

Bankruptcy and Debt Portfolios^{*}

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Abstract

We use a heterogeneous-agent model, in which labor income is risky and markets are incomplete, to analyze consumer debt portfolios of secured and unsecured debt in the US. Compared with previous research, we emphasize the role of durables which not only generate utility but also serve as debt collateral. This allows a meaningful joint analysis of secured and unsecured debt and introduces endogenous bankruptcy costs: durables need to be sold to service secured debt in bankruptcy procedures which implies forgone durable utility since adjusting durables is costly. We solve the model numerically and apply it to understand bankruptcy and consumer debt portfolios in the US and their evolution over time.

Keywords: household debt, durables, collateral, income risk, bankruptcy, risk-sharing.

JEL: E21, D91.

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1 Introduction

Consumer debt has increased substantially in the last decades. In terms of disposable personal income, debt has risen from 60% to above 100% since the 1980s in the US (Dynan and Kohn, 2007) and a similar steep upward trend can be observed for the UK (Tudela and Young, 2005, and Waldron and Zampolli, 2007) and many other developed countries. These higher debt levels have been accompanied by substantial increases of unsecured credit-card debt and consumer bankruptcy filings (although starting from small levels). Unsecured debt as a fraction of disposable income in the US rose from 5 to 9% in the period 1983 to 1998 (Livshits et al., 2007a, Figure 3) at the same time as the bankruptcy incidence of US households increased from 0.2% to 1.5% where households defaulted on approximately \$120 billion or \$1,100 per household each recent year (White, 2006).

In this paper we use a model in which labor income is risky and markets are incomplete to analyze the determinants of the observed increase in debt and bankruptcy incidence in the US. Compared with previous research, we emphasize the role of durables which do not only generate utility but also serve as debt collateral. This allows a meaningful joint analysis of secured and unsecured debt and thus consumer debt portfolios. Moreover, durables introduce an endogenous bankruptcy costs: durables need to be sold to service secured debt in bankruptcy procedures which implies forgone durable utility because adjusting durables is costly.

Since two thirds of bankrupt consumers mention job related problems like wage cuts or unemployment, a natural framework for our analysis is a setting in which uninsurable income fluctuations are the main source of risk which consumers face in the economy. If markets are incomplete, consumers cannot fully insure this risk and differ from each other as they experience different histories of shocks. Micro-founded heterogeneous-agent models with these characteristics have been pioneered by Bewley (1986), Deaton (1991), Aiyagari (1994) and Carroll (1997) and have attracted substantial attention in recent years. For our purposes, such a model allows us to analyze how exogenous changes of the economic environment, which we will discuss in some detail below, alter the steady-state wealth distribution and more interestingly the composition of debt portfolios and bankruptcy filings.

Whereas the classic heterogeneous-agent models assumed that consumers borrow or save in terms of one risk-free asset, recent research by Athreya (2002), Chatterjee, Corbae, Nakajima and Ríos-Rull (2007) and Livshits, MacGee and Tertilt (2007b) has extended these models to unsecured debt. This research differs with respect to how the pricing of unsecured debt and length of the horizon of consumers is modelled (infinite or finite life-cycle)

but in all of the analyses bankruptcy occurs in equilibrium because debt contracts cannot be conditioned on the uncertain realization of labor earnings (see also the survey of Athreya, 2005).¹ Importantly, these models assume that consumers only have access to unsecured debt. In this paper we relax this assumption and allow for an endogenous debt composition: consumers can take on secured debt like mortgages, which are collateralized by durable holdings, *and* unsecured debt like credit-card debt. To the best of our knowledge only Athreya (2006) distinguishes secured and unsecured debt but does not model durable holdings explicitly.² In his model the collateral is exogenous whereas consumers in our model endogenously accumulate durable collateral which also generates utility. This modeling of durables is closest to Fernández-Villaverde and Krueger (2005) who, however, do not allow for equilibrium bankruptcy and unsecured debt (see also Kiyotaki, Michaelides and Nikolov, 2007).

Our analysis of the evolution of the debt portfolio is most closely related to the analyses of unsecured-debt trends by Athreya (2004), Livshits, MacGee and Tertilt (2007a) and Mateos-Planas (2007). The main contribution of our paper is that we explicitly model durables which allows us to distinguish secured and unsecured debt. The advantages of analyzing durables, secured and unsecured debt simultaneously are at least threefold. The first advantage is that the model has an additional margin of substitution in the debt portfolio, between secured and unsecured debt. That margin not only adds realism but also allows to distinguish between various explanations for the upward trend in unsecured debt and the bankruptcy incidence. The second advantage is more realism in a key aspect of the analysis: quantitatively, most of consumers' total debt holdings are secured debt. Credit-card debt instead accounts for at most 15% of total debt holdings in the Survey of Consumer Finances (SCF) 2004. Thus, a quantitative model of household debt needs to explain not only the evolution of the aggregate debt position but also the share of secured and unsecured debt. This is particularly important for the predictions of the model concerning consumer bankruptcy because only unsecured debt can be discharged in bankruptcy proceedings. The third advantage is that the explicit modeling of durables introduces an endogenous bankruptcy cost which has been neglected in previous research. Since some of the durable is seized to satisfy creditors' claims and adjusting the durables is costly, that cost depends on the size of the consumers' durable

¹If contracts can condition on the current endowment, no default occurs in equilibrium since the agent with a high earnings draw is made just indifferent between sharing risk and autarky. See Kehoe and Levine (2001) and their references.

²Pavan (2005) estimates a structural model of consumer bankruptcy with an endogenous durable stock but does not distinguish between secured and unsecured debt.

stock and secured debt.

In our quantitative application we find that the observed fall of lending *and* borrowing rates are key ingredients to match the observed upward trend in consumer debt, the stable share of secured debt and the slight fall of gross financial wealth for consumers that are between 20 and 55 years old. Our model has difficulties, however, to match the strong observed upward trend in the bankruptcy incidence.

The rest of this paper is structured as follows. In Section 2 we present empirical facts which are instructive for our analysis. In Section 3 we present the model, its numerical solution and calibration. In Section 4 we then apply the model to study the evolution of debt portfolios and bankruptcy filings in the US. We conclude in Section 5.

2 Empirical facts

In this section we summarize facts on the US consumer bankruptcy law, the wealth and debt composition and bankruptcy filings in the period 1983 to 2004.

2.1 US consumer bankruptcy law

Bankruptcy in the US is regulated by the Federal Bankruptcy Act of 1978, which contains two chapters relevant for non-farming households. Consumers can choose to file for personal bankruptcy under either Chapter 7 or under Chapter 13. We now summarize the main features of these two chapters which are relevant for our analysis. See Sullivan, Warren and Westbrook (1999) for further details.

Under chapter 7 of the bankruptcy act, the debtor can write off his unsecured debts (except alimony, child support, taxes, and student debts) but must surrender all his assets except for specified exempt amounts. Most of the bankruptcy exemptions are in terms of durable wealth, vary across US states and can be substantial: in Texas or Florida, for example, consumers can keep their house if they file for bankruptcy. Secured creditors' claims, however, have priority and override bankruptcy exemptions.

Under Chapter 13, the debtor agrees to a repayment schedule for part or all of the debt and retains his assets. The repayment plan usually is specified for three years but can take up to five years. Importantly, the debtor cannot repay less under chapter 13 than what creditors would get paid under chapter 7. Hence, we focus on chapter 7 in our model since it places a lower bound on

the unsecured-debt claims of the creditors.³ This is not a strong restriction since most consumers which file for bankruptcy do so under chapter 7 (70%) and many of the repayment plans initiated under chapter 13 fail and are later converted into chapter 7. If consumers file for bankruptcy under chapter 7, the procedure, on average, is completed in four month. Consumers then carry a bankruptcy flag in their records for up to ten years and are not allowed to file for bankruptcy again in the next six years.

In the period 1983 to 2004, which is of interest when we match the model to the data, there have been some changes to the legislation in 1984 to make filing for bankruptcy more restrictive. In practice, however, this has had little impact on the workings of the procedure (Sullivan et al., 2000). The only significant reform took effect after the period for which we have data when income testing was introduced at the end of 2005. Hence, the federal bankruptcy regulation has remained roughly unchanged in the period 1983-2004.

2.2 The evolution of debt portfolios and bankruptcy filings

We compare the asset and debt portfolios of US consumers in 1983 and 2004. We have chosen these two dates because they span the time period in which detailed comparable data on consumers' net worth is recorded in the triennial Survey of Consumer Finances (SCF). The SCF has been widely used as it provides the most accurate information on consumer finances in the US. The data collectors of the Federal Reserve System pay special attention in their sampling procedures to accurately capture the right tail of the very right-skewed wealth distribution (see Kennickell, 2003, and the references therein). Both years, 1983 and 2004, are after a trough in the US business cycle (1982 and 2001 according to the NBER definition) so that changes reflect long-term trends rather than cyclical variation.

³If housing wealth is illiquid and adjustment is costly, some debtors may prefer to file under chapter 13 as this allows them to keep their homes. This is especially relevant in US states in which these homes are not (fully) exempt under chapter-7 bankruptcy filings. We abstract from such complications in this paper.

<i>Variable</i>	<i>SCF 1983</i>	<i>SCF 2004</i>	<i>%-change</i>
	(1)	(2)	(3)
Total net worth (fraction of average net lab. earnings)	4.93	5.30	7.5
Non-financial wealth (fraction of average earnings)	4.55	5.37	18.02
Net-financial wealth (fraction of average earnings)	0.38	-0.07	-118.4
Total gross debt (fraction of average earnings)	-1.18	-2.03	72.03
	<i>%-point-change</i>		
Secured debt (in % of total debt)	84.43	84.41	-0.02
Credit-card debt (in % of total debt)	9.52	12.70	3.18
Payment difficulties (in % of sample size)	4.50	10.18	5.68
Bankrupt in previous year (in % of sample size)	-	1.22	-

Table 1: Wealth portfolios of households with a head between age 20 and 55, in 1983 and 2004, respectively. Source: Authors' calculation based on the SCF. Notes: Quantities are normalized by average net labor earnings in the respective sample year. Data on bankruptcy are not available in the SCF 1983.

We largely follow Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) and Díaz-Giménez, Quadrini and Ríos-Rull (1997) in constructing measures for wealth and labor earnings in the US. Net worth is defined as the sum of net financial assets and non-financial assets. We account for differences in household size using an equivalence scale as reported in Krueger and Fernández-Villaverde (2007), Table 1, last column. To make the empirical data comparable with the data generated by the model, we normalize all variables by average net labor earnings in our sample.⁴ More precisely, we use SCF data on gross labor earnings and the NBER tax simulator described in Feenberg and Coutts (1993) to construct a measure for disposable labor earnings after taxes and transfers for each household in 1983 and 2004.⁵ Arguably, after-tax rather than pre-tax earnings matter for households' consumption decisions since some of the uninsurable labor earnings risk may be eliminated by redistributive taxes and transfers. More detailed information on how we construct the data is contained in the data appendix.

We focus on households with heads between age 20 and 74, where, as in the model, we divide this age range into 15 three-year age intervals between age 20 and 65 and one last sixteenth interval between age 65 and 74. We compute sample averages for these age intervals which we then regress on a cubic polynomial of the age groups for 1983 and 2004, respectively. The resulting predictions allow us to construct smooth life-cycle profiles.

Table 1 displays the means of these life-cycle profiles until age 55 in 1983 and 2004.⁶ We calibrate the model to match these means because our model abstracts from death before age 74 and this is a good approximation for the data only up to a certain age. Allowing for a positive probability of death in all stages of the life cycle would unnecessarily complicate the modeling of bankruptcy further. We now discuss the means and trends of interest for consumers with age 20-55.

- The increase of gross debt and durable wealth and the fall in net-financial wealth.

Gross household debt increased by 72% between 1983 and 2004 for consumers between age 20 and 55, amounting to twice average net-labor earnings

⁴When computing the statistics in the data, we use the sampling weights provided in the SCF. The normalization by net labor earnings and the use of equivalence scales implies that normalized (aggregate) wealth is about twice the wealth to *output* ratio.

⁵We use the programs provided by Kevin Moore on <http://www.nber.org/~taxsim/> for constructing the SCF data in 1983 and 2004 which are fed into the tax simulator on the NBER website.

⁶These means are not weighed by age-cell size, since the population in the model has the same size over the life-cycle. Hence, we also assign the same weight to each age cell in the data.

in the 2004 sample. Since many households hold positive financial assets and debt at the same time, however, the relevant measure of financial debt is the aggregated net-financial asset position if financial assets and debt have approximately the same liquidity.⁷

As Table 1 shows, net financial assets (in terms of average net labor earnings in the sample) have fallen by more than 100% between 1983 and 2004, thus showing the same trend as gross debt, after aggregating financial assets and liabilities.⁸ Non-financial worth, of which a big component is housing, has increased by about 20% between 1983 and 2004. Since total net worth has increased by 7.5% and much of the debt is collateralized by durables, the increase in gross debt which has received substantial attention (see, for example, Iacoviello, forthcoming), has to be put in perspective.

- The rather stable debt composition over time.

We define secured debt as debt secured by land or housing, installment credit and other consumer loans which may be secured by purchased durables, such as cars or furniture, and exclude only credit-card and other financial debt in the SCF. Table 1 shows that secured debt accounts for 84% of total debt in 1983 and 2004, respectively, and thus does not exhibit a significant time trend. Unsecured credit-card debt as a fraction of total debt has increased by 3 percentage points between 1983 and 2004. As we will see below, this positive trend has been much more pronounced for younger age groups.

- The upward trend in payment difficulties and bankruptcy incidence.

The SCF 1983 does not contain direct information on consumer bankruptcy. However, consumers were asked in 1983 *and* 2004 whether they “had had a request for credit turned down by a particular lender or creditor in the past few years, or had been unable to get as much credit as [they] had applied for.” Moreover, they were asked whether they “had not applied for credit because [they] thought [they] would be turned down. [They were] asked for what reasons [they] thought [they] would be turned down on the most recent occasion when this occurred.” We classify households as having payment difficulties if they answer to either of these two questions that they

⁷The main findings are robust if we aggregate only *liquid* assets and total debt, where liquid assets are money-market, savings and checking accounts, individual retirement accounts, certificates of deposit, thrift accounts and saving bonds.

⁸For consumers up to age 74 average net-financial wealth has increased by 20%, however, showing an opposite trend. This can be seen in the top-right panel of Figure 1 below, in which the life-cycle profiles of net-financial wealth in 1983 and 2004 cross at a certain age.

were turned down because of “credit records/history from other institutions; other loans or charge accounts; previous payment records or bankruptcy.” Of course, this measure of payment difficulties is far from perfect but it allows us to look at time trends in payment difficulties in the SCF since there is no information about bankruptcy filings in the SCF 1983. Interestingly, the trend of this measure of payment difficulties is similar to measures of Sullivan et al. (1999, 2000) who used administrative data on bankruptcy filings in 10 judicial districts in 1981 and 16 districts in 1991.⁹

We find that, according to our measure, 4% of households had payment difficulties in 1983 and until 2004 this figure more than doubled to 10%. These percentages are significantly larger than the 1.2% of households in 2004 who reported that they filed for bankruptcy in the last year (this information is not available for 1983).¹⁰ Complementing this with evidence of Sullivan et al. (2000) that about 0.2% of US consumers filed for bankruptcy in the beginning of the 1980s and that this number increased to 0.5% beginning of the 1990s, this shows a significant upward trend in bankruptcy filings and payment difficulties in the US between 1983 and 2004 (see also White, 2006, and references therein).

- Further facts on bankrupts from the literature.

The broad picture of bankrupt consumers which emerges from Sullivan et al. (1999, 2000) is similar to what we find in SCF data for those consumers with payment difficulties. Compared with the population, Sullivan et al. (1999, 2000) report that bankrupt consumers are earnings poor, have less assets than the average population but a significant amount, are quite often homeowners (more than half of bankrupt consumers own a home), have negative net worth on average whereas the population holds a positive amount.

⁹Sullivan et al. (1999, 2000) accessed debtors’ files at bankruptcy courts to collect data on the financial and demographic situation of bankrupt US consumers and compare them with the average US population. Their data are a snapshot of the bankrupt consumers in the early 1980s and 1990s and thus not representative of the population but have the advantage to be highly accurate as the consumers faced the threat of being sued for perjury if not reporting accurately. In 1981 they analyzed 1,529 bankruptcy cases in ten different judicial districts, of which three were in Pennsylvania, three in Illinois and four in Texas. In the second study in 1991 they analyzed 2,400 cases adding four districts in California and two in Tennessee.

¹⁰The figures for 2004 are also similar to those reported in Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) for the SCF 1998. They classified households as having financial trouble if they delayed payments for more than 2 month (this was true for 6% of the households). Moreover, they report that 1.8% of the whole sample had filed for bankruptcy.

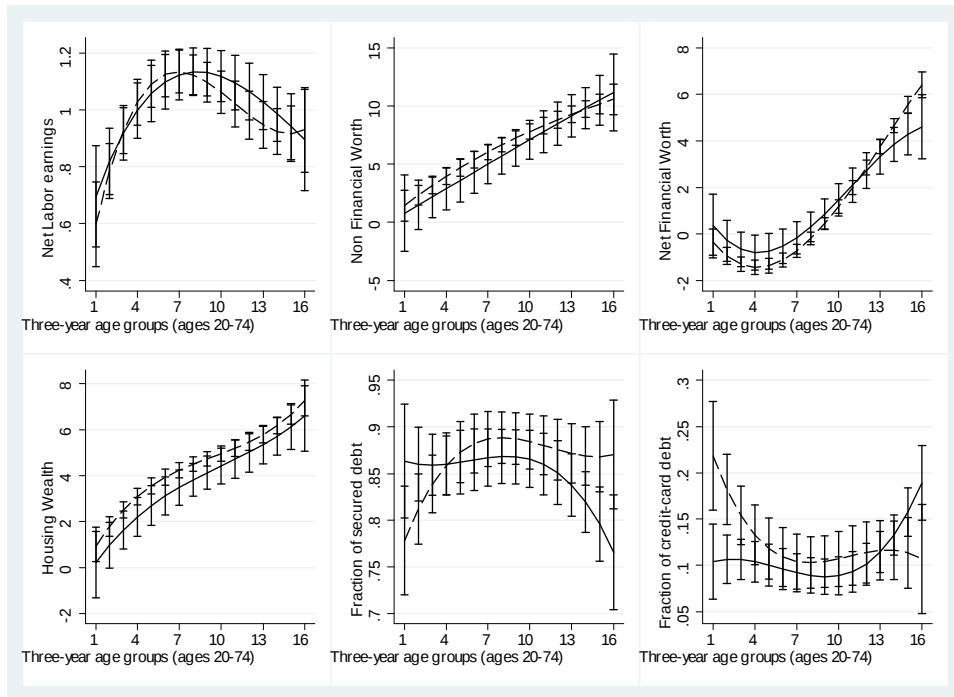


Figure 1: The wealth portfolio of consumers over the life cycle in 1983 and 2004. Source: Authors' calculations based on the SCF. Notes: Solid line: 1983 data; dashed line: 2004 data; 95% confidence intervals displayed for each data point in 1983 and 2004.

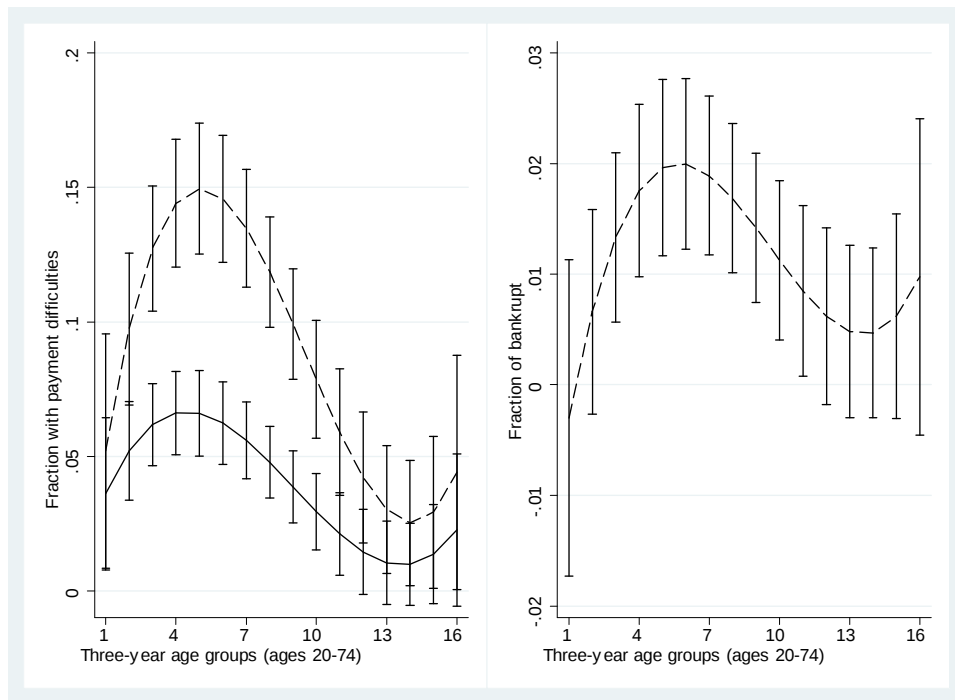


Figure 2: Payment difficulties and bankruptcy incidence over the life cycle in 1983 and 2004. Source: Authors' calculations based on the SCF. Notes: Solid line: 1983 data; dashed line: 2004 data; 95% confidence intervals displayed for each data point in 1983 and 2004. Data for bankruptcy are not available for 1983.

The mean debt-income ratio of bankrupt consumers is about 3 and the median debt-income ratio of bankrupt consumers has increased from 1.4 in 1981 to 2.4 in 1997.

- The life-cycle dimension.

Figures 1 and 2 show how labor earnings, the wealth portfolio and bankruptcy incidence vary over the life-cycle. Each graph plots the predictions from a regression of the sample averages (of each of the 16 age groups in 1983 and 2004, respectively) on a cubic polynomial of the age groups. We display the 95% confidence bounds for each predicted point.

Figure 1 shows that labor earnings have the well-known hump shape over the life cycle where these earnings peak between age 40 and 50 at about 20% higher labor earnings than the average in the sample. We find that the earnings profile has not changed significantly over time. Concerning the wealth portfolio, Figure 1 shows that young consumers start their life with very little wealth, if any. They first borrow to accumulate durables, of which a substantial part is housing, and their net-financial asset position is negative. After age 40, average net-financial wealth starts to be positive (see Fernández-Villaverde and Krueger, 2005, for documenting similar patterns in the SCF 1995). These patterns have remained remarkably stable in the time period 1983-2004, although there is some indication that young consumers incur more debt to purchase durables in 2004: note that the dashed lines for non-financial wealth or housing of young consumers in 2004 are above the solid lines for 1983, whereas the opposite is true for net-financial assets. Finally, the age-profile of the debt portfolio has remained rather stable over time, although young consumers hold much more credit card debt in 2004 than they used to do in 1983: credit-card debt of 20-year old consumers accounted for 20% of their total debt in 2004, an increase of 10 percentage points compared with 1983.

How are these patterns in the wealth and debt portfolio associated with the incidence of bankruptcy? Figure 2 shows that bankruptcy incidence and our measure for payment difficulties have a very similar hump-shape over the life cycle, consistent with the evidence on bankrupts reported in Sullivan et al. (2000), Figure 2.1. Payment difficulties and bankruptcy incidence peak if consumers reach their late 30s and the increase in payment difficulties between 1983 and 2004 (thus possibly also bankruptcy incidence) has been most significant for consumers between age 30 and 50.

2.3 Reasons for bankruptcy.

Previous research has identified the following reasons for bankruptcy (see Sullivan et al., 1999, 2000):

Two thirds of the bankrupt consumers mention job related problems like wage cuts, unemployment, or lower earnings due to health problems. Another fifth of these consumers mentions health problems which are 75% of the time related to job problems. Further reasons for bankruptcy include divorce or the motive to “save” housing property by writing off unsecured debt. Under chapter 7 this is a possibility in states where housing wealth is exempt in bankruptcy procedures.

In this paper we focus on earnings uncertainty (which may be related to health shocks) and the motive to keep the durable. We abstract from medical expense shocks to contain the computational burden given that we consider durable wealth in our model (see Chatterjee et al, 2007, or Livshits et al., 2007b, for models with health expense shocks). After laying out the empirical facts, we now construct a model which we then calibrate to match these facts.

3 The model

There is a continuum of consumers who have access to risk-free secured debt $a^s \leq 0$, which is backed by collateral and bears an interest rate r^s , and risk-free financial assets $a^u \geq 0$ which earn interest r^a . There is a borrowing spread ($r^s > r^a$) due to a fixed cost of financial intermediation. Consumers can also borrow unsecured debt $a^u < 0$. This debt does not need to be backed by collateral so that agents possibly default on that debt depending on their income draw. Creditors of secured debt have priority for the payment of their debt principal and interest and creditors of unsecured debt do not receive full payment if consumers file for bankruptcy. This risk of unsecured debt is priced actuarially fairly by a risk-neutral intermediary which perfectly diversifies the idiosyncratic risk applying the law of large numbers. We will derive the price of unsecured debt below.

Demographics. As in the overlapping-generations model of Livshits et al. (2007b), we assume that households live for 18 periods, where each period j has a length of three years. Life begins at age 20 and the first 15 periods (until age 65) are working periods in which people receive income shocks, while households are in retirement in the last three periods and face no uncertainty. Life ends at age 74. Each generation has a size of measure 1.

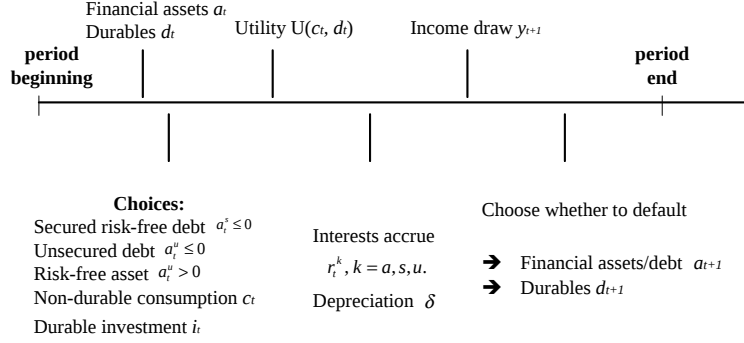


Figure 3: Timing in the model

Timing. Figure 3 illustrates the time line. Given net financial assets a_t and durable stock d_t at the beginning of the period, households determine their planned durable stock $q^d d_{t+1}^*$, non-durable consumption c_t and their financial asset portfolio $(q_t^s a_{t+1}^{*s}, q_t^k a_{t+1}^{*u})$. We assume that the adjustment of durables is costly, as discussed in more detail below. We denote the choices by asterisks since they may differ from the realized values due to the bankruptcy procedure. The prices are $q_t^s = (1 + r_t^s)^{-1}$, $q_t^k = (1 + r_t^k)^{-1}$ and $r_t^k = r_t^a$ if $a_{t+1}^{*u} > 0$ and $r_t^k = r_t^u$ otherwise, and $q^d = (1 - \delta)^{-1}$. Writing the choices above in a discounted way allows us to express wealth in the next period without interest factor which turns out to be convenient when solving the model numerically. The risk-free interest rates r_t^s and r_t^a are taken as given (as in a small-open economy) whereas the risk-premium and thus the interest rate for unsecured debt, r_t^u , is determined endogenously.

After the consumption and portfolio decisions, the consumers enjoy utility before the interest for the financial assets accrues and the durable depreciates. Then uncertain income is drawn before agents decide whether to declare bankruptcy. This determines the net financial assets and durables available for consumers tomorrow. Note that the timing implies that the targeted durable stock d_{t+1}^* is predetermined after the investment decision i_t .¹¹ Of

¹¹The timing for durables is similar to Fernández-Villaverde and Krueger (2005). Note that durables are a state variable for two reasons in our model: (i) the durable stock is

course, the realized durable stock in the next period d_{t+1} depends on the bankruptcy decision and thus is uncertain. Importantly, the filing decision matters for the wealth composition, since consumers' only wealth in the period after bankruptcy are durables. We now are more specific about the adjustment costs, the collateral constraint and the evolution of wealth over time depending on whether the consumer files for bankruptcy or not.

Adjustment costs. Households face costs when adjusting their durable stock. This assumption generates realistic lumpy investment patterns for durables. Moreover, it makes the distinction between durables and non-durables in our model more meaningful as adjustment costs are one key difference between these two types of goods. Since the most important component of durables in our model is housing, the costs can be thought of as moving costs or fees for real estate agents which are typically proportional to the value of the house. Hence, we specify the costs as

$$\phi(d_t, q^d i_t) = \begin{cases} c_f^+ d_t + c_p^+ q^d i_t & \text{if } i_t > 0 \\ c_f^- d_t - c_p^- q^d i_t & \text{if } i_t < 0 \\ 0 & \text{if } i_t = 0 \end{cases}.$$

This flexible specification allows for a fixed component $c_f d_t$ and a variable component $c_p i_t$ (see, for example, Caballero, 1999). Both components are possibly asymmetric, depending on the direction of the adjustment. The fixed component is expressed in terms of the durable stock to ensure that this component remains relevant as d increases. This component generates lumpy adjustment for which there exists overwhelming empirical evidence. The variable component $c_p i_t$ depends on the investment flow instead. If $c_p > 0$, it may be optimal that households adjust, but do not adjust fully to the durable stock that would be optimal if $c_p = 0$.¹²

predetermined and preferences in durables and non-durables are non-separable and (ii) consumers face costs for adjusting the durable. Both assumptions imply that there is an endogenous bankruptcy cost: the utility derived from durables is smaller in the bankruptcy period since at least some durables need to be sold to service debt. This is why durables also matter for the pricing of unsecured debt below.

¹²Note that there could be a role for a rental market of durables in our model due to adjustment costs. To simplify matters, we assume that adjustment costs for owners and renters are the same. For example, at least some components of moving costs do not depend on whether consumers are renters or owners. Then, renting durables is (weakly) dominated by owning them since ownership also provides collateral.

Collateral constraint. The collateral constraint¹³ allows consumers to hold secured debt of at most

$$a_{t+1}^{*s} \geq -\max\{d_{t+1}^* - \phi(d_{t+1}^*, a_{t+1}^{*s}), 0\} . \quad (1)$$

Note that consumers can only use the value of the durable net of the adjustment cost as collateral for secured debt where the relevant adjustment cost is evaluated at d_{t+1}^* and $i_t = a_{t+1}^{*s} < 0$.¹⁴ In case of bankruptcy, the relevant durable stock is d_{t+1}^* of which the amount $|a_{t+1}^{*s}|$ needs to be divested to service secured debt. The maximum operator ensures that for very high adjustment costs $a_{t+1}^{*s} = 0$: consumers then have no access to secured debt since the high adjustment costs make durables useless as collateral.

Using the explicit functional form for adjustment costs introduced above, the collateral constraint becomes

$$a_{t+1}^{*s} \geq -\max\{d_{t+1}^* - c_f^- d_{t+1}^* + c_p^- a_{t+1}^{*s}, 0\}.$$

For the interesting case in which $d_{t+1}^* - c_f^- d_{t+1}^* + c_p^- a_{t+1}^{*s} > 0$, this can be simplified to

$$a_{t+1}^{*s} \geq -\frac{1 - c_f^-}{1 + c_p^-} d_{t+1}^* .$$

Both the fixed and proportional component of adjustment costs tighten the collateral constraint. Note that compared with the literature which sometimes assumes that only a certain fraction of the durable can be used as collateral for secured debt, this is determined endogenously by the parameters of the adjustment cost function in our model.

Budget constraint. The consumer's budget constraint is

$$q_t^s a_{t+1}^{*s} + q_t^j a_{t+1}^{*u} + c_t + \underbrace{q^d d_{t+1}^* - d_t}_{q^d i_t} + \phi(d_t, q^d i_t) \leq a_t + y_t , \quad j = a, u . \quad (2)$$

Evolution of assets.

¹³We assume that consumers cannot collateralize part of their labor income. Otherwise, consumers would be able to use the lower bound of the support of possible labor income realizations as collateral. In practice, this lower bound is very close to zero (expressed in terms of average labor earnings in the population). Moreover, federal law allows garnishment of at most 25% of wage income (and thus also of that lower bound) and this only if the consumer has not filed for bankruptcy. Hence, we abstract from labor income as collateral for simplicity.

¹⁴If the consumer borrows, there are no other financial assets which could be used for paying the adjustment cost and income cannot be collateralized (see the previous footnote).

Repayment. If the consumer decides to repay all his debt obligations we denote the variables with an asterisk. The evolution of durable and financial wealth is then¹⁵

$$d_{t+1}^* = (1 - \delta)d_t + i_t .$$

and

$$a_{t+1}^* = a_{t+1}^{*s} + a_{t+1}^{*u} .$$

The only non-standard feature is that the interest for financial wealth is included in a_{t+1}^{*s} and a_{t+1}^{*u} since we define the choice at the beginning of the period in terms of values discounted by the interest factor. This is done for convenience when solving the model numerically.

Bankruptcy. Under chapter 7 of the US bankruptcy law, bankruptcy filers can keep durable wealth up to a specified exempt level. The most important exemption in many US states is the homestead exemption on which we focus in our model.¹⁶ The exempt level of durable wealth $d^\dagger$ is protected from *unsecured* debt claims. *Secured* debt instead has priority and needs to be paid irrespective of whether the durable wealth falls below the exempt level or not.

At the time of bankruptcy filing the consumer is obliged by law to reveal his financial and income status to the bankruptcy judge. In particular, the judge knows the composition of financial debt, $a_{t+1}^{*s} < 0$ and $a_{t+1}^{*u} < 0$,¹⁷ durable wealth d_{t+1}^* (net of depreciation) and the exemption level $d^\dagger$, and the current income draw y_{t+1} . The judge then first services the secured debt a_{t+1}^{*s} , selling as much of the durable stock as needed. Since the collateral constraint allows consumers to hold secured debt of at most $a_{t+1}^{*s} \geq -\max\{d_{t+1}^* - \phi(d_{t+1}^*, a_{t+1}^{*s} (1 + c_p^-) / (1 - c_f^-), 0\}$, the remaining durable stock may be positive and is denoted by

$$d_{t+1}^+ = \max \left\{ d_{t+1}^* - \phi \left(d_{t+1}^*, \frac{1 + c_p^-}{1 - c_f^-} a_{t+1}^{*s} \right), 0 \right\} + a_{t+1}^{*s} ,$$

¹⁵This follows from $q^d d_{t+1}^* = d_t + q^d i_t$ after accounting for depreciation by multiplying with $(1 - \delta)$.

¹⁶Many states also allow small additional exemptions for other durables like cars, jewelry, household goods or tools of trade. One can interpret the durable d as a composite of these durables and $d^\dagger$ as the corresponding total exemption across all these goods. When we turn to the calibration of the model we will focus on housing and the respective median exemption across US states. See also Athreya (2006).

¹⁷The consumer will never file for bankruptcy if $a_{t+1}^{*u} \geq 0$. In this case there is no benefit as no debt obligations are discharged but there is a strictly positive cost.

where $a_{t+1}^{*s} \leq 0$. Note that $d_{t+1}^+ \leq d^\dagger$, by definition, that is the remaining durable collateral is less than the exempt level. Otherwise the remaining durables which can be used as collateral, and are above the exemption, would guarantee repayment with certainty.¹⁸

The bankruptcy judge then continues to service the outstanding unsecured debt using a fraction $\gamma \geq 0$ of labor earnings y_{t+1} . The parameter γ shall capture the general expectation of “good faith” in bankruptcy procedures so that consumers typically sacrifice some of the labor earnings when filing for bankruptcy (see Livshits et al., 2007b). Thus, at the end of the procedure the remaining labor earnings in the next period are

$$y_{t+1}^+ = (1 - \gamma)y_{t+1} + \max\{\gamma y_{t+1} + a_{t+1}^{*u}, 0\} ,$$

and the judge sets $a_{t+1}^u = 0$. Note that $a_{t+1}^s = 0$ by definition since that debt is secured.

For convenience we summarize the evolution of the assets which depend on the bankruptcy decision. Durable wealth is given by

$$d_{t+1} = \begin{cases} d_{t+1}^* \equiv (1 - \delta)d_t + i_t & \text{if do not file} \\ d_{t+1}^+ \equiv \max\left\{d_{t+1}^* - \phi\left(d_{t+1}^*, \frac{1+c_p^-}{1-c_f} a_{t+1}^{*s}\right), 0\right\} + a_{t+1}^{*s} & \text{if file} \end{cases} . \quad (3)$$

If the consumer files, we know that $a_{t+1}^{*u} < 0$ and $d_{t+1}^+ \leq d^\dagger$.

Financial assets evolve according to

$$a_{t+1} = \begin{cases} a_{t+1}^* \equiv a_{t+1}^{*s} + a_{t+1}^{*u} & \text{if do not file} \\ 0 & \text{if file} \end{cases} . \quad (4)$$

The labor earnings are given by

$$y_{t+1} = \begin{cases} y_{t+1}^* \equiv y_{t+1} & \text{if do not file} \\ y_{t+1}^+ \equiv (1 - \gamma)y_{t+1} + \max\{\gamma y_{t+1} + a_{t+1}^{*u}, 0\} & \text{if file} \end{cases} . \quad (5)$$

Preferences. We assume that preferences are a non-separable function of non-durable and durable consumption $U(c, d)$. For the quantitative application of the model we assume a CRRA utility function with risk aversion σ ,

$$U(c, d) = \frac{\Psi(c, d)^{1-\sigma} - 1}{1 - \sigma},$$

¹⁸Hence, we conjecture that consumers hold no unsecured debt unless the collateral constraint is binding. For the intuition, suppose that the constraint does not hold with equality. Since financial intermediaries know the consumers portfolio, any additional unit of debt would be secured *de facto* by the remaining durable stock (unless that falls below the exemption level $d^\dagger$) and the consumer would be forced to sell the durable, incurring the adjustment costs.

where the consumption basket Ψ denotes a CES aggregator

$$\Psi(c, d) = [\xi c^\alpha + (1 - \xi)(d + \underline{d})^\alpha]^\frac{1}{\alpha}.$$

The constant $\underline{d} > 0$ is assumed small and positive so that some consumers do not hold durables, as is observed in the data.¹⁹ The static elasticity of substitution between c and d is $1/(1 - \alpha)$. Given the rather mixed results for the estimates of that elasticity in the literature, we follow Fernández-Villaverde and Krueger (2005) and use the Cobb-Douglas case as benchmark, i.e., $\alpha = 0$.

The pricing of unsecured debt. The interest rates of risk-free assets r^a and secured debt r^s are taken as given, as in a small open economy. We allow for a borrowing spread $\tau \geq 0$ due to transaction costs so that $r^s = r^a + \tau$. The price of unsecured debt instead is determined by perfectly competitive financial intermediaries which observe the current income state s and thus current labor earnings $y_t(s)$. Moreover, they know the portfolio choice $\nu_t \equiv (a_{t+1}^{*u}, a_{t+1}^{*s}, d_{t+1}^*)$, which is determined in period t , and the age of the consumer where we denote age with the index $j = 1, \dots, J$. The intermediaries are not able, however, to condition on the draw of future income y_{t+1} . Hence, some consumers file for bankruptcy ex post although they were expected to repay ex ante with strictly positive probability. The intermediaries take this into account when they price the debt and assess the repayment probability forming an expectation about future income conditional on the current income state, age and the consumer's portfolio ν_t . Thus, the price of unsecured debt $q_j^u(\nu_t, y_t(s_t))$. There is no cross-subsidization across consumers and consumers with different portfolios, age or income state receive a different interest quote.

Define the probability of default as $\pi_j(\nu_t, s_t)$ and the probability of income state s_{t+1} conditional on the current state s_t as $\theta(s_{t+1}|s_t)$.²⁰ Then, the zero-profit condition implies that the price for unsecured debt ($a_{t+1}^{*u} < 0$) is given by

¹⁹The assumption also ensures that utility is well defined at the beginning of life when consumers have no durables.

²⁰More precisely, the probability of default also depends on d^+ and d^* since the difference between these two values matters for the cost of bankruptcy in terms of forgone durable utility. Hence, the parameters of the adjustment cost function also matter for the probability to default.

$$\begin{aligned}
q_j^u(\nu_t, s_t) = & \quad (6) \\
& (1 - \pi_j(\nu_t, s_t)) q^s \\
& + \pi_j(\nu_t, s_t) q^s \Sigma_{s_{t+1}} \theta(s_{t+1}|s_t) \min \left\{ 1, \frac{\gamma y_{t+1}(s_{t+1})}{|a_{t+1}^{*u}|} \right\}.
\end{aligned}$$

The minimum operator does compare the amount of full repayment of one unit of outstanding unsecured debt with the resources that unsecured creditors receive from the labor earnings $\gamma y_{t+1}(s)$ as a fraction of the total outstanding unsecured debt (transformed into a positive number). If the probability of bankruptcy $\pi(\nu_t, s_t, \alpha) = 0$ or no unsecured debt is discharged if consumers file,²¹ then there is no risk premium on unsecured debt: $q_j^u(\nu_t, s_t) = q^s$.²²

The program. Substituting the budget constraint (2) in the Bellman equation of the consumer at stage j of the life cycle, we get

$$\begin{aligned}
V_j(a_t, d_t, y_t) = & \quad (7) \\
\max_{a_{t+1}^{*s}, a_{t+1}^{*u}, d_{t+1}^*} [& \underbrace{U(a_t + y_t + d_t - q_t^s a_{t+1}^{*s} - q_t^j a_{t+1}^{*u} - q^d d_{t+1}^* - \phi(d_t, q^d d_{t+1}^* - d_t), d_t)}_{c_t} \\
& + \beta E_{\nu, y} \max[V_{j+1}(a_{t+1}^*, d_{t+1}^*, y_{t+1}), V_{j+1}^B(a_{t+1}^+, d_{t+1}^+, y_{t+1}^+) - \psi]].
\end{aligned}$$

Note that there are three costs of bankruptcy in the consumer problem (7):

- an endogenous cost due to the smaller durable stock and the wasted adjustment cost: for a positive amount of secured debt, the durable stock that can be kept under bankruptcy, d_{t+1}^+ , is (weakly) smaller than the target level, d_{t+1}^* . Consumers have to bear adjustment costs and a utility cost for at least one period since the durable stock is predetermined and cannot be immediately adjusted. This is a realistic feature of the model since bankruptcy procedures last 4 month on average with a substantial variation around that mean duration (Sullivan et al., 2000).

²¹In this case $\Sigma_{s_{t+1}} \theta(s_{t+1}|s_t) \min \{1, \gamma y_{t+1}(s_{t+1}) / |a_{t+1}^{*u}|\} = 1$.

²²In the calibration below we allow the exogenous intermediation costs to be higher for unsecured than secured debt. This implies that we replace q^s by $1/(1+\underline{r}^u)$, where $\underline{r}^u$ is the base rate for unsecured debt without the risk premium.

- a cost of not being able to declare bankruptcy in the period following the bankruptcy filing. This captures the US bankruptcy code which forbids consumers to file for bankruptcy in the six years after a bankruptcy filing under chapter 7. Since a period has a length of three years in our model, no bankruptcy can be declared in the following period. This implies that we have to define a different value function V_j^B for that period where

$$V_j^B(a_t, d_t, y_t) = \max_{a_{t+1}^{*s}, a_{t+1}^{*u}, d_{t+1}^*} [U(c(a_{t+1}^{*s}, a_{t+1}^{*u} \geq 0, d_{t+1}^*), d_t) + \beta E_y V_{j+1}(a_{t+1}^*, d_{t+1}^*, y_{t+1})] . \quad (8)$$

For consistency, we assume that households do not have access to unsecured debt in that period.

- an exogenous cost of bankruptcy ψ which can be interpreted as psychological pain or stigma (see Athreya, 2006). We set $\psi = 0$ in our benchmark calibration.

The Bellman equations (7) and (8) together with the equations (3), (4), (5) for the evolution of assets and earnings and the constraints (1), (2), $d_{t+1}^* \geq 0$ and $a_{t+1}^{*s} \leq 0$ complete the set-up of the program.

Equilibrium definition. Dropping time indices and denoting the next period with a prime, a steady state is characterized by the policy functions for non-durable consumption $c_j(a, d, y)$, durable investment $i_j(a, d, y)$, the accumulation equations $a'_j(a, d, y)$, $d'_j(a, d, y)$ so that for given prices $\{r^a, r^s\}$ of risk-free assets and secured debt:

- (i) the envelope of value functions $V_j(a, d, y)$ and $V_j^B(a, d, y)$ attains its maximal value.
- (ii) the price for unsecured debt $q_j^u(\nu, s)$ satisfies the arbitrage equation (6).
- (iii) the distribution measure over the state space $A \times D \times Y$ of agents $\mu(A, D, Y)$ is stationary.

3.1 Calibration and numerical results

The discrete nature of the bankruptcy decision and the presence of adjustment costs imply that we cannot use numerical algorithms which rely on the differentiability of the value function and the first-order conditions to solve the model. Thus, we discretize the state space of the two endogenous

state variables a and d and the exogenous state variable y . We choose 554 gridpoints for $a \in [-3; 7]$ and 30 gridpoints for $d \in [0; 10]$ where the grid is finer for values of $d < 0.75$, with distances of 0.08 to 0.10, and includes the exemption value 0.25 which we calibrate below. For $d > 0.75$ the distance between the grid points is 0.5. Finally, we allow for 5 Markov states of the stochastic component of labor earnings y . For this specification of the grids the model is solved in 2,5 days on a PC of the current computing vintage.

3.1.1 Numerical algorithm

We start with the last period J . In that period the consumer sells all assets to consume them before death. We then compute the available resources with and without filing for bankruptcy on the state space $A \times D \times Y$. This allows us to compute the value functions V_{J-1} and V_{J-1}^B and the set of choices and future income states which imply that consumers declare bankruptcy, $V_{J-1}^B - \psi > V_{J-1}$. We then calculate the price of unsecured debt conditional on the current income state, debt portfolio and durable stock. We proceed to solve the maximization problem of the consumer to determine the optimal debt portfolio and durable investment and continue with analogous computations for the previous period $J - 2$ and so on until the beginning of life. We then use the value and policy functions to simulate the model for a population of 10,000 consumers, so that we can compute model statistics which can be matched to the data.

3.1.2 Calibration

We now discuss the calibration of the income process and other parameters.

The income process. We calibrate the life-cycle income profile similar to Livshits et al. (2007b). Labor earnings are determined by

$$y_j = \phi_j \lambda_j,$$

where ϕ_j is the stochastic productivity of the household in period j of the life cycle and λ_j is the deterministic labor endowment which is hump-shaped over the life cycle. We calibrate the deterministic component using the income means by age group reported in Figure 1. Concerning the stochastic component, we assume that it follows a five-state Markov chain. For calibrating the stochastic component we purge net labor earnings of life-cycle effects focussing on households with a head between 20 and 55 years of age. For this sample we regress net labor earnings on an age polynomial and compute

the quintile means of the residual distribution around the mean income in the SCF 1983. This results in

$$y_{1983} = [0.265, 0.593, 0.851, 1.186, 2.104].$$

We approximate the distribution as log normal $\log y_{1983} \sim \mathcal{N}(-0.0947, 0.1894)$ with a normalized mean of 1. This variance is about 50% of the raw cross-sectional variance which is roughly in line with results of Cunha and Heckman (2007) about the unforecastable fraction of the variance of labor earnings. We then assume an AR(1) process with first-order correlation of 0.95 and use Tauchen (1986)'s method to compute the transition matrix for the triennial periods as

$$\Gamma_{1983} = \begin{bmatrix} 0.8396 & 0.1602 & 0.0002 & 0 & 0 \\ 0.0215 & 0.6784 & 0.2793 & 0.0208 & 0 \\ 0.0004 & 0.2035 & 0.5495 & 0.2427 & 0.0039 \\ 0 & 0.0192 & 0.2694 & 0.6337 & 0.0777 \\ 0 & 0 & 0.0034 & 0.2297 & 0.7669 \end{bmatrix}.$$

The productivity of households in the first period of life is drawn from the stationary distribution

$$\pi_{1983} = [0.0332, 0.2409, 0.3268, 0.2953, 0.1038].$$

Although the Markov chain with five states approximates the log-normally distributed AR(1) process very well, we implement a bias correction which ensures that the discrete Markov chain implies *exactly* the same mean and variance.²³

Benchmark parameters. Table 2 displays the parameter values which we use for our numerical solution. We assume that the aggregator of durable and non-durable consumption goods is Cobb-Douglas ($\alpha = 0$) which is within the range of existing estimates. We then set $\xi = 0.79$ which implies that the expenditure for non-durable consumption is 7 times as high as the expenditure for durables, for consumers with an age between 20 and 55. This is only slightly above the long-run average 6.2 for the US (Fernández-Villaverde and Krueger, 2005). The value for risk aversion, $\sigma = 2$, and the discount factor, $\beta = 0.956$, are within the range of commonly assumed values. We set $\underline{d} = 0.01$, a small and quantitatively negligible value, which allows that some

²³The idea is to choose the standard deviation which we use to compute the transition matrix so that the implied standard deviation of the Markov chain is exactly equal to the one in the data.

<i>Parameters</i>			
<i>Preferences</i>	σ	2	
	α	0	
	β	0.8737	annual: 0.956
	ξ	0.79	
	$\underline{d}$	0.01	
<i>Technology</i>	δ	0.06	annual: 0.02
	c_f^+, c_f^-	0.05	
	c_p^+, c_p^-	0.1	
<i>Bankruptcy</i>	$d^\dagger$	0.25	
	γ	0	
	ψ	0	
<i>Interest rates</i>	r^a	0.0927	annual: 0.03
	r^s	0.2423	annual: 0.075
	$\underline{r^u}$	0.6640	annual: 0.185

Table 2: Benchmark parameters for the numerical solution

consumers do not hold durables. Finally, we adjust the utility of households for changes in their composition using an age-dependent equivalence scale based on Fernández-Villaverde and Krueger (2007), Table 1, last column.

For the technology parameters we assume that durables depreciate at an annual rate of $\delta = 0.02$ which is a good approximation for consumer durables which mostly consist of housing. The adjustment cost parameters are specified symmetrically for upward and downward adjustments as a starting point and are of similar size as the 5-10% of fees which are typically charged by real-estate brokers in the US (Díaz and Luengo-Prado, forthcoming). The parameters imply that 86% of the durable can be collateralized.

The parameters for the bankruptcy procedure are set as follows. We assume that the value of the exempt durables amounts to a quarter of average labor earnings, which, as in Athreya (2006), p. 2063, is the homestead exemption for the median consumer in the US. We assume that none of the labor earnings can be used to service unsecured debt in bankruptcy proceedings, $\gamma = 0$ and that there is no utility cost of declaring bankruptcy in our benchmark calibration, $\psi = 0$.

<i>Variable</i>	<i>SCF data 1983</i> (1)	<i>Model 1983</i> (2)
Housing wealth (as fraction of net lab. earnings)	2.97	1.64
Net-financial wealth (as fraction of net lab. earnings)	0.38	0.34
Gross financial wealth (as fraction of net lab. earnings)	0.64	0.63
Gross financial debt (as fraction of net lab. earnings)	-0.26	-0.29
Secured debt (in % of total debt)	84.43	86.48
Bankrupt (in % of sample)*	0.2	~0

Table 3: Sample statistics in the data and the model. Source: Authors' calculations based on the SCF and the model. Notes: *Data on bankruptcy are not available in the SCF 1983. Statistics from Sullivan et al. (2000).

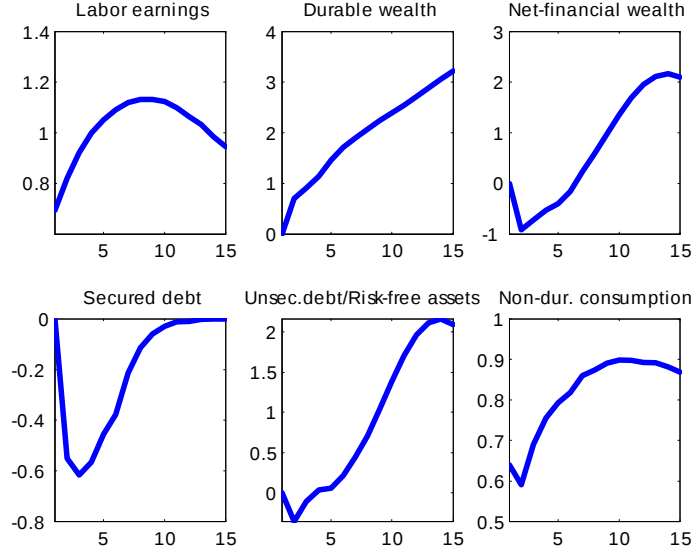


Figure 4: Life-cycle profiles for the model benchmark 1983

Finally, we set the annual risk-free lending rate to 3% and the secured borrowing rate to 7.5%, similar to Athreya (2006) and Livshits et al. (2007a). We further assume that intermediation costs are larger for unsecured borrowing where we assume that the basis rate $\underline{r}^u$ is 18.5%. This base rate for unsecured credit is at the high end of commonly calibrated rates for credit cards and necessary to match the fraction of unsecured versus secured debt in the debt portfolio. The pricing equation in our model implies an additional risk premium for unsecured credit where we restrict the overall rate to be below 100%. This risk premium turns out to be negligible for the chosen equilibrium debt levels in our calibration.

3.1.3 Results

Table 3 shows that the calibrated model matches most target statistics quite closely. The exception is housing wealth which is the best counterpart for the durables in the model. Consumers accumulate housing wealth more slowly in the model than in the data. One reason may be that our model does not include bequests which may be an important source of durable wealth accumulation up to age 55.

Figures 4 and ?? display the life-cycle profiles for the main variables of interest simulating the model for a population of 10,000 consumers who start

without durables and financial wealth at the beginning of their life. The profiles of the model match the data profiles in Figures 1 and 2 remarkably well. Young consumers accumulate durables financing part of their investment with debt. That debt is mostly secured but at the beginning of life, when many consumers are collateral constrained, consumers also take on unsecured debt which, for certain age groups, grows up to a third of average population labor earnings. As the labor earnings of consumers grow on average over the life cycle, consumers repay their debts as they age and eventually start to accumulate financial wealth. On average, consumers have a positive net-financial asset position after age 40. Since unsecured debt is much more costly than secured debt, consumers first repay their unsecured debts and eventually also their secured debts. Whereas very few consumer hold unsecured debt after age 30, substantially more consumers hold secured debt until retirement.

As in the data for 1983, the bankruptcy incidence in the model is negligibly small. Whereas we find no consumer who files for bankruptcy in our simulations for 10,000 consumers, there is a positive (tiny) probability of bankruptcy at the end of the first period of life since the price for unsecured debt in the first period is slightly smaller due to a positive risk premium.

A few other features of the figures deserve discussion. Non-durable consumption falls slightly after the first period and is then hump-shaped over the life-cycle, as in Fernández-Villaverde and Krueger (2005). Whereas the hump shape of non-durable consumption is a realistic feature of our model which is borne out in the data, the initial fall of consumption is just due to our simplifying assumptions that all consumers enter the economy without durables together with the non-separability of durable and non-durable consumption. Since $\sigma > 1 - \alpha$, the cross derivative of the utility function is negative so that consumers mitigate the implied lower utility without durables in the first period with more non-durable consumption. Furthermore, note that all consumers decumulate assets in the last retirement period (which is not included in the figures) because we assume that all consumers die with certainty at age 74 and there is no bequest motive. Since this assumption is a strong simplification and our main interest is consumer debt which occurs early in life, we calibrate our model to match the life-cycle profiles only until age 55.

<i>Variable</i>	<i>Benchmark</i>	<i>No adjustment costs</i>	<i>No durables</i>
	(1)	(2)	(3)
Housing wealth (as fraction of net lab. earnings)	1.64	2.23	0
Net-financial wealth (as fraction of net lab. earnings)	0.34	-0.13	1.53
Gross financial wealth (as fraction of net lab. earnings)	0.63	0.32	1.53
Gross financial debt (as fraction of net lab. earnings)	-0.29	-0.45	0
Secured debt (in % of total debt)	86.48	94.36	-
Bankrupt (in % of sample)	~0	~0	0

Table 4: The role of adjustment costs and durables. Source: Authors' calculations based on the model.

The importance of durables and adjustment costs Durables and their adjustment costs are major new features in our model compared with other studies of consumer bankruptcy. Hence, we gauge their importance by computing the model solution if, starting from the benchmark in 1983, we eliminate adjustment costs or durables from our model. The results are reported in columns (2) and (3) of Table 4, respectively.

The collateral constraint (1) shows that, without adjustment costs, a larger fraction of the durable can be used as collateral. Moreover, the cost of accumulating durables at beginning of life falls without adjustment costs. Hence, the wealth portfolio shifts towards housing and secured debt in column (2) whereas net financial wealth falls. One may have expected that the bankruptcy incidence increases without adjustment costs since adjustment costs imply a financial burden if consumers have to adjust their durable stock in bankruptcy proceedings. The results in column (2) show, however, that the bankruptcy incidence remains negligible due to the laxer collateral constraint and the corresponding shift towards secured debt. This result already highlights that allowing for shifts in the debt portfolio imposes further discipline when searching for exogenous changes which explain the observed trends in the data. Lower adjustment costs (possibly due to a more efficient real-estate market) may help to quantitatively explain the overall increase in debt levels but not the observed rise in bankruptcies.

In column (3) we report results if the utility weight for non-durable consumption is $\theta = 0.99$ and the depreciation $\delta = 0.99$. In this case, consumers hold no durables but also no debt. This shows that durables are essential to generate realistic levels of consumer debt, unsecured debt and (in rare cases also) bankruptcy in our benchmark calibration for 1983.

4 The evolution of bankruptcy and debt portfolios

After characterizing the benchmark solution for 1983, we now investigate whether our model can quantitatively explain the evolution of bankruptcy and debt portfolios which are observed in the period 1983-2004. We consider the following candidate explanations, where some of these explanations have been analyzed in models without durables (for example, Athreya, 2004, and Livshits et al., 2007a):

- The fall of the real interest rate and the borrowing spread for secured and unsecured debt.

- The increase in labor earnings risk.

We focus on the previous two changes in our quantitative study of the evolution of debt portfolios and bankruptcy incidence due to data quality. We also consider the following two changes in the model since their qualitative implications are of interest to compare our model with the previous literature:

- Financial innovations which change the extent to which durables can be used to secure debt.
- Less stigma, that is a smaller utility cost of declaring bankruptcy (see Gross and Souleles, 2002, or Fay, Hurst and White, 2002).

The consensus based on models with unsecured debt (but without secured debt) is that the improvement in the technology of financial intermediaries, which reduced the borrowing spread, quantitatively explains some of the observed increase in unsecured debt and bankruptcy filings (Athreya, 2004; Livshits et al., 2007a). We reconsider these findings and investigate whether they are consistent with the observed composition of the debt portfolio in terms of secured and unsecured debt. This is not obvious since consumers in our model have an additional margin of substitution between secured and unsecured debt and the borrowing spread has fallen for both types of debt. We now briefly discuss the calibration of the considered changes in the period 1983-2004.

Interest rates in 2004. We use evidence by Caporale and Grier (2000) and Caballero, Farhi and Gourinchas (2008) which indicates that the real interest rate in the US has fallen by about 2 percentage points since 1983. We thus reduce r^a from 3% to 1.5%. Moreover, historical data from the Federal Reserve suggests that the borrowing spread between interest rates on secured debt and rates on treasury bills has decreased between 1/2 and 1 percentage point in the same time period. Together with the overall decrease in rates by 1.5 percentage points, we thus reduce r^s by 1/2 a percentage point more than r^a , from 7.5% to 5.5%. Finally, Davis, Kubler and Willen (2006) show that the spread between unsecured borrowing costs and risk-free returns is about 10 percentage points in 2001. We thus calibrate $\underline{r}^u$ as 11.5% in 2004. This implies a fall in the borrowing spread for unsecured credit due to smaller intermediation costs by 7 percentage points.²⁴

²⁴The time-varying spreads reported in Davis, Kubler and Willen (2006), Table 1, do not provide much support for a change in the overall spread for unsecured borrowing. This may not be inconsistent with the fall in intermediation costs which we consider. The

The income process for 2004. We keep the deterministic component of the life-cycle income profile constant since the income means by age group reported in Figure 1 do not exhibit any significant change. Concerning the stochastic component, we use the same procedure detailed above where we keep the income grid constant (to avoid that changes in the results are due to a different discretization) but use the approximation for the income distribution for 2004, $\log y_{2004} \sim \mathcal{N}(-0.1146, 0.2292)$, which implies a 10 percent increase in the standard deviation of log-income. The size of this increase is line with evidence by Cunha and Heckman (2007). Moreover, Hintermaier and Koeniger (2008) show that US data on the net-worth distribution in 1983 and 2004 are consistent with a simple buffer-stock saving model if the observed increase in cross-sectional standard deviation is about 10 percent. Using the new income distribution for 2004 implies the transition matrix

$$\Gamma_{2004} = \begin{bmatrix} 0.8146 & 0.1846 & 0.0008 & 0 & 0 \\ 0.0370 & 0.6468 & 0.2820 & 0.0340 & 0.0002 \\ 0.0014 & 0.2334 & 0.4981 & 0.2584 & 0.0087 \\ 0 & 0.0338 & 0.2809 & 0.5843 & 0.1010 \\ 0 & 0.0001 & 0.0083 & 0.2544 & 0.7372 \end{bmatrix}$$

and the stationary distribution

$$\pi_{2004} = [0.0528, 0.2522, 0.2995, 0.2784, 0.1171].$$

Adjustment costs and the debt portfolio. One might argue that real-estate markets have become more efficient over time. In our model this would be captured by lower adjustment costs for durables which relax the collateral constraint. Since we are not aware of precise data which would allow us to calibrate this change, we refer to our qualitative results in the previous section where we have seen that the debt portfolio shifts towards secured debt and the wealth portfolio shifts towards housing wealth if we reduce adjustment costs to zero.

overall interest on unsecured credit is the sum of the base rate $\underline{r}^u$ and the endogenous risk premium and the fall in the base rate may well have been offset by the increase in the average risk premium.

Variable	Benchmark		Interest rates				Income	Model		Data
	1983	(1)	r^a :1.5%	(2)	r^s :5.5%	(3)	r^u :11.5%	risk:	2004	2004
							(4)	(5)	(6)	(7)
Housing wealth		1.64		1.75		1.71	1.62	1.68	1.83	3.66
Net-financial wealth		0.34		-0.02		0.15	0.14	0.51	-0.17	-0.07
Gross financial wealth		0.63		0.28		0.60	0.59	0.75	0.32	0.53
Gross financial debt		-0.29		-0.30		-0.45	-0.45	-0.24	-0.49	-0.61
Secured debt (% of total debt)		86.48		86.90		89.95	65.94	86.34	78.73	84.41
Bankrupt (% of sample)		~0		~0		~0	0.04	0	0	1.22

Table 5: The evolution of debt portfolios and bankruptcy incidence. Source: Authors' calculations based on the model. Notes: Quantities are normalized by average net labor earnings in the respective year.

Changes in stigma. We have set the utility cost of bankruptcy $\psi = 0$ in our benchmark calibration for 1983. The advantage of our model with durables is that we can afford to neglect such *ad hoc* costs in our calibration. Since there has been a debate on whether a decrease in stigma in the last decades is the cause of the higher observed bankruptcy incidence, we check how the model solution changes for a higher ψ to compare the qualitative findings of our model with the literature. We illustrate the effect of changes in the parameter ψ in 2004. Since the size of the changes in ψ are arbitrary and we are just interested in the qualitative response of the model equilibrium, we increase ψ from 0 to 1. As can be seen in the Bellman equation (7) this shifts the continuation value under bankruptcy down compared with the continuation value under repayment.

4.1 Results

Table 5 displays the results for our candidate explanations. The main findings are summarized as follows:

- The fall of the real interest rate in column (2) reduces gross financial wealth but has little effect on consumer debt or the bankruptcy incidence. Thus, also net financial wealth falls substantially. Since accumulation of risk-free assets becomes relatively less attractive and the opportunity cost of housing wealth falls, housing wealth increases by 7%.
- Smaller costs of financial intermediation for secured debt, and thus a lower interest rate for secured debt in column (3), increase consumer debt by 50% which is about half of the increase observed in the data. Moreover, the share of secured debt in the debt portfolio increases by 3 percentage points. The wealth portfolio shifts slightly towards housing wealth.
- The smaller costs of financial intermediation for *unsecured* debt considered in column (4) also increase consumer debt by 50% but imply a fall of the share of secured debt in the debt portfolio by 21 percentage points. The incidence of bankruptcy increases slightly to 0.04%. Figure 5 shows that the bankruptcy incidence for young consumers rises to above 0.15%, implying still a rather small risk premium for unsecured debt. Furthermore, cheaper unsecured credit implies that more consumers hold unsecured debt at later stages in their life cycle.

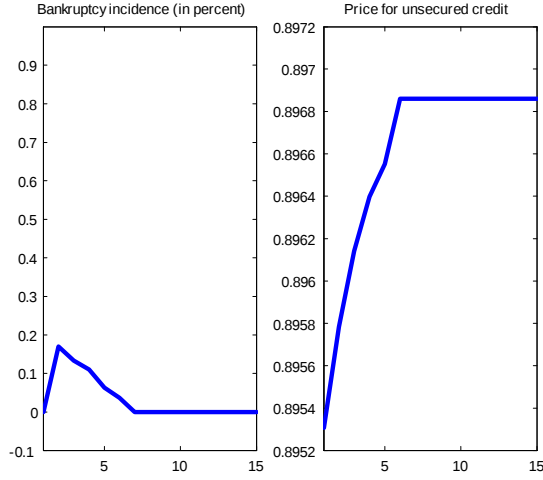


Figure 5: Bankruptcy over the life cycle with cheaper unsecured credit

- The higher labor income risk in column (5) reduces consumer debt by 17% and increases both financial and housing wealth. Especially gross financial wealth increases (by 20%) as consumers have a precautionary motive (see Nakajima, 2005, for similar findings in a general equilibrium model without bankruptcy). Due to this prudent behavior, the probability of bankruptcy is zero (that is, there is not even a tiny positive probability of declaring bankruptcy at the beginning of life).
- In column (6) we consider the changes of columns (2) to (5) together. Concerning financial wealth and debt, the effect of the lower interest rates dominates the effect of higher labor income risk. The model predicts 2/3 of the debt increase observed in the data but overpredicts the fall of gross financial wealth. Hence, the model predicts that net-financial wealth falls to -0.17 in 2004 instead of -0.07 observed in the data. Overall, the model captures the main trends of financial wealth and debt although the downward trend of gross financial wealth is predicted to be larger and the upward trend of debt is predicted to be smaller than in the data. Furthermore, the model predictions for housing wealth are still well below the corresponding value in the data (for the same reasons as in the benchmark for 1983) but the increase of housing wealth by 12% corresponds to more than half of the observed percentage increase observed in the data. Finally, the fall of the prices for secured and unsecured debt imply a small shift of the debt port-

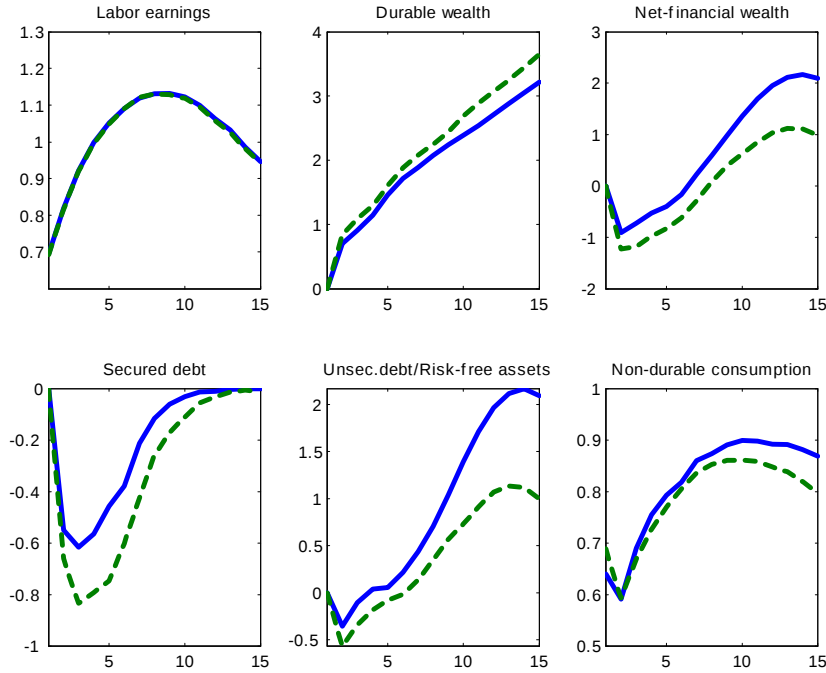


Figure 6: Life-cycle profiles of the model in 1983 (solid graph) and 2004 (dashed graph)

folio towards unsecured debt by 7 percentage points. The bankruptcy incidence which the model predicts for 2004, however, is zero, notwithstanding cheaper unsecured debt, higher debt levels and income risk. The higher income risk possibly increases the cost of bankruptcy in terms of the resulting exclusion from credit markets for 6 years (see Krueger and Perri, 2006).

Figure 6 displays the life-cycle profiles for the model benchmark in 1983 (the solid graph) and the model predictions for 2004 from column (6) in Table 5 (the dashed graph). Comparing the results with the data in Figure 1 shows that the model captures the observed changes in the life-cycle profiles rather well. In particular, the model predicts an increase of unsecured debt for young collateral-constrained consumers. The higher debt levels at the beginning of the life-cycle allow consumers to accumulate more housing wealth and result in lower financial wealth levels at later stages of the life cycle. Hence, the non-durable consumption profile is lower later in life when consumers derive substantial utility from durables. The model fails, however, to capture the crossing of the profiles of net-financial wealth which is observed at age 53-

56 in the data. Most importantly, it seems difficult to generate consumer bankruptcy in our life-cycle model with durables since unsecured debt is held mostly by young collateral-constrained consumers who borrow to accumulate durable wealth. These consumers have little interest to declare bankruptcy in which case all their durable wealth above the exemption level is seized.

Overall, the fall of the lending and borrowing rates is an essential ingredient to explain important observed changes in the data:

- A fall in the risk-free lending rate helps to match the fall in gross financial wealth.
- Cheaper debt, in terms of interest rates for both secured and unsecured debt, helps to explain the rise in consumer debt and to keep the share of secured debt roughly constant over time. In our experiments above the changes of these rates in column (3) or (4) explain about half of the observed increase in consumer debt.
- Cheaper unsecured debt helps to explain the increase of bankruptcy incidence. Our experiments above show, however, that we explain only a very small fraction of observed consumer bankruptcy in 2004. Given the substantial computational burden in our current model with durables and adjustment costs, adding additional shocks which possibly increase the bankruptcy incidence, such as the medical expense shocks considered in Chatterjee et al. (2007) and Livshits et al. (2007a,b), remains a challenge for future research. An alternative would be to consider different pricing assumptions for unsecured debt which we discuss further in the conclusions.

How do our results compare with those in the previous literature? We find that a fall in transaction costs which reduces the cost of borrowing may explain some of the increase in consumer debt but not much of the increase in bankruptcy filings. This result is similar to the life-cycle model with expense shocks, but without durables and secured debt, of Livshits et al. (2007a). In the infinite-horizon framework of Athreya (2004) instead bankruptcy filings are more elastic to changes in transaction costs (see also Mateos-Planas, 2007).

An important value added of our analysis is that we distinguish between secured and unsecured debt. Whereas cheaper borrowing rates imply more unsecured debt in Livshits et al. (2007a) and Athreya (2004), and thus possibly also more bankruptcy incidence, our analysis has shown that it is hard to generate a substantial increase in bankruptcies if both secured and unsecured debt become cheaper which is necessary if one wants to match the

rather constant share of secured debt in the debt portfolio observed in the data. We now give further examples for the importance of analyzing debt *portfolios* by discussing whether lower adjustment costs or utility costs of bankruptcy (stigma ψ) can improve the model predictions.

The effect of lower adjustment costs. The qualitative results of Table 4, column (2), show that lower adjustment costs (for example, due to more efficient real-estate markets) may help to quantitatively explain the overall increase in debt levels but not the observed rise in bankruptcies. Lower adjustment costs imply a shift towards secured debt in the debt portfolio which, however, is not observed in the data.

The effect of less stigma. Livshits et al. (2007a) argue that a fall in stigma may explain some of the upward trend in bankruptcy incidence whereas Athreya (2004) highlights the importance of supply-side responses which tighten access to unsecured credit. We also find that the supply-side response is quantitatively important, as a higher stigma ψ cheapens unsecured credit and thus implies a shift of the debt portfolio towards unsecured debt. Hence, a fall in ψ would shift the debt portfolio towards secured debt which does not fit the basic facts on debt portfolios which are observed in the data.

5 Conclusion

We have studied and solved a model with non-durable and durable consumption which allows us to analyze debt portfolios and bankruptcy decisions. We have found that the observed fall of lending and borrowing rates are key ingredients to match the observed upward trend in consumer debt, the stable share of secured debt and the slight fall of gross financial wealth for consumers between 20 and 55 years of age. We have emphasized the importance of durables to distinguish secured and unsecured debt which helps to impose further discipline in the search for plausible explanations of recent trends in debt portfolios and bankruptcy incidence.

Further research may consider another interesting change in technology concerning developments in the pricing of unsecured debt. Empirical evidence of Edelberg (2006) suggests that banks started to use large databases for the pricing of debt only in the 1990s. Hence, banks may have conditioned the price of unsecured credit less accurately on consumer portfolios in 1983 than in 2004. Modeling such pricing imperfections as in Athreya, Tam and Young (2008) is thus an interesting avenue for future research. A considerable

challenge for this research is the computational burden due to the additional fixed-point problem with interactions in the pricing of unsecured debt across agents.

Data appendix

The variables used in the paper are constructed in the following way:

Gross *labor income* is the sum of wage and salary income. As in Budría Rodríguez et al. (2002) we add a fraction of the business income where the fraction is the average share of labor income in total income in the SCF. *Disposable labor income* is computed using the NBER tax simulator. We use the programs provided by Kevin Moore provided on <http://www.nber.org/~taxsim/> to construct disposable labor earnings for each household in the SCF 1983 and 2004. Following the standardized instructions on the NBER website, we feed the following required data of the SCF into the NBER tax simulator: the US state (where available, otherwise we use the average of the state tax payments across states), marital status, number of dependents, taxpayers above age 65 and dependent children in the household, wage income, dividend income, interest and other property income, pensions and gross social security benefits, non-taxable transfer income, rents paid, property tax, other itemized deductions, unemployment benefits, mortgage interest paid, short and long-term capital gains or losses. We then divide the resulting federal and state income tax payments as well as federal insurance contributions of each household by the household's gross total income in the SCF. This yields the implicit average tax rate for each household in 1983 and 2004. The mean of that average tax rate for consumers in the SCF is 24% in 1983 and 23% in 2004. Finally, we compute household net labor earnings as $(1 - \text{household average tax rate}) * \text{household gross labor earnings (including taxable transfers)}$ and then add non-taxable transfers.

Financial assets are defined as the sum of money in checking accounts, savings accounts, money-market accounts, money-market mutual funds, call accounts in brokerages, certificates of deposit, bonds, account-type pension plans, thrift accounts, the current value of life insurance, savings bonds, other managed funds, other financial assets, stocks and mutual funds.

Secured debt is defined as the sum of mortgage and housing debt, other lines of credit and debt on residential and nonresidential property, debt on non-financial business assets, installment credit and other consumer loans which may be secured by purchased durables, such as cars or furniture.

Unsecured debt is defined as credit-card and other financial debt in the SCF.

Net-Financial assets are defined as *financial assets - gross secured and unsecured debt*.

Non-financial assets are defined as the sum of the value of homes, residential and non-residential property, vehicles, other durables like jewelry or antiques, owned non-financial business assets.

Housing is defined as the sum of the value of homes, residential and non-residential property.

Net worth is defined as the sum of net-financial and non-financial assets.

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