

International Trade and Aggregate Fluctuations in Granular Economies*

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Abstract

This paper proposes a new channel through which international trade affects macroeconomic volatility. We study a multi-country model with heterogeneous firms that are subject to idiosyncratic firm-specific shocks. When the distribution of firm sizes follows a power law with exponent sufficiently close to -1 , the idiosyncratic shocks to large firms have an impact on aggregate volatility. Opening to trade increases the importance of large firms to the economy, thus raising macroeconomic volatility. We next explore the quantitative properties of the model calibrated to data for the 50 largest economies in the world. Our simulation exercise shows that the contribution of trade to aggregate fluctuations depends strongly on country size: in an economy such as the U.S., that accounts for one-third of world GDP, international trade increases volatility by about 3.5%. By contrast, trade increases aggregate volatility by some 30% in a small open economy, such as Belgium or Poland. The model performs well in matching the elasticity of macroeconomic volatility with respect to country size observed in cross-country data.

JEL Classifications: F12, F15, F41

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1 Introduction

Macroeconomic volatility varies substantially across economies. Over the past 35 years, the standard deviation of per capita GDP growth is 2.5 times higher in non-OECD countries compared to the OECD countries. Understanding the sources of these differences is important, as aggregate volatility itself has an impact on a wide variety of economic outcomes.¹ In particular, it has been suggested that openness to trade may contribute to macroeconomic volatility.² Indeed, greater volatility and increased insecurity are often invoked as a negative consequence of globalization (Rodrik 1997, ILO 2004). As world trade experienced dramatic growth in recent decades, understanding the impact of trade openness on macroeconomic fluctuations has become increasingly important.

This paper proposes a new link between trade openness and macroeconomic volatility, which focuses on the role of large exporters. Empirical evidence reveals that the distribution of firm size is very fat-tailed – the typical economy is dominated by a few large firms (Axtell 2001). In a recent contribution, Gabaix (2005) demonstrates that under these conditions, idiosyncratic shocks to individual firms do not average out and can instead generate aggregate fluctuations (see also Delli Gatti et al., 2005). The economy is “granular,” rather than smooth. Gabaix (2005) provides both statistical and anecdotal evidence that even in the largest and most diversified economy in the world – the United States – the biggest firms can appreciably affect macroeconomic fluctuations.

How would international trade affect macroeconomic volatility in such economies? Seminal contributions by Melitz (2003) and Bernard, Eaton, Jensen and Kortum (2003) build models of production and trade in which the unit of observation is a firm. The main prediction of these models is that only the largest and most productive firms export. When a country opens to trade, the largest firms become even larger, as they take advantage of export opportunities and grow to a size not attainable in autarky. On the other hand, smaller ones shrink or disappear.

The main idea of this paper is that trade openness increases volatility by making the economy more granular. This is the straightforward consequence of the effect described above: after trade opening, the biggest firms become even larger relative to the size of the economy, thus contributing more to the overall GDP fluctuations.

¹Numerous studies identify its effects on long-run growth (Ramey and Ramey 1995), welfare (Pallage and Robe 2003, Barlevy 2004), as well as inequality and poverty (Gavin and Hausmann 1998, Laursen and Mahajan 2005).

²A number of empirical studies show that trade openness is associated with higher volatility in a cross-section of countries (Easterly, Islam and Stiglitz 2001, Kose, Prasad and Terrones 2003), as well as at the industry level (di Giovanni and Levchenko 2007). Theoretical contributions include Newbery and Stiglitz (1984) and Kraay and Ventura (2007).

Anecdotal evidence on the importance of large firms for aggregate fluctuations abounds. Here, we would like to describe two examples in which the role of international trade is especially evident. In New Zealand one firm, Fonterra, is responsible for a full one-third of global dairy exports (it is the world's single largest exporter of dairy products). Such a large exporter from such a small country would clearly matter for the macroeconomy. Indeed, Fonterra accounts for 20% of New Zealand's overall exports, and 7% of its GDP. Two additional points about this firm are worth noting. First, 95% of Fonterra's output is exported. Thus, international trade clearly plays a prominent role in making Fonterra as large as it is. And second, the distribution of firm size in the dairy sector is indeed highly skewed. The second largest producer of dairy products in New Zealand is 1.3% the size of Fonterra.³ This phenomenon is not confined to commodity exporting countries. In Korea, a larger manufacturing-based economy, the 10 biggest business groups account for 54% of GDP and 51% of total exports. Even among the top 10, the distribution of firm size and total exports is extremely skewed. The largest one, Samsung, is responsible for 23% of exports and 14% of GDP (see Figure 1).⁴

To formally illustrate the idea that trade increases the importance of large firms in macroeconomic fluctuations, and assess its quantitative relevance, we build and calibrate a multi-country model of trade with heterogeneous firms in the spirit of Melitz (2003) and Chaney (2008). Monopolistically competitive producers differ in their productivity, and face fixed costs to both setting up production and exporting. A firm decides to enter the domestic market if its variable profits cover the fixed costs of production, and it begins exporting if the variable export profits cover fixed costs of accessing a foreign market. As is the case in virtually every application of this type, only the largest/most productive firms export. Most of the recent implementations of heterogeneous firms models assume that the distribution of firm productivity is Pareto (see, e.g., Helpman, Melitz and Yeaple 2004, Ghironi and Melitz 2005, Chaney 2008, Arkolakis 2008). It turns out that the Pareto distribution of productivity implies that distribution of firm sales follows a power law, which is a requirement derived by Gabaix (2005) for the economy to exhibit granular features. In this sense, the link between these two literatures is even more natural than first expected.

We calibrate a model world economy using data for the 50 largest countries in the world. The model matches quite well the overall and bilateral trade volumes for the countries in

³These figures are obtained from <http://www.maf.govt.nz/mafnet/rural-nz/profitability-and-economics/contribution-of-land-based-industries-nz-economic-growth/contribution07.htm> and <http://tvnz.co.nz/view/page/423466/146647>.

⁴It turns out that the size distribution of firms is quite skewed even within business groups. For instance, breaking Samsung down into its constituent firms reveals that the sales of Samsung Electronics alone accounted for 7% of GDP and 15.5% of Korea's exports in 2006. We would like to thank Wonhyuk Lim of KDI for providing us with data on Korean firm and business group sales and exports.

the sample. In addition, it matches the elasticity of aggregate volatility with respect to country size found in the literature (Canning, Amaral, Lee, Meyer and Stanley 1998). We show that international trade plays a crucial role in generating this relationship: without it, the model yields a much flatter relationship between country size and volatility. We then use our calibrated model to perform counterfactual exercises. The first reveals the contribution of international trade to aggregate volatility. We compute what aggregate volatility would be for each country in autarky, and compare it to the volatility under the current trade regime. This exercise shows that the importance of trade for aggregate volatility varies greatly depending on country characteristics. In the largest economies like the U.S. or Japan, aggregate volatility is only 3.5-4% higher than it would have been in complete autarky. In small, but remote economies such as South Africa or New Zealand, trade raises volatility by 15% compared to autarky. Finally, in small, highly integrated economies such as Belgium or Poland, international trade raises aggregate volatility by some 30%.

We then compute the change in aggregate volatility that would occur if trade costs decreased below their current levels. Our simulations show that a 50% reduction in international trade costs will lead to an average 21% increase in aggregate volatility in our set of countries, but the implied impact varies a great deal. Small, remote countries such as New Zealand or Chile are predicted to experience an increase in volatility of as much as 30% as a result of further trade opening. By contrast, the largest countries such as the U.S. or Japan show the lowest increase, of as little as 9%.

The theoretical link between trade openness and volatility studied in this paper has not previously been proposed. Traditional explanations have focused on the propagation of global demand or supply shocks (Newbery and Stiglitz 1984), or on the notion that a more open economy is specialized in fewer sectors. Kraay and Ventura (2007) argue that developing countries are more volatile than developed ones because their comparative advantage is in goods with more elastic product demand and factor supply. We show that trade can increase volatility even if the nature of shocks affecting the firms is unchanged upon opening. Our model also reveals that what matters is not only diversification across sectors, but also across firms within the economy. Finally, our mechanism does not rely on cross-country differences in elasticities in goods and factor markets.

This paper is part of a small but growing literature that studies the relationship between international trade, the production structure, and the macroeconomy. In our previous work (di Giovanni and Levchenko 2007), we use sector-level data to demonstrate that trade openness has a robust positive effect on sector-specific volatility, and that it results in greater sectoral specialization. In di Giovanni and Levchenko (2008), we argue that countries spe-

cializing in especially risky sectors will experience higher macroeconomic volatility than countries exporting in less volatile sectors. While the results in these two papers are informative, the use of sector-level data implies that we cannot be precise about the specific mechanisms at work in the trade-volatility link. Canals, Gabaix, Vilarubia and Weinstein (2007) analyze sector-level export data and demonstrate that exports are highly undiversified, both across sectors and across destinations. Furthermore, they show that this feature of export baskets can explain why aggregate macroeconomic variables cannot account for much of the movements in the current account. Ghironi and Melitz (2005) use the heterogeneous firms model to help account for several puzzles in international finance. Our paper differs from this contribution in its focus on aggregate volatility instead of the behavior of prices. In addition, while the Ghironi and Melitz model analyzes the consequences of a persistent aggregate shock, in this paper we study firm-level idiosyncratic shocks instead. Finally, while Ghironi and Melitz build a two-country model and confirm its quantitative relevance by generating impulse response functions matched to U.S. data, our multi-country model and quantitative approach seek to explain cross-country differences.⁵

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 simulates the model economy and presents the main quantitative and empirical results. Section 4 presents robustness checks and results based on model perturbations. Section 5 concludes.

2 Theoretical Framework

Consider a model in the spirit of Melitz (2003), but with a discrete number of goods as in Krugman (1980). The world is comprised of N countries, indexed by $i, j = 1, \dots, N$. In country i , consumers maximize

$$\begin{aligned} \max \left[\sum_{k=1}^{J_i} c_i(k)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ s.t. \\ \sum_{k=1}^{J_i} p_i(k) c_i(k) = X_i, \end{aligned}$$

where $c_i(k)$ is consumption of good k in country i , $p_i(k)$ is the price of this good, X_i is total expenditure in the economy, and J_i is the number of varieties consumed in country i ,

⁵Alessandria and Choi (2007) provide another analysis of the impact of firm entry and exit decisions on the macroeconomy. They build a two-country model calibrated to the U.S. data, and analyze the impact of an aggregate shock on the extensive margin of exports. In their model, firm-specific productivity displays no persistence, and the economy is not granular: idiosyncratic shocks to firms average out completely.

coming from all countries. It is well known that demand for variety k is equal to

$$c_i(k) = \frac{X_i}{P_i^{1-\varepsilon}} p_i(k)^{-\varepsilon} \quad (1)$$

in country i , where P_i is the ideal price index in this economy,

$$P_i = \left[\sum_{k=1}^{J_i} p_i(k)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (2)$$

There is one factor of production, labor, with country endowments given by L_i , $i = 1, \dots, N$. Each country has a fixed number of potential (but not actual) entrepreneurs $\bar{I}_i$.⁶ Each potential entrepreneur can produce a unique CES variety, and thus has some market power: it faces the demand for its variety given by (1). There are both fixed and variable costs of production and trade. The timing in this economy is given in Figure 2. At the beginning of the period, each potential entrant $k = 1, \dots, \bar{I}_i$ in each $i = 1, \dots, N$ learns its type, which is the marginal cost $a(k)$. On the basis of this cost, each entrepreneur in country i decides whether or not to pay the fixed cost of production f_{ii} , and which, if any, export markets to serve. To start exporting from country j to country i , a firm must pay the fixed cost f_{ij} , and an iceberg per-unit cost of $\tau_{ij} > 1$.⁷ We normalize the iceberg cost of domestic sales to one: $\tau_{ii} = 1$. Having paid the fixed costs of entering these markets, the firm learns the realization of transitory shock $z(k)$.⁸ We assume that $z(k)$ are i.i.d. across firms. Once all of the uncertainty has been realized, each firm produces with a marginal cost $a(k)z(k)$, markets clear, and consumption takes place.

Note that the assumptions we put on the timing of events, namely that the decision to enter markets takes place before $z(k)$ is realized, implies that the realization of the firm-specific transitory shock does not affect the equilibrium number of firms in each market. This simplification lets us analyze the equilibrium production allocation as an approximation around a case in which the variance of z is zero. That is, we abstract from the extensive margin of exports, and entry and exit of firms in response to transitory shocks.⁹ This simplification delivers substantial analytical convenience, while it is unlikely to affect the results. This is because the focus of the paper is on the role of the largest firms in generating aggregate volatility, and the largest firms are inframarginal: their entry decision is

⁶For analytical convenience, we adopt a fixed number of potential entrants as in Chaney (2008) rather than modeling free entry as in Melitz (2003). This is because with free entry, one would have to solve jointly for the wages and the equilibrium number of firms in each country, doubling the number of equations and unknowns.

⁷That is, the firm in country j must ship $\tau_{ij} > 1$ units to country i in order for one unit of the good to arrive there.

⁸The assumption that $z(k)$ is transitory is not crucial for the basic qualitative results in this paper. We adopt it mainly for analytical convenience.

⁹The adjustment in the extensive margin in response to shocks has been studied by Ghironi and Melitz (2005) and Alessandria and Choi (2007).

unlikely to be affected by the realization of the transitory shock. Note also that this timing assumption implies that our analytical approach is akin to the common one of analyzing the response to shocks in deviations from a non-stochastic steady state.

Let w_j be the wage paid to workers in country j . Firm k from country j selling to country i faces a demand curve given by (1), and has a marginal cost $\tau_{ij}w_j a(k)z(k)$ of serving this market. As is well known, profit maximizing price is a constant markup over marginal cost, $p_i(k) = \frac{\varepsilon}{\varepsilon-1} \tau_{ij}w_j a(k)z(k)$, the quantity supplied is equal to $\frac{X_i}{P_i^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ij}w_j a(k)z(k) \right)^{-\varepsilon}$, and the total ex-post variable profits are:

$$\pi_{ij}^V(a(k)z(k)) = \frac{X_i}{\varepsilon P_i^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ij}w_j a(k)z(k) \right)^{1-\varepsilon}. \quad (3)$$

Note that these are variable profits of a firm in country j from selling its good to country i only. These expressions are valid for each country pair i, j , including domestic sales: $i = j$.

The production structure of the economy is pinned down by the number of firms from each country that enter each market. In particular, there is a cutoff marginal cost a_{ij} , above which firms in country j do not serve market i . We assume (and later verify in the calibration exercise), that all firms that decide to export abroad are sufficiently productive to also serve their domestic markets. On the other hand, there is a range of productivities for which firms serve their domestic markets, but choose not to export. In this case, firms with marginal cost above a_{ii} in country i do not operate at all. Assuming that the firm maximizes expected profits, the cutoff a_{ij} is given by the following condition:

$$\mathbb{E} [\pi_{ij}^V(a(k)z(k)) - w_j f_{ij} \mid a = a_{ij}] = 0.$$

To go forward with the analysis, we make the following two assumptions:

Assumption 1 *The marginal entrepreneur is small enough that it ignores the impact of its own realization of $z(k)$ on the total expenditure X_i and the price level P_i in all potential destination markets $i = 1, \dots, N$.*

Assumption 2 *The marginal entrepreneur treats X_i and P_i as fixed (non-stochastic).*

The first assumption is not controversial, and has been made in the literature since Dixit and Stiglitz (1977) and Krugman (1980). The second assumption allows us to take X_i and P_i outside of the expectation operator. It amounts to assuming that the entrepreneur ignores the volatility of GDP and inflation when deciding to enter a market.¹⁰ Under these

¹⁰This assumption may be questioned given the focus on large firms in this paper. However, what is crucial in solving for equilibrium is the behavior of the marginal firm, which is small.

two assumptions, plugging in the variable profits (3) and taking the expectation over z , the zero cutoff profit condition for serving market i from country j reduces to:

$$a_{ij} = \frac{\varepsilon - 1}{\varepsilon} \frac{P_i}{\tau_{ij} w_j} \left(\frac{X_i}{\varepsilon w_j f_{ij}} \right)^{\frac{1}{\varepsilon-1}} [\mathbb{E}_z (z^{1-\varepsilon})]^{\frac{1}{\varepsilon-1}} = \frac{\varepsilon - 1}{\varepsilon} \frac{P_i}{\tau_{ij} w_j} \left(\frac{X_i}{\varepsilon w_j f_{ij}} \right)^{\frac{1}{\varepsilon-1}}, \quad (4)$$

where the second equality comes from normalizing the transitory shocks z such that $\mathbb{E}_z (z^{1-\varepsilon}) = 1$.

Closing the model involves finding expressions for a_{ij} , P_i , and w_i for all $i, j = 1, \dots, N$. The price level for country i can be expressed as follows:

$$P_i^{1-\varepsilon} = \sum_{j=1}^N \sum_{k=1}^{I_{ij}} \left[\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} w_j a(k) z(k) \right]^{1-\varepsilon},$$

where I_{ij} is the number of varieties exported from country j to country i . As an approximation, we solve for the equilibrium production allocation and the price levels ignoring firm-specific transitory shocks. Taking the expectations over $a(k)$ and $z(k)$, and using the fact that $\mathbb{E}_z (z^{1-\varepsilon}) = 1$, the price level becomes:

$$P_i^{1-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{1-\varepsilon} \sum_{j=1}^N I_{ij} (\tau_{ij} w_j)^{1-\varepsilon} \mathbb{E}_a (a(k)^{1-\varepsilon} \mid a < a_{ij}). \quad (5)$$

In order to solve the model, we must make a distributional assumption on the a 's:

Assumption 3 *Labor productivity, $1/a$, is Pareto(b, θ), where b is the minimum value labor productivity can take, and θ regulates dispersion.*

It is then straightforward to show that the marginal cost, a , has a distribution function $G(a) = (ba)^\theta$. Furthermore, following Helpman et al. (2004), we define $V(y) = \int_0^y a^{1-\varepsilon} dG(a) = \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} y^{\theta - (\varepsilon - 1)}$. The expression $V(a_{ij})$ is useful for writing the price levels and total profits in the economy. This implies that $\mathbb{E}_a (a(k)^{1-\varepsilon} \mid a < a_{ij}) = \frac{V(a_{ij})}{G(a_{ij})}$.

The number of actual entrants into market i from market j is $I_{ij} = \bar{I}_j G(a_{ij})$. As a result, the price level can be written as:

$$P_i^{1-\varepsilon} = \sum_{j=1}^N \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} w_j \right)^{1-\varepsilon} \bar{I}_j \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} a_{ij}^{\theta - (\varepsilon - 1)},$$

which, after plugging in the expression for a_{ij} in (4), becomes:

$$P_i = \frac{1}{b} \left[\frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left(\frac{X_i}{\varepsilon} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \left(\sum_{j=1}^N \bar{I}_j \left(\frac{1}{\tau_{ij} w_j} \right)^\theta \left(\frac{1}{w_j f_{ij}} \right)^{\frac{\theta - (\varepsilon - 1)}{\varepsilon - 1}} \right)^{-\frac{1}{\theta}}. \quad (6)$$

Having expressed P_i , and a_{ij} in terms of X_i and w_i , for all $i, j = 1, \dots, N$, it remains to close the model by solving for the X_i 's and w_i 's. To do this, we impose balanced trade for each country, and use the convenient property that total profits in the economy are a constant multiple of X_i .

Proposition 1 *Total profits of firms based in country i are a constant multiple of total expenditure: $\Pi_i = \frac{\varepsilon-1}{\theta\varepsilon} X_i$.*

Proof: See Appendix A. ■

Since by definition $X_i = w_i L_i + \Pi_i$, each country's GDP is a constant multiple its total labor income:

$$X_i = \frac{1}{1 - \frac{\varepsilon-1}{\varepsilon\theta}} w_i L_i. \quad (7)$$

The total sales from country i to country j can be written as:

$$X_{ji} = \frac{X_j}{P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i \right)^{1-\varepsilon} \bar{I}_i \frac{b^\theta \theta}{\theta - (\varepsilon-1)} a_{ji}^{\theta-(\varepsilon-1)}.$$

Using the expression for a_{ji} in (4), and P_j in (6), the total exports from i to j become:

$$X_{ji} = \frac{\bar{I}_i X_j \left(\frac{1}{\tau_{ji} w_i} \right)^\theta \left(\frac{1}{w_i f_{ji}} \right)^{\frac{\theta-(\varepsilon-1)}{\varepsilon-1}}}{\sum_{l=1}^N \bar{I}_l \left(\frac{1}{\tau_{jl} w_l} \right)^\theta \left(\frac{1}{w_l f_{jl}} \right)^{\frac{\theta-(\varepsilon-1)}{\varepsilon-1}}}.$$

Using the trade balance conditions, $X_i = \sum_{j=1}^N X_{ji}$ for each $i = 1, \dots, N$, as well as the expression for total GDP X_i leads to the following system of equations in w_i :

$$w_i L_i = \sum_{j=1}^N \frac{\bar{I}_i w_i^{-\left(\frac{\theta\varepsilon}{\varepsilon-1}-1\right)} \tau_{ji}^{-\theta} f_{ji}^{-\frac{\theta-(\varepsilon-1)}{\varepsilon-1}}}{\sum_{l=1}^N \bar{I}_l w_l^{-\left(\frac{\theta\varepsilon}{\varepsilon-1}-1\right)} \tau_{jl}^{-\theta} f_{jl}^{-\frac{\theta-(\varepsilon-1)}{\varepsilon-1}}} w_j L_j, \quad (8)$$

$i = 1, \dots, N$. There are $N - 1$ independent equations in this system, which can be solved numerically for wages in $N - 1$ countries given a numéraire wage in the remaining country. We will solve it numerically in order to carry out the main quantitative exercise in this paper.

2.1 Power Law in Firm Size, Granularity, and Aggregate Volatility

This economy is granular, that is, idiosyncratic shocks to firms result in aggregate fluctuations, if the distribution of firm size follows a power law with the exponent sufficiently close to 1 in absolute value. Denote by lower case $x(a(k), z(k))$ the sales of an individual firm k . Firm sales x in the economy must conform to:

$$\Pr(x > s) = cs^{-\zeta}, \quad (9)$$

where ζ is close to 1.

It turns out that the baseline Melitz-Pareto model delivers a power law in firm size. In this section, we demonstrate the power law in an autarkic economy, and then discuss how the distribution of firm size is affected by international trade. In our model, the expected sales of a firm as a function of its marginal cost are: $x(a) = Ca^{1-\varepsilon}$, where the constant C reflects the size of domestic demand, and we drop the country subscripts. Under the assumption that $1/a \sim \text{Pareto}(b, \theta)$, $\Pr(1/a < y) = 1 - \left(\frac{b}{y}\right)^\theta$. Therefore, the power law follows:

$$\begin{aligned} \Pr(x > s) &= \Pr(Ca^{1-\varepsilon} > s) = \Pr\left(a^{1-\varepsilon} > \frac{s}{C}\right) = \\ \Pr\left(\left(\frac{1}{a}\right)^{\varepsilon-1} > \frac{s}{C}\right) &= \Pr\left(\frac{1}{a} > \left(\frac{s}{C}\right)^{\frac{1}{\varepsilon-1}}\right) = \left(\frac{b^{\varepsilon-1}C}{s}\right)^{\frac{\theta}{\varepsilon-1}} = (b^{\varepsilon-1}C)^{\frac{\theta}{\varepsilon-1}} s^{-\frac{\theta}{\varepsilon-1}} \end{aligned}$$

satisfying (9) for $c = (b^{\varepsilon-1}C)^{\frac{\theta}{\varepsilon-1}}$ and $\zeta = \frac{\theta}{\varepsilon-1}$. This relationship is depicted in Figure 3. In addition, this calculation shows that $x \sim \text{Pareto}\left(b^{\varepsilon-1}C, \frac{\theta}{\varepsilon-1}\right)$. Thus, our economy will be granular if $\frac{\theta}{\varepsilon-1}$ is close enough to 1, which appears to be the case in practice (see Axtell 2001).

We now derive the expression for aggregate volatility in our economy. Note that there are no aggregate shocks in the model, only the firm-specific idiosyncratic shocks. A firm with marginal cost a and realization of transitory shock z has sales of

$$x(a, z) = \frac{X}{P^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1}az\right)^{1-\varepsilon} = \frac{X}{P^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1}a\right)^{1-\varepsilon} \tilde{z}, \quad (10)$$

where $\tilde{z} \equiv z^{1-\varepsilon}$. We already assumed that $E_z(\tilde{z}) = 1$, and now we further suppose that $\text{Var}_z(\tilde{z}) = \sigma^2$. Expected sales for the firm with productivity a are:

$$E_z[x(a, z)] = \frac{X}{P^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1}a\right)^{1-\varepsilon}. \quad (11)$$

The total sales in the economy is defined by:

$$X = \sum_{k=1}^I x(a(k), z(k)). \quad (12)$$

Appendix B shows that the variance of the growth rate of aggregate sales, or more precisely of the deviation from the expected aggregate sales, is equal to:

$$\text{Var}_z\left(\frac{\Delta X}{E_z(X)}\right) = \sigma^2 h, \quad (13)$$

where h is the Herfindahl index of production shares of firms in this economy, $h = \sum_{k=1}^I h(k)^2$. This is a familiar expression for the variance of a sum of random variables, and is the same as the one used by Gabaix (2005).

Note that granular volatility only has a chance to be quantitatively relevant if the distribution of firm size is sufficiently fat-tailed. In this case, Gabaix (2005) shows that the conventional Law of Large Numbers fails, and asymptotics are instead governed by a version of Levy’s Theorem, with much slower convergence as the number of firms increases. This implies that idiosyncratic shocks to individual (large) firms average out much more slowly. In the simulation below, we calibrate the exponent of the power law distribution in firm size to available estimates. Equation (13) forms the basis of the quantitative exercise below. We will simulate the world economy with a large number of firms in each country, and calculate how the Herfindahl indices change due to international trade. This will reveal the contribution of international trade to aggregate (granular) volatility of individual countries as a function of their characteristics.

How does international trade affect the distribution of firm size and therefore aggregate volatility? As first demonstrated by Melitz (2003), the distribution of firm size becomes more unequal under trade: compared to autarky, the least productive firms exit, and only the most productive firms export abroad. Due to competition from foreign varieties, domestic sales and profits decrease. Thus, as a country opens to trade, sales of most firms shrink, while the largest firms grow larger as a result of exporting.¹¹ Figure 3 depicts this effect. In the two-country case, there is a single productivity cutoff, above which firms export abroad. Compared to autarky, there is a higher probability of finding larger firms above this cutoff. In the N -country case, with multiple export markets there will be cutoffs for each market, with progressively more productive firms exporting to more and more markets and growing larger and larger relative to domestic GDP. Thus, if the distribution of firm sales follows a power law and the economy is granular, international trade has the potential to increase the size of the largest firms, in effect creating a “hyper-granular” economy, with clear implications for the relationship between trade openness and aggregate volatility. While qualitatively this result is a straightforward consequence of the baseline model of trade with heterogeneous firms, the key question is how important this mechanism is quantitatively. This is what we turn to in the next section.

Before describing the simulation results, it is important to discuss the empirical validity of the assumption embedded in equation (13), namely that the volatility of the proportional change in firm sales, σ , is invariant to the firm size x . If the volatility of sales decreases sufficiently fast in firm size, larger firms will be so much less volatile that they will not impact

¹¹Firm-level studies of dynamic adjustment to trade liberalization appear to find empirical support for these predictions. Pavcnik (2002) provides evidence that trade liberalization led to a shift in resources from the least to the most productive firms in Chile. Bernard and Peter Schott (2003) show that a fall in trade costs leads to both exit by the least productive firms and entry by firms into export markets. In addition, existing exporters ship more abroad.

aggregate volatility. In fact, an economy in which larger firms are just agglomerations of smaller units each subject to i.i.d. shocks is not granular: shocks to firms cannot generate aggregate fluctuations. Several papers estimated the relationship between firm size and firm volatility of the type $\sigma = Ax^{-\xi}$ using Compustat data (see, e.g., Stanley et al. 1996 and Sutton 2002). The benchmark case in which larger firms are simply collections of independent smaller firms would imply a value of $\xi = 1/2$, and the absence of granular fluctuations. Instead, the typical estimate of this parameter is about $1/6$, implying that larger firms are not that much less volatile than smaller ones. Gabaix (2005) argues that these estimates may not be reliable, since they are obtained using only data on the largest listed firms. In addition, it is not clear whether estimates based on the U.S. reliably reflect the experience of other countries. Hence, our baseline analysis sets $\xi = 0$, and a value of σ based on the largest 100 listed firms in the U.S. In other words, we assume that all firms in the economy have a volatility as low as largest firms in the economy. However, in the robustness Section 4, we perform our analysis under the assumption that $\xi = 1/6$, and show that it does not appreciably affect our results.¹²

3 Quantitative Evidence

3.1 A Model with Symmetric Countries

The solution to the multi-country model above is particularly simple when the countries are symmetric. Suppose that, for simplicity, $L_i = L$, $\bar{L}_i = \bar{L}$, and the domestic costs of entry are all the same as well: $f_{ii} = f$. Further, suppose that the trade costs are identical between each country-pair: $\tau_{ij} = \tau \forall i, j$, and $f_{ij} = f^X \forall i, j$, with $\tau_{ii} = 1$ as a normalization. This is the setup considered in the original Melitz (2003) paper.

When countries are symmetric, wages in all countries are the same, and we can normalize all of them to 1: $w_i \equiv 1 \forall i$. As a consequence, the total market size $X = \frac{1}{1 - \frac{\varepsilon-1}{\varepsilon\theta}} wL$ is obviously the same in all countries as well. The price level in each country is then given by:

$$P_i = P = \frac{1}{b} \left[\frac{\theta - (\varepsilon - 1)}{\theta} \right]^{\frac{1}{\theta}} \frac{\varepsilon}{\varepsilon - 1} \left(\frac{X}{\varepsilon} \right)^{-\frac{\theta - (\varepsilon - 1)}{\theta(\varepsilon - 1)}} \bar{L}^{-\frac{1}{\theta}} \times \left[f^{-\frac{\theta - (\varepsilon - 1)}{(\varepsilon - 1)}} + (N - 1)\tau^{-\theta} (f^X)^{-\frac{\theta - (\varepsilon - 1)}{(\varepsilon - 1)}} \right]^{-\frac{1}{\theta}}. \quad (14)$$

¹²A related point concerns multi-product firms: if large firms sell multiple imperfectly correlated products, then the volatility of the total sales for multiproduct firms will be lower than the volatility of single product firms. Evidence suggests, however, that even in multiproduct firms the bulk of sales and exports is accounted for by a single product line. Sutton (2002) provides evidence that in large corporations, the constituent business units themselves follow a power law, with just a few very large business units and many much smaller ones. Along similar lines, Adalet (2008) shows that in the census of New Zealand firms, only about 6.5% to 9.5% of output variation is explained by the extensive margin (more products per firm), with the rest explained by the intensive margin (greater sales per product).

A number of things are worth noting about this expression. First, the only difference between the multi-country price level and the autarky price level is the second term in the square brackets, $(N-1)\tau^{-\theta} (f^X)^{-\frac{\theta-(\varepsilon-1)}{(\varepsilon-1)}}$. Thus, since it is raised to a negative power, trade opening lowers the price level compared to autarky. Since the nominal GDP expressed in units of numéraire is unchanged, there are gains from trade. Note that the price level is ever lower, and the gains from trade ever greater, as we increase the number of equal-sized countries in the world, N .

Armed with this expression for the price level, we can proceed to the key outcomes for us, which are the cutoffs for domestic production and exporting. For domestic production, in each country i ,

$$a_{ii} \equiv a_D = \frac{\varepsilon-1}{\varepsilon} \times P \times \left(\frac{X}{\varepsilon f} \right)^{\frac{1}{\varepsilon-1}}, \quad (15)$$

while the exporting cutoff is:

$$a_{ij} \equiv a_X = \frac{\varepsilon-1}{\varepsilon} \times \frac{P}{\tau} \times \left(\frac{X}{\varepsilon f^X} \right)^{\frac{1}{\varepsilon-1}}. \quad (16)$$

Note that since the nominal expenditure expressed in terms of wages is unchanged – $X = \frac{1}{1-\frac{\varepsilon-1}{\varepsilon\theta}}L$ – the only thing that changes the cutoffs as the country opens to trade or as the number of countries increases is the price level P . As we argued above, P falls as the country opens to trade, lowering the cutoff for domestic production. Since a_D is the marginal cost, this implies that the least productive firms stop operating when the country opens to trade – the well known Melitz effect. More importantly for us, a_D keeps falling – the domestic production cutoff becomes more and more stringent – as the number of countries increases.

Finally, $a_X < a_D$: only the most productive firms export. In the symmetric case, the expression for the share of operating firms that export, $G(a_X)/G(a_D)$, is particularly simple. Plugging in a_D and a_X from (15) and (16), it becomes:

$$\frac{G(a_X)}{G(a_D)} = \tau^{-\theta} \left(\frac{f}{f^X} \right)^{\frac{\theta}{\varepsilon-1}}. \quad (17)$$

This relation will become useful when we calibrate the fixed cost of exporting f^X relative to the domestic entry cost f .

The world economy characterized by equations (14), (15), and (16) is particularly easy to implement numerically. We proceed as follows. For a choice of parameter values (described below), we obtain the cutoffs for producing and exporting. Then, we draw a marginal cost a for each of the $\bar{I}$ firms in the economy, and compute its expected sales, checking whether the firm with marginal cost a produces and/or exports. Based on the figures for expected

sales of each firm, we then compute the Herfindahl index of the economy:

$$h = \sum_{k=1}^I h(k)^2,$$

where $h(k)$ is the share of firm k in total sales in the economy. Assuming the shocks to firms are uncorrelated – there is no aggregate shock – the aggregate volatility in this economy is given by $\sigma\sqrt{h}$, where, as above, σ is the standard deviation of the idiosyncratic shock to the firm, $\tilde{z}$. This is Gabaix’s (2005) “granular” volatility. Since the simulation involves drawing firm productivities randomly from a highly skewed distribution, we repeat the exercise 1001 times, and take the medians of the values of interest.

We simulate the economy under the following parameter values (see Table 1 for a summary). The elasticity of substitution is $\varepsilon = 6$. Anderson and van Wincoop (2004) report available estimates of this elasticity to be in the range of 3 to 10, and we pick a value close to the middle of the range. The key parameter is θ , as it governs the slope of the power law. As described above, in this model firm sales follow a power law with the exponent equal to $\frac{\theta}{\varepsilon-1}$. In the data, firm sales follow a power law with the exponent close to 1. Axtell (2001) reports the value of 1.06, which we use to find θ given our preferred value of ε : $\theta = 1.06 \times (\varepsilon - 1) = 5.3$. We set $\tau = 2.86$, which is the average of τ_{ij} ’s calculated using estimates from Helpman, Melitz and Rubinstein (2008).¹³

We set the number of potential entrants $\bar{I}$ as follows. The simulation with $N = 3$ corresponds to a case in which each country accounts for 1/3 of world GDP, as does the United States in the data. When $N = 3$, we thus set $\bar{I} = 10,000,000$, that is, there are ten million potential firms in each country the size of the U.S. In this calibration it implies that there are about 9,500,000 operating firms there. According to the 2002 U.S. Economic Census, there were 6,773,632 establishments with a payroll in the United States. There are an additional 17,646,062 business entities that are not employers, but they account for less than 3.5% of total shipments. Thus, while the U.S. may have many more firms than what we assume here, ten million is a number sufficiently high as to let us consider consequences of granularity. That is, an economy with 9,500,000 equal-sized firms has a Herfindahl index of about 0.0000001, leaving no possibility for generating granular fluctuations. As we increase the number of countries, we do so without increasing the overall number of potential firms in the world. That is, in all simulations with $N > 3$, $\bar{I}$ is set to $10,000,000 \times 3/N$. For simplicity, we set L to the same value as $\bar{I}$ in each instance.

¹³Though quite high, this value of τ is in the same ballpark as the existing estimates of trade costs derived from gravity models. It is a well-known puzzle that trade cost estimates based on volumes of trade are always found to be larger than trade cost estimates based on price differences across locations (for a discussion, see Anderson and van Wincoop 2004). We choose gravity-based estimates because as we will see below, our quantitative model reproduces observed trade volumes quite successfully when we use these trade costs.

In order to ensure an interior solution, we must make sure that the least productive firm does not produce. This restriction boils down to

$$f > \frac{L}{\bar{I}} \frac{\theta - (\varepsilon - 1)}{\theta \varepsilon - (\varepsilon - 1)}. \quad (18)$$

We set f to be 0.003 above that. This restriction implies that f equals about 1.2% of per capita GDP. This is a low value of f . According to the World Bank’s Doing Business Indicators Database (The World Bank 2007a), setting up a business entails a bureaucratic cost of 0.5% of per capita GDP in the United States. In most countries, however, this cost is considerably higher. Certainly, higher levels of f will result in less diversification and higher granular volatility. In this application, however, we choose to ignore the variation in fixed costs of entry, so that we can focus more squarely on the role of international trade. Given the values of τ and f , we calibrate f^X to match the well known stylized fact that only about 20% of firms export (Bernard, Jensen, Redding and Schott 2007). Equation (17) allows us to set f^X . It is not clear how to pick b , which is the minimum value that productivity can take. We set $b = 0.1$, which implies that roughly 95% of potential entrepreneurs enter in autarky.

We simulate the model economy using these parameters for various numbers of countries, and compare volatilities to the autarky case. (This is an exercise in the spirit of Alvarez and Lucas 2007.) The results are presented in Table 2. The first column reports the number of countries; the second, the median square root of the Herfindahl index among the 1,001 simulations, and the third column reports the ratio of aggregate volatilities under trade compared to autarky. The last column assumes that the variance of firm-level shocks does not change as the country opens to trade, in order to focus exclusively on the effects of increased granularity.

As noted above, when $N = 3$ each country accounts for about 1/3 of the world GDP, which is roughly the size of the United States relative to the rest of the world. The (square root of) sales Herfindahl in the U.S., as reported in Gabaix (2005), is 0.062, only slightly lower than the 0.067 implied by our parameterized model with $N = 3$, but very close to that value. Thus, in spite of the assumption of only 10,000,000 firms, the model can reproduce quite well the observed concentration of firm size.

It is clear that as the number of countries increases – that is, as the economy gets smaller relative to the rest of the world – the distribution of firms becomes more and more concentrated. The the square root of the median Herfindahl with three countries is about 0.067. With five countries, it rises by about 10% to 0.072. With 50 countries, it is 0.09, and with 200 countries, 0.12, or nearly twice as high as with $N = 3$. A striking feature of the results is that the contribution of international trade to macroeconomic volatility increases

dramatically as a country gets smaller relative to the rest of the world. Our model implies that in a country like the U.S., macroeconomic volatility is only about 0.2% higher than what it would be in complete absence of trade. By contrast, a country that accounts for 1% of world GDP – which could be Belgium, South Korea, or Argentina – experiences volatility that is 1.1 times higher than what it would have been in autarky. A country that accounts for 0.5% of GDP – Norway, Poland, or South Africa – experiences macroeconomic volatility that is 1.18 times higher than without trade.

How do these results compare to the data? We estimated the relationship between macroeconomic volatility and the share of world GDP in a large sample of countries. We took real, local currency GDP per capita from the World Bank’s World Development Indicators database (The World Bank 2007b), and computed the standard deviation of the annual growth rate of this variable over the period 1970-2006. The share of a country’s GDP in world GDP is constructed based on the nominal US dollar total GDP series from the same database. We compute each country’s share in world GDP for each year, and then take the average of this share over 1970-2006.

It turns out that doing the estimation in logs produces a better fit (see also Canning et al. 1998). The results are reported in Table 3. The univariate regression of log volatility on log world GDP share yields an R^2 of 19%, and the coefficient is significant at the 1% level, with a t -statistic of 4.66. To ensure that we are not just picking up the well known negative relationship between per capita income and volatility, we condition on log per capita income in column 2.¹⁴ This allows us to control for many variables outside the model, such as the role of macroeconomic policy and institutional quality, in a parsimonious way. It has been shown that these variables are important in explaining macroeconomic volatility (see, e.g., Acemoglu, Johnson, Robinson and Thaicharoen 2003), and are highly correlated with per-capita GDP.¹⁵ The overall country size is still highly significant, though the magnitude of the coefficient is somewhat lower.

For maximum comparability to the non-symmetric multi-country simulation below, the above regressions are estimated on the sample of 49 largest countries by total GDP. To check that the results are not unique to this particular sample, we re-estimated our specification on the largest 30, 75, and 100 economies by total GDP, as well as on the largest number of countries for which the data were available, 143. Columns 3-6 of Table 3 show that results are robust. In addition, we carried out non-parametric estimation to allow for a variable slope of this relationship, and the results were quite similar.

¹⁴See Acemoglu and Zilibotti (1997), among others, for explanations of this negative relationship.

¹⁵Alternatively, we controlled for other possible covariates of macroeconomic volatility directly in addition to per capita income. We included measures of economic and political institutions, government size, inflation, financial development, and schooling. The results were robust to the inclusion of these additional controls.

We can now compare the relationship between volatility and country size found in the model and the data. Figure 4 plots the estimated relationship between country size and volatility in the data (after conditioning for per-capita GDP), as well as the same relationship implied by the model. In both cases, the values are normalized by the implied volatility of the largest country, which is the U.S. in the data, and the country accounting for 1/3 of world output in the model.

It is clear that the model matches over four-fifths of the elasticity of volatility with respect to country size found in the data. In the model, this elasticity is -0.121 (σ_T), compared to -0.139 (σ_{GDP}) in the data. In addition, this elasticity is quite close to what has been found in the literature. Canning et al. (1998) report an estimate of -0.15 ± 0.03 . Our model implies that a country accounting for 0.5% of world GDP has volatility about 1.75 times that of the U.S.

The negative relationship between volatility and country size is an intuitive consequence of trade opening in this model. When the economy opens to trade, the largest and most productive firms in both large and small economies will begin exporting and grow in size, making the economy more granular. However, firms based in large countries such as the U.S. already face a large home market, while in a small country the potential foreign market is far larger in size relative to the domestic demand. Thus, a typical large firm in a small country will expand production by more in response to trade opening than a corresponding firm located in a large country. As a result of this, the same trade opening increases granularity by more in smaller countries.

More interesting are the counterfactual experiments we can perform with this model. In particular, as we increase the number of countries, the country size as measured by both $\bar{I}$ and L decreases. Thus, the simulation conflates the role of country size *per se* and the role of openness. While there is no way to separate the two in the data, in the model we can calculate how much volatility would increase for smaller countries without any trade. Figure 4 plots the change in aggregate volatility as a function of country size in autarky. We can see that without trade, the model cannot match the observed relationship in the data. Instead of the elasticity of -0.121 in the model with trade, without trade the relationship is much flatter, -0.086 (σ_A). Without trade, the country that accounts for 0.5% of world GDP has volatility that is only 1.5 times higher than the U.S. Thus, it appears that trade plays an important role in generating the relationship between country size and volatility that we see in the real world.

To summarize, the model with a role for both country size and international trade matches quite well the estimated relationship between country size and volatility found in the data.

3.2 Multi-Country Model

Though the results obtained with symmetric countries are informative, we would like to exploit the rich heterogeneity among the countries in the world. In order to do this, we numerically implement the general multi-country model laid out in Section 2. We use information on country size and trade barriers to solve the model, and then simulate the random draws of firm productivity to compute the Herfindahl indices of firm sales in each country. This will allow us to examine the relationship between granular volatility and various country characteristics in the model, as well as to evaluate the contribution of international trade to aggregate volatility in each country.

In order to fully solve the model numerically, we must find the wages for each country, w_i , using the system of equations (8). To solve this system, we must calibrate the values of L_i , $\bar{I}_i$, τ_{ij} , and f_{ij} for each country and country pair. For finding the values of L_i , we follow the approach of Alvarez and Lucas (2007). First, we would like to think of L not as population per se, but as “equipped labor,” to take explicit account of TFP and capital endowment differences between countries. To obtain the values of L that are internally consistent in the model, we start with an initial guess for L_i for all $i = 1, \dots, N$, and use it to solve for the wages w_i using equation (8). Given this vector of wages, we update our guess for L_i for each country in order to match the ratio of total GDPs between each country i and the U.S.. Using the resulting values of L_i , we solve for the new set of wages, and iterate to convergence (for more on this approach, see Alvarez and Lucas 2007). Thus, our procedure generates vectors w_i and L_i in such a way as to match exactly the relative total GDPs of the countries in the sample. In practice, the results are extremely close to simply equating L_i to the relative GDPs of the countries. In this procedure, we must normalize the population of one of the countries. We thus set L_{US} to its actual value of 291 million as of 2003, and compute L_i of every other country relative to this U.S. value. Finally, we set $\bar{I}_i$ in proportion to L_i . That is, the country’s endowment of entrepreneurs is simply proportional to its “equipped labor” endowment. An important consequence of this assumption is that countries with higher TFP and capital abundance will have a greater number of potential productivity draws, all else equal. This is an assumption adopted by Alvarez and Lucas (2007) and Chaney (2008). We set $\bar{I}_{US} = 10,000,000$, that is, there are ten million potential firms in the U.S. As discussed in the symmetric calibration above, this is the right order of magnitude when compared to the number of firms in the U.S. Economic Census, and high enough to rule out aggregate fluctuations unless the distribution of firm size is sufficiently skewed.

Next, we must calibrate the values of τ_{ij} for each pair of countries. Rather than posit the

same value of τ across all country pairs as we did in the simulation of the symmetric case, we use available gravity estimates to obtain values of τ_{ij} that differ across country pairs. We use two sets of gravity estimates. The first comes from the empirical model of Helpman et al. (2008). That is, we combine geographical characteristics such as bilateral distance, common border, common language, whether the two countries are in a currency union and others, with the coefficient estimates reported by Helpman et al. (2008) to calculate values of τ_{ij} for each country pair. As a robustness check, we also compute τ_{ij} using the estimates of Eaton and Kortum (2002). The advantage of the Helpman et al. (2008) estimates is that they are obtained in an empirical model that accounts explicitly for both fixed and variable costs of exporting, and thus corresponds most closely to the theoretical structure in our paper. Note that in this formulation, $\tau_{ij} = \tau_{ji}$ for all i and j .

Finally, we must take a stand on the values of f_{ii} and f_{ij} . We do not have a good way to calibrate these parameters. In addition, they cannot be calibrated separately from b , the minimum productivity of the firms in the economy. Thus, as in the symmetric simulation above, we proceed to set f_{ii} to a level just high enough to ensure an interior solution, which is (18) plus 0.003. As mentioned above this value of f_{ii} is a rather low one, implying that some 95% of potential entrepreneurs produce in the U.S.. Note that since $\bar{L}_i$ is the same multiple of L_i in every country, f_{ii} is the same for all i in the baseline calibration. Further, we set $f_{ij} = f_{ii} + f_{jj}$. That is, to enter the export market the firm must pay the entry cost for both its own and the foreign market. This is a very simple configuration of fixed costs of entry and exporting. We opt for simplicity mainly because we do not have a way to model a rich variation in f_{ij} between individual country pairs. We then check that the bilateral trade volumes obtained by the model match up well with observed trade data. Finally, we use the same values for ε , θ , and b as the ones in the symmetric simulation (see the third column Table 1 for a summary of the parameters used in the non-symmetric case).

We carry out the analysis on the sample of the largest 49 countries by total GDP, plus the 50th that represents the rest of the world. These 49 countries together cover 97% of world GDP. We exclude entrepot economies of Hong Kong and Singapore, both of which have total trade well in excess of their GDP, due to significant re-exporting activity. Thus, our model is not intended to fit these countries. (We do place them into the rest-of-the-world category). The country sample, sorted by total GDP, is reported in Table 4.

3.3 Model Fit

As described above, our iterative procedure ensures that the ratio of total GDPs in the model, given by $\frac{1}{1-\frac{\varepsilon-1}{\varepsilon\theta}}w_iL_i$ for country i , for any two countries matches exactly the ratio of the total GDPs in the data. However, since the object of the paper is to examine the role

of trade openness in aggregate volatility, it is more important that the model matches well the bilateral and overall trade volumes observed in the data. Comparing bilateral trade patterns generated by the model to the actual data is a good test of the model’s success in describing the world economy, since the calibration procedure does not use any information on actual trade patterns, only country GDPs and estimated bilateral trade costs.

Figure 5 reports the scatterplot of bilateral trade ratios $\pi_{ij} = X_{ij}/X_i$. On the horizontal axis is the natural log of π_{ij} that comes from the model, while on the vertical axis is the corresponding value of that bilateral trade flow in the data. Hollow dots represent exports from one country to another, π_{ij} , $i \neq j$. Solid dots, at the top of the scatterplot, represent sales of domestic firms as a share of domestic absorption, π_{ii} . For convenience, we added a 45-degree line. It is clear that the trade volumes implied by the model match the actual data well. Most observations are quite close to the 45-degree line. It is especially important that we get the overall trade openness ($1 - \pi_{ii}$) right, since that will drive the contribution of trade to the granular volatility in each country. Figure 6 plots the actual values of $(1 - \pi_{ii})$ against those implied by the model, along with a 45-degree line. We can see that though the relationship is not perfect, is it quite close.

Table 5 compares the means and medians of π_{ii} and π_{ij} ’s for the model and the data, and reports the correlations between the two. The correlation between domestic shares π_{ii} the model and the data for this sample of countries is around 0.48. The means and the medians look very similar as well, with the countries in the model slightly more open on average than the data. The correlation between export shares, π_{ij} , is actually higher at 0.67.¹⁶

Overall, though the model calibration does not use any information on trade volumes, it fits bilateral trade data quite well, suggesting that it will be informative about the role of trade in aggregate volatility. This is what we turn to next.

3.4 Trade and Granular Volatility: a Quantitative Analysis

Having solved the model given the data on country GDPs and trade costs, we now simulate it using random productivity draws for each firm in each economy. Namely, in each country i , we draw $\bar{I}_i$ productivities from a $\text{Pareto}(b, \theta)$ distribution. For each firm, we use the cutoffs a_{ji} for serving each market j (including its own market $j = i$) given by equation (4) to determine whether the firm operates, and which, if any, foreign markets it serves. We

¹⁶We also experimented with increasing the number of countries in the simulation to 60. The model fit the data well, though it over-predicted the overall average trade openness of countries by slightly more than the 50-country model. In addition, there are more zeros in bilateral trade data in the 60-country sample compared to the 50-country one. (With 50 countries, among the 2500 possible unidirectional bilateral trade flows, only 18 are zeros.) For these reasons we stick with the largest 49 countries in our analysis.

then calculate the total sales of each firm as the sum of its sales in each market, and compute the Herfindahl index of firm sales in country i . Since the distribution of firm productivities gives rise to a highly skewed distribution of firm sales, there is variation in the Herfindahl index from simulation to simulation, even though we draw as many as 10 million firms in a given country. We thus repeat the exercise 1001 times, and take the median values of the Herfindahl index in each country. In parallel, we also compute the Herfindahl index of firm sales in autarky for each country, given all the parameters. This counterfactual exercise allows us to gauge the contribution of international trade to aggregate volatility. Given these values of the Herfindahl index h , we can then construct each country's granular volatility under trade and in autarky using the formula for total variance (13) and a realistic value of σ . Following Gabaix (2005), we set $\sigma = 0.1$, though since in this paper we will not exploit any variation in σ across countries, none of the results will be driven by this choice.

Since the number of firms $\bar{I}_i$ is a parameter in our model, chosen to be at most 10,000,000 due to computational constraints, we must check that the resulting Herfindahl indices are not too high. In our simulation, the square root of the Herfindahl index for the U.S. is $\sqrt{h_{US}} = 0.068$, which is quite close to the value of 0.062 reported in Gabaix (2005). Thus, choosing a value of $\bar{I}_i$ that is unrealistically low relative to the total number of firms in the U.S. economy does not result in an economy that is too undiversified compared to the data.

How well does the model predict the actual GDP volatility found in the data? Table 6 presents regressions of actual volatility of GDP growth over the period 1970-2006 against the one predicted by the model (σ_T). Column (1) includes no controls. We can see that the fit is not perfect ($R^2 = 0.15$), but the relationship is clearly positive and significant. Interestingly, the slope coefficient in this specification is virtually 1. The second column includes GDP per capita. The fit of the model improves, and though the coefficient on the model volatility drops, it remains significant at the 1% level. The next two columns include measures of export structure volatility and production specialization, since opening to trade can impact aggregate volatility through a change in these variables. Column (3) adds the *risk content of exports*, which captures the overall riskiness of a country's export structure.¹⁷ The model volatility remains significant, and the R^2 of the regression is now 0.40. Finally, a measure of production specialization for the manufacturing sector (Herfindahl of production shares) is included in the fourth column.¹⁸ The number of observations drops to 35 due to limited data availability, but the model volatility still remains significant.

As would be expected, the level of granular volatility is lower than what is observed in

¹⁷This measure is sourced from di Giovanni and Levchenko (2008). A country's export structure can be volatile due to a lack of diversification and/or exporting in sectors that are more volatile.

¹⁸This measure is calculated using the UNIDO database of sectoral production, and sourced from di Giovanni and Levchenko (2007).

the data. Column 1 of Table 7 reports the ratio of the granular volatility implied by the model to the actual GDP volatility found in the data. It ranges between 0.12 and 0.70, with a value of 0.35 for the United States, almost identical to what Gabaix (2005) finds using a very different methodology. Note that the variation in aggregate volatility in the model across countries is generated by differences in country size as well as variation in bilateral trade costs. That gives us a glimpse of how important those two things are to aggregate volatility, when applied through the granular channel.

How much of the elasticity of the aggregate volatility with respect to country size can the model explain? We now return to the exercise performed in the symmetric simulation, and plot the predicted volatility as a function of country size in the data and the model. Figure 7 reports the results. Note that since the level of aggregate volatility in the model does not match up with the level in the data, this graph is only informative about the comparison of slopes, not intercepts. As estimated above, in the data the elasticity of GDP volatility with respect to country size is -0.139 (σ_{GDP}). Our calibrated model produces an elasticity of -0.137 (σ_{T}), which is extremely close to the one in the data though slightly below it in absolute terms. We can also calculate what this relationship would look like in the absence of trade. Figure 7 reports the volatility-size relationship in autarky. We conclude that country size alone cannot account for what we observe in the data. The elasticity of volatility with respect to country size in autarky is just -0.097 (σ_{A}), considerably lower than the -0.139 in the data.

3.4.1 Counterfactual I: The Impact of Trade Openness on Volatility

We now assess the contribution of international trade to the aggregate granular volatility in our sample of countries. Our model yields not only the predicted granular volatility in the simulated trade equilibrium, but also the granular volatility of autarky. Table 7 reports the ratio of the two in each country in the sample. In the table, countries are ranked by overall size, in descending order. We can see that international trade contributes very little to overall GDP volatility in the U.S.. The country is so large and trade volumes are so low that its volatility under trade is only 1.037 times higher than it would be in complete absence of trade. By contrast, smaller countries experience substantially higher volatility as a result of trade openness. For instance, in a country like Ireland, the volatility under trade is some 32 percent higher than it would be in autarky.

3.4.2 Counterfactual II: The Impact of Further Reduction in Trade Costs in All Countries

Having computed what granular volatility would be in the absence of trade, we next carry out the opposite counterfactual experiment: a reduction in trade costs. It would not be very informative to consider totally free trade ($\tau_{ij} = 1 \forall i, j$), since it is unrealistic to model a case in which distance between countries does not affect trade costs, for instance. In this section, we simulate a halving of ad valorem trade costs. This is still a substantial reduction in barriers, that leads to a dramatic increase in the volume of trade. When the model was calibrated to the data, the median domestic sales as a share of domestic absorption π_{ii} was equal to 0.72, which matches the actual data reasonably well. When trade costs decrease by 50%, π_{ii} drops to 0.23, representing a threefold increase in world trade as a share of absorption.

Column 3 of Table 7 reports, for each country, the percentage increase in granular volatility resulting from this reduction in trade costs. We would expect that a further expansion of trade is likely to increase volatility as countries become progressively more specialized. The model predicts that this expansion of trade leads to an increase in granular volatility of 21% on average in this sample of countries. This effect is most pronounced in small and remote countries, as a reduction in trade costs affects them most strongly. The maximum increase, 29%, is predicted to occur in New Zealand, followed by Chile, Thailand, and the Philippines. By contract, the impact is much more muted in countries of similar size but not remote: Ireland and Belgium are both in the bottom five countries, experiencing an increase of 13 and 15% respectively. Predictably, the largest economies experience the smallest increase in granular volatility as a result of further trade opening. The bottom two countries by estimated impact are U.S. and Japan. Note, however, that even for those countries, the impact of the further reduction in trade barriers is more dramatic than the impact of trade so far. For both U.S. and Japan, we calculated that trade increases aggregate volatility by about 3.5-4% compared to autarky. The 50% reduction in trade barriers implies a further increase of 8.5-10% over the status quo.

To better summarize these range of country experiences and possible trade cost reductions, Columns (1) through (4) of Table 8 report the distribution of increases in granular volatility in our set of countries coming from a 10%, 25%, 50% and a 75% reduction in τ_{ij} . Not surprisingly, in this range of trade costs, greater reductions lead to uniform increases in granular volatility throughout the distribution of countries.

3.4.3 Counterfactual III: The Impact of Further Reduction in Trade Costs in a Single Country

It is also informative to compute the volatility impact of a reduction in trade costs for an individual country, holding all third-country trade costs constant. From an individual country's perspective, this counterfactual experiment could be a proxy for a comprehensive liberalization program, or for joining a free trade agreement such as the WTO. To do this exercise, we run the simulation reducing trade costs by 50% for each individual country one-by-one. Column (4) of Table 7 reports the results. The main conclusion from this exercise is that the increase in volatility experienced by a country when its trade costs decrease is similar in magnitude, but always slightly *larger* than the increase in granular volatility coming from a full multilateral liberalization. This result is quite striking, and the intuition for it is as follows. When a country's trade costs decrease, domestic large firms find it easier to penetrate foreign markets. This familiar effect acts to make the largest firms larger, thereby increasing granular volatility. When the liberalization is multilateral, there is also an effect that acts in the opposite direction: firms from all other countries also find it easier to penetrate each market. Thus, in a multilateral liberalization, the price level decreases dramatically in each country, making it more competitive and reducing demand for each firm's good. However, when a liberalization is unilateral, this second effect is absent. As trade costs drop, the size of the domestic exporters increases by more than it would under a multilateral reduction in trade costs. Therefore, the impact on granular volatility is that much greater. Columns (5) through (8) of Table 8 show the increases in aggregate volatility due to single-country trade cost reductions of various sizes, from 10% to 75%. Once again, we see that the magnitudes involved are uniformly greater than those in a multilateral liberalization.

4 Robustness Checks and Model Perturbations

4.1 Volatility Varying with Firm Size

An assumption that makes the analysis above possible is that the volatility of the proportional change in sales, σ , does not change in firm size x . As discussed at the end of Section 2, if firm-level volatility decreases sharply enough in size, shocks to large firms will not generate aggregate volatility. In practice, however, the negative relationship between firm size and volatility of its sales is not very strong. Stanley et al. (1996) and Sutton (2002) estimate the relationship of the type $\sigma = Ax^{-\xi}$, and find a value of $\xi = 1/6$. That is, firm-level volatility does decrease with size, but this elasticity is quite low. To check robustness of our results, we allow the firm-specific volatility to decrease in firm size as estimated by

these authors. In that case, aggregate (granular) variance is given by

$$\text{Var}_z \left(\frac{\Delta X}{E_z(X)} \right) = \sum_{k=1}^I \left(A x(k)^{-\xi} h(k) \right)^2,$$

where, once again, $x(k)$ are sales of firm k , while $h(k)$ is the share of firm k 's sales in total output in the economy.

The rest of the simulation remains unchanged. Since we are not matching the level of aggregate volatility, just the contribution of trade, we do not need to posit a value of the constant A . However, it would be easy to calibrate to match the volatility of the top 100 firms in the U.S. as reported by Gabaix (2005), for example. Note that compared to the baseline simulation, modeling a decreasing relationship between country size and volatility is a double-edged sword: while larger firms may be less volatile as a result, smaller firms are actually more volatile. This implies that the impact of international trade will not necessarily be more muted when we make this modification to the basic model.

Table 9 reports the results of this robustness check in column 1. We can see that the impact of allowing large firms to have lower volatility is minimal. The contribution of trade to granular volatility is virtually the same: while in the baseline simulation trade increases aggregate volatility by 21%, when volatility is allowed to vary by firm, the corresponding increase is 24%. Somewhat surprisingly, therefore, allowing volatility to decrease in firm size implies a larger contribution of trade to aggregate volatility, not a smaller one. In fact, this is the case in every country in the sample save the U.S.

4.2 Robustness to Parameter Changes

We now discuss two further sensitivity checks we performed on the model. The first is to use an alternative parametrization of bilateral iceberg costs τ_{ij} . Instead of using the Helpman et al. (2008) empirical model we use instead that of Eaton and Kortum (2002). The latter employs fewer explanatory variables, and breaks up the bilateral distance variable into discrete intervals rather than using it as a continuous variable. The resulting estimated iceberg trade costs are considerably lower than those implied by the baseline model. While the mean τ_{ij} resulting from the Helpman et al. (2008) estimates is 2.86, the average iceberg cost implied by Eaton and Kortum (2002) is 2.18. Table 9, column 2 reports the contribution of trade to aggregate volatility implied by the model with Eaton and Kortum (2002) trade costs. Predictably, since the trade costs are lower, the contribution of trade to aggregate volatility is higher. In fact, the average increase implied by these alternative trade costs is 41%, about twice the 21% value in the baseline simulation.

Finally, we check how sensitive our results are to the elasticity of substitution ε . Column

3 of Table 9 reports the contribution of trade when $\varepsilon = 4$, while column 4 reports it for $\varepsilon = 8$. We can see that the elasticity of substitution matters a fair bit for our conclusions about the impact of trade on aggregate volatility. With a lower elasticity, the average increase in aggregate volatility due to trade is 48% in our sample of countries, far greater than the baseline. By contrast, when the elasticity of substitution is high, the contribution of trade is much smaller, with an average of just 5.4%. Varying the elasticity of substitution has a large impact on the magnitudes. However, in the context of our calibration the value of ε affects θ , since we match the ratio $\theta/(\varepsilon - 1)$ to a fixed number based on existing empirical estimates. In addition, changes in ε also affect the values of all τ_{ij} , since to back out those from gravity estimates requires one to take a stand on ε . As such, it is difficult to draw firm conclusions regarding how the elasticity of substitution changes our results, since we cannot vary this parameter independently of other key ones in this quantitative exercise.

5 Conclusion

Recent literature in both macroeconomics and international trade has focused attention on the role of firms. We now know that international trade is mostly carried out by the largest firms (Bernard et al. 2007), and these firms are relatively more important in the overall economy because of their exporting activity (Melitz 2003). It turns out that large firms also matter for the macroeconomy. Gabaix (2005) demonstrates that if the distribution of firm size follows a power law with an exponent close to -1 – which appears to be the case in the data – shocks to the largest firms can lead to aggregate fluctuations, which are dubbed “granular.”

This paper argues that openness to international trade can have an impact on aggregate fluctuations by increasing the relative importance of large firms and in effect making the economy more granular. While anecdotal evidence on the macro importance of large exporting firms abounds, our main goal is quantitative. We calibrate and simulate a multi-country model of firm-level production and trade that can generate granular fluctuations. The model matches quite well both the observed bilateral trade volumes, as well as the elasticity of GDP volatility with respect to country size. The counterfactual exercises reveal that the contribution of international trade to aggregate volatility varies a great deal depending on country characteristics. While it is minimal in large, relatively closed economies like the U.S. or Japan, trade increases volatility by as much as 30-40% in small open economies such as Belgium, Poland, or Romania. In addition, our calculations show that a further reduction in trade costs could increase aggregate volatility by as much as a further 30% in some countries.

Recent research incorporates heterogeneous firms into fully dynamic general equilibrium macroeconomics models, focusing on the impact of persistent aggregate shocks and firm entry and exit (Ghironi and Melitz 2005, Alessandria and Choi 2007). The importance of firm-specific idiosyncratic shocks for macroeconomic volatility via the granular channel emphasized in this paper should be viewed as complementary to this work. Future research incorporating these different mechanisms, as well as bringing disaggregated data to the models, will help provide an even more complete picture of the macroeconomic impact of trade integration.

Appendix A Proof of Proposition 1

Proof: The total variable profits from selling to country j from country i are:

$$\Pi_{ji}^V = \sum_{k=1}^{I_{ji}} \frac{X_j}{\varepsilon P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i a(k) z(k) \right)^{1-\varepsilon}.$$

The total sales from i to j are:

$$X_{ji} = \sum_{k=1}^{I_{ji}} \frac{X_j}{P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i a(k) z(k) \right)^{1-\varepsilon}.$$

Therefore, $\Pi_{ji}^V = \frac{X_{ji}}{\varepsilon}$.

The total fixed costs paid by firms in country i to enter market j are equal to $f_{ji} w_i \bar{I}_j (ba_{ji})^\theta$.

We need to show that this quantity is also a constant multiple of X_{ji} . To do so, write

$$\begin{aligned} X_{ji} &= \frac{X_j}{P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i \right)^{1-\varepsilon} \sum_{k=1}^{I_{ji}} (a(k) z(k))^{1-\varepsilon} \\ &= \frac{X_j}{P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i \right)^{1-\varepsilon} \bar{I}_j \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} a_{ji}^{\theta - (\varepsilon - 1)} \\ &= \frac{X_j}{P_j^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{ji} w_i \right)^{1-\varepsilon} \bar{I}_j \frac{b^\theta \theta}{\theta - (\varepsilon - 1)} a_{ji}^{\theta} w_i f_{ji} \frac{\varepsilon P_j^{1-\varepsilon}}{X_j} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \\ &= \frac{\theta}{\theta - (\varepsilon - 1)} \varepsilon \bar{I}_j (ba_{ji})^\theta w_i f_{ji}. \end{aligned}$$

Therefore, the total fixed costs paid by firms in i to export to j are a constant multiple of X_{ji} :

$$\bar{I}_j (ba_{ji})^\theta w_i f_{ji} = \frac{\theta - (\varepsilon - 1)}{\theta} \frac{X_{ji}}{\varepsilon}.$$

Therefore, the total profits from selling to j from country i are:

$$\begin{aligned} \Pi_{ji} &= \Pi_{ji}^V - \frac{\theta - (\varepsilon - 1)}{\theta} \frac{X_{ji}}{\varepsilon} \\ &= \frac{X_{ji}}{\varepsilon} \left(1 - \frac{\theta - (\varepsilon - 1)}{\theta} \right) \\ &= X_{ji} \frac{(\varepsilon - 1)}{\varepsilon \theta}. \end{aligned}$$

This means that the total profits from selling to all countries equal:

$$\Pi_i = \sum_{j=1}^N \Pi_{ji} = \frac{(\varepsilon - 1)}{\varepsilon \theta} \sum_{j=1}^N X_{ji}.$$

Since in equilibrium total income equals total expenditure in each country, $X_i = \sum_{j=1}^N X_{ji}$, leading to the result that $\Pi_i = \frac{(\varepsilon - 1)}{\varepsilon \theta} X_i$. ■

Appendix B Aggregate Volatility Derivation

Given the expression for the actual sales of the firm with a transitory shock z in (10), and the expected sales of the firm with productivity a in (11), the actual sales as an approximation around $E_z [x(a, z)]$ are:

$$x(a, z) = E_z [x(a, z)] + \left. \frac{dx}{d\tilde{z}} \right|_{\tilde{z}=1} \Delta\tilde{z}$$

Therefore, the proportional change in $x(a, z)$, or the growth rate, is given by:

$$\frac{\Delta x(a, z)}{E_z (x(a, z))} = \tilde{z} - 1,$$

and the variance of this growth rate is:

$$\text{Var}_z \left(\frac{\Delta x(a, z)}{E_z (x(a, z))} \right) = \sigma^2.$$

The total sales in the economy are given by (12), thus the change in the total sales relative to the non-stochastic steady state (the growth rate) is:

$$\frac{\Delta X}{E_z X} = \frac{\sum_{k=1}^I \Delta x(a(k), z(k))}{E_z X} = \sum_{k=1}^I \frac{\Delta x(a(k), z(k))}{E_z [x(a(k), z(k))]} \frac{E_z [x(a(k), z(k))]}{E_z X}.$$

This means that the aggregate volatility is

$$\begin{aligned} \text{Var}_z \left(\frac{\Delta X}{E_z X} \right) &= \text{Var}_z \left(\sum_{k=1}^I \frac{\Delta x(a(k), z(k))}{E_z [x(a(k), z(k))]} \frac{E_z [x(a(k), z(k))]}{E_z X} \right) \\ &= \sum_{k=1}^I \text{Var}_z \left(\frac{\Delta x(a(k), z(k))}{E_z [x(a(k), z(k))]} \right) \left(\frac{E_z [x(a(k), z(k))]}{E_z X} \right)^2 = \\ &= \sigma^2 \sum_{k=1}^I \left(\frac{E_z [x(a(k), z(k))]}{E_z X} \right)^2 \\ &= \sigma^2 \sum_{k=1}^I h(k)^2, \end{aligned}$$

where $h(k)$ is the share of the firm k 's sales in total sales. As expected, the volatility of total output in the economy is equal to the volatility of an individual firm's output times the Herfindahl index of production shares.

Appendix C Data Description and Sources

Data on total GDP come from the World Bank's World Development Indicators database (The World Bank 2007b). In order to compute the share of each country in world GDP, we compute shares of each country in world GDP expressed in nominal U.S. dollars in each

year over the period 1970-2006, and take the average share over this period. To compute the GDP volatility, we compute the yearly growth rates of GDP expressed in constant local currency units, and take the standard deviation of that growth rate over 1970-2006. We also use the real PPP-adjusted per capita GDP figures from The World Bank (2007b) to control for the overall level of development in Section 3.

To obtain values τ_{ij} following the estimates of Helpman et al. (2008) and Eaton and Kortum (2002), we use data on bilateral distance, common border, whether the country is an island or landlocked, common language, and colonial ties from Centre d'Etudes Prospectives et Informations Internationales (CEPII). Data on legal origins come from La Porta, Lopez-de Silanes, Shleifer and Vishny (1998). Finally, information on currency unions and free-trade areas come from Rose (2004), and supplemented by internet searches whenever needed.

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Table 1. Parameter Values for Symmetric and Non-Symmetric Country Simulations

Parameter	Symmetric*	Non-Symmetric**
ε	6	6
θ^a	5.3	5.3
b	0.1	0.1
N	[3, 200]	50
L	$10e6 \times 3/N$	endogenous ^e
$\bar{I}$	$10e6 \times 3/N$	$\propto L^f$
τ_{ii}	1	1
τ_{ij}^b	2.86	[1.42, 3.71]
f_{ii}^c	0.0142	0.3287
f_{ij}^d	0.00034	0.6574
σ	0.1	0.1

Notes:

^a $\theta = 1.06(\varepsilon - 1)$, where 1.06 is the power law exponent estimated for U.S. firm sales by Axtell (2001).

^b $\tau_{ij} = \tau_{ji}$.

^c f_{ii} is set to insure an interior solution in equilibrium, so that we set $f_{ii} = \frac{L}{\bar{I}} \frac{\theta - (\varepsilon - 1)}{\theta \varepsilon - (\varepsilon - 1)} + 0.003$.

^d $f_{ij} = f_{ji}$. Symmetric case: f_{ij} set so that $\frac{G(a_X)}{G(a_D)} = \tau_{ij}^{-\theta} \left(\frac{f}{f^X} \right)^{\frac{\theta}{\varepsilon - 1}} = 0.2$. Non-Symmetric Case: $f_{ij} = f_{ii} + f_{jj}$.

^e L is solved endogenously with w . We solve relative to the U.S., where $L_{US} = 291,000,000$.

^f $\bar{I}$ is set proportional to L . We solve relative to the U.S., where $\bar{I}_{US} = 10,000,000$.

* This column represents the baseline case. The “same-size” symmetric case holds L and $\bar{I}$ constant at $10e6$ for each country, regardless of the size of N .

** This column represents the baseline case. Robustness checks include (1) σ varying with firm sales: $\sigma = Ax^{-\xi}$, where $\xi = 1/6$; (2) τ_{ij} based on estimates from Eaton and Kortum (2002), so that $\tau_{ij} \in [1.39, 2.55]$; (3) ε is set to 4 or 8, which in turn implies that θ equals 3.18 or 7.42, respectively.

Table 2. Symmetric Model Simulation Results

Number of countries	$\sqrt{h}$	Ratio of Volatilities Trade/Autarky
3	0.0672	1.002
5	0.0724	1.005
10	0.0768	1.011
15	0.0804	1.016
20	0.0803	1.022
40	0.0854	1.044
50	0.0895	1.055
60	0.0912	1.065
70	0.0925	1.075
100	0.0991	1.102
150	0.1105	1.141
200	0.1177	1.175

Notes: This table reports the square root of the Herfindahl index of firm shares ($\sqrt{h}$) for trade equilibrium, and the ratio of volatility under trade to the volatility under autarky. The values are medians based on 1001 simulations of the symmetric model for each number of countries.

Table 3. GDP Volatility and Country Size Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var: GDP Volatility						
Share	-0.177** (0.038)	-0.139** (0.044)	-0.090+ (0.045)	-0.209** (0.035)	-0.180** (0.027)	-0.142** (0.023)
Income		-0.157* (0.069)	-0.261** (0.070)	-0.049 (0.057)	-0.019 (0.045)	0.018 (0.037)
Constant	-4.352** (0.190)	-2.696** (0.763)	-1.533+ (0.773)	-4.010** (0.601)	-4.154** (0.473)	-4.291** (0.410)
Observations	49	49	30	75	100	143
R^2	0.192	0.273	0.337	0.328	0.296	0.225

Notes: Robust standard errors in parentheses; + significant at 10%; * significant at 5%; ** significant at 1%. All variables are in natural logarithms. The dependent variable is the log of the standard deviation of per capita GDP growth over the period 1970-2006. Share is a country's GDP relative to world GDP; Income is PPP-adjusted per capita income. All right-hand side variables are averages over 1970-2006.

Table 4. Top 49 Countries and the Rest of the World in Terms of 2004 GDP

Country	GDP/ World GDP	Country	GDP/ World GDP
United States	0.300	Indonesia	0.006
Japan	0.124	South Africa	0.006
Germany	0.076	Norway	0.006
France	0.054	Poland	0.005
United Kingdom	0.044	Finland	0.005
Italy	0.041	Greece	0.004
China	0.028	Venezuela, RB	0.004
Canada	0.026	Thailand	0.004
Brazil	0.021	Portugal	0.003
Spain	0.020	Colombia	0.003
India	0.017	Nigeria	0.003
Australia	0.016	Algeria	0.003
Russian Federation	0.015	Israel	0.003
Mexico	0.015	Philippines	0.003
Netherlands	0.015	Malaysia	0.002
Korea, Rep.	0.011	Ireland	0.002
Sweden	0.010	Egypt, Arab Rep.	0.002
Switzerland	0.010	Pakistan	0.002
Belgium	0.009	Chile	0.002
Argentina	0.008	New Zealand	0.002
Saudi Arabia	0.007	Czech Republic	0.002
Austria	0.007	United Arab Emirates	0.002
Iran, Islamic Rep.	0.007	Hungary	0.002
Turkey	0.007	Romania	0.002
Denmark	0.006	Rest of the World	0.027

Notes: Ranking of top 49 countries and the rest of the world in terms of 2004 U.S.\$ GDP. We include Hong Kong, POC, and Singapore in Rest of the World. Source: The World Bank (2007b).

Table 5. Bilateral Trade Shares: Data and Model Predictions for the 50-Country Sample

	model	data
Domestic sales as a share of domestic absorption (π_{ii})		
mean	0.7222	0.7555
median	0.7244	0.7982
corr(model, data)	0.4761	
Export sales as a share of domestic absorption (π_{ij})		
mean	0.0056	0.0051
median	0.0026	0.0012
corr(model, data)	0.6741	

Notes: This table reports the means and medians of domestic output (top panel), and bilateral trade (bottom panel), both as a share of domestic absorption, in the model and in the data. Source: International Monetary Fund (2007).

Table 6. GDP and Granular Volatility: Data and Non-Symmetric Trade Model Predictions

	(1)	(2)	(3)	(4)
Dep. Var: GDP Volatility				
σ_T	1.078** (0.285)	0.853** (0.297)	0.617* (0.279)	0.538+ (0.280)
GDP-per-capita		-0.176* (0.067)	-0.164* (0.062)	-0.199** (0.052)
Risk Content of Exports			0.131* (0.061)	— —
Herfindahl of Production				-0.172 (0.252)
Constant	1.399 (1.298)	2.032 (1.315)	1.775 (1.291)	0.218 (1.158)
Observations	49	49	47	35
R^2	0.15	0.25	0.40	0.38

Notes: Robust standard errors in parentheses. + significant at 10%; * significant at 5%; ** significant at 1%. All variables are in natural logarithms. The dependent variable is the standard deviation of per capita GDP growth over the period 1970-2006. σ_T is the granular aggregate volatility implied by the simulated model. GDP-per-capita is the PPP-adjusted per capita GDP. Risk Content of Exports is the measure of the volatility of a country's export pattern sourced from di Giovanni and Levchenko (2008). Herfindahl of Production is the Herfindahl index of production shares, sourced from di Giovanni and Levchenko (2007).

Table 7. Counterfactual Exercises: International Trade and Granular Volatility

Country	(1)	(2)	(3)	(4)	Country	(1)	(2)	(3)	(4)
	Trade/ Actual	Trade/ Autarky	Further Trade Opening All Countries	Single Country		Trade/ Actual	Trade/ Autarky	Further Trade Opening All Countries	Single Country
United States	0.348	1.037	1.086	1.117	Indonesia	0.266	1.141	1.273	1.342
Japan	0.348	1.036	1.100	1.136	South Africa	0.442	1.146	1.268	1.331
Germany	0.526	1.138	1.155	1.206	Norway	0.701	1.243	1.233	1.277
France	0.586	1.157	1.166	1.217	Poland	0.336	1.275	1.204	1.248
United Kingdom	0.472	1.146	1.176	1.229	Finland	0.443	1.323	1.176	1.200
Italy	0.439	1.130	1.189	1.249	Greece	0.371	1.290	1.208	1.242
China	0.240	1.062	1.167	1.221	Venezuela, RB	0.215	1.236	1.267	1.316
Canada	0.423	1.135	1.165	1.208	Thailand	0.281	1.198	1.277	1.332
Brazil	0.222	1.094	1.226	1.299	Portugal	0.334	1.287	1.192	1.222
Spain	0.454	1.164	1.202	1.259	Colombia	0.548	1.223	1.270	1.318
India	0.287	1.113	1.229	1.301	Nigeria	0.233	1.294	1.250	1.292
Australia	0.495	1.070	1.210	1.275	Algeria	0.269	1.366	1.187	1.215
Russian Federation	0.119	1.155	1.247	1.310	Israel	0.462	1.301	1.229	1.269
Mexico	0.261	1.103	1.212	1.269	Philippines	0.359	1.225	1.276	1.324
Netherlands	0.694	1.262	1.173	1.209	Malaysia	0.323	1.232	1.261	1.308
Korea, Rep.	0.265	1.114	1.208	1.264	Ireland	0.458	1.318	1.147	1.173
Sweden	0.599	1.255	1.217	1.263	Egypt, Arab Rep.	0.496	1.363	1.210	1.239
Switzerland	0.480	1.203	1.216	1.266	Pakistan	0.576	1.279	1.240	1.280
Belgium	0.671	1.336	1.129	1.152	Chile	0.232	1.209	1.282	1.334
Argentina	0.187	1.188	1.268	1.321	New Zealand	0.499	1.157	1.291	1.351
Saudi Arabia	0.133	1.190	1.273	1.337	Czech Republic	0.308	1.325	1.198	1.230
Austria	0.663	1.239	1.209	1.251	United Arab Emirates	0.154	1.283	1.249	1.291
Iran, Islamic Rep.	0.156	1.236	1.264	1.314	Hungary	0.361	1.289	1.240	1.277
Turkey	0.258	1.232	1.237	1.285	Romania	0.245	1.370	1.198	1.225
Denmark	0.648	1.292	1.192	1.225					

Notes: ‘Trade/Actual’ reports the ratio of granular volatility implied by the model under trade to the actual volatility of GDP growth. In calculating the granular volatility, this column assumes that the firm-level volatility is equal to $\sigma = 0.1$. ‘Trade/Autarky’ reports the ratio of granular volatility under trade to the granular volatility under autarky for each country. ‘Further Trade Opening’ reports the ratio of granular volatility under a 50% reduction in iceberg trade costs τ_{ij} to the granular volatility as implied by the model under current trade costs. ‘Multi’ applies this reduction to all countries, while ‘Single’ only applies the reduction to a given country and its trading partners.

Table 8. Counterfactual Exercises: Comparison of Reductions in Trade Costs and Granular Volatility

<i>Percentile</i>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Multi-Country Trade Liberalization			Single-Country Trade Liberalization				
	<i>Percentage Reduction in Trade Costs</i>			<i>Percentage Reduction in Trade Costs</i>				
	10%	25%	50%	75%	10%	25%	50%	75%
5	1.018	1.056	1.129	1.190	1.020	1.067	1.152	1.234
10	1.022	1.066	1.160	1.197	1.025	1.079	1.203	1.245
25	1.028	1.078	1.188	1.242	1.032	1.091	1.225	1.289
50	1.039	1.107	1.211	1.268	1.045	1.127	1.265	1.337
75	1.044	1.124	1.250	1.326	1.051	1.145	1.308	1.389
95	1.049	1.135	1.277	1.374	1.056	1.158	1.337	1.444
min	1.009	1.029	1.086	1.163	1.010	1.034	1.117	1.208
max	1.050	1.137	1.291	1.386	1.057	1.160	1.351	1.486

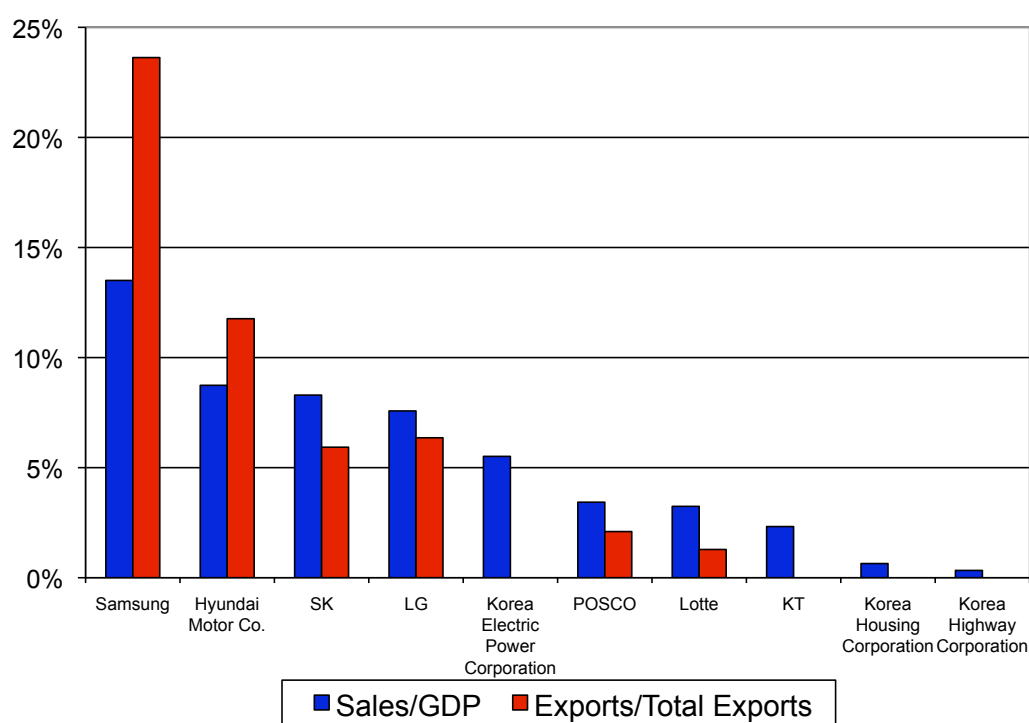
Notes: This table reports percentiles and minimum and maximums of the ratio of granular volatility under a four reductions in iceberg trade costs τ_{ij} to the granular volatility as implied by the model under current trade costs. Multi-Country applies this reduction to all countries, while Single-Country only applies the reduction to a given country and its trading partners.

Table 9. Sensitivity Checks: The Impact of Trade on Granular Volatility

Country	(1)		(2)		(3)		(4)	
	Volatility Decr. in Firm Size	Eaton-Kortum Trade Costs	Volatility Decr. in Firm Size	Eaton-Kortum Trade Costs	Volatility Decr. in Firm Size	Eaton-Kortum Trade Costs	Volatility Decr. in Firm Size	Eaton-Kortum Trade Costs
United States	1.032	1.074	1.136	1.008	1.183	1.269	1.486	1.025
Japan	1.043	1.092	1.162	1.006	1.191	1.305	1.486	1.024
Germany	1.144	1.256	1.317	1.049	1.277	1.492	1.525	1.062
France	1.167	1.281	1.372	1.055	1.296	1.513	1.525	1.080
United Kingdom	1.150	1.272	1.356	1.046	1.348	1.524	1.570	1.112
Italy	1.135	1.277	1.337	1.033	1.315	1.517	1.554	1.085
China	1.079	1.164	1.246	1.010	1.275	1.451	1.569	1.044
Canada	1.155	1.239	1.344	1.040	1.239	1.444	1.560	1.033
Brazil	1.124	1.201	1.344	1.016	1.320	1.487	1.551	1.087
Spain	1.181	1.311	1.398	1.044	1.290	1.449	1.580	1.045
India	1.141	1.285	1.387	1.021	1.336	1.570	1.614	1.056
Australia	1.104	1.190	1.305	1.010	1.397	1.602	1.616	1.092
Russian Federation	1.180	1.395	1.434	1.034	1.335	1.600	1.592	1.064
Mexico	1.135	1.219	1.353	1.020	1.278	1.480	1.573	1.039
Netherlands	1.267	1.424	1.463	1.100	1.274	1.435	1.563	1.046
Korea, Rep.	1.144	1.270	1.358	1.024	1.344	1.502	1.529	1.134
Sweden	1.278	1.470	1.512	1.077	1.396	1.644	1.660	1.087
Switzerland	1.230	1.465	1.491	1.052	1.333	1.528	1.604	1.059
Belgium	1.340	1.479	1.497	1.153	1.271	1.399	1.580	1.039
Argentina	1.222	1.350	1.539	1.037	1.215	1.378	1.541	1.021
Saudi Arabia	1.226	1.467	1.498	1.036	1.350	1.576	1.598	1.080
Austria	1.271	1.488	1.514	1.068	1.326	1.528	1.606	1.062
Iran, Islamic Rep.	1.271	1.525	1.587	1.044	1.333	1.602	1.601	1.063
Turkey	1.260	1.480	1.527	1.062	1.414	1.621	1.653	1.097
Denmark	1.320	1.508	1.542	1.092				

Notes: This table reports the contribution of international trade to aggregate volatility (ratio of volatility under trade to the volatility in autarky) under alternative assumptions. Column (1) reports the results of a simulation in which the firm-specific volatility decreases in firm size. Column (2) reports the results of applying values of τ_{ij} based on Eaton and Kortum (2002). Columns (3) and (4) reports the results of applying the elasticity of substitution of 4 and 8, respectively.

Figure 1. Korean Business Groups Sales As a Share of GDP and Total Exports



Notes: This table reports the sales of the top 10 Korean business groups, as a share of Korean GDP (blue/dark bars) and total Korean exports (red/light bars). Source: Korean Development Institute.

Figure 2. The Timing of the Economy

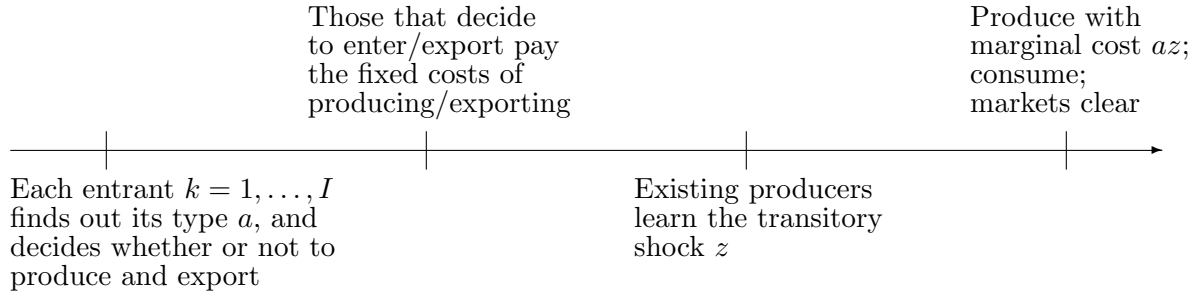


Figure 3. The Analytical Power Law in the Melitz-Pareto Model

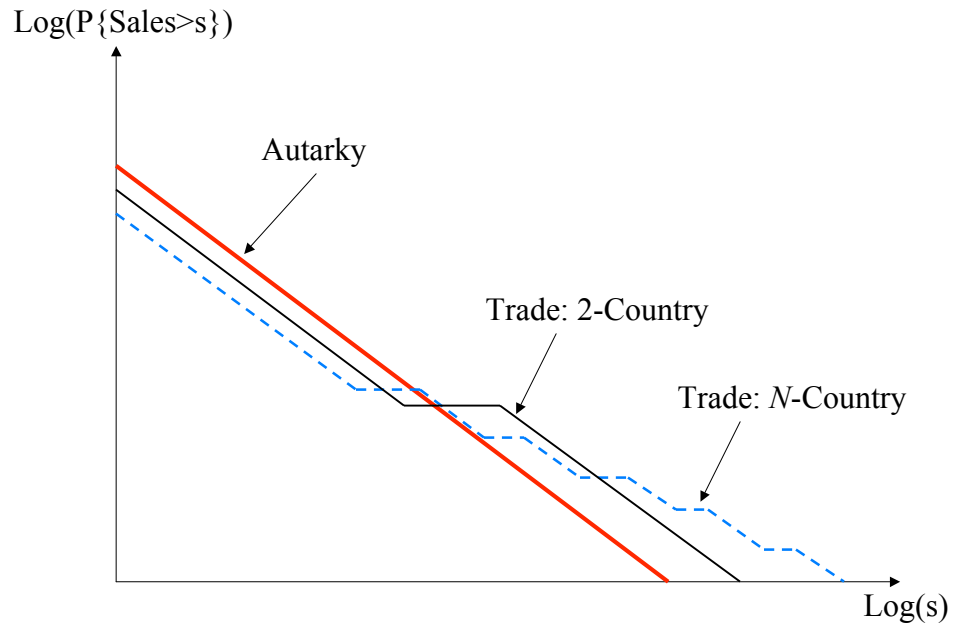
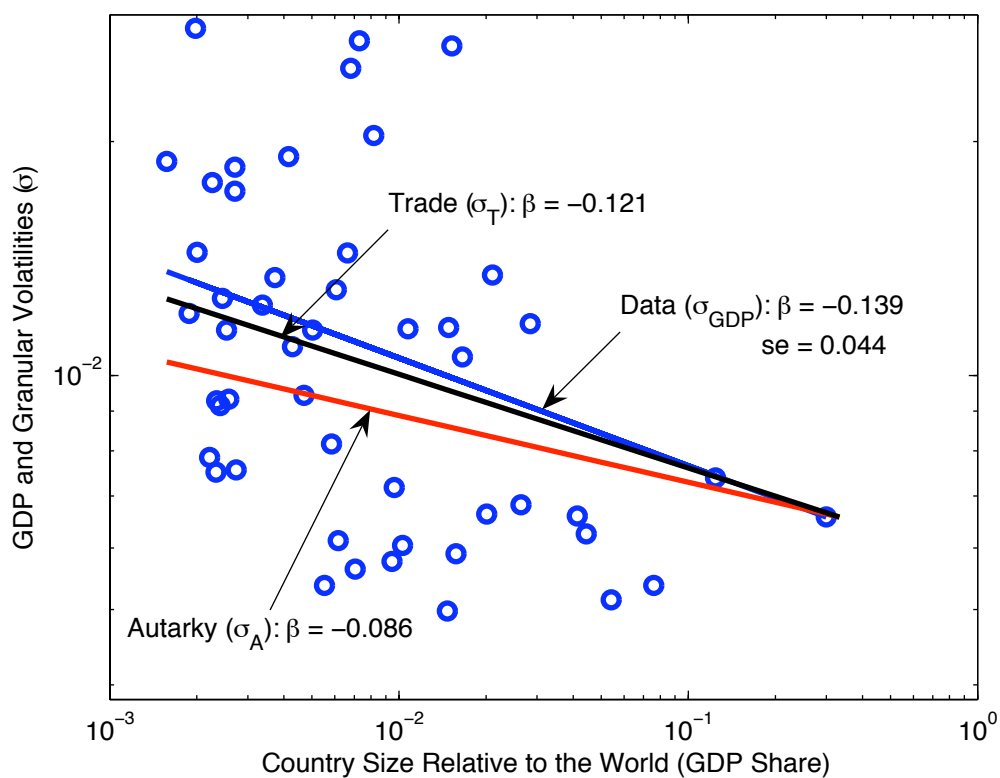
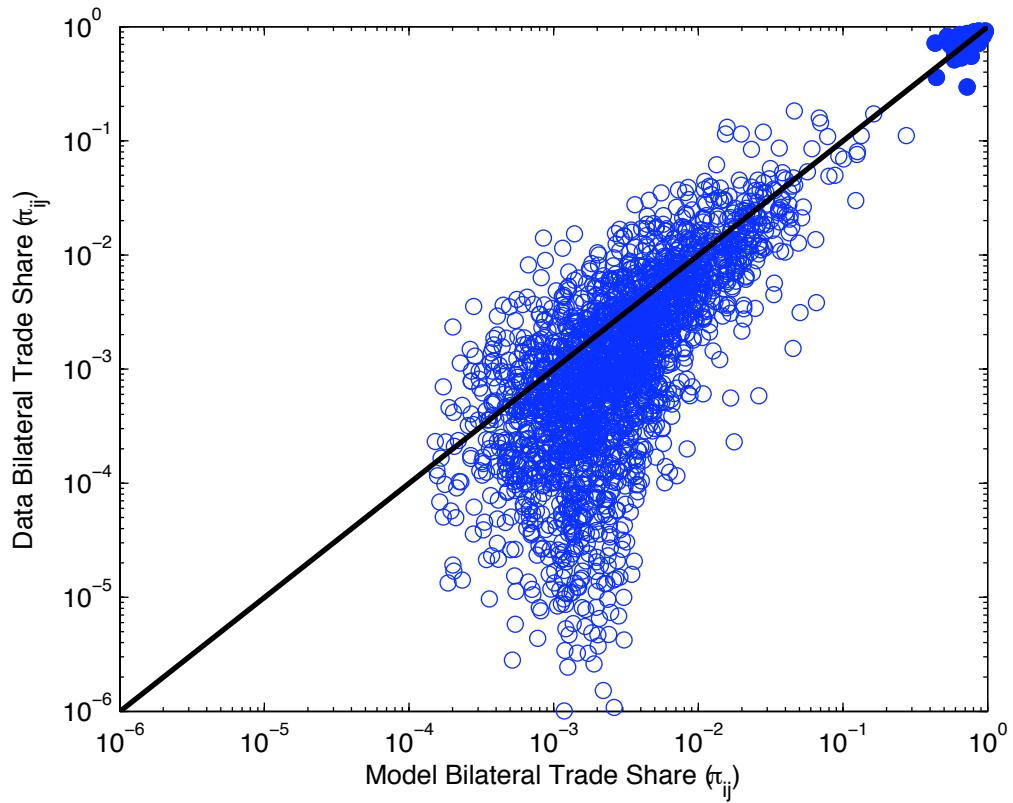


Figure 4. Volatility and Country Size: Data and Symmetric Model Predictions



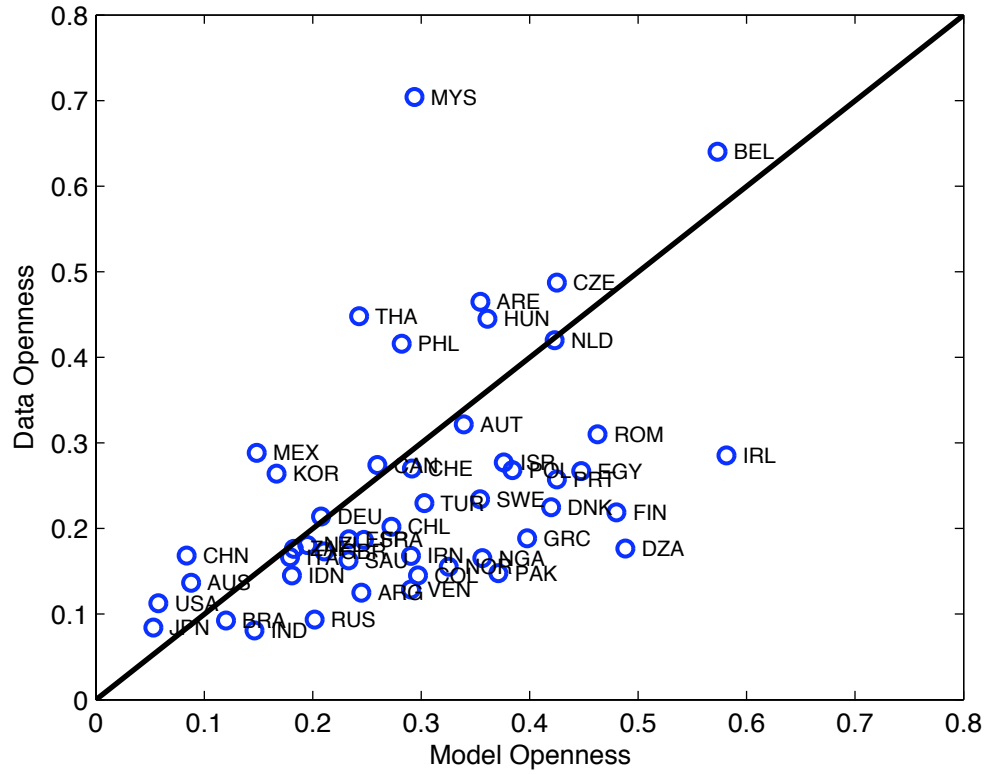
Notes: This figure plots the relationship between country size and aggregate volatility implied by the data (conditioning for per-capita GDP), the model with symmetric countries under trade, and the model with symmetric countries in autarky. The dots represent actual observations of volatility. Note that the data points and regression line are shifted by a constant for ease of visual comparability with the model regressions lines. Source: The World Bank (2007b).

Figure 5. Bilateral Trade Shares: Data and Model Predictions



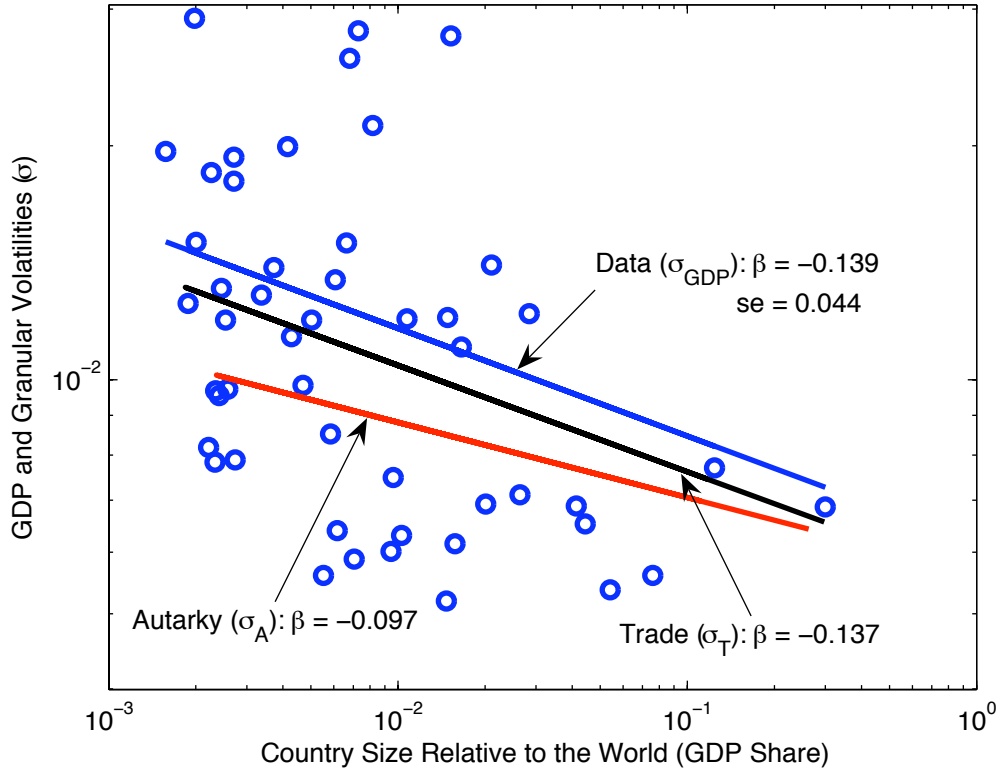
Notes: This figure reports the scatterplot of domestic output (π_{ii}) and bilateral trade (π_{ij}), both as a share of domestic absorption. The values implied by the model are on the horizontal axis. Actual values are on the vertical axis. Solid dots represent observations of π_{ii} , while hollow dots represent bilateral trade observations (π_{ij}). The line through the data is the 45-degree line.

Figure 6. Trade Openness: Data and Model Predictions



Notes: This figure reports total imports as a share of domestic absorption ($1 - \pi_{ii}$). The values implied by the model are on the horizontal axis. Actual values are on the vertical axis. The line through the data is the 45-degree line.

Figure 7. Volatility and Country Size: Data and Non-Symmetric Model Predictions



Notes: This figure plots the relationship between country size and aggregate volatility implied by the data (conditioning for per-capita GDP), the model with non-symmetric countries under trade, and the model with non-symmetric countries in autarky. The dots represent actual observations of volatility. Note that the data points and regression line are shifted by a constant for ease of visual comparability with the model regressions lines. Source: The World Bank (2007b).