

In the Wrong Hands: Complementarities, Resource Allocation, and Aggregate TFP*

JOB MARKET PAPER

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Abstract

I explore mismatch between firm quality and firm management as a mechanism for variations in total factor productivity (TFP) across countries. In my calibrated model, even minor deviations from efficient (assortative) matching have sizeable effects on output and productivity. Underlying this result is the finding that the aggregate implications of matching frictions are highly sensitive to the degree of complementarity between firm and manager attributes. In addition, the relative dispersion of firm and managerial attributes is also key to quantifying the aggregate effects of matching frictions. The key model parameters are pinned down by calibrating the model to U.S. observations on the firm-size distribution and the level and distribution of managerial compensation. My results imply that “crony capitalism”, where key managerial positions are allocated on the basis of political connections rather than talent, imposes a substantial burden on economic welfare.

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1 Introduction

One of the most striking facts in macroeconomics is the variation of income per capita across countries. A factor of almost thirty separates GDP per capita in the most advanced OECD members from countries in sub-Saharan Africa.¹ Recent work suggests that the lion's share in these differences can be explained by cross-country variations in total factor productivity (TFP) rather than variations in human and physical capital accumulation.² The aim of this paper is to explore one particular channel, namely mismatch between firm management and firm quality, to explain observed variations in aggregate productivity across countries.

I construct a model where production requires a project (firm) and a manager in addition to labor and capital. Managers of different abilities seek out projects of commensurate quality in a frictionless over-the-counter market. Together, ability and quality determine the firms' span of control over capital and labor, both of which are traded in competitive factor markets. Decreasing returns to scale in capital and labor ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes. The equilibrium also features assortative matching between the managers' ability and the firms' quality.

Using a combination of firm-level and aggregate data, I calibrate the model to match key moments of the U.S. manufacturing sector. Most importantly, I identify the unobserved distributions of managerial talent and firm quality. I find that the parameters of the distributions are sensitive to the choice of the model's technology parameters. In particular, imposing unit elasticity of substitution between talent and quality turns out to be a potentially restrictive assumption as far as the implied distribution of managerial ability is concerned.

The parameters of the calibrated model suggest that matching frictions in the market for firm and management attributes have considerable effects on output and productivity. I find that plausible degrees of mismatch can replicate observed variations in the distribution of manufacturing firms

¹At purchasing power parity (*Global Purchasing Power Parities and Real Expenditures: 2005 International Comparison Program*, 2008).

²Recent contributions to this literature include, among many others: Klenow and Rodriguez-Clare (1997); Prescott (1998); Hall and Jones (1999); Howitt (2000); Restuccia et al. (2008); Jones (2008).

across countries, provided the elasticity of substitution between ability and quality is low enough. Moreover, the distorted allocations generate economically large variations in measured TFP.

The paper is motivated by considerable evidence that mismatch of managers and projects can explain variations in the efficiency of production. A rich event study literature finds that managerial competence is a quantitatively important factor of efficiency at the firm level. Theoretical macro models, on the other hand, suggest that management quality plays a limited role in accounting for differences in aggregate productivity. My model reconciles these seemingly contradictory conclusions.

My research is related to recent work on the misallocation of resources across plants and firms. [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2007a\)](#) find that the elimination of plant-specific taxes and subsidies can yield TFP gains of 30% or more. My paper formally models a complementary source of distortions, for which there is ample historical and contemporaneous support: crony capitalism. Indeed, there is abundant evidence to suggest that price distortions à la [Restuccia and Rogerson \(2008\)](#) are correlated with attributes of the firm or the manager (or a combination of both) in countries where cronies play an important economic and political role.

Finally, my work revisits the potential of complementarities to shed light on the vast income differences we observe across countries. The idea was first formulated in Kremer’s *O-ring* paper ([Kremer, 1993](#)). [Jones \(2008\)](#) explores the role of complementarities in a model with intermediate goods. I explore matching frictions and complementarities as a source of inefficiency in the production of a single good, rather than a chain of intermediate goods (or tasks). It is, however, natural to interpret my mechanism as a source of “bottlenecks” in a production process involving multiple steps (intermediate goods or tasks) that are complementary to one another, as in [Kremer \(1993\)](#) or [Jones \(2008\)](#).

The rest of this paper proceeds as follows. Section 2 reviews the empirical literature on the role of management in two broad classes of models. Section 3 sets up the benchmark model. I describe how latent firm and manager attributes are inferred from observable payment data and I define the undistorted competitive equilibrium. In section 4, I describe the aggregate and firm-level data and estimate relevant parameters. The model is

calibrated to fit U.S. manufacturing data in section 5. Matching frictions and the associated allocative distortions are discussed in section 6. Section 7 summarizes and concludes.

2 The Importance of Management: Micro vs. Macro Evidence

There is a vast literature in corporate finance – among others – on the contribution of management to efficiency at the firm level. Many of these studies conclude that managerial competence is an important ingredient for productivity and profitability.

One body of evidence suggests that the replacement of incompetent management is a key factor in productivity gains after privatizations in developing countries. [La Porta and Lopez-De-Silanes \(1999\)](#) find that bringing in fresh managers was key to productivity gains after the privatization of state-owned Mexican enterprises in the 1980s and 90s. Real sales per worker increased, on average, by 100% over this period. Similarly, [Garcia et al. \(2001\)](#) and [Cole et al. \(2005\)](#) report a fourfold increase in labor productivity growth in Chilean copper mines following the denationalization in 1990. Growth in real output per worker jumped from 3.5% (1970-1990) to 14% (1990-2000). [Cole et al. \(2005\)](#) argue that some, though not all, of the gains were realized thanks to improved expertise in both new and incumbent mines.

In a study of the Korean automotive industry, the McKinsey Global Institute found that total factor and labor productivity reached only 48% of U.S. levels in 1995.³ The study argued that two of the most important sources of inefficiency were (1) management’s inability to implement lean production and (2) the adoption of needlessly complex manufacturing processes. The latter reflected management’s inability to take manufacturing and assembly aspects into account at the vehicle design stage and was often mentioned as a source of low-quality output and costly rework.⁴

There is also a large corporate finance literature on the economic role of management. In an innovative empirical study, [Bennedsen et al. \(2007\)](#) doc-

³[Productivity-led Growth for Korea \(1998\)](#)

⁴A more comprehensive and detailed account of South Korea’s development experience – and the government’s heavy hand in the process – is in Appendix B.

ument the performance effects of exogenous distractions of a firm's CEO. Based on data for Danish limited liability firms, they find that the death of a CEO or a close family member is associated with significant declines in profitability as well as sales and investment growth. The death of a CEO causes, on average, a 1.7 percent point decline in operating returns on assets; the death of a family member triggers a 0.7 point decline. All events combined are associated with a 0.9 percentage point or 11% decline in operating returns. Moreover, the results suggest that managerial input is a particularly important aspect of performance in skill and capital-intensive firms.⁵

[Bebchuk and Cohen \(2005\)](#) find that arrangements, which protect incumbent management from removal are associated with an economically significant reduction in firm value. Tobin's Q is, on average, 17 points lower for firms with so-called staggered boards in a panel of U.S. public companies in 1995-2002. These results suggest that replacing "entrenched" management yields economically sizeable gains in firm value.

In contrast, recent macro studies on CEO compensation find a very limited role for management. [Gabaix and Landier \(2008\)](#) and [Terviö \(2008\)](#), for instance, find that managerial talent is very narrowly dispersed and argue that replacing CEOs of large US corporations by an arbitrarily chosen colleague hardly affects the market value of those firms. In this class of models, mismatch between firm quality and managerial ability is a trifling source of efficiency losses.

One contribution of this paper is to show how assumptions about the degree of complementarity drive the results found in [Gabaix and Landier \(2008\)](#) and [Terviö \(2008\)](#). A key assumption in these models is that the elasticity of substitution between the attributes of the firm (which I will call firm *quality*) and those of senior management (*ability*) is set to unity. A model with more complementarity between quality and ability undermines the finding that variations in managerial input are economically unimportant. I show that with more complementarity, even small deviations from efficient allocations have a sizeable impact on output and aggregate TFP. Relaxing the unit elasticity assumption in my model is what allows

⁵The effect of shocks is, in fact, concentrated in industries with high R&D expenditures, high investment rates and those exhibiting fast growth. This suggests that the degree of complementarity between the firm and senior management may vary across industries.

me to reconcile the empirical results from the micro studies with those in [Gabaix and Landier \(2008\)](#) and [Terviö \(2008\)](#).

3 The Economic Environment

In recent work, [Hsieh and Klenow \(2007a\)](#) and [Restuccia and Rogerson \(2008\)](#) suggested that idiosyncratic price distortions – due to taxes or subsidies levied at the level of the firm – generate variations in the marginal product of variable inputs across firms. Here, I formalize the idea that two fixed factors, namely management ability and firm quality, need to be combined with capital and labor to produce output. In this model, mismatch of firm and management attributes is another type of allocative distortion.

I assume, for the time being, that there are no taxes and subsidies in the markets for labor and capital and hence rule out productivity losses from input price distortions. Instead, I focus on the assignment of managers with ability a to firms with quality q and I distinguish efficient from inefficient matches. Later, I will assess the aggregate productivity effect of mismatches with various degrees of severity to ascertain whether “crony capitalism” thusly defined can provide an explanation for observed per capita income and productivity differences.

3.1 Population and Firms

The model is populated by a continuum of identical households of measure one and each household has a measure N of members. Each member $i \in [0, N]$ is endowed with a single unit of labor and management ability a_i . Ability is distributed with c.d.f. $F_a(\cdot)$. Households own a measure N of firms with quality q_j , $j \in [0, N]$. Quality follows a distribution with c.d.f. $F_q(\cdot)$.

It is important to keep in mind that households are identical to one another in that every single one of them is endowed with a full support of management abilities and project qualities, in addition to a single unit of labor per period and household member.

Members of each household make an occupational choice between supplying a single unit of labor in their capacity as workers or to run a firm in their capacity as managers. If they choose the latter, a manager with ability a_i is

paired with a firm of quality q_j , with $i, j \in [0, N]$. The manager and firm characteristics are aggregated and jointly determine the span of control of the production, similar to [Lucas \(1978\)](#). I will describe the occupational choice in more detail in section [3.5](#).

3.2 Benchmark Model

3.2.1 Preferences

Households do not value leisure and order their preferences over consumption streams $\{C_t\}$ of the final good by:

$$\sum_{t=0}^{\infty} \beta^t N_t U\left(\frac{C_t}{N_t}\right)$$

where $U(\cdot)$ satisfies $U' > 0$, $U'' < 0$ and the Inada conditions $U'(0) = \infty$ and $U'(\infty) = 0$.⁶ Henceforth, let $c_t = \frac{C_t}{N_t}$.

At every point in time, each of the N_t members of the household consumes an equal share of the household's aggregate consumption bundle C_t . The household as a whole sums the valuations of each of its members.

3.2.2 Technology

Firms produce the final good by combining the fixed factors managerial ability (a) and firm quality (q) with the variable inputs labor (l) and capital (k). The production technology exhibits decreasing returns to scale with respect to the variable inputs, both individually and jointly (i.e. $\alpha, \gamma \in (0, 1)$).

Firm- (i, j) output is

$$y = h(a_i, q_j)^{1-\gamma} (k^\alpha l^{1-\alpha})^\gamma \quad (1)$$

where $h(a_i, q_j) = \psi[\nu_a a_i^\rho + \nu_q q_j^\rho]^{\frac{1}{\rho}}$ and ψ is a normalizing constant (see section [3.4.2](#)). I will drop the subscripts whenever the context is sufficiently self-explanatory.

The elasticity of substitution between a and q is denoted by $\sigma = \frac{1}{1-\rho}$. The

⁶Since I am only interested in the steady state of the economy, I need not make any further assumptions about $U(\cdot)$ at this time.

owners of the two fixed factors are joint residual claimants and factor payments always exhaust the (value of) output.⁷

3.3 Household's Problem

Members of a household are infinitely lived and maximize their lifetime utility subject to a budget constraint and the law of motion for capital. Assume that the members are rank-ordered by their management ability and indexed by i (high i is associated with high ability). Firm qualities are indexed analogously.

$$\max_{c_t, k_t} \sum_{t=0}^{\infty} \beta^t N_t U(c_t) \quad (2)$$

subject to

$$rk_t + \int_0^1 (\omega_t[i] + \pi_t[i]) di + \kappa w \geq c_t + x_t \quad (3)$$

$$x_t = k_{t+1}(1 + \eta) - (1 - \delta)k_t \quad (4)$$

where $k_t = \frac{K_t}{N_t}$ is the capital stock per household member, $x_t = \frac{X_t}{N_t}$ denotes investment per capita, and $\eta = \frac{N_{t+1} - N_t}{N_t}$ is the growth rate of household size. κ denotes the endogenous measure of household members who supply one unit of labor inelastically. For simplicity, I assume $\eta = 0$ unless noted otherwise.

Members who supply one unit of labor inelastically are paid the competitive wage rate w . Those who work as managers earn flow compensation $\omega[i]$ in equilibrium. Only a subset of household members will chose the latter occupation. In the budget constraint, this implies that $\omega[i] = 0$ for some i . Firm qualities that are matched with a manager are paid $\pi[i]$; idle firms earn no quasi-rents (that is, $\pi[i] = 0$ for some i). I describe the determination of the managers' compensation in detail in section 3.4.2.

3.4 Firm's Problem

Firms face two distinct, yet simultaneous, problems in their production decision:

⁷In this model, factor payments exhaust output because a and q are residual claimants. In fact, the aggregator $h(\cdot, \cdot)$ need not exhibit constant returns to scale for the production technology to be homogeneous of degree one.

1. They must hire a manager and pay her the equilibrium wage, which, of course, depends on her type (ability).
2. They hire labor and capital in factor markets, with competitive prices w and r , respectively.

Jointly, a firm's quality, say q' , and its manager's ability, say a' , determine the span of control over variable inputs. In particular, the span of control is characterized by $h(a', q')$.

For a given ability-quality match, the solution to an individual firm's problem with respect to the variable inputs labor and capital depends not only on its own quality and that of its manager, but also on the equilibrium distribution of all other ability-quality pairs, taking into account the occupational choices of households, and the prices of capital and labor.

Assume, for one moment, that *all* firms have been matched with a manager. At this point, it is worth anticipating an equilibrium result of the assignment problem. Efficiency and – thanks to the welfare theorems – the competitive equilibrium entail positive assortative matching between q and a as long as ability and quality are complements. One can think of assortative matching as the outcome of a random initial match followed by a “period” of frictionless search. In this over-the-counter market, trading is instantaneous and comes to a halt endogenously as soon as all pairs are matched assortatively.

This simplifies the analysis in that the universe of active firms can be determined from the occupational choice of the household. With this in mind, it is straightforward to characterize the hiring decision for each firm, given the competitive factor prices for capital and labor, the distribution of ability-quality pairs, the household's occupational cutoff and hence the pool of available workers.⁸ The total capital stock is endogenous.

In what follows, I shall describe these two problems more formally.

3.4.1 Variable Inputs

In this basic model, we ignore the possibility of incentive problems between managers and owners of firm quality. Instead, they agree to maximize the

⁸This is, in essence, a span-of-control problem with a single fixed factor, $h(a, q)^{1-\gamma}$. Lucas (1978) outlines a solution algorithm for this type of problem.

joint quasi-rent of the firm. How these rents are apportioned to the manager and the owner is determined by the solution to the assignment problem, which I will describe shortly.

With this simplifying assumption, the firm simply solves:

$$\max_{k[a,q], l[a,q]} h(a, q)^{1-\gamma} g(k[a, q], l[a, q])^\gamma - rk[a, q] - wl[a, q]$$

where

$$\begin{aligned} h(a, q) &= \psi f(a, q) \\ f(a, q) &= (\nu_a a^\rho + \nu_q q^\rho)^{\frac{1}{\rho}} \\ g(k[a, q], l[a, q]) &= k[a, q]^\alpha l[a, q]^{1-\alpha} \end{aligned}$$

Moreover, $\alpha, \gamma \in (0, 1)$, $k[\cdot], l[\cdot] \geq 0$, and $\psi \in \mathbb{R}^{++}$. The technology need not exhibit constant returns to scale in all four inputs. In particular, since a and q are residual claimants, the payments to production factors always exhaust (the value of) output exactly, regardless of the values for ν_a and ν_q .⁹

The first-order conditions of this concave problem are:¹⁰

$$h(a, q)^{1-\gamma} \gamma (k[a, q]^\alpha l[a, q]^{1-\alpha})^{\gamma-1} \alpha \left(\frac{k[a, q]}{l[a, q]} \right)^{\alpha-1} = r \quad (5)$$

$$h(a, q)^{1-\gamma} \gamma (k[a, q]^\alpha l[a, q]^{1-\alpha})^{\gamma-1} (1-\alpha) \left(\frac{k[a, q]}{l[a, q]} \right)^\alpha = w \quad (6)$$

Dividing (5) by (6) one can see immediately that the capital-labor ratio is equalized across firms. Firms run by different ability-quality pairs simply differ in the *scale* of operation, but not in the factor intensity of production.

Next, let me turn to the assignment problem of fixed inputs.

3.4.2 Fixed Factors

Recall that managerial ability and firm quality follow distributions with c.d.f. $F_a(\cdot)$ and $F_q(\cdot)$ respectively. It will turn out to be convenient to keep

⁹For ease of interpretation, I assume $\nu_a = \nu_q$ and $\nu_a + \nu_q = 1$. The aggregator f can then be interpreted as a power mean of a and q (Jones, 2008).

¹⁰For a given cutoff $\underline{h}$, the problem is indeed concave for all $h(a, q) \geq \underline{h}$. Under positive assortative matching, there is a one-to-one correspondence between $\underline{h}$ and the occupational rank cutoff κ .

track of a manager's (firm's) rank, rather than her ability (quality). For this purpose, let me define the inverse cumulative distribution functions:

$$\begin{aligned} a[i] &= F_a^{-1}(i) \\ q[i] &= F_q^{-1}(i) \end{aligned}$$

Similarly, I keep track of equilibrium management compensation and payments to the owners of firm quality by indexing them by rank. In other words, if a manager with ability a is in the i^{th} percentile of the distribution, her equilibrium compensation is denoted by $\omega[i]$. Analogously, the equilibrium payments to the owners of the equally ranked firm quality are denoted by $\pi[i]$.

Since firms hire managers at the *extensive* margin – and managers choose firms in the same way – this problem does not have necessary first-order conditions. Instead, the hiring decision satisfies sorting and participation constraints.¹¹

I can indeed “partial out” the assignment problem from the firms' hiring decisions for labor and capital if we choose ψ conveniently. In particular, if

$$\psi = \left(\frac{w}{\gamma}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha} \frac{r}{w}\right)^{\frac{\alpha\gamma}{1-\gamma}} \left(\frac{1}{1-\alpha}\right)^{\frac{\gamma}{1-\gamma}} \frac{1}{1-\gamma}$$

then the (net) surplus generated by $f(a, q)$ – and split between managers and owners of firm quality – is consistent with the gross surplus net of payments to the variable inputs labor and capital. That is,

$$f(a, q) = h(a, q)^{1-\gamma} g(k[a, q], l[a, q])^\gamma - rk[a, q] - wl[a, q]$$

Clearly, the owners of firm quality and the managers are joint residual claimants. The solution to the assignment problem characterizes the split of the (net) surplus between those two fixed factors of production. Moreover, the complementarity of a and q combined with the fact that all firms produce a single homogeneous final good implies that efficiency requires positive assortative matching.

The assignment problem's sorting constraints are:

$$\begin{aligned} f(a[i], q[i]) - \omega[i] &\geq f(a[j], q[i]) - \omega[j] & \forall i, j \in [\kappa, 1] \\ f(a[i], q[i]) - \pi[i] &\geq f(a[i], q[j]) - \pi[j] & \forall i, j \in [\kappa, 1], \end{aligned} \tag{7}$$

¹¹This assignment structure follows [Terviö \(2008\)](#).

where κ denotes the (endogenous) occupational cutoff for managers, and – by assortative matching – firms.

The first line of (7) states that any firm owner prefers to hire a manager with equally ranked ability and pay her the equilibrium wage over any other manager (when she is paid the equilibrium wage). Similarly, according to the second line, a manager prefers to be matched with a firm of equally ranked quality when both owners and managers are paid their equilibrium compensation.

The participation constraints are:

$$\begin{aligned}\omega[i] &\geq w & \forall i \in [\kappa, 1] \\ \pi[i] &\geq \underline{\pi} & \forall i \in [\kappa, 1]\end{aligned}\tag{8}$$

For now, I assume that the outside options of firms and managers are identical for all types. As long as the outside option does not increase “too fast” with ability (quality), the constraint only binds for the lowest types.¹²

Next, substitute $j = i - \epsilon$ in equation (7), rearrange, and divide by ϵ to obtain:

$$\frac{f(a[i], q[i]) - f(a[i - \epsilon], q[i])}{\epsilon} \geq \frac{\omega[i] - \omega[i - \epsilon]}{\epsilon}$$

Taking the limit $\epsilon \rightarrow 0$ yields the slope of the wage profile:

$$\omega'[i] = f_a(a[i], q[i])a'[i]\tag{9}$$

Proceeding analogously for the manager’s sorting constraint, that is, the second line in (7), yields the slope of the payment profile for the owners of firm quality:

$$\pi'[i] = f_q(a[i], q[i])q'[i]\tag{10}$$

Integrating over active managers, the payment profiles for managers and firms are, respectively:

$$\omega[i] = w + \int_{\kappa}^i f_a(a[j], q[j])a'[j]dj\tag{11}$$

$$\pi[i] = \underline{\pi} + \int_{\kappa}^i f_q(a[j], q[j])q'[j]dj\tag{12}$$

¹²The slope of the outside payment profile plays a more subtle role in the counterfactual economy, where the assignment between managers and firms is no longer positive assortative. I will return to this point later.

The assumption of continuously distributed managerial ability and firm quality simplifies the analysis. Differential rent problems such as this one satisfy the no-surplus condition spelled out in [Ostroy \(1980, 1984\)](#). This satisfies an alternative definition of a competitive equilibrium and eliminates the need to establish a bargaining protocol between firm owners and managers.¹³

Assuming that the payment profiles $\omega[i]$ and $\pi[i]$ are known, equations (11) and (12) form a system of two ordinary differential equations in $a[i]$ and $q[i]$, with initial values $\underline{a}, \underline{q}$ satisfying;

$$f(\underline{a}, \underline{q}) = w + \underline{\pi} \quad (13)$$

3.5 Equilibrium

An equilibrium in this economy is defined as:

Definition 1 *An equilibrium consists of prices $\omega[i]$, $\pi[i]$, r , w , an occupational rank cutoff κ (and hence a measure of active firms $1 - \kappa$), per capita income c , factor inputs $k[., .]$ and $l[., .]$ such that, for given prices:*

1. *Household members maximize utility subject to the budget constraint.*
2. *Firms maximize payments to fixed factors subject to the technology constraints.*
3. *The labor market clears:*

$$\kappa = \int_{\kappa}^1 l(a[j], q[j])dj$$

where the left hand side is the measure of labor supplied by households, namely all those who prefer not to take on management responsibility.

4. *The capital market clears:*

$$k = \int_{\kappa}^1 k(a[j], q[j])dj,$$

*where k denotes the aggregate capital holdings of households.*¹⁴

¹³See a similar discussion in [Terviö \(2008\)](#). The standard reference for differential rent models is [Sattinger \(1979, 1993\)](#).

¹⁴As long as we assume that there is a unit measure of households, the aggregate and average capital holdings are identical.

5. Managers prefer their assigned firm to any other assignment at the equilibrium wage profile $\omega[\cdot]$.¹⁵
6. Agents weakly prefer their occupational choice between being a worker and being a manager. The lowest ranked manager (of rank κ) is indifferent between the two occupations in equilibrium; higher-ranked managers strictly prefer their job to being a worker. Formally, the marginal manager is paid:

$$\begin{aligned}\omega[\kappa] &= h(a[\kappa], q[\kappa])^{1-\gamma} g(k(a[j], q[j]), l(a[j], q[j]))^\gamma \\ &\quad - rk(a[j], q[j]) - wl(a[j], q[j]) - \pi[\kappa] \\ &= w\end{aligned}$$

7. Firms owners prefer their assigned manager to any other at the equilibrium profit profile $\pi[\cdot]$.
8. The owner of the marginal firm quality is paid the outside option:

$$\begin{aligned}\pi[\kappa] &= h(a[\kappa], q[\kappa])^{1-\gamma} g(k(a[j], q[j]), l(a[j], q[j]))^\gamma \\ &\quad - rk(a[j], q[j]) - wl(a[j], q[j]) - \omega[\kappa] \\ &= \underline{\pi}\end{aligned}$$

3.6 Total Factor Productivity

To compare the competitive equilibrium to allocations where firm quality and managers are not matched assortatively, we need to adopt a working definition of measured TFP, both at the level of the firm and in the aggregate.

The most natural definition is to account for all observable factor inputs and to collect the unobservable ability and quality inputs in the term for firm-level TFP, $h(a, q)^{1-\gamma}$. Ideally, one would need to correct for the single unit of (observable) labor that I ignore by treating management ability as unobservable.

Note, however, that the average firm size is decreasing in distortions at the extensive margin. Therefore, I tend to overestimate labor inputs in models with efficient allocations relative to those with mismatches between ability and quality. Put differently, some of the output gains from improvements

¹⁵Recall that managers and owners are residual claimants. They split the surplus between $\omega[\cdot]$ and $\pi[i]$ as specified in section 3.4.2.

in the allocation of firms to managers are attributed to factor accumulation, although the gains stem from increases in TFP.¹⁶ In any event, we err on the side of caution as far as the TFP effects from misallocation of ability and quality are concerned.

In the aggregate, it is not quite sufficient to keep track of idiosyncratic TFP levels alone. Rather, to accurately account for the weight of each firm in the economy, aggregation requires that we integrate the relative firm-size weighted TFP levels:¹⁷

$$\text{TFP} = \int_{\underline{h}}^1 (\psi h)^{1-\gamma} \frac{l(a[h], q[h])}{l} dh \quad (14)$$

where $\underline{h}$ denotes the smallest firm-manager pair. It can be characterized by κ in the competitive equilibrium. In the distorted allocation, the characterization of $\underline{h}$ is more intricate and will be discussed in section 6.

4 Data and Estimation

4.1 Preliminaries

In the model, firm size is determined endogenously by aggregating firm and management attributes through the function $f(\cdot, \cdot)$. The fundamental arguments of the function, however, are unobservable. The firm-level data in **CompuStat North America** contains information on payments to senior managers and to the owners of firm quality and the assignment framework from section 3.4.2 can then be deployed to infer the underlying distribution(s) of attributes.

In the special case of log-additive aggregation, the solution to the system of differential equations (9) and (10) is a pair of reduced form expressions that characterize the profile of talent and quality exceedences, $\frac{a[i]}{a[\kappa]}$ and $\frac{q[i]}{q[\kappa]}$ respectively.¹⁸ For a more general CES aggregator with substitution elasticity different from one, however, I cannot characterize the distribution of

¹⁶Since labor supply is perfectly inelastic and workers are homogeneous there is, of course, no such thing as accumulation of labor. The growth accounting adopted here, however, looks as if labor inputs increased when in fact the aggregate supply of labor is normalized to a unit measure.

¹⁷Since the capital-labor ratio is constant across firms, we can either use the employment share of the firm or capital input share to weigh the firm-specific productivities.

¹⁸See Appendix A for the derivation and details.

a and q without identifying the cutoffs $a[\kappa]$ and $q[\kappa]$ at the same time. That is, I cannot solve the system of differential equations based on the payment profiles $\omega[i]$ and $\pi[i]$ alone. I need additional structure.

The span-of-control model with endogenous occupational choice from section 3 provides such a framework. Namely, I can reverse the solution algorithm used in Terviö (2008). I use the fully specified general equilibrium model to inform the parameters of the talent and quality distributions. To carry out this exercise, I need to identify empirical targets with counterparts that the model can be calibrated to.

4.2 Data

I use a comprehensive data set containing the largest manufacturing firms by market capitalization in the United States. I start with the three thousand largest firms listed in the **Russell 3000®** index, year-by-year. I extract corporate data from **CompuStat North America** database for all those index members that are listed in the database. I then further restrict the sample to firms that are identified with two-digit NAICS codes 31-33, that is, corporations whose main line of business is identified as manufacturing.¹⁹

I also collect data on executive compensation from the **CompuStat ExecuComp** database. Coverage of executive compensation is somewhat limited, especially in the early 1990s. For that reason, the sample of firms with sufficient corporate and compensation data shrinks yet more. **ExecuComp** reports compensation details for as many as 13 corporate officers. 89% of firms report on at least five officers and 96% report on their top four. I retain the five most highly compensated officers in all firms. For robustness, I verify that the estimated coefficients are indeed insensitive to the inclusion of firms reporting on fewer than five executives.

In the data set, I identify all relevant payment streams and asset transfers from the firm to its executives and its securities holders. From the firms' perspective, payments to firm quality and management ability can be categorized into flow payments or stock transfers. Cash dividends, stock repurchases, and interest payments on corporate debt, for instance, are flow

¹⁹I follow standard practice in excluding financial firms and utilities. The former are excluded as they are seen as fundamentally different from corporations in the real economy. The latter are – or used to be – regulated. Although manufacturing firms produce innumerable differentiated goods, they are sufficiently homogeneous for the present argument.

payments to the firms' securities holders. Salaries, bonuses, long-term incentive pay, company match to 401(k) contributions, and reimbursement of travel expenses are all examples of flow payments to executive officers. Stock transfers to the firms' owners consist of capital gains. Managers receive such transfers in the form of stock options and stock grants.

In equation (1), as long as $\gamma < 1$ and since a and q are (joint) residual claimants, payments to the four factor inputs always exhaust the (value of) output. Compensation for current output in the form of stock grants or options to securities holders and corporate officers must therefore be recorded at their flow value, that is, their perpetuity value.²⁰ Recording those transfers at their fair asset value would contaminate the true contribution of a and q to current output using the "attribution mechanism" in 3.4.2. The next section describes how I compute flow payments from available data.

4.3 Computation of Flow Payments

Let Π_t denote the firm's profit after all factors of production have received their flow compensation, but prior to the distribution of financial assets.

$$\Pi[i]_t = y[i]_t - r_t k[i]_t - w_t l[i]_t - \omega[i]_{F,t} - \pi[i]_{F,t}$$

where it is understood that firm i is of quality $q[i]$ and run by a manager of ability $a[i]$. $\omega[i]_{C,t}$ and $\pi[i]_{C,t}$ denote current payments – C for "current" – to managers and owners of firm quality, respectively. They consist of dividends and (net) interest payments to owners, on the one hand, and salaries, bonuses, etc. to managers, on the other hand.

Let $V[.]^*$ denote the market value of the firm and recall that

$$\begin{aligned} V[i]_t^* &= k[i]_t^* + \sum_{s=t+1}^{\infty} (1 + r_t)^{-s} \Pi[i]_s \\ &= k[i]_t^* + V[i]_t \end{aligned} \tag{15}$$

²⁰Terviö (2008) approaches the problem from a different angle and capitalizes all flow payments. His approach simplifies the calculation of payments to the owners of firm quality somewhat. On the other hand, I can avoid making assumptions about future flow payments to the firms' top managers. My problem is thus essentially a static problem and I can focus on the manager's contemporaneous contribution to output.

Owners and managers claim shares of $V[i]_t$ and receive them as stock options, stock grants, or in the form of capital gains.²¹ The split reflects their respective contributions to current output and is determined by equations (9) and (10). That is,

$$V[i]_t = \omega[i]_{V,t} + \pi[i]_{V,t}$$

The flow cost to the firm of compensating $a[i]$ and $q[i]$ with financial assets such as stock options, stock grants, and capital gains at the end of the period consists simply of the perpetuity values $\omega[i]_{P,t}$ and $\pi[i]_{P,t}$ of $\omega[i]_{V,t}$ and $\pi[i]_{V,t}$, respectively:

$$\begin{aligned}\omega[i]_{V,t} &= \sum_{s=t+1}^{\infty} (1+r_t)^{-s} \omega[i]_{P,t} \\ \pi[i]_{V,t} &= \sum_{s=t+1}^{\infty} (1+r_t)^{-s} \pi[i]_{P,t}\end{aligned}$$

Assuming a constant net real interest rate r , which – in steady state – is determined by the fundamental parameters of the model, it follows that

$$\begin{aligned}\omega[i]_{P,t} &= r\omega[i]_{V,t} \\ \pi[i]_{P,t} &= r\pi[i]_{V,t}\end{aligned}$$

The total flow payments to $a[i]$ and $q[i]$ can then be computed as:

$$\begin{aligned}\omega[i]_t &= \omega[i]_{C,t} + \omega[i]_{P,t} \\ \pi[i]_t &= \pi[i]_{C,t} + \pi[i]_{P,t}\end{aligned}$$

I add the flow compensation for the five most senior executives. The sum compensates the collective ability input provided by the most senior officers. I remain agnostic about the functional form of the aggregation of managerial talent and I do not attempt to identify the individual contributions. Formally,

$$a[i] = \Gamma(a[i]_{\text{rank } 1}, \dots, a[i]_{\text{rank } 5})$$

where Γ is an unknown function. $\omega[i]$ is the compensation paid to $a[i]$.

²¹Current profits are distributed to owners by means of dividends and to managers through bonuses. Since these are current payments, we need not worry about them here.

[Garicano and Rossi-Hansberg \(2007\)](#) assume a hierarchical management structure where, in equilibrium, more able individuals are closer to the top of the management pyramid and different strata interact in well-defined ways. Moreover, higher quality firms will feature “higher” management pyramids. Here I abstract from a richer interpretation of management teams to keep the model tractable. Empirically, smaller firms report compensation detail for fewer corporate officers. In this sense, my measure captures the reduced managerial input both in terms of less competent executives at the very top of the firm-specific executive pyramid and in terms of “flatter” executive hierarchies.

As far as payments to the owners of firm quality are concerned, they can be characterized as total payments to the firms’ securities holders net of payments to the owners of the physical capital stock. This is formally captured by equation (15). For lack of a better alternative I use the book value in corporate reports as a proxy for the physical capital stock.

The interested reader may want to refer to Appendix C for a detailed description of the individual components in the owners’ and managers’ flow compensation.

4.4 Empirical Evidence for Assortative Matching

Under the assumption that firms produce a single homogeneous final good and provided the technology exhibits sufficient complementarity between the firm and management attributes, the Pareto-efficient allocation exhibits positive assortative matching between ability and quality. In our model, equations (9) - (12) suggest this is equivalent to assortative matching between (flow) payments to ability and quality.

Year-by-year, the Spearman rank correlation between total flow payments to the top-five executives and payments to firm quality fluctuates between 0.57 (in 2000) and 0.78 (in 1998) for those firms with positive payments to quality. In the sub-sample of firms with negative payments to quality, the Spearman rank correlation varies between -0.49 (in 1997) and -0.62 (in 1996). The rank correlation between the absolute value of payments to quality and managerial compensation lies in the 0.50 (in 2000) to 0.63 (in 2004) range.

These correlations are close to those reported in [Terviö \(2008\)](#). The simi-

larity of these results – despite methodological differences – confirms the robustness of assortative matching in the U.S. economy.²²

One possible reason for deviations from perfect assortative matching (PAM) can be measurement error. However, I am reluctant to attribute the full departure to measurement error alone. For one, manufacturing does not produce a single homogeneous good and variations in relative demand for these differentiated goods can be a source for less-than-perfect rank correlations. Moreover, the model is stylized and does not claim to capture the many factors on which top management’s impact on firm productivity depends. [Bertrand and Schoar \(2003\)](#), for instance, describe the effects of different management styles on firm performance. Similarly, [Bloom and Van Reenen \(2007\)](#); [Bloom et al. \(2007\)](#) document multiple dimensions of management ability that affect profitability, productivity, and survival rates.

4.5 Small and Medium-Sized Firms: Inference Problem

The sample of firms described in previous sections does not, of course, include small or medium-sized firms. The US manufacturing sector, however, consists of a large number of such firms. To fix ideas about the relative importance of small, medium, and large firms in US manufacturing, it is worth looking at US Census data. To illustrate the point, I use data for 2000.

According to the US Census Bureau there were almost five million firms (and more than 6 million establishments) with at least one employee in the United States in 2000.²³ Of those, 283,602 enterprises (and more than 330,000 establishments) were classified as manufacturing firms.

²²There payments to the firm are capitalized and payments to the CEO are taken directly from the **ExecuComp** database (variable `tdc1`). This variable lumps together flow compensation and asset transfers in the form of stock grants and options. Here, I attempt to construct a more coherent measure of executive compensation. The rank correlation reported in [Terviö \(2008\)](#) is between the market value of the firm and `tdc1`.

²³Firms and establishments are reported as having no employees if they have no one on payroll during the mid-March pay period, but with employees on payroll for at least one other pay period during the entire 2000 calendar year. Firms in this category can be seasonally operational firms, firms that ceased to exist after January 1, 2000 but prior to the pay period including mid-March, or new firms that started operating after the pay period including mid-March, 2000. In 2000, 726,862 firms and 767,912 establishments were reported as having no employees.

Unfortunately, there is no firm-level data that would allow me to identify payments to management ability and firm quality in smaller firms. Instead, one has to assume that the data generating process for a and q is the same for **all** manufacturing firms and that the “observed” (a, q) pairs for large firms are the right tails of that unified process.²⁴ The challenge at hand, then, is to characterize the distribution of large firms as sharply as possible and to identify a distribution whose tails fit that characterization tightly.

The model in this paper provides sufficient structure to make plausible inferences about the distribution of firms and managers across the full size spectrum of manufacturing firms. The task is disciplined (1) by the observable firm size distribution in the U.S. and (2) by the relationship between payments and unobserved characteristics in the right tail of that distribution. In section 4.6, I identify features of this relationship that can then be used as calibration targets in section 5.

4.6 Estimation

One of the “best documented empirical regularit[ies] regarding levels of executive compensation” is what [Gabaix and Landier \(2008\)](#) call “Robert’s Law”, according to which executive compensation is proportional to (firm size) ^{τ} .²⁵

In Table 1, I report the coefficients of the following regression:

$$\log(\omega[i]) = \alpha + \tau \log(\text{firm size}) + \epsilon \quad (16)$$

I run the regression with two different proxies for firm size: (1) market capitalization and (2) the number of employees. Each regression is run twice, with and without industry fixed effects. For brevity, I only report the estimates for τ . The coefficients confirm “Robert’s Law” and in the calibration I will target a value of 0.3.

For theoretical reasons that will become apparent in section 5, I need one additional target. The distribution of firm and management attributes completely determines the size distribution of firms in the model. The size dis-

²⁴I do not, of course, observe a and q . Instead, they are inferred from the differential equations (9) and (10).

²⁵The original quote is from [Baker et al. \(1988\)](#).

	log(executive compensation)			
	(I)	(II)	(III)	(IV)
log(market cap)	.291 (.008) (.006)	.292 (.008) (.006)		
log(# of employees)			.278 (.008) (.007)	.287 (.004) (.007)
Industry Fixed Effects	No	Yes	No	Yes
<i>N</i>	6875	6875	6915	6915
<i>R</i> ²	.53	.57	.46	.52

Table 1: Panel Regression: Executive Compensation and Firm Size

Explanation: The panel covers manufacturing firms listed in the Russell 3000 index between 1994 and 2006. Only firms with sufficient data on executive compensation are included. The first row of standard errors (in parentheses) is clustered by year. The second row of standard errors (in parentheses) is clustered at the firm level. The industry fixed effects are based on 3-digit NAICS codes.

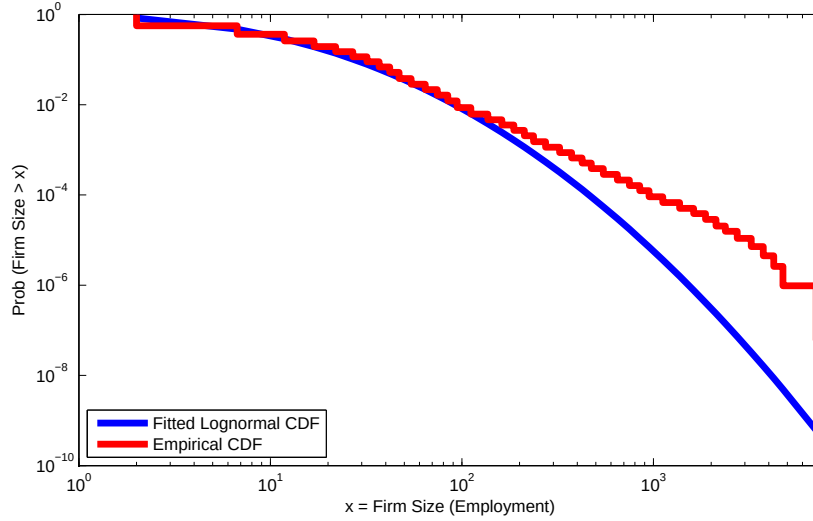


Figure 1: Distribution of U.S. Manufacturing Firms (2000)

tribution, together with the remaining primitives pins down the competitive equilibrium. To match all the targets, it is thus imperative that I pick distributional parameters for a and q that satisfy “Robert’s Law” as well as the size distribution of firms.

Figure 1 illustrates the distribution of U.S. manufacturing firms in 2000. In addition to the size bins (red stairs), the figure shows the maximum likelihood fit for the log-normal distribution, since this is the parametrization I will be working with in the calibration. Visual inspection reveals that it underestimates the measure of very large firms and I thus err on the side of caution.²⁶ In the calibration, I choose the 98.5th percentile of U.S. manufacturing firms as the empirical target. Below this cutoff all firms have 499 or fewer employees. This guarantees that I have a sufficient measure of large firms in the model.

Arguably, one of the key parameters in the model is the elasticity of substitution (σ) between firm quality and managerial competence. The model itself does not provide enough structure to estimate the elasticity empir-

²⁶I err on the side of caution since in span-of-control style models all the “action” in terms of output and productivity is driven by the tails of the distribution.

ically. The reason is that any information about complementarity is contained in the cross-derivative of the aggregator $f(\cdot, \cdot)$, which plays no role in the determination of the competitive equilibrium. Appendix C outlines a promising estimation strategy based on a *Simulated Method of Moments* (SMM), which I invite the interested reader to take a look at.

5 Calibration

In this section, I calibrate the model to US manufacturing data. I treat the United States as an economy with no deviations from assortative matching. Some of the parameters have analogues in the growth model and I choose their values using standard procedures. Since the model features a non-degenerate firm-size distribution, I need to calibrate additional parameters for the model to match its targets.

I adopt standard values for the household's discount factor β and for the depreciation rate of capital δ . Moreover, I assume that household size is constant over time ($\eta = 0$). Together, the three parameters determine the real interest rate in steady state. I assume that one model period corresponds to one year in the data. The share parameter α is chosen to match the factor income shares in the US national income and product accounts. The parameter values are summarized in Table 2.

The extent of decreasing returns is an important parameter in the model. [Atkeson and Kehoe \(2005\)](#) among others find a value for $\gamma = 0.85$.

Project qualities q that are not deployed in production are traded at a price of zero. It is thus intuitive to normalize payments to the marginal quality: $\pi[\kappa] = 0$. It is worth emphasizing here that the marginal manager's flow compensation $\omega[\kappa] = w$ cannot be normalized. It is determined endogenously.

The depreciation rate δ is calibrated to match the average capital-output ratio for 1998-2005 in US manufacturing. Based on the **National Income and Product Accounts**, the average ratio for the period is 1.29. [Gordon \(1971\)](#) argues that the national accounts underestimate the price of capital and hence the capital-output ratio. I therefore follow [Atkeson and Kehoe \(2005\)](#) and assume a capital-output ratio of 1.46. The depreciation rate that

Panel A: Standard Parameters			
Parameter	Value		Target
β	0.96		Real rate of return
δ	0.15		Capital-output ratio
η	0		Constant household size
γ	0.85		From Atkeson and Kehoe (2005)
α	0.36		Factor income share of capital
$\pi[\kappa]$	0		Normalization

Panel B: Distributional Parameters			
Parameter	Value		Target
	$\frac{1}{1-\rho} = 1$	$\frac{1}{1-\rho} = \frac{1}{2}$	
μ_q	0	0	Normalization
μ_a	0	–	Normalization
	–	4.9652	
σ_q	6.986	3.5285	Firm size and compensation targets
σ_a	0.054	3.5200	

Table 2: Benchmark Calibration to US Data

matches the capital-output target is 15%.²⁷

The capital share α is calibrated to match capital’s average income share in manufacturing between 1998 and 2005. I abstract from taxes and set capital’s share to 1 minus labor’s average share of 63.7%.

In section 4.6 I identified two additional calibration targets: (1) the elasticity of management’s compensation relative to the elasticity of firm size, which equals 0.3 for large firms in U.S. manufacturing and (2) the 98.4th percentile of U.S. manufacturing firms in 2000, which corresponds to a firm with 499 employees.²⁸

²⁷The depreciation rate may seem high compared to the values used elsewhere, e.g. [Restuccia and Rogerson \(2008\)](#). However, it is key to remember that I’m not targeting the aggregate capital-output ratio of 2.24, the average ratio for 1998-2005 according to the BEA accounts.

²⁸In the model, I target the 98.4th percentile of the firms that are operating in equilibrium, which corresponds to approximately the 99.96th percentile of all potential projects.

For the purposes of calibration, I hypothesize that both managerial ability a and firm quality q follow log-normal distributions with parameters (μ_a, σ_a) and (μ_q, σ_q) , respectively. Since a and q are combined using a CES aggregator, I can normalize $\mu_q = 0$ without loss of generality. Note also that the share parameters ν_a and ν_q are not separately identified from the means of $\log a$ and $\log q$ (that is, μ_a and μ_q) respectively.²⁹ For given ν_a and ν_q , this leaves three parameters (σ_a, σ_q and $\mu_a - \mu_q = \mu_a$) to meet my targets.

In the special unit elasticity case we can further normalize $\mu_q = 0$. With Cobb-Douglas aggregation, the means of the logarithms of a and q are factored out as multiplicative constants. This, in turn, implies that we can match two targets – the firm size distribution and the relative elasticity – using the variances σ_a and σ_q .³⁰ In this calibration, management’s payment share in large firms is 0.71%. I did not target the share but the calibration is fairly close to the empirical counterpart of 1%. I keep the share target for the cases with non-unit elasticity, where we have one additional parameter – μ_a – to meet it.³¹

Remarkably, the relative dispersion of a and q is highly sensitive to the substitution elasticity $\frac{1}{1-\rho}$. Panel B of Table 2 reports the log-normal parameters for a Cobb-Douglas aggregator ($\rho = 0$) and a CES aggregator with more complementarity ($\rho = -\frac{1}{3}$). The dispersion of managerial talent in the former case is very small and firm size is mostly driven by variations in q . The latter case, on the other hand, suggests that both firm and management attributes play a more important role in the determination of size. Importantly, for both elasticity parameters we match the firm size, elasticity, and payment share targets exactly. Figures 2 and 3 illustrate the distributions of managerial ability and firm quality as well as the calibrated payment profiles.

With unit elasticity of substitution, the model confirms the conclusions of

²⁹The combination of log-normal distributions for both fixed factors and CES aggregation allows for this normalization. One cannot, in general, adopt the same normalization for more general classes of aggregators.

³⁰In the unit elasticity case, the variances σ_a and σ_q are not identified separately from the Cobb-Douglas exponents ν_a and ν_q , respectively.

³¹An alternative strategy would be to normalize output to unity by moving $\mu_q = \mu_a$ in the Cobb-Douglas case and $\mu_q - \mu_a$ in the non-unit elasticity cases. This would simply scale the model up or down but would not affect the competitive equilibrium.

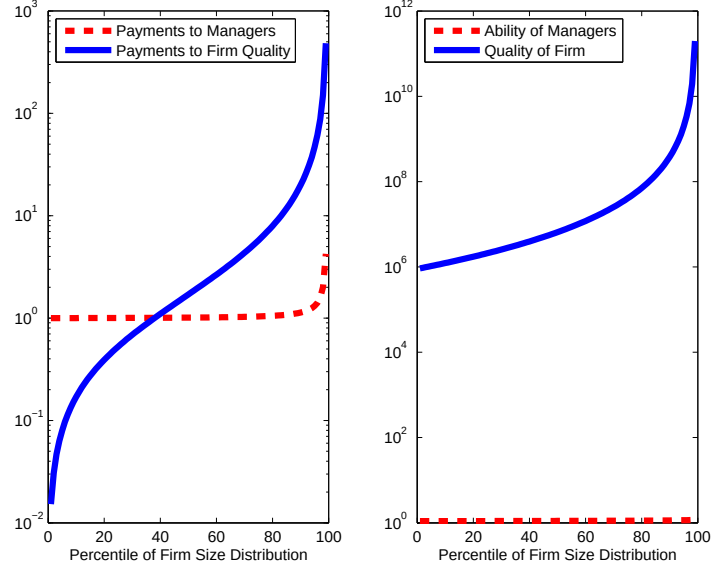


Figure 2: Payments and Fundamental Attributes ($\frac{1}{1-\rho} = 1$)

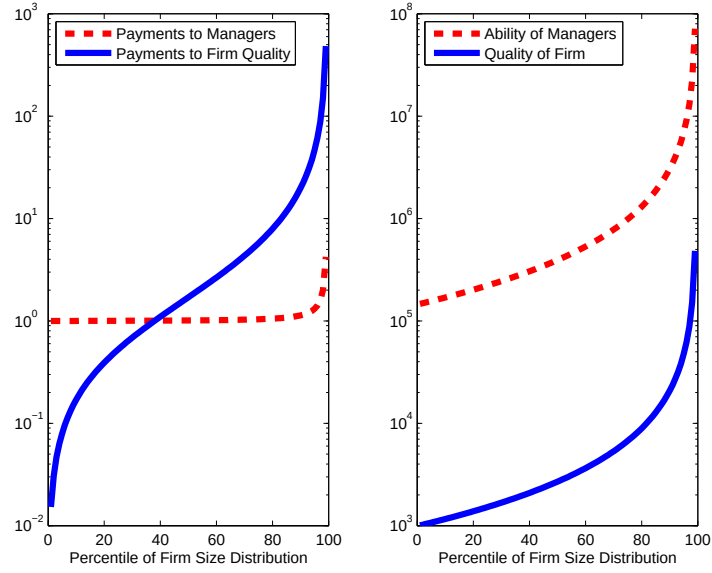


Figure 3: Payments and Fundamental Attributes ($\frac{1}{1-\rho} = \frac{1}{2}$)

Terviö (2008) and Gabaix and Landier (2008) that there is little variation in managerial talent. In fact, my model extends their finding from the sample of large publicly-traded firms to the universe of all (manufacturing) firms. This, truth be told, is not particularly surprising. For commonly used distributions, the partial derivative of the inverse distribution function (denoted here by $a'[i]$ and $q'[i]$) is highest in the tails. So if one was to find dispersion anywhere on the rank interval $[0, 1]$ it would indeed be in the neighborhood of the upper bound. The talent spacing in lower percentiles is typically tighter. My model differs from earlier work precisely in that it has a “complete” universe of firms, rather than just the tails of the distribution.

The change in relative dispersion of firm and management attributes as a function of the substitution elasticity suggests that mismatch of these two fixed factors may indeed be a source of productivity differences. In section 6, I analyze the effect of different types of mismatch on output, productivity, and the shape of the firm size distribution.

6 Quantitative Analysis of “Crony Capitalism”

In this section, I consider two broad classes of matching frictions. The first category of policies locks the occupational choice of household members in the competitive equilibrium into place. Firms and managers are then matched non-assortatively in a subset of the firm and management distributions. The second class of frictions affects occupational choices in that some individuals who would be supplying labor inelastically in the benchmark model run firms instead, and vice versa. Analogously, firm qualities that would idle in the competitive benchmark may be put into operation while others may be forced into “hibernation”. The mismatch is across the complete range of abilities and qualities, and I use evidence on variations in the firm size distribution across countries to discipline the extent of “crony capitalism”.

6.1 Matching Frictions: Occupational Choice Fixed

First, I consider matching frictions between firm qualities and managers confined to certain percentile ranges of the firm size distribution. Rather than being matched assortatively, (a, q) -pairs are formed randomly, provided they are “located” between the same two critical percentiles, say $\underline{i}$

and $\bar{i}$, of their respective quality and ability distributions. For instance, if $\underline{i} = \kappa$ and $\bar{i} = \frac{1+3\kappa}{4}$, then firm qualities and managers in the first quartile of their respective distributions are matched randomly. However, no manager in the first quartile is ever paired with a firm quality in the second, third, or fourth quartile.

Clearly, the severity of the matching friction is a function of the number, say I , of percentile ranges, spaced evenly between κ and 1. In the limit, as $I \rightarrow +\infty$, the assignment is positive assortative. When $I = 1$, matching between all firm qualities and managers above the occupational cutoff rank κ is completely random.

With a continuum of qualities and abilities, matching frictions of this nature affect the distribution of firm sizes. In particular, the distribution is less dispersed, since non-assortative matching tends to “eliminate” both very small and very large firms. Note, however, that the size support within each percentile – and hence in the full distribution of firms – remains unchanged compared to the competitive benchmark.

Recall that under this policy experiment, I fix the occupational cutoff at the competitive level κ . The change in the distribution of ability-quality pairs due to the matching frictions affects the employment level and aggregate capital stock in the distorted allocation for given factor prices r and w . Remember also I restrict myself to comparing steady-state allocations, which implies that the interest rate r is pinned down by β and the depreciation rate δ . Therefore, restoring full employment requires that the real wage w adjust. This, in turn, affects the capital-labor ratio, which is proportional to $\frac{w}{r}$.

To evaluate the effect of such matching frictions, I randomly pair N evenly rank-spaced firm qualities with equally many evenly rank-spaced managers.³² I then approximate the distribution of firms by a piece-wise linear function with N grid points and compute aggregate output, employment and the economy’s capital stock.³³ To compute aggregate TFP, I divide out-

³²Consider an arbitrary percentile range with lower and upper rank bounds $\underline{i}$ and $\bar{i}$. Evenly rank-spaced firm qualities (managers) are separated by the rank distance $\frac{\bar{i}-\underline{i}}{N-1}$ from their neighbors to the right and left.

³³The piece-wise linear approximation implies that quality and ability are distributed uniformly between the grid points. The points themselves lie exactly on their respective distribution functions. The approximation tends to overestimate the number of high-ability

put by a composite input, where the latter is a Cobb-Douglas aggregate of the capital stock and employment using their respective income shares α and $1 - \alpha$.

Table 3 reports the effects of matching frictions on output and TFP for $I = \{10, 4, 2, 1\}$ (in four separate panels) and three values of the substitution elasticity: 1, $\frac{1}{2}$, and $\frac{1}{20}$. The results confirm earlier findings that under the assumption of unit elasticity aggregation, mismatch of fixed factors is a negligible source of efficiency and output losses. With higher degrees of complementarity, on the other hand, the losses increase dramatically. In Panel A ($I = 10$), the output loss widens from .02% under Cobb-Douglas aggregation to 11% when the elasticity of substitution is $\frac{1}{20}$; the decline in TFP jumps from .01% to 7.5%.

Remarkably, even under more severe matching frictions, output and TFP decline minimally – less than .4% – if the elasticity of substitution is set to unity (first row in panels B, C, and D, respectively). On the other hand, as the elasticity of substitution approaches the Leontief case (third row in each panel), aggregate output falls by more than 25% and TFP by almost 19% under the assumption that $I = 1$ (Panel D).

It is, of course, not surprising that matching frictions are more harmful economically when ability and quality are complements. What has been overlooked thus far is that the underlying distributions of ability and talent are sensitive to the assumed degree of complementarity. Together, they imply that “crony capitalism” can depress output and TFP far more than previously thought. If, to the contrary, managers hardly differed in their abilities regardless of the value of $\frac{1}{1-\rho}$, then the size distribution would be driven almost exclusively by attributes of the firm and mismatch would indeed not be a meaningful source of inefficiency.

6.2 Matching Frictions: Endogenous Occupational Choice

If distortions in the market for managerial talent and firm quality are of a nature to generate mismatch between the full range of the two fixed factors, one would expect the enterprise size distribution to have a very different shape compared to the efficient benchmark. More specifically, the distribution in the distorted case must have thinner tails as high quality firms are

managers (high-quality firms).

Panel A: Intra-Decile Random Matching ($I = 10$)		
$\frac{1}{1-\rho}$	$\frac{Y}{Y^*}$	$\frac{TFP}{TFP^*}$
1	.998	.999
$\frac{1}{2}$	.989	.992
$\frac{1}{20}$	.890	.925
...		
Panel B: Intra-Quartile Random Matching ($I = 4$)		
$\frac{1}{1-\rho}$	$\frac{Y}{Y^*}$	$\frac{TFP}{TFP^*}$
1	.998	.998
$\frac{1}{2}$	.974	.983
$\frac{1}{20}$	.839	.889
...		
Panel C: Random Matching Above/Below Median ($I = 2$)		
$\frac{1}{1-\rho}$	$\frac{Y}{Y^*}$	$\frac{TFP}{TFP^*}$
1	.997	.998
$\frac{1}{2}$	.956	.971
$\frac{1}{20}$	.791	.855
...		
Panel D: Random Matching ($I = 1$)		
$\frac{1}{1-\rho}$	$\frac{Y}{Y^*}$	$\frac{TFP}{TFP^*}$
1	.996	.997
$\frac{1}{2}$	.931	.953
$\frac{1}{20}$	.734	.813

Table 3: Matching Frictions Within Various Ranges of Distribution

not always matched with competent managers. In fact, some large firms may be run by managers who, in a frictionless economy, would be supply-

ing labor to any one of the firms in the distribution inelastically. Very large enterprises account for a smaller share of all firms.

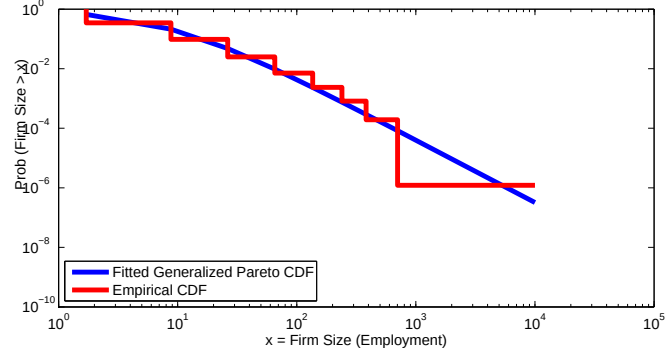


Figure 4: US Manufacturing (2000), $\hat{k} = 0.4767$, $\hat{\sigma} = 3.8293$

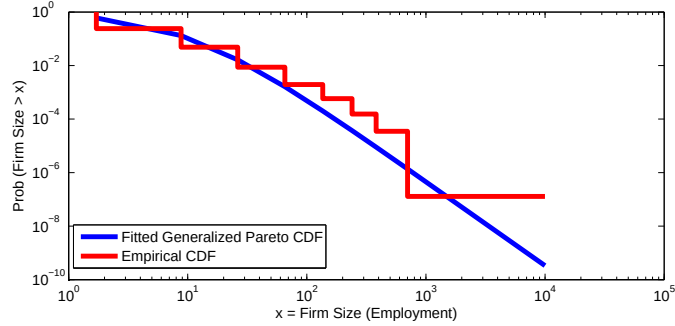


Figure 5: Korean Manufacturing (1998), $\hat{k} = 0.3179$, $\hat{\sigma} = 3.0944$

Figures 4 and 5 highlight the shape difference between US and Korean manufacturing. For ease of comparison, US manufacturing firms are sorted into the same size bins as those reported by the National Statistical Office of Korea and the x and y (log) scales are identical in the two figures.³⁴

The parameter of interest in the Generalized Pareto Distribution (GPD) is

³⁴The US firm size data is much less granular. The US Census tabulates data in as many as 44 size bins.

$\hat{k}$, which characterizes the shape of the tail. A smaller number indicates a more concave plot in log-log space, and hence a more rapid decay in the probability mass as firms become larger. Both the mean and variance of the distribution are increasing in $\hat{k}$ and in the scale parameter $\hat{\sigma}$.³⁵

Since I assume the unobservable firm and talent attributes to be distributed log-normally, I can parametrize the degree of mismatch by the correlation coefficient between a and q (denoted by ρ_{aq}). In U.S. manufacturing, 96.50% of firms with at least five employees have 499 employees or less. Among Korean manufacturing firms, the corresponding share is 99.54%. A very small share of Korean firms has 500 or more employees.

Starting from the calibrated distributions of firm and management attributes in the U.S., I pick the ρ_{aq} that lowers the variance by just enough to shift the 499 employee cutoff from the 96.50th to the 99.54th percentile of the firm size distribution. It is then a straightforward (though time-consuming) numerical exercise to compute the aggregate productivity decline relative to the efficient U.S. benchmark.

I do not mean to suggest that the variation in the shape of the firm size distribution is fully accounted for by frictions in the markets for fixed factors. Restuccia and Rogerson (2008) have demonstrated that idiosyncratic distortions in the market for variable inputs also affect the size distribution. In Korea, for instance, there is ample evidence for capital subsidies of a considerable magnitude. Krueger (2002) estimates capital subsidies in the order of ten percent of GDP in the late sixties and early seventies, and thus captures the effect of industrial policies put in place in the run-up to the so-called “heavy and chemical industries drive”.³⁶

By calibrating to firm size distributions without taking into account distortions at the intensive margin for capital and labor, I characterize a lower bound on the importance of crony capitalism. Capital subsidies targeted at large enterprises – as was the case in Korea – tend to fatten the right tail of the firm size distribution, thus offsetting the effect non-assortative matching between fixed factors, which thins that same tail. Unfortunately, there is no micro data on payments to management and firm quality to disentangle

³⁵Note that the GPD slightly overestimates the probability of very large firms. For the current exercise this is of no consequence.

³⁶This episode is commonly referred to by the acronym *HCI drive*.

the two effects.

7 Conclusion

In this framework, mismatch of firms and managers generates productivity (output) losses of almost 20% (27%), provided the degree of complementarity between a and q is sufficiently high. The effect is driven by two – complementary – mechanisms.

First, the inferred dispersion of managerial competence is sensitive to the substitution elasticity with firm quality. In the Cobb-Douglas case, managers across the size distribution of firms are virtually identical. However, departures from unit elasticity in the Leontief direction imply much more variation in the managers' ability. The second mechanism is mismatch itself. When managers are sufficiently homogeneous, mismatch has only a minor effect, since the size distribution is driven mostly by the distribution of fundamental attributes of the firm. On the other hand, when firm size is determined by heterogeneity in both firms' and managers' attributes, then mismatch is far more detrimental. Moreover, exactly how damaging mismatch is, depends on the substitution elasticity. This is, in essence, the "power mean" effect discussed in [Jones \(2008\)](#).

Here, both mechanisms move in the same direction. An elasticity of less than unity is associated with more dispersion in managerial talent (relative to the dispersion in firm attributes). The "power mean" property of the CES aggregator function then amplifies the effect of mismatch. Together, they explain how departures from Cobb-Douglas yield results that are very different from those reported in previous research and suggest that matching frictions can indeed impose substantial burdens on economic welfare.

Incidentally, a version of this model where the participation constraint depends on the manager's ability offers insights into the effects of industry-specific salary caps: high-ability managers prefer their outside option, which enables less competent executives to take their place. The "flight" of managerial talent lowers, for each firm and in the aggregate, productivity and output. Such restrictions are currently discussed in the United States in the context of bail-out plans for the financial and automotive industries.

An important question for future research is the exact degree of comple-

mentarity between firm and management attributes. The evidence presented in Section 2 suggests a substitution elasticity significantly smaller than unity. Appendix D outlines a promising estimation procedure based on *Simulated Method of Moments* (SMM). The procedure uses empirical deviations from assortative matching and assumptions about measurement error on payments to inform the elasticity of substitution.³⁷

In future theoretical work I plan to investigate some micro foundations of matching frictions. In ongoing research with Guillermo Ordoñez we ask under what conditions endogenously long-lived political coalitions are formed and how they affect the allocation of resources. This project is motivated by our belief that interactions between political and economic agents are key to understanding aggregate inefficiencies and we expect to break some new ground in the class of dynamic political economy models.

In a model similar to [Shimer and Smith \(2000\)](#) and [Kiyotaki and Lagos \(2007\)](#), search frictions generate equilibria with non-assortative matching between managers and firms. Variations in search frictions capture the stylized fact that markets for skilled individuals are shallow in developing countries compared to those in highly industrialized economies. A version with a richer participation structure also features “brain-drain” of high-ability individuals.

³⁷I wish to thank Jesus Fernandez-Villaverde, Jin Hahn, Dan Ackerman, and Lee Ohanian for suggesting this approach.

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A Reduced Form Solution to System of Ordinary Differential Equations

Under the assumption that $a[i]$ and $q[i]$ are combined by a Cobb-Douglas aggregator, the system of differential equations formed by 9 and 10 has a reduced form solution. The two equations can, in fact be rewritten as:

$$\frac{a'[i]}{a[i]} = \omega'[i] \nu_a^{-1} a[i]^{-\nu_a} q[i]^{-\nu_q} \quad (17)$$

$$\frac{q'[i]}{q[i]} = \pi'[i] \nu_q^{-1} a[i]^{-\nu_a} q[i]^{-\nu_q} \quad (18)$$

Let

$$z[i] = a[i]^{-\nu_a} q[i]^{-\nu_q} \underbrace{=}_{\text{since payments exhaust surplus}} (\omega[i] + \pi[i])^{-1}$$

Moreover, let

$$\begin{aligned} \tilde{z}[i] &= \ln z[i] \\ -\tilde{z}[i] &= \ln (\omega[i] + \pi[i]) \end{aligned}$$

Since $\omega[i] + \pi[i] = \exp(-z[i])$, it follows that

$$\begin{aligned} \tilde{z}'[i] &= \frac{1}{\exp(-z[i])} (\omega'[i] + \pi'[i]) \\ \tilde{a}'[i] &= \frac{d \ln a[i]}{di} = \omega'[i] \nu_a^{-1} z[i] \\ \tilde{q}'[i] &= \frac{d \ln q[i]}{di} = \pi'[i] \nu_q^{-1} z[i] \end{aligned}$$

I can then solve for

$$\begin{aligned} \ln \left(\frac{a[i]}{a[\kappa]} \right) &= \tilde{a}[i] - \tilde{a}[\kappa] \\ &= \int_{\kappa}^i \tilde{a}'[j] dj \\ &= \int_{\kappa}^i \omega'[j] \nu_a^{-1} z[j] dj \\ &= \int_{\kappa}^i \omega'[j] \nu_a^{-1} (\omega[j] + \pi[j])^{-1} dj \end{aligned}$$

Exponentiating both sides of the equation, I get

$$\frac{a[i]}{a[\kappa]} = \exp \left(\int_{\kappa}^i \frac{\omega'[j]}{\nu_a(\omega[j] + \pi[j])} dj \right) \quad (19)$$

Similarly, I can characterize the exceedences of $q[i]$ over a threshold as

$$\frac{q[i]}{q[\kappa]} = \exp \left(\int_{\kappa}^i \frac{\pi'[j]}{\nu_q(\omega[j] + \pi[j])} dj \right) \quad (20)$$

For substitution elasticities different from unity, the system of ordinary differential equations does not have an analogous reduced form solution, which relies on multiplicative separability of the aggregator. For general CES aggregators, additional structure is needed to characterize the cutoffs $a[\kappa]$ and $q[\kappa]$ and the distribution of ability and talent above the rank cutoff κ .

B Evidence for the Effects of Mismatch: South Korea

South Korea is a well documented case of heavy-handed government intervention, especially in the manufacturing sector. The economic and political constellation in the aftermath of the Korean War was unique and provided the impetus for a strong government hand in economic development and industrial policy.

Although Korea has been described as a development miracle, there is reason to be circumspect. It is, in fact, far from clear whether Korea experienced several decades of unprecedented growth because or in spite of the government's heavy involvement in all economic matters. Given that Korea's infrastructure was severely impaired as a result of the Korean War and given also that the southern half of the peninsula was predominantly agricultural, it is not really surprising that capital accumulation was such a powerful engine of growth between the 1960s and early 1990s. What is remarkable is that despite a presumably high marginal product of capital combined with massive investment subsidies, many industries that were targeted for preferential treatment by the government struggled to generate positive returns on investment. This raises the specter of resource misallocation on a rather broad scale. Moreover, there is evidence for inefficiency at the firm level in a number of industries, many of which had been singled out for preferential treatment under the government's industrial policies.

In what follows I give some historical background, shed some light on the business-government nexus, and eventually take a closer look at two important industries: automotive and steel. While the latter has been a success story from day one, Korean car manufacturing has been troubled by a panoply of issues for most of its existence.

B.1 Korea's Political and Economic/Business Elites in 1961

B.1.1 Business Elite

In 1961, Korea's business sector was dominated by a relatively small number of wealthy individuals. They had risen to prominence in the 1950s under the Syngman Rhee government and accumulated (some of) their wealth by appropriating US aid for private use.³⁸

³⁸In the 1950s, US aid covered between 60 and 90% of central government expenditures. The amount of aid provided ample opportunities for diversion of funds for private use.

Most of these businessmen made their fortune in the three “whites”: sugar, flour, and cotton. These (light) industries depended heavily on the US for raw materials and there was widespread racketeering of US aid (Lie, 1998).

Their wealth turned out to be a political and social concern in a country with a per capita GNP of approximately \$100 (current dollars), or less than 10% of US GDP per capita during the 1950s and most of the 1960s. This was particularly true in the context of recurring political instability in the early 1960s.

B.1.2 Political Leadership

Park Chung-hee took power in the 1961 military coup that ousted the short-lived Chang Myon (who took over from Syngman Rhee in 1960). He was to be Korea’s president until 1979 (when he was assassinated).

Park’s primary goal was to solidify his political leadership. In the particular context of post-war Korea this required that he appear to crack down on political corruption (which was one of the triggers of the 1960 political upheaval) and to improve national security against Soviet- and Chinese-backed North Korea. Park concluded that national security required a co-ordinated economic development effort.

It’s important to remember that after partition, North Korea held on to most of the peninsula’s (heavy) industrial assets and natural resources. The south, on the other hand, was essentially an agrarian economy with some light industries (textiles and some basic consumer goods). In Park’s view of the world, standing firm against the North required – in addition to a powerful standing army – the development of (heavy) industries deemed critical to the war effort.

B.2 New Relationship between Business and Government

Park was in a position to realign the relationship between business elites and government using a “carrot and stick” policy. Given widespread resentment against political corruption, Park was in a strong position vis-à-vis the business community.

In 1961, Korean banks were nationalized. The government exploited this

Industry	Assigned Corporation
Cement	Kŭmsŏng Textile (Ssangyong)
Fertilizer	Samsung / Samho Textile
Electricity	Taehan Milling
Iron	Taehan Cement / Tongyang Cement / Kŭkdong Marine / Taehan Industry
Synthetic fiber	Hwasin / Chosŏn Silk Mill / Han'guk Glass

Source: [Jones and SaKong \(1980\)](#)

Table 4: Industry Assignments in Early 1960s

strategic position to allocate loans on preferential terms and grant access to scarce foreign exchange (to purchase imported goods and services) selectively. At the same time, the government threatened to confiscate assets that were – in its view – accumulated “illicitly”. This “carrot and stick” policy was key to a redefined relationship between business and government.

Park ultimately refrained from seizing assets in return for political and financial support from the business community. Moreover, he offered financing on preferential terms to those who agreed to implement the president’s industrial policies. This, in essence, enabled the business elite to accumulate additional assets in the name of national security, economic development, and political stability rather than through outright corruption.

For reasons that are not entirely clear, leading businessmen were assigned to strategic industries in which they had no prior experience. Table 4 shows a sample of industry assignments..

Two aspects of this episode were remarkable:

1. Park selected a group of businessmen with a track record of rent-seeking on a large scale rather than entrepreneurial success to implement his ambitious industrial policy.
2. The most prominent among the “illicit profiteers” founded the *Federation of Korean Industries* (FKI) in 1962 and coordinated the assignment of individuals to target industries (of which there were six: the five in table 4 plus oil refining). Some have argued that the deliberate “mismatch” (illustrated in Table 4) ensured a level playing field among

the key players.

In 1973, the government launched the “heavy and chemical industries (HCI) drive”. The declaration merely formalized the promotion of key heavy industries set in motion in the late 1960s. It added the industrial machinery, shipbuilding, automotive, electronics, steel, and petrochemicals industries to the original six targets.

While this industrialization drive did not alter the relationship between government and business elites immediately, it did offer new opportunities to extract rents. The *chaebol* expanded aggressively into these new target industries and thereby planted the seeds for a shifting power balance with the government (see [B.3.3](#) below).

B.3 Mutual Dependence

While the government was in a dominant position in the 1960s, and possibly in the 1970s, the relationship was somewhat more balanced in the late eighties and early nineties. Early in the process, the government was in a position to punish those who did not hold up their end of the bargain. Prior to the democratic transition in 1987, the government actually cracked down on non-cooperative business groups on at least two occasions. Later on, some of the *chaebol* had already become too big to fail and the government was far more constrained in its ability to rein in corporate excesses (in the form of unusually high debt-to-equity ratios).

B.3.1 1971: Liquidation of *Samhak*

The owner of *Samhak* – the largest producer of *soju* liquor at that time – backed the opposition candidate Kim Dae Jung in the 1971 presidential election. Shortly after the election, which Kim lost, he was convicted of tax evasion and forced into bankruptcy ([Lie, 1998](#)).³⁹

B.3.2 1985: Liquidation of *Kukje*

Kukje was one of the ten largest *chaebol* in the early 1980s. Like many other business groups, *Kukje* was diversified and highly leveraged; in fact, *Kukje* was the most highly leveraged group in 1985 and cash flow from operations was insufficient to service outstanding debt.

³⁹See also: *Far Eastern Economic Review*, November 30, 1995, p. 71-2 for more details.

Kukje was poorly managed. Senior management positions were filled with the chairman's – allegedly – unqualified sons-in-law and the group built new headquarters in downtown Seoul at a time when it could hardly service its debt. Construction was financed by raising additional debt on the curb market (Graham, 2003).

Importantly, *Kukje*'s chairman openly supported an opposition candidate in National Assembly elections earlier in the year and refused to make significant donations to quasi-governmental organizations (which were routinely used to channel political slush funds). Observers believe that this may have played a role in the government's unwillingness to come to its rescue and honor its checks (Graham, 2003; Kang, 2002a,b)

Within weeks *Kukje* was forced to declare bankruptcy and was liquidated. The pieces were picked up at fire sale prices by the well-connected *Hanil* and *Dongkuk* groups. The liquidation was, in fact, so opaque that the Constitutional Tribunal ruled it unconstitutional in 1993 (Graham, 2003).

Kukje was not the only group in difficulty at the time. *Daewoo* was also highly leveraged. Its chairman was summoned home from a trip to Tokyo and warned about the excessive accumulation of debt. At the same time, however, the finance minister assured *Daewoo*'s chairman that *Kukje* alone would be forced into bankruptcy. *Daewoo* continued to rely on debt to expand and diversify until it declared bankruptcy in 2000 (Graham, 2003).

The point of this episode is that being leveraged and poorly managed is not – per se – sufficient for a firm's demise. The straw that broke the camel's back was that Yang Chung-mo backed a high-profile opposition politician and did not replenish the government's political slush funds through the usual channels, thereby cutting himself off from a potentially vital lifeline in the form of government-arranged financing (i.e. policy loans, subsidies).

B.3.3 Too Big To Fail

While the two previous examples illustrate how the government was able to “revoke” special status, the *Daewoo* episode illustrates another key feature. Some *chaebol* had become so large – namely in terms of employees and financial liabilities– that they were simply too big to fail. The government had no choice but to extend additional financing (on preferential terms)

to avoid bankruptcy, even when cash flow from operations barely covered debt service.

Generally speaking, the larger the *chaebol* became, the smaller was the government's bargaining weight. A number of poorly performing business groups had become too big to fail and the government had no choice but to come to their rescue.

It was not until the aftermath of the Asian financial crisis in 1997 that the business sector went through a long-overdue phase of restructuring. Among the most high-profile failures were *Hanbo* in June 1997 (which was no longer able to service \$6 billion worth of debt), *Kia* in October 1997 (which was nationalized and later taken over by Hyundai), and the breakup of *Daewoo* under a \$73 billion debt burden in the fall of 1999.⁴⁰

B.4 TFP vs. Factor Accumulation

The stylized facts for Korea suggest that the government-business relations were of a nature that preserved "inefficient" matches, i.e. incompetent managers were in charge of high-quality plants (that is, plants with a large natural efficient scale of production such as shipbuilding or petrochemicals).

My model contends that inefficient assignments undermine total factor productivity (TFP).

In a detailed country study, the McKinsey Global Institute decomposes average GDP per capita growth for two periods: 1970-82 and 1982-95.

The aggregate data for the first period support my model quite nicely. TFP did not contribute to GDP per capita growth at all. Instead, growth was fueled by factor accumulation. In the 1980s and 90s, on the other hand, labor and capital inputs contributed far less to GDP per capita compared to the first period, while TFP accounted for nearly 50% of average GDP per capita growth.

An in-depth look at individual manufacturing industries reveals remark-

⁴⁰GM eventually bought *Daewoo Motor Company* (DMC) for \$400 million in May 2002. GM originally offered \$4 billion, but lowered the bid after due diligence uncovered previously unknown liabilities. GM lowered the bid yet more to the final price of \$400 million after labor unrest further delayed the transaction.

(per capita)	1970-82	1982-95
Labor input	31%	10%
Capital input	75%	46%
TFP	-6%	44%
GDP	100%	100%
Average GDP growth	6.7%	7.7%

Table 5: McKinsey Decomposition (1998)

ably different patterns of productivity. To illustrate the point, I will describe the steel and automotive industries in more detail.

The two industries made significant contributions to aggregate economic activity. Together, they accounted for 4.9% of Korean value added and 2.3% of total employment in 1995.⁴¹

Moreover, they highlight how government interference can affect productivity levels through precisely the channel I emphasize in my model. Firms and industries with minimal government involvement – other than receiving preferential loans – exhibit TFP levels similar to corresponding US and Japanese levels, which are commonly held to represent best practice.

B.4.1 Steel Industry

By most accounts, the Korean steel industry is the most successful example of Korea's industrial policy emphasizing heavy and high-tech industries. In particular, the state-owned *Pohang Iron and Steel Company* (POSCO), Korea's only integrated steel mill, has been one of the most efficient steel producers worldwide since the 1990s.

POSCO was established in 1968 and became operational in 1972. The first chairman of POSCO was a highly respected former general who accepted the job under three conditions:

1. no government influence in the procurement of equipment, goods,

⁴¹The corresponding figures for the US are: 1.7% (value added) and 0.8% (employment). In Japan the two industries accounted for 2.9% of value added and 2.1% of employment (McKinsey Global Institute, 1998).

and services;

2. no government influence in the hiring and management of POSCO personnel; and, most importantly,
3. no political donations allowed from POSCO.

Korea's steel output grew from 500,000 tons in 1968 to 37 million tons in 1995 and POSCO's share jumped from nothing to 57% over the same 27-year period.

Table 6 summarizes industry productivity in Korea and Japan. Relative

	Korea	Japan	United States
TFP	111	110	100
Capital Productivity	115	101	100
Labor Productivity	108	121	100
Integrated Mills	123	121	n.a.
Mini Mills	80	122	n.a.

Table 6: Industry Productivity in 1995 (McKinsey Global Institute, 1998)

to the the US in 1995 (US = 100), Korea is among the most efficient steel producers in the world: Korea's industry TFP was 111, compared to 110 for Japan's. In terms of labor productivity, Korea was 8% more efficient than the US, but 13 percentage points less so than Japan. On the other hand, Korea's capital productivity exceeded both Japan's and the United States' by about 14 percent.

The labor productivity gap between Korea and Japan is almost entirely attributable to mini mills. Compared to Japan's integrated mills, POSCO is somewhat more productive and is thus certainly among the most efficient steel producers world-wide.

One of the sources of Korea's high capital productivity is a high capacity utilization rate close to 100%, compared to Japan's 66% in 1995. This is in sharp contrast to Korea's automotive industry, which has been plagued by unusually low capacity utilization rates.

B.4.2 Automotive Industry

The Korean automotive industry alone accounted for almost three percent of total value added in 1995.⁴² The industry's employment share was two percent.

Unlike the steel industry, the Korean automotive industry lagged behind the US and Japan in terms of labor and total factor productivity throughout the 1980s and 90s. Table 7 shows Korean and Japanese productivity levels relative to the US in 1995. Table 8 shows that productivity growth

	Korea	Japan	United States
TFP	48	127	100
Labor Productivity	48	144	100
Capital Productivity	48	99	100

Table 7: Industry Productivity in 1995 (McKinsey Global Institute, 1998)

in Korea exceeded the corresponding growth rates in Japan and the US between 1985 and 1995. Importantly, though, if one adjusts for the (estimated) 22-year lag between the "birth" of the Japanese and Korean auto industries, the latter underperformed in terms of labor productivity.

Korea's automotive industry has also been plagued by low capacity utilization rates, especially compared to Japan. Kang (2002b) reports capacity utilization rates for Hyundai Motors of 49% in 1969, 25.8% in 1972, and 61.8% in 1979. In that year, Hyundai was the top performer in terms of capacity utilization. Saehan/Daewoo used 24.6% of its capacity in 1979 while Kia used 55.4%. The industry average was 48.6%.

Not surprisingly, the cost of debt has exceed the return on invested capital between 1975 and 1995, except for a short period in the mid-1980s.

By comparison, the US automotive industry used 79.6% of installed capacity in 1972 and 79.7% in 1979.⁴³ Conservative estimates of Japanese utiliza-

⁴² Among major car producing countries, only Germany's automotive sector contributed more to GDP than Korea's.

⁴³ The average capacity utilization rate in the US automotive industry between 1972 and 2008 was 77.2%. The lowest rate was 52.0% in 1984; the highest rate was 91.1% in 1978.

Panel A			
Compounded Annual Growth Rate: 1985-1996			
	Korea	Japan	United States
TFP	11.5%	2.7%	2.3%
Capital Productivity	2.3%	-1.5%	1.7%
Labor Productivity	16.3%	4.8%	4.5%
...			
Panel B			
Twenty-Year Labor Productivity Growth Rate			
	Toyota	Nissan	Hyundai
Labor Productivity	13.8%	12.9%	8.6%
...			
Panel C			
Physical Labor Productivity at End of Twenty-Year Period			
	Toyota	Nissan	Hyundai
Year	1974	1974	1996
Vehicles per Employee	44.7	35.5	27.9

Table 8: Productivity Growth (McKinsey Global Institute, 1998)

tion rates in 1979 are in the neighborhood of 79%.⁴⁴

In sum, in 1979 the Korean car industry lagged both the US and Japanese industries by approximately 30 percentage points in terms of capacity utilization. Although the US utilization rate fluctuated quite a bit during the 1970s with a low of 64% in 1975 and a high of 91% in 1978 (and a 1972-1979 average of 80.5%), the Hyundai figures for 1972 suggest that Korea was lagging behind the US throughout the decade.⁴⁵

The McKinsey productivity report argues that Korea's low capacity utilization rates are partly driven by the aggressive introduction of new models, each of which is based on a new platform and requires its own production infrastructure. In 1996, annual production for large passenger cars and SUVs were 16,000 and 35,000 respectively. Best-practice suggests a volume of at least 100,000 physical units to produce profitably.

The relatively low level of TFP in Korea's automotive industry has multiple sources, two of which are clearly related to management ability:

1. Inability to implement lean production; and
2. Needlessly complex manufacturing processes due to insufficient consideration of manufacturing and assembly aspects in the design process.

The second factor is often mentioned as a possible source of low-quality output and costly rework. In 1997, Kia had the worst quality record among 38 surveyed manufacturers. Hyundai was ranked 34th in the same survey. As recently as 2008, Hyundai and especially Kia lag most leading car brands in different quality categories.⁴⁶

The Korean automotive industry is a nice illustration of firm-management

⁴⁴The Japanese Ministry of Economy, Trade and Industry (METI) reports a so-called *operating ratio* instead of capacity utilization rates. The ratio is normalized to 100 for capacity utilization in 2005. The estimate is based on the assumption that the average capacity utilization rate in the automotive industry (which corresponds to NAICS code 33611 in the United States and encompasses passenger cars, trucks, and buses in Japan) is identical in the US and in Japan between 1978 and 2007. Most sources assume that the average Japanese utilization rate is higher than the corresponding US rate, particularly in the automotive industry. I err, therefore, on the side of caution.

⁴⁵Unfortunately, the Japanese operating ratio is not available prior to 1978.

⁴⁶J.D. Power and Associates, 2008 Initial Quality Study

mismatch. Car manufacturing has a large efficient scale of production and requires considerable managerial know-how. Korean manufacturers have put in place a modern production infrastructure thanks to cooperation agreements with some of the world's most efficient car producers. However, they appear to struggle to implement best practices from design for manufacturing to quality control along the production cycle. Improvements in management practices alone could improve efficiency through higher capacity utilization (which may require leaving market segments with low annual output) and more closely coordinated design and manufacturing decisions.

Although Korea's car industry has managed to gain a foothold on world markets in a fairly short period of time, a close comparison with best practices around the world suggests that invested capital has not been managed efficiently and that for extended periods of time, the return on investment did not cover the cost of debt, despite access to financial markets on preferential terms.

C Firm-Level Data for U.S. Manufacturing: 1994-2006

This appendix describes the sources and individual components of firm-level data as well as executive compensation in a panel of U.S. manufacturing firms. I rely on two sources for most of the data: **CompuStat** and **ExecuComp**, both published by Standard & Poor's.

C.1 Management Compensation

The **ExecuComp** database reports several elements of executive compensation, which allows me to classify them as flow payments or asset transfers. In particular, I can identify the following components for up to thirteen executives per firm and year:

1. Salary;
2. Bonus;
3. Value of stock options at grant date;
4. Long term incentive payouts; and
5. Other miscellaneous payments such as:
 - severance payments;
 - debt forgiveness;
 - imputed interest on preferential loans to executive officers;
 - tax reimbursements;
 - signing bonuses;
 - 401(k) contributions; and
 - life insurance premiums.

From the firm's perspective all of these are flow payments, with the exception of stock grants and stock options. **ExecuComp** reports Black-Scholes values at grant date for options and fair values for stocks so we need not worry about pricing the assets correctly for now. We do, however, need to convert the asset prices to flow payments using the perpetuity formula in the previous section.

I compute total flow compensation for all executives in the dataset. I then

rank the managers by total flow compensation for each firm, year-by-year.⁴⁷ Since the vast majority of firms report on at least five officers, I limit myself to the top-five for **all** firms.⁴⁸

Including long-term incentive payouts (LTIPs) in managerial pay is potentially problematic, as managers are compensated for inputs that are not contemporaneous. However, as a share of current compensation, LTIPs account for only 4.7% among top-ranked executives (standard deviation: 0.132), 4.1% among second-ranked executives (standard deviation: 0.120), and less than 4% for lower-ranked managers. Omitting LTIPs does not affect the estimation results in section 4 significantly.

The mix of financial assets and current compensation in executive pay varies by rank. Stocks and stock options account, on average, for 42.2% of total compensation (consisting of current compensation plus the value of financial assets) among top-ranked executives (standard deviation: 0.298). The corresponding shares for lower-ranked executives are 39.8% (second ranked), 38.8% (third ranked), 38.0% (fourth-ranked), 36.7% (fifth-ranked).

C.2 Payments to Owners of Firm Quality

I proceed similarly for payments to the owners of firm quality. First, I identify all cash flows between the firm itself and its securities holders. [Hall \(2001\)](#) identifies four distinct streams:

1. Dividends paid, net of dividends received;
2. Repurchases of equity and purchases of equity in other corporations, net of equity issued;
3. Interest paid on outstanding debt less interest received on holdings of debt; and

⁴⁷I ignore job titles in this procedure. The vast majority of top-ranked officers carry the titles CEO, president, chairman, and vice-chairman, as well as various combinations of the four. In some cases, the most highly compensated officer is the COO, typically in conjunction with any one of the four job titles mentioned earlier.

⁴⁸In general, smaller firms report on fewer managers. This pattern holds regardless of the measure of firm size (number of employees, sales, total assets). This is not much of a concern here, but I will get back to this point later. For larger firms, the cutoff means that we ignore managerial compensation for lower-ranked executives. Similarly, in a study on management styles, [Bertrand and Schoar \(2003\)](#) analyze the five most senior executives in each firm.

4. Repayment of debt obligations less acquisitions of debt instruments.

Firms cannot hide or park cash anywhere else, so this is a complete account of cash flows. **CompuStat** reports data on all necessary items, except net purchases of equity in other corporations. Stock prices are all taken from the **Center for Research in Security Prices** (CRSP) database.

In addition to cash flows, I record one-year ex-dividend holding returns. Based on these holding returns, we can compute aggregate capital gains to the firms' shareholders. The holding period is each firm's fiscal year, so as to synchronize the dates for all prices and quantities.

As before, I am interested in the contribution of $q[i]$ to (the value of) firm i 's current output, not the market value of the firm. For that reason, I compute the flow (perpetuity) values of the capital gains and add them to the cash flows. Together, they describe the total flow payments from the firm to its securities holders, denoted by $\phi[i]$ for each firm i .

Next, we need to identify payments to the owners of k and those to the owners of q . Assuming that the economy is in steady-state (or on a balanced growth path, for that matter), securities holders require a return r on their capital stocks. In steady state, the real interest rate is pinned down by the agents' discount factor β :

$$r = \frac{1 - \beta}{\beta}$$

In the data, I need to take inflation into account and the payments to capital holders are:

$$(r + \varphi_t) \times (\text{book value of firm})$$

where φ_t denotes the inflation rate between time t and $t + 1$.⁴⁹ The book value itself is the difference between a firm's total assets and its intangibles.

Payments to the owners of firm quality, denoted by $\pi[i]$, are determined residually as:

$$\pi[i] = \phi[i] - (r + \varphi_t) \times (\text{book value of firm})$$

Flow payments, unlike market values, are not bounded by zero below. This has implications for our empirical assessment of assortative matching discussed in section 4.4.

⁴⁹I adopt the convention that capital owners are compensated at the end of the period.

D Future Empirical Work: Simulated Method of Moments

This appendix contains a brief outline of future empirical work.⁵⁰ While certain elements of the procedure are already in place, additional time and effort is required to implement the algorithm.

I hypothesize that the tails of managerial talent and firm quality follow Generalized Pareto Distributions (GPD) with CDF:

$$F(x) = 1 - \left(1 + \frac{kx}{\sigma}\right)^{-\frac{1}{k}} \quad (21)$$

where k is the shape parameter and σ is the scale parameter. Note that the two-parameter GPD assumes the data consist of exceedences. If they weren't one would simply subtract a threshold μ (i.e. the smallest value in the sample) from every observation to obtain exceedences from the raw data. μ is the so-called location parameter in a three-parameter GPD. Here I assume $\mu = 0$.

Following are step-by-step descriptions of the simulation procedure:

1. Draw a $2 \times N$ matrix of uniform random variables. Denote the two columns by I_1 and I_2 . To guarantee stochastic equicontinuity, I draw the matrix only once (McFadden, 1989).
2. Draw a $2 \times N$ vector of standard normal random variables. Denote the columns by ϵ and u .
3. Fix seven parameters: $\theta = (\rho, k_A, \sigma_A, k_Q, \sigma_Q, \sigma_\epsilon, \sigma_u)$, where σ_ϵ and σ_u are measurement errors associated with management compensation and payments to firm quality, respectively.
4. Compute $\hat{\omega}[i_1]$ and $\hat{\pi}[i_2]$ using θ as well as equations (11) and (12). Each i_1 corresponds to an element in I_1 and analogously for i_2 . Denote the vectors by $\hat{\Omega}$ and $\hat{\Pi}$, respectively.
5. Compute

$$\begin{aligned} X &= \hat{\Omega} + \sigma_\epsilon \epsilon \\ Y &= \hat{\Pi} + \sigma_u u \end{aligned}$$

⁵⁰I would like to thank Jesus Fernandez-Villaverde, Dan Akerberg, Jin Hahn, as well as my advisor Lee Ohanian for suggesting this line of inquiry.

6. Compute the following seven sample moments:

$$\begin{array}{ll} \frac{1}{N} \sum_{i=1}^N x[i] & \frac{1}{N} \sum_{i=1}^N y[i] \\ \frac{1}{N} \sum_{i=1}^N x^2[i] & \frac{1}{N} \sum_{i=1}^N y^2[i] \\ \frac{1}{N} \sum_{i=1}^N x^3[i] & \frac{1}{N} \sum_{i=1}^N y^3[i] \\ \frac{1}{N} \sum_{i=1}^N x[i]y[i] & \end{array}$$

7. Pick θ that minimizes the weighted “distance” between the simulated and empirical moments:

$$\left| \begin{array}{ll} \frac{1}{N} \sum_{i=1}^N x[i] & - \frac{1}{N} \sum_{i=1}^N \omega[i] \\ \frac{1}{N} \sum_{i=1}^N x^2[i] & - \frac{1}{N} \sum_{i=1}^N \omega^2[i] \\ \frac{1}{N} \sum_{i=1}^N x^3[i] & - \frac{1}{N} \sum_{i=1}^N \omega^3[i] \\ \frac{1}{N} \sum_{i=1}^N y[i] & - \frac{1}{N} \sum_{i=1}^N \pi[i] \\ \frac{1}{N} \sum_{i=1}^N y^2[i] & - \frac{1}{N} \sum_{i=1}^N \pi^2[i] \\ \frac{1}{N} \sum_{i=1}^N y^3[i] & - \frac{1}{N} \sum_{i=1}^N \pi^3[i] \\ \frac{1}{N} \sum_{i=1}^N x[i]y[i] & - \frac{1}{N} \sum_{i=1}^N \omega[i]\pi[i] \end{array} \right|$$

The weighting matrix Σ is determined iteratively. The initial matrix is the I -matrix. The initial $\hat{\theta}_0$ is then used to construct $\hat{\Sigma}_0$ and so forth until the estimates $\hat{\theta}$ and $\hat{\Sigma}$ converge.