The Case for a Financial Approach to Money Demand

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Abstract

The distribution of money across households is much more similar to the distribution of financial assets than to that of consumption levels. This is a puzzle for theories which directly link money demand to consumption, such as cash-in-advance (CIA), money-in-the-utility function (MIUF) or shopping-time models. This paper shows that the joint distribution of money and financial assets can be explained by an incomplete-market model in which frictions are introduced into financial markets. Money demand is modeled as a portfolio choice with a fixed transaction cost in financial markets.

JEL codes: E40, E50.

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1 Introduction

Why do households hold money? Various theories of money demand answer this question by focusing on the transaction role money plays in goods markets (e.g., shopping-time and cash-in-advance (CIA) models), transaction costs in financial markets (Allais, 1947; Baumol, 1952; Tobin, 1956) or simply assuming a liquidity role for money, as models with the money in the utility function (MIUF). These theories are observationally equivalent in aggregate data: they can be realistically calibrated to match various estimates, such as the interest elasticity of money demand. In this paper, I show that microeconomic data can be used to discriminate between these different theories: Household data unambiguously reject standard models of money demand, such as the CIA, baseline MIUF or shopping-time models, while theories based on financial frictions are better able to reproduce realistic distributions of money, consumption and wealth.

In both Italian and US data, the distribution of money (M1) is similar to that of financial wealth, and much more unequally distributed than is that of consumption (as measured by the Gini coefficient, for example). In 2004, the Gini coefficients are around 0.3 for consumption, 0.5 for income, 0.8 for net wealth and 0.8 for money. This stylized fact, further detailed below, holds for different definitions of money, various time periods, and after controlling for life-cycle effects. This distribution of money cannot be understood in standard macroeconomic models where money demand is introduced via CIA, MIUF or shopping-time considerations. In these models, real money balances are proportional to consumption, and the distributions of money holdings and consumption should be equally distributed among households (i.e. have the same Gini coefficient). As shown below, this property holds even when we consider more general transaction technologies on the goods market, which may produce scale economies. In addition to its theoretical interest, the ability to reproduce the distribution of money is crucial for the assessment of the real and welfare effects of inflation. When money holdings are widely dispersed, inflation can be expected to have significant distributional effects.

We show that a realistic joint distribution of consumption, money and financial assets can be reproduced via transaction costs in financial markets only, without any assumption regarding frictions in the goods market. Money demand is modeled as a portfolio choice between money and a riskless interest-bearing asset. Money holdings can be freely adjusted, but there is a transaction cost of adjusting the quantity of financial assets. This foundation of money demand was introduced by Allais, Baumol and Tobin in their inventory approach to monetary theory. Jovanovic (1982) and Romer (1986) provided general-equilibrium extensions, but without focusing on the properties of the equilibrium distributions. Contrary to these papers, I assume, following Heller (1974) and Chatterjee and Corbae (1994), that households do not need money to make purchases, and do not face a cash-in-advance constraint. I thus assume that the payment system is well organized, so that the transaction demand for money is close to zero. This financial approach to money demand makes it possible to consider money as a special asset in the theory of asset prices with transaction costs in financial markets (see Lo et al. 2004 for references). In this literature, transaction costs reflect differences in liquidity between assets (Huang, 2003, for example).

This portfolio choice is introduced into a production economy where infinitely-lived agents face uninsurable income fluctuations and borrowing constraints, a framework often described as the "Bewley-Huggett-Aiyagari" environment. In this type of economy, households choose between two assets with different returns, but also different transaction costs, in order to smooth idiosyncratic income fluctuations. Due to the transaction cost, households participate only infrequently in financial markets to rebalance their portfolio, as documented by Vissing-Jorgensen (2002) among others. This type of economy does not introduce life-cycle considerations and is thus well-suited for the analysis of heterogeneity within generations. The model is calibrated to reproduce the idiosyncratic income fluctuations faced by US households. The transaction cost is chosen to match the average quantity of money held by households in the US economy.

This model generates a realistic joint distribution of money and financial assets, with the transaction cost being the only deviation from the baseline heterogenous-agents model. This result is robust to various changes in the model parameters, and to modeling choices. In particular, although the participation cost in the financial market affects the average amount of money held, it does not significantly change the dispersion of the distribution of money holdings. This is because households hold money to smooth consumption without paying transaction costs in financial markets. They only participate in financial markets to increase their financial savings when their money holdings are high, and to dis-save when their money holdings are low. Between these two boundaries, which depend on household wealth, money is used as an asset to smooth consumption. In consequence, although the amount saved in money is on average much less than that invested in financial markets, the dispersion of the distributions of money and assets remain similar to each other.

Related Literature

To my knowledge this paper is the first to reproduce a realistic distribution of money and wealth. The paper belongs first to the literature on money demand, and more specifically to the Allais-Baumol-Tobin model in general equilibrium. In a recent paper, Alvarez et al. (2002) introduce both a fixed transaction cost and a cash-in-advance constraint in a general-equilibrium setting. To simplify their analysis of the short-run effect of money injections, they assume that markets are complete and, in consequence, that all agents have the same financial wealth. As my goal is to introduce only essential departures from the benchmark settings to reproduce the joint distribution of money and wealth, I only assume a transaction cost in financial markets and do not introduce a cash-in-advance constraint. Heller (1974) has shown that the fixed transaction cost in financial markets suffices for money to have positive value in equilibrium.

Second, this paper belongs to the literature on money demand in economies with idiosyncratic shocks and incomplete markets. The initial papers in this literature considered money as the only available asset for self-insurance against idiosyncratic shocks (Bewley, 1980 and 1983; Scheinkman and Weiss 1986; Imohoroglu, 1992). More recent papers have introduced another financial asset with some additional frictions to justify positive money demand. Imrohoroglu and Prescott (1991) use a per-period cost, so that households hold either money or financial assets, but never both, and consider the real effects of various monetary arrangements. Erosa and Ventura (2002) introduce a cash-in-advance constraint and a fixed cost of withdrawing money from financial markets to study the inflation tax. Akyol (2004) analyses an endowment economy where the timing of market openings implies that only high-income agents hold money. Bai (2005) also introduces transaction costs in financial markets, but in the context of an endowment economy to study the real effect of inflation on the real interest rate (the so-called Mundell-Tobin effect) and on welfare. Recent papers in the search-theoretic literature (Molico, 2006) also explain the co-existence of money and financial assets by introducing centralized financial markets and decentralized goods markets (Chiu and Molico, 2007). None of these papers describes or reproduces the joint distribution of money and financial assets. Finally, Heer et al. (2007) conclude that standard monetary models fail to reproduce the money-age distribution, but do not provide an alternative model.

This paper is also related to the empirical work which has estimated money demand using household data. Mulligan and Sala-i-Martin (2000) introduce a fixed adoption cost of the technology to participate in financial markets, in addition to a shopping-time constraint. They estimate the adoption cost via various economic and econometric models using US household data. Attanasio et al. (2002) estimate a shopping-time model *à la* McCallum and Goodfriend (1987), using Italian household data. Finally, Alvarez and Lippi (2007) use Italian household data to estimate a model where households face a cash-in-advance constraint, a fixed transaction cost and a stochastic cost of withdrawing money. They show that this stochastic component improves the outcome of the model as compared to a deterministic

Baumol-Tobin framework. Although I also use household data, my goal is different: I reproduce a realistic joint distribution of money, wealth, and consumption as a general-equilibrium outcome, and show that a simple friction in financial markets suffices to generate these results.

The paper is organized as follows. Section 2 presents empirical facts about the distribution of money in Italy and the US. Section 3 shows that the usual assumptions regarding money demand fail to reproduce these facts. Section 4 describes the fixed transaction-cost model, and the parameterization appears in Section 5. Section 6 presents the results and the distribution of money and assets, and Section 7 discusses some robustness tests. Finally, Section 8 concludes.

2 The Distribution of Money

This section presents some empirical facts about the distribution of money and assets in Italy and the US. Although the model below will be calibrated using US data, I use Italian data to verify that the properties of the distribution of money are similar across countries. In the following, I use a narrow definition of money, M1, to emphasize the distinction between money and other financial assets. The stylized fact is that the distribution of money is similar to the distribution of assets. The same analysis has been carried out for various monetary aggregates and the results are quantitatively similar. As a summary of the following analysis, Fig. 1 depicts the Lorenz curves of money, income and net worth[1] distributions using the 2004 Survey of Consumer Finance, and those of the consumption, income, net worth, and money distributions using Italian data from the 2004 Survey of Households' Income and Wealth. In both cases, I only consider households whose head is aged between 35 and 44 to avoid life-cycle effects. Money is more unequally distributed than income and net wealth in both countries.

[1] As is fairly usual, I use net worth as a summary statistic for all types of assets. The Lorenz curve of financial assets is very similar to that of net wealth.

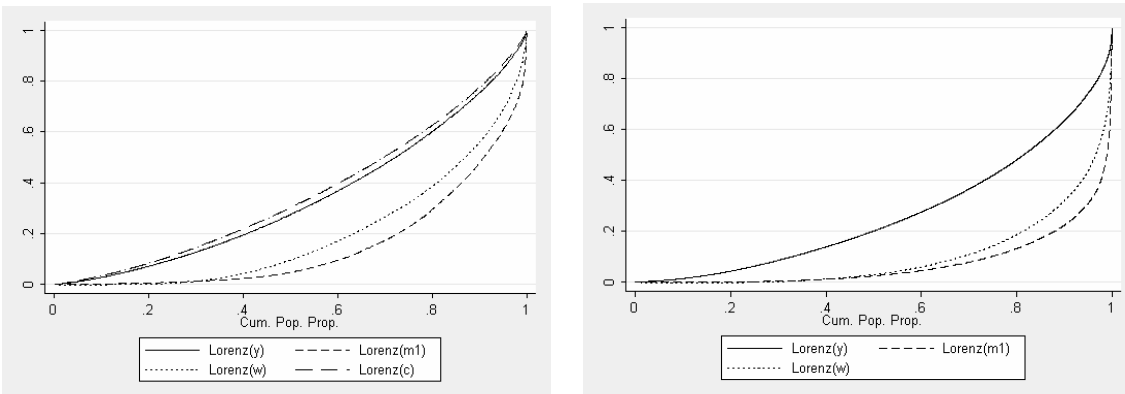


Figure 1: Lorenz Curves of Income (y), Money (m1), Wealth (w) and Consumption (c), in Italy (left) and the US (right), for households whose head is aged between 35 and 44.

2.1 Italian Data

This section uses the 2004 Italian Survey of Households' Income and Wealth to examine the distribution of money. This periodic survey provides data for various deposit accounts, currency, income and wealth in the Italian population. Each survey is conducted on a sample of about 8,000 households, and provides representative weights. A number of recent papers have used this data set to analyse money demand at the household level (Attanasio, et al. 2002; Alvarez and Lippi 2007, amongst others).

Table 1: Distribution of Money and Wealth, Italy 2004

Gini coefficient of	Cons.	Income	Net W.	Money
Total Population	.30	.35	.59	.68
Popn., 35≤Age≤44	.29	.32	.61	.70
Popn., 35≤Age≤44, 99%.	.27	.31	.57	.63

Table 1 shows the Gini coefficient of the distributions of consumption, income, net worth and money (in columns) for three different types of households (in rows). The first column presents the Gini coefficient for total consumption, and the first row shows the figure for the whole population. This is fairly low, at .30. To avoid life-cycle effects the second line focuses on households whose head is aged between 35 and 44. The Gini coefficient is almost unchanged at .29. The second column shows the results for the distribution of income. The Gini coefficeint is a little higher than that of consumption at .35, falling to .32 for the 35-44 age group. The third column performs the same exercise for the distribution of net wealth. This is more dispersed than consumption or income: the Gini coefficient for net worth is .59, increasing slightly to .61 for the 35-44 age group.

I use Italian data to construct the quantity of money (M1) held by each households, as the sum of the amount held in currency and in checking accounts. Although checking accounts are interest-bearing in Italy, the interest rate is low enough for this aggregation to be relevant: the average interest rate on checking accounts is below 1%, whereas the average yearly yield of Italian 10-year securities was over 4% in 2004. The last column of Table 1 shows the distribution of money. The Gini coefficient is very high here, at .68, and increases to .70 for the 35-44 age group. As a robustness check, I consider the distribution of money without including the 1% of the households who hold the most money. Some households may hold money in their checking accounts for a few days prior to buying very expensive durable goods (such as houses). If the survey interview occurs during this period, we will observe high levels of money balances that are not relevant.[2] The Gini coefficient on money holdings falls from .70 to .63 after this exclusion, thus remaining high.

The distribution of money is thus similar to that of net wealth, and is very different from that of consumption. For space reasons, this section has characterized the distribution by the Gini coefficient. However, other measures of inequality yield the same results. This can be seen graphically in Figure 1, which shows the four Lorenz curves for the population aged between 35 and 44.

[2] I carry out this exercise even though it is problematic to justify the exclusion of this 1% of households. If households keep money to buy a house over a period of one week, and buy a new house as often as every five years, the probability that they will be observed with this money the day of the interview is only $(1/52) * (1/5) = 0.4\%$.

Table 2 presents the empirical correlations between money holdings, consumption levels, income and wealth. Money is positively correlated with consumption, income and wealth, with a coefficient of between .2 and .3. The correlation between the ratio of money over total financial assets and wealth is negative. That is, the share of money in the financial portfolio falls with wealth. This property of the money/wealth distribution had already been noted in the US economy by Erosa and Ventura (2002).

Table 2: Empirical Correlations, Italy 2004, 35≤age≤44

Money & Income	.21
Money & Consumption	.27
Money & Net Wealth	.30
(Money/Fin. W.) & Net .W.	-0.13

2.2 US Data

US data do not allow us to carry out the same detailed analysis: Income, money and financial wealth come from by the Survey of Consumer Finance (SCF), and the distribution of consumption can be found in the survey of Consumer Expenditures (CE). Hence, we cannot calculate the correlation between consumption and money. I use a conservative definition of money, which is the amount held in checking accounts. This is the only fraction of M1 which is available in the data. I also provide statistics for the amount held in all transaction accounts, which corresponds to the M2 aggregate.

The distribution of income in the SCF[3] 2004 is given in the first column of Table 3. The Gini coefficient is .54 and decreases to .47 (second row) if one considers the households whose head is aged between 35 and 44. It decreases further to .41 if one excludes the 1% money-richest households (third row).

[3] The same exercise can be carried out for a number of years of the SCF. The results are quantitatively similar.

The results for the distribution of net wealth are given in column 2. The values of the Gini coefficient are very similar between specifications, and range between .81 and .73.

The Gini coefficient of the distribution of money held in checking accounts is given in the column 3. This is very high at .81. Excluding life-cycle effect in the second row, the Gini coefficient increases to .83. Finally the third row excludes the 1% money-richest households: the Gini coefficient falls, but is still high at .75. The fourth column performs the same analysis for money held in all transaction accounts, such as checking, savings and money market accounts. The Gini coefficient here is of the same order of magnitude, and falls from .85 to .79. excluding the 1% money-richest households. The Gini coefficient for money is higher than that of the distribution of income for all definitions of money and for all sets of households. As a result, the distribution of money is much closer to the distribution of net wealth than to the distribution of income.

Table 3: Distribution of Money and Wealth

Gini coefficient of	Income	Net. W	Check. Acc.	Trans.Acc.
Total Population	.54	.81	.81	.85
Pop., 35≤Age≤44	.47	.80	.83	.85
Pop., 35≤Age≤44, 99%.	.41	.73	.75	.79

The correlation between money (checking account), income and other assets is presented in Table 4. Money is positively correlated with both income and net wealth: richer households hold more money on average. The last line of Table 4 shows the correlation between the ratio of money in financial wealth and total net wealth. This correlation is negative. As in the Italian data, richer households hold more money but as a smaller fraction of their financial wealth.

Table 4: Empirical Correlations

US, 2004, $35 \leq \text{age} \leq 44$

Money & Income	.12
Money & Net Wealth	.17
(Money/Fin. W.) & Net Wealth	-0.08

Table 5 below presents some additional properties of the joint distribution of money and assets in the US economy, which will be used to illustrate the model's outcome. The table shows the fraction of total wealth and total money held by the richest 1% of the population (line 1), the richest 10% (line 2), the richest 20% (line 3) and the poorest 40% (line 4). First, the richest households hold a significant fraction of money, whereas the 40% poorest households hold a much lower fraction. Second, we can check that the proportion of money in total wealth is higher for the poorest households than for the richest households. Poor households hold relatively more money than financial assets, but they hold a smaller fraction of the total quantity of money.

Table 5: Asset Holding Distribution

US, $35 \leq \text{Age} \leq 44$

	Fract. of Wealth	Fract. of Checking
Wealth 99-100	32.7%	10.9%
Wealth 90-100	70.0%	60.8%
Wealth 80-100	82.4%	72.6%
Wealth 0-40	1.03%	5.4%

Finally, the distribution of consumption can be obtained from the survey of Consumer Expenditures (CE). Krueger and Perri (2002) note that the distribution of consumption is much less unequally distributed than that of income. The consumption Gini coefficient is around 0.27 and changes only little over time. I calculate the same Gini coefficient for total consumption using the NBER extract of the Consumer Expenditures survey in 2002, which is the latest year available. I find a Gini coefficient of .28. There is substantial empirical debate about the quality of the data and the estimated changes in consumption inequality (Attanasio et al. 2004, for example). The consensus view is that consumption levels are less unequally distributed than income. As a result, the distribution of money is much closer to the distribution of total wealth than to the distribution of consumption.

To summarize these US and Italian findings: 1) inequality in money holdings is more similar to inequality in net wealth and very different from inequality in consumption; 2) money is positively correlated with wealth, income and consumption levels; and 3) the ratio of money over financial assets falls with wealth.

3 Some Difficulties in Linking Money and Consumption

Simple models of money demand. Simple models of money demand cannot reproduce the shape of the distribution of money, when they link money demand to consumption. They assume that the real money holdings of a household i, m^i, are simply proportional to consumption, c^i

$$m^i = Ac^i$$

where A is a scaling factor, the same for all households, which may depend on the nominal interest rate, real wages and preference parameters. This form is used for instance in Cooley and Hansen (1989) to assess the welfare cost of inflation. It also results in all models with money-in-the utility function (MIUF) where the utility function is homothetic in money and consumption in the sense of Chari et al. (1996), which is the benchmark case in this literature. It is also obtained in a simple specification of the shopping-time model (McCallum and Goodfriend 1987).

In this case, the distributions of money and consumption are homothetic, and their Gini coefficients are equal. This is at odd with the data, as shown in section 2.

Economies of scale in the transaction technology. Some authors have noted that the share of money holdings in total wealth falls with total wealth and have concluded that the transaction technology exhibits scale economies: Richer households, even if they consume more, need less money because they buy more goods via credit. Dotsey and Ireland (1996) provide a microfoundation of this transaction technology, which uses the flexibility provided by the definition of cash and credit goods in Stokey and Lucas (1987). Erosa and Ventura (2002) use this formulation of a heterogenous-agents setting. This implies that the quantity of money and the consumption level of household i satisfy the following relationship:

$$\frac{m^i}{c^i} = A\left(c^i\right)^{-\theta} \text{ with } \theta > 0 \tag{1}$$

However, this specification is not able to reproduce a realistic distribution of money. With moderate returns (a low value of θ), the distribution of money is more equally distributed than the distribution of consumption, because the households with higher consumption levels hold fewer real balances. A more dispersed distribution of money can only be obtained with a implausibly high increasing returns in the transaction technology. In this case, households who consume the most hold almost no money, whereas households who consume little hold higher levels of money balances. However, one implication of this assumption is that consumption and money sould be *negatively* correlated, as higher consumption implies lower money holdings and vice versa. This correlation is rejected by the data.

To illustrate, I consider the distribution of consumption of Italian households aged between 35 and 44. I generate fictitious money distributions with various transaction technologies, using the general form of the transaction technology (1) for various values of θ. I finally analyze the distributional properties of the joint distribution of money and consumption.

Table 6: Properties of the Distribution of Money for Different Transaction Technologies

Values of θ	Data	0	0.5	1	2	3.7
Gini of consumption	.29	.29	.29	.29	.29	.29
Gini of Money	.70	.29	14	0	0.30	.70
Corr. Money Consumpt.	.27	1	.97	0	-0.63	-0.36

Table 6 presents the value of the Gini coefficient and the correlation between money and consumption for various values of θ. For θ less than 1, the distribution of money is more concentrated than the distribution of consumption. To obtain a wider dispersion of money, the returns on the transaction technology must be higher than 1, but the correlation between money and consumption then becomes negative, which is at odd with the data.

The same type of experiment can be carried out with the US Data. Using the distribution of money, I generate a fictitious distribution of consumption using (1). I determine the value of θ for which the distribution of consumption is realistic in terms of the Gini coefficient. Again, we need a value of θ of over 3 to obtain a Gini coefficient under .47, which is the Gini coefficient on income.

Finally, note that the microfoundation of money demand with scale economies in Dotsey and Ireland (1996) requires increasing returns to scale to obtain the correct sign on the interest elasticity of money demand. Erosa and Ventura (2002), who do not explicitly reproduce the distribution of money, use a value of θ of over 3.

To summarize, economies of scale in the transaction technology can not alone generate a realistic distribution of money. This is because money is at the same time positively correlated with consumption and much more dispersed than consumption levels. The following model proves than we can obtain a realistic distribution of money by focusing on transaction technologies in the *financial* market and not in the *goods* markets. The correlation between money and consumption will appear as an outcome, rather than as a specific utility function imposed on the households.

4 The Model

The economy is populated by a unit mass of households, a representative firm and the Government. There is a consumption-investment good and there are two assets: money and a riskless asset issued by firms. The Government finances a public good via the inflation tax and distorting taxes on labor and capital.

Time is discrete and $t = 0, 1, ..$ denotes the period. There is no aggregate uncertainty, but households face idiosyncratic productivity shocks. These shocks are not insurable, and households can partially self-insure by holding money or riskless assets. The crucial assumption is that households must pay a fixed cost λ in terms of the final good[4] to enter the financial market in order to adjust their financial position, and pay no cost to adjust their monetary holdings.

4.1 Households

There is a continuum of length 1 of infinitely-lived households who enjoy utility from consumption c and disutility from hours worked n. For simplicity only, I follow Greenwood, Hercowitz and Huffman (1988) and Domeij and Heathcote (2004) in assuming the following functional form for the period utility function (see also Heathcote, 2005, for a discussion of the properties of this functional form):

$$u(c, n) = \frac{1}{1-\gamma}\left[\left(c - \psi\frac{n^{1+\frac{1}{\varepsilon}}}{1+1/\varepsilon}\right)^{1-\gamma} - 1\right]$$

In this specification, ε is the Frisch elasticity of labor supply, ψ scales labor supply, and γ is the risk-aversion coefficient. In each period, a household i can be in one of three states according its labor market status. Its productivity e_t^i is then either e^1, e^2

[4]The results do not significantly change if we assume that this cost is paid in labor, and thus affects labor supply.

or e^3. For instance, a household whose productivity is e^1 and which works n_t hours earns labor income of $e^1 n_t w_t$, where w_t is the after-tax wage by efficiency unit. Labor productivity e_t^i follows a three-state first order Markov chain with transition matrix denoted T. $N_t = [N_t^1, N_t^2, N_t^3]'$ is the distribution vector of households according to their state on the labor market in period $t = 0, 1....$ The distribution in period t is $N_0 T^t$. Given standard conditions, which will be fulfilled here, the transition Matrix T has an unique ergodic set $N^* = \{N_1^*, N_2^*, N_3^*\}$ such that $N^* T = N^*$. To simplify the dynamics, I assume that the economy starts with the distribution N^* of households.

The variables a_t^i and m_t^i denote respectively the *real* quantity of financial assets and money held at the end of period $t-1$, and r_t is the after-tax real interest rate on the riskless asset. P_t denotes the money price of one unit of the investment-consumption good, and $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate between periods $t-1$ and t. The real income at the beginning of period t of a household holding a_t^i and m_t^i is thus $\frac{m_t^i}{\Pi_t} + (1 + r_t)\, a_t^i$.

Households pay proportional taxes on capital and labor income: τ_t^{cap} is the tax rate on capital and τ_t^{lab} is the tax rate on labor. The variables $\tilde{w}_t$ and $\tilde{r}_t$ are respectively the real wage and the real interest rate before taxes:

$$
\begin{aligned}
w_t &= \left(1 - \tau_t^{lab}\right) \tilde{w}_t \\
r_t &= \left(1 - \tau_t^{cap}\right) \tilde{r}_t
\end{aligned}
$$

In period t, each household can choose to participate or not in the financial market. If she participates, she pays a cost of λ and can freely use her total monetary and financial resources $\frac{m_t^i}{\Pi_t} + (1 + r_t)\, a_t$ to consume the amount c_t^i, and to save a quantity a_{t+1}^i in financial assets and a quantity m_{t+1}^i in money. If the household does not participate, she can only use its monetary revenue m_t^i / Π_t to consume c_t^i and to keep a fraction m_{t+1}^i in money. It is assumed that her financial wealth is reinvested in financial assets[5]: $a_{t+1}^i = (1 + r_t)\, a_t^i$. This transaction choice is summarized by the

[5]This is the standard assumption made by Romer (1986) for instance. The quantitative results

dummy variable I_t^i, which equals 1 when the household participates and 0 otherwise.

No private households can issue money $m_t^i \geq 0$, and households face a simple borrowing limit when participating in financial markets: $a_t^i \geq 0$, for $t = 0, 1..$ and $i \in [0, 1]$.

The program of household i can be summarized as follows:

$$\max_{\{m_{t+1}^i, a_{t+1}^i, c_t^i, n_t^i, I_t^i\}_{t=0,1..}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, n_t^i\right)$$

subject to

$$\begin{aligned}
c_t^i + m_{t+1}^i + I_t^i \left(a_{t+1}^i - (1 + r_t)\, a_t^i + \lambda\right) &= e_t^i w_t n_t^i + \frac{m_t^i}{\Pi_t} \\
\left(1 - I_t^i\right)\left(a_{t+1}^i - (1 + r_t)\, a_t^i\right) &= 0 \\
c_t^i, n_t^i, m_{t+1}^i, a_{t+1}^i &\geq 0, I_t^i \in \{0, 1\} \\
a_0^i, m_i^0 &\text{ given}
\end{aligned}$$

Recursive Formulation

The program of the households can be written in a recursive way as follows (see Bai 2005, for a proof of the existence of Bellman equations in this type of economy). Define $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ as the maximum utility that a household with productivity e_t^i can reach at period t if she participates in financial markets at period t and if she holds an amount m_t^i and a_t^i of monetary and financial wealth respectively; $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ is the analogous utility if the household does not participate.

The Bellman value $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ satisfies

$$\begin{aligned}
V_t^{par}\left(m_t^i, a_t^i, e_t^i\right) = &\max_{a_{t+1}^i, m_{t+1}^i, n_t^i, c_t^i} \{u\left(c_t^i, n_t^i\right) \\
&+\beta E_t \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}\}
\end{aligned}$$

$$\text{subject to } m_{t+1}^i + a_{t+1}^i + c_t^i = w_t e_t^i n_t^i + \frac{m_t^i}{\Pi_t} - \lambda + (1 + r_t)\, a_t^i$$

$$c_t^i, n_t^i, a_{t+1}^i, m_{t+1}^i \geq 0$$

do not change if interest is paid in money.

The value $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ satisfies

$$\begin{aligned} V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right) &= \max_{m_{t+1}^i, n_t^i, c_t^i} \{u\left(c_t^i, n_t^i\right) \\ &\quad + \beta E \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}\} \\ \text{subject to } m_{t+1}^i + c_t^i &= w_t e_i^t n_t^i + \frac{m_t^i}{\Pi_t} \\ a_{t+1}^i &= a_t^i\left(1+r_t\right) \\ c_t^i, n_t^i, m_{t+1}^i &\geq 0 \end{aligned}$$

Note first that the definition of the Bellman equations differs according to the budget constraints of the household: either the household faces one budget constraint, or the consumption choice is made facing both monetary and financial budget constraints. Second the household anticipates that her current portfolio choice may affect her participation choice tomorrow.

Finally, the maximum utility that a household with productivity e_t^i and assets m_t^i and a_t^i can reach is

$$V_t\left(m_t^i, a_t^i, e_t^i\right) = \max\{V_t^{par}\left(m_t^i, a_t^i, e_t^i\right), V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right)\}$$

According to this last maximization, the household either chooses to participate, in which case $I_t^i = 1$, or not, $I_t^i = 0$.

The solution of the household's problem produces a set of optimal decision rules defined over the productivity set $E = \{e^1, e^2, e^3\}$ and the set of assets:

$$\left.\begin{aligned} c_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ a_{t+1}(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ m_{t+1}(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ n_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1] \\ I_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \{0,1\} \end{aligned}\right\} t = 0, 1, ...$$

4.2 Firms

The consumption-investment good is produced by a representative firm in a competitive market. Capital depreciates at a rate of δ and is installed one period before production. We denote by K_t and L_t aggregate capital and aggregate effective labor used in production in period t . Output Y_t is given by

$$Y = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1$$

Effective labor supply is:

$$L_t = e^1 L_t^1 + e^2 L_t^2 + e^3 L_t^3$$

where L_t^1, L_t^2 and L_t^3 is the aggregate labor supply of workers of productivity $1, 2$ and 3 respectively. Profit maximization yields the following relationships

$$\tilde{w}_t = F_L'(K_t, L_t) \tag{2}$$

$$\tilde{r}_t + \delta = F_K'(K_t, L_t) \tag{3}$$

where $\tilde{w}_t$ and $\tilde{r}_t$ are before-tax real wages per efficient unit and the real interest rate.

4.3 Monetary Policy

At each period t, monetary authorities create an amount of new money Δ_t. Let M_t be the total amount of nominal money in circulation at the *end* of period t. The law of motion of the nominal quantity of money is thus

$$M_t = M_{t-1} + \Delta_t \tag{4}$$

The real value of the inflation tax is thus Δ_t / P_t.

I focus below on stationary equilibria where monetary authorities create a quantity of money proportional to the total nominal quantity of money of the previous period, with a coefficient of π. In this case $\Delta_t = \pi M_{t-1}$ and the revenue from the inflation tax is $\pi M_{t-1} / P_t$.

4.4 Government

The Government finances a public good, which costs G_t units of goods in period t. It receives the inflation tax Δ_t / P_t and the proportional taxes on capital and labor income, with coefficients τ_t^{cap} and τ_t^{lab} respectively. It is assumed that the Government does not issue any debt. Its budget constraint is

$$G_t = \tau_t^{cap}\tilde{r}_t K_t + \tau_t^{lab}\left(N_t^1 L_t^1 e_t^1 + N_t^2 L_t^2 e_t^2 + N_t^3 L_t^3 e_t^3\right)\tilde{w}_t + \frac{\Delta_t}{P_t} \tag{5}$$

where N_t^1, N_t^2 and N_t^3 are the number of type $1, 2$ and 3 households respectively.

4.5 Market Clearing

Denote $\Phi_t : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1]$ as the joint distribution of households over financial assets, money holdings and productivity in period t. Money and capital market equilibria state that money is held by households at the end of each period, and that financial savings are lent to the representative firm. These can be written as, for $t \geq 0$:

$$M_t = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} P_t m_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{6}$$

$$K_{t+1} = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} a_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{7}$$

The goods-market equilibrium requires that the amount produced is either consumed by the State, invested in the firm, consumed by the households, or destroyed in the transaction cost. This can be written as

$$\begin{aligned} G_t + K_{t+1} + \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} c_t(a,m,e)\, d\Phi_t(a,m,e) \\ + \lambda \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} I_t(a,m,e)\, d\Phi_t(a,m,e) = F(K_t, L_t) + (1-\delta) K_t \end{aligned} \tag{8}$$

4.6 Equilibrium

For a given path of Government spending $\{G_t^{cap}\}_{t=0..\infty}$ and money creation $\{\Delta_t\}_{t=0..\infty}$, an equilibrium in this economy is a sequence of decision rules $c_t(.,.,.)$, $a_t(.,.,.)$, $m_t(.,.,.)$, $n_t(.,.,.)$ $I_t(.,.,.)$ defined over $\mathbb{R}^+ \times \mathbb{R}^+ \times \{e^1, e^2, e^3\}$ for $t = 0..\infty$, sequences of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and sequences of taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$ such that:

1. The functions $c_t(.,.,.)$, $a_t(.,.,.), m_t(.,.,.), n_t(.,.,.)$ $I_t(.,.,.)$ solve the household's problem for a sequence of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$.

2. The joint distribution Φ_t over productivity and wealth evolves according to the decision rules and the transition matrix T.

3. Factor prices are competitively determined by firm optimal behavior (2)-(3).

4. The quantity of money in circulation follows the law of motion (4).

5. Markets clear: equations (6)-(8).

6. Tax rates $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau_t^{cap}\}_{t=0..\infty}$ are such that the government budget (5) is balanced.

A *stationary* equilibrium is an equilibrium where the nominal money growth rate, the values $G, \tau^{lab}, \tau^{cap}$, r, w, the gross inflation rate $\Pi = \frac{P_t}{P_{t-1}}$, the joint distribution Φ and the decision functions $c(.,.,.)$, $a(.,.,.), m(.,.,.), n(.,.,.)$ $I(.,.,.)$ are time-invariant. In such an equilibrium, the aggregate real variables are constant whereas the nominal variables all grow at the same rate.

5 Parameterization

The model period is one quarter. Table 7 summarizes the parameter values at a quarterly frequency in the stationary benchmark equilibirum

Table 7: Parameter Values

α	β	ψ	σ	ε	τ^{cap}	τ^{lab}	δ	π	λ
0.36	0.99	117	1	0.3	0.397	0.296	0.015	0.007	0.13

Preference and fiscal parameters

The preference and technology parameters have been set to standard values. The capital share is fixed at $\alpha = 0.36$ (Cooley and Prescott, 1995) and the depreciation rate is $\delta = 0.015$, such that the annual depreciation rate is 6% (Stokey and Rebelo, 1995). The discount factor β is set to 0.99, to obtain a realistic capital-output ratio of around 3. The risk-aversion parameter, σ, is set to 1. The Frisch Elasticity of labor supply ε is estimated to be between 0.1 and 1. I use a conservative value of 0.3. Given this value, ψ is set such that aggregate effective labor supply is close to 0.33. The fiscal parameters are calibrated to match the actual tax distortions in the US economy. Following Domeij and Heathcote (2004), the average tax rate on capital income τ^{cap} is 39.7 percent, whereas the average tax rate on labor income τ^{lab} is 26.9 percent. The implied government consumption to annual output ratio is 0.24, which is a little higher than, but not too dissimilar to, the U.S. average of 0.19 over the 1990-1996 period.

The household productivity process

Different models of the income process are now used in the literature. Our modeling strategy is to use a simple process which yields realistic distributions for consumption, income and wealth. I consequently use that in Domeij and Heathcote (2004), with endogenous labor used at a quarterly frequency. They estimate a three-state Markov process, which reproduces the process for logged labor earnings using PSID data. The Markov chain is estimated under two constraints: (i) The first-order autocorrelation in annual labor income is 0.9; and (ii) The standard deviation of the residual in the wage equation is 0.224. These values are consistent with estimations found in the literature (Storesletten, Telmer and Yaron, 2007; Floden and Lindé, 2001, amongst others).

The transition matrix is

$$T = \begin{bmatrix} 0.974 & 0.026 & 0 \\ 0.0013 & 0.9974 & 0.0013 \\ 0 & 0.026 & 0.974 \end{bmatrix}$$

The three productivity levels are $e^1 = 4.74$, $e^2 = 0.848$, $e^3 = 0.17$. The long-run distribution of productivity across the three states is $N^* = [0.045 \;\; 0.91 \;\; 0.045]'$. This parameterization yields a realistic distribution of both wealth and consumption, which is very useful for the issue that we address.

Monetary Parameters

The remaining parameters concern monetary policy and the transaction cost. First, I take the US annual inflation rate in 2004, 2.8 percent. Consequently, the quarterly inflation rate is $\pi = 0.007$. I calibrate the transaction cost $\lambda = 0.13$ to reproduce the ratio of money over income of households between 35 and 44 years old. Money is defined as above as the amount in checking accounts in SCF 2004. The ratio of money over income is 8%. I find a value of 0.19 for λ, which corresponds to 3% of households' average annual income. Scaling by average income per capita in the US of \$43000, we find an annual transaction cost to financial markets for the riskless asset of around \$1450. To my knowledge, there is no consensus in the empirical literature regarding the level of such costs. The empirical strategy of Mulligan and Sala-i-Martin (2000) and Paiella (2001) only provides the median cost or the lower bound of the participation cost. Some insights can be obtained from the literature which estimates the cost of participating in the risky-asset market. Vissing-Jorgensen (2002) estimates this participation cost to be as high as 1100 dollars in order to understand the transaction decisions of 95% of non-participants, whereas a cost of 260 dollars suffices to explain the choices of 75% of non-participants. In consequence, although our cost is towards the top-end of these estimates, it is not inconsistent with current empirical results. I show below the results of the model at a monthly frequency. This estimation of the transaction cost is robust to changes in the definition of the period.

6 Results

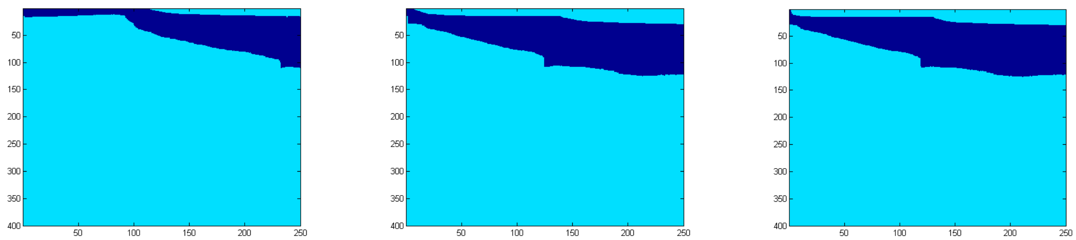
This section first presents the household policy rules and then the properties of the distribution of consumption, income, money and total wealth.

6.1 Participation Decisions

The economic behavior of households is best described by the participation decision in financial markets. Fig. 8 presents the household decision rule according to their productivity. The x-axis measures the quantity of financial assets held at the begin-

Table 8: Participation Decisions

Type 1 Households | Type 2 Households | Type 3 Households



ning of the period, and the y-axis shows the quantity of money held at the beginning of the period. Each point in the squares is a begining-of-period portfolio. For each household type, the dark area represents the portfolio for which households choose not to participate in financial markets. The lighter area represents portfolios for which households choose to participate in financial markets. Households holding a high quantity of money and a small quantity of financial assets (in the South-West corner) and households holding a small quantity of money and high quantity of assets (in the North-East corner) both participate in financial markets; the households who are in between do not participate. Households with a large amount of money and few assets (NE) participate to save in financial assets and dis-save money. Households with little money and many assets participate to dis-save in financial assets and save in money. It can be seen that the lower is productivity the bigger is the area where households dis-save, and the smaller is the area where households save. These portfolio choices summarize the same trade-offs as the ones obtained in (S, s) models first studied in Grossman and Laroque (1990) with one asset. Households thus hold both money and financial assets in equilibrium, although the (marginal) return on money is lower than that on financial assets.

6.2 The Distribution of Money and Financial Assets

The distribution of consumption, income, financial wealth and money in the benchmark economy is summarized in Table 9, which presents the associated Gini coefficients.

Table 9: Gini coefficients

	Consumption	Income	Money	Wealth
US Data (Age 35-44)	.28	.47	.83	.80
Model	.32	.37	.85	.81

First note that the ranking of the Gini coefficients is the same as that in the data, and that the model performs quantitatively well in reproducing the inequality in the distribution of consumption, income, money and wealth. The Gini of the total wealth distribution is 0.81. The Gini coefficient for money is 0.85, which is very similar to that actually observed in the US economy. The Gini coefficient for consumption is a little higher than its empirical counterpart, whereas the Gini coefficient for income is lower than that found in the data.

Table 10 below investigates the distributional properties of the model. As in Table 5 above, the table shows the fraction of wealth and money held by various subpopulations, ranked by their wealth. The right-hand side of the table presents the values produced by the model. For ease of comparison, the left-hand side reproduces the empirical counterparts in the US in 2004. The model performs relatively well in reproducing the wealth and money holdings of the poorest households. However, it does not reproduce a realistic distribution of wealth for the richest 1% of households. This difficulty is known in this class of models (see DeNardi, 2004 for instance).

Table 11 presents the correlation between money, income and financial wealth generated by the model. The left-hand side shows values in US data for the relevant

Table 10: The Distribution of Asset Holdings

	US Data, 35≤Age≤44		Model	
	Fract. of Wealth	Fract. of Checking	Fract. of Wealth	Fract. of Money
Wealth 99-100	32.7%	10.9%	14.0%	0.3%
Wealth 90-100	70.0%	60.8%	68.7%	43.2%
Wealth 80-100	82.4%	72.6%	84.7%	79.4%
Wealth 0-40	1.0%	5.4%	1.5%	2.1%

age group, and the right-hand side shows the model results. All of the model correlations have the right signs. Further, the correlations between money & consumption and money & wealth are higher than those in the data. There are therefore other sources of heterogeneity in money holdings which are not captured by the model. Finally, the model is able to reproduce the average negative correlation between wealth and the ratio of money to total wealth.

Table 11: Empirical Correlations,

	US Data	Model
Money & Income	.12	.40
Money & Consumption	-	.45
Money & Wealth	.17	.39
(Money/ Wealth) & Wealth	-0.08	-0.07

7 Robustness Checks

The model thus yields a realistic joint distribution of money and financial assets. Three robustness checks are considered in this section. First, the model is parameterized with different values for transaction costs, to produce changes in the distribution of money and wealth. Second, the model is parameterized at a monthly frequency. Third, we check whether the results are robust to an alternative labor-income process.

7.1 The Effect of Transaction Costs

To assess the effect of transaction costs on the inequality in the money distribution, Table 12 presents the change in relative money holdings as the cost λ varies. To simplify the exposition, this cost is presented as a percentage of the benchmark value.

The first line shows the change in financial wealth, and the second represents the change in money holdings. All figures are expressed as a percentage of the value of the financial wealth held in the benchmark case. For example, in the benchmark case ($\lambda = 100\%$), households hold money equal to 2% of their amount held in financial wealth. The third and fourth lines present the Gini coefficients for money and financial wealth respectively.

Table 12: Share of Money Holdings

λ	0	50	100	200	∞
Fin. Wealth	278	117	100	80	0
Money	0	1.26	2.02	2.67	12.5
Gini Money	–	0.81	0.82	0.84	0.94
Gini Fin. W.	0.63	0.79	0.77	0.79	–

When $\lambda = 0$, there is no transaction cost in entering financial markets, and as a result money is a dominated asset which is not held in equilibrium. This is the benchmark from the models of Hugget (1993) and Aiyagari (1994). When $\lambda = \infty$, the transaction cost of entering the financial market is too high and households only hold money. This corresponds to the Bewley (1983), Scheinkman and Weiss

(1986) and Imrohoroglu (1992) models. For values of λ between these two extremes, households hold both money and financial assets in equilibrium. The quantity of money (financial assets) held decreases (increases) with the transaction cost. In all cases, the Gini coefficient for money remains high, and actually rises with the transaction cost. The greater the quantity of money in the economy, the more unequally it is distributed.

7.2 A Monthly Specification

As a second robustness check, the model is re-calibrated at a monthly frequency. The discount factor, the financial market transaction cost, and the preference for leisure have been re-calibrated to match roughly the same capital-output ratio, the same money-output ratio and the same labor supply. The modified parameters are listed below.

ψ	β	λ
210	0.9968	0.75

The labor process has been modified accordingly. The new transition matrix T^* is such that $T = (T^*)^3$.

The results are very similar to those from the quarterly specification. For instance the Gini coefficient for money is 0.82 and that for wealth is 0.82. The transaction cost λ is 2.3% of annual average income, whereas it was 3% in the quarterly calibration. The results of the model are therefore roughly invariant to the definition of the period. I use the quarterly specification to increase the converge speed of the algorithm.

7.3 An Alternative Income Process

I now check that its ability to reproduce the distribution of money is not due to the particular values of the parameters retained in the calibration exercise. I here follow the calibration described in Diaz, Pijoan-Mas and Rios-Rull (2003) which is a simplified version of Castaneda, Diaz-Gimenez and Rios-Rull (2003). I use this calibration at a quarterly frequency. Here, labor supply is exogenous and households face the earning process

$$\{e_1, e_2, e_3\} = \left\{ \begin{array}{ccc} 1.000, & 5.290, & 46.550 \end{array} \right\}$$

$$T = \begin{bmatrix} 0.998 & 0.002 & 0 \\ 0.0022 & 0.9949 & 0.0029 \\ 0 & 0.0215 & 0.9785 \end{bmatrix}$$

$$N^* = \begin{bmatrix} 0.4922 & 0.4475 & 0.064 \end{bmatrix}$$

I calibrate the discount factor β and the transaction cost λ to obtain the same capital-output ratio and the same money-output ratio as in the benchmark economy. This produces

σ	β	λ
2	0.981	0.2

The Gini coefficients are as follows.

Gini coefficient of		
Income	Money	Wealth
0.61	0.65	0.7

As before, we have the correct ranking of the Gini coefficients, and a high Gini coefficient for money. In this specification, however, consumption appears to be too dispersed, with a Gini coefficient of 0.52, which is greater than the value found in US data (Krueger and Perri, 2005). Nevertheless, the fact that the money distribution is more dispersed than are consumption and income is a robust outcome of the formalization of money demand in this paper.

8 Conclusion

This paper uses a remarkable fact about household money holdings to discuss the relevance of various theories of money demand: the distribution of money across households is similar to the distribution of financial assets, and very different from the distribution of consumption. The theories which provide a foundation of money demand with frictions in the goods market cannot reproduce this fact with realistic assumptions. This paper provides a model where a simple transaction cost in financial markets generates a realistic joint distribution of money and financial assets. Due to this cost, the average expected return to money can be higher than that on financial assets without any other frictions. Of course, this paper does not claim that the existence of a cash-in-advance constraint for a range of goods is unimportant for some aspects of money demand, for example for currency holdings. But it does seem that a financial approach to money demand is more relevant when we consider broader money aggregates, such as $M1$. The analysis of the real and welfare effects of various types of shocks in this framework is a useful avenue for future research.

A Appendix

A.1 Equilibrium Values

The following table provides the equilibrium values of the model.

K	M	Y	C	L	r	w
15.7992	0.4231	1.2944	0.6694	0.3169	0.0087	1.9108

A.2 Computational Strategy

The computational strategy for the stationary equilibrium of the type of model used in this paper is now well-defined. The stationary value functions are solved by iteration over the value function. For a given real interest rate r and a fixed inflation rate, the other six value functions $\{V_j^{par}(.,.,e^1), V_j^{par}(.,.,e^2), V_j^{par}(.,.,e^3), V_j^{ex}(.,.,e^1),$ $V_t^{ex}(.,.,e^2), V_t^{ex}(.,.,e^3)\}$ are iterated using standard techniques. The value function iteration avoids any assumptions regarding the differentiability of the value function, which is not guaranteed here. After convergence, I determine the stationary distribution to compute aggregate financial savings and the total real quantity of money. I then adjust the real interest rate until the market for the riskless financial assets clears. Note that no equilibrium condition is necessary for the real quantity of money, as the ratio of the nominal to the real quantity of money endogenously determines the equilibrium money price of the final good.

To speed up convergence, I compute the level of the value function on a first grid where maxima are found by a hill-climbing algorithm. To obtain precise value functions, I extrapolate this first grid via a second grid. One important point is that this must be a *square* grid (and not rectangular), so as not to favour an asset in term of portfolio adjustment. The first grid has 30 values for money and 700 values for financial assets. The second grid decomposes each increment into 50 separate points for both money and financial assets. Value maximization is thus carried out for 35000 different values of the financial asset and 1500 values for money. Finally, the code is done in FORTRAN.

References

Aiyagari, S. Rao and Ellen R. McGrattan. 1998. "The optimum quantity of debt", *Journal of Monetary Economics*, 42(3): . 447-469.

Akyol, Ahmet. 2004. "Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk", *Journal of Monetary Economics*, 51(6): 1245-1269.

Allais, Maurice. 1947. *Economie et Intérêt*, Paris: Imprimerie Nationale.

Alvarez, Fernando, Andrew Atkeson and Patrick J. Kehoe. 2002. "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets.", *Journal of Political Economy*, 110(1): 73-112.

Alvarez, Fernando and Francesco Lippi. 2007. "Financial Innovation and the Transactions Demand for Cash", NBER Working Paper Series no. 13416; Cambridge: National Bureau of Economic Research.

Attanazio, Orazio, Luigi Guiso and Tulio Jappelli. 2002. "The Demand for

Money, Financial Innovation and the Welfare Cost of Inflation: An Analysis with Household Data", *Journal of Political Economy*, 110(2): 317-351.

Attanasio, Orazio, Erich Battistin and Hidehiko Ichimura. 2004. "What really happened to consumption inequality in the US?", NBER Working Paper 10228.

Bai, Jinhui. 2005. "Stationary Monetary Equilibrium in a Baumol-Tobin Exchange Economy: Theory and Computation", Georgetown University Working Paper.

Baumol, William. 1952. "The Transaction Demand for Cash: An Inventory Theoretic Approach", *The Quarterly Journal Of Economics*, 66(4): 545-556.

Bewley, Truman. 1980. "The optimum quantity of money", In: Kareken, J.H, Wallace, N., (Eds), *Models of Monetary Economies.*

Bewley, Truman. 1983. "A difficulty with the optimum quantity of money", *Econometrica*, 51(5): 1485-1504.

Castaneda, Ana, Javier Diaz-Gimenez and Jose-Victor Rios-Rull. 2003. "Accounting for earnings and wealth inequality", *Journal of Political Economy*, 111(4): 818-857.

Chatterjee, Satyajit and Dean Corbae. 1992. "Endogenous market participation and the general equilibrium value of money", *Journal of Political Economy* 100(3): 615-646.

Chari, V. V., Lawrence J. Christiano, and Patrick J.Kehoe. 1996. "Optimality of the Friedman rule in economies with distorting taxes," *Journal of Monetary Economics*, vol. 37(2-3): 203-223.

Chiu, Jonathan and Miguel Molico. 2007. "Liquidity, Redistribution, and the Welfare Cost of Inflation", Working paper, Bank of Canada.

Cooley, Thomas F. and Gary D. Hansen. 1989. "The Inflation Tax in a Real Business Cycle Model", *American Economic Review*, 79(4): 733-48.

Cooley, Thomas F. and Edward C. Prescott. 1985. "Economic Growth and Business Cycles", in T. Cooley ed., Frontiers of Business Cycle Research, Princeton University Press.

Diaz, Antonia, Josep Pijoan-Mas, and Rios-Rull, Jose-Victor. 2003. "Precautionary savings and wealth distribution under habit formation preferences," *Journal of Monetary Economics*, vol. 50(6): 1257-1291

De Nardi, Mariacristina. 2004. "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, vol. 71(7), pages 743-768.

Domeij, David and Jonathan Heathcote. 2004. "On the distributional effects of reducing capital taxes", *International Economic Review*, 45(2): 523–554.

Dotsey, Michael and Peter Ireland. 1996. "The Welfare Costs of Inflation in General Equilibrium", *Journal of Monetary Economics* 37(1): 29-47.

Erosa, Andres and Gustavo Ventura. 2002. "On Inflation as a Regressive Consumption Tax", *Journal of Monetary Economics*, 49(4): 761-795.

Floden, Martin and Jesper Lindé. 2001. "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?", *Review of Economic Dynamics*, 4(2): 406–437.

Gomes Francisco and Michaelides, Alexander. 2007. "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents", *Review of Financial Studies*, forthcoming.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, vol. 78(3): 402-17.

Grossman, Sanford and Guy Laroque. 1990. "Asset Pricing and Optimal Porfolio Choice in the Presence of Illiquid Durable Consumption Goods", *Econometrica*, vol. 58(1): 25-51.

Heathcote, Jonathan. 2005. "Fiscal Policy with Heterogenous Agents and Incomplete Markets", *Review of Economic Studies*, 72(1): 161-188.

Heer, Burkhard, Alfred Maussner, and Paul McNelis. 2007. "The money-age distribution: Empirical facts and limited monetary models", CESifo Working paper 1917.

Heller, Walter. 1974. "The Holding of Money Balances in General Equilibrium", *Journal of Economic Theory,* 7(1): 93-108.

Huang, Ming. 2003. "Liquidity shocks and equilibrium liquidity premia". *Journal of Economic Theory*, 109:104–129.

Huggett, Mark. 1993. "The risk-free rate in heterogenous-agent incomplete-insurance economies", *Journal of Economic Dynamics and Control*, 17(5-6): 953-69.

Hoffman, Dennis .L., Robert.H. Rasche, and Margie A. Tieslau. 1995. "The stability of Long-Run Money Demand in Five Industrial Countries", *Journal of Monetary Economics*, 35(2): 317-339.

Imrohoroglu Ayse and Edward Prescott. 1991. "Seigniorage as a Tax: A Quantitative Evalutation", *Journal of Money Credit and Banking*, 23(3): 462-475.

Imrohoroglu, Ayse. 1992. "The welfare cost of inflation under imperfect insurance", *Journal of Economic Dynamics and Control*, 16(1): 79-91.

Jovanovic, Boyan. 1982. "Inflation and Welfare in the Steady State", *Journal of Political Economy*, 90: 561-77.

Kehoe, Timothy.J., David.K. Levine, and Michael Woodford. (1992), "The optimum quantity of money revisited", in *Economic Analysis of Markets and Games*, ed. by Dasgupta, D. Gale, O. Hart, and E. Maskin, Cambridge, MA: MIT press, 501-526.

Kiyotaki, Nobuhiro, and Randall Wright. 1989. "On Money as a Medium of Exchange", *Journal of Political Economy*, 97(4): 927-954

Krueger, Dirk, and FabrizioPerri. 2005. "Does income inequality lead to Consumption inequality? Evidence and Theory", Forthcoming *Review of Economic Studies.*

Lo, Andrew.W., Harry Mamaysky, and Jiang Wang. 2004. "Asset Prices and Trading Volume under Fixed Transactions Costs", *Journal of Political Economy,* 112(5): 1054-1090.

McCallum, Bennet. T. and Marvin.S. Goodfriend. 1987. "The Demand for Money: Theoretical Studies", P Newman, M. Milgate, and J. Eatwell (Eds), The New Palgrave Dictionary of Economics, Houndmills, UK, 775-781.

Mulligan, Casey, and Xavier Sala-i-Martin. 2000., "Extensive Margins and the

Demand for Money at Low Interest Rates", *Journal of Political Economy*, 108(5): 961-991

Molico, Miguel. 2006. "The Distribution of Money and Prices in Search Equilibrium," *International Economic Review,* 47(3): 701-722.

Paiella, Monica. 2001. "Limited Financial Market Participation: A transaction cost-based Explanation", IFS working paper 01/06.

Romer, David. 1986. "A Simple General Equilibrium Version of the Baumol-Tobin Model", *The Quarterly Journal of Economics*, Vol. 101(4): 663-686.

Scheinkman, Jose A. and Laurence Weiss. 1986. "Borrowing constraints and aggregate economic activity", *Econometrica*, 54(1): 23-45.

Stokey, Nancy L., and Robert Lucas. 1987. "Money and Interest in a Cash-in-advance Economy", *Econometrica*, 55(3): 491-514.

Stokey, Nancy L. and Sergio Rebelo. 1995. "Growth Effects of Flat-Rate Taxes", *Journal of Political Economy*, 103(3): 519-550.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron. 2007. "Asset pricing with idiosyncratic risk and overlapping generations" , *Review of Economic Dynamics*, vol. 10 (4): 519-548.

Tobin, James. 1956. "The Interest Elasticity of Transactions Demand for Cash", *The Review of Economics and Statistics*, 38(3): 241-247.

Vissing-Jorgensen, Annette. 2002. "Towards an explanation of the households portfolio choice heterogeneity: Non financial income and participation cost structure", NBER Working Paper 8884.