

The Macroeconomics of Real Estate*

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Abstract

Is it possible to explain the house price to GDP ratio and the house price to stock price ratio as being generally constant, deviating from its respective mean only because of shocks to productivity? We build a two-sector RBC model for residential and non-residential capital with adjustment costs to capital in both sectors. We show that an anticipated future shock to productivity growth in the non-residential sector leads to a large increase in house prices in the present. We use this property of the model to explain the current house price behavior in the U. S., the U. K., Japan and Germany.

JEL-classification: R21, R31, R33

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1 Introduction

Many practitioners and researchers are puzzled by the recent developments in the real estate market prior to the downturn in 2006. While countries such as the U.S., Spain or the U.K. experienced high growth rates in real estate prices, real estate prices did not grow at all or even fell in countries like Japan, Germany and Switzerland. This may explain the growing interest of researchers in understanding the real estate market in recent years. In the second edition of his bestseller book on irrational exuberance, Shiller (2005; *Irrational Exuberance* Book by Princeton UP) especially focuses on the development of house prices. In one of his graphs, he shows that the development in house prices over the last 120 years can be explained neither by building costs nor by population nor by interest rates. We replicate this graph in Figure 1. So he assumes a sizeable degree of irrationalism in the market for real estate. But is this really the only valid conclusion? In Figure 1 we look at the period from 1980 onwards only and compare his data that he kindly provides on his webpage¹ to ours.² We find it to be very similar. And we compare it to real GDP, to find that the evolution of house prices does not look that odd once we compare it to the evolution of real GDP. So the task seems be much simpler: instead of attempting to explain the dramatic increase in house prices, the question is only why house prices increased relative to GDP over the last years before the downturn. To put this into context, we also look at the evolution of the house price to stock price ratio in order to see whether there are similarities.

After resorting to the data, we then lay out our theory to simultaneously explain movements in GDP, house prices and stock prices. Our assumption is that all three variables feature the same balanced growth path. But temporary shocks may lead to deviations in the house price to GDP and the house price to stock price ratio. The main point of this paper is then to show how expectations about the prospects of the economy can explain an immediate rise in house prices relative to stocks and GDP. We make this point by building a two-sector RBC model for non-residential capital and housing capital. Both sectors are subject to concave adjustment costs, but only the real estate sector features a constant factor of production. Because of the latter assumption, the relative price of real estate has a positive trend. Related models are Piazzesi et al. (2007), Benhabib et al. (1991) and Greenwood and Hercowitz (1991). While all papers have a consumption side that is comparable to our model, the first and the second paper lack an explicit production function, whereas the second and the

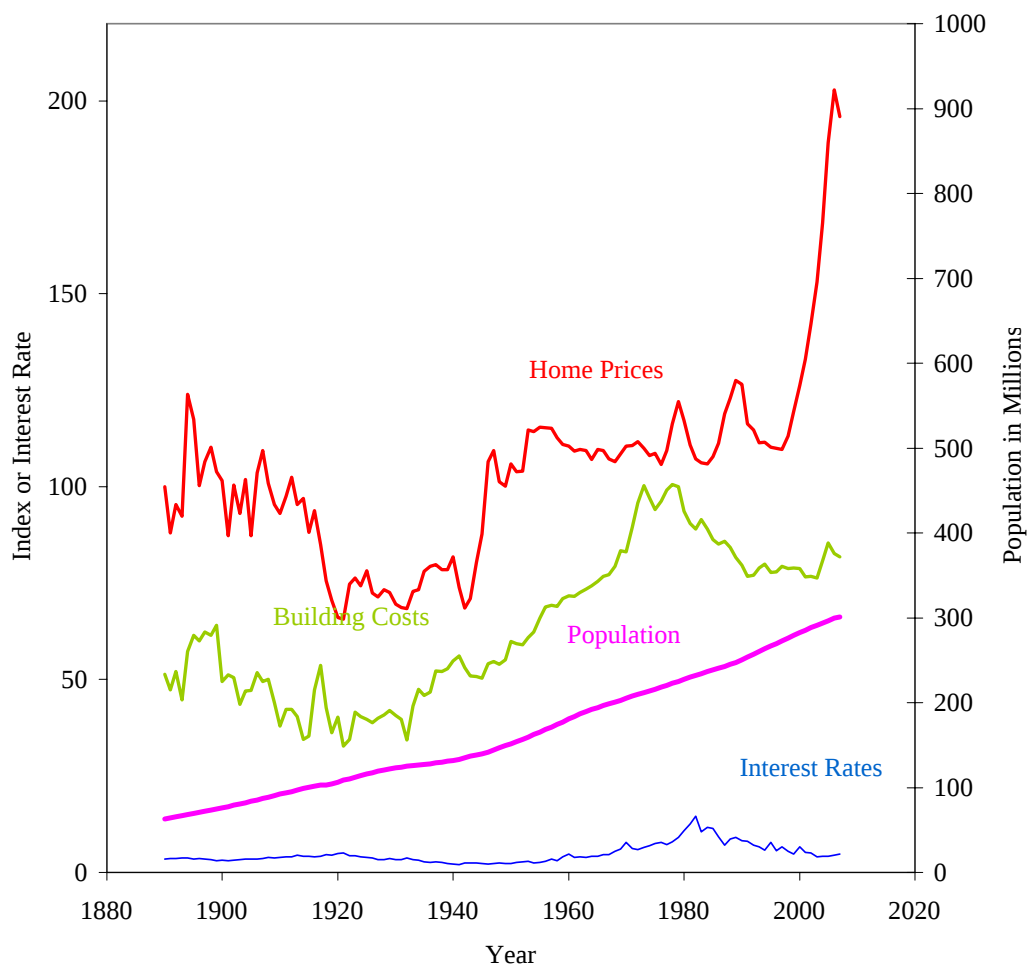
¹See <http://www.econ.yale.edu/~shiller/books.htm>.

²We have chosen to log the data and to index it so that the mean between 1908 and 1995 equals zero.

third paper answer a different question: They analyze the implications of non-market activities by households. Nonetheless, the modeling strategy is similar in spirit.

Of course, house price movements have already been addressed in many papers. We will only briefly mention some of them here. Case et al. (2000), Campbell and Cocco (2005) and Iacoviello (2005) all explore the connections between real estate and consumption and their possible implications for economic (fiscal or monetary) policy. The models presented in Gammoudi and Mendes (2005), Iacoviello (2005), Ortalo-Magné and Rady (2002) as well as

Figure 1: House Prices development in the long run: Figure 2.1 from Shiller (2005)



Yang (2005) all feature household heterogeneity with respect to borrowing constraints, homeownership or age. Lustig and Van Nieuwerburgh (2005) investigate the effect of changing house prices on the housing wealth to human wealth ratio in a model of housing collateral.

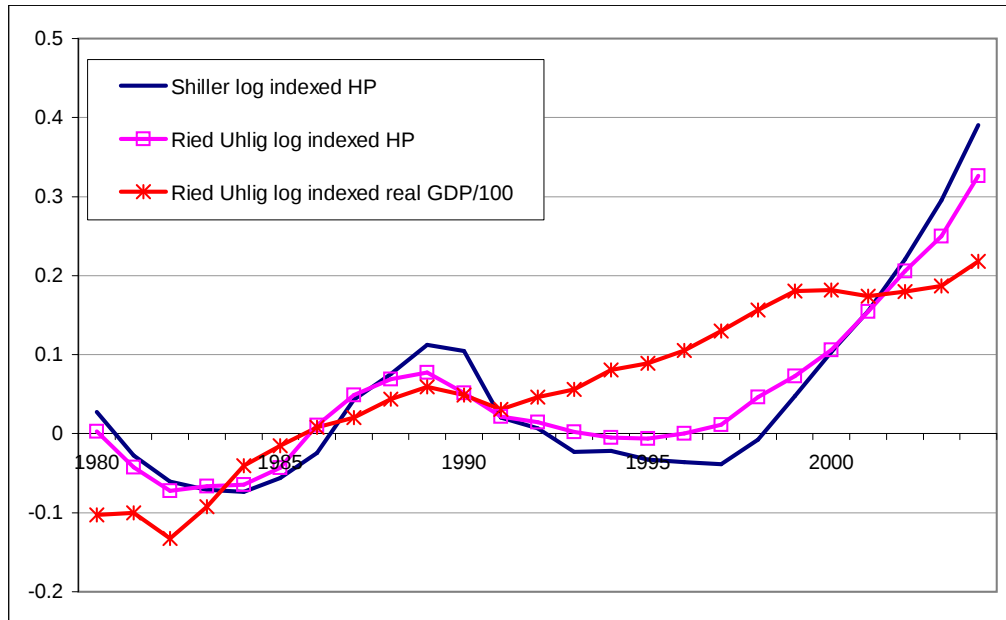
The effect of inflation on house prices – either as an effective tax subsidy to owner occupied housing or through money illusion – is analyzed in a series of papers by Poterba (1984, 1991, 1992) and, more recently, Brunnermeier and Julliard (2008).

The paper is structured as follows. Section 2 summarizes the empirical findings. Section 3 presents the main idea in a simplified endowment economy. An elaborated production economy, its solution and its results then follow in section 4. Section 5 concludes.

2 Some Facts

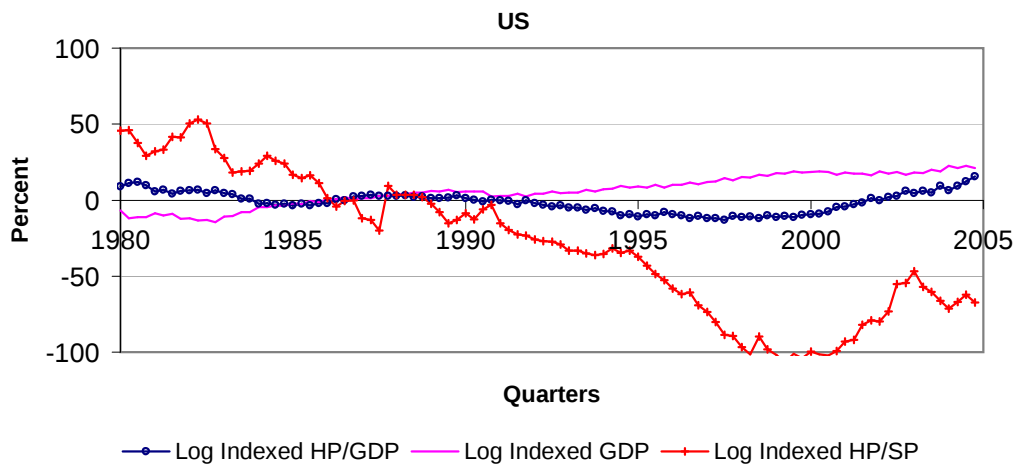
Figures 3 to 5 present the evolution of (log) real GDP, the ratio of (log) real house prices (HP) to real GDP and the ratio of (log) real house prices to (log) real stock

Figure 2: Comparison of Shiller data with our data, log indexed house prices from 1980 onwards: not too many differences, and less dramatic



prices (SP) for the U. S., the U. K., Japan and Germany.³ We see that the house price to GDP ratio seems to be relatively stationary. It is most volatile in the U. K. and in the U. S., but much less volatile than the house price to stock price ratio. Relative to the own country's stock market, in the last ten years houses have become more expensive in the U. K. and Japan, but less expensive in the U. S. and Germany.

Figure 3: GDP, House Prices to GDP Ratio and Stock Prices to GDP Ratio for the U. S.



³All series have been indexed such that the average of 1980:I - 1995:IV equals zero. For more details, the reader is referred to the appendix.

Figure 4: GDP, House Prices to GDP Ratio and Stock Prices to GDP Ratio for the U. K., Japan and Germany

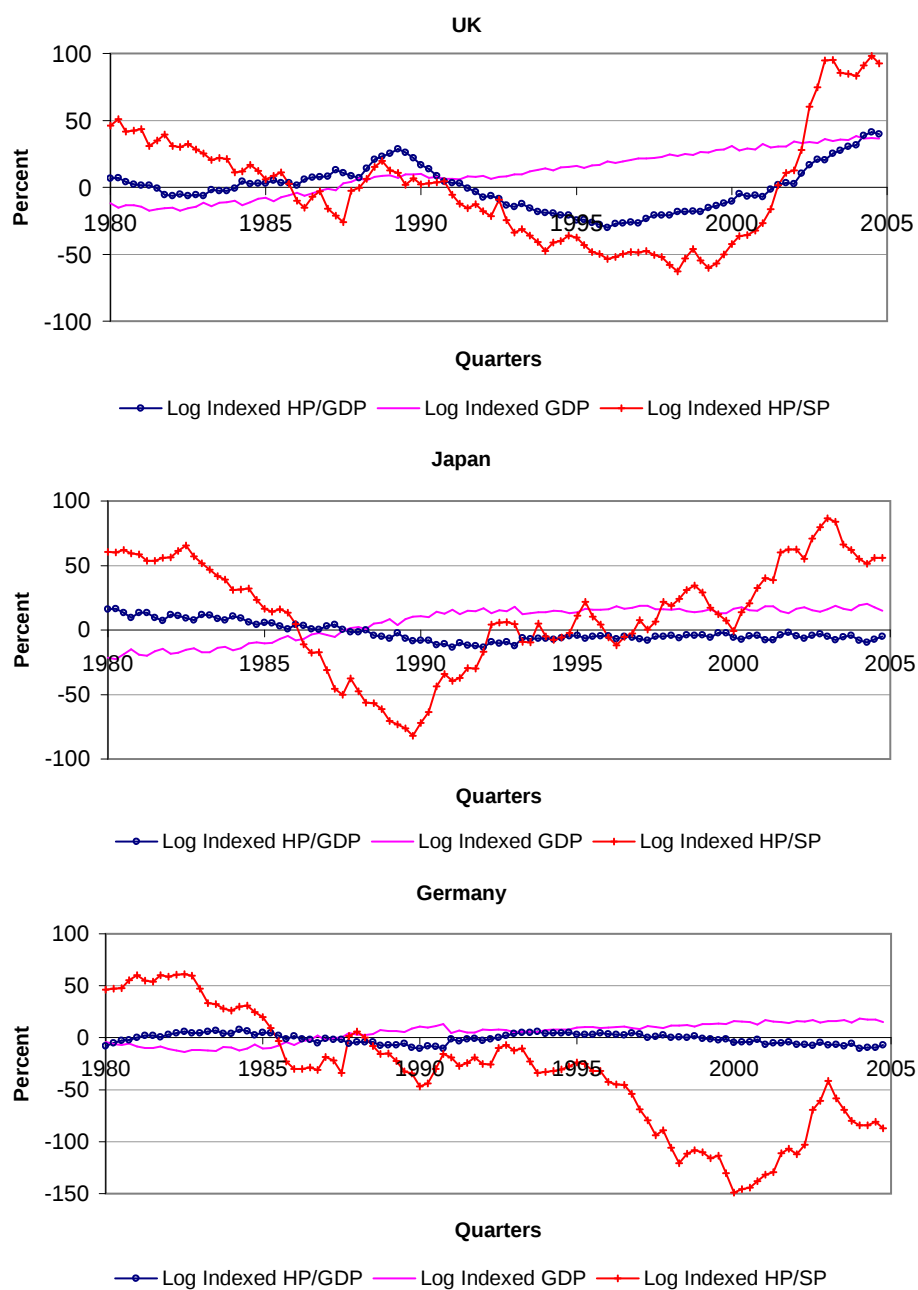
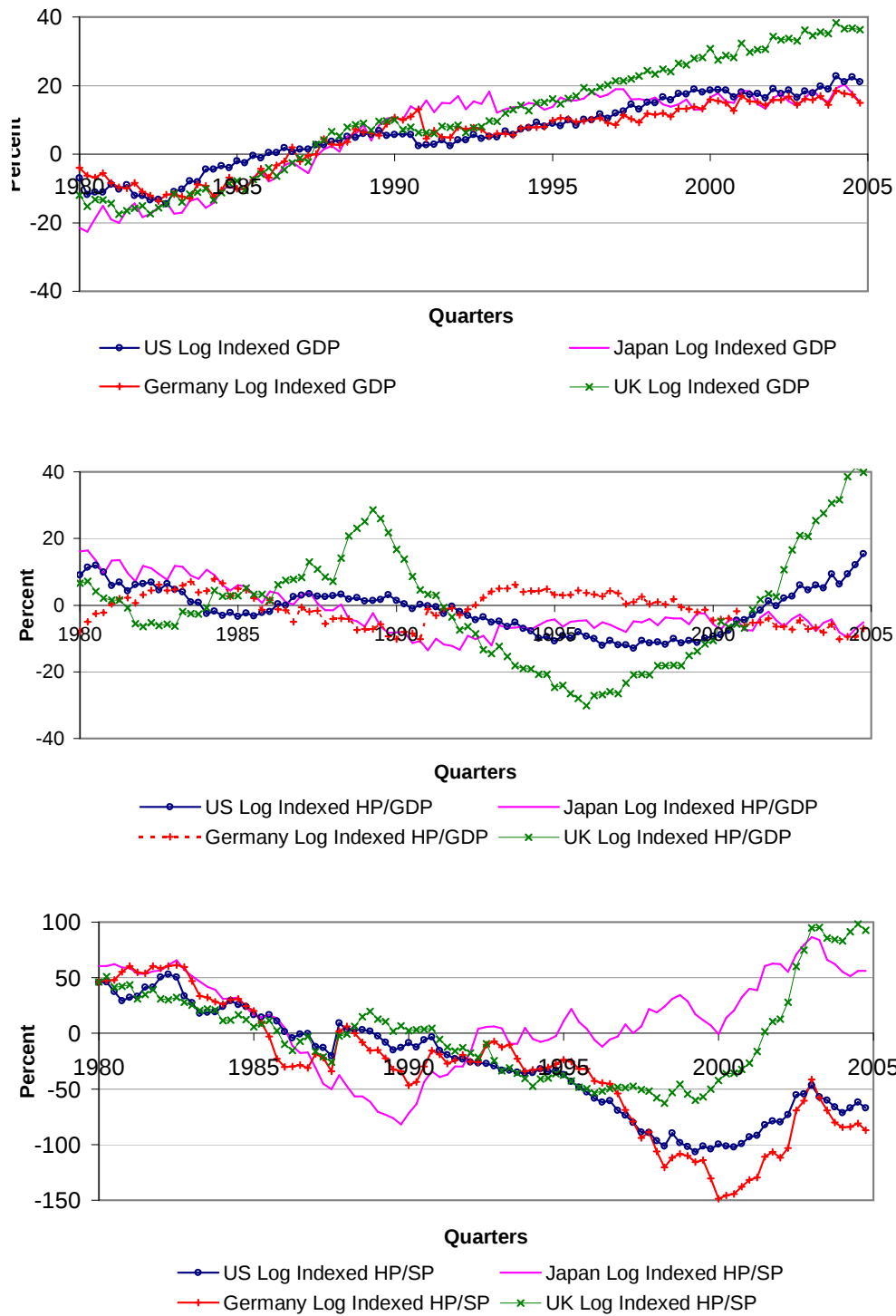


Figure 5: Comparison between the US, the UK, Japan and Germany:
GDP, House Prices to GDP and House Prices to Stock Prices Ratio



3 An Endowment Economy

Let us start with a stripped down endowment economy to preview the gist of the main model laid out in the following section. A representative agent has preferences of the form

$$\max_{c_t, s_t, a_t, b_t} E_t \sum_{t=0}^{\infty} \beta^t (\theta \log c_t + (1 - \theta) \log s_t) \quad (1)$$

for consumption good c_t and housing services s_t . There is a constant stock of real estate, the usage of which is determined by $s_t \in [0, 1]$. Consumption is determined by an exogenous source z_t : $c_t = z_t$. Households trade in the ownership a_t of the source of the consumption good and ownership b_t of real estate, resulting in one unit of housing services. The according prices are p_t and q_t , respectively. Given the relative price of housing services r_t , the budget constraint of the household is:

$$c_t + r_t s_t + p_t a_t + q_t b_t = (p_t + z_t) a_{t-1} + (q_t + r_t) b_{t-1} . \quad (2)$$

Market clearing requires $a_t = b_t = s_t = 1$ and $c_t = z_t$. For the production function this means that $c_t = 1$ as well as $s_t = 1$. Defining output as the sum of consumption and housing services,

$$y_t \equiv c_t + r_t s_t , \quad (3)$$

in equilibrium the following equation holds:

$$c_t + r_t = z_t + r_t = y_t . \quad (4)$$

Optimal allocation results in the following expenditure shares:

$$c_t = \theta y_t \quad \text{and} \quad r_t = (1 - \theta) y_t . \quad (5)$$

For the rental rate r_t , this can be transformed to show the relation to productivity:

$$r_t = \frac{1 - \theta}{\theta} z_t . \quad (6)$$

As further first-order necessary conditions one obtains two asset pricing equations for stocks and real estate:

$$p_t = \beta E_t \left[\frac{c_t}{c_{t+1}} (z_{t+1} + p_{t+1}) \right] \quad (7)$$

$$q_t = \beta E_t \left[\frac{c_t}{c_{t+1}} (r_{t+1} + q_{t+1}) \right] . \quad (8)$$

Solving equation (7) backward results in

$$\frac{p_t}{z_t} = \frac{\beta}{1 - \beta} \quad (9)$$

which can be transformed by using $z_t = \theta y_t$ to give a stock price to GDP ratio

$$\frac{p_t}{y_t} = \theta \frac{\beta}{1 - \beta} . \quad (10)$$

Equivalently, the house price to GDP ratio is given by

$$\frac{q_t}{y_t} = (1 - \theta) \frac{\beta}{1 - \beta} \quad (11)$$

and the house price to stock price ratio

$$\frac{q_t}{p_t} = \frac{1 - \theta}{\theta} . \quad (12)$$

In this setup, movements in GDP should be directly reflected in movements in stock and real estate prices, where the size of the movement in prices depends on the size of the utility parameter θ . If this parameter were modelled as a random variable, changes in current GDP have time-variant (positive) effects on stock and real estate prices.

3.1 Balanced Growth Path

If real estate is constant, but the exogenous income source is assumed to grow according to

$$z_t = \gamma^t, \gamma > 1, \quad (13)$$

the price of housing services r_t is growing with the exogenous income source z_t . According to equation (6), it is

$$r_t = \frac{1 - \theta}{\theta} \frac{c_t}{s_t} = \frac{1 - \theta}{\theta} \gamma^t . \quad (14)$$

If z_t is interpreted as productivity, then house prices grow proportionately with productivity.

Hence this stylized endowment economy features constant house price to GDP and house price to stock price ratios. Given a constant stock of real estate, trend growth in productivity and thus GDP translate into trend growth of the same size in house prices. We use this building blocks for the more elaborated production economy of the next section. There, prices and productivity will be specified with more detail.

4 A Production Economy

In this model, there are two sectors: consumption goods are produced using consumption good capital and labor. Real estate is produced using real estate capital and constant land. Both types of capital are subject to adjustment costs. The model bears similarity to Uhlig (2004), with the extension for two sectors.

The representative agent maximizes utility over consumption c_t , housing services s_t and leisure $1 - n_t$ as follows:

$$\max_{c_t, s_t, n_t, x_{c,t}, x_{s,t}, k_{c,t}, k_{s,t}} E_t \sum_{t=0}^{\infty} \beta^t (u(c_t, z_{s,t}, s_t) - A n_t) , \quad (15)$$

where $z_{s,t}$ denotes a preference shock and n_t denotes labor, subject to the budget constraint, two production functions and two equations for capital accumulation. The utility function $u(\cdot)$ is assumed to be additively separable⁴ and of the form

$$u(c_t, z_{s,t}, s_t) = \theta \log c_t + (1 - \theta) z_{s,t} \log s_t. \quad (16)$$

The agent is endowed with one unit of time per period and L units of land. The budget constraint is given by

$$c_t + x_{c,t} + x_{s,t} = y_{c,t} , \quad (17)$$

where $x_{c,t}$ is investment in the consumption good production $y_{c,t}$ and $x_{s,t}$ is investment in housing. These constructs are normalized so that every real quantity is measured in units of consumption.

The production technology in the consumption good sector is given by the Cobb-Douglas function in consumption good capital and productive labor $z_{c,t} n_t$,

$$y_{c,t} = k_{c,t-1}^{\alpha_c} (z_{c,t} n_t)^{1-\alpha_c} , \quad (18)$$

Investment in housing increases real estate production and also housing services. For simplicity we assume real estate production to be equal to the service flow it generates, hence, the words *housing services* and *real estate production* are used interchangeably. Real estate production has the Cobb-Douglas functional form in the arguments real estate capital and constant stock of land,⁵

$$s_t = y_{s,t} = k_{s,t-1}^{\alpha_s} L^{1-\alpha_s} . \quad (19)$$

⁴In a similar model with two different growth patterns, Greenwood et al. (1997) report that in the class of constant elasticity of substitution models, balanced growth is only permitted in the simple case like the one considered here.

⁵The fixed factor in real estate production may also be excluded by setting $\alpha_s = 1$.

In this setting, the agent builds real estate by investing in $x_{s,t}$, both measured in units of consumption, out of the income she receives from producing output in the consumption good industry $y_{c,t}$. Once real estate is built, it provides housing services s_t that do not have to be paid for additionally.⁶ Note that there is no such thing as “income” from owning a house, as it is owner-occupied. Instead, the house provides utility through housing services. One could imagine an alternative model in which the representative household rents a house or flat from real estate firms. These real estate firms work in a competitive market with zero profits and pay the marginal product to each production factor. As both production factors are owned by the representative household, this setting renders equivalent results. Capital accumulation in both sectors is subject to a concave adjustment cost function whose details will be specified later:

$$k_{i,t} = (1 - \delta_i)k_{i,t-1} + g_i \left(\frac{x_{i,t}}{k_{i,t-1}} \right) k_{i,t-1}, \quad i \in \{c, s\}. \quad (20)$$

The exogenous process for housing preferences $z_{s,t}$ is assumed to be AR(1), while productivity $z_{c,t}$ is a unit root process with stochastic trend growth μ_t and an additional shock:

$$\log z_{s,t} = \rho_s \log z_{s,t-1} + \epsilon_{s,t} \quad (21)$$

$$\log z_{c,t} = \log \mu_t + \log z_{c,t-1} + \epsilon_{c,t}, \text{ where} \quad (22)$$

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + (1 - \rho_\mu) \log \bar{\mu} + \nu_t. \quad (23)$$

While the shock $\epsilon_{c,t}$ influences the level of productivity, ν_t influences the level of trend productivity and hence the growth rate of productivity. Below, we will focus on the effects of both shocks. We will put special attention to the question what happens after an anticipated future shock to the growth rate of productivity, ν_{t+i} . With this, the model is closed.

It will prove useful to define the four marginal products (wage rate and divi-

⁶“Selling” real estate can be done by disinvesting in $x_{s,t}$.

dends) and the two returns to investments in capital:

$$d_{c,t} = \alpha_c \frac{y_{c,t}}{k_{c,t-1}}, \quad (24)$$

$$w_t = (1 - \alpha_c) \frac{y_{c,t}}{n_t} \quad (25)$$

$$d_{s,t} = \alpha_s p_t \frac{y_{s,t}}{k_{s,t-1}}, \quad (26)$$

$$d_{L,t} = (1 - \alpha_s) p_t \frac{y_{s,t}}{L}, \quad (27)$$

$$R_{i,t+1} = g'_i \left(\frac{x_{i,t}}{k_{i,t-1}} \right) \left(d_{i,t+1} + \frac{1 - \delta_i + g_i \left(\frac{x_{i,t+1}}{k_{i,t}} \right)}{g'_i \left(\frac{x_{i,t+1}}{k_{i,t}} \right)} - \frac{x_{i,t+1}}{k_{i,t}} \right), i \in \{c, s\}. \quad (28)$$

All of these variables are measured in units of the consumption good. The marginal products of real estate production are translated into units of the consumption good by using the relative price

$$p_t = \frac{(1 - \theta) z_{s,t}}{\theta} \frac{c_t}{s_t} = \frac{\lambda_{ys,t}}{\lambda_{bc,t}}, \quad (29)$$

which is the ratio of marginal utility of housing services to marginal utility of consumption, which itself is equal to the ratio of the Lagrange multipliers for budget constraint and real estate production.

For the purpose of a comparison between the model and the stylized facts presented in section 2, we have to define output, house prices and stock prices. Output is defined as the sum of consumption good production and housing services (or real estate production),⁷ both measured in units of the consumption good:

$$y_t = y_{c,t} + p_t y_{s,t}. \quad (30)$$

Following Hayashi (1982) and Hall (2001), the value of consumption good capital is the product of the shadow value of installed consumption good capital and the quantity of consumption good capital,

$$V_{c,t} = q_{c,t} k_{c,t}, \text{ where } q_{c,t} = \frac{\lambda_{bc,t} + \lambda_{kc,t}}{\lambda_{bc,t}}. \quad (31)$$

The shadow value of installed capital, also known as Tobin's q , is the value of a unit of capital in units of the consumption good. Hence, like the relative price

⁷To digest this, one may again think of a model with real estate firms that produce and sell real estate, as mentioned above. Then, real estate production is clearly part of overall output.

of real estate, it can be written in terms of the Lagrange multipliers for the capital accumulation equation $\lambda_{bc,t} + \lambda_{kc,t}$ and the budget constraint. This value of consumption good capital is the total market value of the non-real estate capital stock. It is equal to all stocks and all corporate bonds outside the real estate sector. This variable will serve as the theoretical counterpart to the stock price.

Our theoretical counterpart to real estate prices we call value of housing stock. It is the sum of the value of real estate capital (calculated as before) and the value of land,

$$V_{s,t} = q_{s,t}k_{s,t} + V_{L,t}, \text{ where } q_{s,t} = \frac{\lambda_{bc,t} + \lambda_{ks,t}}{\lambda_{bc,t}}. \quad (32)$$

Lastly, the value of land is the sum of all expected discounted future imputed land dividend payments,

$$V_{L,t} = E_0 \left[\sum_{j=0}^{\infty} R_{s,t+j}^{-j} d_{L,t+j} \right] = d_{L,t} + \frac{\beta}{\bar{\gamma}} E_t [V_{L,t+1}], \quad (33)$$

where the latter equality makes use of the fact that steady state returns $\bar{R}_s = \frac{\bar{\gamma}}{\bar{\beta}}$ are equal to the ratio of steady state productivity growth $\gamma_t = z_{c,t}/z_{c,t-1}$ to the discount factor.

4.1 Balanced Growth Path

To obtain a solution to the model it is convenient to reformulate it in terms of stationary variables. The appropriate transformation features the same growth rate for consumption, output of consumption goods, both types of investment and both types of capital. However, due to the fixed factor in real estate production, housing services grow more slowly.⁸ This bears an important implication. As real estate is produced using one finite input factor, its balanced growth is smaller than balanced labor productivity growth. Thus, the price for real estate production and accordingly the price for housing services will rise as a result of this scarcity. Finally, land and labor feature zero growth.

Given a conjectured growth rate for each variable, we transform the model into a stationary one. Dividing all variables except for housing services s_t and the already stationary variables labor n and land L by the respective labor productivity $z_{c,t-1}$ and using a tilde to denote the detrended variables, e.g. $\tilde{c}_t \equiv c_t/z_{c,t-1}$, but $\tilde{s}_t \equiv s_t/z_{c,t-1}^{\alpha_s}$, $\tilde{p}_t \equiv p_t/z_{c,t-1}^{1-\alpha_s}$, $\tilde{k}_{i,t} \equiv k_{i,t}/z_{c,t}$, $i \in \{c, s\}$ and

⁸For a comparable model with two differing growth patterns, see Greenwood et al. (1997).

$\tilde{n}_t \equiv n_t$, $\tilde{L} \equiv L$,⁹ we obtain the following representation:

$$\max_{\tilde{c}_t, \tilde{s}_t, \tilde{n}_t, \tilde{x}_{c,t}, \tilde{x}_{s,t}, \tilde{k}_{c,t}, \tilde{k}_{s,t}} E_t \sum_{t=0}^{\infty} \beta^t [\theta \log \tilde{c}_t + (1-\theta) z_{s,t} \log \tilde{s}_t - A \tilde{n}_t + Z_t] , \quad (34)$$

where $Z_t = (\theta + (1-\theta) z_{s,t}) \log z_{c,t-1}$ is a stochastic scaling factor.¹⁰ The maximization is subject to

$$\tilde{y}_{c,t} = \tilde{c}_t + \tilde{x}_{c,t} + \tilde{x}_{s,t} , \quad (35)$$

$$\tilde{y}_{c,t} = \tilde{k}_{c,t-1}^{\alpha_c} \tilde{n}_t^{1-\alpha_c} , \quad (36)$$

$$\tilde{y}_{s,t} = \tilde{s}_t = \tilde{k}_{s,t-1}^{\alpha_s} \tilde{L}^{1-\alpha_s} , \quad (37)$$

$$\tilde{\gamma}_t \tilde{k}_{i,t} = (1-\delta_i) \tilde{k}_{i,t-1} + g_i \left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}} \right) \tilde{k}_{i,t-1} , i \in \{c, s\}, \quad (38)$$

where $\tilde{\gamma}_t$ is labor productivity growth,¹¹

$$\tilde{\gamma}_t = \frac{z_{c,t}}{z_{c,t-1}}.$$

The adjustment cost functions are specified such that for the detrended variables, the first-order behavior of the capital accumulation is the same as in the no-adjustment-cost case:

$$g_i \left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}} \right) = \frac{\tilde{\delta}_i^{1/\xi_i}}{1-1/\xi_i} \left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}} \right)^{1-1/\xi_i} + \frac{\tilde{\delta}_i}{1-\xi_i} , i \in \{c, s\}.$$

The parameter $\tilde{\delta} = \delta + \bar{\gamma} - 1$ is chosen to match the investment to capital ratio in the steady state no-adjustment cost capital accumulation equation $\tilde{\gamma} \tilde{k}_i = (1-\delta) \tilde{k}_i + \tilde{x}_i$. Thus, the adjustment cost function satisfies

$$g(\tilde{\delta}) = \tilde{\delta}, \quad g'(\tilde{\delta}) = 1, \quad \xi = -\frac{g'(\tilde{\delta})}{\tilde{\delta} g''(\tilde{\delta})},$$

⁹For the detrending procedure, it has to be taken into account that $k_{i,t-1}, i \in \{c, s\}$ is the capital stock that is valid at the beginning of period t and is thus to be divided by $z_{c,t-1}$ as well, while $k_{i,t}, i \in \{c, s\}$ is valid at the beginning of period $t+1$ and is thus to be divided by $z_{c,t}$.

¹⁰Note that the discount factor β is not affected, as the utility specification is logarithmic. See King et al. (1988; JME: Prod, Growth and BC I. The Basic Neoclassical Model; p. 203).

¹¹Note that in order to have zero steady state adjustment costs, as measured by a non-binding Lagrange multiplier $\bar{\lambda}_{ki} = 0$, we have to divide all equations by the same level of productivity, i.e. $z_{c,t-1}$. Therefore, $\tilde{\gamma}_t$ has to show up on the left hand side in the capital accumulation equations.

and $\xi > 0$ is the elasticity of the investment-capital ratio with respect to Tobin's q .¹²

The exogenous process for technology given in equation (22) is transformed to

$$\log \tilde{\gamma}_t = \log \mu_t + \epsilon_{c,t}, \quad (39)$$

while the other exogenous processes remain as in equations (21) and (23).

The household's optimal choice of consumption, housing services, labor and the two kinds of each investment and capital is obtained by setting up the usual Lagrange function. For simplification, we have replaced $y_{c,t}$ by equation (36), so that we are left with four constraints. Denoting the Lagrange multipliers by $\lambda_{bc,t}$ for the budget constraint, $\lambda_{ys,t}$ for the constraint on production of real estate, and $(\lambda_{bc,t} + \lambda_{kc,t})$, $(\lambda_{bc,t} + \lambda_{ks,t})$ for the two types of capital accumulation.¹³ $\lambda_{kc,t}$ and $\lambda_{ks,t}$ measure the difference between the Lagrange multiplier on the budget constraint and the one on the respective capital accumulation equation. For linear adjustment cost functions, these multipliers are zero.

The first-order conditions for consumption, housing services, labor, the two

¹²See Jermann (1998; JME, Asset pricing in prod econ).

¹³In balanced growth, the Lagrange multiplier for the budget constraint $\lambda_{bc,t}$ and the ones for the two capital accumulation equations $(\lambda_{bc,t} + \lambda_{ki,t})$ are trending with $z_{c,t-1}^{-1}$, while the detrending of the Lagrange multiplier for real estate production is according to $\tilde{\lambda}_{ys,t} = \lambda_{ys,t} / z_{c,t-1}^{-\alpha}$.

kinds of investment and the two kinds of capital are:¹⁴

$$\frac{\theta}{\tilde{c}_t} = \tilde{\lambda}_{bc,t} \quad (40)$$

$$\frac{(1-\theta)z_{s,t}}{\tilde{s}_t} = \tilde{\lambda}_{ys,t} \quad (41)$$

$$A = \tilde{\lambda}_{bc,t}(1-\alpha_c)\frac{\tilde{y}_{c,t}}{\tilde{n}_t} \quad (42)$$

$$\tilde{\lambda}_{bc,t} = (\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t})g'_c\left(\frac{\tilde{x}_{c,t}}{\tilde{k}_{c,t-1}}\right) \quad (43)$$

$$\tilde{\lambda}_{bc,t} = (\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t})g'_s\left(\frac{\tilde{x}_{s,t}}{\tilde{k}_{s,t-1}}\right) \quad (44)$$

$$\begin{aligned} \tilde{\gamma}_t(\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t}) &= \beta E_t[\tilde{\lambda}_{bc,t+1}\alpha_c\frac{\tilde{y}_{c,t+1}}{\tilde{k}_{c,t}} + (\tilde{\lambda}_{bc,t+1} + \tilde{\lambda}_{kc,t+1}) \\ &\quad \left(1 - \delta_c - g'_c\left(\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}}\right)\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}} + g_c\left(\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}}\right)\right)] \end{aligned} \quad (45)$$

$$\begin{aligned} \tilde{\gamma}_t(\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t}) &= \beta E_t[\tilde{\lambda}_{ys,t+1}\alpha_s\frac{\tilde{y}_{s,t+1}}{\tilde{k}_{s,t}} + (\tilde{\lambda}_{bc,t+1} + \tilde{\lambda}_{ks,t+1}) \\ &\quad \left(1 - \delta_s - g'_s\left(\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}}\right)\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}} + g_s\left(\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}}\right)\right)] . \end{aligned} \quad (46)$$

¹⁴Note that the derivative of the adjustment cost function with respect to $\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}$ equals $g'_i\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right) = \tilde{\delta}_i^{1/\xi_i}\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right)^{\frac{-1}{\xi_i}}, i \in \{c, s\}$.

As additional equations we get:¹⁵

$$\tilde{d}_{c,t} = \alpha_c \frac{\tilde{y}_{c,t}}{\tilde{k}_{c,t-1}}, \quad (47)$$

$$\tilde{w}_t = (1 - \alpha_c) \frac{\tilde{y}_{c,t}}{\tilde{n}_t}, \quad (48)$$

$$\tilde{d}_{s,t} = \alpha_s \tilde{p}_t \frac{\tilde{y}_{s,t}}{\tilde{k}_{s,t-1}}, \quad (49)$$

$$\tilde{d}_{L,t} = (1 - \alpha_s) \tilde{p}_t \frac{\tilde{y}_{s,t}}{\tilde{L}}, \quad (50)$$

$$\tilde{R}_{i,t+1} = \tilde{\gamma}_t^{-1} g'_i \left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}} \right) \left(\tilde{d}_{i,t+1} + \frac{1 - \delta_i + g_i \left(\frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}} \right)}{g'_i \left(\frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}} \right)} - \frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}} \right), i \in \{c, s\}, \quad (51)$$

$$\tilde{p}_t = \frac{\tilde{\lambda}_{ys,t}}{\tilde{\lambda}_{bc,t}}, \quad (52)$$

$$\tilde{y}_t = \tilde{y}_{c,t} + \tilde{p}_t \tilde{y}_{s,t}, \quad (53)$$

$$\tilde{V}_{c,t} = \tilde{\gamma}_t \tilde{q}_{c,t} \tilde{k}_{c,t}, \quad \text{where } \tilde{q}_{c,t} = \frac{\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t}}{\tilde{\lambda}_{bc,t}}, \quad (54)$$

$$\tilde{V}_{s,t} = \tilde{\gamma}_t \tilde{q}_{s,t} \tilde{k}_{s,t} + \tilde{V}_{L,t}, \quad \text{where } \tilde{q}_{s,t} = \frac{\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t}}{\tilde{\lambda}_{bc,t}}, \quad (55)$$

$$\tilde{V}_{L,t} = \tilde{d}_{L,t} + \frac{\beta}{\tilde{\gamma}} E_t [\tilde{\gamma}_t \tilde{V}_{L,t+1}]. \quad (56)$$

The detrended model is given by the 20 equations (35) to (38), (40) to (56) and the three exogenous processes (21), (23) and (39). After the detrending procedure, the calculation of the unique steady state of the model is straightforward. Two issues shall be stressed here. First, if $\bar{\mu} = 1$, this implies $\bar{\gamma} = 1$ and hence no growth in productivity. Steady state growth is given by $\bar{\mu} > 1$. Second, note that $\bar{\lambda}_{ki} = 0, i \in \{c, s\}$, i.e., in steady state the shadow price of adjustment costs is zero. This means that Tobin's q is equal to unity, and the market value equals the replacement cost of installed capital.

¹⁵Wage, land dividend, overall output and the three values have to be detrended by $z_{c,t-1}$. The usual detrending procedure does not apply to the two dividends and Tobin's q for capital, as they are a function of already detrended variables only. Returns are obtained by replacing the capital accumulation Lagrange multipliers with the first-order conditions for investment. Obviously, returns do not trend, so $\tilde{R}_{i,t+1} = R_{i,t+1}$; compare Talmain (2003) and Greenwood et al. (1997).

4.2 Loglinearized Model

We will approximate this system of equations around the nonstochastic steady state of the detrended variables, using the method of log-linearization. Denoting the nonstochastic steady state of the detrended variable by a bar and the log deviation of a variable from its steady state by a hat, e.g. $\hat{c}_t = \log \tilde{c}_t - \log \bar{c}$, the system of equations is given in Table 4.2. The equations in Table 4.2 have the interpretation of budget constraint, production of consumption good and of real estate, capital accumulation in both sectors, first order conditions for consumption, housing services, labor, two types of investment¹⁶ and two types of capital, i.e. the Euler equations. Then follow the equations for wage, dividends and returns, the relative price of housing services to consumption goods and overall output. The next three equations determine the values of consumption good capital, housing stock and land. Finally, the three stochastic processes for productivity, trend productivity and preferences for housing services close the system.

Notice that in loglinear terms Tobin's q is equal to $\lambda_{ki,t}$, the Lagrangean multiplier difference. Both sizes measure the shadow price of installed capital or the change in the ratio of market value of installed capital and the intrinsic asset value, see Hall (2001).

4.3 Results

Our baseline calibration is given in Table 2, in most cases it is in line with the usual values used in the real business cycle literature, see Cooley (1995; Frontiers book). We set the steady state labor share to one third, the capital share in consumption good production to 0.36. The capital share in real estate production is set using the balanced growth properties of the model. In the model, the relative price of housing to non-housing grows more slowly than real GDP, so that the relative price is detrended $\tilde{p}_t = p_t / z_{c,t-1}^{1-\alpha_s}$. Hence, we know that relative house price growth to the power of $(1 - \alpha_s)$ equals productivity and real GDP growth. Using data on average real GDP growth and growth in the CPI expenditure category housing above the growth rate of all other non-housing CPI categories, we find $\alpha_s = 0.05$ and thus the capital share in real estate production to be 95 percent. The depreciation rate for consumption good capital is set to 2.5 percent per quarter, that for real estate capital to half of that, as real estate is typically more durable. The share of consumption in the utility function determines also the share of expenditures on consumption relative to housing

¹⁶For these loglinear approximations, note that $\hat{\lambda}_{kc}$ and $\hat{\lambda}_{ks}$ cannot be defined in the usual way, as their steady state value equals zero. Instead, we define e.g. $\hat{\lambda}_{ks} = \frac{\lambda_{ks} - \bar{\lambda}_{ks}}{\bar{\lambda}_{bc} + \bar{\lambda}_{ks}} = \frac{\lambda_{ks}}{\bar{\lambda}_{bc}}$.

Table 1: The loglinearized Model

$$\hat{y}_{c,t} = \frac{\bar{c}}{\bar{y}_c} \hat{c}_t + \frac{\bar{x}_c}{\bar{y}_c} \hat{x}_{c,t} + \frac{\bar{x}_s}{\bar{y}_c} \hat{x}_{s,t} \quad (57)$$

$$\hat{y}_{c,t} = \alpha_c \hat{k}_{c,t-1} + (1 - \alpha_c) \hat{n}_t \quad (58)$$

$$\hat{y}_{s,t} = \hat{s}_t = \alpha_s \hat{k}_{s,t-1} \quad (59)$$

$$\bar{\gamma} \hat{k}_{c,t} = (1 - \delta_c) \hat{k}_{c,t-1} + \bar{\delta}_c \hat{x}_{c,t} - \bar{\gamma} \hat{\gamma}_t \quad (60)$$

$$\bar{\gamma} \hat{k}_{s,t} = (1 - \delta_s) \hat{k}_{s,t-1} + \bar{\delta}_s \hat{x}_{s,t} - \bar{\gamma} \hat{\gamma}_t \quad (61)$$

$$\hat{c}_t = -\hat{\lambda}_{bc,t} \quad (62)$$

$$\hat{s}_t = -\hat{\lambda}_{ys,t} + \hat{z}_{s,t} \quad (63)$$

$$\hat{n}_t = \hat{\lambda}_{bc,t} + \hat{y}_{c,t} \quad (64)$$

$$\hat{\lambda}_{kc,t} = \hat{q}_{c,t} = \frac{1}{\xi_c} (\hat{x}_{c,t} - \hat{k}_{c,t-1}) \quad (65)$$

$$\hat{\lambda}_{ks,t} = \hat{q}_{s,t} = \frac{1}{\xi_s} (\hat{x}_{s,t} - \hat{k}_{s,t-1}) \quad (66)$$

$$0 = E_t[\hat{\lambda}_{bc,t+1} - \hat{\lambda}_{bc,t} - \hat{\gamma}_t + \hat{R}_{c,t+1}] \quad (67)$$

$$0 = E_t[\hat{\lambda}_{bc,t+1} - \hat{\lambda}_{bc,t} - \hat{\gamma}_t + \hat{R}_{s,t+1}] \quad (68)$$

$$\hat{d}_{c,t} = \hat{y}_{c,t} - \hat{k}_{c,t-1} \quad (69)$$

$$\hat{w}_t = \hat{y}_{c,t} - \hat{n}_t \quad (70)$$

$$\hat{d}_{s,t} = \hat{p}_t + \hat{y}_{s,t} - \hat{k}_{s,t-1} = (\alpha_s - 1) \hat{k}_{s,t-1} \quad (71)$$

$$\hat{d}_{L,t} = \hat{p}_t + \hat{y}_{s,t} \quad (72)$$

$$\hat{R}_{c,t} = \frac{\bar{R}_c - 1 + \delta_c}{\bar{R}_c} \hat{d}_{c,t} - \hat{\lambda}_{kc,t-1} + \frac{\bar{\gamma}}{\bar{R}_c} \hat{\lambda}_{kc,t} \quad (73)$$

$$\hat{R}_{s,t} = \frac{\bar{R}_s - 1 + \delta_s}{\bar{R}_s} \hat{d}_{s,t} - \hat{\lambda}_{ks,t-1} + \frac{\bar{\gamma}}{\bar{R}_s} \hat{\lambda}_{ks,t} \quad (74)$$

$$\hat{p}_t = \hat{\lambda}_{ys,t} - \hat{\lambda}_{bc,t} \quad (75)$$

$$\hat{y}_t = \frac{\bar{y}_c}{\bar{y}} \hat{y}_{c,t} + \frac{\bar{p} \bar{y}_s}{\bar{y}} \hat{p}_t + \frac{\bar{p} \bar{y}_s}{\bar{y}} \hat{s}_t \quad (76)$$

$$\hat{V}_{c,t} = \hat{q}_{c,t} + \hat{k}_{c,t} + \hat{\gamma}_t \quad (77)$$

$$\frac{\bar{V}_s}{\bar{V}_s - \bar{V}_L} \hat{V}_{s,t} = \hat{q}_{s,t} + \hat{k}_{s,t} + \hat{\gamma}_t + \frac{\bar{V}_L}{\bar{V}_s - \bar{V}_L} \hat{V}_{L,t} \quad (78)$$

$$0 = E_t[-\hat{V}_{L,t} + \beta \hat{V}_{L,t+1} + \beta \hat{\gamma}_t + (1 - \beta) \hat{p}_t + (1 - \beta) \hat{s}_t] \quad (79)$$

$$\hat{\gamma}_t = \hat{\mu}_t + \epsilon_{c,t}, \quad (80)$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \nu_t, \quad (81)$$

$$\hat{z}_{s,t} = \rho_s \hat{z}_{s,t-1} + \epsilon_{s,t}. \quad (82)$$

Table 2: Baseline Calibration

Parameter	Value	Explanation
$\bar{N}$	0.333	Steady state employment is $\frac{1}{3}$ of total time endowment
α_c	0.36	Capital share in consumption good production
α_s	0.95	Capital share in real estate production
δ_c	0.025	Depreciation rate for capital
δ_s	0.012	Depreciation rate for real estate capital
β	1/1.01	Discount factor
θ	0.58	Share of consumption in the household's utility
ξ_c	0.23	Capital adjustment cost curvature parameter (0=capital fixed, ∞ =no cost)
ξ_s	0.12	Real estate capital adjustment cost curvature parameter
ρ_s	0.95	Autocorrelation of preference shock
$\bar{\mu}$	1.006	Steady state technology trend
ρ_μ	0.99	Autocorrelation of trend technology
σ_{ϵ_c}	0.712	Standard deviation of technology shock in percent
σ_{ϵ_s}	0.407	Standard deviation of housing preference shock in percent
σ_{ϵ_v}	0.712	Standard deviation of trend technology shock in percent

services. In U.S. CPI data housing has a relative importance of 42 percent of all expenditures, so we set $\theta = 0.58$. For the capital adjustment cost parameter, we follow Jermann (1998; JME, Asset pricing in prod econ) and set $\xi_c = 0.23$ for the consumption good sector, whereas for real estate we assume higher adjustment cost curvature of $\xi_s = 0.12$. The preference shock is assumed to have an AR(1) coefficient of .95 and a standard deviation of 0.407, the unit root technology process has standard deviation of 0.712 percent, like the trend technology process, which has an AR(1) coefficient of 0.99. The mean of the trend technology growth is set to 1.006, which implies a steady state real quarter to quarter growth of 0.6 percent.

The model is solved using standard algorithms, see Uhlig (1999). The results of the model are presented in two ways: First, we show the behavior of the detrended variables. Second, we re-transform the variables to again include the specific trend that has been removed earlier, along the way proposed in Uhlig (2003? How well do we understand...).

For the discussion of results we focus on impulse responses of GDP, relative prices and the values of land, housing and consumption good capital. The impulse responses of these variables in detrended form to preference and the various productivity shocks are given in Figures 6 to 9. A shock in housing preferences makes housing services more valuable. Thus, both the relative price of

Figure 6: Impulse response to a shock in housing preferences

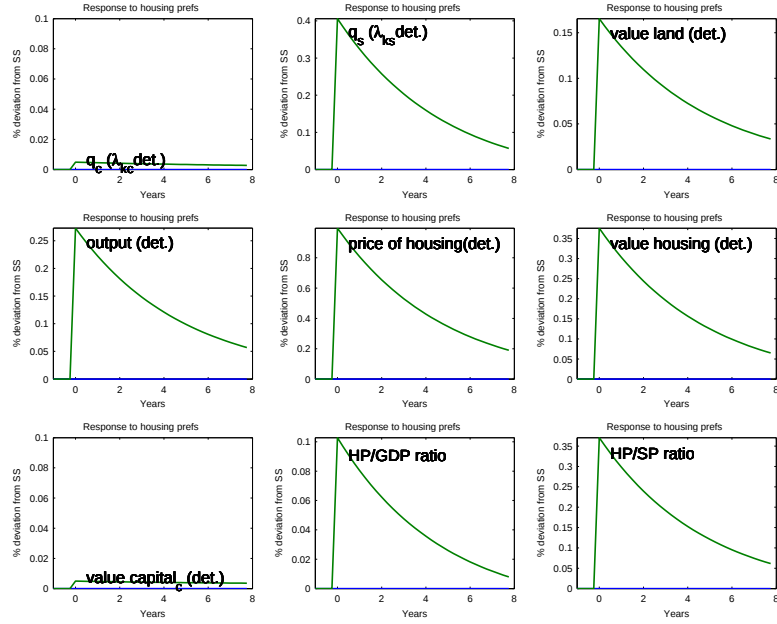


Figure 7: Impulse response to a shock in the level of technology

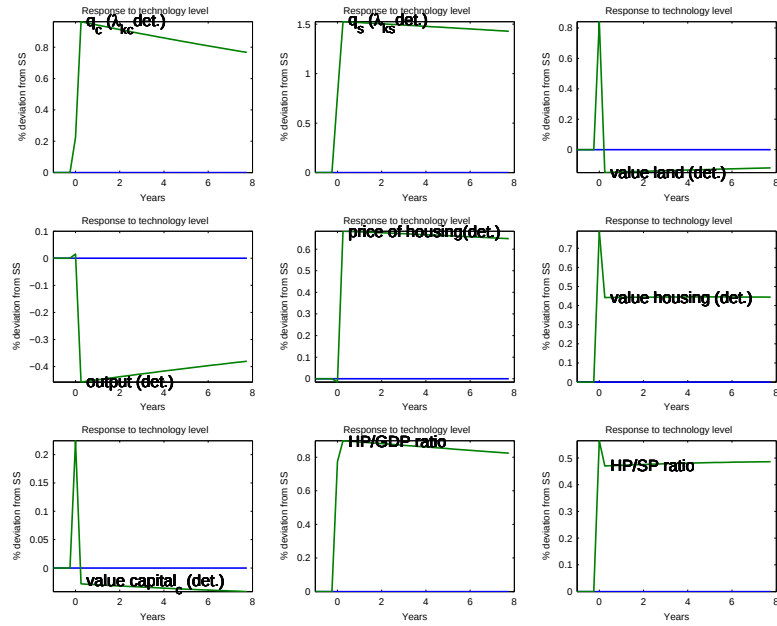


Figure 8: Impulse response to a shock in technology growth

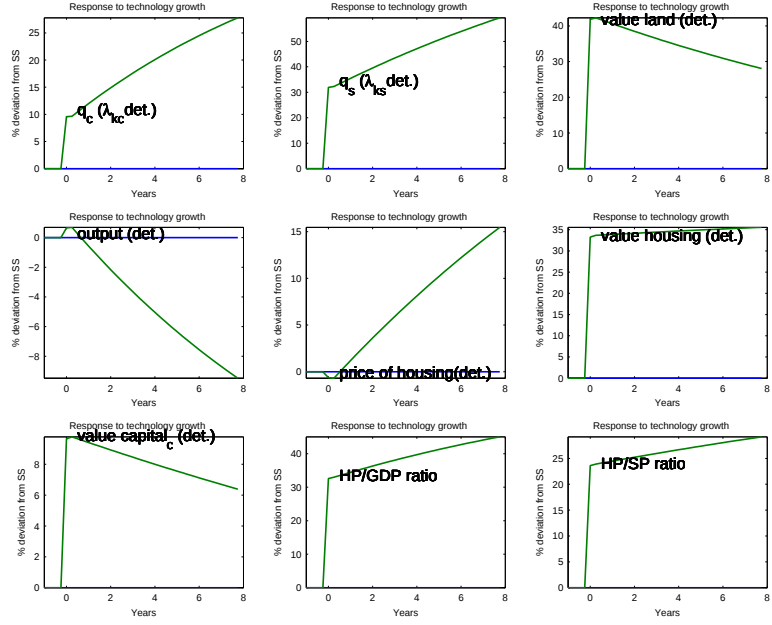
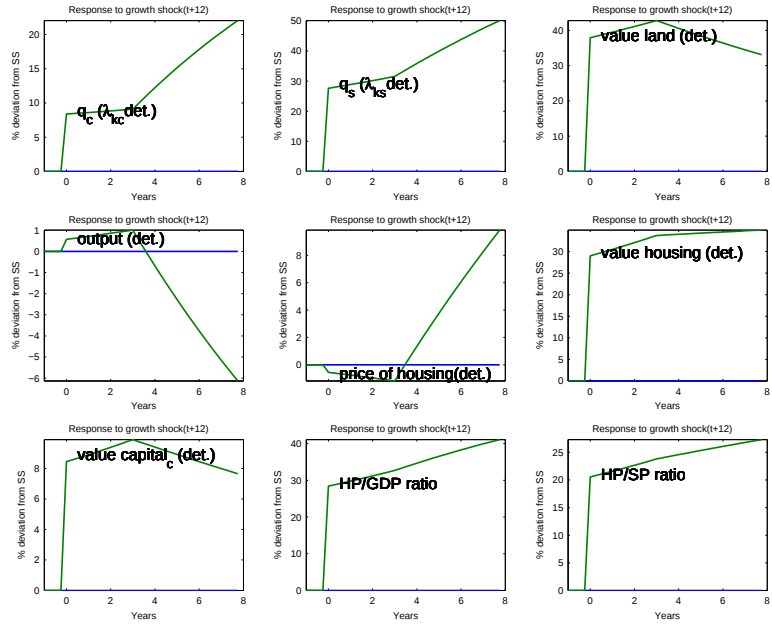


Figure 9: Impulse response to a shock in technology growth twelve periods ahead



housing services p_t and Tobin's q for housing rise on impact. And accordingly, the values of land and of housing rise. As future real estate production can be financed only by means of consumption good production, Tobin's q for consumption good capital and overall output rise as well, but to a lesser extent. So the ratios of housing value to GDP and of housing value to the value of the capital stock in the consumption good sector both rise on impact and then fall to return to their steady state.

A shock to the level of technology leads to highly persistent responses. The market values of both types of capital and hence both types of Tobin's q increase significantly. But as capital adjustment is costly and thus takes place only gradually, the value of non-housing capital and hence output grow only slowly, slower than the trend. Hence, the detrended variables fall on impact. But as the adjustment costs in the real estate sector are assumed to be bigger, Tobin's q is higher in that sector, and the relative price as well as the value of housing are above their steady state for a long time. Hence, the two ratios are again both positive, just like after the preference shock.

A shock to technology growth, i. e., a positive v_t , implies a higher trend growth rate for a long time. Hence, the positive effect on Tobin's q is big, increasing and prolonged compared to the level shock. After an initial increase, output falls below steady state, whereas all three values are above their steady state. Note in particular that the steady-state deviation of the housing-related values are sizeable: The value of land rises by 40 percent on impact, the value of housing by more than 30 percent. Hence, the house price to GDP ratio and the house price to stock price ratio on impact increase by more than 30 and more than 20 percent, respectively. These numbers get quite a way along what we have seen in the data in section 2.

If a future shock to technology growth is anticipated today, the responses on impact are nearly as big as if the shock hit today. The responses then increase until the shock hits. As an example, Figure 13 depicts a technology growth shock anticipated three years in advance. Here, the increases in the house price to GDP and the house price to stock price ratios are between 30 and 20 percent.

The impulse responses of the different types of output, the two capital stocks, the relative price of housing and the three values with the trend again added are given in Figures 10 to 13. The figures show the generally positive effect of all shocks on these trending variables.

As we have seen, in this model future productivity growth leads to an immediate boom in the value of housing and the value of the capital stock in the consumption good sector. Using stock prices and house prices as proxies for these values, we find a stock price boom and a house price boom today in response to an expected future technology growth shock. Put it the other way round, a situation in which both house price to GDP ratio and house price to

Figure 10: Impulse response to a shock in housing preferences, including trend

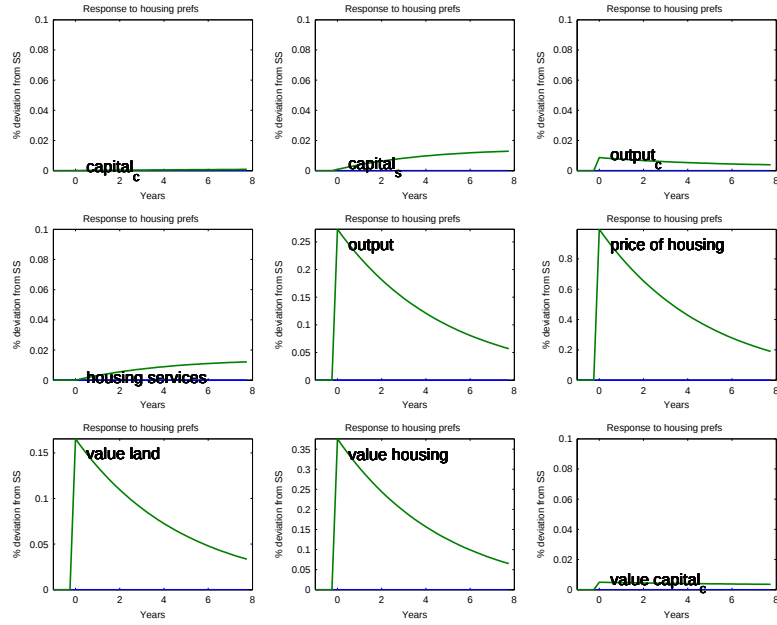


Figure 11: Impulse response to a shock in the level of technology, including trend

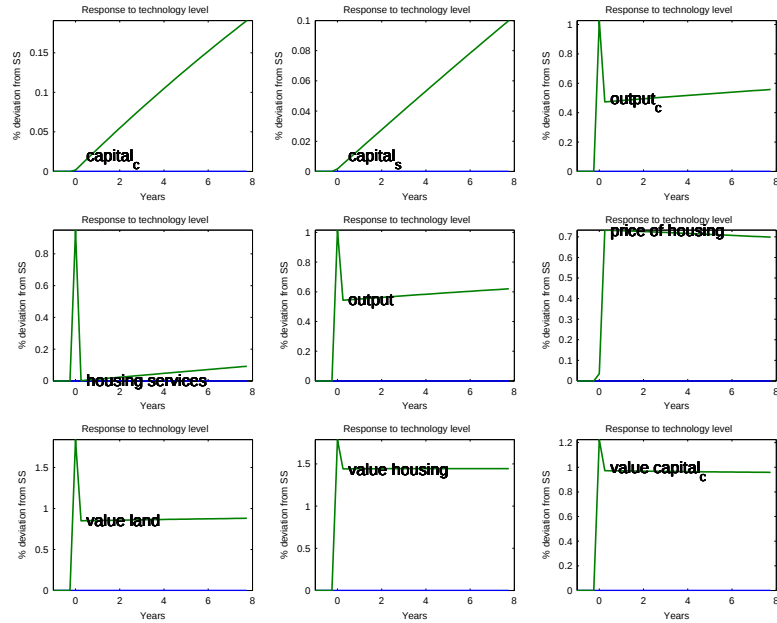
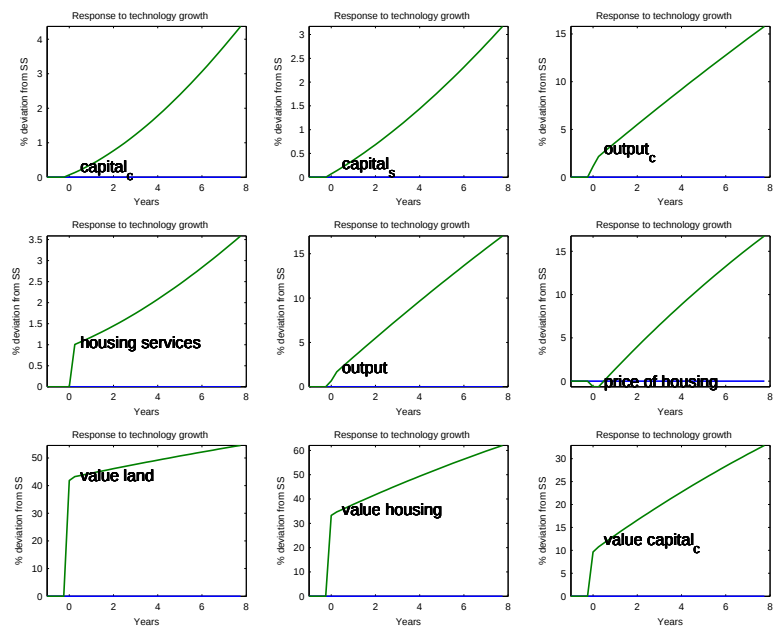
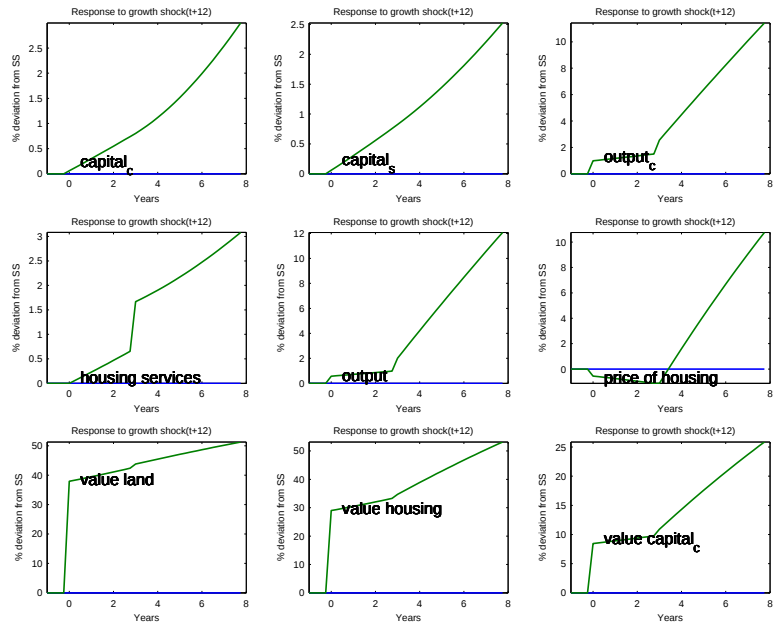


Figure 12: Impulse response to a shock in technology growth, including trend



stock price ratio move upward could be the result of an increase in the expected future technology growth. In other words, positive economic forecasts may effect house prices more than proportionately.

Figure 13: Impulse response to a shock in technology growth twelve periods ahead, including trend



5 Conclusion

A simple two sector RBC model can be used to address issues in real estate economics. Assuming real estate production to be restricted by the finite factor land, whereas the production of other goods is not restricted in this way, the relative price of real estate has to increase over time as the economy grows. In particular, we have shown that a sudden increase in the value of real estate can be explained by a shock to present or expected future productivity growth. The result of this kind of shock is a simultaneous and big increase in the house price to GDP ratio and in the house price to stock price ratio. The implied positive correlation between the two ratios is generally confirmed by the data for the four countries U. S., U. K., Japan and Germany.

One may be tempted to argue that the high house price to GDP ratio in the U. S. and the U. K. until recently, as documented in the middle panel of Figure 5 were the result of high expected future productivity growth rates in these countries, whereas the low ratios in Japan and Germany came from less optimistic expectations. But this conclusion may be a bit daring, as the evolution of house price to stock price ratio plotted below in that very figure does not allow for the same dichotomy of countries. As it seems, our model is not rich enough to allow for both effects simultaneously. We shall leave this for future research.

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A Data Appendix

In our empirical analysis, we use quarterly data from 1980:1 to 2004:4 for the U.S., U.K., Japan and Germany. The series are, with name, source and mnemonic in EcoWin:

- For the U.S.:
 - United States, House Prices, NATIONAL, Index; The Office for Federal Housing Enterprise Oversight; ew:usa11750
 - United States, National Income Account, Gross Domestic Product, Overall, Total, Current Prices, USD; US Department of Commerce; ew:usa01151
 - United States, NYSE, Composite Index, End of Period, USD; EcoWin AB; ew:usa15755
 - United States, Consumer Price Index, All items, Total; OECD; oecd:usa_cpaltt01_ixobq
 - United States, Working-age population; OECD; oe:usa_poptq
- For the U.K.:
 - United Kingdom, House Prices, Nationwide, United Kingdom, all properties, Index; Nationwide(?); ew:gbr03621
 - United Kingdom, Expenditure Approach, Gross Domestic Product, Total, Current Prices, GBP; Office for National Statistics (ONS); ew:gbr01022
 - United Kingdom, FTSE, 30, Ordinary Share Index, Average, GBP; FTID; ew:gbr15105
 - United Kingdom, Consumer Price Index, All items, Total; OECD; oecd:gbr_cpaltt01_ixobq
 - United Kingdom, Working-age population; OECD; oe:gbr_poptq
- For Japan:
 - Japan, Consumer Prices, Nationwide, subgroup, housing excluding imputed rent, Index; Ministry of Internal Affairs and Communications; ew:jpn11802
 - Japan, Expenditure Approach, Gross Domestic Product, Total, Current Prices, JPY; Economic and Social Research Institute (ESRI); ew:jpn01320

- Japan, Nikkei, 225, Index, End of Period, JPY; EcoWin AB; ew:jpn15005
- Japan, Consumer Price Index, All items, Total; OECD; oecd:jpn_cpaltt01_ixobq
- Japan, Working-age population; OECD; oe:jpn_poptq
- For Germany:
 - Germany, House Prices, Total, excl. cellar; Federal Statistical Office Germany; ew:deu11500
 - Germany, Expenditure Approach, Gross Domestic Product, Total (linked), Current Prices, EUR; EcoWin AB; ew:deu01994
 - Germany, Deutsche Boerse, DAX 30, Index, Total Return, End of Period, EUR; EcoWin AB; ew:deu15005
 - Germany, Consumer Price Index, All items, Total; OECD; oecd:deu_cpaltt01_ixobq
 - Germany, Working-age population; OECD; oe:deu_poptq, together with the semiannual series oe:deu_popts for pre-1991 data, averaged to quarterly frequency

The series are - if necessary - transformed to quarterly frequency, deseasonalized and deflated using the CPI. The real GDP is divided by the working age population to obtain the appropriate measure for real GDP per capita. We then index the series so that the average of 1980-1995 equals zero, and take (natural) logarithms. Alternatively, we subtracted a linear trend, as proposed in ECB (2003) because of the observed long cycles in house prices. Other detrending methods like the Hodrick-Prescott filter lead to different results, especially for the correlations between house prices and stock prices.¹⁷

¹⁷See Canova (1998) for a detailed analysis of the effects of different detrending methods.