

FISCAL POLICY AND THE LABOUR MARKET: THE EFFECTS OF PUBLIC SECTOR EMPLOYMENT AND WAGES*

Pedro Gomes[†]

London School of Economics

June 3, 2009

Abstract

I build a dynamic stochastic general equilibrium model with search and matching frictions and two sectors, in order to study the labour market effects of public sector employment and wages. I discuss what is the public sector wage that achieves the social planner's solution and how it varies with different labour market parameters. Public sector wage and employment shocks have mixed effects on unemployment. A wage shock raises unemployment rate while a reduction in the separations reduces it. Hiring more people can increase or decrease unemployment depending on the level of wages. All shocks increase the wage and crowd out employment in the private sector because they induce more unemployed to search for public sector jobs. I then discuss the optimal policy in response to technology shocks. It consists on a counter-cyclical vacancies posting and a procyclical public sector wage. Deviations from the optimal policy can increase the volatility of unemployment rate significantly. In the empirical part, I start by reviewing the micro evidence on public sector wages and the endogeneity of the sector choice. Then, I employ Bayesian techniques to estimate the parameters of the model for the US, using quarterly data on government employment and wages, unemployment rate, private sector wages, job separation rate and job finding rate. I find evidence that the direct search mechanism plays a role over the business cycle and that public sector vacancies are mildly counter-cyclical while public sector wages are acyclical. To complete the empirical study, I estimate a structural VAR model. I find that public sector employment and wages positively affect private sector wage. The negative effect on private sector hours is only significant over the last 20 years. Both government employment and wages do not respond to innovations in productivity.

JEL Classification: E24; E32; E62; J31; J45.

Keywords: Public sector employment; public sector wages; unemployment; fiscal shocks.

*I would like to thank participants at the London School of Economics, University Pompeu Fabra, Lisbon Technical University, European Central Bank and Bank of England seminars; and the 40th MMF conference, the PEUK PhD workshop, the EDP-Jamboree, the RES conference and the IZA summer school. I want to thank in particular to António Afonso, Thijs Van Rens, Davide Debortoli, Jordi Gali, Francesco Caselli, Rachel Ngai and Bernardo Guimarães, Stephen Millard, Luís Costa, Clare Leaver, Ad van Riet, Mathias Trabandt, Wouter den Haan and Albert Marcet. Pedro Gomes acknowledges financial support from FCT.

[†]Corresponding author. London School of Economics, STICERD, Houghton Street, WC2A 2AE, London, United Kingdom. Tel:+44(0)2078523546, Fax:+44(0)2079556951, Email: p.gomes@lse.ac.uk.

1 Introduction

Traditionally, macroeconomists have studied the aggregate effects of government spending in the form of goods bought to the private sector.¹ However, the main element of government consumption is compensation to employees. As shown in Table 1, it represents between 50 to 60% of final government consumption expenditures in most OECD countries. But public sector employment is not just an important aspect of fiscal policy; it is also a sizable element of the labour market. In OECD economies, public sector employment ranges from 7% to 30% of total employment. Given its relevance, it seems reasonable that part of the transmission mechanism of fiscal policy occurs through the labour market.

Compared to the theoretical research that focusses on government spending as buying part of the production of the economy, the literature that studies the effects of public sector employment and wages is relatively scarce. Early references include Rotemberg and Woodford (1992) and Finn (1998) that find that, contrary to government purchases of goods and services, the purchase of hours raises real wages and reduces private output. More recently, Pappa (2005) and Cavallo (2005) also concluded that, in a perfectly competitive labour market, after an increase in government employment, private sector hours and output goes down. Ardagna (2007) and Algan, Cahuc, and Zylberberg (2002) study the issue in a unionized economy. In their setting, an increase in public sector employment, public sector wage or unemployment benefits, raises the wage in the private sector and therefore unemployment.

However, to fully understand the transmission mechanisms of fiscal policy through the labour market it is crucial to model the existing frictions. There have been some attempts to do it in a search and matching environment. In Holmlund and Linden (1993) an increase in public employment has a direct negative effect in unemployment but crowds out private employment due to an increase in wages. But, for all realistic calibrations the direct effect of reducing unemployment is always stronger than the indirect effect through wages. Quadrini and Trigari (2007) examine the impact of public sector employment on business cycle volatility and find that the presence of public sector increases the volatility of private and total employment. Hörner, Ngai, and Olivetti (2007) study the effect of public enterprises and conclude that a country with public sector firms has higher unemployment than if the companies were privately managed.

The aim of this work is to provide a comprehensive, yet simple, framework to study the macroeconomic effects of public sector employment and public sector wages. I build a dynamic stochastic general equilibrium model with search and matching frictions along the lines of Pissarides (1988) with both public and private sectors. The model shares many features with Quadrini and Trigari (2007). One of the main difficulties with the model is the calibration of the friction parameters,

¹For example: Barro (1990), Baxter and King (1993), Ludvigson (1996) study the effects of government spending in a Neo-Classical setting. Linnemann and Schabert (2003) extend the analysis to the standard New Keynesian model to fiscal policy and Galí, López-Salido, and Vallés (2007) introduces rule of thumb agents. These papers share the feature of considering government spending as goods bought to the private sector. Most of them do not explore the effects of government spending *per se*, but focus on the effects of alternative forms of financing.

Table 1: Public sector and the labour market

	Public wage bill (% gov. consumption)	Public Employment (% total employment)	Unemployment rate	Correlation (u_t, l_t^g)
Australia	52.2%	14.1%	6.3%	0.51
Austria	53.4%	13.1%	4.7%	0.34
Belgium	53.8%	17.9%	6.9%	0.91
Canada	59.8%	20.5%	6.8%	0.55
Denmark	67.8%	30.5%	4.4%	0.78
Finland	63.2%	24.8%	9.9%	0.76
France	58.4%	22.5%	9.4%	0.95
Germany	41.5%	11.6%	7.5%	0.82
Iceland	60.0%	19.0%	2.3%	0.74
Ireland	57.0%	12.7%	4.3%	0.84
Italy	55.6%	16.9%	10.7%	-0.40
Japan	37.7%	8.4%	4.7%	0.35
Luxembourg	49.1%	15.0%	2.6%	0.88
Netherlands	42.2%	10.9%	2.6%	0.80
Norway	63.1%	33.6%	3.4%	0.82
Portugal	72.8%	14.3%	4.0%	0.22
Spain	59.2%	14.1%	11.4%	0.13
Sweden	59.2%	31.1%	4.7%	0.33
United Kingdom	53.3%	18.0%	5.5%	0.19
United States	66.5%	15.2%	4.1%	0.66
Average	56.3%	18.2%	5.9%	0.49

Note: Public wage bill, public employment and unemployment rate refer to the year 2000. The correlation between public sector employment and the unemployment rate is computed from quarterly data (1970 to 2007). Source: OECD.

particularly for the public sector. In order to do it I gather information from the Job Openings and Labor Turnover Survey for the United States, the Labour Force Survey for the United Kingdom, as well as from other sources.

In a first stage, I solve the social planner's problem to find the optimal allocation. I then solve the decentralized equilibrium and determine the level of public sector wage consistent with the optimal steady state allocation. The optimal public sector wage premium depends only on the differences of the labour market frictions parameters of the public sector relative to the private sector. For the chosen calibration the optimal wage is 2.5% lower than in the private sector. If the government sets a higher wage, it raises private sector wages and induces too many unemployed to search for public sector jobs, reducing private sector job creation and increasing unemployment. Conversely, if it sets a lower wage, few unemployed want a public sector job and the government faces recruitment problems.

The model allows us to disaggregate fiscal shocks into wage and employment shocks and the latter into separation and hiring shocks. The response of the variables to the three shocks is different. Paying more to public sector workers raises unemployment through two channels. On the one hand, more unemployed direct their search towards the public sector. On the other hand, the public sector wage spillover to the private sector wage as it increases the value of unemployment.

The two channels lead to a reduction of job creation in the private sector. These two channels are also in place under a separation or hiring shock, but they are counter balanced by the direct effect of increasing public employment. In general, a separation rate shock always reduces unemployment, but a hiring shock can have opposite effects on unemployment, depending on the level of public sector wages. If the government wages are high, when the government increases the number of vacancies it induces much more unemployed to search for these new jobs, which enhances the crowding out effect in the private sector.

The opposite effects of the different components of fiscal policy is one of the key results of the paper. The extensive empirical literature that evaluates the macroeconomic effects of government spending tend to find mixed effects on private consumption, real wage or employment.² As a consequence, the center of the debate has been on the technical methodology, particularly on the identification of fiscal shocks. I believe that the mixed evidence is more related to the data, rather than the methodological strategy used. Fiscal policy shocks can have different effects depending on the type of expenditure we are considering: employment, wages, purchases of privately produced goods, government investment or transfers. By including all components together, some in particular or using different samples in which the composition of spending has changed, we cannot expect to identify properly one type of fiscal shocks. This hypothesis is consistent with recent evidence from Caldara and Kamps (2008) that, using the same variables and sample, concluded that different identification strategies yield similar results.

This argument is not entirely new. Finn (1998) distinguishes between the purchase of goods and services, and employment compensation components of spending and find that they have opposite effects on private output, private employment and private investment. I go a step further and show that disaggregating employment compensation into shocks in employment and in the per-employee wage have different effects on unemployment. To strengthen my argument I do a simple modification to the model, substituting public employment for services bought directly to the private sector. In such an economy, increases in the government purchase of goods decreases the real wage and increases private employment, contrary to shocks in employment and wage.

Finally, I examine the properties of the model when subject to technology shocks. The optimal policy consists of a countercyclical public vacancy posting and a procyclical public sector wage. If the government follows the optimal policy, the presence of the public sector reduces unemployment volatility. Deviations from optimal policy can entail significant welfare losses. If, for instance, public sector wage does not respond to the cycle, unemployment volatility increases more than twice relative to the scenario under optimal policy.

Many of the model's results, particularly on public sector wages, are driven by the assumption that the unemployed can direct their search towards the private or public sector; so the purpose of most of the empirical part of the paper is to convince that this is an important mechanism. First, I review the evidence from the microeconomic studies on public sector wages, showing

²See Caldara and Kamps (2008) for an overview.

evidence that supports the endogeneity of the sector choice. Then, I employ Bayesian techniques to estimate the parameters of the model for the US, between 1948 and 2007, using quarterly data on: government employment and wages, unemployment rate, private sector wages, job-separation rate and job-finding rate. I add a parameter that measures the relevance of the fluctuations on the share of unemployed searching for public sector jobs, and find that the mechanism is clearly supported by the data. The mechanism was less important during the period of the “great moderation” and more relevant during periods with stronger volatility. Additionally, I find that the government follows a mildly counter-cyclical vacancy policy but that public sector wages are acyclical.

To complete the empirical part of the paper I estimate a structural VAR. I find that government employment and wages do not respond to innovations in productivity. Both public sector employment and wages positively affect private sector wage. The negative effect on private sector hours is only significant over the last 20 years.

The paper proceeds as follows. Section 2 describes the model and Section 3 its calibration. Section 4 discusses how the government can attain the social planner’s solution in steady-state and what are the costs of moving away from the efficient allocation. Section 5 examines the response to fiscal shocks while Section 6 addresses the optimal fiscal policy over the business cycle. Section 7 discusses two extensions of the model. In the empirical part of the paper, Section 8 reviews the existing micro literature, Section 9 explains the details and displays the results of the model estimation and Section 10 shows the impulse responses from the VAR. Section 11 concludes.

2 Model

2.1 General setting

The model is a dynamic stochastic general equilibrium model with public and private sectors. The only rigidities present in the model are due to search and matching frictions. Public sector variables are denoted with superscript $\{g\}$ while private sector variables are denoted by $\{p\}$. Time is denoted by $t = 0, 1, 2, \dots$

The labour force consists of many individuals $j \in [0, 1]$. Part of them are unemployed (u_t), while the remaining are working either in the public (l_t^g) or in the private (l_t^p) sectors.

$$1 = l_t^p + l_t^g + u_t \quad (1)$$

Total employment is denoted by l_t . The presence of search and matching frictions in the labour market prevents some unemployed from finding jobs. The evolution of public and private sector employment depends on the number of new matches m_t^p and m_t^g and on separations in each sectors. I consider that, in each period, a constant fraction of jobs are destroyed. This fraction might be different in the two sectors.

$$l_{t+1}^i = (1 - \lambda^i)l_t^i + m_t^i, \quad i = p, g \quad (2)$$

I assume the unemployed choose which sector they want to search in, so u_t^i represents the number of unemployed searching in sector i . The number of matches formed in each period is determined by two matching functions.

$$m_t^i = m^i(u_t^i, v_t^i), \quad i = p, g \quad (3)$$

The matching functions are assumed to have constant returns to scale and to be homogeneous of degree 1. An important part of the subsequent analysis focuses on the behaviour of the share of unemployed search for a public sector job, defined as: $s_t = \frac{u_t^g}{u_t}$.

From the matching functions we can define the probabilities of vacancies being filled q_t^i , the job-finding rates conditional on searching in a particular sector, p_t^i , and the unconditional job-finding rates, f_t^i :

$$q_t^i = \frac{m_t^i}{v_t^i}, \quad p_t^i = \frac{m_t^i}{u_t^i}, \quad f_t^i = \frac{m_t^i}{u_t}, \quad i = p, g$$

The assumption of directed search implies that the number of vacancies posted in one sector only affects contemporarily the probability of filling a vacancy in the other sector through the endogenous reaction of s_t .

2.2 Households

In the presence of unemployment risk we would observe consumption differences across different individuals. As in Merz (1995), I assume all the income of the members is pooled so the private consumption is equalised across members.

The household is infinitely-lived and has preferences over private consumption good, c_t , and a public consumption good g_t . The household also has a disutility from working $\nu(l_t)$, which captures the foregone leisure, home production or some type of unemployment compensation.

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, g_t) - \nu(l_t)] \quad (4)$$

Where $\beta \in (0, 1)$ is the discount factor. The budget constraint in period t is given by:

$$c_t + B_t = (1 + r_{t-1})B_{t-1} + w_t^p l_t^p + w_t^g l_t^g + \Pi_t \quad (5)$$

Where r_{t-1} is the real interest rate from period $t-1$ to t , B_{t-1} are the holdings of one period bonds. $w_t^i l_t^i$ is the total wage income from the members working in sector i . Finally, Π_t encompasses all lump sum transfers from the firm and taxes paid to the government.

The household chooses c_t and B_t to maximize the expected lifetime utility subject to the sequence of budget constraints, taking the public consumption good as given. The solution is the consumption Euler equation:

$$u_c(c_t, g_t) = \beta(1 + r_t)E_t[u_c(c_{t+1}, g_{t+1})] \quad (6)$$

2.3 Workers

The value of each member to the household depends on their current state. The value of being employed is given by:

$$W_t^i = w_t^i - \frac{\nu_l(l_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} [(1 - \lambda^i) W_{t+1}^i + \lambda^i U_{t+1}], \quad i = p, g \quad (7)$$

Where $\beta_{t,t+k} = \beta^k \frac{u_c(c_{t+k}, g_{t+k})}{u_c(c_t, g_t)}$ is the stochastic discount factor. The value of being employed depends on the current wage and the disutility of working, as well as, the continuation value of the job, that depends on the separation probability in each sector. Under the assumption of directed search the unemployed agents are either searching in the private or in the public sector, with value functions given by:

$$U_t^i = E_t \beta_{t,t+1} [p_t^i W_{t+1}^i + (1 - p_t^i) U_{t+1}], \quad i = p, g \quad (8)$$

The value of being unemployed and searching in one sector, depends on the probabilities of finding a job and the value of working in that particular sector. Optimality implies that there are movements between the two segments that guarantee there is no additional gain of searching in one sector *vis-à-vis* the other:

$$U_t^p = U_t^g = U_t \quad (9)$$

This equality determines the share of unemployed searching in each sector. The expression is given implicitly by:

$$\frac{m_t^p E_t [W_{t+1}^p - U_{t+1}]}{(1 - s_t)} = \frac{m_t^g E_t [W_{t+1}^g - U_{t+1}]}{s_t} \quad (10)$$

An increase of the value of being employment in the public sector, driven by either an increase in the wage, an increase in the probability of being hired or a decrease in the separation rate, leads to an endogenous increase in s_t , until there is no extra gain from searching in that sector. Under the directed search assumption the public sector wage plays a key role in determining s_t . If we consider that search is random, public sector wages would not affect any decision of households.

2.4 Firms

The private sector representative firm hires labour to produce the private consumption good. The production function is linear in labour, but part of the resources produced have to be used to pay the cost of posting vacancies $\varsigma^p v_t^p$.

$$y_t = a_t^p l_t^p - \varsigma^p v_t^p \quad (11)$$

At time t , the level of employment is predetermined and the firm can only control the number of vacancies it posts. The value of opening a vacancy is given by:

$$V_t = \beta_{t,t+1} [q_t^p J_{t+1} + (1 - q_t^p) V_{t+1}] - \varsigma^p \quad (12)$$

Where J_t is the value of a job for the firm that is given by:

$$J_t = a_t^p - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda^p) J_{t+1}] \quad (13)$$

Free entry guarantees that in equilibrium the value of posting a vacancy is zero ($V_t = 0$), so we can combine these equations into the following expression:

$$\frac{\varsigma^p}{q_t^p} = E_t \beta_{t,t+1} [a_{t+1}^p - w_{t+1}^p + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p}] \quad (14)$$

The condition states that the expected cost of hiring a worker must equal its expected return. The benefits of hiring an extra worker is the discounted value of the expected difference between its marginal productivity and its wage and the continuation value, knowing that with some probability λ^p the match can be destroyed.

Finally, I consider the private sector wage is the outcome of a Nash bargaining between workers and firms. The sharing rule is given by:

$$(1 - b)(W_t^p - U_t) = bJ_t \quad (15)$$

2.5 Government

The government produces its consumption good using a linear technology on labour. As in the private sector, the costs of posting vacancies are deduced from production.

$$g_t = a_t^g l_t^g - \varsigma^g v_t^g \quad (16)$$

It sets a lump sum tax to finance the wage bill:

$$\tau_t = w_t^g l_t^g \quad (17)$$

Finally, the government follows a policy for the sequence of public sector vacancies and public sector wage $\{v_t^g, w_{t+1}^g\}_{t=0}^\infty$. I assume the government sets the wage one period in advance, at the time it posts the vacancies. As s_t is determined based on the expected value of both public and private sector wage in $t + 1$, the current period public sector wage does not affect any variable in the model. I assume the government commits to a certain future path for wages. There is no time inconsistency problem because, as taxes are lump sum, the government does not gain from setting a different current wage than promised.

Throughout the paper I contrast two types of policies from the government: exogenous policies to help understanding the functioning of the model and the optimal policy - the one arising from the social planner's problem.

2.6 Competitive equilibrium

Definition 1 A competitive equilibrium is a sequence of prices $\{r_t, w_t^p\}_{t=0}^\infty$ such that, given a sequence of government vacancies and wages $\{v_t^g, w_{t+1}^g\}_{t=0}^\infty$, the household chooses a sequence of consumption and the fraction of unemployed members searching in the public sector $\{c_t, s_t\}_{t=0}^\infty$ and firms choose private sector vacancies $\{v_t^p\}_{t=0}^\infty$ such that: (i) the household maximises its lifetime utility; (ii) the allocation of searching workers, s_t is such that the values of searching in the two sectors equalize (equation 10); (iii) private sector vacancies satisfy the free entry condition (14); (iv) the private wage w_t^p solves the bargaining condition (15); (v) the private goods market clears: $c_t = y_t$; and (vi) the lump sum taxes τ_t are chosen to balance the government budget (equation 17).

2.7 Social planner's solution

Up until now, I have described the competitive equilibrium. As a benchmark for the analysis of the model we are going to consider the first best solution. The social planner's problem is to maximize the consumers lifetime utility (4) subject to the labour market and technology constraints (1)-(3), (11) and (16). The first order conditions are given by:

$$\frac{\varsigma_t^p}{q_t^p} = \beta E_t \left\{ \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \left[(1 - \eta^p) a_{t+1}^p - (1 - \eta^p) \frac{\nu_l(l_{t+1})}{u_c(c_{t+1}, g_{t+1})} + (1 - \lambda^p) \frac{\varsigma_{t+1}^p}{q_{t+1}^p} - \frac{\eta^p \varsigma_{t+1}^p v_{t+1}^p}{(1 - s_{t+1}) u_{t+1}} \right] \right\} \quad (18)$$

$$\frac{\varsigma_t^g}{q_t^g} = \beta E_t \left\{ \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} \left[(1 - \eta^g) a_{t+1}^g - (1 - \eta^g) \frac{\nu_l(l_{t+1})}{u_g(c_{t+1}, g_{t+1})} + (1 - \lambda^g) \frac{\varsigma_{t+1}^g}{q_{t+1}^g} - \frac{\eta^g \varsigma_{t+1}^g v_{t+1}^g}{s_{t+1} u_{t+1}} \right] \right\} \quad (19)$$

$$\frac{u_g(c_t, g_t) \varsigma_t^g v_t^g \eta^g}{(1 - \eta^g) s_t} = \frac{u_c(c_t, g_t) \varsigma_t^p v_t^p \eta^p}{(1 - \eta^p)(1 - s_t)} \quad (20)$$

Where η^i is the matching elasticity with respect to unemployment in sector i . Conditions (18) and (19) describe the optimal level of private and public sector vacancies. On the left hand side we have the expected cost of hiring an extra worker. The right hand side give us the marginal social benefit of hiring an additional worker that consists on its marginal productivity minus the utility cost of working, weighted by the matching elasticity with respect to vacancies, plus, given that the match is kept with probability $(1 - \lambda^i)$, its continuation value. The last element, that enters negatively in the expression, reflects the fact that hiring an additional worker makes it harder to recruit a worker in the future for both sectors.

The optimal share of unemployed searching in a particular sector, pinned down in (20), depends positively on the marginal utility of consumption of the respective good, on the number of vacancies and their cost, and on the matching elasticity with respect to unemployment in the sector.

3 Calibration

To solve the model, we have to make assumptions on the functional form of the utility function and the matching function. I assume the utility function has a CES form which allow us to address different elasticities of substitution between the private and public consumption good. As typical in the literature, the matching function is a Cobb-Douglas. The disutility of labour as the form of constant relative risk aversion preferences.

$$m_t^i = \bar{m}^i u_t^{i\eta^i} v_t^{i1-\eta^i}$$

$$u(c_t, g_t) = \frac{1}{\gamma} \ln[c_t^\gamma + \zeta g_t^\gamma]$$

$$\nu(l_t) = \chi \frac{l_t^{1+\iota}}{1+\iota}$$

One of the caveats of the model is that the quantitative predictions and policy prescriptions of the model are closely tied to a number of parameters for which not much evidence exists, namely the friction parameters in the public sector ($\bar{m}^g$, η^g , λ^g and ς^g). Therefore the starting point of this section is to review evidence from several sources for the United States and the United Kingdom.

3.1 Evidence for the United States

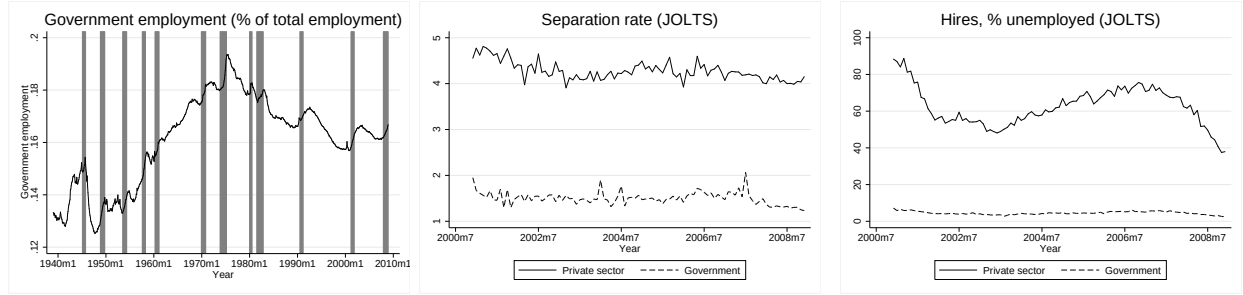
The first graph of Figure 1 shows the monthly government employment as a share of total employment since 1940. The data are taken from the Employment, Hours, and Earnings from the Current Employment Statistics survey. On average, government employment corresponds to 16% of total employment. There are strong counter-cyclical fluctuations but these are due to fluctuations in private employment, rather than fluctuations in public employment.³

On the second graph we have the monthly separation rate for the two sectors, taken from the Job Opening and Labour Turnover Survey (JOLTS). The separation rate in the private sector is almost 3 times than in the government: 4.3% against 1.5%. The third graph plots the new hires of each sector as a share on the total unemployed, which can be seen as a proxy for the job-finding rate. The probability of finding a job in the government is only 4.5% whilst 62.5% in the private sector. Most of the business cycle fluctuations occur in the private sector.

To estimate the matching elasticity with respect to vacancies I regress the log of the job-finding rate on the log of tightness of each sector. Tightness is defined as the ratio between job openings and unemployment. The estimated coefficient for the private sector is 0.63, in line with estimates from the literature (Petrongolo and Pissarides (2001)). The coefficient for the public sector is much higher, around 0.79, suggesting the vacancies are more important determinants of matches in the public sector.

³The correlation between the HP detrended series of the log of government employment with the unemployment rate is only -0.18 comparing with -0.93 of the private sector employment.

Figure 1: Evidence for the United States



Relative to the duration of vacancies in both sectors, we can gain some insights from a recent paper by Davis, Faberman, and Haltiwanger (2009). They use JOLTS data to study the behaviour of vacancies and hiring. They find that the probability of filling a vacancy per month is 0.8 for the government and 1.4 for the private sector. After adjusting the data, they estimate that the duration of a vacancy is 30 days for the government and 20 for the private sector.

3.2 Evidence on the United Kingdom

An alternative to establishment level data is to look at household surveys. I explore data from the UK Labour Force Survey (LFS). The LFS samples around 60,000 households for five successive quarters. The sample is split into five waves. Every quarter one wave leaves the survey and a new wave enters. In this way, we can observe the changes in the labour market status of 80% of the households that took part in the survey and obtain the gross labour market flows.⁴

LFS employment can be decomposed into private sector and government. Government employment includes employment from local and central government, health authorities, universities and armed forces.⁵ Figure 2 shows the government employment, the job-separation and job-finding rates.

Close to 22% of total employment is government employment. As in the United States, the separation rate is almost three times as high in the private sector as in the public sector (1.6% against 0.6%). In contrast, both the unemployed and inactive are seven times more likely to find a job in the private sector than in the public sector (23.6% against 3.4%).

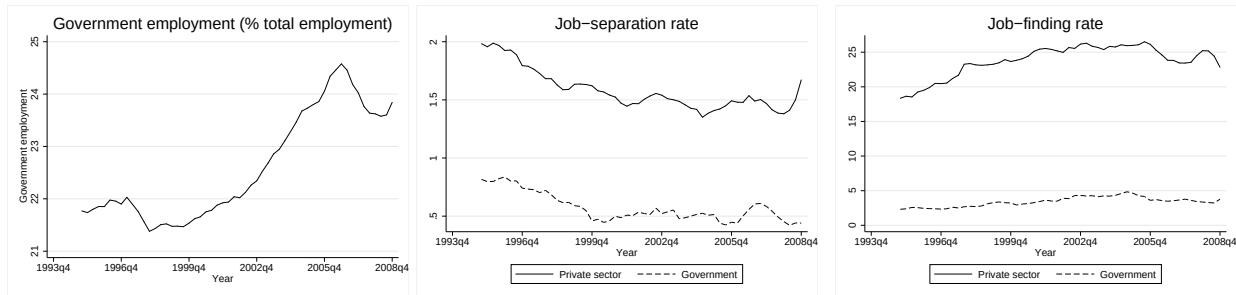
As there is no available information on the vacancies by sector, I estimate the matching elasticity with respect to unemployment by simply regressing the log of the matches by sector on the log of unemployment. The estimated coefficient is 0.4 for the private sector and 0.2 for the government. If we consider a constant returns to scale matching technology these values are in line with the estimates from the United States.

There are other sources of information from the United Kingdom. Recently, the National Audit Office (2009) published a study: “Recruiting Civil Servants efficiently”. They analyse the

⁴See Gomes(2009) for a detailed study on UK labour market flows.

⁵As the LFS is a household survey, the split is based on individuals self reporting whether they work in the public sector and is therefore they are prone to misclassification error.

Figure 2: Evidence for the United Kingdom



recruitment practices of the six largest organizations in central government between 2007 and 2008. They find that, on average it takes 16 weeks to recruit a new member of staff. This number contrasts with one of 12 weeks for the whole economy, found by a study from the Chartered Institute of Personnel and Development (2009). This later study also estimate that the median direct cost of recruiting is £4600 while the estimated total cost of labour turnover is around £5800. This corresponds to between 10 to 12 weeks of the median income in United Kingdom.⁶

3.3 Final calibration

Table 2 shows the baseline calibration and the steady-state values for some of the variables. I calibrate the model at a quarterly frequency to be close to the US economy. I normalize the technology in both sectors to 1 and the discount factor to 0.99.

I set public employment to be 0.15 of the labour force and the public sector wage to be 2% higher than private sector wage ($\pi \equiv \frac{\bar{w}^g}{\bar{w}^p} = 1.02$). This value is in the lower bound of the values found in literature that range between 0 and 10% (Gregory and Borland (1999)). I set the quarterly separation rate in the private sector at 0.06 and in the public sector at 0.03. The private sector matching elasticity with respect to unemployment, η^p , is set to 0.5 while in the public sector is 0.2. I calibrate the matching efficient in the two matching function to target the average time to fill a vacancy of 20 days in the private sector and 30 days in the public sector.

As there is not much evidence on the cost of recruiting in both sector, I consider ς^i to be equal across sectors. Given that the duration of vacancy is higher in the public sector, this assumption implies that the average cost of recruiting is higher than in the private sector. I set it equal to 2, which implies that the sum of the recruitment costs correspond to 3 percent of the total labour costs, value found in Russo, Hassink, and Gorter (2005). It also implies that the cost of recruiting per person is 1.5 months of wages for the private and 2.4 months for the public sector, numbers that consistent with the evidence presented above for the United Kingdom and also with the study by Boca and Rota (1998).

I give the value of 0.5 to the worker's share in the Nash bargaining, so the model satisfies the Hosios condition. The value of leisure in the utility function is calibrated, such that the

⁶ £479 according to the ONS.

unemployment rate in steady-state is 0.06, which implies an outside option equivalent to 43 percent of the average wage. The parameter ι is set to 0.

The empirical evidence relative to the substitution elasticity between private and government consumption is not conclusive. Evans and Karras (1998) find that private consumption and non-military expenditure are in general substitutes while private consumption and military expenditure are complements. Fiorito and Kollintzas (2004), on the other hand, disaggregate expenditure into “public goods” (defence, public order, and justice) and “merit goods” (health, education, and other services) and find that the goods in the first category are substitutes and in the second one are complements to the private sector consumption. As it is hard to select one value for γ , I distinguish three cases: if the goods are substitutes ($\gamma = 0.5$), complements ($\gamma = -0.5$) and one where the elasticity of substitution of 1 ($\gamma = 0.0$). In each case, the parameter ζ is chosen such that the optimal level of public sector employment is 0.15.

Table 2: Calibration

a^p	1	η^p	0.5	ς^p	2.0	$\bar{m}^p$	1.71	λ^p	0.06
a^g	1	η^g	0.2	ς^g	2.0	$\bar{m}^g$	1.97	λ^g	0.03
b	0.5	β	0.99	ι	0.0	χ	0.46	π	1.02
$\gamma = 0$			$\gamma = 0.5$			$\gamma = -0.5$			
$\zeta = 0.18$			$\zeta = 0.43$			$\zeta = 0.08$			
l^g	0.150	q^g	2.5	f^g	0.075	$\frac{\sum \varsigma^i v^i}{\sum w^i l^i}$	0.031	s	0.20
u	6.00	q^p	3.9	f^p	0.79	$\frac{\nu}{u_c w^g}$	0.43	$\frac{W^g - U}{W^p - U}$	2.69

4 Steady-state analysis

4.1 Attaining the steady-state efficient allocation

The efficient steady-state allocation consists of a triplet of $\{\bar{v}^p, \bar{v}^g, \bar{s}\}$.⁷ In order to achieve it, the government can post a number of vacancies consistent with the optimal allocation directly, but it has still to induce an optimal share of the unemployed searching for public sector jobs. The government can do so by choosing an appropriate level of the public sector wage. The Nash bargaining in the private sector, provided that Hosios condition is satisfied, insures an efficient posting of private sector vacancies.⁸

⁷The first natural question to ask is whether efficiency can be achieved if the government privatises the sector. There are two possible answers depending on whether we consider the frictions in the public sector as intrinsic to the sector itself or to the fact that they are publicly run. Consider first, that the frictions are different across sectors because one of them is run by the government. If the sector is privatised, it is not clear that, with a different set of friction parameters, the representative household would be better off. On the other hand, if we consider the frictions are intrinsic to the sector itself, even if the Hosios condition was satisfied in both sectors, the resulting wages would not necessarily be consistent with the optimal s_t , so the competitive equilibrium might not be optimal.

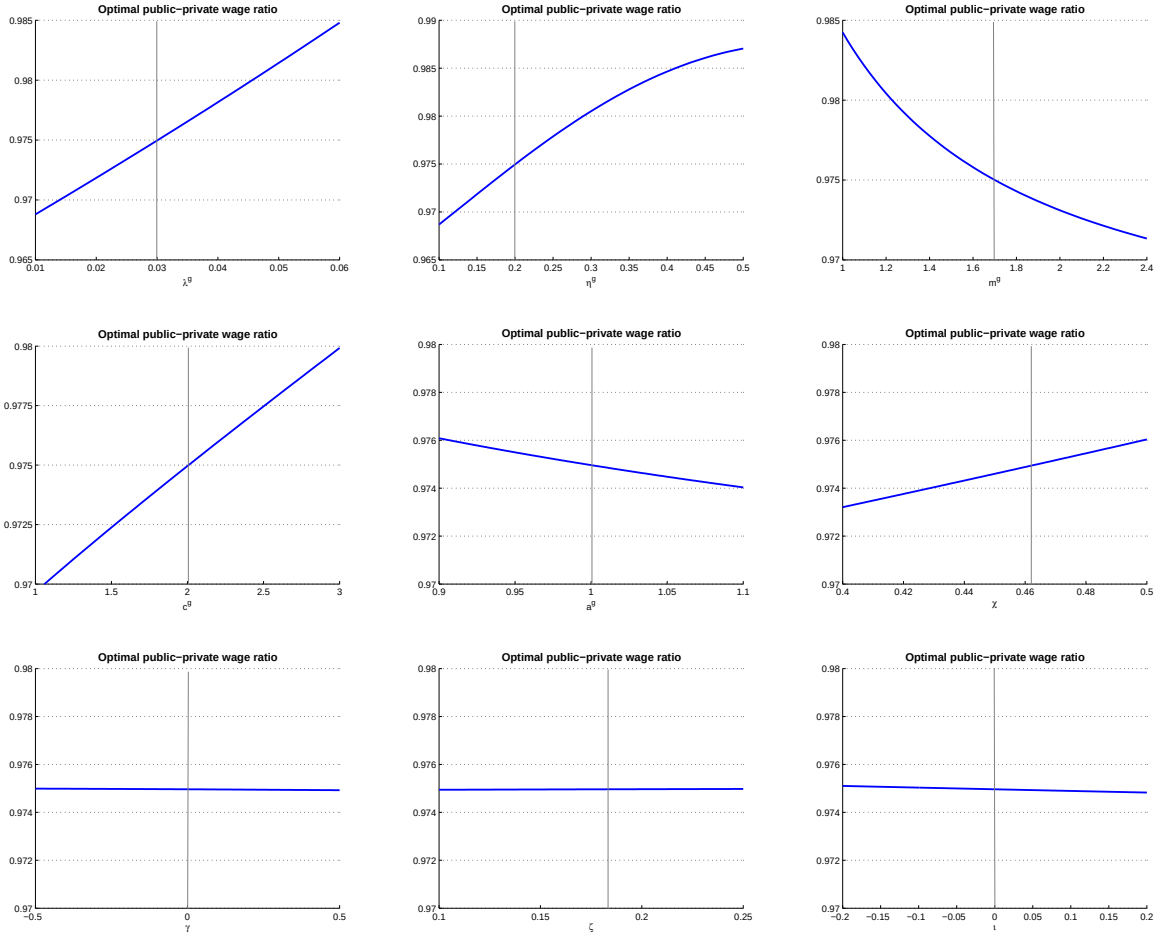
⁸If the government sets its vacancies optimally and a wage such that the share of unemployed searching for public jobs is optimal, the decentralized equilibrium level of private sector vacancies, if the Hosios condition is satisfied, will be efficient. Although I do not prove this statement analytically in the context of this model, I confirmed this in every simulation run.

Let us assume that the government sets the public sector wage, giving a premium over the private sector wage:

$$\bar{w}^g = \pi \bar{w}^p$$

Even though we cannot find an analytical solution for the optimal ratio as a function of the model's parameters, we can find it numerically. If the friction parameters are equal across sectors, the optimal public-private wage ratio is 1 independently of the parameters of the utility function or technology. For the baseline calibration the optimal public sector wage should be 2.5% lower than in the private sector. Figure 3 shows how this value depends on the different parameters.

Figure 3: Optimal public-private wage ratio



As the separation rate in the public sector decreases, the value of working in that sector increases so the unemployed increase the search of public jobs above optimum. The public sector wage has to decline relative to the private sector to ensure that the search effort is optimal.

If the cost of posting vacancies is higher it is better, from a social point of view, to have less vacancies and more people searching for public sector jobs so the optimal wage is higher. The same argument goes through when the public sector matching process depends less on vacancies than the

private sector (higher η^g); it is more efficient to have more unemployed searching for public sector jobs so the government should pay more to its workers. This is also the case when the matching efficiency is lower in the public sector.

From the rest of the graphs we can see that the optimal wage does not depend on γ , ζ and ι but, although in small magnitude, it depends on χ and on the productivity of the public sector. Higher χ , raises the value of employment in the private sector relative to the public sector, because people are more likely to have another spell of unemployment there. All things equal, higher χ induces more unemployed to search in the private sector, so the government should pay higher wages to correct it. If the government is less productive, this makes the relative cost of posting vacancies higher, so the social planner prefers that the matching is driven by the unemployment side, therefore the optimal wage is higher.

Estimates of public sector wage premium have proved quite sensitive to the country choice, empirical specification, education and sex of worker or even the sub-sector of the government. In general, people have found a positive wage premium in the public sector ranging from 0% and 10% (Gregory and Borland (1999)). These values are hard to reconciled with the optimal wage so there is a clear possibility that governments are paying more to their employees than they should.

To investigate the welfare consequences of paying more to public sector employees, I compare the unemployment rate and household's welfare when the public sector wage is optimal (a gap of 2.5%), with the baseline case where the government sets exogenously a premium of 2%. Table 3 shows the optimal level of unemployment, the optimal public-private wage ratio, the unemployment in the baseline case and the welfare gains of moving to the optimal allocation.

Table 3: Welfare costs of high public sector wage

	Optimal	Baseline steady-state		
	steady-state	$\gamma = 0$	$\gamma = 0.5$	$\gamma = -0.5$
Public-private wage ratio	0.975	1.02	1.02	1.02
Unemployment rate	0.051	0.060	0.060	0.060
Share of unemployed searching in public sector	0.051	0.203	0.203	0.203
Potential welfare gains	0%	0.50%	0.65%	1.13%

Note: the welfare gains as a percentage of baseline steady state private consumption good.

The level of unemployment in the baseline case was calibrated to 6%. When the government sets its wage optimally, the unemployment rate drops to 5.1%, independently of the scenarios. This happens because many unemployed that were searching for public sector jobs, now find it more attractive to search in the private sector boasting job creation. We can conclude that the public sector wage is an important determinant of equilibrium unemployment. In terms of welfare, moving to the optimal wage generates a gain of between 0.5% to 1.13% of steady-state consumption.

5 The effects of fiscal shocks

This framework allows us to disentangle the effects of increasing public sector employment from increasing the public sector wage. Within a public employment shock we can distinguish between a shock in hiring or in separations. The fiscal shocks can be represented as

$$\ln(\lambda_t^g) = \ln(\lambda^g) + \epsilon_t^l, \quad w_t^g = \bar{w}^g \quad v_t^g = \bar{v}^g$$

$$\ln(v_t^g) = \ln(\bar{v}^g) + \epsilon_t^v, \quad w_t^g = \bar{w}^g$$

$$\ln(w_t^g) = \ln(\bar{w}^g) + \epsilon_t^w, \quad l_t^g = \bar{l}^g$$

where the shocks ϵ_t^i follow an AR(1) process with autocorrelation coefficient of 0.8. When one of the decision variables is out of equilibrium we have to make an assumption on how the other is set. Under the wage shock I assume the government sets the level of employment as constant.⁹ Under the vacancies shock I assume the government holds the public sector wage constant at its steady-state level.¹⁰ Finally, under the separation shock I consider that both the wage and vacancies are constant at their steady-state level.

I consider a vacancies and separation rate shocks that generates an increase of 6.6 percent public sector employment which corresponds to 1 percentage point of the labour force. The peak in government employment is attained 10 quarters after the shock. For comparison purposes I consider a shock to wages of 6.6%. Figure 4 shows the response of the unemployment rate to the three shocks under the baseline and the optimal steady states. The response of all variables to the three shocks are in appendix.

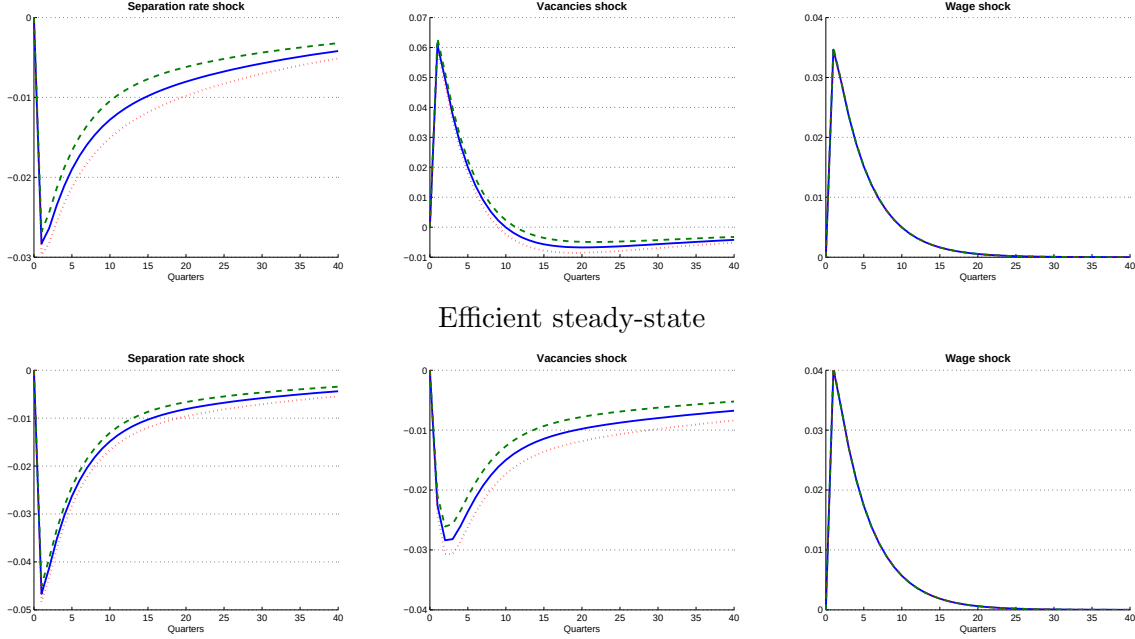
Both separation and vacancy shocks increase public sector employment but crowd out private sector employment through three channels. First, as there are less unemployed available, the cost of hiring an extra worker increases. Second, more unemployed search for public sector jobs because either the probability of getting a job is higher or because the separation rate is lower, which further reduces the probability of filling a vacancy for the firm. Finally, as the overall job-finding probability increases so does the value of being unemployed, which increases the wage in the private sector through the wage bargaining.

Now, the question is whether the crowding out of private employment is partial or whether it outweighs the increase in public employment and raises unemployment. Under a separation shock the effect on unemployment is always negative, but not under a vacancy shock. Starting from the efficient steady-state a shock to hiring reduces unemployment but in the baseline scenario with

⁹I could alternatively assume that the government keeps the level of vacancies constant. If it sets the level of public employment, the government posts less vacancies to attract the same number of workers. If it maintains the same number of vacancies, as more unemployed search for government jobs, public sector employment increases after a wage shock. Under this policy, the shock to wages also incorporates a shock to employment. Although this does not change the results qualitatively, the overall positive effect on unemployment is not as strong.

¹⁰I have also analysed the case where the government sets the premium over the private sector wage as constant which would allow the public sector wage to move after the shock. The results are very similar.

Figure 4: Response of unemployment rate to a fiscal shocks
Baseline steady-state



Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). Variables in logs.

higher steady-state wages, a vacancies shock actually raises unemployment. When the government increases the number of openings, if the wage rate is very high, it makes many more unemployed search in the public sector, thus enhancing the crowding out effect on private sector job creation. In fact, any fiscal shock has a higher crowding out effect when the steady-state wages are higher.

The qualitative results do not depend on γ , and even quantitatively there are small differences between the three cases. If the goods are substitutes the effect on unemployment is smaller. Households reduce more the consumption of the private good, leading to a bigger crowding out of the private sector employment. If the goods are complements the overall effect on unemployment is stronger, as the increase in government services raises the marginal utility of private consumption.

An increase in public sector wage leads to a reduction of private sector employment *via* two channels. On the one hand, the increase of the public sector wage spillover to the private sector wage. The elasticity of the private sector wages with respect to public sector wages is around 0.05. On the other hand, it induces more unemployed to search for a job in the government, which reduces the probability of filling a vacancy for firm in the private sector. As a consequence, the firm posts less vacancies and unemployment rises.

All the fiscal shocks increase the real wage. This happens even in the presence of a negative wealth effect. They crowd out private production, they raise the marginal utility of private consumption, lowering the relative value of leisure. The increase in the probability of finding a job in the public sector or its value is large enough to counterbalance this effect. In terms of magnitude, the effect of the shocks on unemployment is small, between -0.4 to 0.4 percentage points for such

strong fiscal shocks.¹¹

The opposite effects of the different types of fiscal shocks on unemployment is one of the main results of the paper. The extensive literature that tries to understand the macroeconomic effects of fiscal policy finds contradictory effects on real wages, employment and private consumption. Rotemberg and Woodford (1992) find that after a military expenditure shock (government military purchases and military employment) real wages go up but Edelberg, Eichenbaum, and Fisher (1999) and Ramey and Shapiro (1998) find that after a government military purchases shock real wages go down. Blanchard and Perotti (2002), Fatás and Mihov (2001) find that private consumption increases after a government consumption shock but Mountford and Uhlig (2008), Ramey (2008) and Tenhofen and Wolff (2007) report a negative or zero response of private consumption. Most of the discussion has focused on the technical methodology, particularly on the identification of fiscal shocks.

In light of my results, I believe the mixed evidence is not due to methodological issues, but it is related to the data used. Fiscal policy shocks can have different effects depending on the type of expenditure we are considering. The macroeconomic effects of government spending are different if the government increases 1% the wage of all employees or if it increases employment by 1%. The model even suggests that the effects on unemployment can be different depending on how the increase in employment takes place. If we include all components together or have different samples, as in most studies, we are mixing the effects.

6 Public sector policies and the business cycle

6.1 Response under the optimal rule

One of the main conclusions of the Real Business Cycle literature is that governments should not try to pursue active business cycle policies. Although the model presented is, in essence, a real business cycle model with only real frictions, the policy prescription is quite different. Let's examine the effects of a persistent negative private technology shock on the economy, under different government policies. I again consider an AR(1) shock with autoregressive coefficient of 0.8.

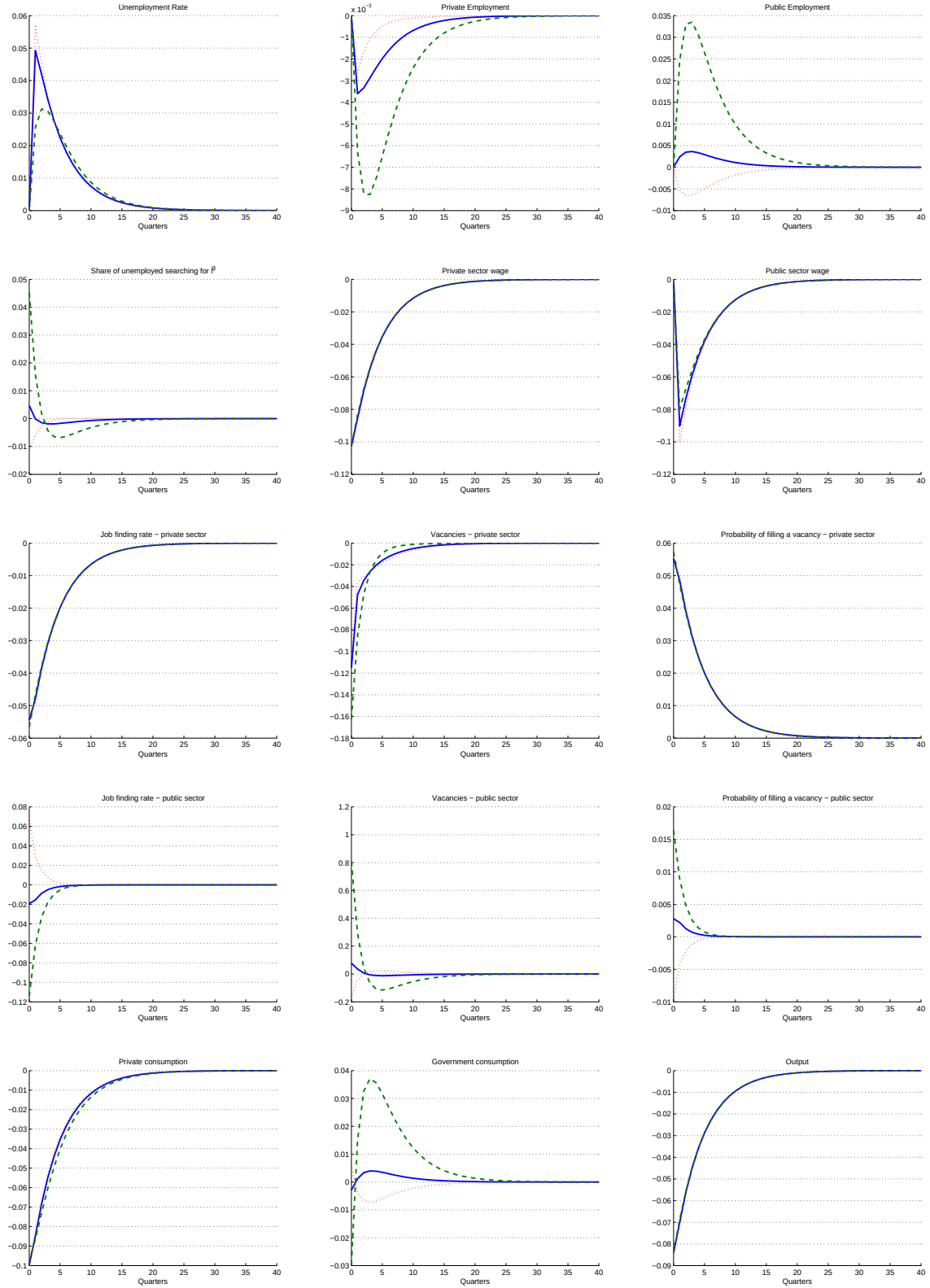
$$\ln(a_t^p) = \ln(\bar{a}^p) + \epsilon_t^a$$

The impulse responses when the government follows the optimal rule are shown in Figure 5. After the negative productivity shock, the private sector posts less vacancies, the probability of finding a job in the private sector decreases so the unemployed increase their search of public sector jobs. The unemployment rate increases.

The optimal response of the government is to increase the number of public sector vacancies and consequently of employment. This result is overturned only in the case where the goods are strong

¹¹As pointed out by Shimer (2005), the search and matching models are not able to generate enough fluctuations on unemployment. This seems to also be the case under fiscal shocks.

Figure 5: Response to a private sector technology shock under the optimal policy



Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.5$) and dotted line ($\gamma = -0.5$). Variables in logs.

complements. In that case, the government should decrease its vacancies. The argument for hiring more people in recessions to work in the public sector is different from the traditional demand argument (the famous metaphor of digging holes and covering them). If the private sector has lower productivity, it is better for the economy to absorb part of the unused labour force into the public sector. If, for example, the government jobs were not productive it would not be optimal to hire anyone in the first place.

On the other hand, under the three scenarios, public sector wage should follow the decline of the private sector wage. In recessions, if public sector wage is constant, it becomes more attractive relative to the private sector wage, therefore increasing the share of unemployed searching for public sector jobs. This further dampens private job creation and amplifies the business cycle. The optimal fiscal policy is, therefore, to have a leaning-against-the-wind countercyclical vacancies policy and a procyclical wage policy.

In their paper, Quadrini and Trigari (2007) concluded that the best policy to stabilize total employment is to have a procyclical public sector employment. This appears counter-intuitive and it is clearly at odds with the optimal policy.¹² In their model, the government does not choose vacancies and wages optimally. Instead, it sets a wage premium exogenously. As we have seen previously, this potentially generates significant deviations from the efficient steady-state and it distorts the response of the model to shocks. Under a high public sector wage premium, the crowding out of private sector employment after a shock to public sector employment can be more than complete, so it can increase unemployment. This switch in the effects of public sector employment on unemployment completely alters the policy recommendations.

6.2 Response under simple rules

Existing evidence by Lane (2003) and Lamo, Pérez, and Schuknecht (2007) suggest that public sector wages are less procyclical than private sector wages, which implies that governments might not be following the optimal policy. I now compare the responses to a technology shock when the government follows simple rules for vacancies and public sector wage of the type:

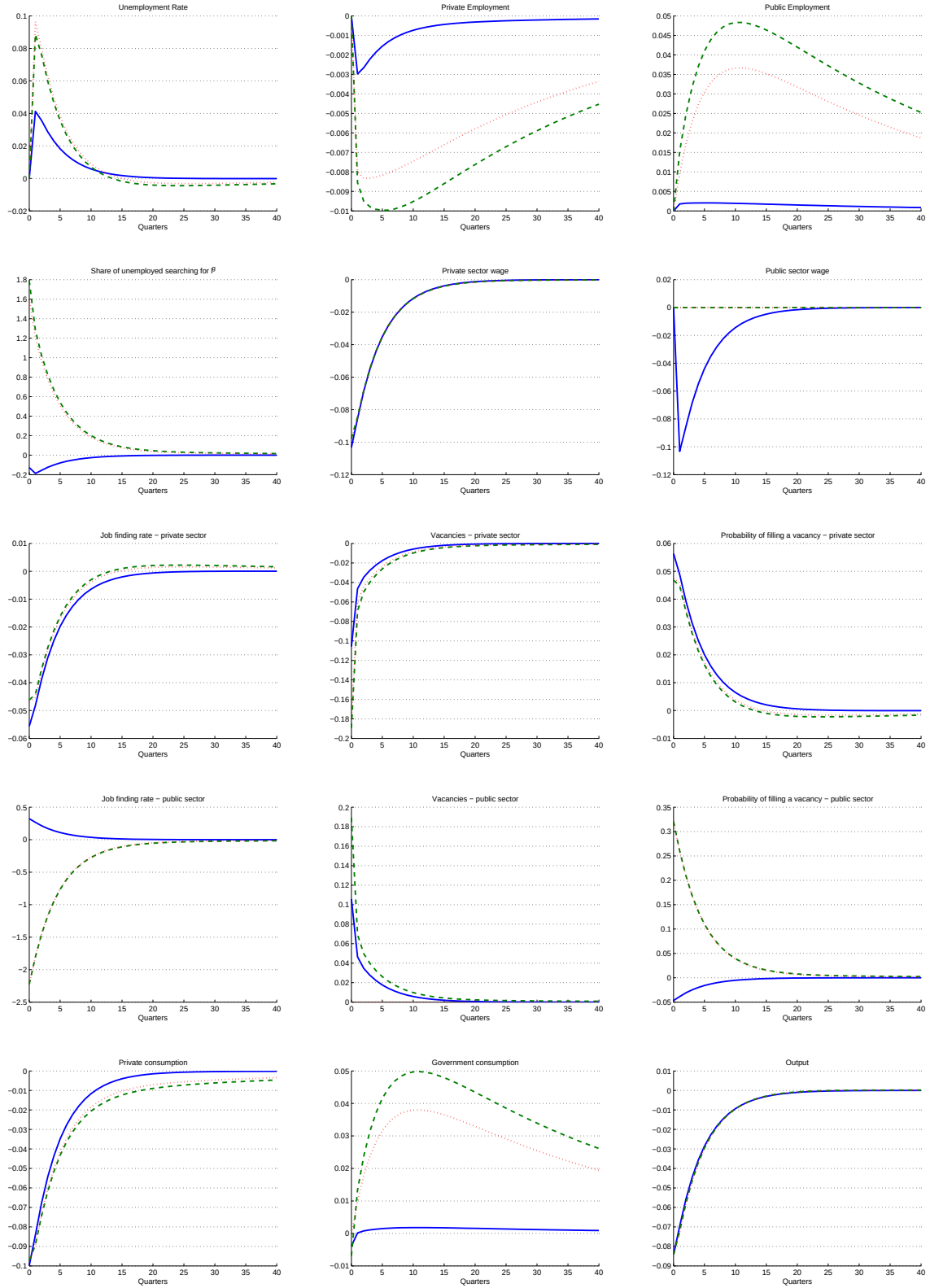
$$\log(v_t^g) = \log(\bar{v}^g) + \psi^v [\log(v_t^p) - \log(\bar{v}^p)] \quad (21)$$

$$\log(w_t^g) = \log(\bar{w}^g) + \psi^w [\log(w_t^p) - \log(\bar{w}^p)] \quad (22)$$

I consider three rules. The first where the public sector vacancies decline proportionally to the private sector vacancies and the public sector wage moves one to one with private sector wages ($\psi^v = -1$ and $\psi^w = 1$), which should mimic closely the optimal policy when $\gamma = 0$. The second rule is a countercyclical public sector vacancies but with constant public sector wage ($\psi^v = -1$ and $\psi^w = 0$). Finally, the case where the governments do not respond to the cycle ($\psi^v = 0$ and $\psi^w = 0$). Figure 6 shows the impulse response functions.

¹²Unless there are strong complementarities between the two goods, which was not their case.

Figure 6: Response to a private sector technology shock under alternative rules



Note: Solid line (countercyclical vacancies and procyclical wages), dash line (countercyclical vacancies and constant wages) and dotted line (constant vacancies and wages). Variables in logs.

When the government does not adjust its wages, the response of unemployment is much stronger. The increase of the search of public sector jobs is much higher than under the optimal policy which amplifies the negative effect on private sector job creation.

Another conclusion of Quadrini and Trigari (2007) was that the presence of public employment in the economy increases the volatility of unemployment. In our model this is not necessarily true. Table 4 compares the standard deviation of the key variables under the different policies, as well as, when there is no public employment. Under the optimal rule, the presence of public sector employment decreases the volatility of unemployment relative to the case where the public sector is absent. This also happens under the simple rule of countercyclical vacancies and procyclical wages. Nevertheless, under the two policies where the public sector wages are acyclical, the volatility of unemployment increases by twofold. The effects of the presence of public sector employment on the volatility of unemployment depends crucially on the government policy.

Table 4: Business cycle properties under the different policies

	Policy	Standard deviations				Correl	Welfare
		l_t^p	l_t^g	u_t	w_t^p	(l_t^g, u_t)	cost
$\zeta = 0$		0.006	—	0.094	0.176	—	1.437%
$\gamma = 0.0$	Opt.	0.007	0.008	0.087	0.176	0.94	1.214%
	Rule 1	0.006	0.010	0.072	0.176	0.564	1.224%
	Rule 2	0.052	0.265	0.151	0.172	0.234	2.397%
	Rule 3	0.041	0.198	0.161	0.173	0.288	2.108%
$\gamma = 0.8$	Opt.	0.019	0.077	0.071	0.174	0.94	1.278%
	Rule 1	0.006	0.011	0.074	0.175	0.557	1.332%
	Rule 2	0.052	0.262	0.155	0.172	0.279	2.156%
	Rule 3	0.040	0.190	0.164	0.172	0.331	2.001%
$\gamma = -1.0$	Opt.	0.004	0.015	0.093	0.179	-0.91	1.545%
	Rule 1	0.006	0.010	0.071	0.178	0.570	1.563%
	Rule 2	0.054	0.269	0.147	0.173	0.185	3.530%
	Rule 3	0.042	0.207	0.157	0.174	0.239	3.030%

The last column of the table gives the welfare costs of business cycles under different rules.¹³ When the public sector is absent, the welfare costs of fluctuations are very small. This is a well known result from the literature. When the public sector is present and the government follows the optimal policy the welfare costs of fluctuations are lower than when the public sector is absent. However, when the government does not follow the optimal policy, the welfare cost of fluctuations increases significantly, particularly if the public sector wages are constant. The cost is around two times higher compared to the optimal policy scenario.

¹³See appendix for details.

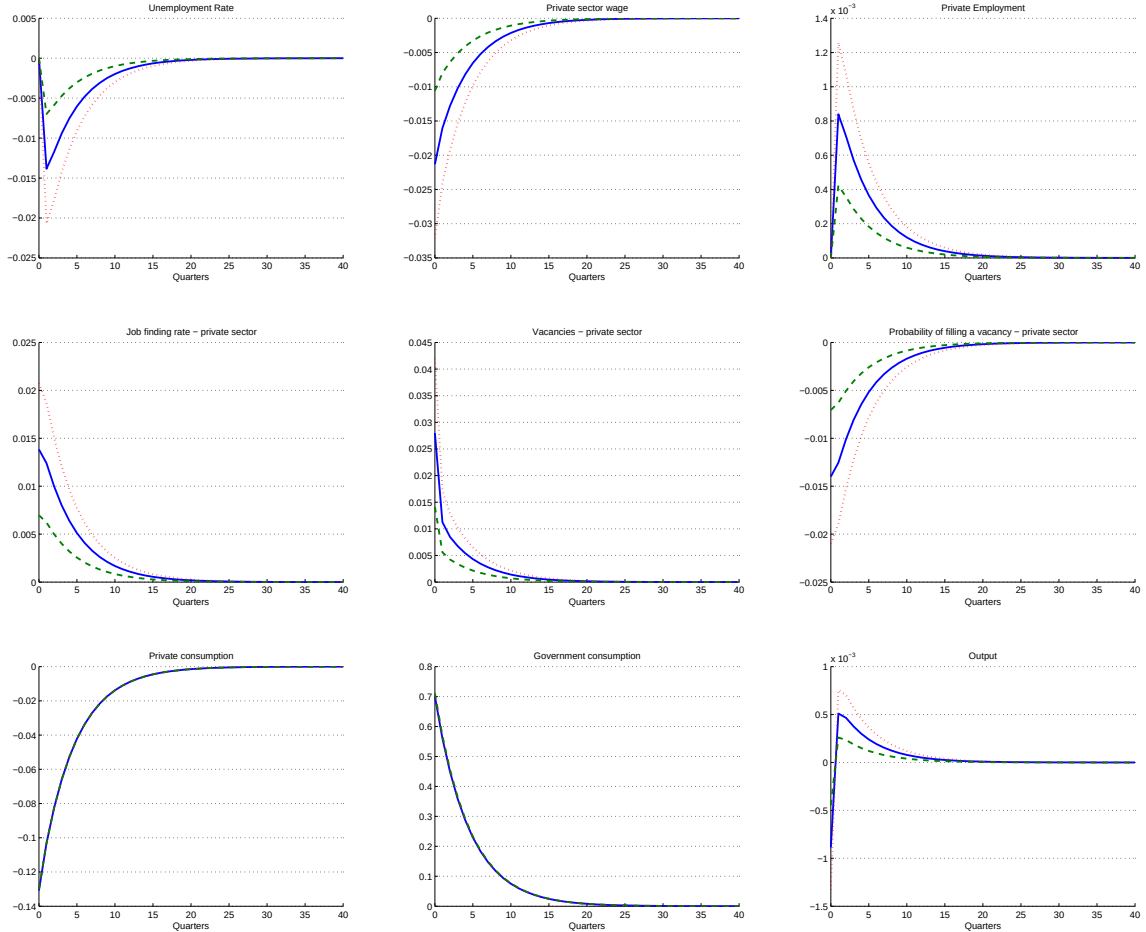
7 Extensions

7.1 Government consumption as goods bought to the private sector

To compare the results of the model with the ones from a typical model of government consumption I construct an extension where there is no public sector employment ($l_t^g = 0$), but where the government can buy its consumption good from the private sector ($c_t + g_t = y_t$). I am particularly interested in the response of the variables to a government consumption shock (Figure 7) and the optimal response of government consumption to a negative technology shock (shown in appendix).

There are two main differences of a government consumption shock relative to an employment or wage shock. Notice that the effects on wages and private employment are the opposite from the baseline model. On the one hand private sector wages go down because the reduction of private consumption raises the marginal utility of consumption, decreasing the value of unemployment. On the other hand, private employment goes up and total unemployment goes down. But the differences are also visible in the optimal business cycle policy. In this setting, in a recession government should buy less goods to the private sector.

Figure 7: Response to a government consumption shock



Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

7.2 Productive public employment

Two papers by Finn (1998) and Cavallo (2005) that have studied the effects of government employment found that they generate negative responses of private employment. This fact is consistent with the model. However, a recent paper by Linnemann (2009) finds, in the context of a VAR, that government employment shocks generate positive responses of private employment. The purpose of this extension is to see if it is possible to generate this positive response of private sector employment within this model, if we consider that public sector employment also affects the productivity of the private sector. I consider that technology in the private sector is as follows:

$$\ln(a_t^p) = \ln(\bar{a}^p) + \alpha[\ln(l_t^g) - \ln(\bar{l}^g)] \quad (23)$$

In appendix I show the response of unemployment and private employment to a separation and vacancies shock for different values of α . For higher levels of α the crowding out effect on private employment is lower and therefore it has a bigger negative impact on unemployment. However even with a value as high as 0.2, the crowding out is still substantial.

8 Review of micro evidence

The theoretical model has one important policy prescription - governments should keep track of the private sector wages over the business cycle. If not, the volatility of unemployment is higher because of the fluctuation in the share of unemployed searching for public sector jobs. It is clear, however, that this result is entirely driven by the directed search assumption. The aim of this section is to review the existing micro evidence on public sector wages that supports it.

Gregory and Borland (1999) undertake the most detailed survey of the literature on public sector labour markets, summarising its main facts. First, most of the studies have found a positive wage premium for public sector employees, although there is not an agreement on the magnitude - it ranges between 0 and 10%. The premium is much higher for females than males, it is higher for veterans and minorities and it is higher for federal government employees than state or local government employees. The authors also report a number of studies that have found the existence of queues for federal public jobs. For example, one study by Venti (1985) finds that for each federal government job opening, there are 2.8 men and 6.1 times as many women that want the job.

Another fact, also mentioned in Gregory and Borland (1999) but with slight less emphasis, is that there are also differences across education levels. Particularly in the last twenty years, more educated individuals tend to be paid less in the public sector while individuals with less education tend to receive a higher premium. This was documented, for instance in Katz and Krueger (1991), who also find that blue collars are willing to queue to obtain public sector jobs whereas highly-skilled workers are hard to recruit and retain in the public sector. A recent study for the United Kingdom from Postel-Vinay and Turon (2007) also reach similar conclusions. They find that once selection has been accounted for, the average value of a job for life is slightly higher in the public sector than

in the private sector, although it is close to zero among workers categorised as high-employability individuals based on their low unobserved propensity to become unemployed. Additionally, they find some evidence of job queuing for public sector jobs among low-employability individuals, whom face larger potential premia from working there.

Most of the studies that estimate the public sector wage premium use switching regression models. The idea is that unemployed can self-select to work in the sectors in which they have more advantages. Blank (1985) finds that sectoral choice is influenced by wage comparison but also by other factors. Heitmueller (2006) is able to quantify this effect and finds that an increase in 1 percent in the wages in the public sector relative to the private sector increases the probability of choosing public sector employment by 1.3 for men and 2.9 percent for women. Pfeifer (2008) finds that more risk averse individuals also sort into public sector employment.

Besides the mechanism of directed search, another important question is how pro-cyclical are public sector wages. As mentioned earlier, macro evidence suggests they are less procyclical than private sector wages. One study by Devereux and Hart (2006) uses micro data to estimate the procyclicality of wages and find that, for job movers, in the private sector the wages are procyclical but for the public sector they are acyclical.

9 Bayesian Estimation of the model

Although the micro evidence supports the endogeneity of the sector choice, it does not necessarily imply that, from a macroeconomic perspective, the mechanism plays a significant role over the business cycle. As a further contribution I estimate a log-linearized version of the model using Bayesian methods as in Smets and Wouters (2007) and Sala, Söderström, and Trigari (2008). Besides evaluating the direct search mechanism, I can also assess the cyclicity of the public sector wages and employment, as well as get estimates for some of the key friction parameters. As in the theoretical section, I assume two simple rules for public sector wages and vacancies where each variable responds to a moving average of the private counterpart:

$$\begin{aligned}\ln(v_t^g) &= \ln(\bar{v}^g) + \psi^v \left[\frac{\sum_{i=0}^3 \ln(v_{t-i}^p)}{4} - \ln(\bar{v}^p) \right] + \ln(\omega_t^v) \\ \ln(w_t^g) &= \ln(\bar{w}^g) + \psi^w \left[\frac{\sum_{i=0}^3 \ln(w_{t-i}^p)}{4} - \ln(\bar{w}^p) \right] + \ln(\omega_t^w)\end{aligned}$$

Following one of the extensions, I allow the private sector technology to depend partially on the level of public sector employment.

$$\ln(a_t^p) = \ln(\bar{a}^p) + \alpha(\ln(l_t^g) - \ln(\bar{l}^g)) + \ln(\omega_t^a)$$

In order to test the relevance of the directed search mechanism, I modify the equation determining the endogenous search of public sector jobs (10). The log-linearised expression is:

$$\tilde{s}_t = \kappa(1 - \bar{s})E_t(\tilde{x}_{t+1}^g - \tilde{x}_{t+1}^p - \tilde{m}_t^p + \tilde{m}_t^g) \quad (24)$$

From the original expression, I have added the parameter κ that measures the significance of the mechanism. If it is close to 0, means that the data does not support the assumption and that the share of unemployed searching for a job in the public sector does not fluctuate over the business cycle.

I use quarterly data on US from 1948:1 to 2007:1 for 6 variables: (1) unemployment rate, (2) government employment (% of labour force), (3) government per employee real wage, (4) private sector per hour real wage, (5) aggregate job-separation rate and (6) aggregate unemployed job-finding rate. The series of government per employee real wage is calculated by dividing total compensation of government workers from NIPA tables, by total government employment. The monthly job-finding and job-separation rates are taken from Shimer (2007) and they are transformed into quarterly, by allowing for multiple transitions between the two states within the quarter.¹⁴ All other variables are taken from the Bureau of Labor Statistics.

I include 6 different shocks: a shock to government vacancies, to government wages, to private and public separation rates, private sector bargaining power and to technology. These shocks are described in the following equations:

$$\begin{aligned}\ln(\omega_t^v) &= \rho^v \ln(\omega_{t-1}^v) + \epsilon_t^v \\ \ln(\omega_t^w) &= \rho^w \ln(\omega_{t-1}^w) + \epsilon_t^w \\ \ln(\omega_t^a) &= \rho^a \ln(\omega_{t-1}^a) + \epsilon_t^a \\ \ln(\lambda_t^g) &= (1 - \rho^{lg}) \ln(\bar{\lambda}^g) + \rho^{lg} \ln(\lambda_{t-1}^g) + \epsilon_t^{lg} \\ \ln(\lambda_t^p) &= (1 - \rho^{lp}) \ln(\bar{\lambda}^p) + \rho^{lp} \ln(\lambda_{t-1}^p) + \epsilon_t^{lp} \\ \ln(b_t) &= (1 - \rho^b) \ln(\bar{b}) + \rho^b \ln(b_{t-1}) + \epsilon_t^b\end{aligned}$$

With the exception of the wages, all other variables are stationary. I take advantage of this and use four different transformations of the data to estimate the model. In the first version the stationary variables enter in levels and the wages enter in log differences. In the second version all variables enter in log differences. In the third version all variables are HP filtered and in the last version they are detrended by regressing the log variables on a linear, quadratic and cubic trend. All the equations of the model in its log-linearized form and the relation of the observable variables with the model variables in each version can be found in appendix.

I calibrate β to 0.99, the utility function parameter ζ to be equal to 1.8 and I normalise technology in both sector to 1. In each iteration, the steady-state public sector vacancies are set such that in equilibrium the public employment rate is 0.15. The steady-state public sector wages are set as a premium over the private sector wages. Exploratory studies suggested that two parameters cannot be identified. The cost of posting a vacancy in the public sector only affects the model through the the stochastic discount factor and the scale parameter for the public

¹⁴For example, the job-finding rate is calculated as $f^q = f^m(1 - \lambda^m)^2 + f^m \lambda^m f^m + (1 - f^m)^2 f^m(1 - f^m) f^m(1 - \lambda^m)$

sector matching function cannot be pinned down because the public sector vacancies target public employment. In each iteration I set the matching efficiency such that the duration of a vacancy is 30 days and the cost of recruiting equal to the private sector. I estimate all other parameters.

I assume the matching elasticity with respect to unemployment, the productivity of government employment, the steady-state bargaining power of the unemployed and the autoregressive coefficients of the shock process, to have a Beta distribution. I assume that the standard deviations of the shock process have an inverse gamma distribution. To be completely agnostic relative to the direct search mechanism, I assume that κ is uniformly distributed between 0 and 1. All other parameters are assumed to be normally distributed.

Given the strong evidence presented in Section 3 the prior mean for the separation rates is 0.06 for the private and 0.03 for the public sector. However, as the values for the matching elasticity in the public sector were given by a *back-of-the-envelope* calculation, I start with the prior that the means and standard deviation are the same across sectors. The prior distribution of the business cycle policy parameters is centered around 0.

I estimate the model with Bayesian methods (see An and Schorfheide (2007) for a review). The likelihood function of the model is combined with the prior distribution of the parameters, to obtain the posterior distribution. Then, 500,000 draws of the posterior are generated with the Metropolis Hastings algorithm, where the step size is chosen such that the acceptance rate is equal to 1/3. The draws are divided into five chains of 100,000 draws each in order to evaluate the stability of the sample. Table 5 and 6 report the prior distribution and the mean, the 5th and the 95th percentile of the posterior distribution of parameters.

The posterior distributions are close across the 4 alternative versions for most of the parameters. The elasticity of the matching function with respect to unemployment is much lower in the public sector. The estimated mean value for the private sector is around 0.65, but only 0.2 in the public sector. The efficiency of the private sector matching function seems to be close to the prior but the mean of the posterior distribution of the cost of posting a vacancy is lower.

The mean of the posterior distribution of κ , is between 0.43 and 0.57 depending on the transformation of the data used, suggesting that although s_t does not fluctuate as much as the model predicts, the mechanism still has explanatory power.

On the policy side, there is a mild countercyclical policy in vacancies with an estimated mean between -0.2 and -0.45 . On the other hand, evidence on public sector wage policy is mixed. Two versions estimate ψ^w around 0.2 but the other two estimate the mean around zero. It seems that public sector wages are, at most, slightly procyclical.

The estimated mean for the bargaining power is around 0.85, slightly lower than the value found in Sala, Söderström, and Trigari (2008). The posterior distribution of the flow value of unemployment is between 0.30 and 0.47. The posterior distribution of γ suggests that there are

Table 5: Prior and posterior distribution of structural parameters

		Prior distribution	Levels	Posterior distribution		Det.
				Differences	HP	
a) Structural parameters						
Elasticity of substitution (public and private goods)	γ	Normal (0,0.1)	-0.024 (-0.135,0.091)	0.068 (-0.112,0.213)	0.143 (-0.012,0.265)	0.159 (0.008,0.301)
Disutility of work	χ	Normal (0.5,0.1)	0.361 (0.228,0.517)	0.399 (0.289,0.510)	0.353 (0.188,0.527)	0.310 (0.192,0.433)
Labour supply elasticity	ι	Normal (0.0,0.05)	-0.052 (-0.125,0.032)	-0.021 (-0.086,0.053)	-0.024 (-0.087,0.052)	-0.001 (-0.069,0.064)
Cost of posting vacancy	ς^i	Normal (2,0.5)	1.860 (1.361, 2.402)	1.366 (0.502,2.028)	1.111 (0.470,1.698)	1.287 (0.628,1.876)
Matching efficiency (private sector)	m^p	Normal (2,0.2)	1.884 (1.631,2.153)	1.985 (1.765,2.199)	2.186 (1.913,2.428)	2.162 (1.925,2.398)
Matching elasticity w.r.t unemployment - private sector	η^p	Beta (0.5,0.10)	0.761 (0.700,0.819)	0.713 (0.648,0.780)	0.623 (0.560,0.729)	0.583 (0.513,0.658)
Matching elasticity w.r.t unemployment - public sector	η^g	Beta (0.5,0.10)	0.2442 (0.146,0.338)	0.214 (0.119,0.320)	0.215 (0.140,0.284)	0.161 (0.097,0.221)
Separation rate - private sector	λ^p	Normal (0.06,0.005)	0.057 (0.048,0.066)	0.061 (0.054,0.068)	0.048 (0.041,0.056)	0.051 (0.041,0.059)
Separation rate - public sector	λ^g	Normal (0.03,0.005)	0.031 (0.022,0.039)	0.028 (0.023,0.032)	0.038 (0.032,0.044)	0.035 (0.029,0.042)
Importance of direct search	κ	Uniform [0,1]	0.481 (0.377,0.586)	0.433 (0.333,0.535)	0.571 (0.478,0.659)	0.543 (0.442,0.649)
Productivity of public employment	α	Beta (0.1,0.2)	0.134 (0.000,0.251)	0.205 (0.087,0.321)	0.043 (0.000,0.133)	0.174 (0.000,0.285)
Bargaining power	b	Beta (0.5,0.10)	0.880 (0.838,0.921)	0.868 (0.825,0.911)	0.865 (0.813,0.911)	0.856 (0.811,0.899)
Public sector wage premium	π	Normal (1.02,0.005)	1.023 (1.0166,1.031)	1.020 (1.014,1.027)	1.019 (1.013,1.026)	1.019 (1.012,1.025)
Business cycle response of public sector wages	ψ^w	Normal (0.0,0.2)	0.155 (-0.062,0.385)	0.225 (0.021,0.467)	0.017 (-0.204,-0.191)	-0.043 (-0.234,0.162)
Business cycle response of public sector vacancies	ψ^v	Normal (0.0,0.2)	-0.218 (-0.571,0.203)	-0.445 (-0.753,0.017)	-0.256 (-0.449,-0.053)	-0.387 (-0.626,-0.139)

no particular complementarity or substitutability between the two goods in the utility function. The estimates of the posterior distribution of α are centered around 0.15. This value suggests that public employment might increase the productivity of the private sector or, alternatively it might be capturing demand effects that are absent from the model.

To evaluate the stability of the crucial parameters, I estimate the models for three subsamples of roughly 20 years: 1948:1 to 1967:3, 1967:4 to 1987:2 and 1987:3 to 2007:1. The results are summarized graphically in appendix. The parameter κ was important in the first two periods (posterior mean close to 0.6), but its importance has somewhat diminished during the period of the great moderation. The parameter α was also quite high during the first period, suggesting that there were strong complementarities in the production function. These complementarities have disappeared in the last two decades, as the estimated mean was close to zero.

Regarding the policy parameters, ψ^w seems to have become more procyclical in the last subsample, according to the estimation in levels and in differences, while ψ^v does not show any particular trend. The elasticity of the matching with respect to unemployment in the public sector is quite

Table 6: Prior and posterior distribution of shock parameters

		Prior distribution	Levels	Posterior distribution		
				Differences	HP	Det.
b) Autoregressive parameters						
Productivity	ρ^a	Beta	0.987	0.986	0.668	0.887
		(0.5,0.15)	(0.979,0.994)	(0.979,0.994)	(0.580,0.743)	(0.846,0.928)
Public sector wage	ρ^w	Beta	0.981	0.979	0.790	0.943
		(0.5,0.15)	(0.970,0.993)	(0.966,0.992)	(0.741,0.848)	(0.921,0.966)
Public sector vacancies	ρ^v	Beta	0.603	0.491	0.196	0.340
		(0.5,0.15)	(0.418,0.972)	(0.361,0.621)	(0.107,0.274)	(0.238,0.444)
Private sector separation rate	ρ^{lp}	Beta	0.956	0.942	0.771	0.896
		(0.5,0.15)	(0.932,0.981)	(0.913,0.971)	(0.705,0.844)	(0.855,0.938)
Public sector separation rate	ρ^{lg}	Beta	0.412	0.506	0.366	0.548
		(0.5,0.15)	(0.127,0.802)	(0.284,0.741)	(0.182,0.561)	(0.317,0.773)
Bargaining power	ρ^b	Beta	0.933	0.934	0.802	0.875
		(0.5,0.15)	(0.882,0.984)	(0.890,0.984)	(0.739,0.873)	(0.834,0.918)
c) Standard deviations						
Productivity	σ^a	IGamma	0.008	0.008	0.007	0.008
		(0.05,0.15)	(0.007,0.008)	(0.007,0.009)	(0.006,0.007)	(0.007,0.008)
Public sector wage	σ^w	IGamma	0.011	0.011	0.010	0.011
		(0.05,0.15)	(0.011,0.012)	(0.010,0.012)	(0.009,0.010)	(0.010,0.012)
Public sector vacancies	σ^v	IGamma	0.193	0.258	0.158	0.179
		(0.05,0.15)	(0.048,0.279)	(0.196,0.325)	(0.125,0.192)	(0.136,0.218)
Private sector separation rate	σ^{lp}	IGamma	0.078	0.067	0.064	0.066
		(0.05,0.15)	(0.063,0.098)	(0.061,0.072)	(0.059,0.068)	(0.061,0.071)
Public sector separation rate	σ^{lg}	IGamma	0.046	0.010	0.011	0.012
		(0.05,0.15)	(0.002,0.080)	(0.002,0.019)	(0.002,0.020)	(0.002,0.023)
Bargaining power	σ^b	IGamma	0.014	0.014	0.011	0.010
		(0.05,0.15)	(0.008,0.019)	(0.008,0.019)	(0.006,0.015)	(0.006,0.014)
Log density			4154.8	2950.4	3149.8	3041.8
Log density (Random Search model)			4134.4	2782.0	3047.8	3034.3

stable between 0.2 and 0.4, while in the private sector, the estimated mean in the first period is around 0.7, while in the two subsequent periods, it is slightly above 0.5.

10 Evidence from structural VAR

In this section I estimate a VAR model to understand how does the economy respond to exogenous shocks in public sector employment and wages, and whether or not the response of the government policy to a technology shocks is close to the optimal policy.

10.1 VAR Setting

The structural VAR is given by

$$A\mathbf{Y}_t = C(L)\mathbf{Y}_{t-1} + B\mathbf{v}_t$$

$$\mathbf{Y}_t = \begin{bmatrix} \pi_t \\ l_t^g \\ w_t^g \\ w_t^p \\ h_t^p \end{bmatrix}$$

Where $\mathbf{Y}_t$ is the vector of macroeconomic variables. It includes five variables: private sector hours h_t^p , private sector wage w_t^p , government wage w_t^g , government employment l_t^g and productivity π_t . Matrix A describes the contemporaneous relation among the variables and $C(L)$ is a matrix finite-order lag polynomial. $\mathbf{v}_t$ is the vector of structural disturbances and matrix B reflect the disturbances variance and possible covariance. From the reduce form estimation residuals $\boldsymbol{\mu}_t$ by imposing restrictions on matrices A and B we can back up the structural innovations.

$$A\boldsymbol{\mu}_t = B\mathbf{v}_t$$

I consider the Cholesky recursive decomposition where:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 1 & 0 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 & 0 \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix}$$

As we are interested in identifying productivity, government employment and government wages shocks, only the ordering on the first three variables matter. I order government wages after employment, because a shock to employment affects the average wage by an definition. I allow both of them to respond contemporaneously to a productivity shock.

The data are taken mostly from the Bureau of Labour Statistics and the Bureau of economic analysis. The baseline variables are “business sector: hours of all persons” h_t^p , “business sector: compensation per hour” w_t^p , “government per employee nominal wage” w_t^g , government employment l_t^g and “business sector: output per hour of all persons” π_t .

The overall sample starts in 1950. Under the baseline case I take the natural logarithm of the variables and I HP filter the data so all variables enter as percentage deviations from trend. I estimate the VAR with two lags (which was pointed by final prediction error, Akaike’s, Schwarz’s Bayesian and the Hannan and Quinn information criterion).

10.2 Impulse responses

Figure 8 shows the impulse responses to the three shocks: productivity shock, government employment shock and government wage shock. The error bands correspond to a 90% confidence interval.

After a productivity shock, private sector wage increase contemporaneously (with an elasticity of 1/6) but the effect dies out quickly. The contemporaneous response of private hours is not statistically significant. However, it picks up in the following quarters and stay above zero for more than 2 years. Government variables do not respond significantly to productivity shocks.

Government employment increases after a productivity shock but the error bands are quite large. Government wage, if any, responds negatively after the shock.

A government employment shock has the expected positive significant effect on private sector wage. Nevertheless, the effect on private hours is not negative as expected - it is statistically not different from zero. One possible explanation for this is that the government services produced with public employment are complement to the private sector. Either because the goods are complements in the utility function or because public employment raises the productivity of the private sector. The positive response of productivity to a government employment shock gives some support for this hypotheses. Alternatively, there might be a demand channel which pushes the private sector production. At the time of the government employment shock, average government wage tends to diminished but soon picks up and stays above level for around 4 years.

A government wage shock does impinge into private sector wage, with an elasticity of around 0.11%. It has also a negative effect on private sector total hours although it is not statistically significant. Overall, the model's responses to both public employment and wages seem consistent with the data.

10.3 Robustness check

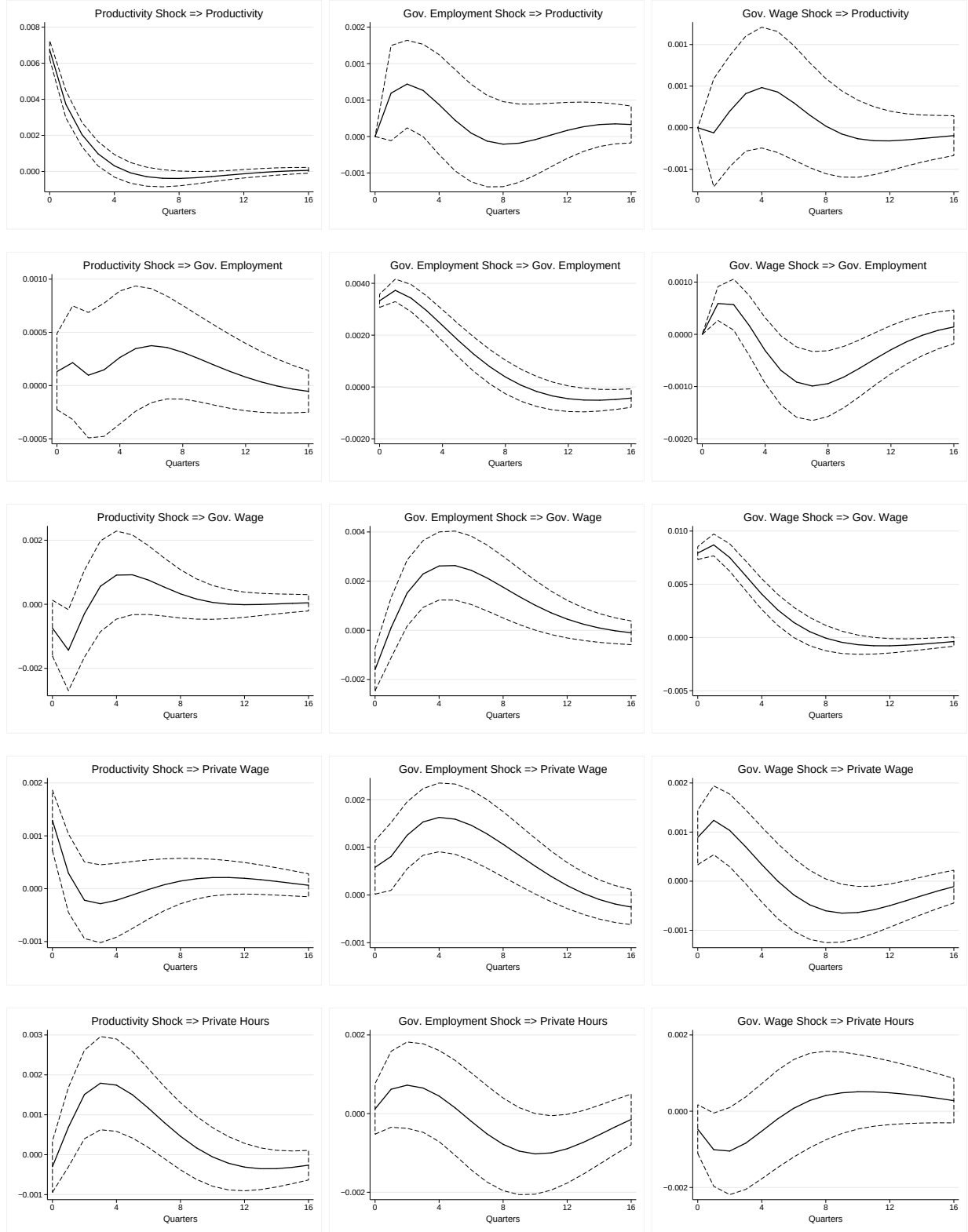
To access the robustness of the results I run several alternative models. I use alternative variables: real wages instead of nominal wages and, as an alternative to total private hours, I use both unemployment rate and vacancies. I consider different subsamples: splitting the sample in two, as well as restricting it to the last 20 years. I try a different ordering of the variables, with productivity lining up after government employment and government wages. As an alternative to the HP filter, I use other detrending methods: I estimate the VAR with the variables in first differences and compute the cumulative impulse responses and I use a linear and quadratic detrending of the variables. Finally, I have also estimated the VAR with different lag lengths. The impulse responses of selected variables are shown in Appendix.

In general, the results are in line with those of the baseline case. There are, however, some results worth mentioning. The effect of a government employment shock on private real wages is not statistical different from zero, suggesting that the increase in nominal wage from the baseline case is somehow deteriorated by increasing inflation. As in the baseline case, government real wage shock impinges on private sector real wages but the effect on hours in not significant.

When we restrict the sample to last 20 year, total hours decline after both a government employment and wage shocks, which is more in line with the predictions of model. The properties of government employment might be intrinsically different now from the ones in previous decades. Also the effect of government wage on private wage is much stronger (elasticity of around 0.5%).

When we use the unemployment rate, instead of private hours, it goes down after both productivity and a government employment shock. If we use vacancies, they increase after a government

Figure 8: Baseline VAR



employment shock, and also after a government wage shock. This mixed evidence on the effect of public sector employment and wages on private sector might indicate that there are other channels playing a role.

Estimating the VAR with different lag lengths, running it in differences or after different detrending methods and ordering of the variables does not alter the main results.

11 Conclusion

This paper sheds some light on the links between public and private sectors focussing in the labour market. I have built a dynamic stochastic general equilibrium model with search and matching frictions, to analyse the effects of fiscal policies in the labour market and to determine the optimal employment and wage policy. I want to highlight three main conclusions.

First, the public sector wage plays an important role in archiving the steady-state efficient allocation. If the government sets a very high wage, more unemployed direct their search towards public sector jobs, private sector wage is higher and the job creation lower. As a consequence, equilibrium unemployment is higher entailing a welfare loss for the representative consumer.

The second conclusion is that the response of unemployment to fiscal shocks depends on whether it is a shock to employment or to wages. A shock to public sector employment tends to decrease unemployment while a shock to public sector wages always increases it. Both shocks increase the wage and crowd out employment in the private sector. These effects contrast with the case where the government buys goods to the private sector. In such scenario, a fiscal shock increases private employment and decreases the private sector wage. The mixed effects of the different components of government consumption on the labour market, might be one reason why many empirical studies on the macroeconomic effects of government spending find ambiguous results.

Finally, the optimal fiscal response to business cycles is to have a “leaning-against-the-wind” public employment policy and a procyclical public sector wage. Empirical evidence on the US suggest that, particularly, public sector wages do not follow the optimal policy. Deviations from the optimal policy can substantially increase unemployment volatility and the welfare costs of fluctuations.

Recent research has emphasised the importance of the interaction between labour market friction and nominal rigidities for the understanding of business cycles and the effects of monetary policy (for instance: Thomas (2008), Krause, Lopez-Salido, and Lubik (2008), Sveen and Weinke (2008) and Blanchard and Gali (2008)). I show that the presence of labour market frictions also increases the scope of action for governments and the effects of fiscal policy, even in the absence of nominal rigidities.

The model’s key mechanism is the endogenous choice of sector in which the unemployed are searching. Although the estimation suggested that the importance of the mechanism has diminished

during the period of the great moderation, current events lead me to believe that the mechanism is playing significant role during the current recession. A casual look in the newspaper gives the impression that unemployed are turning to the public sector to look for a job, but also that the wages there have not suffered as much as in the private sector. Albeit great praise for their reactions against the economic crises, governments can still do better.

References

- ALGAN, Y., P. CAHUC, AND A. ZYLBERBERG (2002): “Public employment and labour market performance,” *Economic Policy*, 17(34), 7–66.
- AN, S., AND F. SCHORFHEIDE (2007): “Bayesian Analysis of DSGE Models,” *Econometric Reviews*, 26(2-4), 113–172.
- ARDAGNA, S. (2007): “Fiscal policy in unionized labor markets,” *Journal of Economic Dynamics and Control*, 31(5), 1498–1534.
- BARRO, R. J. (1990): “Government Spending in a Simple Model of Endogenous Growth,” *Journal of Political Economy*, 98(5), S103–26.
- BAXTER, M., AND R. G. KING (1993): “Fiscal Policy in General Equilibrium,” *American Economic Review*, 83(3), 315–34.
- BLANCHARD, O., AND J. GALI (2008): “Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment,” NBER Working Papers 13897, National Bureau of Economic Research, Inc.
- BLANCHARD, O., AND R. PEROTTI (2002): “An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output,” *The Quarterly Journal of Economics*, 117(4), 1329–1368.
- BLANK, R. M. (1985): “An analysis of workers’ choice between employment in the public and private sectors,” *Industrial and Labor Relations Review*, 38(2), 211–224.
- BOCA, A. D., AND P. ROTA (1998): “How Much Does Hiring and Firing Cost? Survey Evidence from a Sample of Italian Firms,” *LABOUR*, 12(3), 427–449.
- CALDARA, D., AND C. KAMPS (2008): “What are the effects of fiscal shocks? A VAR-based comparative analysis,” Working Paper Series 877, European Central Bank.
- CAVALLO, M. (2005): “Government employment expenditure and the effects of fiscal policy shocks,” Working Paper Series 2005-16, Federal Reserve Bank of San Francisco.
- CHARTERED INSTITUTE OF PERSONNEL AND DEVELOPMENT (2009): “Recruitment, retention and turnover survey report,” Available at <http://www.cipd.co.uk/subjects/recruitmen/general/>.

- DAVIS, S. J., R. J. FABERMAN, AND J. HALTIWANGER (2009): “The flow approach to labor markets: new data sources and micro-macro links,” Discussion paper.
- DEVEREUX, P. J., AND R. A. HART (2006): “Real wage cyclicalities of job stayers, within-company job movers, and between-company job movers,” *Industrial and Labor Relations Review*, 60(1), 105–119.
- EDELBERG, W., M. EICHENBAUM, AND J. D. FISHER (1999): “Understanding the Effects of a Shock to Government Purchases,” *Review of Economic Dynamics*, 2(1), 166–206.
- EVANS, P., AND G. KARRAS (1998): “Liquidity Constraints and the Substitutability between Private and Government Consumption: The Role of Military and Non-military Spending,” *Economic Inquiry*, 36(2), 203–14.
- FATÁS, A., AND I. MIHOV (2001): “The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence,” CEPR Discussion Papers 2760, C.E.P.R. Discussion Papers.
- FINN, M. G. (1998): “Cyclical Effects of Government’s Employment and Goods Purchases,” *International Economic Review*, 39(3), 635–57.
- FIORITO, R., AND T. KOLLINTZAS (2004): “Public goods, merit goods, and the relation between private and government consumption,” *European Economic Review*, 48(6), 1367–1398.
- GALÍ, J., J. D. LÓPEZ-SALIDO, AND J. VALLÉS (2007): “Understanding the Effects of Government Spending on Consumption,” *Journal of the European Economic Association*, 5(1), 227–270.
- GREGORY, R. G., AND J. BORLAND (1999): “Recent developments in public sector labor markets,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3 of *Handbook of Labor Economics*, chap. 53, pp. 3573–3630. Elsevier.
- HEITMUELLER, A. (2006): “Public-private sector pay differentials in a devolved Scotland,” *Journal of Applied Economics*, 0, 295–323.
- HOLMLUND, B., AND J. LINDEN (1993): “Job matching, temporary public employment, and equilibrium unemployment,” *Journal of Public Economics*, 51(3), 329–343.
- HÖRNER, J., L. R. NGAI, AND C. OLIVETTI (2007): “Public Enterprises And Labor Market Performance,” *International Economic Review*, 48(2), 363–384.
- KATZ, L. F., AND A. B. KRUEGER (1991): “Changes in the Structure of Wages in the Public and Private Sectors,” NBER Working Papers 3667, National Bureau of Economic Research, Inc.
- KRAUSE, M. U., D. LOPEZ-SALIDO, AND T. A. LUBIK (2008): “Inflation dynamics with search frictions: A structural econometric analysis,” *Journal of Monetary Economics*, 55(5), 892–916.
- LAMO, A., J. J. PÉREZ, AND L. SCHUKNECHT (2007): “The cyclicalities of consumption, wages and employment of the public sector in the euro area,” Working Paper Series 757, European Central Bank.

- LANE, P. R. (2003): “The cyclical behaviour of fiscal policy: evidence from the OECD,” *Journal of Public Economics*, 87(12), 2661–2675.
- LINNEMANN, L. (2009): “Macroeconomic effects of shocks to public employment,” *European Economic Review*.
- LINNEMANN, L., AND A. SCHABERT (2003): “Fiscal Policy in the New Neoclassical Synthesis,” *Journal of Money, Credit and Banking*, 35(6), 911–29.
- LUDVIGSON, S. (1996): “The macroeconomic effects of government debt in a stochastic growth model,” *Journal of Monetary Economics*, 38(1), 25–45.
- MERZ, M. (1995): “Search in the labor market and the real business cycle,” *Journal of Monetary Economics*, 36(2), 269–300.
- MOUNTFORD, A., AND H. UHLIG (2008): “What are the Effects of Fiscal Policy Shocks?,” NBER Working Papers 14551, National Bureau of Economic Research, Inc.
- NATIONAL AUDIT OFFICE (2009): “Recruiting Civil Servants efficiently,” Available at http://www.nao.org.uk/publications/0809/recruiting_civil_servants_effi.aspx.
- PAPPA, E. (2005): “New-Keynesian or RBC Transmission? The Effects of Fiscal Shocks in Labour Markets,” CEPR Discussion Papers 5313, C.E.P.R. Discussion Papers.
- PETRONGOLO, B., AND C. A. PISSARIDES (2001): “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 39(2), 390–431.
- PFEIFER, C. (2008): “Risk Aversion and Sorting into Public Sector Employment,” IZA Discussion Papers 3503, Institute for the Study of Labor (IZA).
- PISSARIDES, C. A. (1988): “The Search Equilibrium Approach to Fluctuations in Employment,” *American Economic Review*, 78(2), 363–68.
- POSTEL-VINAY, F., AND H. TURON (2007): “The Public Pay Gap in Britain: Small Differences That (Don’t?) Matter,” *Economic Journal*, 117(523), 1460–1503.
- QUADRINI, V., AND A. TRIGARI (2007): “Public Employment and the Business Cycle,” *Scandinavian Journal of Economics*, 109(4), 723–742.
- RAMEY, V. A. (2008): “Identifying Government Spending Shocks: Its All in the Timing,” Discussion paper.
- RAMEY, V. A., AND M. D. SHAPIRO (1998): “Costly capital reallocation and the effects of government spending,” *Carnegie-Rochester Conference Series on Public Policy*, 48(1), 145–194.
- ROTEMBERG, J. J., AND M. WOODFORD (1992): “Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity,” *Journal of Political Economy*, 100(6), 1153–1207.

- RUSSO, G., W. H. J. HASSINK, AND C. GORTER (2005): “Filling vacancies: an empirical analysis of the cost and benefit of search in the labour market,” *Applied Economics*, 37(14), 1597–1606.
- SALA, L., U. SÖDERSTRÖM, AND A. TRIGARI (2008): “Monetary policy under uncertainty in an estimated model with labor market frictions,” *Journal of Monetary Economics*, 55(5), 983–1006.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- (2007): “Reassessing the ins and outs of unemployment,” NBER Working Papers 13421.
- SMETS, F., AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97(3), 586–606.
- SVEEN, T., AND L. WEINKE (2008): “New Keynesian perspectives on labor market dynamics,” *Journal of Monetary Economics*, 55(5), 921–930.
- TENHOFEN, J., AND G. B. WOLFF (2007): “Does anticipation of government spending matter? Evidence from an expectation augmented VAR,” Discussion paper.
- THOMAS, C. (2008): “Search and matching frictions and optimal monetary policy,” *Journal of Monetary Economics*, 55(5), 936–956.
- VENTI, S. F. (1985): “Wages in the Federal and Private Sectors,” NBER Working Papers 1641, National Bureau of Economic Research, Inc.

Appendix I - Derivations

Social planner’s problem

The social planner’s problem is to maximize the consumers lifetime utility (4) subject to the technology constraints (16) and (11) and the labour market conditions: (1)-(3). Setting up the lagrangean:

$$\begin{aligned}
& \sum_{k=0}^{\infty} \beta^{t+k} \{ u(a_{t+k}^p l_{t+k}^p - \varsigma^p v_{t+k}^p, a_{t+k}^g l_{t+k}^g - \varsigma^g v_{t+k}^g) - \nu(l_{t+k}) \\
& - \Omega_{t+k}^1 [l_{t+k+1}^p - (1 - \lambda^p) l_{t+k}^p - m((1 - s_{t+k})(1 - l_{t+k}^p - l_{t+k}^g), v_{t+k}^p)] \\
& - \Omega_{t+k}^2 [l_{t+k+1}^g - (1 - \lambda^g) l_{t+k}^g - m(s_{t+k}(1 - l_{t+k}^p - l_{t+k}^g), v_{t+k}^g)] \}
\end{aligned}$$

The first order conditions are given by:

$$\begin{aligned}
v_t^c &: u_c(c_t, g_t) \varsigma^p = \Omega_t^1 (1 - \eta^p) q_t^p \\
v_t^g &: u_g(c_t, g_t) \varsigma^g = \Omega_t^2 (1 - \eta^g) q_t^g \\
s_t &: \frac{\Omega_t^2 \eta^g m_t^g}{s_t} = \frac{\Omega_t^1 \eta^c m_t^c}{1 - s_t} \\
l_{t+1}^p &: \Omega_t^1 = \beta \{ a_{t+1}^p u_c(c_{t+1}, g_{t+1}) - \nu_l(l_{t+1}) + \Omega_{t+1}^1 (1 - \lambda^p) - \Omega_{t+1}^1 \eta^p \frac{m_{t+1}^c}{u_{t+1}} - \Omega_{t+1}^2 \eta^g \frac{m_{t+1}^g}{u_{t+1}} \} \\
l_{t+1}^g &: \Omega_t^2 = \beta \{ a_{t+1}^g u_g(c_{t+1}, g_{t+1}) - \nu_l(l_{t+1}) + \Omega_{t+1}^2 (1 - \lambda^g) - \Omega_{t+1}^1 \eta^c \frac{m_{t+1}^c}{u_{t+1}} - \Omega_{t+1}^2 \eta^g \frac{m_{t+1}^g}{u_{t+1}} \}
\end{aligned}$$

Plugging the first two equations on the third gives the implicit expression for optimal level of search in the each sector:

$$\frac{u_g(c_t, g_t) \varsigma^g \eta^g v_t^g}{(1 - \eta^g) s_t} = \frac{u_c(c_t, g_t) \varsigma^p \eta^p v_t^p}{(1 - \eta^p) (1 - s_t)}$$

If we rewrite the third first order condition as $\Omega_t^2 \eta^g M_t^g + \Omega_t^1 \eta^c M_t^c = \frac{\Omega_t^2 \eta^g M_t^g}{s_t} = \frac{\Omega_t^1 \eta^c M_t^c}{1 - s_t}$ we can use it to simplify the last two conditions and get:

$$\begin{aligned}
\frac{\varsigma^p}{q_t^p} &= \beta \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \left\{ (1 - \eta^p) a_{t+1}^p - (1 - \eta^p) \frac{\nu_l(l_{t+1})}{u_c(c_{t+1}, g_{t+1})} + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p} - \frac{\eta^p \varsigma^p v_{t+1}^p}{(1 - s_{t+1}) u_{t+1}} \right\} \\
\frac{\varsigma^g}{q_t^g} &= \beta \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} \left\{ (1 - \eta^g) a_{t+1}^g - (1 - \eta^g) \frac{\nu_l(l_{t+1})}{u_g(c_{t+1}, g_{t+1})} + (1 - \lambda^g) \frac{\varsigma^g}{q_{t+1}^g} - \frac{\eta^g \varsigma^g v_{t+1}^g}{s_{t+1} u_{t+1}} \right\}
\end{aligned}$$

Welfare costs of high public sector wages

Let $\{c_{opt}, g_{opt}, l_{opt}\}$ be the steady state private and government consumption under the optimal public sector wage and $\{\bar{c}, \bar{g}, \bar{l}\}$ the allocation under an exogenous public sector wage. We want to find out what is the welfare gain as a percentage of steady state private consumption of having public sector wage moving towards the optimum. This is given by x that solves the following equation:

$$u(c_{opt}, g_{opt}) - \nu(l_{opt}) = u((1 + x)\bar{c}, \bar{g}) - \nu(\bar{l})$$

Using the utility functions,

$$x = \frac{[\exp[\ln(c_{opt}^\gamma + \zeta g_{opt}^\gamma) + \gamma \chi(\bar{l} - l_{opt})] - \zeta \bar{g}^\gamma]^{\frac{1}{\gamma}}}{\bar{c}} - 1, \gamma \neq 0$$

If $\gamma = 0$, the utility function of consumption is not defined, so I use the equivalent $u(c_t, g_t) = \ln(c_t) + \zeta \ln(g_t)$, so the welfare cost in terms of steady state consumption is given by:

$$x = \frac{\exp[\ln(c_{opt}) + \zeta(\ln g_{opt} - \ln \bar{g}) + \chi(\bar{l} - l_{opt})]}{\bar{c}} - 1, \gamma = 0$$

Welfare costs of business cycles

I want to calculate the welfare costs of business cycles, when the economy is subject to technology shocks, under different policies for $\{v_t^g, w_t^g\}$. Let us start by defining the variables in log deviations from the steady state:

$$\begin{aligned}\tilde{c}_t &= \log\left(\frac{c_t}{\bar{c}}\right) & c_t &= \bar{c} \exp(\tilde{c}_t) \\ \tilde{g}_t &= \log\left(\frac{g_t}{\bar{g}}\right) & g_t &= \bar{g} \exp(\tilde{g}_t) \\ \tilde{l}_t &= \log\left(\frac{l_t}{\bar{l}}\right) & l_t &= \bar{l} \exp(\tilde{l}_t)\end{aligned}$$

If we do a second order approximation to the variables around the steady state $\{\bar{c}, \bar{g}, \bar{l}\}$

$$\begin{aligned}c_t &= \bar{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) + o(3) \\ g_t &= \bar{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) + o(3) \\ l_t &= \bar{l}(1 + \tilde{l}_t + \frac{1}{2}\tilde{l}_t^2) + o(3)\end{aligned}$$

The second order approximation of the utility function gives:

$$\begin{aligned}U(c_t, g_t, l_t) &= U(\bar{c}, \bar{g}, \bar{l}) + U_c(\bar{c}, \bar{g}, \bar{l})[c_t - \bar{c}] + U_g(\bar{c}, \bar{g}, \bar{l})[g_t - \bar{g}] + U_l(\bar{c}, \bar{g}, \bar{l})[l_t - \bar{l}] + \\ &\quad \frac{1}{2}U_{cc}(\bar{c}, \bar{g}, \bar{l})[c_t - \bar{c}]^2 + \frac{1}{2}U_{gg}(\bar{c}, \bar{g}, \bar{l})[g_t - \bar{g}]^2 + \frac{1}{2}U_{ll}(\bar{c}, \bar{g}, \bar{l})[l_t - \bar{l}]^2 + \\ &\quad U_{cg}(\bar{c}, \bar{g}, \bar{l})[c_t - \bar{c}][g_t - \bar{g}] + U_{cl}(\bar{c}, \bar{g}, \bar{l})[c_t - \bar{c}][l_t - \bar{l}] + U_{gl}(\bar{c}, \bar{g}, \bar{l})[g_t - \bar{g}][l_t - \bar{l}] + o(3)\end{aligned}$$

But for it to be a correct second order approximation we have to plug in the second order approximation of the variables. As we assume additive separability of the utility functions, we can drop the cross terms of the consumption goods with employment.

$$\begin{aligned}U(c_t, g_t, l_t) &= U(\bar{c}, \bar{g}, \bar{l}) + U_c(\bar{c}, \bar{g}, \bar{l})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)] + U_g(\bar{c}, \bar{g}, \bar{l})[\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)] + U_l(\bar{c}, \bar{g}, \bar{l})[\bar{l}(\tilde{l}_t + \frac{1}{2}\tilde{l}_t^2)] \\ &\quad + \frac{1}{2}U_{cc}(\bar{c}, \bar{g}, \bar{l})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)]^2 + \frac{1}{2}U_{gg}(\bar{c}, \bar{g}, \bar{l})[\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)]^2 + \frac{1}{2}U_{ll}(\bar{c}, \bar{g}, \bar{l})[\bar{l}(\tilde{l}_t + \frac{1}{2}\tilde{l}_t^2)]^2 \\ &\quad + U_{cg}(\bar{c}, \bar{g}, \bar{l})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)][\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)] + o(3)\end{aligned}$$

Collecting terms and substituting the derivatives,

$$\begin{aligned}U(c_t, g_t, l_t) &= U(\bar{c}, \bar{g}, \bar{l}) + u_c \bar{c} \tilde{c}_t + u_g \bar{g} \tilde{g}_t - \nu_l \bar{l} \tilde{l}_t + \\ &\quad \frac{\bar{c}}{2}(\bar{c}u_{cc} + u_c)\tilde{c}_t^2 + \frac{\bar{g}}{2}(\bar{g}u_{gg} + u_g)\tilde{g}_t^2 - \frac{\bar{l}}{2}(\bar{l}\nu_{ll} + \nu_l)\tilde{l}_t^2 + u_{cg}(\bar{c}, \bar{g})\bar{c}\bar{g}\tilde{c}_t\tilde{g}_t + o(3)\end{aligned}$$

and take the unconditional expectation, we can write the welfare loss in terms of the moments of the variables:

$$\begin{aligned}E[u(c_t, g_t) - \nu(l_t) - u(\bar{c}, \bar{g}) + \nu(\bar{l})] &\approx u_c \bar{c} E[\tilde{c}_t] + u_g \bar{g} E[\tilde{g}_t] - \nu_l \bar{l} E[\tilde{l}_t] + \frac{\bar{c}}{2}(\bar{c}u_{cc} + u_c)E[\tilde{c}_t^2] + \\ &\quad \frac{\bar{g}}{2}(\bar{g}u_{gg} + u_g)E[\tilde{g}_t^2] - \frac{\bar{l}}{2}(\bar{l}\nu_{ll} + \nu_l)E[\tilde{l}_t^2] + u_{cg}(\bar{c}, \bar{g})\bar{c}\bar{g}E[\tilde{c}_t\tilde{g}_t] \equiv \Xi\end{aligned}$$

I solve the model up to a second order, using perturbation methods, simulate it for 50000 periods and compute the moments of the variables to find the value of Ξ . If we want to express the welfare

costs in terms of percentage of steady state consumption we solve the following equation:

$$u((1-x)\bar{c}, \bar{g}) - u(\bar{c}, \bar{g}) = \Xi$$

For the CES function, the derivatives are given by:

$$\begin{aligned} u_c(\bar{c}, \bar{g}) &= \frac{\bar{c}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\ u_g(\bar{c}, \bar{g}) &= \frac{\zeta \bar{g}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\ u_{cc}(\bar{c}, \bar{g}) &= \frac{(\gamma-1)\bar{c}^{\gamma-2}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} - \frac{\gamma \bar{c}^{2\gamma-2}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \\ u_{gg}(\bar{c}, \bar{g}) &= \frac{(\gamma-1)\zeta \bar{g}^{\gamma-2}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} - \frac{\zeta^2 \gamma \bar{g}^{2\gamma-2}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \\ u_{cg}(\bar{c}, \bar{g}) &= \frac{-\gamma \zeta \bar{g}^{\gamma-1} \bar{c}^{\gamma-1}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \\ \nu_l(\bar{l}) &= -\chi \bar{l}^\iota \\ \nu_{ll}(\bar{l}) &= -\chi \iota \bar{l}^{\iota-1} \end{aligned}$$

And the expression for the welfare cost is:

$$\begin{aligned} \frac{1}{\gamma} \ln[(1-x)\bar{c}^\gamma + \zeta \bar{g}^\gamma] - \frac{1}{\gamma} \ln[\bar{c}^\gamma + \zeta \bar{g}^\gamma] &= \Xi \\ x = 1 - \frac{\{\exp[\gamma \Xi + \ln(\bar{c}^\gamma + \zeta \bar{g}^\gamma)] - \zeta \bar{g}^\gamma\}^{\frac{1}{\gamma}}}{\bar{c}}, \gamma \neq 0 \end{aligned}$$

If $\gamma = 0$ the solution is given by:

$$x = 1 - \frac{\exp\{\Xi + \ln \bar{c}\}}{\bar{c}}$$

Extension: Government Consumption

The lagrangean of the social planner's problem is:

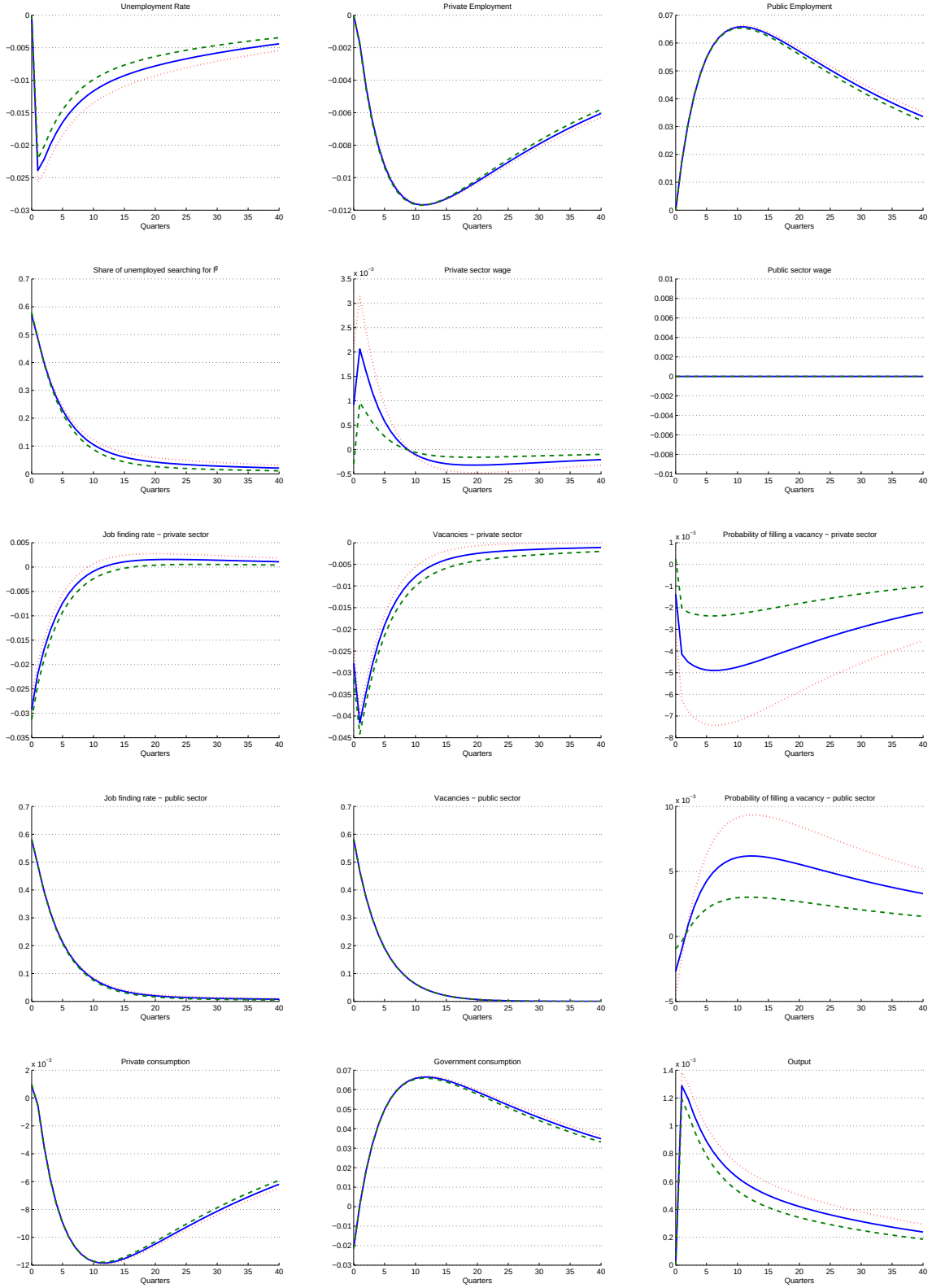
$$\sum_{k=0}^{\infty} \beta^{t+k} \{u(a_{t+k}^p l_{t+k}^p - \varsigma^p v_{t+k}^p - g_{t+k}, g_{t+k} - \nu(l_{t+k}^p) - \Omega_{t+k}^1 [l_{t+k+1}^p - (1-\lambda^p)l_{t+k}^p - m(1-l_{t+k}^p, v_{t+k}^p)])\}$$

The first order conditions are given by:

$$\frac{\varsigma^p}{q_t^p} = \beta E_t \left\{ \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} [(1-\eta^p)a_{t+1}^p + (1-\eta^p) \frac{\nu_l(l_{t+1})}{u_c(c_{t+1}, g_{t+1})} + (1-\lambda^p) \frac{\varsigma^p}{q_{t+1}^p} - \frac{\eta^p \varsigma^p v_{t+1}^p}{u_{t+1}}] \right\} \quad (25)$$

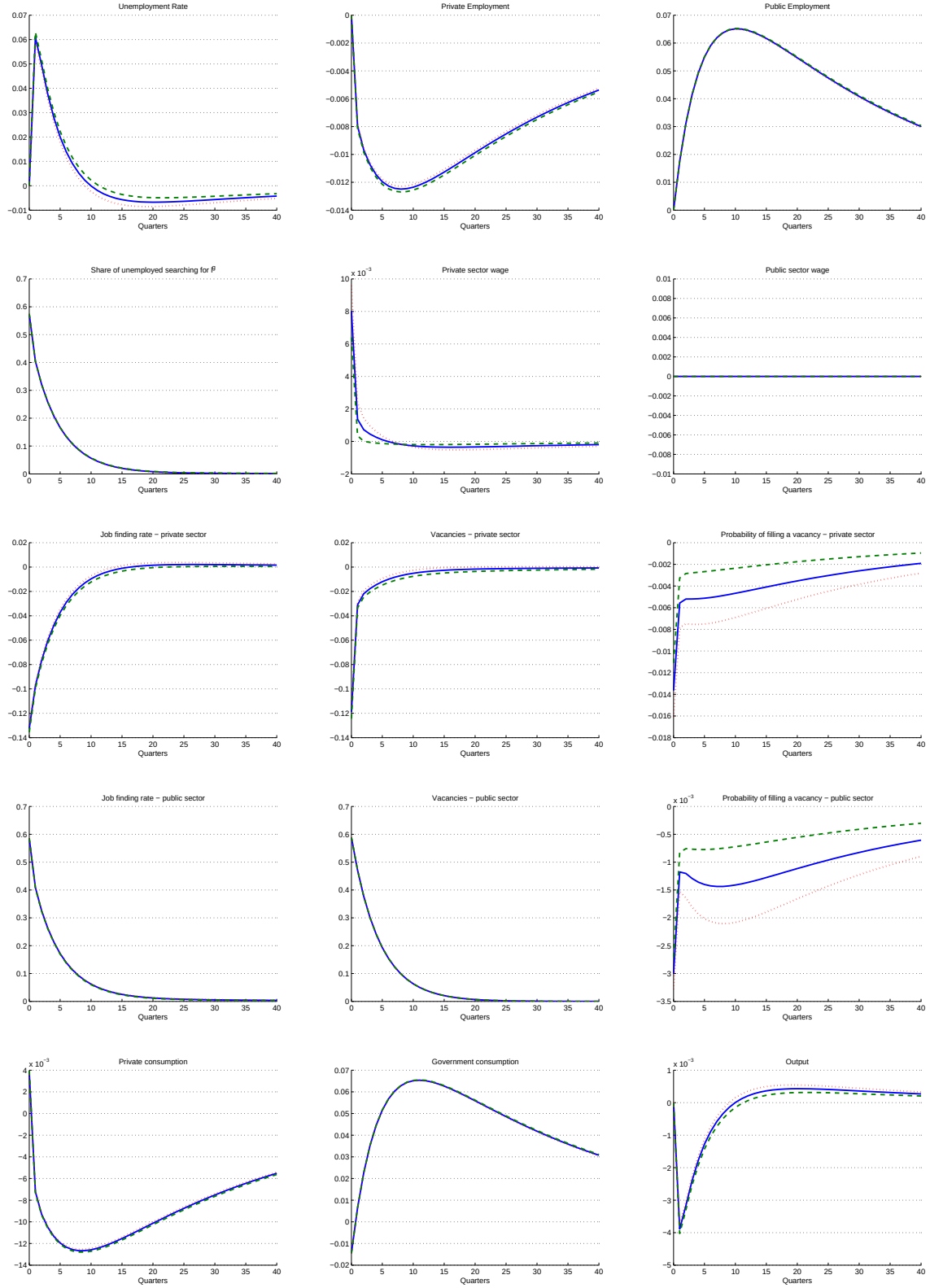
$$u_c(c_t, g_t) = u_g(c_t, g_t) \quad (26)$$

Figure A1: Response to a public sector vacancies shock (Efficient steady-state)



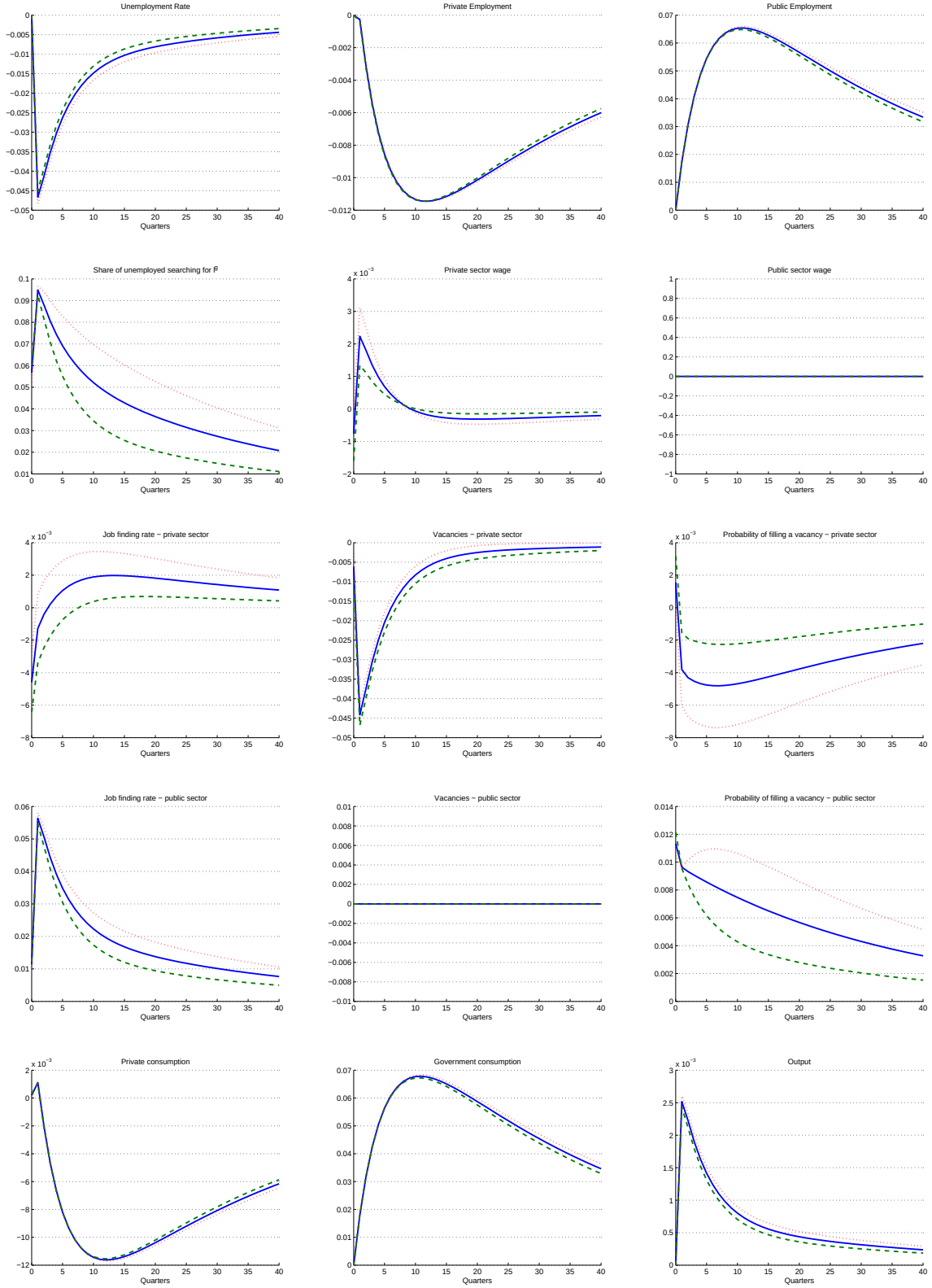
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A2: Response to a public sector vacancies shock (Baseline steady-state)



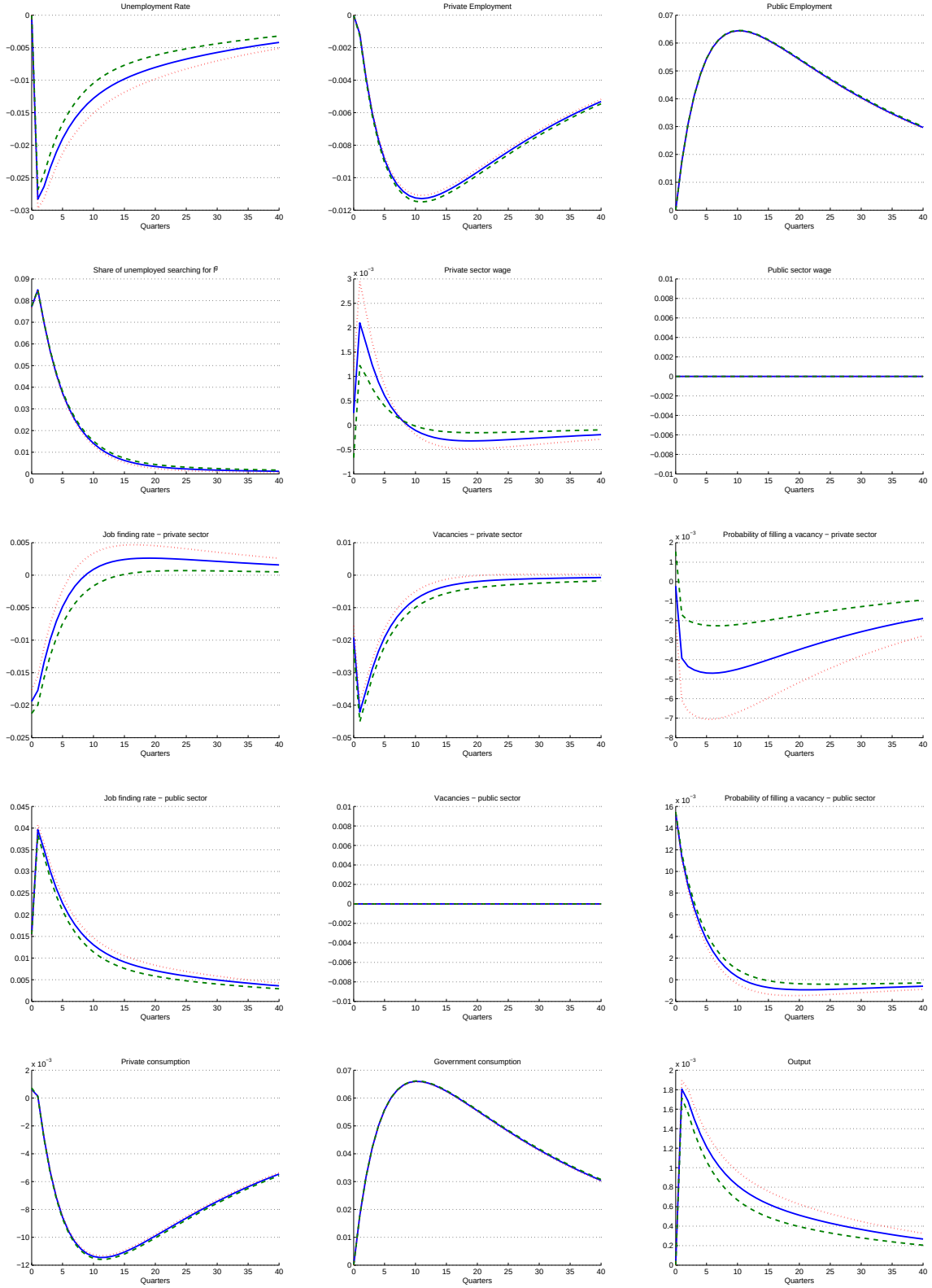
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A3: Response to a public sector separation shock (Efficient steady-state)



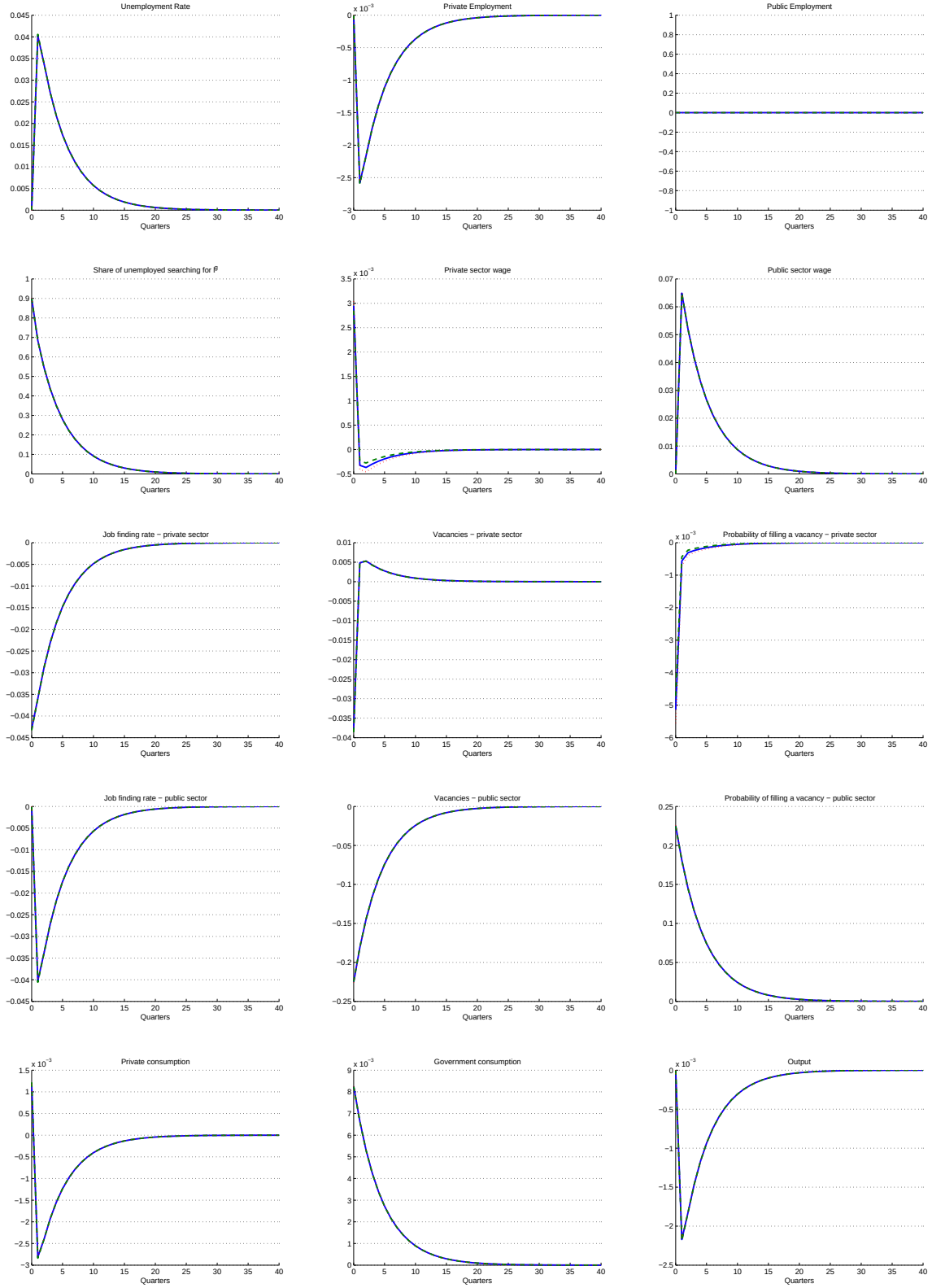
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A4: Response to a public sector separation shock (Baseline steady-state)



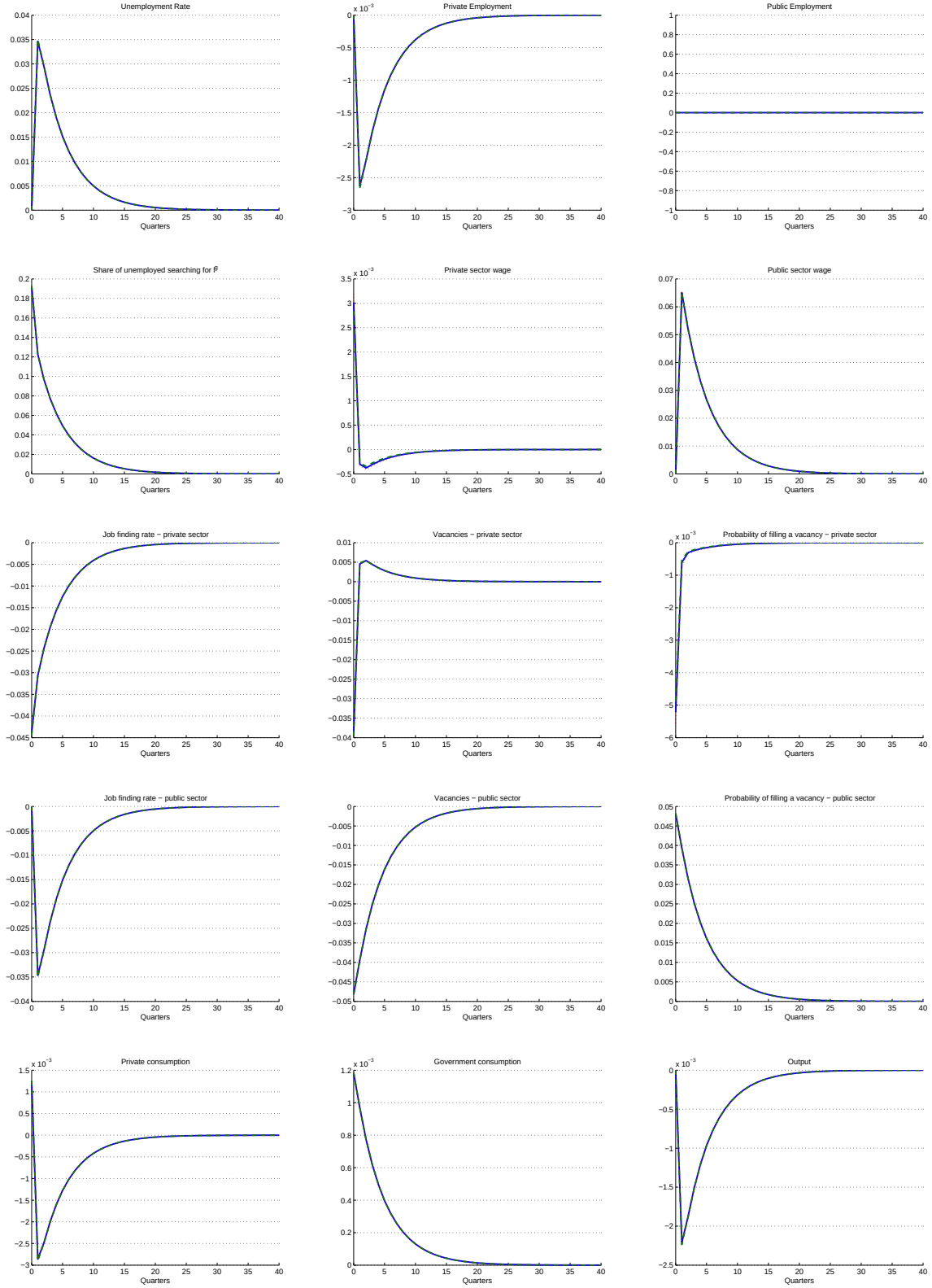
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A5: Response to a public sector wage shock (Efficient steady state)



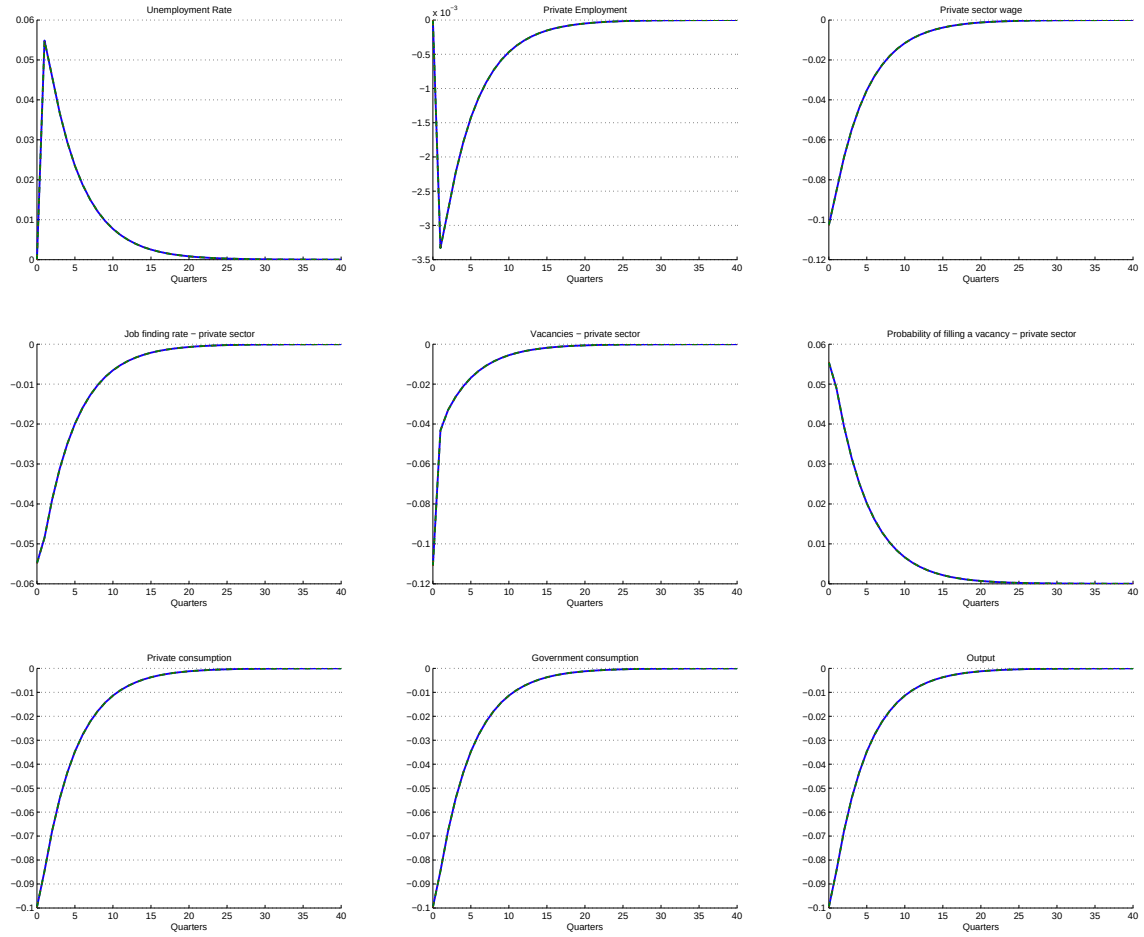
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A6: Response to a public sector wage shock (Baseline steady state)



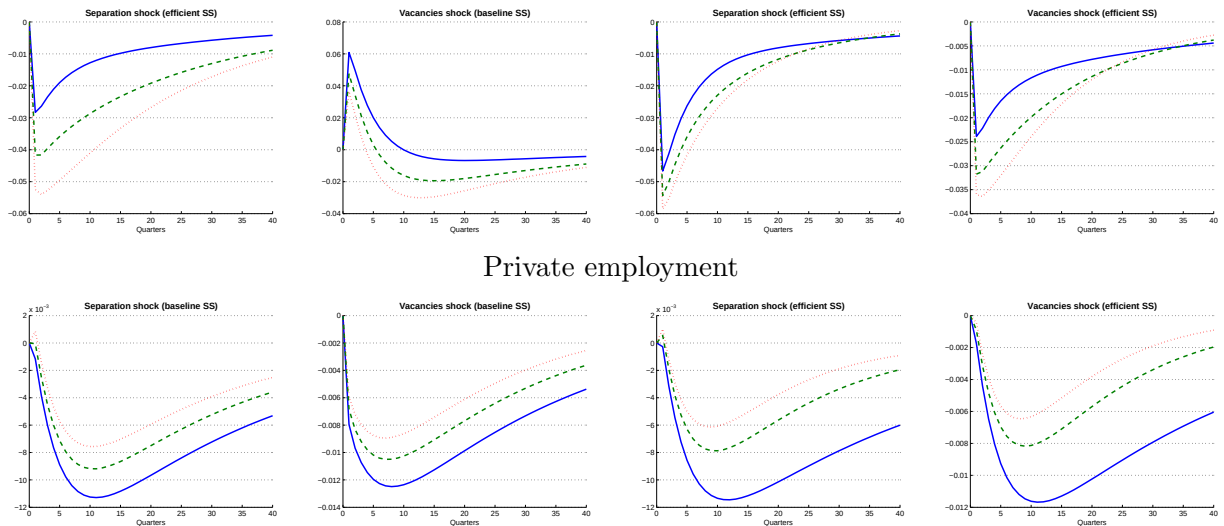
Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A7: Optimal policy with government consumption



Note: Solid line ($\gamma = 0.0$); dash line ($\gamma = 0.8$) and dotted line ($\gamma = -1.0$). Variables in logs.

Figure A8: Response to a public employment shock
Unemployment rate



Note: Solid line ($\alpha = 0.0$); dash line ($\alpha = 0.1$) and dotted line ($\alpha = 0.2$). Variables in logs.

Appendix II - Bayesian estimation

Estimated model in levels

The labour market is described by the following equations:

$$1 = l_t^p + l_t^g + u_t \quad (\text{A1})$$

$$l_{t+1}^p = (1 - \lambda_t^p)l_t^p + m_t^p \quad (\text{A2})$$

$$l_{t+1}^g = (1 - \lambda_t^g)l_t^g + m_t^g \quad (\text{A3})$$

$$m_t^p = \mu^p((1 - s_t)u_t)^{\eta^p} (v_t^p)^{1-\eta^p} \quad (\text{A4})$$

$$m_t^g = \mu^g(s_t u_t)^{\eta^g} (v_t^g)^{1-\eta^g} \quad (\text{A5})$$

$$q_t^p = \frac{m_t^p}{v_t^p} \quad (\text{A6})$$

$$p_t^p = \frac{m_t^p}{(1 - s_t)u_t} \quad (\text{A7})$$

$$p_t^g = \frac{m_t^g}{s_t u_t} \quad (\text{A8})$$

The marginal utility of consumption and the stochastic discount factor.

$$u_c(c_t, g_t) = \frac{c_t^{\gamma-1}}{c_t^\gamma + \zeta g_t^\gamma} \quad (\text{A9})$$

$$\nu_l(l_t) = \chi(l_t^p + l_t^g)^\iota \quad (\text{A10})$$

$$\beta_{t,t+1} = \beta \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \quad (\text{A11})$$

I define a new variable that is the difference between the value of working and being unemployed x_t^i . I use it to re-write the equation that pins down the s_t and the Nash bargaining equation.

$$x_t^p = W_t^p - U_t^p = w_t^p - \frac{\nu_l(l_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} (1 - \lambda_t^p - p_t^p) x_{t+1}^p \quad (\text{A12})$$

$$x_t^g = W_t^g - U_t^g = w_t^g - \frac{\nu_l(l_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} (1 - \lambda_t^g - p_t^g) x_{t+1}^g \quad (\text{A13})$$

$$J_t = a_t^p - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda_t^p) J_{t+1}] \quad (\text{A14})$$

$$\frac{m_t^p E_t [x_{t+1}^p]}{(1 - s_t)} = \frac{m_t^g E_t [x_{t+1}^g]}{s_t} \quad (\text{A15})$$

$$(1 - b_t)(x_t^p) = b_t J_t \quad (\text{A16})$$

Finally we have the production functions, the equation that determines the firm's optimal vacancy posting and the policy equations.

$$c_t = a_t^p l_t^p - \varsigma^p v_t^p \quad (\text{A17})$$

$$g_t = a_t^g l_t^g - \varsigma^g v_t^g \quad (\text{A18})$$

$$\frac{\varsigma^p}{q_t^p} = E_t \beta_{t,t+1} [a_{t+1}^p - w_{t+1}^p + (1 - \lambda_t^p) \frac{\varsigma^p}{q_{t+1}^p}] \quad (\text{A19})$$

$$\ln(v_t^g) = \ln(\bar{v}^g) + \psi^v \left[\frac{\sum_{i=0}^3 \ln(v_{t-i}^p)}{4} - \ln(\bar{v}^p) \right] + \ln(\omega_t^v) \quad (\text{A20})$$

$$\ln(w_t^g) = \ln(\bar{w}^g) + \psi^w \left[\frac{\sum_{i=0}^3 \ln(w_{t-i}^p)}{4} - \ln(\bar{w}^p) \right] + \ln(\omega_t^w) \quad (\text{A21})$$

$$\ln(a_t^p) = \ln(\bar{a}^p) + \alpha (\ln(l_t^g) - \ln(\bar{l}^g)) + \ln(\omega_t^a) \quad (\text{A22})$$

I include 6 different shocks: a shock to government vacancies, to government wages, to private and public separation rates, private sector bargaining power and to technology. These shocks are described in the following equations:

$$\ln(\omega_t^v) = \rho^v \ln(\omega_{t-1}^v) + \epsilon_t^v \quad (\text{A23})$$

$$\ln(\omega_t^w) = \rho^w \ln(\omega_{t-1}^w) + \epsilon_t^w \quad (\text{A24})$$

$$\ln(\omega_t^a) = \rho^a \ln(\omega_{t-1}^a) + \epsilon_t^a \quad (\text{A25})$$

$$\ln(\lambda_t^g) = (1 - \rho^{lg}) \ln(\bar{\lambda}_t^g) + \rho^{lg} \ln(\lambda_{t-1}^g) + \epsilon_t^{lg} \quad (\text{A26})$$

$$\ln(\lambda_t^p) = (1 - \rho^{lp}) \ln(\bar{\lambda}_t^p) + \rho^{lp} \ln(\lambda_{t-1}^p) + \epsilon_t^{lp} \quad (\text{A27})$$

$$\ln(b_t) = (1 - \rho^b) \ln(\bar{b}) + \rho^b \ln(b_{t-1}) + \epsilon_t^b \quad (\text{A28})$$

Finally, I define that overall separation rate and job finding rates:

$$f_t = \frac{m_t^p + m_t^g}{u_t} \quad (\text{A29})$$

$$\Lambda_t = \frac{\lambda_t^p l_t^p + \lambda_t^g l_t^g}{l_t^p + l_t^g} \quad (\text{A30})$$

Estimated model - steady state

I determine that the government employment in steady state is 0.15. As there is no recurrent way to write the steady state values of the equations, they solve the following non linear system of equations:

$$\begin{aligned} \bar{l}^g &= 0.15 \\ \bar{l}^p &= 1 - \bar{l}^g - \bar{u} \\ \bar{m}^p &= \lambda^p \bar{l}^p \\ \bar{m}^g &= \lambda^g \bar{l}^g \\ \bar{m}^p &= \mu^p ((1 - \bar{s})\bar{u})^{\eta^p} (\bar{v}^p)^{1-\eta^p} \\ \bar{m}^g &= \mu^g (\bar{s}\bar{u})^{\eta^g} (\bar{v}^g)^{1-\eta^g} \\ p_t^p &= \frac{\bar{m}^p}{(1 - \bar{s})\bar{u}} \\ p_t^g &= \frac{\bar{m}^g}{(\bar{s})\bar{u}} \\ \bar{q}^p &= \frac{\bar{m}^p}{v^p} \\ \bar{x}^g &= \frac{\bar{w}^g - \frac{\nu_l}{u_c}}{1 - \beta(1 - \lambda^g - \bar{p}^g)} \\ \bar{x}^p &= \frac{\bar{w}^p - \frac{\nu_l}{u_c}}{1 - \beta(1 - \lambda^p - \bar{p}^p)} \\ \bar{m}^p \bar{x}^p \bar{s} &= \bar{m}^g \bar{x}^g (1 - \bar{s}) \\ (1 - b)(\bar{x}^p) &= b\bar{J} \\ \bar{J} &= \frac{\bar{a}^p - \bar{w}^p}{1 - \beta(1 - \lambda^p)} \\ \frac{\varsigma^p}{\bar{q}^p} (1 - \beta(1 - \lambda^p)) &= \beta(\bar{a}^p - \bar{w}^p) \\ \bar{w}^g &= \pi \bar{w}^p \end{aligned}$$

$$\begin{aligned}
\bar{c} &= \bar{a}^p \bar{l}^p - \varsigma^p \bar{v}^p \\
\bar{g} &= \bar{a}^g \bar{l}^g - \varsigma^g \bar{v}^g \\
u_c(\bar{c}, \bar{g}) &= \frac{\bar{c}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\
\nu_l &= \chi(\bar{l}^p + \bar{l}^g)^\iota \\
\bar{f} &= \frac{\bar{m}^p + \bar{m}^g}{\bar{u}} \\
\bar{\Lambda} &= \frac{\lambda^p \bar{l}^p + \lambda^g \bar{l}^g}{\bar{l}^p + \bar{l}^g}
\end{aligned}$$

Estimated log-linearized model

The variables with tilde are expressed in deviations from steady state.

$$0 = \bar{l}^p \tilde{l}_t^p + \bar{l}^g \tilde{l}_t^g + \bar{u} \tilde{u}_t \quad (\text{L1})$$

$$\tilde{l}_{t+1}^p = (1 - \bar{\lambda}^p) \tilde{l}_t^p - \bar{\lambda}^p \tilde{\lambda}_t^p + \bar{\lambda}^p \tilde{m}_t^p \quad (\text{L2})$$

$$\tilde{l}_{t+1}^g = (1 - \bar{\lambda}^g) \tilde{l}_t^g - \bar{\lambda}^g \tilde{\lambda}_t^g + \bar{\lambda}^g \tilde{m}_t^g \quad (\text{L3})$$

$$\tilde{m}_t^p = \eta^p (\tilde{u}_t - \frac{\bar{s}}{1 - \bar{s}} \tilde{s}_t) + (1 - \eta^p) \tilde{v}_t^p \quad (\text{L4})$$

$$\tilde{m}_t^g = \eta^g (\tilde{u}_t + \tilde{s}_t) + (1 - \eta^g) \tilde{v}_t^g \quad (\text{L5})$$

$$\tilde{q}_t^p = \tilde{m}_t^p - \tilde{v}_t^p \quad (\text{L6})$$

$$\tilde{p}_t^p = \tilde{m}_t^p + \frac{\bar{s}}{1 - \bar{s}} \tilde{s}_t - \tilde{u}_t \quad (\text{L7})$$

$$\tilde{p}_t^g = \tilde{m}_t^g - \tilde{s}_t - \tilde{u}_t \quad (\text{L8})$$

$$\tilde{u}_c(\tilde{c}_t, \tilde{g}_t) = \tilde{c}_t(\gamma - 1 - \frac{\gamma \bar{c}^\gamma}{\bar{c}^\gamma + \zeta \bar{g}^\gamma}) - \tilde{g}_t(\frac{\zeta \gamma \bar{g}^\gamma}{\bar{c}^\gamma + \zeta \bar{g}^\gamma}) \quad (\text{L9})$$

$$\tilde{\nu}_l(\tilde{u}_t) = -\frac{\iota \bar{u}}{1 - \bar{u}} \tilde{u}_t \quad (\text{L10})$$

$$\tilde{\beta}_{t,t+1} = E_t[\tilde{u}_c(\tilde{c}_{t+1}, \tilde{g}_{t+1}) - \tilde{u}_c(\tilde{c}_t, \tilde{g}_t)] \quad (\text{L11})$$

$$\tilde{x}_t^p = \frac{\bar{w}^p}{\bar{x}^p} \tilde{w}_t^p - \frac{\bar{\nu}_l}{\bar{x}^p \bar{u}_c} (\tilde{\nu}_l - \tilde{u}_c) - \beta(\bar{\lambda}^p \tilde{\lambda}_t^p + \bar{p}^p \tilde{p}_t^p) + \beta(1 - \bar{\lambda}^p - \bar{p}^p) E_t(\tilde{x}_{t+1}^p + \tilde{\beta}_{t,t+1}) \quad (\text{L12})$$

$$\tilde{x}_t^g = \frac{\bar{w}^g}{\bar{x}^g} \tilde{w}_t^g - \frac{\bar{\nu}_l}{\bar{x}^g \bar{u}_c} (\tilde{\nu}_l - \tilde{u}_c) - \beta(\bar{\lambda}^g \tilde{\lambda}_t^g + \bar{p}^g \tilde{p}_t^g) + \beta(1 - \bar{\lambda}^g - \bar{p}^g) E_t(\tilde{x}_{t+1}^g + \tilde{\beta}_{t,t+1}) \quad (\text{L13})$$

$$\tilde{J}_t = \frac{\bar{a}^p}{\bar{J}} \tilde{a}_t^p - \frac{\bar{w}^p}{\bar{J}} \tilde{w}_t^p + \beta E_t((1 - \bar{\lambda}^p) \tilde{\beta}_t + (1 - \bar{\lambda}^p) \tilde{J}_{t+1} - \bar{\lambda}^p \tilde{\lambda}_t^p) \quad (\text{L14})$$

To test the relevance of the directed search assumption I add the parameter κ to the log-linearized equation that determines $\tilde{s}_t$

$$\tilde{s}_t = \kappa(1 - \bar{s}) E_t(\tilde{x}_{t+1}^g - \tilde{x}_{t+1}^p - \tilde{m}_t^p + \tilde{m}_t^g) \quad (\text{L15})$$

$$\tilde{J}_t + \frac{1}{1 - \bar{b}} \tilde{b}_t = \tilde{x}_t^p \quad (\text{L16})$$

$$\tilde{c}_t = \frac{\bar{a}^p \bar{l}^p}{\bar{c}} (\tilde{a}_t^p + \tilde{l}_t^p) - \frac{\varsigma^p \bar{v}^p}{\bar{c}} \tilde{v}_t^p \quad (\text{L17})$$

$$\tilde{g}_t = \frac{\bar{a}^g \bar{l}^g}{\bar{g}} (\tilde{a}_t^g + \tilde{l}_t^g) - \frac{\varsigma^g \bar{v}^g}{\bar{g}} \tilde{v}_t^g \quad (\text{L18})$$

$$-\frac{\varsigma^p}{\bar{q}^p} \tilde{q}_t^p = \beta[\bar{a}^p \tilde{a}_{t+1}^p - \bar{w}^p \tilde{w}_{t+1}^p - (1 - \bar{\lambda}^p) \frac{\varsigma^p}{\bar{q}^p} \tilde{q}_{t+1}^p - \bar{\lambda}^p \frac{\varsigma^p}{\bar{q}^p} \tilde{\lambda}_t^p + (\bar{a}^p - \bar{w}^p + (1 - \bar{\lambda}^p) \frac{\varsigma^p}{\bar{q}^p}) \tilde{\beta}_t] \quad (\text{L19})$$

$$\tilde{v}_t^g = \psi^v \frac{\tilde{v}_t^p + \tilde{v}_{t-1}^p + \tilde{v}_{t-2}^p + \tilde{v}_{t-3}^p}{4} + \tilde{\omega}_t^v \quad (\text{L20})$$

$$\tilde{w}_{t+1}^g = \psi^w \frac{\tilde{w}_t^p + \tilde{w}_{t-1}^p + \tilde{w}_{t-2}^p + \tilde{w}_{t-3}^p}{4} + \tilde{\omega}_t^w \quad (\text{L21})$$

$$\tilde{a}_t^p = \alpha \tilde{l}_t^g + \omega_t^a \quad (\text{L22})$$

$$\tilde{\omega}_t^v = \rho^v \tilde{\omega}_{t-1}^v + \epsilon_t^v \quad (\text{L23})$$

$$\tilde{\omega}_t^w = \rho^w \tilde{\omega}_{t-1}^w + \epsilon_t^w \quad (\text{L24})$$

$$\tilde{\omega}_t^a = \rho^a \tilde{\omega}_{t-1}^a + \epsilon_t^a \quad (\text{L25})$$

$$\tilde{\lambda}_t^g = \rho^{lg} \tilde{\lambda}_{t-1}^g + \epsilon_t^{lg} \quad (\text{L26})$$

$$\tilde{\lambda}_t^p = \rho^{lp} \tilde{\lambda}_{t-1}^p + \epsilon_t^{lp} \quad (\text{L27})$$

$$\tilde{b}_t = \rho^b \tilde{b}_{t-1} + \epsilon_t^b \quad (\text{L28})$$

$$f_t = \tilde{m}_t^p \frac{\bar{m}^p}{\bar{m}^p + \bar{m}^g} + \tilde{m}_t^g \frac{\bar{m}^g}{\bar{m}^p + \bar{m}^g} - \tilde{u}_t \quad (\text{L29})$$

$$\tilde{\Lambda}_t = (\tilde{\lambda}_t^p + \tilde{l}_t^p) \frac{\bar{\lambda}^p \bar{l}^p}{\bar{\lambda}^p \bar{l}^p + \bar{\lambda}^g \bar{l}^g} + (\tilde{\lambda}_t^g + \tilde{l}_t^g) \frac{\bar{\lambda}^g \bar{l}^g}{\bar{\lambda}^p \bar{l}^p + \bar{\lambda}^g \bar{l}^g} + \tilde{u}_t \frac{\bar{u}}{1 - \bar{u}} \quad (\text{L30})$$

Definition of observable variables

Model 1	Model 2	Model 3 and 4
$\begin{bmatrix} l_t^{gOb} & \bar{l}^g(1 + \tilde{l}_t^g) \\ u_t^{Ob} & \bar{u}(1 + \tilde{u}_t) \\ w_t^{gOb} & \tilde{w}_t^g - \tilde{w}_{t-1}^g \\ w_t^{pOb} & \tilde{w}_t^p - \tilde{w}_{t-1}^p \\ \Lambda_t^{Ob} & \bar{\Lambda}(1 + \tilde{\Lambda}_t) \\ f_t^{Ob} & \bar{f}(1 + \tilde{f}_t) \end{bmatrix} =$	$\begin{bmatrix} l_t^{gOb} & \tilde{l}_t^g - \tilde{l}_{t-1}^g \\ u_t^{Ob} & \tilde{u}_t - \tilde{u}_{t-1} \\ w_t^{gOb} & \tilde{w}_t^g - \tilde{w}_{t-1}^g \\ w_t^{pOb} & \tilde{w}_t^p - \tilde{w}_{t-1}^p \\ \Lambda_t^{Ob} & \tilde{\Lambda}_t - \tilde{\Lambda}_{t-1} \\ f_t^{Ob} & \tilde{f}_t - \tilde{f}_{t-1} \end{bmatrix} =$	$\begin{bmatrix} l_t^{gOb} & \tilde{l}_t^g \\ u_t^{Ob} & \tilde{u}_t \\ w_t^{gOb} & \tilde{w}_t^g \\ w_t^{pOb} & \tilde{w}_t^p \\ \Lambda_t^{Ob} & \tilde{\Lambda}_t \\ f_t^{Ob} & \tilde{f}_t \end{bmatrix} =$

Model with random search

A1 to A3 are the same

$$m_t^p + m_t^g = \mu^p(u_t)^{\eta^p} (v_t^p + v_t^g)^{1-\eta^p} \quad (\text{B4})$$

$$v_t^g m_t^p = v_t^p m_t^g \quad (\text{B5})$$

$$p_t^p = \frac{m_t^p}{u_t} \quad (\text{B7})$$

$$p_t^g = \frac{m_t^g}{u_t} \quad (\text{B8})$$

A6, A9-A11 are the same.

$$x_t^p = w_t^p - \frac{\nu_l(l_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} ((1 - \lambda_t^p - p_t^p) x_{t+1}^p - p_t^g x_{t+1}^g) \quad (\text{B12})$$

$$x_t^g = w_t^g - \frac{\nu_l(l_t)}{u_c(c_t, g_t)} + E_t \beta_{t,t+1} ((1 - \lambda_t^g - p_t^g) x_{t+1}^g - p_t^p x_{t+1}^p) \quad (\text{B13})$$

We drop equation A15 and all the other equations are the same. For the log linearized model the matching functions are

$$\tilde{m}_t^p - \tilde{m}_t^g = \tilde{v}_t^p - \tilde{v}_t^g \quad (\text{93})$$

$$\tilde{m}_t^p - \tilde{v}_t^p = \eta^p \tilde{u}_t - \eta^p \left(\frac{\bar{m}^p}{\bar{m}^p + \bar{m}^g} \tilde{v}_t^p + \frac{\bar{m}^g}{\bar{m}^p + \bar{m}^g} \tilde{v}_t^g \right) \quad (94)$$

$$\tilde{x}_t^p = \frac{\bar{w}^p}{\bar{x}^p} \tilde{w}_t^p - \frac{\bar{v}_l}{\bar{x}^p \bar{u}_c} (\tilde{v}_l - \tilde{u}_c) - \beta (\bar{\lambda}^p \tilde{\lambda}_t^p + \bar{p}^p \tilde{p}_t^p) + \beta (1 - \bar{\lambda}^p - \bar{p}^p) E_t(\tilde{x}_{t+1}^p + \tilde{\beta}_{t,t+1}) \quad (95)$$

Appendix III - Data

Table A1: Data - definition and sources

	Variable	Definition and source	Availability
π_t	Productivity	Business Sector: Output Per Hour of All Persons (BLS)	1947q1 2008q2
L_t^g	Government employment	All Employees: Government (BLS)	1939q1 2008q3
w_t^g	Government per employee nominal wage	Government consumption expenditures: Compensation of general government employees / government employees (BEA-NIPA Tables and own calculation)	1947q1 2008q2
w_t^p	Business sector hourly nominal wage	Business Sector: Compensation Per Hour (BLS)	1947q1 2008q2
H_t^p	Total business sector hours	Business Sector: Hours of All Persons (BLS)	1947q1 2008q2
u_t	Unemployment rate	Civilian Unemployment Rate (BLS)	1948q1 2008q3
v_t	Vacancies	Index of Help Wanted Advertising in Newspapers (The Conference Board)	1951q1 2006q2
wr_t^g	Government per employee real wage	Government per employee nominal wage deflated by CPI (BEA-NIPA Tables and own calculation)	1947q1 2008q2
wr_t^p	Business sector hourly real wage	Business Sector: Real Compensation Per Hour (BLS)	1947q1 2008q2
Λ_t	Separation rate	Job-separation rate (Shimer, own calculation for quarterly aggregation)	1948q1 2007q1
f_t	Job finding rate	Job-finding rate (Shimer, own calculation for quarterly aggregation)	1948q1 2007q1

Figure A9: Looking at the data

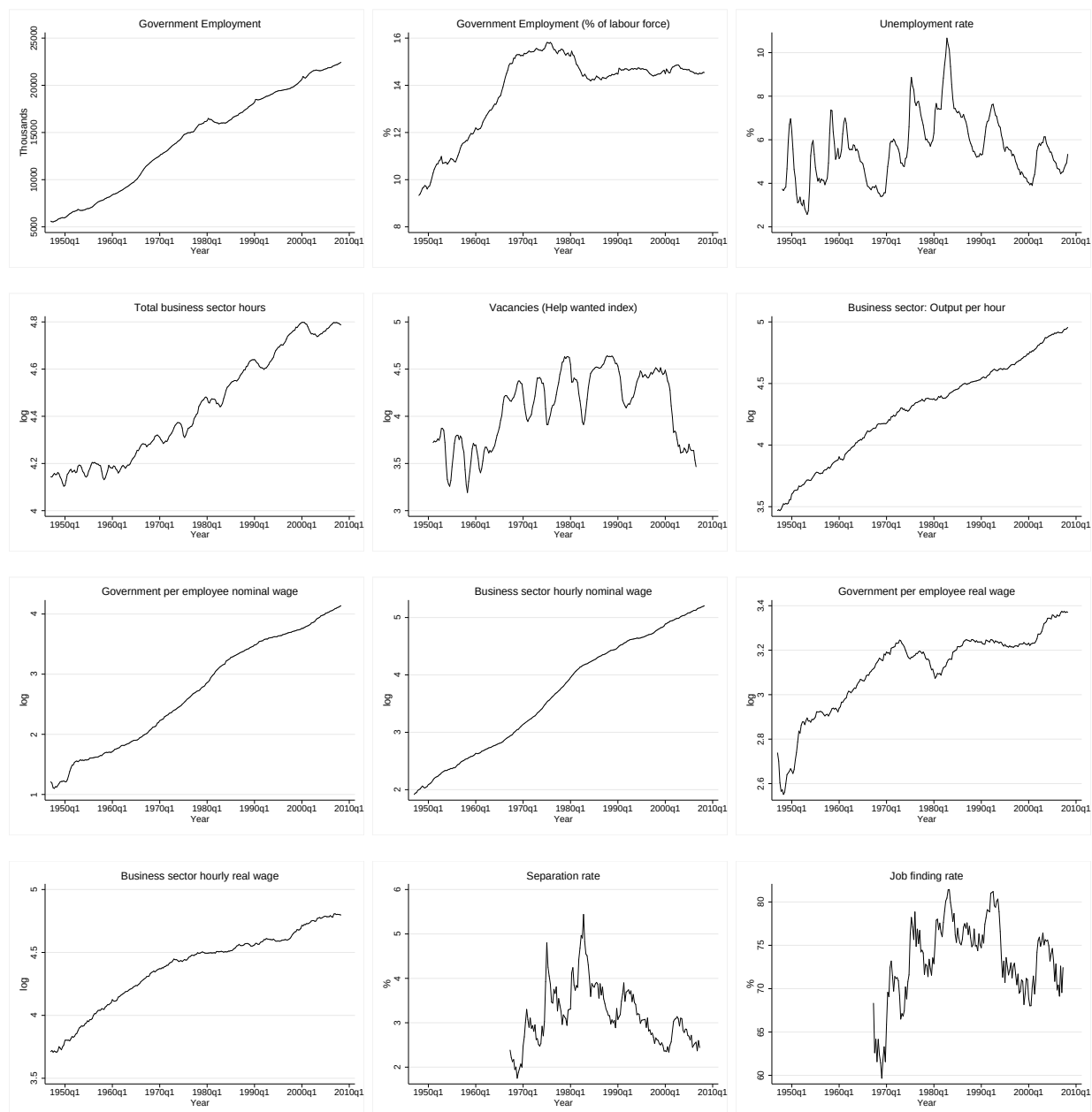
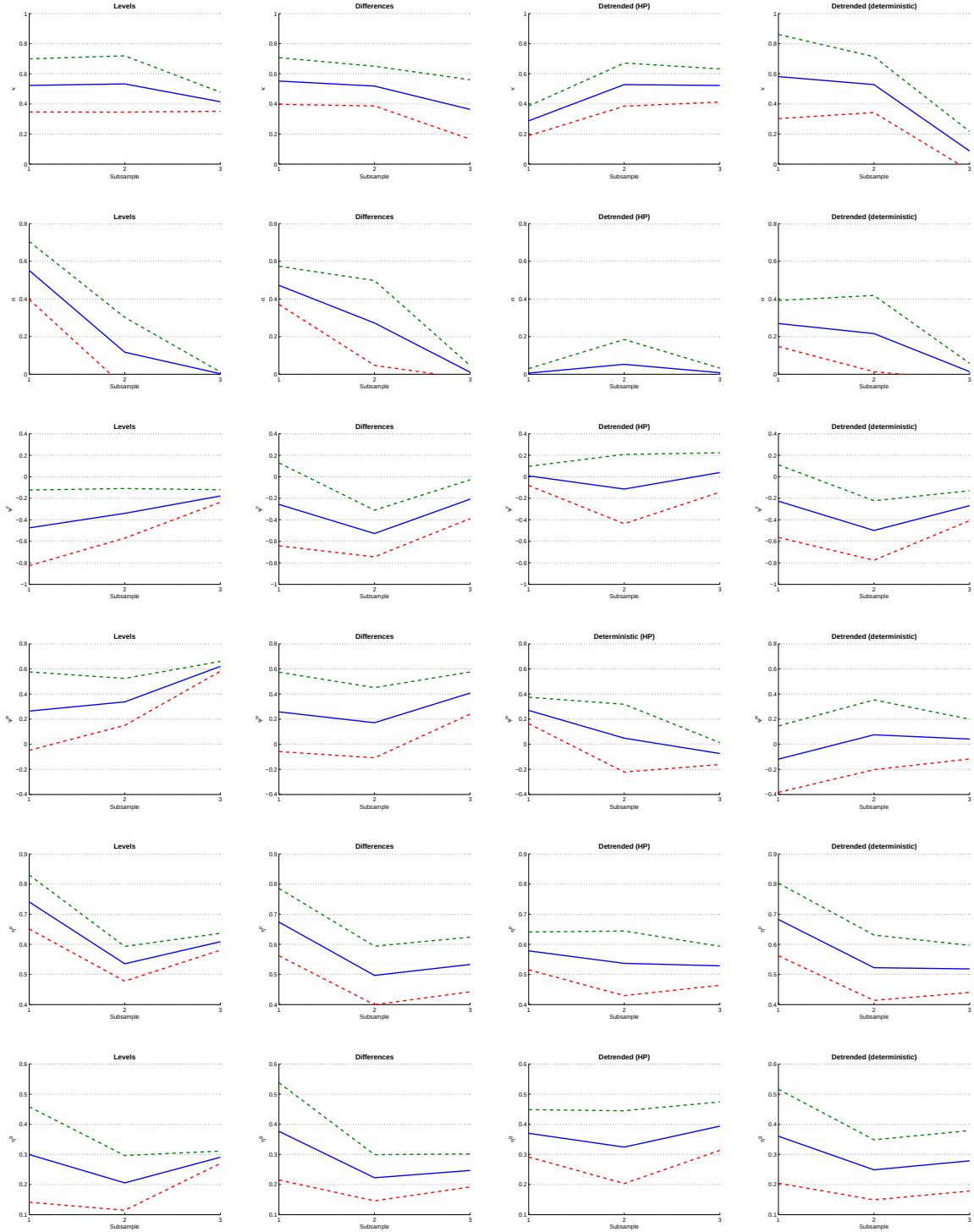


Figure A10: Subsample stability of selected estimated parameters



Note:.

Figure A11: VAR with real wages

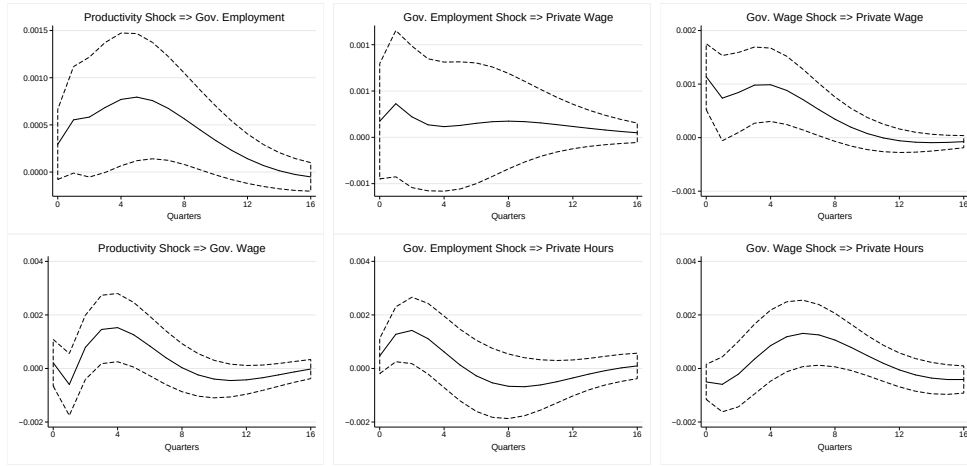


Figure A12: VAR with unemployment rate

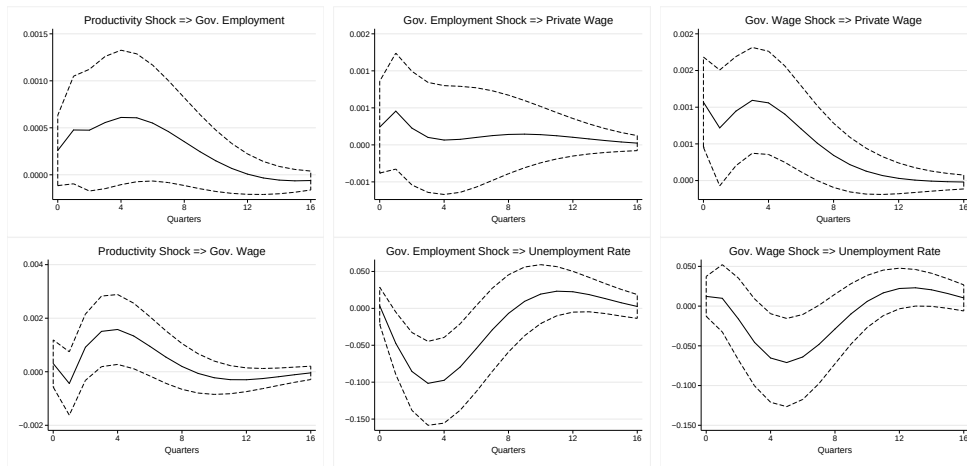


Figure A13: VAR with vacancies

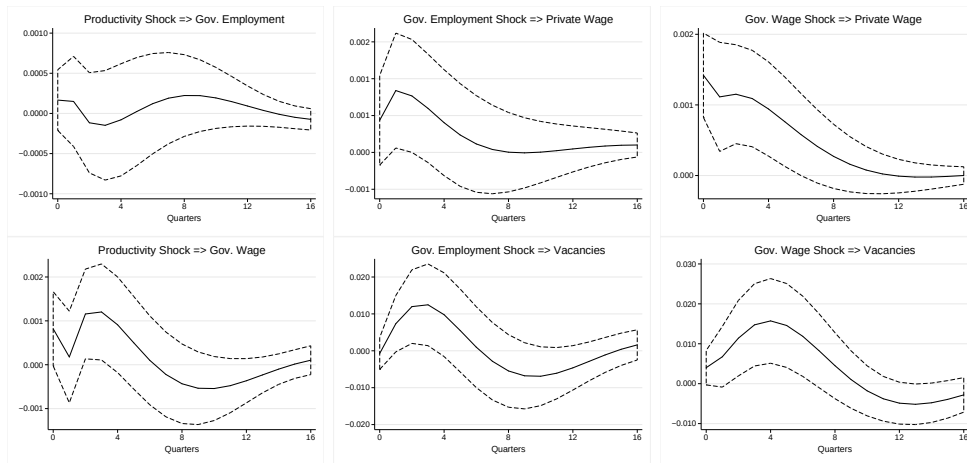


Figure A14: VAR over last 20 year

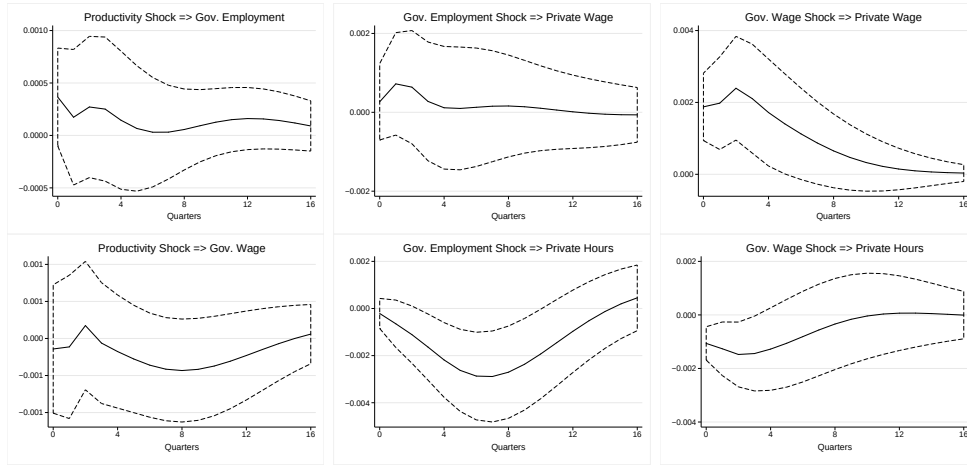


Figure A15: VAR in differences

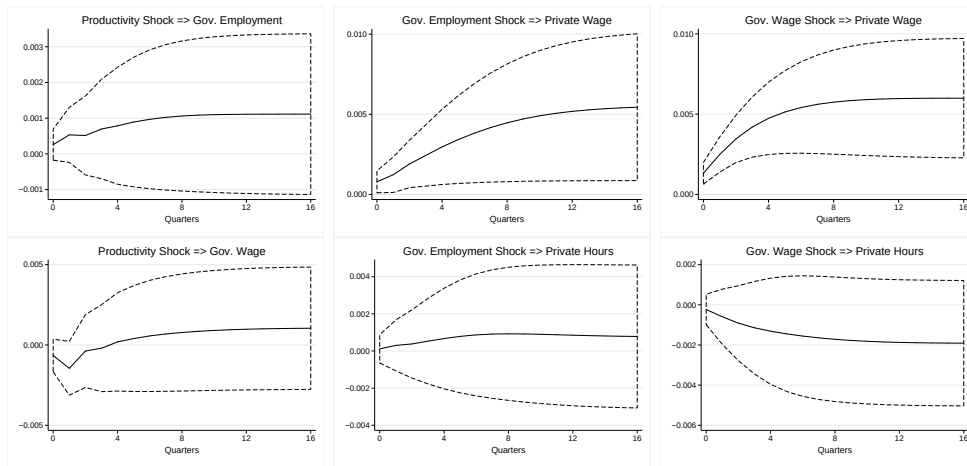


Figure A16: Fig: VAR with alternative ordering

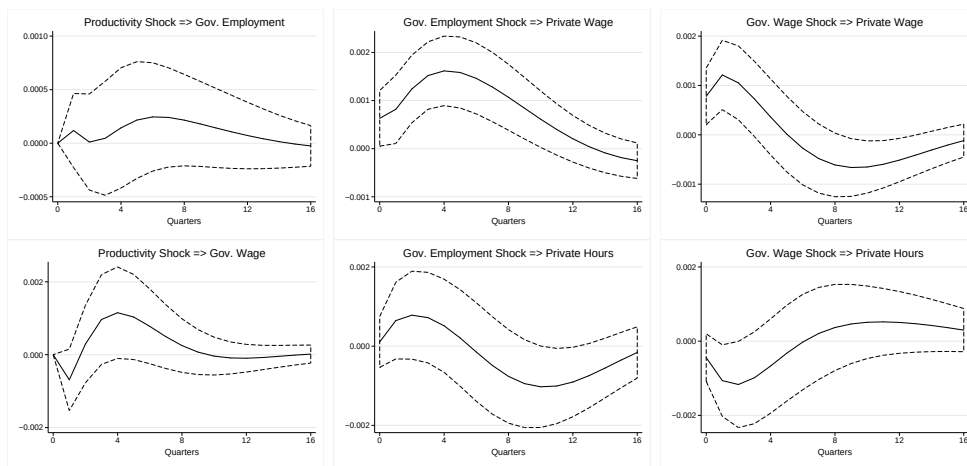


Figure A17: VAR with different detrending

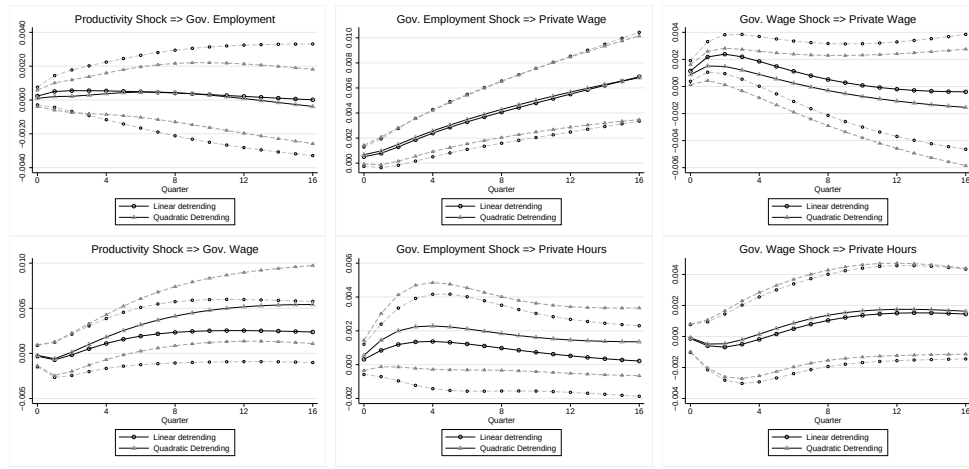


Figure A18: VAR with different lag length

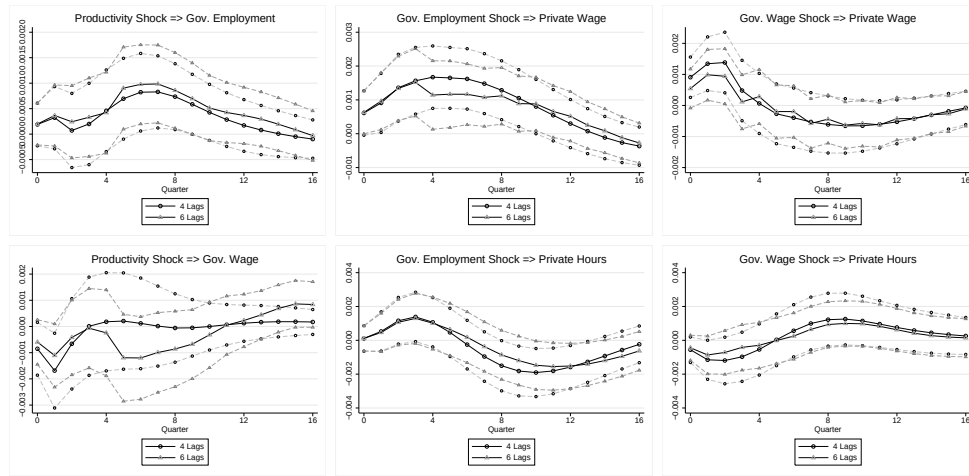


Figure A19: VAR over different sub-samples

