by fluctuations in productivity when $\sigma \neq 1$. The quantitative impact of alternative values of σ will be investigated in Section 3.

Welfare Criterion.

The transparency of the model allows for analytical expressions for the loss function. We first note that zero is the optimal steady-state inflation rate. The subsequent two results provide analytical expressions for the welfare function.

Proposition 1: The gross inflation rate that maximizes a present value welfare criterion is unity in the steady state regardless of whether subsidies exist that make the steady state efficient or not.

Proof: See appendix.

Proposition 2: A quadratic approximation to the welfare function around a steady state with zero inflation is given by:

$$\frac{U(t) - U^*(t)}{U_c(ss)c_{ss}} \approx -\frac{1}{2} \left[\frac{(\varepsilon - 1)}{\lambda} \hat{\pi}_t^2 + (\sigma + \theta)(\hat{y}_t^g)^2 + \frac{\alpha(1 - \alpha)}{1 + \theta} \hat{\phi}_t^2 \right]$$

Here, $U^*(t)$ denotes welfare in a first best economy without agency or price adjustment costs. Consequently, the approximation expresses the welfare gap between a first best economy without both agency and price adjustment costs and our model economy in percentage of steady state consumption.

Proof: See appendix.

Agency costs add the variance of $\hat{\phi}_t$ to the loss function which captures the tightness of the collateral constraint. Recall that $\hat{\phi}_t$ has the alternative interpretation as a risk premium on an intra-temporal loan for the L-input wage bill. Thus, the traditional concern by central banks about credit market tightness and the volatility of risk premia have a counterpart in our welfare-based loss function. When $\alpha = 0$, the constrained labor input is not needed for production and

the collateral constraint is not operative. Hence our loss function collapses to the standard loss function as in Woodford (2003).⁹

Proposition 3: Using (17), we can write the welfare function in terms of the two underlying distortions:

$$\frac{U(t) - U^*(t)}{U_c(ss)c_{ss}} \approx -\frac{1}{2} \left[\frac{(\varepsilon - 1)}{\lambda} \hat{\pi}_t^2 + \frac{\alpha[\theta + \alpha + \sigma(1 - \alpha)]}{(1 + \theta)(\sigma + \theta)} \hat{\phi}_t^2 + \frac{1}{(\sigma + \theta)} \hat{z}_t^2 - \frac{2\alpha}{(\sigma + \theta)} \hat{\phi}_t \hat{z}_t \right]$$

Proof: By inspection.

Proposition 3 has several important implications. First, the objective function implies a preference for the two underlying distortions, $\hat{\phi}_t$ and $\hat{z}_t$, to positively co-vary. This is not surprising as $\hat{\phi}_t$ acts as a tax and $\hat{z}_t$ like a subsidy. Second, the importance of stabilizing inflation is quite clear. For typical calibrations, the coefficient on inflation is at least an order of magnitude larger than the coefficients on the risk premium and marginal cost. For example, for our baseline calibration, the weight on inflation is roughly 1000 times larger than the weight on the risk premium! Hence, the loss function implies a strong preference for stabilizing inflation, even at the cost of allowing the other distortions to vary. This is the principle reason that zero inflation is close to the optimal policy in a quantitative sense.

⁹ The standard DNK welfare criterion assumes subsidies are paid to the firm to make marginal cost equal to unity in the steady-state. This implies that the welfare weight on inflation is $\frac{\varepsilon}{\lambda}$. The weight here differs slightly because marginal cost is less than unity in the steady state. Instead, the subsidies we introduce to render the steady-state efficient are paid directly to households to boost labor supply. See Keen and Wang (2004) for details of the Rotemberg approximation.

3. Optimal Policy and Simple Rules

In this section, we provide some analytical and quantitative results on optimal policy and the welfare losses of alternative policy rules. Under commitment, the policy maker maximizes

$$\max_{\{\hat{\pi}_{t+j}, \hat{\phi}_{t+j}, \hat{y}_{t+j}^g, \hat{e}_{t+j}\}_{j=0}^{\infty}} -\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(\varepsilon-1)}{\lambda} \hat{\pi}_{t+j}^2 + (\sigma + \theta)(\hat{y}_{t+j}^g)^2 + \frac{\alpha(1-\alpha)}{1+\theta} \hat{\phi}_{t+j}^2 \right], \quad (30)$$

subject to the constraints (expressed in terms of the variables in the loss function)

$$\hat{\pi}_t = \lambda(\sigma + \theta)\hat{y}_t^g + \alpha\lambda\hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} + \lambda\epsilon_t^\pi, \quad (31)$$

$$\hat{e}_t = \frac{1}{\beta} \{ \hat{e}_{t-1} - (1 - \beta)(1 + m)(\sigma + \theta)\hat{y}_t^g - [(1 - \beta)\alpha(1 + m) - 1]\hat{\phi}_t + n_t \}, \quad (32)$$

$$\begin{aligned} \hat{\phi}_t &= (1 + \theta)(E_t \hat{y}_{t+1}^g - \hat{y}_t^g) - (1 - \alpha)(E_t \hat{\phi}_{t+1} - \hat{\phi}_t) \\ &\quad - \frac{(\sigma-1)(1+\theta)}{\sigma+\theta} (E_t \hat{a}_{t+1} - \hat{a}_t) - n_{t+1} \end{aligned} \quad (33)$$

Where (33) is (24b) written in terms of the output gap. Under the timeless perspective, the Lagrangian of this problem is modified such that it is written in a time invariant form. See Woodford (2003) for details.

Proposition 4: The optimal inflation target under commitment from a timeless perspective is given by:

$$\begin{aligned} \hat{\pi}_t &= \left[\frac{\alpha\varepsilon(1 - \beta) - 1}{m} \right] \Delta \hat{y}_t^g - \left[\frac{\alpha(1 + m)(1 - \beta)(1 - \alpha)}{m(1 + \theta)} \right] \Delta \hat{\phi}_t \\ &\quad - \left[\frac{\alpha(1 + m)(1 - \beta)(\sigma - 1) + 1 + \theta}{m(\sigma + \theta)} \right] \hat{X}_t \\ &\quad + \left[\frac{(1 + m)(1 - \beta)[\alpha(\sigma - 1) - (\sigma + \theta)] + 1 + \theta}{\beta m(\sigma + \theta)} \right] \hat{X}_{t-1} \end{aligned}$$

The variable $\hat{X}_t$ is a lag-polynomial in $\Delta\hat{\phi}_t$ and $\Delta\hat{y}_t^g$ defined via the recursion:

$$\hat{X}_t = \frac{\alpha\beta(1-\sigma)}{(\sigma + \theta + \alpha(1-\sigma))} E_t \hat{X}_{t+1} + E_t \Xi_{t+1}$$

$$\Xi_t \equiv \frac{\alpha\beta(\sigma + \theta)[(1 + \theta)\Delta y_t^g - (1 - \alpha)\Delta\hat{\phi}_t]}{(1 + \theta)(\sigma + \theta + \alpha(1 - \sigma))}$$

Proof: (This relationship is derived from the first-order conditions of the LQ problem by substituting out the Lagrange multipliers in the spirit of a targeting criterion suggested by Giannoni and Woodford (2003). Proof to be added.)

Several comments are in order. First, optimal policy relates inflation to the *first difference* of the remaining variables in the loss function rather than to the levels. This feature is shared with the targeting criterion in the simple DNK model. Note that for $\alpha = 0$ this condition is simply the targeting criterion from the standard DNK model, $\pi_t = -\frac{1}{\epsilon-1}\Delta y_t^{gap}$

Second, the criterion is particularly simple if $\sigma = 1$.

$$\hat{\pi}_t = -\frac{1}{m}\Delta\hat{y}_{t+1}^g - \frac{(1-\alpha)}{m}\Delta\hat{y}_t^g + \frac{\alpha\beta(1-\alpha)}{m(1+\theta)}\Delta\hat{\phi}_{t+1} - \frac{\alpha(1-\alpha)}{m(1+\theta)}\Delta\hat{\phi}_t$$

In this case, the inflation rate responds negatively to $\Delta\hat{y}_t^g$, $\Delta\hat{\phi}_t$, and $E_t\Delta\hat{y}_{t+1}^g$, but positively to $E_t\Delta\hat{\phi}_{t+1}$. At first glance the “tightening” of policy when agency costs or the risk premium increases is surprising. This comes about because agency costs manifest themselves as mark-up shocks in equation 25. But this tightening is reversed next period as inflation responds positively to expected changes in the risk premium.

Third, as we move away from $\sigma = 1$, the recursion in $\hat{X}_t$ implies that policy will be either forward-looking or backward-looking depending on the size of $\frac{\alpha\beta(1-\sigma)}{(\sigma+\theta+\alpha(1-\sigma))}$. This term is in the unit circle if and only if $\alpha(1+\beta)(\sigma-1) < (\sigma+\theta)$. If this inequality holds then policy is forward-looking with

$$\hat{X}_t = \sum_{j=0}^{\infty} \left[\frac{\alpha\beta(1-\sigma)}{(\sigma+\theta+\alpha(1-\sigma))} \right]^j E_t \Xi_{t+1+j}$$

Note that these coefficients are all positive only if $\sigma < 1$. The sum of the coefficients on the future component is given by

$$\sum_{j=1}^{\infty} \left[\frac{\alpha\beta(1-\sigma)}{(\sigma+\theta+\alpha(1-\sigma))} \right]^j = \left[\frac{\sigma+\theta+\alpha(1-\sigma)}{\sigma+\theta+\alpha(1-\beta)(1-\sigma)} \right] - 1 \approx \frac{\alpha(1-\sigma)}{\sigma+\theta}$$

For $\sigma = 1$, this sum is (of course) unity as all weight is placed on the one-period-ahead forecast. But for other values of σ this sum remains close to unity. Hence, the degree of forward-looking behavior beyond one period is modest. If instead $\alpha(1+\beta)(\sigma-1) > (\sigma+\theta)$, policy is backward-looking with

$$\hat{X}_t = - \sum_{j=0}^{\infty} \left[\frac{\sigma+\theta+\alpha(1-\sigma)}{\alpha\beta(1-\sigma)} \right]^{j+1} \Xi_{t-j}$$

The sum of these coefficients on current and lagged values is (again) given by

$$\sum_{j=0}^{\infty} \left[\frac{\sigma+\theta+\alpha(1-\sigma)}{\alpha\beta(1-\sigma)} \right]^{j+1} = \left[\frac{\sigma+\theta+\alpha(1-\sigma)}{\sigma+\theta+\alpha(1-\beta)(1-\sigma)} \right]$$

Again, the degree of backward-looking behavior is modest.

Calibration.

Parameters are set to standard values. The discount factor β is set 0.99. We set the elasticity in the CES aggregator $\epsilon = 10$, consistent with a markup of roughly 11%. We calibrate the Rotemberg price adjustment cost parameter such that the slope of the Phillipcurve is 0.052. This resulting value for φ is 173.08. In a log-linearly equivalent Calvo price setting model such a slope implies that prices are fixed for 5 quarters on average. The share of constrained labor α is set to 0.5. We think of this as the share of intermediate goods firms that are collateral constrained. Our baseline calibration for the preference parameters follows Woodford (2003, page 172): $\sigma = 0.16$ and $\theta = 0.47$, implying large intertemporal and intratemporal elasticities. This implies that there are strategic complementarities in the model. Woodford argues that in a model without physical capital and investment spending, one needs a high intertemporal elasticity to match the interest-sensitivity of GDP. For sensitivity analysis we also consider the case of lower elasticities: $\sigma = \theta = 2$. We assume that the total factor productivity follows an AR(1) process for autoregressive coefficient 0.95, whereas net worth and markup processes are assumed to slightly less persistent with AR coefficients 0.9.

Impulse Response Functions.

Figures 1A-1B present the impulse responses to a technology shock for the two preference assumptions under two different versions of monetary policy: a policy that stabilizes inflation, and the optimal monetary policy under commitment. For the case of $\sigma = 0.16$ and $\theta = 0.47$, inflation behavior under optimal policy is essentially zero inflation (the scale is 10^{-3}). This occurs despite a sizeable output gap opening up for the first 4 quarters. This illustrates how

agency costs break the divine coincidence present in the standard DNK model according to which both inflation and output gap can be stabilized simultaneously with technology shocks. Our model embeds a policy trade-off even for technology shocks, but this tradeoff is largely resolved in favor of stabilizing inflation. Under both policies, the nominal rate increases slightly, a peak of about 6 basis points. Similarly, under both monetary policies the response of the risk premium and net worth are nearly identical. Consistent with (29), the technology shock increases the risk premium and net worth increases and responds in a hump-shaped manner. This is quite similar to the results in Carlstrom and Fuerst (1997). The basic mechanism is that the technology shock drives up the demand for credit faster than net worth can respond. Since the risk premium rises by more than the increase in marginal cost, the output gap declines.

For the alternative calibration of $\sigma = \theta = 2$ in Figure 1B, the technology shock causes the risk premium to fall, and thus the output gap to rise (both are opposite the previous case). Optimal policy quickly eliminates this behavior so that the risk premium, marginal cost, and the gap are quickly driven back to zero. This is accomplished by a sharp decline, then equally sharp increase in the nominal (and real) interest rate. Again, the inflation behavior is nearly identical under the two policies.

Figures 2A-2B present the complementary impulse responses to a net worth shock. The immediate impact of the shock is to sharply lower the risk premium. This decline leads to a sizeable increase in output and the gap. As suggested by the welfare criterion, optimal policy calls for marginal cost and the risk premium to positively co-vary. This decline in marginal cost mitigates the expansionary effect of the decline in the risk premium so that the gap moves by less under optimal policy. This is accomplished by the cumulative real rate being higher under optimal policy than under zero inflation. (Recall that in this DNK model, the gap is proportional

to the sum of all future real rates (see (21)). This is particularly clear in Figure 2A. Finally, note that when $\sigma = 0.16$ and $\theta = 0.47$, strategic complementarities implies that the Phillips curve trade-off between inflation and the output gap is very steep compared to our alternative calibration. Even a small fall in inflation implies that optimal policy can close about one third of the output gap that opens up in the first few quarters under a policy of full inflation stabilization. When $\sigma = 2$ and $\theta = 2$, the Phillips curve is not nearly as flat and the inflation-gap tradeoff is weak. Consequently, the output gap under optimal policy behaves more similarly to the gap under inflation stabilization.

Quantifying Welfare Costs of Alternative Policies.

In Tables 1A and 1B we quantify the welfare losses for four different specifications of policy. Note that these losses are scaled by the variance of the innovations into the exogenous stochastic processes and are thus very small for typical calibrations of these variances, e.g., $\sigma_a^2 = (0.007)^2$.

Along with the zero-inflation and optimal policy under commitment, we calculate the welfare losses for alternative interest rate rules. We consider simple rules of the form $\hat{R}_t = \tau \hat{\pi}_t + \tau_g \hat{y}_t^g$. We set $\tau = 1.5$ and $\tau_g = 0.5$ (“with gap”) or $\tau_g = 0$ (“without gap”). We do not take a stand on the relative importance of the structural shocks, so we compute the welfare loss based on unconditional variances separately for each shock. Again, several comments are in order.

First, the agency cost distortions imply that the central bank cannot eliminate all welfare losses arising from technology shocks. Further, the presence of agency costs magnifies the welfare costs of all shocks, regardless of monetary policy. Second, as already suggested, inflation stabilization comes quite close to achieving the welfare level of the optimal monetary

policy for all three shocks. Figure 3 demonstrates how quickly welfare losses decline as the central bank increases the interest rate response to inflation.

Third, the two Taylor rules do significantly worse than the zero inflation policy. As noted earlier, the welfare function places great weight on inflation, and under either Taylor rule the volatility of inflation is sharply increased.

Fourth, the importance of the slope of the Phillips curve for the output gap inflation tradeoff mentioned earlier is confirmed when comparing the welfare losses under net worth shocks in Tables 1A and 1B. For the calibration in Table 1A, the Phillips curve is flat and optimal monetary policy improves significantly upon full inflation stabilization. The parameterization for Table 1B implies that the trade off is weak and optimal policy does not improve upon inflation stabilization (up to rounding).

Fifth, and most interestingly, whether including the gap in the Taylor rule is efficacious depends upon the nature of the shock and the degree of agency costs. If agency costs are relatively unimportant (the bottom row in each entry), then responding to the gap is welfare-improving if there are technology shocks because an increase in the gap-response is equivalent to an increase in the inflation coefficient. However, this is not the case when there are mark-up shocks. When there are mark-up shocks, responding to the gap causes the volatility of inflation to increase relative to a rule that does not respond to the gap.

But agency costs add two interesting wrinkles to this result. First, agency costs manifest themselves as endogenous mark-up shocks when the Phillips curve is expressed in terms of the gap (see (25)). Hence, even if there are only technology shocks, the presence of agency costs can make responding to the gap a bad idea (see Table 1A). Second, for the case of net worth

shocks, responding to the gap is always a bad idea, and the welfare cost can be quite large depending upon the variance of net worth shocks and how important agency costs are. Thus, the endogenous mark-up shocks coming from agency costs can be quite important.

Taylor Rules with a Response to Asset Prices.

As additional sensitivity analysis, we consider Taylor rules in which the central bank responds directly to asset prices. In particular, we consider rules of the form:

$$\hat{R}_t = \tau \hat{\pi}_t + \tau_q \hat{q}_t.$$

Figures 4-5 present the welfare costs of such a policy rule (with $\tau = 1.5$) under both types of shocks. For this numerical example, we have set variance of the innovations to 0.007². In both cases, the welfare costs are lowest when τ_q is slightly negative (for the case of net worth shocks, the minimizer is -0.02 and -0.06; for technology shocks, -0.02 and -0.11). As we move away from this optimum, the welfare costs are asymmetric. A negative coefficient has little effect, but positive responses can quickly become very costly. The intuition is as follows. Responding to share prices means that the central bank responds to dividends (see (A4)). But in this model with endogenous dividends, responding positively to dividends means that the central bank responds negatively to marginal cost (see (A10)). Using this logic we can write the policy rule approximately like this:

$$\hat{R}_t = \tau \hat{\pi}_t + \tau_q \hat{q}_t \approx \tau \hat{\pi}_t - \tau_z \hat{z}_t = \left(\tau - \frac{\tau_z}{\lambda}\right) \hat{\pi}_t + \frac{\tau_z}{\lambda} \beta \hat{\pi}_{t+1}.$$

Hence, responding to share prices implies that the response to current inflation is substantially reduced (although the policy is determinate because of the strong response to future inflation).¹⁰ Thus, such a policy will increase inflation variability, and given the large weight of inflation in the welfare function, is quite costly. This discussion suggests, and numerical results confirm, that this result is largely independent of the size of α .¹¹

Uncertainty about credit frictions and optimal policy.

In practice, central banks typically face considerable uncertainty about the strength and the exact channels through which credit frictions impact the economy. What outcomes would policymakers achieve if they were to set policy optimally under commitment but given an imperfect assessment of the importance of agency costs for the economy?

We present two crude ways to answer this question. In the first scenario, the central bank ignores the implications of credit frictions for the loss function, but is aware of how credit frictions change the structural model. This is simply optimal policy under commitment with a zero weight on the risk premium term. In the second scenario, the policymaker ignores how these frictions change the loss function *and* the structural model. Optimal policy is then fully described by the targeting criterion¹²

$$\hat{\pi}_t = -\omega(\hat{y}_t^g - \hat{y}_{t-1}^g) \quad (30)$$

¹⁰ For the values reported here, there is always a unique equilibrium. However, as in Carlstrom and Fuerst (2007), if τ_q is too large, there is equilibrium indeterminacy.

¹¹ However, numerical results imply that the welfare losses for α very small (but nonzero) are substantially larger than the DNK model without agency costs. This occurs because the collateral constraint is always binding in the agency cost model for all values of α , while there is no such constraint in the basic DNK model.

¹² Here, ω equals the relative of weight on the output gap versus inflation divided by the slope of the Phillips curve.

For both cases, equilibrium outcomes are determined by the credit friction model plus a description of policy.

We examine only net worth shocks as these types of shocks are often associated with financial distress. We assume that the central bank is able to compute $\hat{y}_t^g$ correctly and set policy accordingly. The impulse responses in Figure 6 show that the three descriptions of policy bring about qualitatively similar paths for the economy that are reasonably close to the optimal policy under commitment given the true model structure. Inflation behaves nearly identical across all policies. The policymaker that ignores the weight on the risk premium in the loss function stabilizes the output gap by too much in the initial two periods, but essentially replicates the path for the output gap under optimal policy from period three onwards.

When the simple targeting criterion (30) is implemented, the output gap dynamics differs significantly from the optimal path for the gap only in the initial period. It appears remarkable that the targeting criterion (30) performs reasonably well in the context of our model. After all, it is derived from a framework in which the dynamics of net worth and agency costs are absent. This finding mirrors a similar result in Curdia and Woodford (2008) who also argue that the targeting criterion derived from the standard DNK model performs well in their credit friction model.

Less weight on inflation: an ad hoc welfare criterion.

A recurring theme throughout the paper is that although the output gap and the risk premium enter into the welfare criterion of Proposition 2, a policy that ignores the gap and the risk premium but achieves zero inflation comes quite close to the optimal policy. As already noted, this is because the weight on inflation in the welfare function is several orders of magnitude larger than the weights on the gap and risk premium. Hence as a final form of

sensitivity analysis, we ask how “optimal” policy would look if the weight on inflation was much lower.

We now consider an ad hoc welfare function that is not motivated by the primitives of the model. In particular, Figure 7 assumes that the weight on inflation is the same as the weight on the output gap. Figure 7 shows the model response to a net worth shock under three different policy scenarios: zero inflation, the true optimal policy, and the optimal policy assuming the ad hoc loss function. The smaller weight on inflation under the ad hoc criterion leads the central bank to allow greater inflation variability and thus drive the gap (and net worth) back to zero more quickly. Such a policy exacerbates the risk premium distortion, although this is true only for the first two periods. Hence, this ad hoc criterion implies little difference in the behavior of agency costs, but sharply different behavior in inflation and the gap. Inflation is significantly more volatile while the gap is significantly less persistent and thus less volatile in a unconditional sense.

4. Conclusion.

This paper has developed a tractable model of credit frictions in an otherwise standard DNK model. The model provides a rationale for the central bank to consider credit frictions when setting policy. In particular, a measure of credit market tightness which we interpret as a risk premium enters directly into the central bank welfare criterion derived here.

A key implication of our model is that agency costs manifest themselves as endogenous perturbations entering the output-gap version of the Phillips curve. Consequently, there generally is a trade-off between stabilizing inflation, the output gap and the risk premium,

regardless of the type of shock hitting the economy. However, the DNK nature of the model implies that fluctuations in inflation are much more costly in welfare terms than variability of the output gap or the risk premium. Consequently, stabilizing inflation is near-optimal even if agency costs are quite severe.

An interesting question is the sensitivity of these results to the assumption that subsidies render the steady state efficient. We have numerically computed optimal policy with a distorted steady state obtained via the first-order conditions from the fully nonlinear Ramsey problem and found that they are similar quantitatively. The likely explanation is that inflation stabilization again remains the dominating objective for the policymaker given the large weight attached to inflation in the loss function.

Table 1A ($\sigma = 0.16, \theta = 0.47$)

Welfare Losses under Different Monetary Policies

The 3 entries in each cell correspond to $\alpha = 0.5$, $\alpha = 0.25$, and $\alpha = 0.01$.

	Technology shocks	Net Worth Shocks	Markup Shocks
Optimal Commitment	$32\sigma_a^2$ $19\sigma_a^2$ $1\sigma_a^2$	$122\sigma_n^2$ $53\sigma_n^2$ $2\sigma_n^2$	$316\sigma_\pi^2$ $324\sigma_\pi^2$ $326\sigma_\pi^2$
Inflation Stabilization	$33\sigma_a^2$ $19\sigma_a^2$ $1\sigma_a^2$	$140\sigma_n^2$ $58\sigma_n^2$ $2\sigma_n^2$	$388\sigma_\pi^2$ $413\sigma_\pi^2$ $425\sigma_\pi^2$
Taylor Rule with gap	$383\sigma_a^2$ $91\sigma_a^2$ $16\sigma_a^2$	$1485\sigma_n^2$ $394\sigma_n^2$ $2\sigma_n^2$	$6134\sigma_\pi^2$ $5915\sigma_\pi^2$ $5772\sigma_\pi^2$
Taylor Rule without gap	$177\sigma_a^2$ $120\sigma_a^2$ $101\sigma_a^2$	$166\sigma_n^2$ $61\sigma_n^2$ $2\sigma_n^2$	$777\sigma_\pi^2$ $528\sigma_\pi^2$ $431\sigma_\pi^2$

(We assume that the TFP follows an AR(1) process with autoregressive coefficient 0.95, whereas net worth and markup processes are assumed to slightly less persistent with AR coefficients 0.9. Welfare losses are scaled by the variance of the innovation in the exogenous processes for TFP (σ_a^2), net worth (σ_n^2) and markup (σ_π^2), respectively. Each table entry is multiplied by 100.)

Table 1B ($\sigma = 2, \theta = 2$)

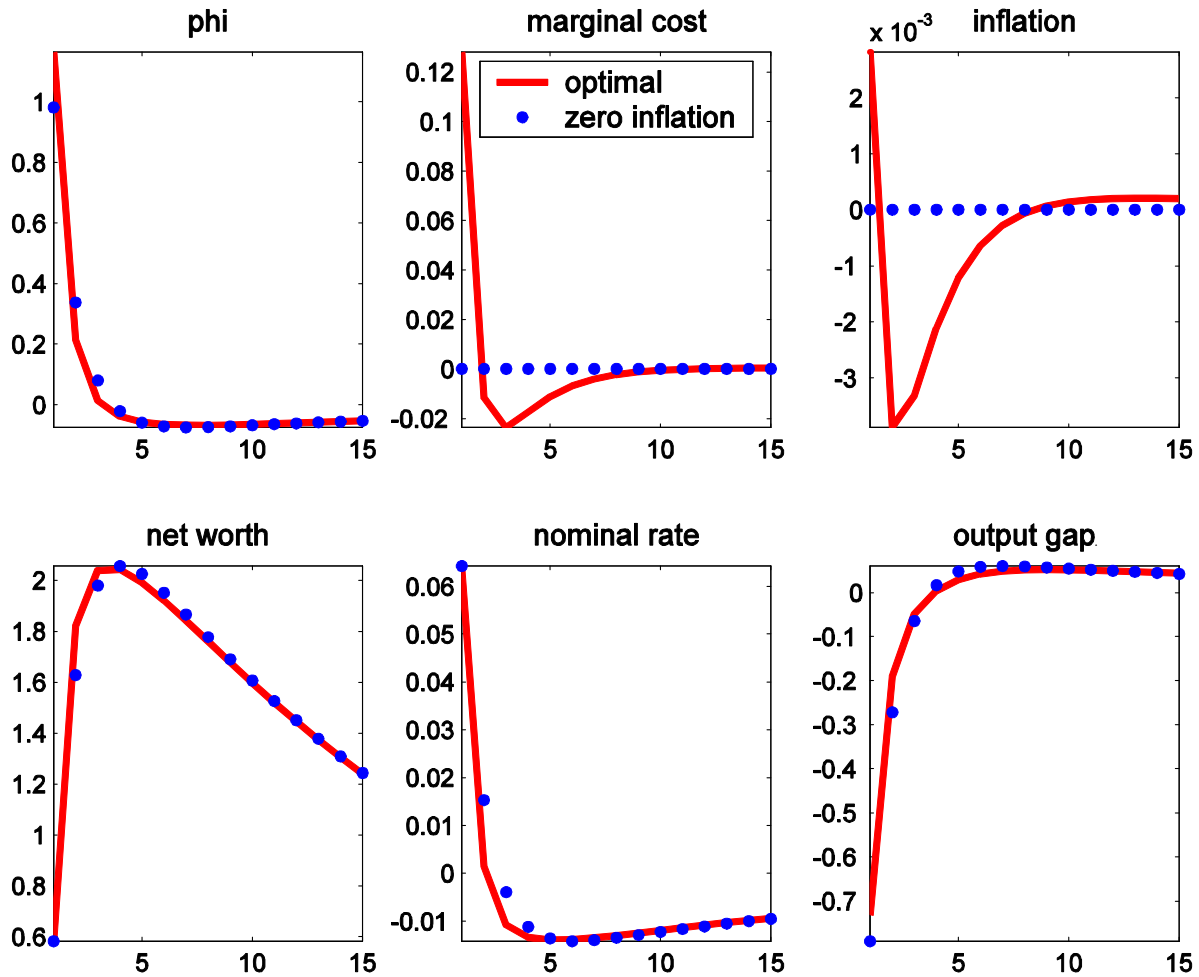
Welfare Losses under Different Monetary Policies

The 3 entries in each cell correspond to $\alpha = 0.5$, $\alpha = 0.25$, and $\alpha = 0.01$.

	Technology shocks	Net Worth Shocks	Markup Shocks
Optimal Commitment	$4\sigma_a^2$ $2\sigma_a^2$ $0\sigma_a^2$	$39\sigma_n^2$ $21\sigma_n^2$ $1\sigma_n^2$	$69\sigma_\pi^2$ $64\sigma_\pi^2$ $61\sigma_\pi^2$
Inflation Stabilization	$4\sigma_a^2$ $2\sigma_a^2$ $0\sigma_a^2$	$39\sigma_n^2$ $21\sigma_n^2$ $1\sigma_n^2$	$75\sigma_\pi^2$ $70\sigma_\pi^2$ $66\sigma_\pi^2$
Taylor Rule with gap	$1022\sigma_a^2$ $941\sigma_a^2$ $960\sigma_a^2$	$418\sigma_n^2$ $111\sigma_n^2$ $1\sigma_n^2$	$1696\sigma_\pi^2$ $1533\sigma_\pi^2$ $1518\sigma_\pi^2$
Taylor Rule without gap	$1630\sigma_a^2$ $1512\sigma_a^2$ $1495\sigma_a^2$	$100\sigma_n^2$ $34\sigma_n^2$ $1\sigma_n^2$	$521\sigma_\pi^2$ $284\sigma_\pi^2$ $277\sigma_\pi^2$

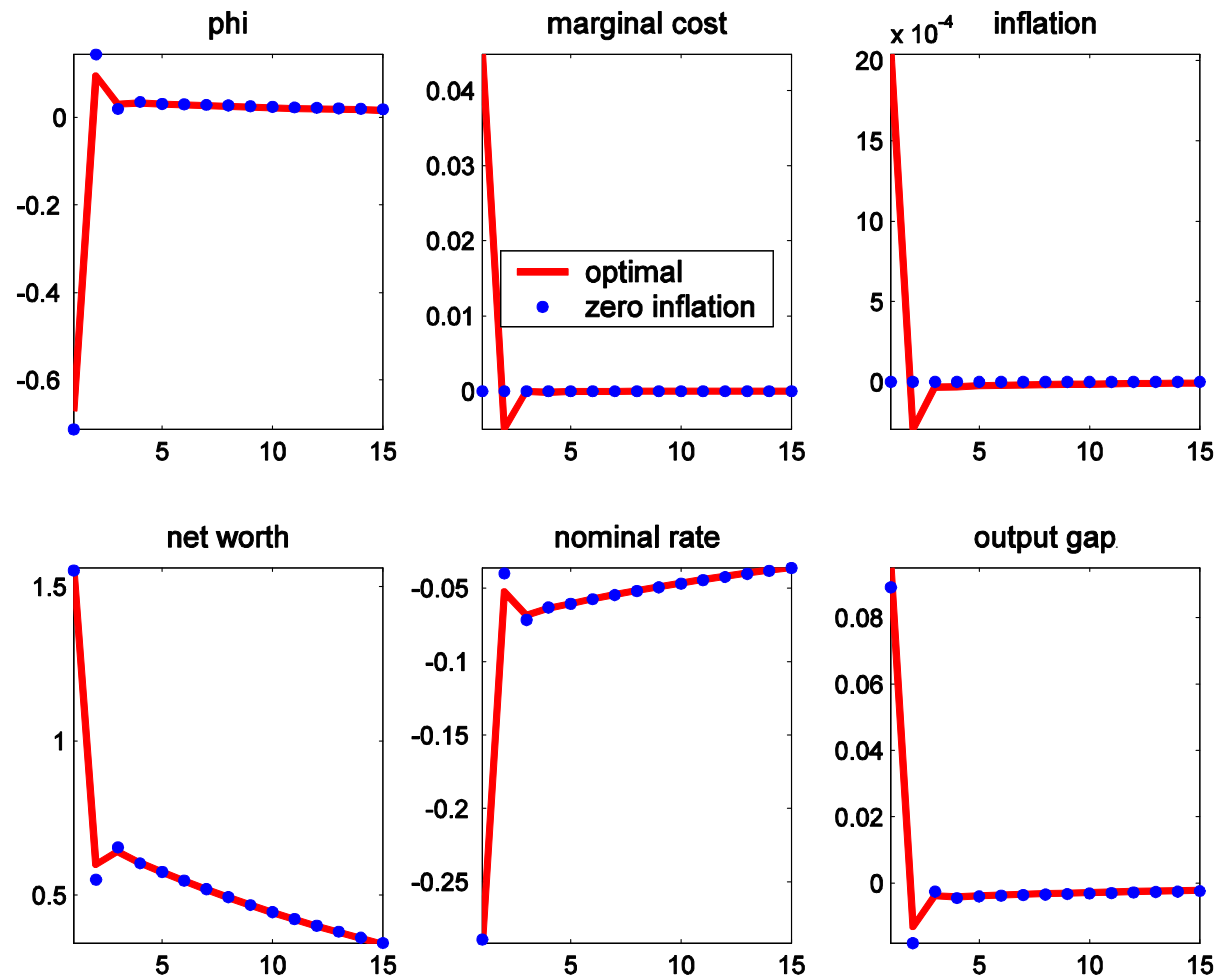
(We assume that the TFP follows an AR(1) process with autoregressive coefficient 0.95, whereas net worth and markup processes are assumed to slightly less persistent with AR coefficients 0.9. Welfare losses are scaled by the variance of the innovation in the exogenous processes for TFP (σ_a^2), net worth (σ_n^2) and markup (σ_π^2), respectively. Each table entry is multiplied by 100.)

Figure 1A: Impulse response to a technology shock for $\sigma = 0.16$, $\theta = 0.47$.



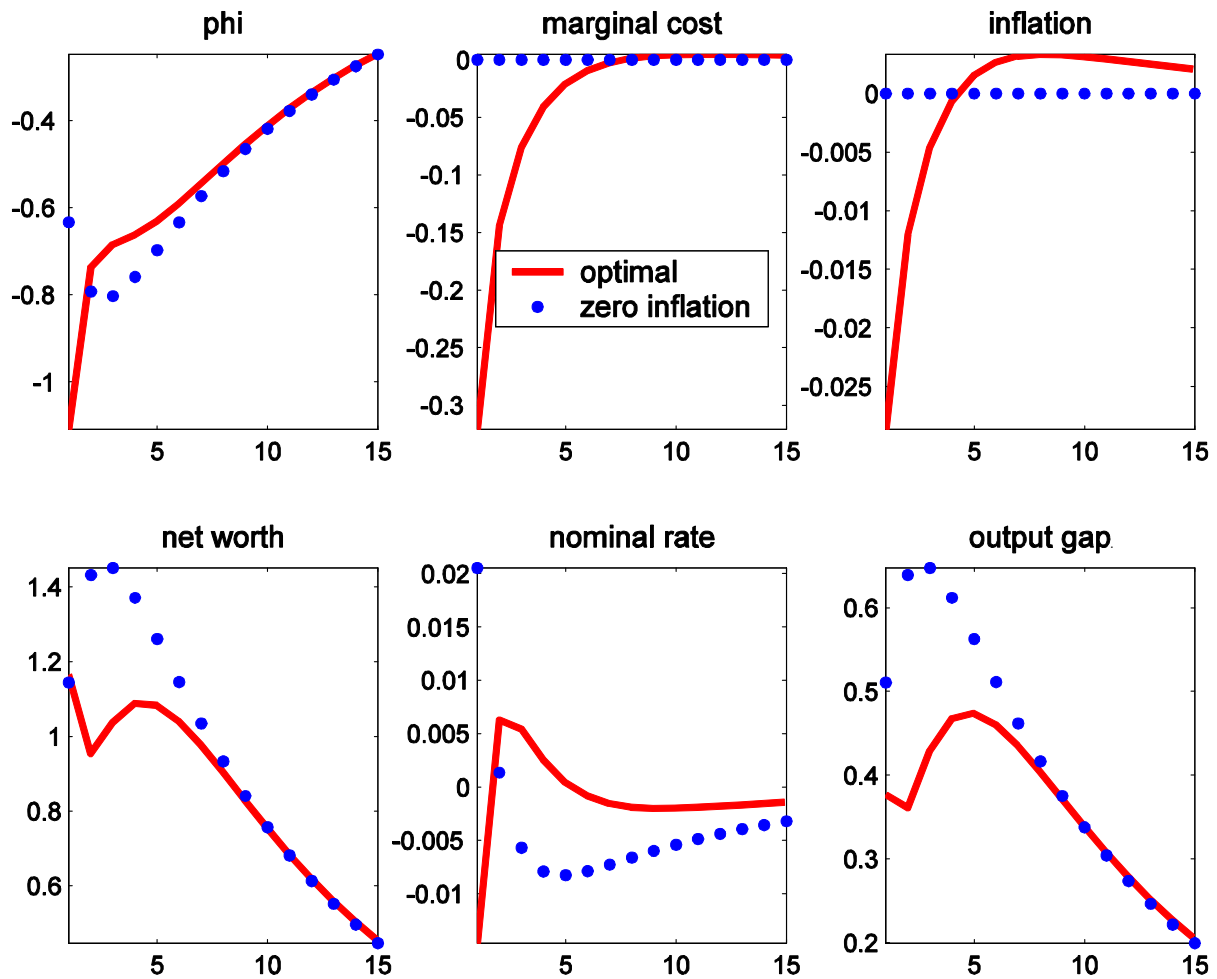
Note: The x-axis measure quarters after the shock. The y-axis measures percentage deviations from the steady state.

Figure 1B: Impulse response to a technology shock for $\sigma = 2$, $\theta = 2$.



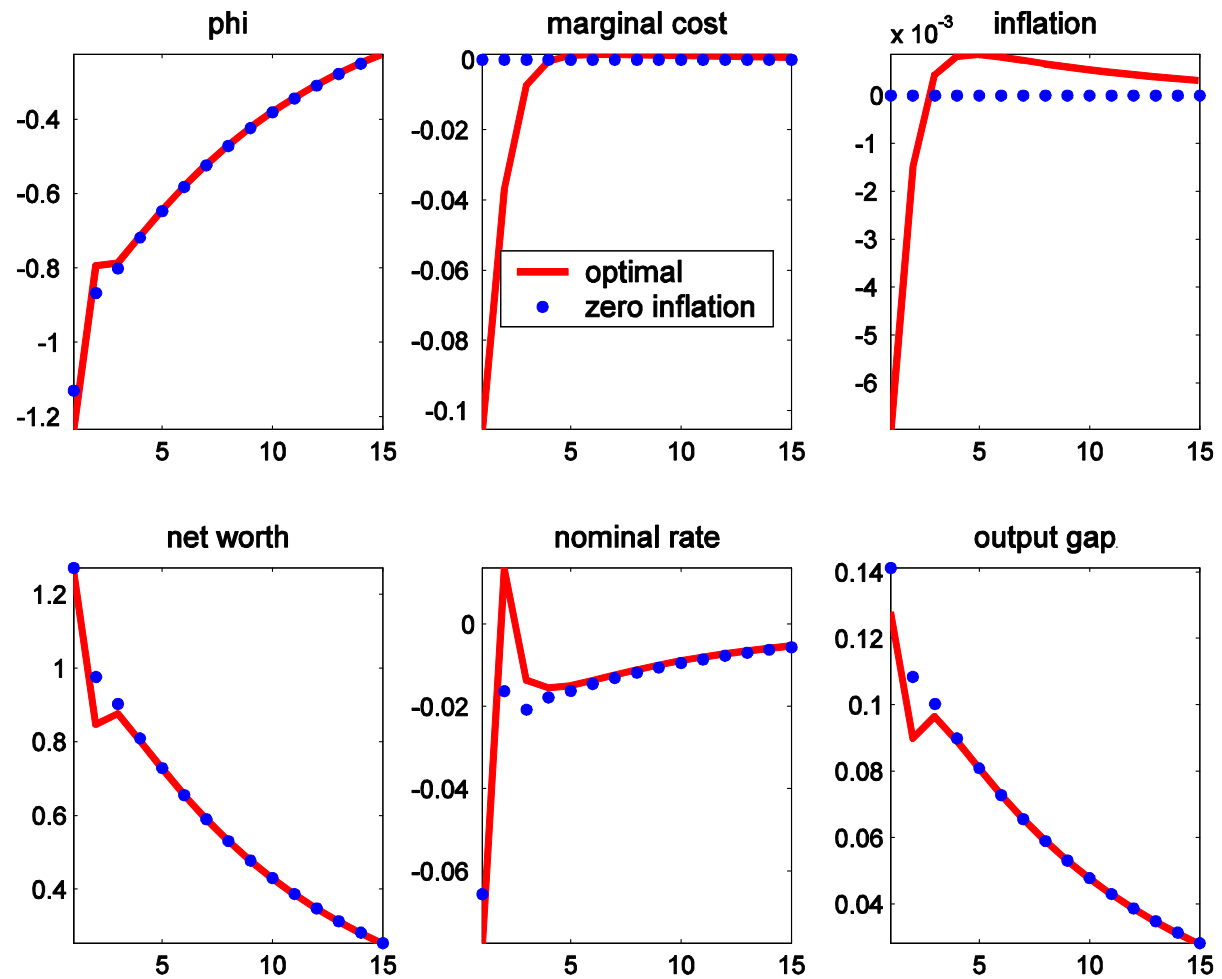
Note: See Figure 1A.

Figure 2A: Impulse response to a net-worth shock for $\sigma = 0.16$, $\theta = 0.47$.



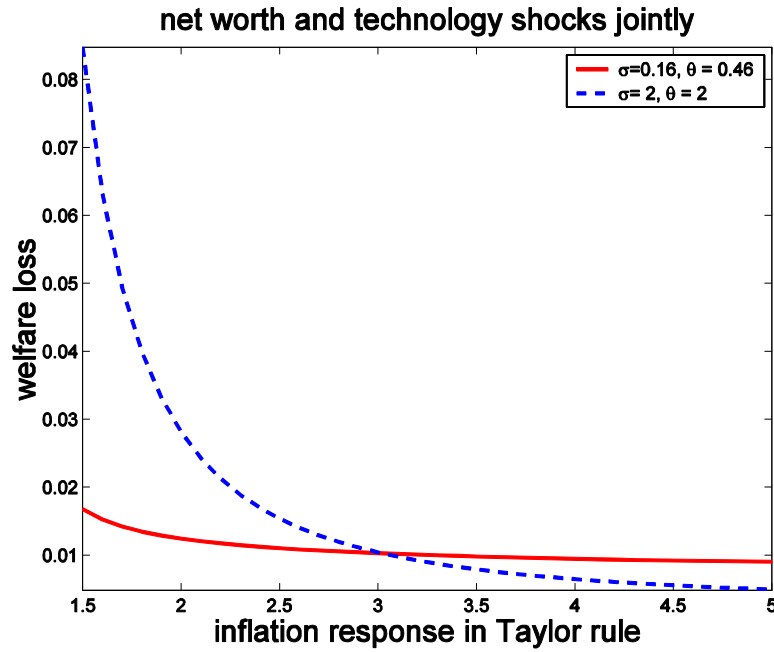
Note: See Figure 1A.

Figure 2B: Impulse response to a net-worth shock for $\sigma = 2$, $\theta = 2$.



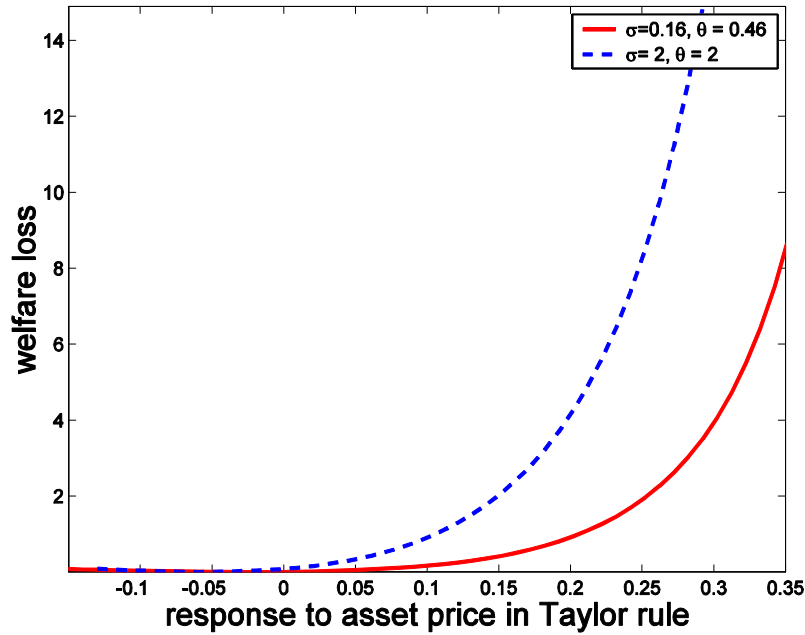
Note: See Figure 1A.

Figure 3: Welfare as a function of the inflation response in the Taylor rule



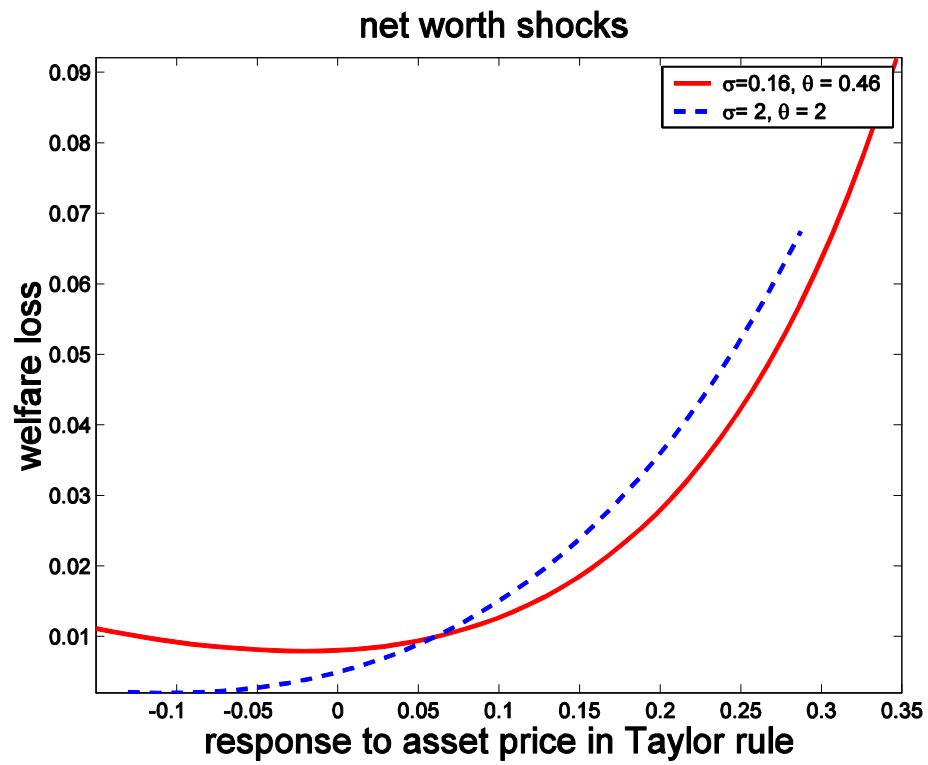
Note: The innovations into the processes for technology and net worth are assumed to have standard deviation 0.007. The $AR(1)$ coefficients are 0.95 and 0.9, respectively. The Taylor rule is of the form $\hat{R}_t = \tau \hat{\pi}_t$, where the inflation response τ is plotted on the x-axis. Welfare losses have been multiplied by 100.

Figure 4: Welfare as a function of the asset price response in the Taylor rule: TFP shocks technology shocks



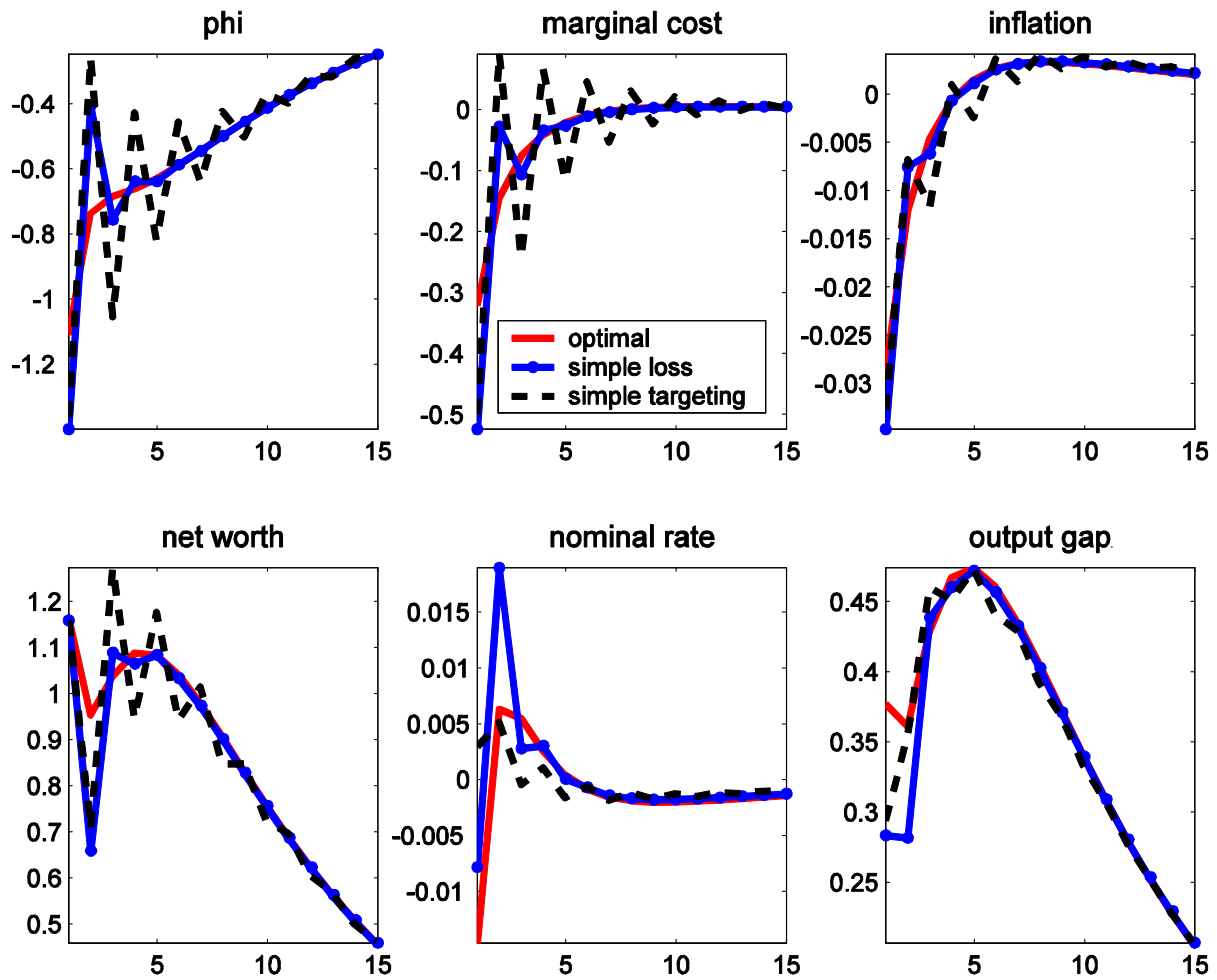
Note: The Taylor rule is of the form $\hat{R}_t = 1.5\hat{\pi}_t + \tau_q \hat{q}_t$, where the asset price response τ_q is plotted on the x-axis. Welfare losses have been multiplied by 100.

Figure 5: Welfare as a function of the asset price response in the Taylor rule: net worth shocks



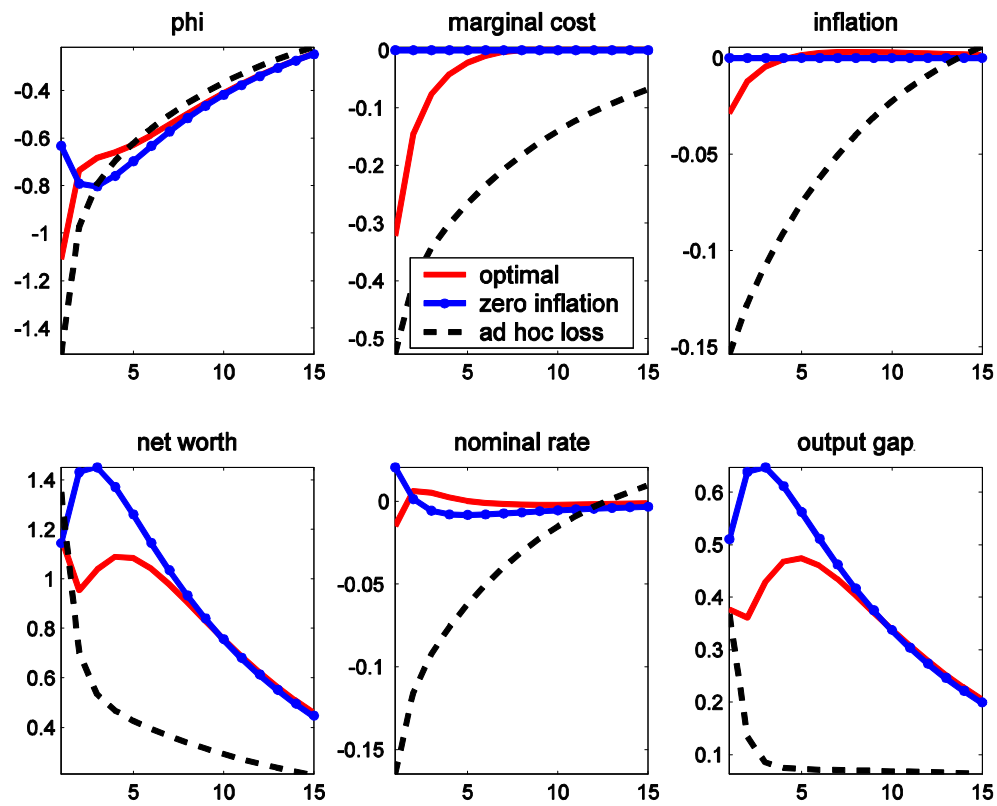
Note: See Figure 3

Figure 6: Alternative loss functions: response to net worth shocks ($\sigma = 0.16$, $\theta = 0.47$).



Note: *Optimal* refers to optimal monetary policy as presented in the previous section. *Simple loss* refers to optimal policy under commitment given the correct model and zero weight on the risk premium in the loss function. *Simple targeting* refers to policy implementing the targeting criterion given by (30)

Figure 7: Ad hoc loss function: response to net worth shocks ($\sigma = 0.16$, $\theta = 0.47$).
 (ad hoc loss function imposes common weight on inflation and the gap)



Note: See Figure 1A

APPENDIX

1. The model in log-deviations.

$$\sigma \hat{y}_t + \theta \hat{L}_t = \hat{w}_t \quad (\text{A1})$$

$$\sigma \hat{y}_t + \theta \hat{u}_t = \hat{r}_t \quad (\text{A2})$$

$$\sigma(E_t \hat{y}_{t+1} - \hat{y}_t) = \hat{R}_t - E_t \hat{\pi}_{t+1} \quad (\text{A3})$$

$$\hat{q}_t = \beta E_t \hat{q}_{t+1} + (1 - \beta) E_t \hat{d}_{t+1} - \sigma(E_t \hat{y}_{t+1} - \hat{y}_t) \quad (\text{A4})$$

$$\hat{e}_t + \hat{q}_t = \hat{z}_t + \hat{y}_t \quad (\text{A5})$$

$$\hat{w}_t + \hat{L}_t = \hat{e}_{t-1} + \beta \hat{q}_t + (1 - \beta) \hat{d}_t + \hat{n}_t \quad (\text{A6})$$

$$\hat{z}_t + \hat{y}_t = \hat{r}_t + \hat{u}_t \quad (\text{A7})$$

$$\hat{y}_t = \hat{a}_t + (1 - \alpha) \hat{u}_t + \alpha \hat{L}_t \quad (\text{A8})$$

$$\hat{\pi}_t = \lambda \hat{z}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \epsilon_t^\pi \quad (\text{A9})$$

$$\hat{d}_t = \hat{y}_t - \left(\frac{z}{1-z} \right) \hat{z}_t. \quad (\text{A10})$$

$$\hat{\phi}_t = \hat{z}_t + \hat{y}_t - \hat{w}_t - \hat{L}_t \quad (\text{A11})$$

where we have used the fact that $\hat{p}_t = \hat{z}_t + \hat{a}_t$ and $\hat{c}_t = \hat{y}_t = \hat{a}_t + \hat{x}_t$. The system is closed with a policy rule for the nominal interest rate.

2. Steady-state and subsidies.

In the steady state we will choose subsidies for L-wages (w_{sub}) and u-wages (r_{sub}) to achieve the efficient steady-state. In particular we have:

$$B_1 c_{ss}^\sigma L_{ss}^\theta = w_{ss}(1 + w_{sub})$$

$$B_2 c_{ss}^\sigma u_{ss}^\theta = r_{ss}(1 + r_{sub})$$

$$Q_{ss} = \frac{\beta}{1 - \beta} D_{ss}$$

$$Z_{ss} = P_{ss} = \frac{\varepsilon - 1}{\varepsilon}$$

$$D_{ss} = X_{ss}(1 - Z_{ss})$$

$$P_{ss} MPL_{ss} = w_{ss}(1 + \phi_{ss})$$

$$P_{ss} MPU_{ss} = r_{ss}$$

$$e_{ss} Q_{ss} = \gamma \alpha P_{ss} x_{ss}$$

$$e_{ss}(Q_{ss} + D_{ss}) = w_{ss} L_{ss}$$

$$c_{ss} = x_{ss}$$

We pick the subsidies so that the first-best level of output is achieved:

$$B_1 c_{ss}^\sigma L_{ss}^\theta = MPL_{ss} \frac{Z_{ss}}{(1 + \phi_{ss})} (1 + w_{sub}) = MPL_{ss}$$

$$B_2 c_{ss}^\sigma u_{ss}^\theta = MPU_{ss} Z_{ss} (1 + r_{sub}) = MPU_{ss}$$

Note that the only steady-state value that affects the log-linearized dynamics is the steady state value of z .

3. Deterministic Dynamics.

Flexible prices. In the case of flexible prices the characteristic equation of the deterministic system is quadratic and is given by:

$$h(x) \equiv \beta x^2 - (1 + \beta A)x + A.$$

where $A \equiv \frac{\alpha(1-\sigma)}{\alpha(1-\sigma)+\theta+\sigma}$. The roots are $1/\beta$ and A . Hence, there is determinacy if and only if A is in the unit circle. For $\sigma > 1$, A is in the unit circle as long as α is not too large. Let α_{cut} denote the upper bound for α . We have:

$$\alpha_{cut} = \frac{1}{2} \left(\frac{(\theta+\sigma)}{(\sigma-1)} \right), \text{ for } \sigma > 1.$$

In summary, for flexible prices there is a unique positive root if and only if $\sigma < 1$. If $\sigma > 1$, the system is exactly determined if $\alpha < \alpha_{cut}$. But in this case there are oscillatory dynamics. If $\sigma > 1$, and $\alpha > \alpha_{cut}$, the system is over-determined.

Sticky prices. In this case the characteristic equation of the system is a quartic. With one pre-determined variable, there must be one stable eigenvalue for determinacy. This eigenvalue will determine the deterministic dynamics of the system. Let $f(x)$ denote this quartic. We have the following:

$$f(1) = \frac{1-\beta}{\beta}$$

$$f(0) = \alpha \left(\frac{\sigma(1+\tau\lambda)-\tau\lambda}{\beta D} \right)$$

where $D \equiv \{\tau\lambda(\sigma + \theta) + \sigma[1 - \alpha(1 - \beta)(1 + m)]\}$. We assume that $D > 0$ which will hold for all typical calibrations. We are interested in the nature of the dynamics, so let us assume that we have determinacy. Since $f(1) > 0$, the (assumed-to-be) unique stable root is positive if and only

if $f(0) < 0$ or $\sigma < \frac{\lambda\tau}{1+\lambda\tau}$. This bound is independent of α . In contrast, if $\sigma > \frac{\lambda\tau}{1+\lambda\tau}$, then $f(0) > 0$ so that the stable root is negative.

As with the flexible price model, it is possible for the system to be over-determined. A necessary condition for determinacy is that $f(-1) < 0$. This is satisfied if $\alpha < \alpha_{cut}$, where

$$\alpha_{cut} = \frac{\lambda(\tau+1)(\sigma+\theta)+2\sigma(1+\beta)}{2[\lambda(\tau+1)(\sigma-1)+2\sigma(1+\beta)]} \quad 1 \text{ if } \sigma > \frac{\lambda(\tau+1)}{[\lambda(\tau+1)+2(1+\beta)]}$$

$$\alpha_{cut} = 1 \text{ if } \sigma < \frac{\lambda(\tau+1)}{[\lambda(\tau+1)+2(1+\beta)]}.$$

Numerical simulations suggest that when $\alpha > \alpha_{cut}$ (so that $f(-1) > 0$ and $f(1) > 0$) the model is typically overdetermined. (Note that α_{cut} collapses to the flexible price expression as $\lambda \rightarrow \infty$.) This cut-off is decreasing in σ so that the smallest it can be (as we vary σ) is $1/2$.

It is possible that $\sigma < \frac{\lambda\tau}{1+\lambda\tau}$ (so that $f(0) < 0$) and $\alpha > \alpha_{cut}$ (so that $f(-1) > 0$), so that the system is underdetermined (two roots in the unit circle). However, this indeterminacy does not arise for plausible calibrations. To see this note that α_{cut} is decreasing in σ so that the smallest α_{cut} can be is given by setting $\sigma = \frac{\lambda\tau}{1+\lambda\tau}$ so that we have

$$\alpha_{cut} \left(\text{with } \sigma = \frac{\lambda\tau}{1+\lambda\tau} \right) = \frac{1}{2} + \left(\frac{1+\theta}{2} \right) \left(\frac{(\tau+1)(\lambda\tau+1)}{2\beta\tau+\tau-1} \right),$$

For typical calibrations this is greater than one so that $\alpha > \alpha_{cut}$ will not be satisfied.

In summary, assuming that there is only one root in the unit circle, the dynamics are characterized as follows: (i) if $\sigma < \frac{\lambda\tau}{1+\lambda\tau}$ the single root is positive so that there are non-oscillatory dynamics, (ii) this root is zero if $\sigma = \frac{\lambda\tau}{1+\lambda\tau}$ so that there are no deterministic dynamics,

and (iii) this root is negative if $\sigma > \frac{\lambda\tau}{1+\lambda\tau}$ so that there are oscillatory dynamics. In this latter case, there is an upper bound on the value of α given by α_{cut} , in which the system is typically over-determined if $\alpha > \alpha_{cut}$.

4. Proofs.

Proposition 1:

The gross inflation rate that maximizes a present value welfare criterion is unity in the steady state regardless of whether subsidies exist that make the steady state efficient.

Proof:

The policymaker maximizes $W_t = E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}$, where U is the period utility function, subject to the nonlinear equilibrium conditions. For this proof, we only need to consider those constraints that involve the inflation rate, i.e. the resource constraint and the price setting equation. These are:

$$C_t = A_t L_t^\alpha U_t^{1-\alpha} \left(1 - \frac{\varphi}{2} (\pi_t - 1)^2\right)$$

$$((\epsilon - 1) - \epsilon z_t + \varphi(\pi_t - 1)\pi_t) Y_t C_t^{-\sigma} = E_t \varphi \beta (\pi_{t+1} - 1) \pi_{t+1} Y_{t+1} C_{t+1}^{-\sigma}$$

Here, ϵ is the elasticity of the final goods bundler. Let λ_t^1 and λ_t^2 , respectively, denote the Lagrange multipliers on these constraints. The first-order condition of the Ramsey planner problem for choice of π_t evaluated at the steady state is

$$0 = \lambda_{ss}^1 \varphi (\pi_{ss} - 1) A_{ss} L_{ss}^\alpha U_{ss}^{1-\alpha}$$

The resource constraint is a binding constraint on the policymaker, $\lambda_{ss}^1 > 0$, consequently the optimal steady state gross inflation rate is unity. Since none of the constraints used in this proof depend on subsidies, the results holds regardless of whether subsidies exist that render the steady state efficient. QED.

Proposition 2: A quadratic approximation to the welfare function around the zero net inflation steady state is given by:

$$\frac{U(t) - U^*(t)}{U_c(ss)c_{ss}} \approx -\frac{1}{2} \left[\frac{(\epsilon - 1)}{\lambda} \hat{\pi}_t^2 + (\sigma + \theta)(\hat{y}_t^g)^2 + \frac{\alpha(1 - \alpha)}{1 + \theta} \hat{\phi}_t^2 \right] + tip$$

Proof:

For ease of exposition, we derive the welfare function in terms of absolute deviations and then convert to percentage deviations in the last step. We define $\tilde{X}_t \equiv X_t - X_{ss}$ as absolute deviations from the steady state and $\hat{X}_t \equiv (X_t - X_{ss})/X_{ss}$ as percentage deviations. In a first step, we take a quadratic approximation to the utility functional:

$$U(t) - U(ss) \approx U_c \tilde{c}_t + U_L \tilde{L}_t + U_u \tilde{u}_t + \frac{1}{2} (U_{cc} \tilde{c}_t^2 + U_{LL} \tilde{L}_t^2 + U_{uu} \tilde{u}_t^2) + U_{cL} \tilde{c}_t \tilde{L}_t + U_{cu} \tilde{c}_t \tilde{u}_t + U_{Lu} \tilde{L}_t \tilde{u}_t$$

The resource constraint is given by:

$$c_t = A_t L_t^\alpha u_t^{1-\alpha} \left[1 - \frac{\varphi(\pi_t - 1)^2}{2} \right] \equiv A_t f(L_t, u_t) \left[1 - \frac{\varphi(\pi_t - 1)^2}{2} \right]$$

A quadratic approximation to this expression is given by: (note that $\pi_{ss} = 1$)

$$\tilde{c}_t \approx c_{ss} \tilde{A}_t + f_L(ss) \tilde{L}_t + f_u(ss) \tilde{u}_t + \frac{1}{2} (f_{LL}(ss) \tilde{L}_t^2 + f_{uu}(ss) \tilde{u}_t^2 - c_{ss} \varphi \tilde{\pi}_t^2) + f_{LA}(ss) \tilde{L}_t \tilde{A}_t + f_{uA}(ss) \tilde{u}_t \tilde{A}_t + f_{Lu}(ss) \tilde{L}_t \tilde{u}_t$$

Since we have assumed separable utility, and that the steady-state is efficient, we have:

$$U(t) - U(ss) \approx \frac{1}{2} (U_{cc} \tilde{c}_t^2 + U_{LL} \tilde{L}_t^2 + U_{uu} \tilde{u}_t^2) + \frac{1}{2} U_c (f_{LL}(ss) \tilde{L}_t^2 + f_{uu}(ss) \tilde{u}_t^2 - c_{ss} \varphi \tilde{\pi}_t^2 + f_{LA}(ss) \tilde{L}_t \tilde{A}_t + f_{uA}(ss) \tilde{u}_t \tilde{A}_t + f_{Lu}(ss) \tilde{L}_t \tilde{u}_t) + tip \quad (1)$$

Here, *tip* denotes terms independent of policy. Since all of these terms are of second order, we now only need consider linear terms in the subsequent expansions. The equilibrium choice of u is given by:

$$U_u(t) = U_c(t) f_u(t) P_t$$

Log-linearizing this expression and imposing efficiency in the steady-state, we have:

$$\hat{u}_t = \frac{\alpha(1-\sigma)}{\alpha(1-\sigma) + (\sigma + \theta)} \hat{L}_t + \frac{1}{\alpha(1-\sigma) + (\sigma + \theta)} \hat{z}_t + \frac{(1-\sigma)}{\alpha(1-\sigma) + (\sigma + \theta)} \hat{a}_t$$

We will use this expression to eliminate u from the welfare function (1).

Labor choice can be characterized this way:

$$U_u(t) f_L(t) = U_L(t) f_U(t) (1 + \phi_t)$$

Expanding this up to the first order (and imposing efficiency in the steady-state) we have:

$$\hat{\phi}_t = (\theta + 1)\hat{u}_t - (\theta + 1)\hat{L}_t$$

The efficient response to shocks is easily found by setting $\hat{z}_t = \hat{\phi}_t = 0$.

$$\hat{L}_t^e = \frac{(1-\sigma)}{(\sigma+\theta)} \hat{a}_t$$

$$\hat{u}_t^e = \frac{(1-\sigma)}{(\sigma+\theta)} \hat{a}_t$$

We now define the labor gap to be the gap between actual labor and efficient labor. Substituting this into (1), simplifying and subtracting the same approximations in an efficient economy with flexible prices and no credit frictions (which is independent of policy), we have

$$\frac{U(t) - U^*(t)}{U_c(ss)c_{ss}} \approx -\frac{1}{2} \left[\varphi \hat{\pi}_t^2 + \frac{\alpha(1+\theta)(\sigma+\theta)}{\theta+\alpha+\sigma(1-\alpha)} (\hat{L}_t^g)^2 + \frac{1-\alpha}{\theta+\alpha+\sigma(1-\alpha)} \hat{z}_t^2 \right]$$

Next, we use $\lambda = (\epsilon - 1)/\varphi$ to rewrite the weight on inflation in terms of the slope of the Phillipscurve. Using (16) and (17) to eliminate $\hat{z}_t^2$ and $(\hat{L}_t^g)^2$, we arrive at

$$\frac{U(t) - U^*(t)}{U_c(ss)c_{ss}} \approx -\frac{1}{2} \left[\frac{(\epsilon-1)}{\lambda} \hat{\pi}_t^2 + (\sigma+\theta)(\hat{y}_t^g)^2 + \frac{\alpha(1-\alpha)}{1+\theta} \hat{\phi}_t^2 \right]$$

QED.

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