Dealing with Attrition When Refreshment Samples are Available: An Application to the Turkish Household Labor Force Survey

by

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Abstract

This paper discusses attrition models in the presence of a refreshment sample. In particular, *Missing at Random* (MAR) and *Hausman and Wise* (HW) models are estimated using data from the Turkish HLFS and these models are tested by using the refreshment sample available within the data set. Hirano *et al.* (2001) showed that the refreshment sample permits the estimation of a parametric model of which MAR and HW are special cases. In this paper, a version of this *Additive Non-ignorable* (AN) model is derived and estimated in the context of a three-state labor market application. Futher, strict bounds on the estimated probabilities are calculated using Manski's bounds approach. Empirical findings confirm that attrition is non-ignorable, and neither MAR nor HW adequately model it.

Keywords: attrition, selection, refreshment sample, non-parametric bounds, Additive Non-ignorable model.

JEL codes: J21, C81, C14.

1 Introduction

The increased availability of panel data from household surveys has given researchers the opportunity to examine a wide range of issues that cannot be dealt with using cross-sectional or repeated cross-sectional data. However, the problem of missing data is almost unavoidable in panel data sets. Many, if not all, applied economic analysts ignore this problem and they do one of two things: they use the margins of the unbalanced panel if they are interested in static outcomes (such as labor market status at a point in time) or use the balanced panel if they are interested in dynamic outcomes (such as transition rates between labor market states). This approach is problematic, because the seminal works of Heckman (1979) and Hausman and Wise (1979) have established that ignoring missing observations may induce selectivity and lead to incorrect inference.

Attrition, which is a special case of missing data, occurs if a unit which is scheduled to be re-interviewed, drops out of the sample in subsequent waves even though it responded in the initial wave. As a consequence information on time-varying variables is available only for a subset of the original sample. There are three main approaches to model attrition, which differ in the assumptions about the stochastic nature of the missing data. The first one is called *Missing Completely at Random* (MCAR), in which missing observations constitute a random sample of the original sample. This assumption implies that the remaining observations can be treated as a random sample of the original population. This assumption is akin to ignoring the attrition problem. However, due to its obvious practical advantages, it is very common in empirical studies. The second one is called *Missing at Random* (MAR) and is due to Little and Rubin (1987). MAR assumption implies that conditional on observables, nonresponse is independent of unobserved variables, such as the values of the time-varying variables which would have been observed at the time of the subsequent interview. The third one assumes that conditional on observables, nonresponse is independent of the values of the time-varying variables in the initial round. This assumption originated in Hausman and Wise (1979) and is identified by the names of the authors (*Hausman and Wise*, or HW). Fitzgerald *et al.* (1998) discuss parametric tests of the restrictions implied by the HW model and apply them in the context of attrition in the the Michigan Panel Study of Income Dynamics (PSID). Tunalı (2007) applies them to the Turkish Household Labor Force Survey (HLFS).

Under either MAR and HW, the balanced panel can be reweighted to yield unbiased estimates of the quantities of interest. The disadvantage of MAR is that it is not testable using the panel alone. However, some panel data sets (such as the Turkish HLFS) are supplemented by additional random samples at subsequent rounds. Ridder (1992) refers to these as *refreshment samples*. Hirano *et al.* (2001) show that both MAR and HW assumptions can be tested non-parametrically if a refreshment sample is available. Furthermore, they propose and estimate a model, called the *Additive Non-Ignorable* (AN) model, of which MAR and HW are special cases under the maintained parametric assumptions. Their approach is appealing, because the identifying assumptions of the MAR and HW models can be tested, and in case of rejection, a more general model of attrition can be relied on for reweighting the panel. In the pre-publication version of their paper, Hirano *et al.* (1998) estimate the AN model and test the restrictions implied by MAR and HW. In order to estimate the AN model, they rely on the Markov Chain Monte Carlo (MCMC) simulator and recover the missing data under the maintained assumptions of the AN model. Nevo (2003) examines a structure similar to the AN model and shows that Generalized Methods of Moments (GMM) estimation can be relied upon to produce the weights needed for reweighting the panel. Both Hirano *et al.* (2001) and Nevo (2003) illustrate their methodologies on the same data set, the Dutch Transportation Panel (DTP)[1].

The application in this paper focuses on popular time-varying outcome variables from labor economics, such as the labor force participation rate, the unemployment rate, as well as transition rates between three labor market states, non-participation, employment, and unemployment. Since 2000 the Turkish Statistics Institute (TURKSTAT) has been relying on a rotating sampling frame to conduct the HLFS. Two features of the HLFS data render them interesting for our purposes. Firstly, the sampling frame yields a short panel component which can, in principle, be used for dynamic analyses (however TURKSTAT does not publish estimates of transition rates). Secondly, the sampling frame is address-

[1] The Dutch Transportation Panel is a survey on transportation usage by Dutch households that incorporated refreshment samples in its design; see Ridder (1992) for additional information.

based, and households which leave the address are not followed. Consequently there is attrition to be dealt with. In fact, Tunalı and Baltacı (2004) provide evidence that even static labor market indicators constructed on the cross-sections are biased.

This paper readdresses the problem of attrition in the Turkish HLFS using select rounds of the micro data collected on a quarterly basis over the period 2000-2001. In this application, the interest lies in the distribution of variable Z (namely the labor market status of an individual at a point in time), which takes on three values. By using different assumptions on attrition, the marginal and joint distributions of Z are estimated for different sampling intervals. Using the refreshment samples, the restrictions imposed by MAR and HW models are tested without parametric assumptions. Further, the AN model of which MAR and HW are special cases is estimated by using the balanced sample and the refreshment sample, and the tests are repeated under parametric assumptions.

Not all data sets include a refreshment sample in their design. Therefore, I also report bounds on the quantities of interest by using the approach of Manski (see Manski (1989) and Horowitz and Manski (1998, 2000)). This approach does not need any parametric and/or identification assumptions and there is no need for a refreshment sample.

The next section discusses different attrition models, their identification assumptions and the advantages of the presence of a refreshment sample. Section 3 reframes the discussion in the context of the empirical work undertaken in this paper. This section also derives the strict bounds for the estimated probabilities under different attrition models. Section 4 presents the data set and Section 5 reports the empirical results. In order to account for the sampling variation that may exist in any dataset, Bonferroni confidence intervals and bootstrapped standard errors are calculated. The concluding section summarizes the key findings and points out extensions. The tables are collected at the end of the paper.

2 General Model

This paper considers a two-period panel data set which suffers from attrition. Let Z_{it} be a vector containing all time-varying variables for unit i at time t, $t = 1, 2$, and let

X_i be a vector containing all time-invariant variables for unit i[2]. In the first period, a random sample of size N is drawn from the population of interest. We assume that all units in this period participated in the survey and refer to N as the full or original sample[3]. In the second period, a subset of the original sample, of size N_A, does not participate in the survey. If this subset is also a random sample from the original sample, one can arrive at correct statistical inferences by using the remaining part of the original sample. This remaining part of the original sample, of size $N_B = N_F - N_A$, is called the *balanced panel*. The balanced sample includes observations that responded in both periods. The subset N_A of the original sample for which Z_2 values are missing is called the *attritor subsample*. In the second period, we again draw a random sample of size N_R from the original population. This random sample is called the *refreshment sample*. For observations in the refreshment sample, the values of Z_1 are missing, but this is a delibarate result of the sample design.

In many empirical analyses, the interest lies in the joint distribution of Z_1, Z_2, X (alternatively the conditional distribution of Z_1, Z_2 given X). Note that the probability density function of (Z_1, Z_2, X) can be written as:

$$f(Z_1, Z_2, X) = \frac{f(Z_1, Z_2, X|W = 1) \cdot Pr(W = 1)}{Pr(W = 1|Z_1, Z_2, X)}, \tag{1}$$

where W is the binary indicator denoting the willingness to respond in the second period[4]. $W_i = 1$ implies that unit i, after responding in the first period, also responds in the second period; whereas $W_i = 0$ implies otherwise[5]. One can directly estimate $f(Z_1, Z_2, X|W = 1)$, the joint probability of Z_1, Z_2 and X conditional on being observed in the second period, from the balanced panel N_B. The willingness to respond probability, $Pr(W = 1)$, can be estimated from the full sample N. The term in the denominator is critical for the identification of the joint distribution of (Z_1, Z_2, X). Estimation of $Pr(W = 1|Z_1, Z_2, X)$ hinges on identification assumptions maintained by the various attrition models. Afterwards, one

[2]When it is clear from the context, the subscript i is dropped.

[3]Non-response in the first period can be handled using standard schemes, such as weighting with the inverse probability of response. This requires knowledge of the population distribution of the observables.

[4]To derive eq(1), we use the alternate factorizations of the joint distribution of (Z_1, Z_2, X, W) for $W = 1$: $f(Z_1, Z_2, X, W = 1) = f(Z_1, Z_2, X|W = 1) \cdot Pr(W = 1) = Pr(W = 1|Z_1, Z_2, X) \cdot f(Z_1, Z_2, X)$.

[5]Observe that W is not observed for the individuals in the refreshment sample, drawn in the second period.

can proceed by using imputation or weighting techniques for correct statistical inference (see Hellerstein and Imbens (1999), Brownstone and Valletta (1996)).

Arguably the most commonly used assumption to deal with missing data is *Missing Completely at Random* (MCAR), whereby the probability of responding is independent of Z_1, Z_2 and X. This assumption may be stated as:

$$W_i \perp Z_{i1}, Z_{i2}, X_i \qquad (MCAR). \tag{2}$$

If this assumption is valid, in eq(1) we have$Pr(W = 1) = Pr(W = 1|Z_1, Z_2, X)$ for all i and one can draw correct inferences by using the balanced panel only. Since the individuals who did not respond in the second period are randomly chosen, the subset of individuals who responded in both periods is a random sample of the original population.

The MAR model assumes that conditional on Z_{1i} and X_i, the second period variable Z_{i2} is missing at random, i.e.,

$$W_i \perp Z_{i2} | Z_{i1}, X_i \qquad (MAR). \tag{3}$$

Following the terminology of Little and Rubin (1987), attrition is *ignorable* if the MAR assumption is correct. The MAR assumption is also described as *selection on observables* (Fitzgerald, Gottschalk and Moffit, 1998) since the probability of responding in the second period depends on the first period variables, which are observable for all units in the original sample. Thus, under the MAR assumption, one can write down the attrition probability as

$$Pr(W_i = 1|Z_{i1}, Z_{i2}, X_i) = Pr(W_i = 1|Z_{i1}, X_i). \tag{4}$$

By contrast, the HW model assumes that conditional on the contemporaneous variable Z_{2i} and X_i, the attrition indicator W_i is independent of the first period variable Z_{1i}:

$$W_i \perp Z_{i1} | Z_{i2}, X_i \qquad (HW). \tag{5}$$

The independence assumption of the HW model was originally exploited for handling sample selection in cross-sectional surveys by Heckman (1976,1979) and applied in the context of attrition by Hausman and Wise (1979). The HW assumption is also referred to as *selection on unobservables* (Fitzgerald *et al.*, 1998) since the probability of attrition depends on Z_2 which is unobservable for individuals in N_A:

$$Pr(W_i = 1|Z_{i1}, Z_{i2}, X_i) = Pr(W_i = 1|Z_{i2}, X_i). \tag{6}$$

MAR and HW both ensure identification, but are not testable unless a refreshment sample is available (Hirano *et al.* (2001)). Therefore, the choice between them is based upon theoretical considerations. A priori, the respondent's willingness to participate in subsequent rounds may be assumed to depend on past or contemporary characteristics. In the application part of this paper, Z_{it} denotes the labor market status of individual i at period $t = 1, 2$. An individual's probability of attrition may depend on his/her labor market status in period 1, which is observable for all individuals in the original sample. For example, an unemployed person might expand his job search to areas beyond the local labor market and may not be available for the subsequent interview. Alternatively, the attrition probability may depend on the labor market status in period 2, which is unobservable if that individual is not interviewed in that period. For example, an individual might leave the first period location to accept employment in a different location.

With the help of refreshment samples, one can distinguish between the two models and choose the appropriate one. Since it is representative of the population of interest in the second period, the refreshment sample N_R provides the marginal distribution of Z_2. One can also compute the marginal probabilities using N_B under MAR and HW assumptions, and then test whether the estimates are the same as those obtained from the refreshment sample.

To illustrate the key ideas, we consider an example where the attrition probability can be expressed as a linear function of binary indicators Z_1 and Z_2:

$$W_i = 1\left\{\alpha_0 + \alpha_1 Z_{1i} + \alpha_2 Z_{2i} + \alpha_3 Z_{1i} Z_{2i} + \eta_i > 0\right\}, \tag{7}$$

where $1\{.\}$ is the indicator function and η_i is a zero mean random disturbance whose distribution is independent of Z's. In this example, MAR and HW models are restricted versions of eq(7). In particular, MAR assumption implies that α_2 and α_3 are equal to zero, whereas HW assumption implies α_1 and α_3 are equal to zero. Observe that the only common restriction made by both MAR and HW is that α_3 is equal to zero, i.e., neither model permits the attrition probability to depend on the interaction between Z_{1i} and Z_{2i}.

Hirano *et al.* (2001) show that subject to a distributional assumption on η and the common restriction $\alpha_3 = 0$, one can estimate eq(7) with the help of a refreshment sample. They term their model the *Additive Non-Ignorable* (AN) model to underscore the restriction. Afterwards, one can test whether the available data supports the MAR or the HW model by testing the significance of the parameters α_1 and α_2. Also note that an equation richer in parameters, where X_i as well as interactions of Z_{1i} and X_i or Z_{2i} and X_i are included in the attrition equation, can be estimated. Yet, the presence of refreshment sample does not lead to full non-parametric identification of the attrition process because at most 3 parameters of eq(7) can be recovered.

In the working paper version of their paper, Hirano *et al.* (1998) exploit the nesting property of the AN model to test the HW and the MAR restrictions. Their methodology is based on an imputation approach. In their application, their focus is on the income elasticity of the number of trips, as recorded in the Dutch Transportation Panel (DTP). This elasticity is obtained by regressing the difference of the logarithms of the number of trips in the two periods on the difference of logarithms of household income. It is assumed that differenced variables are multivariate normal. Besides these variables, a dummy variable that shows whether the household lives in a city is added to the AN model. They first use the structure imposed by the AN model to impute the missing values for observations in the attritor and the refreshment samples by using the MCMC method. Then they estimate the parameters of the function $g(.)$ as if there were no missing data problem[6]. They report the parameter estimates for the AN model and conclude that MAR and HW models are inadequate in describing the attrition process.

Nevo (2003) shows that GMM estimation can be relied upon to estimate the AN model rather than using imputation techniques. He uses the same data set, the DTP, and examines the performance of various models in capturing the changes in the total number of trips over time. In our application, we exploit the common structure in Hirano *et al.*

[6]For the attritors, the second period variables are imputed, whereas the first period variables and the willingness to respond variable are imputed for the refreshment sample.

(2001) and Nevo (2003), and show that α's can be recovered from a system of equations which represent the restrictions on the first and second period marginals. As we show below, our approach is computationally simpler and can be implemented by using Excel.

3 Empirical Application

This section describes the empirical context in which the preceding theoretical arguments will be put to use. Our ultimate aim is to recover the 3×3 joint distribution of individuals among three labor market states in two periods. Along the way it is possible to test the restrictions imposed by the MAR and HW models with the help of the refreshment sample. We also derive strict bounds for the estimated probabilities. Finally, we write down the version of the AN model that applies in our case and describe our methodology for recovering the joint distribution of interest.

Let Z_{it} denote the labor market status of individual i at time t with

$$Z_{it} = \begin{cases} 0, & \text{if} \quad \text{non}-\text{participant}; \\ 1, & \text{if} \quad \text{employed}; \\ 2, & \text{if} \quad \text{unemployed}; \end{cases} \tag{8}$$

In the two-wave case, $t = 1, 2$. Let W_i be the attrition indicator such that $W_i = 1$ implies that the individual i responded in both periods, whereas $W_i = 0$ implies otherwise. In what follows, we ignored the vector X to keep the empirical work managable. In this case, the joint distribution of interest can be fully specified as $Pr(Z_1 = z_1, Z_2 = z_2, W = w)$, where $z_1, z_2 = 0, 1, 2$ and $w = 0, 1$. Note that in the presence of attrition, only nine probabilities out of eighteen are identified. In particular, $Pr(Z_1 = z_1, Z_2 = z_2, W = 0)$ with $z_1, z_2 = 0, 1, 2$ are not identified. However, these probabilities are identified under the MAR or HW assumptions. Therefore, the joint distribution of (Z_1, Z_2) is fully identified subject to additional restrictions.

Following Hirano *et al.* (2001), let us define:

$$q_{z_2|z_1 w} = Pr(Z_2 = z_2 | Z_1 = z_1, W = w),$$

$$r_{z_1 w} = Pr(Z_1 = z_1, W = w).$$

With this notation, $q_{z_2|z_1 1}$ and $r_{z_1 w}$ with $z_1, z_2 = 0,1,2$ and $w = 0,1$ are identified. Hence, these probabilities can be estimated by using the balanced panel N_B and full panel N, respectively. The remaining nine probabilities can be identified under additional restrictions given in section 3.1.

3.1 Identification Restrictions

In the absence of a refreshment sample, the independence assumptions for MAR and HW provide the identifying restrictions for $q_{z_2|z_1,0}$, $z_1, z_2 = 0,1,2$. By the independence assumption in eq(3), for $z_2 = 0,1,2$ MAR model implies that

$$\begin{aligned} q_{z_2|00} &= q_{z_2|01}, \\ q_{z_2|10} &= q_{z_2|11}, \\ q_{z_2|20} &= q_{z_2|21}. \end{aligned} \tag{9}$$

The restrictions implied by eq(5) are more complicated[7]. Thus for $z_1 = 0,1,2$, HW model yields:

$$\begin{aligned} \frac{Pr(W=1, Z_2=0)}{Pr(W=0, Z_2=0)} &= \frac{q_{0|z_1 1} \cdot r_{z_1 1}}{q_{0|z_1 0} \cdot r_{z_1 0}}, \\ \frac{Pr(W=1, Z_2=1)}{Pr(W=0, Z_2=1)} &= \frac{q_{1|z_1 1} \cdot r_{z_1 1}}{q_{1|z_1 0} \cdot r_{z_1 0}}, \\ \frac{Pr(W=1, Z_2=2)}{Pr(W=0, Z_2=2)} &= \frac{q_{2|z_1 1} \cdot r_{z_1 1}}{q_{2|z_1 0} \cdot r_{z_1 0}}. \end{aligned} \tag{10}$$

By rewriting these equations for $z_1 = 0,1,2$, we obtain nine equalities. Since there are nine unidentified probabilities, these nine equality restrictions yield a uniqe solution for each unidentified probability.

Observe that the marginal distribution of Z_2 can be expressed as:

$$Pr(Z_2 = z_2) = \sum_{z_1, w} q_{z_2|z_1 w} \cdot r_{z_1 w}. \tag{11}$$

Hence, when missing probabilities are estimated subject to the identification restrictions implied by MAR and HW, one can calculate the marginal distribution of Z_2. Observe that

[7]The derivation is given in the Appendix.

we have an independent estimate of this marginal distribution if a refreshment sample is available at time $t = 2$. Thus, we can use the refreshment sample to test the restrictions implied by MAR and HW.

3.2 Strict Bounds on the Estimated Probabilities

This section derives the bounds which can be used to provide a check on the probabilities estimated under the MAR and HW assumptions. Following Manski (1989) and Horowitz and Manski (1998, 2000), one can derive strict bounds on the unidentified probabilities without making identification assumptions[8]. These strict bounds are consequences of a worst-case scenario and are obtained by setting unidentified probabilities to their extreme values (0 and 1). To see how well MAR and HW models are doing, the estimated probabilities can be compared to the estimated bounds obtained under this worst-case scenario.

Let us define a binary variable Y_{it}, which is equal to 1 if individual i is a participant in the labor market in period t and is equal to zero otherwise. The strict bounds on its marginal probability or transition probabilities between states can be calculated. Note that $Pr(Y_2 = k|Y_1 = j)$, for $j, k = 0, 1$, may be written as:

$$\begin{aligned} Pr(Y_2 = k|Y_1 = j) = Pr(Y_2 = k|Y_1 = j, W = 1) \cdot Pr(W = 1|Y_1 = j) + \\ Pr(Y_2 = k|Y_1 = j, W = 0) \cdot Pr(W = 0|Y_1 = j). \end{aligned} \tag{12}$$

All the terms in the right handside are known except for $Pr(Y_2 = k|Y_1 = j, W = 0)$. Yet, we have $0 \leq Pr(Y_2 = k|Y_1 = j, W = 0) \leq 1$ with certainty since any probability must be between 0 and 1. Assuming in turn, that the unidentified probability takes on its extreme values of 0 and 1, we obtain the lower and upper bounds for the transition probability of interest[9]:

$$\begin{aligned} Pr(Y_2 = k|Y_1 = j, W = 1) \cdot Pr(W = 1|Y_1 = j) \leq Pr(Y_2 = k|Y_1 = j) \leq \\ Pr(Y_2 = k|Y_1 = j, W = 1) \cdot Pr(W = 1|Y_1 = j) + Pr(W = 0|Y_1 = j). \end{aligned} \tag{13}$$

[8]Here, 'strict' means that we obtain the bounds by only using the data with, and need no assumptions on the data generating process or parametric restrictions of form.

[9]Since $Pr(Y_2 = k|Y_1 = j) = \frac{Pr(Y_2=k,Y_1=j)}{Pr(Y_2=k)}$, the joint probability of (Y_1, Y_2) as well as the marginal probability of Y_2 can be bounded.

The same procedure will be applied to the binary variable U_{it}, which is equal to 1 if individual i is unemployed in period t conditional on i being a labor market participant and is equal to zero otherwise. This yields:

$$Pr(U_2 = k|U_1 = j, W = 1) \cdot Pr(W = 1|U_1 = j) \leq Pr(U_2 = k|U_1 = j) \leq$$
$$Pr(U_2 = k|U_1 = j, W = 1) \cdot Pr(W = 1|U_1 = j) + Pr(W = 0|U_1 = j). \qquad (14)$$

3.3 The Additive Non-Ignorable Model

As noted above, one can rely on eq(7) to specify a parametric attrition model. With some abuse of notation, we add a second subscript to Z_1 and Z_2 and redefine the α's, so that our model may be expressed as:

$$Pr(W = 1|Z_1, Z_2) = Pr(W = 1|Z_{11} = z_{11}, Z_{21} = z_{21}, Z_{12} = z_{12}, Z_{22} = z_{22})$$
$$= g(\alpha_0 + \alpha_1 z_{11} + \alpha_2 z_{21} + \alpha_3 z_{21} + \alpha_4 z_{22}), \qquad (15)$$

with $z_{tj} = 0, 1;\ t, j = 1, 2$ where t denotes the time period and j indexes the pair of dummies. According to our revised notation, there are three states (thus, two mutually exclusive dummy variables) in each of two periods. With non-participation as the reference state, $z_{t1} = 1$ implies that the individual is employed in period t and is not employed otherwise. Similarly, $z_{t2} = 1$ implies that the individual is unemployed in period t and is not unemployed otherwise. The function $g(.)$ is a predetermined strictly increasing function satisfying $lim_{a\to-\infty} g(a) = 0$ and $lim_{a\to\infty} g(a) = 1$. The application in this paper relies on probit and logit cdf's, which satisfy the stated conditions, and on the linear probability model, which does not. Although the linear probability model specification can yield probability estimates outside the interval $[0, 1]$, we rarely encountered this problem in our empirical work.

Consider a 3×3 matrix with each cell denoting a joint probability of membership in particular labor market states in subsequent periods. Since there is attrition in the second period, the probability in each cell may be computed using eq(1). The relevant expressions are shown in Table 1. Observe that the joint probabilities in the balanced panel are inflated or deflated by the factor $Pr(W = 1)/g(.)$. The factor is the ratio of the marginal probability of being observed in period 2 (i.e., non-attritor) divided by the conditional probability of non-attrition expressed as a function of Z_1 and Z_2. Row-wise summations yield the marginal distribution of Z_1, and columnwise summations yield the marginal distribution of Z_2. Given the form of $g(.)$, the estimation problem boils down to the choice of the parameters (α's) such that the restrictions on the marginal probabilities are satisfied by the estimated 3x3 matrix. In other words, we solve a system of non-linear equations whereby the estimated joint probabilities are consistent with the information provided by the original panel and the refreshment sample. Letting $P_{z_{11}z_{21}z_{12}z_{22}|k} = Pr(Z_{11} = z_{11}, Z_{21} = z_{21}, Z_{12} = z_{12}, Z_{22} = z_{22}|W = k)$, $k = 0, 1$, the restrictions are the following:

$$\frac{P_{0000|1} \cdot Pr(W=1)}{g(\alpha_0)} + \frac{P_{0100|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_2)} + \frac{P_{0001|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_4)} = Pr(Z_1 = 0);$$

$$\frac{P_{1000|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1)} + \frac{P_{1100|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1+\alpha_2)} + \frac{P_{1001|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1+\alpha_4)} = Pr(Z_1 = 1);$$

$$\frac{P_{0010|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_3)} + \frac{P_{0110|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_2+\alpha_3)} + \frac{P_{0011|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_3+\alpha_4)} = Pr(Z_1 = 2);$$

$$\frac{P_{0000|1} \cdot Pr(W=1)}{g(\alpha_0)} + \frac{P_{1000|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1)} + \frac{P_{0010|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_3)} = Pr(Z_2 = 0);$$

$$\frac{P_{0100|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_2)} + \frac{P_{1100|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1+\alpha_2)} + \frac{P_{0110|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_2+\alpha_3)} = Pr(Z_2 = 1);$$

$$\frac{P_{0001|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_4)} + \frac{P_{1001|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_1+\alpha_4)} + \frac{P_{0011|1} \cdot Pr(W=1)}{g(\alpha_0+\alpha_3+\alpha_4)} = Pr(Z_2 = 2).$$

Two additional issues may arise in empirical implementation. The first one is the incorporation of observed time-invariant characteristics (the vector X) into the problem. When we include X into our model, all three components of the cell probabilities as well as the marginal distributions of Z_1 (from the full sample) and Z_2 (from the refreshment sample) have to be expressed conditional on X. In this case the balanced sample can also be envisoned as consisting of mutually exclusive subsamples obtained for every distinct X combination. The non-parametric approach to estimation entails writing the same system of six equation for each distinct X combination, so that we obtain a different set of five parameters on each subsample. This approach is feasible when X is low dimensional (as is often the case in the reweighting schemes used by statistical agencies) but will become cumbersome when X is high dimensional. Also, particular cells of the 3×3 table may be empty for some X combinations. If a high dimensional X is required, the practical problems may be circumvented by resorting to a parametric specification. This is akin to specifying parametric models for each of the three probability components and is subject to the usual pitfalls of parametric estimation. Recall that components of the AN model must enter the model additively. Polynomials in X and interactions of X with the time-varying variables (Z_1 and Z_2) are within the class of permissable specifications. Nevo's (2003) GMM framework subsumes both the non-parametric and the parametric approaches to the problem.

The second practical issue is handling of sampling weights. Both the original sample and the refreshment sample might have sampling weights that render them representative for the population. The sampling weights can be handled by treating them as the values of an individual specific covariate so that the approaches decribed in the previous paragraph become applicable. Unfortunately TURKSTAT did not provide the sampling weights, so we are unable to tackle this issue in our empirical work.

As a further simplification, we exclude the vector X from our analysis and concentrate on the estimation of the common 3×3 matrix given in Table 1 from which the transition probabilities of interest can be derived. In this case, we do not need to resort to the GMM framework of Nevo (2003) but need to solve a system of non-linear equations. In fact, when $g(.)$ equals the identity function, the equations can be expressed as a system of linear equations. We rely on Microsoft Excel's 'Solver' tool to solve the system. Solver uses the Generalized Reduced Gradient (GRG2) method for nonlinear optimization (see Lasdon *et al.* (1978) for the details). Note that there are six equations, yet there are five parameters to be estimated. However, this system of non-linear equations are just-identified as proven in Hirano *et al.* (2001). Therefore, one equation is redundant in the sense that excluding any of these six equations does not change the solutions.

3.4 Sampling Variation

This paper is mainly concerned with tests of identification restrictions under different attrition models. Since we work with samples, we need to consider the implications of sampling variation. To see the effects of the sampling variation on bounds derived in Section 3.2, the Bonferroni confidence intervals can be reported. For expositional brevity, we consider $Pr(Y_2 = j)$, $j = 0, 1$. The bound on this quantity may be written as $Pr(Y_2 = j|W = 1) \cdot Pr(W = 1) \leq Pr(Y_2 = j) \leq Pr(Y_2 = j|W = 1) \cdot Pr(W = 1) + Pr(W = 0)$. The bounds simplify as follows:

$$Pr(Y_2 = j, W = 1) \leq Pr(Y_2 = j) \leq 1 - Pr(Y_2 \neq j, W = 1). \tag{16}$$

The asymptotic standard errors of the estimates of the lower and upper bounds are given by

$$S_L = \{Pr(Y_2 = j, W = 1) \cdot [1 - Pr(Y_2 = j, W = 1)] / N\}^{1/2}, \tag{17}$$

$$S_U = \{Pr(Y_2 \neq j, W = 1) \cdot [1 - Pr(Y_2 \neq j, W = 1)] / N\}^{1/2}. \tag{18}$$

(see Horowitz and Manski, 1999). The analogus argument can be applied when the quantity of interest is a conditional probability. A Bonferroni asymptotic joint confidence region with a confidence level of at least 95 % is obtained by forming the intersection of individual 97.5 % regions:

$$\text{estimate of lower bound} \mp S_L \cdot (2.24), \tag{19}$$

$$\text{estimate of upper bound} \mp S_U \cdot (2.24). \tag{20}$$

Note that the width of the interval depends on the standard errors and the significance level chosen.

The method we use to estimate the AN model does not yield standard errors of the parameter estimates. Note that we solve a system of non-linear equations and the parameters of the $g(.)$ function are unique subject to the choice of its form. However, closer examination of the system reveals that there is sampling variation in the marginal distributions of Z_1 and Z_2 computed from the original sample and the refreshment sample, respectively. These marginal probabilities are located on the right handside of each equation in our non-linear system. Quantities on the left handside correspond to the attrition process we are trying to model. Therefore, we cannot consider sampling variation in these quantities. In order to account for the sampling variation coming from the original sample and the refreshment sample, we use the bootstrap method. In particular, we draw with replacement 50 samples of the same size from the original sample and the refreshment sample. In this way, we obtain 50 pairs of marginal distributions (of Z_1 and Z_2) and use Excel Solver to obtain a set of parameter estimates for each pair. We use the set of parameter estimates to compute the bootstrapped means and standard errors.

4 Data

The Turkish Household Labor Force Survey (HLFS) conducted between 2000-2002 is used for the empirical analysis. Since 2000, the Turkish Statistics Institute (TURKSTAT) (formerly the State Institute of Statistics, SIS) has been using a rotating sample frame for conducting this survey. According to the (steady-state) rotation plan, a household is interviewed in two subsequent quarters, rested for the next two, and reinterviewed in two additional subsequent rounds. Hence, HLFS has a short-panel component. The rotating sample frame is address-based. In fact, the design is very similar to the Current Population Survey conducted in the United States (Bureau of Labor Statistics, 2002). Tunalı (2007) provides a more detailed discussion of the data set and the attrition patterns it entails. This paper focuses on individual level attrition. Since we are concerned with labor market outcomes, we confine our sample to individuals who are 15 and older.

Table 2 displays the sampling frame of the HLFS for 2000-02. Each survey round includes eight subsamples which are representative of the population (see DİE (2001)). The O(dd) vs. E(ven) distinction is not important for our purposes. The suffix following O or E designates the visit number to a particular subsample. Subsamples with the suffix 1 are not subject to attrition, and may therefore be viewed as refreshment samples. The refreshment sample in each round becomes the first period of a short panel, which follows the 2 in-2 out-2 in quarterly rotation scheme.

The methodology applied in this paper needs a two-period panel. We will use micro data from the first two quarters of 2000 and 2001. In this way, we can study two annual and two quarterly panels. The sample sizes are shown in Table 3. For the quarterly panel in the first quarter of 2000, the total of subsamples E1 and O1 in rotations 2, 4, 6 and 8 are used. This yields an orginal sample with $N = 26,244$ individuals in the panel. Note that the individuals in these subsamples are planned to be reinterviewed in the subsequent quarter, Q2. However, only $N_B = 24,064$ of them participate in the survey in Q2. The total of subsamples E1 and O1 drawn in the second quarter for rotations 3, 5, 7 and 9 is the refreshment sample for this panel and it consists of $N_R = 25,669$ individuals. For the quarterly panel in the first quarter of 2001, we use the total of subsamples E1 and O1 in rotations 10 and 12. This original panel consists of $N = 13,397$ individuals and $N_B = 12,340$ of them participate in the survey in the second quarter. The refreshment sample for this panel is the total of subsamples E1 and O1 drawn in the second quarter of 2001 for rotations 12 and 14. This sample consists of $N_R = 14,201$ individuals.

For the first of the two annual panels, the original panel is composed of subsamples E1 and O1 in rotations 6 and 8. Note that this is a subset of the first quarterly panel. From the original panel of size $N = 12,327$, $N_B = 11,192$ participate in the second round interviews. The refreshment sample is the total of E1 and O1 drawn in the first quarter of 2001 for rotations 10 and 12; it consists of $N_R = 13,397$ individuals. The second annual panel examines the transitions between the second quarters of 2000 and 2001. The original panel is composed of E1 and O1 drawn in the second quarter of 2000 for rotations 7 and 9. This is a subset of the refreshment sample for the first quarterly panel. The size of this panel is $N = 11,984$ and $N_B = 10,980$ of them participate in the survey in the next round. The refreshment sample of this panel is the same as that of the second quarterly panel.

When we compare the attrition rates for these panels, we do not see huge differences. For the first two quarters in 2000, the attrition rate is 0.083, whereas it is 0.078 for the first two quarters in 2001. The annual attrition rate for the period between the first quarter of 2000 and the first quarter of 2001 is 0.092, while it is 0.083 between the second quarters of the same years. The attrition rate for 12 months is expected to be higher than the attrition rate for 3 months since the longer time period makes individual moves more probable. However, the difference is smaller than expected. This is because we do not study 3 month and 12 month attriton in the same panel. In other words, N is different for quarterly and annual panels. Tunalı (2007) also uses the Turkish HLFS and examines the attrition rates at the household level. Unlike our application, he uses the nested form of panels, i.e., N remains the same for the subsequent rounds of attrition. His findings reveal that the attrition rate increases considerably (from about 5 to 12 percent on average) when we move from the 3 month to the 12 month interval.

5 Empirical Results

This section reports the empirical findings. We study attrition at 3 month and 12 month intervals. We use two quarterly panels (2000: Q1-Q2 and 2001: Q1-Q2) and two annual panels (Q1: 2000-2001 and Q2: 2000-2001). We have 10 tables for each panel. The first table displays the data breakdown by labor market status in the balanced panel (N_B), the subsample of attritors (N_A) and the refreshment sample (N_R). The second gives the conditional distribution of Z_1 by attrition status (W). The next group of four tables are concerned with the methodology of Section 3.1: We report directly estimable parameters, model-based estimates under MAR and HW assumptions, the marginal distribution of Z_2 implied by these assumptions, and results for the tests of equality of model-based probabilities to those estimated on the refreshment sample. The results from the bounding approach discussed in Sections 3.2 and 3.3 are reported next, using a pair of tables. Finally, the AN model estimates for the model in Section 3.4 and the implied reweighted 3×3 matrices are given.

To illustrate what each table displays, let us review the table contents for the quarterly panel in 2000. Table 4 displays the data brakdown for this panel; it includes the balanced panel, subsample of attritors and the refreshment sample in the form of seperate tables

(4A-4C). Rowwise summation in the balanced sample yields the marginal distribution of Z_1, and is comparable to the marginal distribution of Z_1 computed on the subsample of attritors. Columnwise summation yields the marginal distribution of Z_2 and is comparable to the marginal disribution of Z_2 computed on the refreshment sample. Table 5 consists of individuals in the full sample and displays the distribution of Z_1 conditional on W, the binary attrition indicator. By examining this table, one can see how attritors and individuals who participate in both periods differ according to first period labor market status.

Remember that the marginal distribution of Z_2 plays an important role in our methodology. We showed that this marginal distribution can be expressed as a sum of probability products (q and r's defined in Section 3). Table 6 displays probabilities which are identified without further restrictions. On the other hand, Table 7 displays the estimates of probabilities which hinge on identification restrictions of MAR and HW models. Since MAR assumption implies that the probability of responding in the second period is independent of the labor market status in the second period, the second row of Table 6 is the same as the first row of Table 7. With the directly estimable probabilities in Table 6 and the model-based estimates in Table 7 in hand, Table 8 displays the marginal distribution of Z_2 computed under MAR and HW assumptions. We then test whether the marginal distributions of Z_2 under MAR and HW assumptions are the same as the one implied by the refreshment sample. The p-values for these tests are displayed in Table 9. Table 10 reports the strict bounds on relevant conditional probabilities and the widths of these bounds derived from the 95% Bonferroni intervals. Since the bounds in these tables are computed without any identifying assumptions, they provide a check on the quantities of interest computed under MAR and HW assumptions.

We turn to our parametric results next. Table 11 displays the parameter estimates of the AN model. Tables 12 gives the weighted joint distributions of (Z_1, Z_2) respectively for LPM, probit and logit specifications. To facilitate comparison, the contents of the balanced panel which is the subject of Table 4.A is reported again, as Table 13. Our discussion of empirical findings is undertaken in two subsections. The first subsection discusses the findings from MCAR, MAR and HW models, whereas the second subsection discusses the findings from the AN models.

5.1 Findings From MCAR, MAR and HW Models

If the MCAR assumption is valid, the sample of attritors is a random sample drawn from the original panel. This implies that the balanced panel is also a random sample of the original sample. Therefore, one can use the balanced panel to obtain unbiased estimates. However, examination of the marginal distribution of Z_1 conditional on W reveals that there are systematic differences between attritors and non-attritors (see Tables 5, 15, 25 35). First of all, unemployed ($Z_1 = 2$) individuals are overrepresented among attritors. For example, for the annual panel Q2: 2000-2001, the probability of being unemployed in the first period is 0.042 according to the balanced panel, whereas it is 0.091 among attritors (see Table 35). An unemployed individual may be tempted to expand his job search process to other areas than local ones, thus he is expected to be more mobile. Also, likelihood of non-participation in the first period ($Z_1 = 0$) is overestimated in the balanced panel although the proportionate differential is much less in comparison to the unemployed. Being employed in the first period ($Z_1 = 1$) is overestimated among the attritors if we examine quarterly panels. Yet, the result reverses if we examine annual panels. Again, the proportionate differentials are smaller in comparison to the unemployed.

One implication of MCAR is that the willingness to respond is independent of the labor market status in the first period. One can test this implication using contingency tables. For example, if this implication is valid for the annual panel Q2: 2000-2001, the marginal distribution of Z_1 (the labor market status in the first period) must be the same for attritors and non-attritors (see the marginal distribution of Z_1 conditional on W in Table 35). The null hypothesis that no association exists between attrition status and labor market state is tested by applying the χ^2 test with two degrees of freedom. The null hypothesis is handily rejected for the quarterly panel in 2001 and both annual panels (p-value$<<$0.01). For the quarterly panel in 2000, the p-value is 0.119. Therefore, the null hypothesis is not rejected at the 10 percent significance level. However, this does not mean that MCAR is valid since this is a test of one implication, not of MCAR itself. Remember that MCAR is a stronger assumption than either MAR or HW. Therefore, if one rejects either MAR or HW, MCAR is also rejected. In this respect, we will provide additional tests for MCAR.

Inspection of Tables 34.A-34.C (also 4.A-4.C, 14.A-14.C and 24.A-24.C for other panels) reveals that the balanced panel and the refreshment sample imply very close probability distributions for Z_2. The differential between the marginal probabilities of Z_2 is less than 0.006 for the annual panel Q2: 2000-2001 (see Table 34). For example, the probability of being employed is 0.416 in the balanced panel and is 0.421 in the refreshment sample. The differential is less than 0.002 for the quarterly panel 2000: Q1-Q2 (see Table 4) and for the annual panel Q1: 2000-2001 (see Table 24). For the annual panel Q1: 2000-2001, the differentials are less than 0.014. Since refreshment samples yield roughly the same probabilities with the Z_2 margins of the balanced panel, it is tempting to use the balanced panel can be used for statistical inference. Yet, we pointed out that the balanced panel and attritors imply very different distributions for Z_1. This might seem puzzling, yet it is entirely possible that the probability of response in the second period does not depend on the second period labor market status, while it depends on the first period labor market status. In other words, this general observation suggests that MAR assumption may be more appealing than the HW assumption in our empirical context.

As discussed earlier, one can compute the marginal probabilities of Z_2 under MAR and HW assumptions, and then compare them to the ones obtained from the refreshment sample. In this way, we can evaluate how well MAR and HW do in modeling attrition. In Table 36 (also in 6, 16, and 26), we report estimates of the direcly estimable parameters. Since parameters $q_{z_2|z_1,1}$ are conditional on being observed in the second period, (i.e. $W = 1$), they are identified without additional assumptions. Parameters $r_{z_1,w}$ are always identified since they are based on the original panel. Parameters $q_{z_2|z_1,0}$ are not identified. The model-based estimates of these probabilities under the respective identifying assumptions of MAR and HW models are reported in Table 37 (also in 7, 17 and 27). We see that the probability of being non-participant in the second period conditional on being an attritor and non-participant in the first period ($q_{0|00}$) is larger for MAR in all four panels. For example, that probability is 0.858 according to MAR and it is 0.816 in HW case (see Table 37). For $q_{0|20}$, MAR estimates are larger than HW estimates and the differential is larger than that for $q_{0|00}$. Estimates for $q_{0|10}$ are very close under both models. This pattern implies that MAR model gives more weight to the probability of being non-participant in the second period in comparison to the HW model. Indeed, model based estimates of the marginal distribution of Z_2 support this claim (see Tables 8, 18, 28 or 38). For $q_{1|10}$ and $q_{1|20}$, MAR always gives a larger estimate, whereas the two models give very close estimates for $q_{1|00}$. Unlike the pattern seen for non-participation, the discrepancies between estimated probabilities are smaller. Therefore, the probability of being employed in the second period computed under MAR assumption is equal or larger than the HW estimate. Since there are three labor market states and probabilities are sum to one, the probability of being unemployed in the second period is overestimated by the HW model in comparison the the MAR model.

When we compare the marginal distributions of Z_2 implied by MAR and HW assumptions, we see that the probability of non-participation is consistently overestimated and the probability of unemployment is consistently underestimated by MAR. In the annual panels, MAR also overestimates the employment probability. Yet, the differences are typically small. Indeed, the χ^2 tests with two degrees of freedom show that the marginal distribution of Z_2 implied by MAR and HW assumptions are statistically the same for all panels except for the annual panel Q2: 2000-2001. For this panel, Table 39 reveals that the p-value is 0.003. Upon comparing the marginal distributions of Z_2 implied by MAR and HW models with the one computed from the refreshment sample, we see that they both perform poorly in modelling attrition. The p-values for the tests (reported in the bottom rows of Tables 9, 19, 29, 39) are very close to zero, therefore we reject the null hypothesis that they are the same. Evidently neither model adequately reflects the attrition process.

Remember that the non-parametric strict bounds on these unidentified probabilities can be computed even when a refreshment sample is not available. These bounds can be used to see how MAR or HW differ as they put weights on the second period probabilities. As discussed earlier, MAR overestimates $q_{0|00}$ and $q_{0|20}$ in comparison to HW. From Table 40 (also from Tables 10, 20 and 30) we see that MAR estimates are closer to the upper bounds, whereas HW estimates are located near the middle of the range. The widths of the estimated bounds obtained from the Bonferronni intervals are reported in the same table. In our application, we found that the width of 95% Bonferroni intervals of the bounds are narrower than the range calculated from bounds for almost all cases. We conclude that the problem of sampling variation is of secondary concern, whereas the identification problem is of primary concern.

5.2 Findings From the AN Models

The estimation results for the AN model are provided for all three specifications (the linear probability model, probit and logit) for each panel. Further, the weighted 3×3 matrix (the joint probability of (Z_1, Z_2)) is reported for each specification. In order to put these estimated weighted matrices in perspective, we report the same matrix for the balanced panel along with the asymptotic standard errors of each cell probability.

Overview of parameter estimates reveals that the signs of the parameters are remarkably consistent across specifications (see Tables 11, 21, 31 and 41). There is only one exception to this pattern: For the quarterly panel in 2000, the sign of estimated α_1 is positive for the LPM, whereas it is negative for probit and logit estimates (see Table 11). Upon close inspection, we see that the magnitudes are tiny, and the sign change does not have substantive consequences. Thus our findings corroborate the conclusion in Hirano *et al.* (2001) that the AN model is robust to functional form choices. On the other hand, the sign patterns differ across panels. One can obtain different sign patterns for different panels since attrition may not be a stationary process. Since we study the labor market status of individuals as a determinant of the attrition process, quarterly and yearly economic shocks can affect the process. Indeed, Turkey experienced adverse shocks in the fourth quarter of 2000 and in the first quarter of 2001 which had consequences for the labor market (Tunalı, 2003). Arguably these economic shocks are responsible for the inconsistent sign patterns across panels.

One practical disadvantage of the LPM is that probability estimates can lie outside the interval $[0,1]$. We encountered this in one case among 36 possible cases: Table 21 reveals that the sum of the estimates for α_0 and α_1 exceeds one. Yet, this does not pose a serious problem since this sum is used along with the the probability of responding ($Pr(W=1)$) to inflate the joint probability coming from the balanced panel. Moving on, observe that the parameter estimates for the logit specification are larger than the probit versions. We expect the logit estimates to be approximately 1.81 times probit estimates.[10]. However, our results yield a factor closer two. This is because our estimated probabilities are near the tails of the distribution and logit has fatter tails than probt. As seen in Table 1, the AN model provides factors ($Pr(W=1)/g(.)$) which we can use to inflate or deflate the joint probabilities provided by the balanced panel. We see that these factors are consistent across parametric specifications. For example, considering the annual panel Q2: 2000-2001, the conditional probabilities of being employed and unemployed in the second period given being unemployed in the first period ($p_{0110|1}$ and $p_{0011|1}$) are inflated by around 1.12, whereas the probability of being non-participant in the second period given being employed in the first period ($p_{1000|1}$) is deflated by around 0.96.

Upon examination of the weighted joint probabilities of (Z_1, Z_2) implied by each specification, one can see that they are very close to each other. For example, we obtain almost identical matrices for the annual panel Q2: 2000-2001 (see Tables 42.A-C). For the other panels, the discrepancy in estimated probabilities is less than 0.002. When we compare these weighted matrices with the relevant balanced panel, they look very close to each other (see Table 43). Indeed, the estimated joint probabilities under the AN model for each specification are always within one standard error of the estimates coming from the balanced panels. Since the AN model is the most general model we can estimate and it gives almost the same estimates as the balanced panel, one can use the balanced panel to capture the aggregate picture of the labor market we study.

[10] Remember that the ratio of the slope estimates of the logistic distribution and the standard normal distribution is $\pi/\sqrt{3}$, i.e., approximately 1.81. The largest ratio observed in our estimates is 1.98 and the smallest is 1.74. Hence, one can say that the differences between the slope estimates across logit and probit estimates are mainly due to scaling.

As Hirano *et al.* (2001) establish, MAR and HW assumptions are nested in the AN model. Therefore, one can test these assumptions by testing the significance of the corresponding parameters estimates. Since our approach does not yield standard errors, we turn to bootstrapping techniques and provide the bootstrapped parameter estimates and their standard errors for the annual panel Q2: 2000-2001. Table 44 reports these statistics for each specification. As we explained in Section 3.4, we bootstrap the marginal distributions of Z_1 and Z_2 coming from the full panel and the refreshment sample, respectively. By imposing these marginals and keeping the joint probabilities coming from the balanced panel, we obtain 50 sets of parameter estimates for our system of equations. Our bootstrapped estimates are very close to the ones estimated from the original data. By using these standard errors, one can test the significance of parameters individually. Since t-ratios are always over 5, p-values are practically zero. Although MAR and HW assumptions imply joint restrictions ($\alpha_2 = \alpha_4 = 0$ and $\alpha_1 = \alpha_3 = 0$), given the individual magnitudes we feel confident rejecting both models. Thus the attrition probability is a function of the labor market status in both periods. In other words, attrition is not ignorable.

This finding might seem puzzling in light of our earlier statement that one can use the balanced panel to study the labor market. Yet these findings are not mutually exclusive. It is one thing to establish that attrition is not ignorable, and another to claim that the balanced panel serves particular purposes. In fact Fitzgerald *et al.* (1998) reach a similar conclusion in the case of the PSID: There is systematic attrition, yet it can be ignored in studying many outcomes of interest. Likewise in their painstaking evaluation of panel data collecetd in Africa, Alderman *et al.* (2001) conclude that many panel data sets suffer from severe attrition, but failure to account for it does not seem to affect the conclusions drawn from the balanced panel.

Furthermore, our vindication of the balanced panel was based on the examination of the joint distribution $f(Z_1, Z_2)$. In many applications, interest lies not in the joint distribution, but in the conditional distributions $f_2(Z_2|Z_1)$ or $f_1(Z_1|Z_2)$. Since the computation of these conditional distributions require knowledge of the appropriate marginals, additional considerations enter the picture. In Tables 45.A-B we report the estimates for these conditional distributions using the balanced panel. In Tables 46.A-B we report the estimates using the average of the logit and probit based estimates of the AN model (in Tables 42.B-C). Recall that the AN model exploits unbiased estimates of the respective marginal distributions $h_1(Z_1)$ and $h_2(Z_2)$ obtained in turn from the full sample, and the refreshment sample. In empirical work, this can usher in additional confidence.

This prudent approach does not appear to be warranted in the case of the annual panel Q2: 2000-2001. Pairwise inspection reveals that the differences between entries of Tables 45.B and 46.B are larger compared to those between Tables 45.A and 46.A. However, the magnitudes of the differences are still small. Chi-squared tests (with two degrees of freedom) show that the null hypotheses that the conditional distributions obtained from the balanced panel are the same as the unbiased versions cannot be rejected at conventional levels. Notably the smallest p-value ($= 0.1005$) is obtained for $f_2(Z_2|Z_1 = 2)$, which captures the annual transitions from unemployment to various states. The second smallest p-value ($= 0.1829$) is obtained for $f_1(Z_1|Z_2 = 1)$, which captures the transitions into employment from various states. Arguably these are important transiton rates from the perspective of labor market analysts. Given the simplicity of obtaining the estimates of the AN model, analysts might prefer to work with the AN model estimates anyway.

6 Conclusion

This paper discussed the main attrition models in the presence of a refreshment sample. As Hirano *et al.* (2001) showed, the presence of a refreshment sample permits one to estimate a richer attrition model (AN) such that the others (MAR and HW) are special cases of it. Beyond, one can test (the identifying assumptions of) MAR and HW by using the refreshment sample in two ways. One way is the non-parametric one: Since the refreshment sample provides an independent estimate of the marginal distribution for the period where attrition occurs, one can test whether the marginal distributions computed on the balanced panel under MAR and HW assumptions are the same as the unbiased one obtained from the refreshment sample. In this way, one can evaluate how well MAR and HW do in modelling attrition. The second way is the parametric one: Since both MAR or HW models are nested in the AN model, one can test the significance of the corresponding parameters to test each assumption in turn.

In the empirical part, we examined the suitability of the various attrition models to the Turkish HLFS. We used data from two quarters in 2000 and two quarters in 2001 to obtain two quarterly and two annual two-period panels. The distribution of interest in our application was $f(Z_1, Z_2)$, where Z_t is the three-way labor market status in period $t = 1, 2$. Both MAR and HW were rejected in our non-parametric tests. Since refreshment samples may not always be available, we also computed strict bounds on various probabilities of interest using Manski's worst-case approach. The ranges obtained from the bounds turned out to be quite wide, and underscored the importance of the identifying assumptions.

The AN model was estimated for three different specifications of the probability of attrition expressed as a function of Z_1 and Z_2: The linear probability model, probit and logit. The signs of parameter estimates were consistent across specifications. Further, the weighted joint distributions were almost identical. Therefore, our findings corroborate those in Hirano *et al.* (2001) that the AN model is robust to functional specification. Because the method we used for estimating the AN model parameters did not yield standard errors, we relied on bootstrapping. Both MAR and HW were rejected in our parametric tests as well.

We conclude that attrition in four panels exists and is not ignorable. Furthermore, neither MAR nor HW models are capable of capturing the labor market state-dependence of attrition. The more general AN specification emerges as the model of choice. Interestingly comparison of the weighted joint distribution(s) based on the AN model with the one obtained from the balanced panel revealed that the differences were negligible. This suggests that one could use the balanced panel for statistical inferences on our data sets. However, differences turned out to be larger in the case of transition probabilities (conditional distributions of $Z_2|Z_1$ and $Z_1|Z_2$). Given the computational simplicity of obtaining the AN model weights, prudent users are likely to be tempted to rely on it.

7 Appendix: Derivation of the HW restrictions

Consider the distribution of the triplet (Z_1, Z_2, W) which is assumed to have finite support and positive mass everywhere. In our application $Z_1 = z_1$ and $Z_2 = z_2$ where $z_1, z_2 = 0, 1, 2$; and $W = k$ where $k = 0, 1$. Alternate factorizations of the joint distribution may be used to obtain:

$$Pr(W|Z_1, Z_2) \cdot Pr(Z_1, Z_2) = Pr(Z_2|Z_1, W) \cdot Pr(Z_1, W). \tag{21}$$

According to the HW assumption, conditional on Z_2, W is independent of Z_1. Thus

$$Pr(W|Z_1, Z_2) = Pr(W|Z_2) = \frac{Pr(W, Z_2)}{Pr(Z_2)}. \tag{22}$$

Substituting eq(22) into eq(21), we obtain:

$$\frac{Pr(W, Z_2)}{Pr(Z_2)} \cdot Pr(Z_1, Z_2) = Pr(Z_2|Z_1, W) \cdot Pr(Z_1, W). \tag{23}$$

Note that the right handside is the product of q's and r's defined in the text. When we rewrite the equation above for $W = 0$ and $W = 1$ and take the ratio of these equalities, we obtain the restrictions in Section 3.1.

8 References

Alderman, H., Behrman, J.R., Kohler, H.P., Maluccio, J.A., Watkins, S.C. (2001) "Attrition in Longitudinal Household Survey Data." *Demographic Research*, 5:4, 80-123.

Becketti, S., Gould, W., Lillard, L., Welch, F. (1988) "The Panel Study of Income Dynamics after Fourteen Years: An Evaluation." *Journal of Labor Economics*, 6:4, 472-492.

Bureau of Labor Statistics, (2002) "Current Population Survey: Design and Methodology." Technical Paper 63RV, U.S. Department of Labor, Bureau of Labor Statistics.

Brownstone, D., Valletta, R.G. (1996) "Modeling Earnings Measurement Error: A Multiple Imputation Approach." *The Review of Economics and Statistics*, 70:4, 705-717.

DİE, (2001) *Hanehalkı İsgücü Anketi: Kavramlar ve Yöntemler [Household Labor Force Survey: Concepts and Methods]*. Devlet İstatistik Enstitüsü Matbaası: Ankara.

Fitzgerald, J., Gottschalk, P., Moffit, R. (1998) "An Analysis of Sample Attrition in Panel Data." *Journal of Human Resources*, 33:2, 251-259.

Hausman, J.A., Wise, D.A. (1979) "Attrition Bias in Experimental Panel Data: The Gray Income Maintenance Experiment." *Econometrica*, 47:2, 455-473.

Heckman, J. (1979) "Sample Selection Bias as a Specification Error." *Econometrica*, 47, 153-361.

Hirano, K., Imbens, G.W., Ridder, G., Rubin, D.B. (2001) "Combining Panel Data Sets with Attrition and Refreshment Samples." *Econometrica*, 69:2, 1645-1659.

Horowitz, J.L., Manski, C.F. (1998) "Censoring of Outcomes and Regressors Due to Survey Nonresponse: Identification and Estimation Using Weights and Imputations." *Journal of Econometrics*, 84, 37-58.

Horowitz, J.L., Manski, C.F. (2000) "Nonparametric Analysis of Randomized Experiments with Missing Covariate and Outcome Data." *Journal of the American Statistical Association*, 95:449, 77-84.

L. S. Lasdon, A. D. Warren, A. Jain, and M. Ratner. (1978) "Design and testing of a generalized reduced gradient code for nonlinear programming" *ACM Trans. Math. Software 4*, 34-50.

Lillard, L.A., Panis, W.A. (1998) "Panel Attrition from the Panel Study of Income Dynamics: Household Income, Marital Status, and Mortality." *Journal of Human Resources*, 33:2, 437-457.

Little, R., Rubin, D. (1987) *Statistical Analysis with Missing Data.* New York: Wiley.

Manski, C.F. (1989) "Anatomy of Selection Problem." *Journal of Human Resources*, 24:3, 343-360.

Nevo, A. (2003) "Using Weights to Adjust for Sample Selection When Auxiliary Information is Available." *Journal of Business and Economic Studies*, 21:1, 43-52.

Ridder, G. (1992) "An Empirical Evaluation of Some Models for Non-random Attrition in Panel Data." *Structural Change and Economic Dynamics*, 3, 337-355.

Tunalı, İ. (2003). "Background Study on Labor Market and Employment in Turkey." Final report/ June 27, 2003 prepared for the European Training Foundation, Ankara.

Tunalı, İ., Baltacı, R. (2004) "Attrition in the New HLFS, 2000-2002." Mimeo.

Tunalı, İ. (2007) "Analysis of Attrition Patterns in the Turkish Household Labor Force Survey, 2000-2002." Mimeo.

Van den Berg, G., Lindeboom, M. (1998) "Attrition in Panel Survey Data and the Estimation of Multi-State Labor Market Models." *Journal of Human Resources*, 33:2, 458-478.

Zabel, J.E. (1998) "An Analysis of Attrition in the Panel Study of Income Dynamics and the Survey of Income and Program Participation with an Application to a Model of Labor Market Behavior." *Journal of Human Resources*, 33:2, 479-506.