International and Intra-national Real Exchange Rates: Theory and Evidence

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Abstract

In this paper we study two long-standing puzzles in the International Finance literature: the fact that the real exchange rate (RER) is very volatile (RER volatility puzzle) and that it covaries negatively with domestic consumption relative to foreign consumption (Backus-Smith puzzle). To understand the two puzzles we depart from the existing literature by focusing on a disaggregated analysis of consumption and RER. First, using region-level data for a large number of developed and developing economies we document the characteristics of the two puzzles in cross-country and within-country data. We then develop a combined model of inter-regional and international trade. The model exhibits variations in inter-regional as well as in inter-national consumption and real exchange rates. We show that with a combination of within country and across country shocks, and endogenous tradability the model can rationalize the two puzzles and does so in both international and intra-national dimensions.

JEL Classification: F3, F4

Keywords: Backus-Smith puzzle, real exchange rate volatility, endogenous non-tradability, regional heterogeneity

1 Introduction

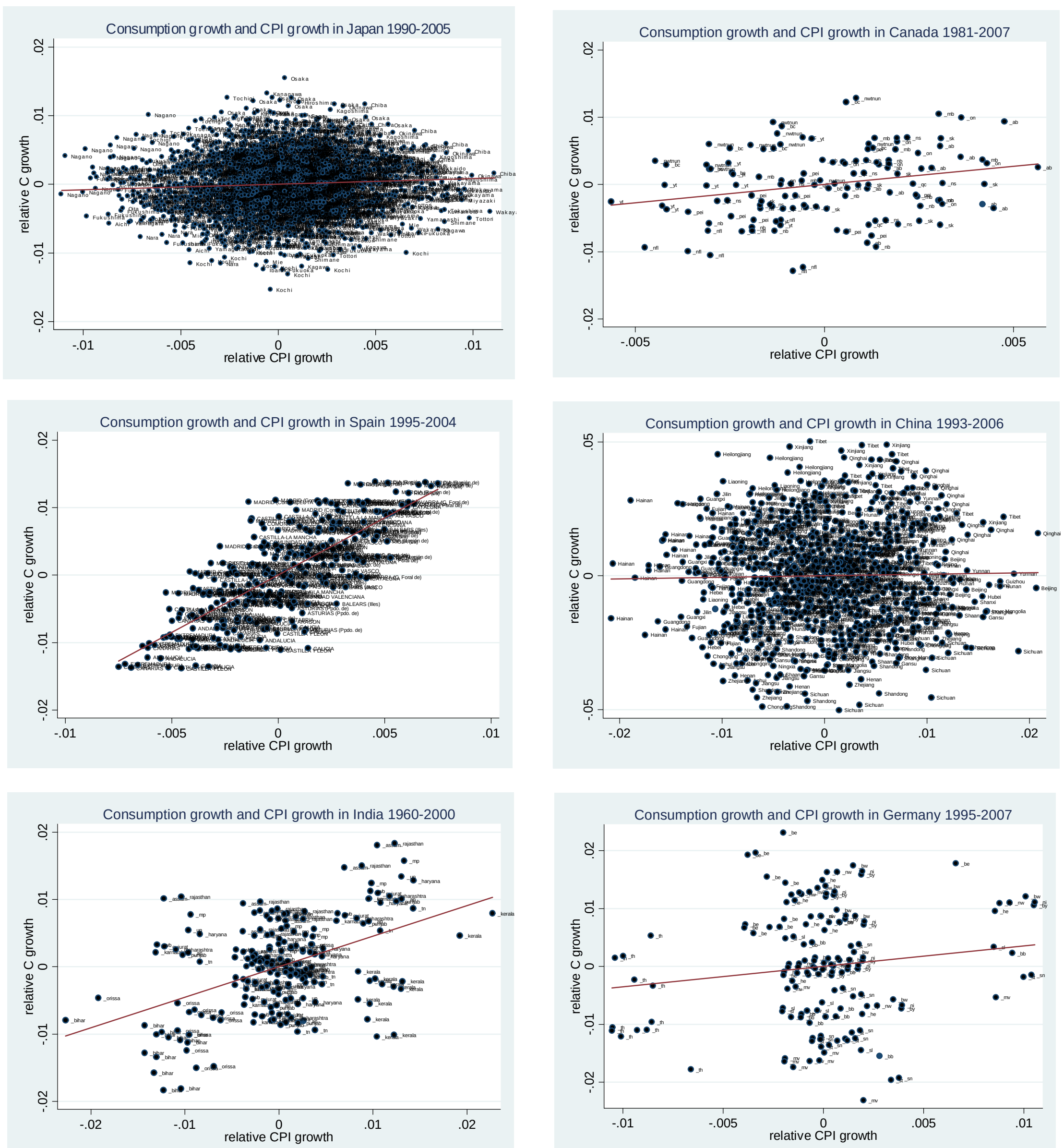
Despite dramatic changes affecting international asset markets, there exists ample evidence that international risk-sharing is far from perfect. One of the well-known corroborations of this fact is the Backus-Smith puzzle, due to Backus and Smith (1993) and Kollmann (1995). The puzzle is the result of a striking disagreement between theory and data. In particular, existing theoretical models with complete asset markets imply a high positive correlation between the cross-country consumption growth rates and the real exchange rate (RER). In the data, however, such positive correlation is forcefully rejected. A related though distinct puzzle in the international finance literature is known as the real exchange rate volatility puzzle. This puzzle says that the volatility of the real exchange rate is very high and significantly exceeds the volatility of consumption and GDP, while the corresponding statistic predicted by the theory is small. Both these puzzles pose significant challenge to the standard open-economy models with complete markets and representative agents.

In this paper we propose a novel explanation for the Backus-Smith and volatility puzzles. Our explanation relies on two central elements: within country (regional) heterogeneity and endogenous non-tradability. We show that each feature implies a lower international consumption-RER correlation, but is not sufficient by itself. Only a combination of regional heterogeneity and endogenous tradability resolves the Backus-Smith puzzle and generates realistic volatility of RER relative to consumption.

Our theoretical analysis is supplemented by a novel empirical study of the of Backus-Smith correlation and real exchange rate volatility. In particular, we depart from the existing studies of the consumption-RER relationship that focus on cross-country analysis and instead develop a more disaggregated analysis using region-level data for a large sample of economies. This analysis helps us to establish the key properties of the real exchange rate and consumption in regional data, a line of research not explored in the existing literature. We then show that our theoretical model can rationalize these regional facts as well.

For this purpose we construct a new dataset of historical series for consumption and prices for a large number of regions in a number of developed and developing countries. To date we have collected regional data for Canada, Japan, Spain, Germany, India and China. Two sets of our empirical findings are particularly interesting. First, the data indicates that the Backus-Smith puzzle is not a purely cross-country phenomenon. In fact, we find that the departures from perfect risk-sharing are significant within countries as well. For within country data, we normally find a positive correlation between relative consumption and real exchange rate. But this correlation is very small, much less than predicted by the theory. For instance, the average correlation between relative consumption and the regional real exchange rate across Canadian provinces is equal to 0.12, a significant departure from a coefficient of 1 predicted by the theory. Similar results characterize the degree of inter-regional risk-sharing in the other countries in our sample. We also find that the degree of risk-sharing varies substantially across regions and over time in a given country. For instance, in Canada, the correlation ranges from -0.3 (between Alberta and Ontario) to 0.55 (between Quebec and Newfoundland). Figure 1 illustrates these findings. For each country in our sample it presents a scatterplot of all bilateral regional average CPI growth and consumption growth pairs over a corresponding sample period. Japan and China exhibit practically no correlation between the two variables, while the correlation is slightly positive for Canada, Spain, Germany and India.

Figure **1**. Backus-Smith correlation across regions



In terms of volatilities we find that real exchange rate is significantly less volatile than consumption in the regional data and significantly more volatile than consumption in the international data. These empirical findings discipline our theoretical work as they provide us with the additional testable implication that our model should comply with. In particular, any explanation for the Backus-Smith puzzle and the real exchange rate volatility puzzle in the international data must also comply with the corresponding properties of the RER and consumption in the regional data. Providing such an explanation is the main objective of our paper.

Our model is a relatively canonical model of international business cycles with incomplete asset markets. We consider a world economy composed of two symmetric countries. Each country consists of two regions. Each region is represented by a continuum of households and a continuum of monopolistically competitive firms who choose whether to sell their goods in the domestic regional market, foreign regional market within the same country, or to export to the foreign country. The split is endogenously determined by a combination of regional and international transportation costs and fixed costs of exporting. Each firm is subject to a sector-specific total factor productivity (TFP) shock and a firm-specific productivity shock. As TFP varies, the mass of firms producing in each sector fluctuates over time, inducing changes in the supply of goods and relative prices. We find that allowing for regional heterogeneity and endogenous international and regional non-tradability generates a negative correlation between RER and relative consumption across countries and a positive correlation across regions within a country, thus allowing us to account for the Backus-Smith puzzle in both international and regional dimension. The two features also generate realistic volatility of RER relative to consumption in both international and regional dimensions.

TFP shocks in our model originate in the internationally and regionally traded and regionally nontraded sectors. Internationally traded goods shocks trigger wealth effects and Balassa-Samuelson effects and thus imply a negative correlation between international RER and relative consumption. Internationally nontraded shocks have the opposite effects. By introducing within country heterogeneity and endogenous tradability, one at a time, we show that the international Backus-Smith correlation can be reduced. Regional heterogeneity is important because agents within a given country can share some of their internationally uninsurable country-specific shocks with their countrymen living in the other regions. This reduces the effects of these shocks on RER and relative consumption. Endogenous tradability is important because it makes the size of each sector cyclical, as new varieties become traded across regions and countries. This amplifies the effects of internationally traded shocks and suppresses the effects of internationally-nontraded shocks. A combination of the two features generates the Backus-Smith correlation consistent with both the intra-national and international data.

This paper contributes to the existing literature along several dimensions. First, it contributes to the literature studying Backus-Smith and real exchange rate volatility puzzle in the context of open economy models. A number of papers have tried to explain the Backus-Smith puzzle using models with incomplete financial market structure. Benigno and Thoenissen (2008), introduce a production economy with non-traded and traded goods sectors. Under the assumption that productivity shocks to the traded goods sector are more persistent and more volatilile than those in the non-traded goods section, they can obtain a negative correlation between relative consumption and the real exchange rate. Corsetti, Dedola, and Leduc (2008) show that with a very low elasticity of substitution between domestic and foreign traded goods, they can account for the Backus-Smith puzzle. By contrast, our model can account for Backus-Smith puzzle while using symmetric sectoral productivity processes and standard values for substitution elasticities. Further, the existing work focuses only on international risk-sharing and does not study the intra-national risk-sharing. They also cannot account for the RER volatility puzzle. Bodenstein (2006) introduces limited contract enforcement into a two-country complete markets model with the production structure of Corsetti, Dedola, and Leduc (2008) to rationalize both the Backus-Smith and RER volatility puzzles. However, he too does not study intra-national risk-sharing. Engel and Wang (2008) introduce durable goods into an otherwise standard business cycle model and show that they can replicate the Backus-Smith correlation in the data under the parametrization similar to that in Benigno and Thoenissen (2008). Our contribution is to extend the existing work by departing from a representative agent framework and incorporating regional heterogeneity within each country. Furthermore, our approach allows sectoral tradability to be endogenously determined as a result of profit maximizing decisions by heterogenous firms.

Studies that do allow for inter-regional risk-sharing are primarily empirical and focus entirely on the comovement between consumption and output growth rates (Sorensen and Yosha, 1998; Crucini and Hess, 1999; Labhard and Sawicki, 2006). Our contribution to this empirical literature is to focus on a measure of risk-sharing due to Backus and Smith (1993) and provide a set of stylized facts about the intra-national Backus-Smith correlation and real exchange rate volatility for a relatively large sample of developed and developing countries. Furthermore, we show that these regional and international facts can be rationalized jointly.

Finally, several recent papers have studied the role of endogenous non-tradability for the international business cycles. Ghironi and Melitz (2005) incorporate Melitz (2003) model of trade into a dynamic two-country model and show that it improves the model performance in terms of several business cycle moments. Alessandria and Choi (2007) find that endogenous firms export decisions have negligible effects on the properties of international business cycles. In a similar framework, Naknoi (2008) studies the dynamics of RER under different exchange rate regimes. Corsetti, Martin, and Pesenti (2008) use endogenous tradability model to study the dynamics of current account adjustment. Bergin and Glick (2005) and Bergin, Glick, and Taylor (2006) use a model of endogenous tradability to study price dispersion and the Balassa-Samuelson effect. Our approach embeds the sectoral tradability decisions into a dynamic international business cycles framework and allows for both international and regional export decisions in the firm's problem.

The rest of the paper is structured as follows. The next section describes the Backus-Smith and real exchange rate volatility puzzles. Section 3 presents our empirical findings on the two puzzles in the international and intra-national data. Section 4 develops the model that we propose to rationalize our empirical results. Section 5 describes how the model is calibrated and solved. The results are presented in Section 6. The last section concludes.

2 Stating the Puzzles

To illustrate the main idea behind the Backus-Smith puzzle, consider a two country international business cycle model. A problem facing a representative household in country $j = 1, .., J$ is as follows:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \delta^i U(C^j_{t+i}, X^j_{t+i}),$$

where δ is the subjective discount factor, C_t denotes a composite consumption good in period t, X_t denotes factors other than consumption that can affect utility (e.g. preference shocks, labor choice, etc.). C_t is defined as a CES aggregator over the consumption of traded, C^{T}_t, and nontraded, C^{N}_t, goods. Let P^j_t denote consumption price index in country j.

Every period households receive stochastic endowments of traded and nontraded goods: Y^{T}_t and Y^{N}_t, respectively. Further, suppose that in this world a complete set of state-contingent securities is available to households in all countries. In this case, the equilibrium can be obtained as a solution to a social planner's problem. In particular,

$$\max \mathbb{E}_t \sum_{j=1}^{J} \lambda_j \sum_{i=0}^{\infty} \delta^i U(C^j_{t+i}, X^j_{t+i})$$

subject to the resource constraints

$$\begin{aligned} \sum_j C^{\text{T}}_{jt} &= \sum_j Y^{\text{T}}_{jt}, \qquad \forall t \\ C^{\text{N}}_{jt} &= Y^{\text{N}}_{jt}, \qquad \forall j, \forall t \end{aligned}$$

The resulting key optimality condition is

$$U_c(C_{i,t}, X_{i,t}) = \frac{P_{i,t}}{P_{j,t}} U_c(C_{j,t}, X_{j,t}),$$

where $U_c(C_{i,t}, X_{i,t})$ denotes the marginal utility of aggregate consumption in country i. Further, let $e_t^{i,j} = P_{i,t}/P_{j,t}$ denote the real exchange rate between country i and j. Then the condition above can be written as

$$U_c(C_{i,t}, X_{i,t}) = e_t^{i,j} U_c(C_{j,t}, X_{j,t}). \tag{1}$$

This equation must hold in every date and state of the world, between any two countries i and j. It says that the social planner allocates consumption between households in countries i and j in such that its marginal utility (converted into the same units using the real exchange rate) is equalized across countries. If utility is of a constant relative risk aversion (CRRA) form, with the coefficient of relative risk aversion ω, equation (1) becomes

$$\left(\frac{C_{i,t}}{C_{j,t}}\right)^{-\omega} = e_t^{i,j},$$

or equivalently in logs

$$-\omega \left(\ln C_{i,t} - \ln C_{j,t}\right) = \ln e_t^{i,j}.$$

These expressions establish the close relationship between *the* real exchange rate and relative consumption in countries i and j. They imply that during the times when the real exchange rate in country i appreciates, the consumption in that country should fall relative to country j's consumption. The expression above must also hold in growth rates:

$$-\omega \left(\Delta \ln C_{i,t} - \Delta \ln C_{j,t}\right) = \Delta \ln e_t^{i,j},$$

where $\Delta \ln X_{i,t} = \ln X_{i,t} - \ln X_{i,t-1}$. Therefore, if markets are complete, the correlation, $\rho_{e,c_j/c_i} = corr(\Delta \ln e_t^{i,j}, \omega \Delta \ln \frac{C_{j,t}}{C_{i,t}})$ should be equal to 1, as pointed out by Backus and Smith (1993). In the data, a number of authors have found this correlation to be very low and often negative (see Backus and Smith, 1993; Kollmann, 1995; Chari, Kehoe, and McGrattan, 2002; Corsetti, Dedola, and Leduc, 2008). Ravn (2001) and Lewis (1996) study the role of preference nonseparabilities in explaining Backus-Smith puzzle and find that the puzzle is robust to them.

A related puzzle is the real exchange rate volatility puzzle. It relates the volatility of the real exchange rate to the volatility of macroeconomic aggregates, such as consumption or output. In the context of the complete markets model presented above, the relative volatility of the real exchange rate and consumption can be written as

$$\begin{aligned}
\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}} &= \frac{\mathbb{V}(\Delta \ln e_t^{i,j})}{\sqrt{\mathbb{V}(\Delta \ln C_{i,t})\mathbb{V}(\Delta \ln C_{j,t})}} \\
&= \omega^2 \frac{\mathbb{V}(\Delta \ln C_{i,t}) + \mathbb{V}(\Delta \ln C_{j,t}) - 2\mathbb{CV}\left(\Delta \ln C_{i,t}, \Delta \ln C_{j,t}\right)}{\sqrt{\mathbb{V}(\Delta \ln C_{i,t})\mathbb{V}(\Delta \ln C_{j,t})}} \\
&= \omega^2 \left(\sqrt{\frac{\mathbb{V}(\Delta \ln C_{i,t})}{\mathbb{V}(\Delta \ln C_{j,t})}} + \sqrt{\frac{\mathbb{V}(\Delta \ln C_{j,t})}{\mathbb{V}(\Delta \ln C_{i,t})}} - 2\rho_{c_i,c_j} \right),
\end{aligned}$$

where $\mathbb{V}(.)$ and $\mathbb{CV}(.)$ denote the unconditional variance and covariance; and $\rho_{c_i,c_j} = corr(\Delta \ln C_{i,t}, \Delta \ln C_{j,t})$. The complete markets model above predicts $\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}$ to be very small, unless ω is very large. In the data, this relative volatility is large (see Chari, Kehoe, and McGrattan, 2002; Bodenstein, 2006 and our estimates in the next section).

3 Empirical analysis

In this section we study the properties of real exchange rate and relative consumption empirically. We start by documenting these properties in the cross-country data, and then develop a more disaggregated analysis that relies on the region-level data for a large sample of economies. To conduct this analysis we construct a novel dataset of historical series for consumption and prices for a large number of regions in a number of developed and developing countries. Our dataset contains regional data for Canada, Japan, Spain, Germany, India and China and covers at least ten years of annual data. A detailed description of the data and its sources is provided in Appendix A.1.

Following Backus and Smith (1993), a number of papers have studied the correlation between consumption and real exchange rates in the international data. Chari, Kehoe, and McGrattan (2002) report the median value for the correlation between real exchange rate and relative consumption in quarterly data between 1973-1994 for a sample of OECD countries to be -0.07. Corsetti, Dedola, and Leduc (2008) in a sample of OECD economies during 1970-2001 find that this correlation is well below one and often is negative when measured in levels or growth rates. Benigno and Thoenissen (2008) re-estimate the correlations between the two variables (both logged and Hodrick-Prescott filtered) in levels and first-differences using the US as a reference country. In their sample of annual data during 1970-2000 they find that the median correlation is -0.288 in levels and -0.168 in the first-differences.

We confirm these findings for our sample of countries in annual data during 1970-2004.

Table 1 summarizes the results. Columns (i)-(iii) report the average, maximum and minimum of the bilateral correlations between the growth rates of real exchange rate and relative consumption for 6 different countries measured relative to OECD economies. For all countries except China, on average the correlation is negative.

Table 1. International C-RER correlation and relative volatility

		$\rho_{e,c_j/c_i}$		$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$
	mean	max	min	mean
	(i)	(ii)	(iii)	(iv)
Canada	-0.308	0.275	-0.686	0.928
Japan	-0.241	0.125	-0.639	1.141
Spain	-0.355	0.176	-0.639	1.177
India	-0.073	0.122	-0.440	1.773
China	0.321	0.520	0.011	2.612
Germany	-0.148	0.331	-0.511	1.111

Column (iv) reports the volatility of the growth rate in the real exchange rate relative to the volatility of consumption growth in country i and OECD economies (j).[1] For all countries in our sample, except Canada, the volatility of real exchange rate exceed the volatility of consumption. The real exchange rate in China exhibits the most volatile relative consumption among the countries in our sample. These numbers are consistent with the available estimates in the literature. For instance, in the quarterly data for US relative to 9 European economies during 1972:1-1993:3, Chari, Kehoe, and McGrattan (2002) find that the volatility of real exchange rate is 4.4 times that of US GDP. For the period of 1978:4-2003:4 Bodenstein (2006) computes that real exchange rate in US relative to the other G7 countries is 3.4 times as volatile as US consumption. Similarly, Johri and Lahiri (2008) report a number for this volatility equal to 5.5. Our findings are for the annual data and for developed economies are consistent with these numbers. In India and especially China, the real exchange rate exhibits even more volatility relative to consumption than in the developed economies.

Next, we re-estimate the correlation and volatility properties of real exchange rate in the intra-national data. Such analysis also allows us to control for the movements in the nominal exchange rate. Table 2 presents the results.

As in the international data, columns (i)-(iii) report the summary statistics on the bilateral correlation of the growth rate in real exchange rate and relative consumption between two regions within a given country. All of the average correlations are well below 1, with

[1] For each OECD economy in our sample, we used the remaining OECD countries as a reference sample.

Table 2. Regional C-RER correlation and relative volatility

	$\rho_{e,c_j/c_i}$			$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$
	mean	max	min	mean
	(i)	(ii)	(iii)	(iv)
Canada	0.123	0.55	-0.35	0.47
Japan	0.121	0.85	-0.69	0.47
Spain	0.196	0.80	-0.70	0.25
India	0.656	0.85	0.34	0.73
China	0.117	0.88	-0.84	0.36
Germany	0.108	0.89	-0.63	0.23

the minimum values well into the negative range. The numbers clearly indicate a failure of full RER-adjusted risk sharing within countries, but provide some evidence of more risk sharing within countries than across countries. Column (iv) reports the relative volatility of real exchange rate growth between two regions within a given country to their consumption growth. All numbers are well below one, suggesting that regional real exchange rate is significantly less volatile than regional consumption.

Overall, two results stand out from our empirical study. First, the Backus-Smith puzzle is not a purely cross-country phenomenon. The puzzle remains in the region/state-level data. Further, the Backus-Smith correlation varies substantially across regions/states in a given country. Second, the RER is more volatile than consumption in the international data and is less volatile than consumption in the intra-national data. We next develop a model to rationalize these findings.

4 Model economy

We work with a conventional two-country model with incomplete asset markets. This type of model has been used extensively in the literature to study the properties of international business cycles, including the Backus-Smith correlation and the volatility puzzle. First, we extend this model to allow for regional heterogeneity within each country and show this model can not account for the properties of international data. We then introduce endogenous non-tradability into this model and show that it too can not produce a realistic Backus-Smith correlation and RER volatility. Only when both regional heterogeneity and endogenous tradability are added simultaneously, can our model generate negative correlation between real exchange rate and relative consumption (in both levels and growth rates) in the international dimension, while predicting a positive correlation between the two variables in the intra-national dimension. Furthermore, the model's predictions for the volatility of real exchange rate and relative consumption across regions and across countries are in accord with the data. We next present our model that allows for both regional heterogeneity and endogenous non-tradability.

4.1 Firms

We consider a world economy consisting of two symmetric economies, home (H) and foreign (F). Each country is populated by a continuum of firms and households who reside in two regions, A and B, within each country. Each region in our economy specializes in the production of a continuum of goods, indexed by $i \in [0,1]$. Each differentiated good i is produced using constant returns to scale technology in labor input, l_i:

$$y_{it} = X_t A_i l_{it}.$$

Here X_t is the total factor productivity (TFP), and A_i is the good-specific productivity. Firms can sell their output in three markets: *the* domestic market, in another region within the same country, and abroad. Exporting to a foreign country is costly. There is a fixed cost of beginning to export, denoted by f_x; and iceberg transportation costs, τ_I. In order to sell goods in another region within the same country producers have to incur iceberg transportation costs τ_R, where $\tau_R < \tau_I$. As in Ghironi and Melitz (2005) and Bergin and Glick (2005), we assume that firms hire domestic labor to cover the fixed costs of exporting. Transportation costs are common to all producers.

Let w denote the wage rate in country H. Then the maximization problem of a firm i operating in region $j = \{\text{A,B}\}$ in country H yields a mark-up pricing rule. In particular, for goods sold on the domestic markets, prices are

$$\begin{aligned} p_{ij} &= \frac{1}{\beta}\frac{w_j}{X_{tj}A_{ij}} \qquad i \in \text{RN}, \\ p_{ij} &= \frac{1}{\beta}\frac{w_j}{X_{tj}A_{ij}} \qquad i \in \text{RT}, \\ p_{ij} &= \frac{1}{\rho}\frac{w_j}{X_{tj}A_{ij}} \qquad i \in \text{IT}. \end{aligned}$$

Here RN is used to denote goods produced in the regional nontraded, RT-regional traded, and IT-international traded sectors. Prices for goods sold in the foreign regional and international markets are

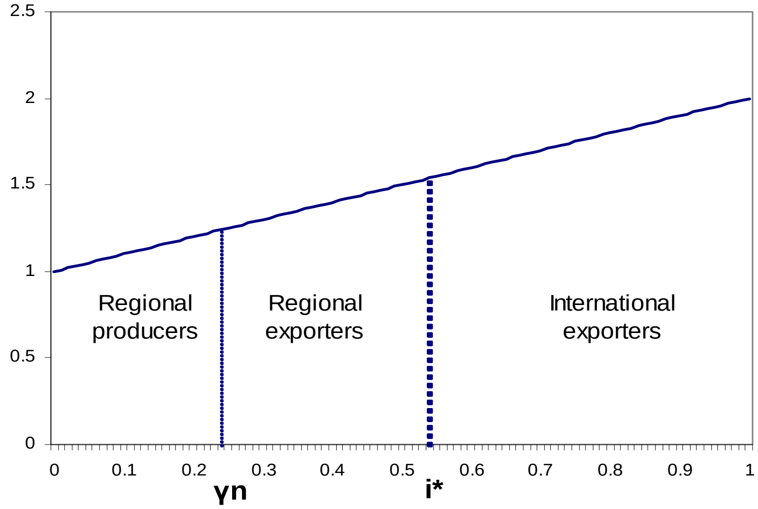
$$
\begin{aligned}
p^*_{i,-j} &= \frac{1}{\beta}\frac{w_j}{X_{tj}A_{ij}}\frac{1}{1-\tau_R} \qquad i \in \text{RT}, \\
p^*_{ij} &= \frac{1}{\rho}\frac{w_j}{X_{tj}A_{ij}}\frac{1}{1-\tau_I} \qquad i \in \text{IT},
\end{aligned}
$$

respectively. Here $\frac{1}{\beta}$ and $\frac{1}{\rho}$ are constant markups. Markups vary with the market in which goods are sold. Due to the fixed costs of exporting, firms with lower productivity levels will choose to sell in the domestic market. When making this decision, a firm computes potential profits from export sales after accounting for the fixed costs of exporting. A firm will export if and only if these profits are non-negative. Let π^*_{ij} denote the profits received by firm i in region j from exporting, given by

$$
\pi^*_{ij} = p^*_{ij}c^*_{ij} - \frac{w_j}{X_{tj}A_{ij}}\frac{1}{1-\tau_I}c^*_{ij} - f_x w_j \qquad i \in \text{IT},
$$

where $c^*_{ij}, i \in \text{IT}$ is consumption demand for H region $j-$produced internationally traded goods sold in country F. A firm for which $\pi^*_{ij} = 0$ will pin down the threshold index i^* of the marginal firm that will export. All firms with productivities below this cutoff will sell in country H, while the firms whose productivities are above the cutoff will sell in country F. Within a group of firms selling in the home country, a share γ_N of firms with the lowest productivity levels will sell goods in the domestic regional market. The remaining share of firms will export goods to the other region within the same country. We assume that the share of firms servicing domestic region, γ_N, is fixed to preserve tractability and to focus on the role played by international non-tradability. Such assumption is also supported empirically (Stockman and Tesar, 1995). Figure **2** summarizes the productivity distribution of firms within region j.

Figure **2**. Firm productivity distribution within a region



Note that the set of firms selling in the foreign regional or foreign international markets will change over time as TFP in different markets change. A similar problem is faced by the firms operating in country F.

Following Melitz (2003) and Bergin and Glick (2005), we define "average" productivity levels – average A – for firms producing different categories of goods in region j:

$$\begin{aligned}
\tilde{A}_{\mathrm{RN},j}^{\frac{\beta}{1-\beta}} &= \frac{1}{\gamma_{\mathrm{N}} i_j^*} \int_0^{\gamma_{\mathrm{N}} i_j^*} A_{ij}^{\frac{\beta}{1-\beta}} di, \\
\tilde{A}_{\mathrm{RT},j}^{\frac{\beta}{1-\beta}} &= \frac{1}{(1-\gamma_{\mathrm{N}})\, i_j^*} \int_{\gamma_{\mathrm{N}} i_j^*}^{i_j^*} A_{ij}^{\frac{\beta}{1-\beta}} di, \\
\tilde{A}_{\mathrm{IT},j}^{\frac{\rho}{1-\rho}} &= \frac{1}{1-i_j^*} \int_{i_j^*}^{1} A_{ij}^{\frac{\rho}{1-\rho}} di, \\
\tilde{A}_{j}^{\frac{\phi}{1-\phi}} &= \int_0^1 A_{ij}^{\frac{\phi}{1-\phi}} di.
\end{aligned}$$

Our focus is on aggregate dynamics, and as shown in Melitz (2003), the average productivities are sufficient to characterize these dynamics. We can now define average goods prices in terms of these productivity averages. In particular, prices of goods sold on the domestic market in region j, country H are given by:

$$R_{jt}^{\mathrm{N}} = \frac{1}{\beta}\frac{w_{jt}}{X_{jt}\tilde{A}_{\mathrm{RN},j}}, \qquad R_{jt}^{\mathrm{A}} = \frac{1}{\beta}\frac{w_{jt}}{X_{jt}\tilde{A}_{\mathrm{RT},j}}, \qquad P_{jt}^{\mathrm{H}} = \frac{1}{\rho}\frac{w_{jt}}{X_{jt}\tilde{A}_{\mathrm{IT},j}}. \tag{2}$$

Prices of goods that originated in region j, country H and are sold in the foreign regional or international markets are given by

$$R^{\text{B}}_{jt} = \frac{1}{\beta} \frac{w_{jt}}{X_{jt} \tilde{A}_{\text{RT},j}} \frac{1}{1 - \tau_R}, \qquad \hat{P}^{\text{H}}_{jt} = \frac{1}{\rho} \frac{w_{jt}}{X_{jt} \tilde{A}_{\text{IT},j}} \frac{1}{1 - \tau_I}, \tag{3}$$

where we used letter R to denote regional prices or indices; and letter P to denote international prices or indices. An analogous set of prices applies to the F country.

Profits received by domestic firms in country H are given as the sum of profits received by domestic firms selling in three different markets:

$$\Pi_j = \int_0^{\gamma_{\text{N}} i^*_j} \pi_{ij} di \,|_{i \in \text{RN}} + \int_{\gamma_{\text{N}} i^*_j}^{i^*_j} \pi_{ij} di \,|_{i \in \text{RT}} + \int_{\gamma_{\text{N}} i^*_j}^{i^*_j} \pi^*_{i,-j} di \,|_{i \in \text{RT}} + \int_{i^*_j}^{1} \pi_{ij} di \,|_{i \in \text{IT}} + \int_{i^*_j}^{1} \pi^*_{ij} di \,|_{i \in \text{IT}}$$

Profits received by firms in country F are defined analogously.

4.2 Households

Households derive utility from consuming two goods: a composite tradable and a composite domestic nontradable. The share of each good in the aggregate consumption basket is endogenously determined. The preferences of H households residing in region j are represented by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \delta^s U(C^{\text{T}}_{j,t+s}, C^{\text{N}}_{j,t+s}), \tag{4}$$

where $0 < \delta < 1$ is the discount factor, and $U(.)$ is a concave sub-utility function. $C^{\text{T}}_{j,t}$ represents a basket of tradable consumption, and $C^{\text{N}}_{j,t}$ denotes H nontradable consumption. The utility function is of CRRA form, with ω being the coefficient of risk aversion, and is given by

$$U(C^{\text{T}}_j, C^{\text{N}}_j) = \frac{1}{1-\omega} \left(\left[\lambda_{\text{T}}(i^*_j)^{1-\phi} \left(C^{\text{T}}_j \right)^{\phi} + \lambda_{\text{N}}(i^*_j)^{1-\phi} \left(C^{\text{N}}_j \right)^{\phi} \right]^{\frac{1}{\phi}} \right)^{1-\omega}.$$

Here $\lambda_{\text{T}}(i^*_j)$ and $\lambda_{\text{N}}(i^*_j) = 1 - \lambda_{\text{T}}(i^*_j)$ are the weights of internationally tradable and nontradable consumption in the aggregate consumption basket; i^*_j, as before, denotes the endogenous cut-off in the continuum of goods produced in region j in country H below which goods are internationally non-traded. In the Appendix A.2 we show $\lambda_{\text{N}}(i^*_j) = i^*_j$ and $\lambda_{\text{T}}(i^*_j) = 1 - i^*_j$. The elasticity of substitution between tradable and nontradable consumption is $(1-\phi)^{-1} > 0$. A composite tradable good, $C^{\text{T}}_{j,t}$, is given by a CES aggregator over tradables produced in the H and F countries:

$$C^{\text{T}}_j = \left[\lambda_{\text{H}}(i^*_j)^{1-\rho} \left(C^{\text{H}}_j \right)^{\rho} + \lambda_{\text{F}}(i^*_j)^{1-\rho} \left(C^{\text{F}}_j \right)^{\rho} \right]^{\frac{1}{\rho}},$$

where $(1-\rho)^{-1} > 0$ is the elasticity of substitution between tradable goods. Here $\lambda_{\text{H}}(i_j^*)$ and $\lambda_{\text{F}}(i_j^*) = 1 - \lambda_{\text{H}}(i_j^*)$ denote the weights that households in country H assign to the consumption of H and F-produced tradable goods. Again, these weights are endogenously linked to the shares of non-traded goods in the two countries as:

$$\lambda_{\text{H}}(i_j^*) = \frac{1-i_j^*}{2-i_j^*-\hat{\imath}^*} \qquad \text{and} \qquad \lambda_{\text{F}}(i_j^*) = \frac{1-\hat{\imath}^*}{2-i_j^*-\hat{\imath}^*} \ ,$$

where $\hat{\imath}^* = 0.5(\hat{\imath}_{\text{A}}^* + \hat{\imath}_{\text{B}}^*)$ is the average nontraded share in country F. Hereafter we will use a hat, "ˆ", over a variable to denote F country variables.

Nontradable consumption of households living in region $j = \{\text{A,B}\}$ is given by a CES aggregator over consumption of tradables produced in regions A and B, Z_j^{A} and Z_j^{B}, respectively, and consumption of nontradables produced in region j within country H, Z_j^{N}:

$$C_j^{\text{N}} = \left[\gamma_{\text{T}}^{1-\alpha}\left[\gamma_{\text{A}}(i_j^*)^{1-\beta}\left(Z_j^{\text{A}}\right)^{\beta} + \gamma_{\text{B}}(i_j^*)^{1-\beta}\left(Z_j^{\text{B}}\right)^{\beta}\right]^{\frac{\alpha}{\beta}} + \gamma_{\text{N}}^{1-\alpha}\left(Z_j^{\text{N}}\right)^{\alpha}\right]^{\frac{1}{\alpha}},$$

where γ_{T} and $\gamma_{\text{N}} = 1-\gamma_{\text{T}}$ represent the weights that households in region j assign to the consumption of regionally tradable and nontradable goods. As mentioned before in the firm's problem, we assume that γ_{N} is exogenously given. The elasticity of substitution between regionally tradable and nontradable goods is given by $(1-\alpha)^{-1} > 0$. Further, $\gamma_{\text{A}}(i_j^*)$ and $\gamma_{\text{B}}(i_j^*) = 1-\gamma_{\text{A}}(i_j^*)$ denote the weights that households in region j assign to the consumption of A- and B−produced tradable goods. These weights are also functions of i_j^* as

$$\gamma_{\text{A}}(i_j^*) = \frac{i_{\text{A}}^*}{i_{\text{A}}^*+i_{\text{B}}^*} \qquad \text{and} \qquad \gamma_{\text{B}}(i_j^*) = \frac{i_{\text{B}}^*}{i_{\text{A}}^*+i_{\text{B}}^*}.$$

The elasticity of substitution between two regionally tradable goods is given by $(1-\beta)^{-1} > 0$. Each consumption good is a CES aggregate of individual goods. For instance, consumption aggregates in region j, country H are given by:

$$\begin{aligned}
\left(Z_j^{\text{N}}\right)^{\beta} &= \int_0^{\gamma_{\text{N}} i_j^*} \left(\frac{1}{\gamma_{\text{N}} i_j^*}\right)^{1-\beta} c_{ij}^{\beta} di & i \in \text{RN} \\
\left(Z_j^{\text{A}}\right)^{\beta} &= \int_{\gamma_{\text{N}} i_j^*}^{i_j^*} \left(\frac{1}{(1-\gamma_{\text{N}})\, i_j^*}\right)^{1-\beta} c_{ij}^{\beta} di & i \in \text{RT} \\
\left(Z_j^{\text{B}}\right)^{\beta} &= \int_{i_j^*}^{i_j^*+(1-\gamma_{\text{N}}) i_{-j}^*} \left(\frac{1}{(1-\gamma_{\text{N}})\, i_{-j}^*}\right)^{1-\beta} \left(c_{i,-j}^*\right)^{\beta} di & i \in \text{RT} \\
\left(C_j^{\text{H}}\right)^{\rho} &= \int_{i_j^*}^{1} \left(\frac{1}{1-i_j^*}\right)^{1-\rho} c_{ij}^{\rho} di & i \in \text{IT} \\
\left(C_j^{\text{F}}\right)^{\rho} &= \int_{1}^{2-\hat{\imath}^*} \left(\frac{1}{1-\hat{\imath}^*}\right)^{1-\rho} \left(\hat{c}_{ij}^*\right)^{\rho} di & i \in \text{IT}
\end{aligned}$$

See Appendix A.2 for derivations. For simplicity, we assume that households treat internationally-traded goods originated in the two regions of the foreign country as perfect substitutes.

Preferences of F households living in region $j = \{\text{A,B}\}$ are similarly defined in terms of tradable consumption basket, $\hat{C}_j^{\text{T}}$, and a nontradable consumption basket, $\hat{C}_j^{\text{N}}$. The tradable consumption bundle in F country is defined symmetrically in terms of H tradables, $\hat{C}_j^{\text{H}}$, and F tradables, $\hat{C}_j^{\text{F}}$, as

$$\hat{C}_j^{\text{T}} = \left[\hat{\lambda}_{\text{H}}(\hat{\imath}_j^*)^{1-\rho}(\hat{C}_j^{\text{H}})^{\rho} + \hat{\lambda}_{\text{F}}(\hat{\imath}_j^*)^{1-\rho}(\hat{C}_j^{\text{F}})^{\rho}\right]^{\frac{1}{\rho}}.$$

Nontradable consumption bundle of F households living in region j is defined in terms of tradables produced in the regions A and B, $\hat{Z}_j^{\text{A}}$ and $\hat{Z}_j^{\text{B}}$, respectively, and consumption of region$-j$ nontradables, $\hat{Z}_j^{\text{N}}$:

$$\hat{C}_j^{\text{N}} = \left[\gamma_{\text{T}}^{1-\alpha}\left[\gamma_{\text{A}}(\hat{\imath}_j^*)^{1-\beta}\left(\hat{Z}_j^{\text{A}}\right)^{\beta} + \gamma_{\text{B}}(\hat{\imath}_j^*)^{1-\beta}\left(\hat{Z}_j^{\text{B}}\right)^{\beta}\right]^{\frac{\alpha}{\beta}} + \gamma_{\text{N}}^{1-\alpha}\left(\hat{Z}_j^{\text{N}}\right)^{\alpha}\right]^{\frac{1}{\alpha}}.$$

Households in each country finance their consumption expenditures with wage income and profits, Π_j, received from domestic firms. Households also have access to international borrowing and lending at interest rate R_t.[2] As a result, asset markets in our model are incomplete both across regions within a country, and across countries. We use this simple asset structure in our model to preserve comparability with the existing models of international business cycles, and to maintain the symmetry across regions and countries in the asset structure dimension.

As noted before, we assume that internationally-traded goods produced in both regions

[2]We assume that bonds are denominated in units of internationally tradable goods produced by country H.

within the same country are perfect substitutes. In combination with no barriers to regional trade in these goods, we get for H country $P^{\chi}_{\text{A},t} = P^{\chi}_{\text{B},t} \equiv P^{\chi}_{t}$, with $\chi = \{\text{H,F}\}$; and for F country $\hat{P}^{\chi}_{\text{A},t} = \hat{P}^{\chi}_{\text{B},t} \equiv \hat{P}^{\chi}_{t}$, with $\chi = \{\text{H,F}\}$.

Therefore, the period$-t$ budget constraint of households living in country H, region j is given by

$$\begin{aligned} P^{\text{H}}_{t} C^{\text{H}}_{j,t} + P^{\text{F}}_{t} C^{\text{F}}_{j,t} + R^{\text{A}}_{j,t} Z^{\text{A}}_{j,t} + R^{\text{B}}_{j,t} Z^{\text{B}}_{j,t} + R^{\text{N}}_{j,t} Z^{\text{N}}_{j,t} + \frac{1}{R_t} P^{\text{H}}_{t} B_{j,t} &\leq P^{\text{H}}_{t} B_{j,t-1} \\ + \left(L^{\text{H}}_{j,t} + L^{\text{J}}_{j,t} + L^{\text{N}}_{j,t} + f_x(1 - i^{*}_{j})\right) w_{j,t} + \Pi_{j,t}, & \end{aligned} \tag{5}$$

where $B_{j,t}$ denotes period$-t$ holdings of international bond, P^{χ}_{t} is the H country price of internationally traded goods produced in country χ, with $\chi = \{\text{H,F}\}$, and $R^{\varkappa}_{t}$ is the price of regional good $\varkappa = \{\text{A,B,N}\}$ as defined in (2)-(3). $L^{\text{H}}_{j,t}, L^{\text{J}}_{j,t}, L^{\text{N}}_{j,t}$ denote aggregate labor employed in the production of internationally tradable, regionally tradable and non-tradable goods, respectively, in region j. Households in the F country, living in region j face an analogous constraint

$$\begin{aligned} \hat{P}^{\text{H}}_{t} \hat{C}^{\text{H}}_{j,t} + \hat{P}^{\text{F}}_{t} \hat{C}^{\text{F}}_{j,t} + \hat{R}^{\text{A}}_{j,t} \hat{Z}^{\text{A}}_{j,t} + \hat{R}^{\text{B}}_{j,t} \hat{Z}^{\text{B}}_{j,t} + \hat{R}^{\text{N}}_{j,t} \hat{Z}^{\text{N}}_{j,t} + \frac{1}{R_t} P^{\text{H}}_{t} \hat{B}_{j,t} &\leq P^{\text{H}}_{t} \hat{B}_{j,t-1} \\ + \left(\hat{L}^{\text{F}}_{j,t} + \hat{L}^{\text{J}}_{j,t} + \hat{L}^{\text{N}}_{j,t} + f_x(1 - \hat{\imath}^{*}_{j})\right) \hat{w}_{j,t} + \hat{\Pi}_{j,t}, & \end{aligned} \tag{6}$$

where $\hat{B}_{j,t}$ denotes the bond holdings of F households in region j. Finally, we choose internationally traded goods produced in country H to be a numeraire in our economy.

4.3 Equilibrium

The first-order conditions for H households living in region j are given by

$$\frac{\partial U_t / \partial C^{\text{F}}_{j,t}}{\partial U_t / \partial C^{\text{H}}_{j,t}} = \frac{P^{\text{F}}_{t}}{P^{\text{H}}_{t}}, \tag{7a}$$

$$\frac{\partial U_t / \partial Z^{\text{A}}_{j,t}}{\partial U_t / \partial C^{\text{H}}_{j,t}} = \frac{R^{\text{A}}_{t}}{P^{\text{H}}_{t}} \tag{7b}$$

$$\frac{\partial U_t / \partial Z^{\text{B}}_{j,t}}{\partial U_t / \partial C^{\text{H}}_{j,t}} = \frac{R^{\text{B}}_{t}}{P^{\text{H}}_{t}} \tag{7c}$$

$$\frac{\partial U_t / \partial Z^{\text{N}}_{j,t}}{\partial U_t / \partial C^{\text{H}}_{j,t}} = \frac{R^{\text{N}}_{j,t}}{P^{\text{H}}_{t}}. \tag{7d}$$

These equations define the relative prices of F traded goods, region A and B-originated traded goods, and region$-j$ nontraded goods in terms of H international tradables as ratios of their respective marginal utilities to the marginal utility of H tradables. Household's consumption demand in region j for individual goods is given by

$$
\begin{aligned}
c_{ij}/Z^{\mathrm{N}} &= \frac{1}{\gamma_{\mathrm{N}} i_j^*}\left[p_{ij}/R^{\mathrm{N}}\right]^{\frac{1}{\beta-1}}, && i \in \mathrm{RN} \\
c_{ij}/Z_j^{\mathrm{A}} &= \frac{1}{(1-\gamma_{\mathrm{N}}) i_j^*}\left[p_{ij}/R_j^{\mathrm{A}}\right]^{\frac{1}{\beta-1}}, && i \in \mathrm{RT} \\
c_{i,-j}^*/Z_j^{\mathrm{B}} &= \frac{1}{(1-\gamma_{\mathrm{N}}) i_{-j}^*}\left[p_{i,-j}^*/R_j^{\mathrm{B}}\right]^{\frac{1}{\beta-1}}, && i \in \mathrm{RT} \\
c_{ij}/C_j^{\mathrm{H}} &= \frac{1}{1-i_j^*}\left[p_{ij}/P^{\mathrm{H}}\right]^{\frac{1}{\rho-1}}, && i \in \mathrm{IT} \\
\hat{c}_{ij}^*/C_j^{\mathrm{F}} &= \frac{1}{1-\hat{\imath}^*}\left[\hat{p}_{ij}^*/P^{\mathrm{F}}\right]^{\frac{1}{\rho-1}}, && i \in \mathrm{IT}.
\end{aligned}
$$

In the bond economy, the first-order conditions also include

$$
P_t^{\mathrm{H}} \partial U_t/\partial C_{j,t}^{\mathrm{H}} = \beta R_t \mathbb{E}_t P_{t+1}^{\mathrm{H}} \partial U_t/\partial C_{j,t+1}^{\mathrm{H}}, \tag{8}
$$

which is the standard pricing equations for the bond. The first-order conditions for F households are symmetric.

In equilibrium, households and firms decisions must also be consistent with the market clearing conditions. The market clearing conditions in the nontraded goods sector in region $j = \{\mathrm{A},\mathrm{B}\}$ are

$$
c_{ijt} = y_{ijt} \qquad \text{and} \qquad \hat{c}_{ijt} = \hat{y}_{ijt}, \qquad i \in \mathrm{RN}.
$$

For the regional traded goods originated within country H, the market clearing requires

$$
c_{i\mathrm{A}t} + c_{i\mathrm{A}t}^* \frac{1}{1-\tau_R} = y_{i\mathrm{A}t} \qquad \text{and} \qquad c_{i\mathrm{B}t} + c_{i\mathrm{B}t}^* \frac{1}{1-\tau_R} = y_{i\mathrm{B}t}, \qquad i \in \mathrm{RT}.
$$

while for those originated within country F, the market clearing is

$$
\hat{c}_{i\mathrm{A}t} + \hat{c}_{i\mathrm{A}t}^* \frac{1}{1-\tau_R} = \hat{y}_{i\mathrm{A}t} \qquad \text{and} \qquad \hat{c}_{i\mathrm{B}t} + \hat{c}_{i\mathrm{B}t}^* \frac{1}{1-\tau_R} = \hat{y}_{i\mathrm{B}t}, \qquad i \in \mathrm{RT}.
$$

Here c_{ij}^* $(\hat{c}_{ij}^*)$, $i \in \mathrm{RT}$ is consumption demand for H (F) country, region $j-$produced regionally-traded goods sold in the other region within H (F) country.

Finally, in equilibrium, the world demand for each internationally-traded good must be equal to its corresponding supply. The internationally traded goods in each country originate in the two regions within the country. Therefore, for the internationally-tradable good produced in the H country, market clearing requires that

$$c_{i\text{A}t} + c_{i\text{B}t} + \left(c^*_{i\text{A}t} + c^*_{i\text{B}t}\right)\frac{1}{1-\tau_I} = y_{i\text{A}t} + y_{i\text{B}t}, \qquad i \in \text{IT}$$

while for the international tradables originated in the F country, market clearing requires

$$\hat{c}_{i\text{A}t} + \hat{c}_{i\text{B}t} + \left(\hat{c}^*_{i\text{A}t} + \hat{c}^*_{i\text{B}t}\right)\frac{1}{1-\tau_I} = \hat{y}_{i\text{A}t} + \hat{y}_{i\text{B}t}, \qquad i \in \text{IT}.$$

Here c^*_{ij} $(\hat{c}^*_{ij})$, $i \in \text{IT}$ is consumption demand for H (F) country, region $j-$produced internationally-traded goods sold in the F (H) country, as defined before.

Labor market clearing in each region within country H requires

$$\int_0^{\gamma_{\text{N}} i^*_j} l_{ijt}di + \int_{\gamma_{\text{N}} i^*_j}^{i^*_j} l_{ijt}di + \int_{i^*_j}^{1} l_{ijt}di + f_x(1-i^*_j) = L^s_{jt}, \qquad j = \{\text{A,B}\},$$

where L^s_j is exogenously given labor supply in region j. Similarly, for each region in country F, labor market clearing implies

$$\int_0^{\gamma_{\text{N}} \hat{i}^*_j} \hat{l}_{ijt}di + \int_{\gamma_{\text{N}} \hat{i}^*_j}^{\hat{i}^*_j} \hat{l}_{ijt}di + \int_{\hat{i}^*_j}^{1} \hat{l}_{ijt}di + f_x(1-\hat{i}^*_j) = \hat{L}^s_{jt}, \qquad j = \{\text{A,B}\},$$

where $\hat{L}^s_j$ is labor supply in region j in country F.

An equilibrium in this economy consists of a sequence of goods prices

$\{P^{\text{H}}_t, P^{\text{F}}_t, \hat{P}^{\text{H}}_t, \hat{P}^{\text{F}}_t, R^{\text{A}}_{j,t}, R^{\text{B}}_{j,t}, \hat{R}^{\text{A}}_{j,t}, \hat{R}^{\text{B}}_{j,t}, R^{\text{N}}_{j,t}, \hat{R}^{\text{N}}_{j,t}\}$ and an interest rate R_t, such that households in both countries make their consumption and bond allocation decisions optimally, taking prices as given; firms in both countries make their profit maximizing decisions; and all markets clear. We also require an asset market clearing condition. We assume that bonds are in zero net supply, so that bond market clearing condition is $B_{\text{A},t} + B_{\text{B},t} + \hat{B}_{\text{A},t} + \hat{B}_{\text{B},t} = 0$.

4.4 Variables of interest

In our economy there is a sequence of price indices that comprise regional and international real exchange rates. As before, we use letter R to denote regional prices or indices; and letter P to denote international prices or indices. To simplify the notation, we omit explicit references to i^*_j in the consumption weights, $\gamma_{\text{A}}, \gamma_{\text{B}}, \lambda_{\text{H}}, \lambda_{\text{F}}, \lambda_{\text{N}}, \lambda_{\text{T}}$.

Let $R^{\text{T}}_{j,t}$ denote the price of regionally traded basket in region $j = \{\text{A, B}\}$. This price index in H country is given by

$$R^{\mathrm{T}}_{\mathrm{A},t} = \left[\gamma_{\mathrm{A}} \left(R^{\mathrm{A}}_{\mathrm{A},t}\right)^{\frac{\beta}{\beta-1}} + \gamma_{\mathrm{B}} \left(R^{\mathrm{B}}_{\mathrm{A},t}\right)^{\frac{\beta}{\beta-1}}\right]^{\frac{\beta-1}{\beta}} \quad \text{and} \quad R^{\mathrm{T}}_{\mathrm{B},t} = \left[\hat{\gamma}_{\mathrm{A}} \left(R^{\mathrm{A}}_{\mathrm{B},t}\right)^{\frac{\beta}{\beta-1}} + \hat{\gamma}_{\mathrm{B}} \left(R^{\mathrm{B}}_{\mathrm{B},t}\right)^{\frac{\beta}{\beta-1}}\right]^{\frac{\beta-1}{\beta}}. \tag{9}$$

Next, let $P^{\mathrm{N}}_{j,t}$ denote the price of aggregate internationally nontraded consumption basket in region j. It is a composite of prices of regionally traded and regionally nontraded goods:

$$P^{\mathrm{N}}_{\mathrm{A},t} = \left[\gamma_{\mathrm{T}} \left(R^{\mathrm{T}}_{\mathrm{A},t}\right)^{\frac{\alpha}{\alpha-1}} + \gamma_{\mathrm{N}} \left(R^{\mathrm{N}}_{\mathrm{A},t}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad P^{\mathrm{N}}_{\mathrm{B},t} = \left[\hat{\gamma}_{\mathrm{T}} \left(R^{\mathrm{T}}_{\mathrm{B},t}\right)^{\frac{\alpha}{\alpha-1}} + \hat{\gamma}_{\mathrm{N}} \left(R^{\mathrm{N}}_{\mathrm{B},t}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\alpha-1}{\alpha}}. \tag{10}$$

Finally, let P^{T}_t denote the price of aggregate internationally traded consumption basket in regions A and B in country H. It is composed of prices of internationally traded goods in country H:

$$P^{\mathrm{T}}_t = \left[\lambda_{\mathrm{H}} \left(P^{\mathrm{H}}_t\right)^{\frac{\rho}{\rho-1}} + \lambda_{\mathrm{F}} \left(P^{\mathrm{F}}_t\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}. \tag{11}$$

The aggregate price index in region j in country H, therefore, is given by

$$P_{j,t} = \left[\lambda_{\mathrm{T}} \left(P^{\mathrm{T}}_t\right)^{\frac{\phi}{\phi-1}} + \lambda_{\mathrm{N}} \left(P^{\mathrm{N}}_{j,t}\right)^{\frac{\phi}{\phi-1}}\right]^{\frac{\phi-1}{\phi}}. \tag{12}$$

Notice that only the price of internationally nontraded goods, $P^{\mathrm{N}}_{j,t}$, is region-specific. The price of internationally traded goods, P^{T}_t, is common across the regions within the same country.

The price indices in the foreign country are symmetrically defined. Thus, the prices of regional traded baskets in regions A and B in country F are given by

$$\hat{R}^{\mathrm{T}}_{\mathrm{A},t} = \left[\gamma_{\mathrm{A}} \left(\hat{R}^{\mathrm{A}}_{\mathrm{A},t}\right)^{\frac{\beta}{\beta-1}} + \gamma_{\mathrm{B}} \left(\hat{R}^{\mathrm{B}}_{\mathrm{A},t}\right)^{\frac{\beta}{\beta-1}}\right]^{\frac{\beta-1}{\beta}} \quad \text{and} \quad \hat{R}^{\mathrm{T}}_{\mathrm{B},t} = \left[\hat{\gamma}_{\mathrm{A}} \left(\hat{R}^{\mathrm{A}}_{\mathrm{B},t}\right)^{\frac{\beta}{\beta-1}} + \hat{\gamma}_{\mathrm{B}} \left(\hat{R}^{\mathrm{B}}_{\mathrm{B},t}\right)^{\frac{\beta}{\beta-1}}\right]^{\frac{\beta-1}{\beta}}. \tag{13}$$

Prices of aggregate internationally nontraded consumption baskets in regions A and B in country F are:

$$\hat{P}^{\mathrm{N}}_{\mathrm{A},t} = \left[\gamma_{\mathrm{T}} \left(\hat{R}^{\mathrm{T}}_{\mathrm{A},t}\right)^{\frac{\alpha}{\alpha-1}} + \gamma_{\mathrm{N}} \left(\hat{R}^{\mathrm{N}}_{\mathrm{A},t}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad \hat{P}^{\mathrm{N}}_{\mathrm{B},t} = \left[\hat{\gamma}_{\mathrm{T}} \left(\hat{R}^{\mathrm{T}}_{\mathrm{B},t}\right)^{\frac{\alpha}{\alpha-1}} + \hat{\gamma}_{\mathrm{N}} \left(\hat{R}^{\mathrm{N}}_{\mathrm{B},t}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\alpha-1}{\alpha}}. \tag{14}$$

Price of aggregate internationally traded consumption basket in regions A and B in country F is given by

$$\hat{P}^{\mathrm{T}}_t = \left[\hat{\lambda}_{\mathrm{H}} \left(\hat{P}^{\mathrm{H}}_t\right)^{\frac{\rho}{\rho-1}} + \hat{\lambda}_{\mathrm{F}} \left(\hat{P}^{\mathrm{F}}_t\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}, \tag{15}$$

while the aggregate price index in region j in country F is

$$\hat{P}_{j,t} = \left[\hat{\lambda}_{\mathrm{T}} \left(\hat{P}^{\mathrm{T}}_t \right)^{\frac{\phi}{\phi-1}} + \hat{\lambda}_{\mathrm{N}} \left(\hat{P}^{\mathrm{N}}_{j,t} \right)^{\frac{\phi}{\phi-1}} \right]^{\frac{\phi-1}{\phi}} . \tag{16}$$

Now we are ready to define RERs. The regional real exchange rate is defined as the price of consumption basket in region B relative to the price of consumption basket in region A. Thus, the regional real exchange rates in country H, RER^{H}_t, and country F, RER^{F}_t, are given by

$$RER^{\mathrm{H}}_t = \frac{P_{\mathrm{B},t}}{P_{\mathrm{A},t}} \qquad \text{and} \qquad RER^{\mathrm{F}}_t = \frac{\hat{P}_{\mathrm{B},t}}{\hat{P}_{\mathrm{A},t}}. \tag{17}$$

International real exchange rate in our model, RER^{I}_t, is given by the ratio of F to H aggregate price indices:

$$RER^{\mathrm{I}}_t = \frac{\frac{1}{2}\hat{P}_{\mathrm{A},t} + \frac{1}{2}\hat{P}_{\mathrm{B},t}}{\frac{1}{2}P_{\mathrm{A},t} + \frac{1}{2}P_{\mathrm{B},t}}, \tag{18}$$

where we exploited the symmetry of the regions within a given country to construct the country-wide aggregate price index as the average of price indices across regions within that country.

The terms-of-trade in the model are defined as a relative price of foreign to domestic internationally-traded goods and are given by $TOT_t = \hat{P}^{\mathrm{F}}_t / P^{\mathrm{H}}_t$.

5 Parameter values and computations

Parameter values for the calibration of our benchmark model are summarized in Table **3**. We consider the world economy as consisting of two symmetric countries, matching the properties of US economy in annual data. Most of the preference parameter values are standard in the literature and, in particular, follow closely those adopted by Stockman and Tesar (1995). In particular, δ is set to 0.96 to obtain the steady-state real interest rate of 4% per annum. Coefficient of relative risk aversion, ω, is set to 2. The values for substitution elasticities are chosen as follows. First, the value for ϕ is set, following Mendoza (1995), to obtain the elasticity of substitution between tradable and nontradable consumption equal to 0.74. Second, the elasticity of substitution between H and F traded goods is set to equal 6 to obtain a 20% mark-up of price over marginal costs, a value commonly used in the literature (Obstfeld and Rogoff, 2000).

We parameterize the fixed cost of exporting parameter, f_x, in both countries to obtain the share of nontradables in aggregate consumption expenditure, λ_{N} and $\hat{\lambda}_{\mathrm{N}}$, equal to 0.55

Table 3. Benchmark Model Parameters				
PREFERENCES				
Subjective discount factor	δ	0.96		
Risk-aversion	ω	2		
	International goods:		Regional goods:	
share of nontraded goods			$\gamma_{\mathrm{N}} = \hat{\gamma}_{\mathrm{N}}$	0.55
elasticity of substitution b/n				
traded and nontraded goods	$1/(1-\phi)$	0.74	$1/(1-\alpha)$	0.74
H and F traded goods	$1/(1-\rho)$	6	$1/(1-\beta)$	6
PRODUCTIVITY				
	International goods:		Regional goods:	
persistence of traded shocks	$a_{ii}^{\mathrm{H}} = a_{ii}^{\mathrm{F}}$	0.9	$a_{ii}^{\mathrm{A}} = a_{ii}^{\mathrm{B}}$	0.9
persistence of nontraded shocks			a_{ii}^{N}	0.9
volatility of				
traded innovations (variance)	$\Omega_e^{\mathrm{H}} = \Omega_e^{\mathrm{F}}$	0.0001	$\Omega_e^{\mathrm{A}} = \Omega_e^{\mathrm{B}}$	0.0001
nontraded innovations (variance)			Ω_e^{N}	0.0001

in the steady state. This number is calculated using OECD STructural ANalysis (STAN) database.[3] We set the shares of home goods in the internationally-traded consumption basket in both countries, λ_{H} and $\hat{\lambda}_{\mathrm{F}}$, to 0.5 in the steady state, so that there is no consumption home bias built in exogenously in the model. Instead we calibrate the international iceberg transportation costs to match the share of international imports to be equal to 10% of output in the steady state.

The regional dimension of the model is more difficult to parametrize as the estimates are not readily available in the literature. Under our benchmark calibration, we assume that preference parameters over regional consumption are symmetric to the preference parameters in the international dimension. That is, we assume that the value of α is chosen to set the elasticity of substitution between regional tradable goods to 0.74, while the share of nontraded regional goods, $\gamma_{\mathrm{N}} = \hat{\gamma}_{\mathrm{N}}$ in set to 0.55. Regional iceberg transportation costs, τ_R, are calibrated to obtain regional trade volume to be 1.25 times the international trade volume in the steady state. The latter number was obtained as an average using interprovincial and international trade data for Canada during 1984-2005. State-level trade data for US is not available, to the best of our knowledge, so we use the trade numbers for Canada to calibrate τ_R parameter in our model. The elasticity of substitution between two regional goods is set to 6 in both countries, which pins down the value for β. As part of our sensitivity analysis

[3]These numbers are similar to the estimates in the literature. For instance, Corsetti et al. (2008) and Dotsey and Duarte (2006) use $\lambda_{\mathrm{N}} = 0.55$, Stockman and Tesar (1995) report λ_{N} close to 0.5; Pesenti and van Wincoop (2002) also argue that 0.5 of consumers budget is allocated to nontradables; Benigno and Thoenissen (2008) assume $\lambda_{\mathrm{N}} = 0.45$.

we also explore how the predictions of the model are affected by variations in the regional preference parameters.

The available estimates for sectoral productivity processes in the literature are very dissimilar, thus we assume independent productivity processes across sectors, across regions and across countries. Each of the productivity processes follows an AR(1) process. The AR(1) coefficients are all set to 0.9. Innovations to all productivity processes have equal variance of 0.0001. We assume that international shocks are common across the regions within the same country, while all other shocks are region-specific.

The model is solved by linearizing the system of equilibrium conditions and solving the resulting system of linear difference equations. To make our bond economy stationary, we introduce small quadratic costs on bond holdings. We study the properties of the model's equilibrium by simulating it over 100 periods. The statistics reported in the next section are derived from 200 simulations.

6 Results

This section presents the main findings from the numerical solution to the model. We start by characterizing the properties of RER in a production economy with no firm heterogeneity due to firm-specific productivity. Such simplification eliminates the endogenous non-tradability feature in our model and reduces the setting to a standard international business cycle economy with a representative firm in each region. Then we allow for firm heterogeneity and consider a production economy with endogenous tradability. Under each scenario we also evaluate the role of within country heterogeneity by studying a version of the model with and without regions. We use an economy without endogenous tradability and with no regional heterogeneity as a benchmark.

6.1 No firm heterogeneity

The properties of real exchange rate and consumption generated by the production bond economy with homogeneous and perfectly competitive firms are summarized in Table **4**. Column (i) reports the volatility of the two variables and the correlation between them across regions, while column (ii) presents the corresponding statistics across countries. We study two versions of this model: without regional heterogeneity, the results for which are reported in the top panel; and with regional heterogeneity, the results for which are summarized in the bottom panel of Table **4**. The latter exercise allows us to isolate the contribution of each new feature that we introduce.

Without the regional dimension, our model has been used extensively in the literature to study international business cycles, terms of trade and real exchange rate movements (see Tesar, 1993; Corsetti, Dedola, and Leduc, 2008; Benigno and Thoenissen, 2008). The international business cycle properties of our model under the benchmark parametrization, therefore, are standard. As expected, it predicts international RER that is positively correlated with consumption across countries, even though asset markets are incomplete. This correlation is positive and high when we consider both levels and growth rates of the two variables (0.73 in levels and 0.75 in growth rates). When regional heterogeneity is re-introduced, the international correlation between RER and relative consumption declines substantially, but remains positive. The correlation between RER and consumption across regions is also positive, consistent with the data, but is significantly above its empirical counterpart. These results re-confirm the consumption-RER puzzle in the international data, and show that within country heterogeneity, while working in the right direction, is not sufficient to resolve the Backus-Smith puzzle.

The model at the same time does better in accounting for the volatility of RER relative to that of consumption. The implied relative volatility of international RER is well above one, conformable with the data. The model, however, predicts too much volatility for the regional RER.

Table 4. RER and relative consumption: representative firm

	Across regions	Across countries
No regional heterogeneity	(i)	(ii)
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$		1.68
Correlation b/n		
RER and $C-\hat{C}$		0.73
growth rate of RER and $C-\hat{C}$		0.75
Regional heterogeneity		
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$	0.97	1.57
Correlation b/n		
RER and $C-\hat{C}$	0.99	0.30
growth rate of RER and $C-\hat{C}$	0.99	0.31

Overall, our benchmark model with heterogenous regions and a single non-state-contingent international bond is not able to account for the differences in consumption-RER relationship across regions and across countries. We next explore the role of endogenous tradability in capturing the link between the two variables.

6.2 Endogenous tradability

Next, we present the results obtained in the production economy with heterogeneous firms. As before, we consider two versions of the model. The top panel of Table **5** reports the statistics obtained in the version of our model without regional heterogeneity, while the bottom panel summarizes the corresponding statistics in the version of the model with regional heterogeneity.

Table **5**. RER and relative consumption: endogenous tradability

	Across regions	Across countries
No regional heterogeneity	(i)	(ii)
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$		1.68
Correlation b/n		
RER and $C - \hat{C}$		0.52
growth rate of RER and $C - \hat{C}$		0.53
Regional heterogeneity		
$\sqrt{\frac{\sigma_e^2}{\sigma_{c_i}\sigma_{c_j}}}$	0.46	1.14
Correlation b/n		
RER and $C - \hat{C}$	0.57	-0.10
growth rate of RER and $C - \hat{C}$	0.57	-0.10

The contribution of endogenous non-tradability to the properties of RER and relative consumption can be isolated by comparing the statistics in the top panels of Tables **4** and **5**. Introducing endogenous tradability into a generic international business cycle model (with no regions) does not affect the relative volatility of RER, but reduces consumption-RER correlation both in levels and growth rates. This correlation, however, remains high and positive (0.52 in levels, 0.53 in growth rates). Therefore, endogenous non-tradability per se is not sufficient to overturn the Backus-Smith puzzle.

The results from our full model that incorporates both regional heterogeneity and endogenous non-tradability are presented in the bottom panel of Table **5**. This model provides the best match to the data. It is able to generate positive correlation between RER and relative consumption across regions and a corresponding negative correlation across countries. It also implies relative volatility of RER to consumption that is well above one across countries and well below one across regions, consistent with the data. We next develop the intuition for these results.

6.3 Discussion

To understand the results above it is useful to decompose the international and regional real exchange rates into their components. In particular, in Appendix A.3 we show that the log international real exchange rate (in deviation from the steady state), can be expressed as

$$rer^{\mathrm{I}}_{\mathrm{A},t} = \left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}\left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right)\kappa_1(\hat{p}^{\mathrm{F}}_t - p^{\mathrm{H}}_t) + \hat{\lambda}_{\mathrm{N}}\kappa_2\left(\hat{p}^{\mathrm{N}}_{\mathrm{A},t} - \hat{p}^{\mathrm{T}}_{\mathrm{A},t}\right) - \lambda_{\mathrm{N}}\kappa_2\left(p^{\mathrm{N}}_{\mathrm{A},t} - p^{\mathrm{T}}_{\mathrm{A},t}\right), \tag{19}$$

where lowercase letters denote the log transformations for all variables in deviations from their steady state values (e.g., $p^{\mathrm{H}}_t \equiv \ln P^{\mathrm{H}}_t - \ln P^{\mathrm{H}}$, etc.); and $\kappa_1^{\frac{\rho-1}{\rho}} = \left(\hat{P}^{\mathrm{F}}/P^{\mathrm{T}}\right), \kappa_2^{\frac{\phi-1}{\phi}} = \left(\hat{P}^{\mathrm{N}}_{\mathrm{A}}/\hat{P}_{\mathrm{A}}\right) = (P^{\mathrm{N}}_{\mathrm{A}}/P_{\mathrm{A}})$ are coefficients that depend on the steady state values of relative prices. For analytical tractability, in this decomposition we focused on the aggregate price indices between regions A in countries H and F. Clearly, the international real exchange rate will depend on the weighted average of the aggregate price indices in regions A and B in the two countries.

The expression in (19) decomposes international real exchange rate into two components: (i) a component associated with the international terms of trade movements, $\left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}\left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right)\kappa_1(\hat{p}^{\mathrm{F}}_t - p^{\mathrm{H}}_t)$; (ii) a component arising due to variations in the relative prices of internationally-nontraded goods (the last two terms in the expression above).[4] If there is consumption home bias in households' preferences, $\left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}\left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right) > 0$, then the improvements in the terms of trade will be associated with the real exchange rate appreciation. Furthermore, any variations in the relative price of nontradable goods in the two countries will also contribute to real exchange rate movements to the extent of the weight of nontradable consumption in the aggregate consumption basket of the two countries, $\lambda_{\mathrm{N}}\kappa_2$ and $\hat{\lambda}_{\mathrm{N}}\kappa_2$.

6.3.1 Role of regional heterogeneity

This decomposition allows us to develop the intuition for the mechanism underlying the model without regions and no endogenous non-tradability. In that model the productivity shocks can originate in two sectors: internationally tradable and nontradable. In response to a positive shock in the internationally tradable sector of country H, its terms of trade deteriorates, which will tend to depreciate the RER. This effect, however, may be counterbalanced by the movements in the relative prices of nontraded goods. The latter will arise from two

[4] A similar decomposition is also derived in Benigno and Thoenissen (2008) in a two-country two-sector model with no iceberg trade costs.

sources: first is due to a standard Balassa-Samuelson effect, where positive productivity shock in the tradable sector will trigger an increase in the real wage, thus driving up relative prices of nontraded goods; second is due to a wealth effect in country H, resulting from productivity improvement in that country. In our model these two effects dominate and, as a result, positive productivity shocks in the internationally-tradable sector are associated with RER appreciation and an increase in relative consumption.

Positive shocks that originate in the internationally-nontradable sector have the opposite effect on the RER. Such shocks lead to a fall in the relative prices of nontraded goods, thus depreciating the RER. The RER depreciation is accompanied by an increase in relative consumption, thus working against the resolution of the Backus-Smith puzzle. As the top panel of Table 4 illustrates, a model with no regions and no endogenous non-tradability implies that productivity shocks to internationally nontraded sector dominate the consumption-RER correlation, predicting large positive numbers for that correlation. When heterogenous regions are introduced, the positive association between relative consumption and RER becomes weaker. Much of the decline is due to attenuated effects of the shocks in the internationally nontraded sector. To understand this result, we must deduce how the relative prices of nontradable goods are determined. Due to the presence of regional heterogeneity in our model, we can write the price of nontraded goods in region A in country H as

$$p^{\mathrm{N}}_{\mathrm{A},t} = r^{\mathrm{A}}_{\mathrm{A},t} + \gamma_{\mathrm{B}} \left(\tfrac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(r^{\mathrm{B}}_{\mathrm{A},t} - r^{\mathrm{A}}_{\mathrm{A},t}\right) + \gamma_{\mathrm{N}}\kappa_4 \left(r^{\mathrm{N}}_{\mathrm{A},t} - r^{\mathrm{T}}_{\mathrm{A},t}\right), \tag{20}$$

where $\kappa_3^{\frac{\beta-1}{\beta}} = (R^{\mathrm{B}}_{\mathrm{B}}/R^{\mathrm{T}}_{\mathrm{A}}) = \left(\hat{R}^{\mathrm{B}}_{\mathrm{B}}/\hat{R}^{\mathrm{T}}_{\mathrm{A}}\right) = (R^{\mathrm{B}}_{\mathrm{B}}/R^{\mathrm{T}}_{\mathrm{B}}) = \left(\hat{R}^{\mathrm{B}}_{\mathrm{B}}/\hat{R}^{\mathrm{T}}_{\mathrm{B}}\right)$ and $\kappa_4^{\frac{\alpha-1}{\alpha}} = (R^{\mathrm{N}}_{\mathrm{A}}/P^{\mathrm{N}}_{\mathrm{A}}) = \left(\hat{R}^{\mathrm{N}}_{\mathrm{A}}/\hat{P}^{\mathrm{N}}_{\mathrm{A}}\right) = (R^{\mathrm{N}}_{\mathrm{B}}/P^{\mathrm{N}}_{\mathrm{B}}) = \left(\hat{R}^{\mathrm{N}}_{\mathrm{B}}/\hat{P}^{\mathrm{N}}_{\mathrm{B}}\right)$. The relative price of nontraded goods contains two components: (i) a component associated with the movements in the relative prices of regionally-produced goods, r^{A}_t, and regional terms of trade movements, $\left(r^{\mathrm{B}}_{\mathrm{A},t} - r^{\mathrm{A}}_{\mathrm{A},t}\right)$; (ii) a component arising due to variations in the relative prices of regionally-nontraded goods (the last term).

When regional heterogeneity is introduced, the internationally nontraded shocks can originate in the regionally-traded sector and in the regionally-nontraded sector. These two shocks will have an effect on $p^{\mathrm{N}}_{\mathrm{A},t}$ analogous to those on $p_{\mathrm{A},t}$ in the model with no regions. For instance, a positive regionally nontraded shock in region A will tend to depreciate $p^{\mathrm{N}}_{\mathrm{A},t}$ through lower $r^{\mathrm{N}}_{\mathrm{A},t}$, but this effect will be partially offset by an increase in the price of regionally traded goods from that region, $r^{\mathrm{A}}_{\mathrm{A},t}$. Similarly, a positive regionally traded shock in region A will depreciate $p^{\mathrm{N}}_{\mathrm{A},t}$ through a drop in $r^{\mathrm{A}}_{\mathrm{A},t}$, but again, this effect will be partially counterbalanced by higher prices of regionally-nontraded goods, $r^{\mathrm{N}}_{\mathrm{A},t}$. As a result, in response to these shocks, $p^{\mathrm{N}}_{\mathrm{A},t}$ will adjust by less, thus leading to smaller movements in the international RER. Splitting the internationally nontraded sector into its regionally-traded and regionally-nontraded components allows agents to share these shocks across regions within the same country, thus moderating their contribution to the relative consumption-RER correlation. As a result, the Backus-Smith correlation falls when regions are present.

Next, we derive the regional RER. In region B in country H, the price of nontraded goods is given by the same two components as in (20):

$$p^{\mathrm{N}}_{\mathrm{B},t} = r^{\mathrm{A}}_{\mathrm{B},t} + \hat{\gamma}_{\mathrm{B}}\kappa_3\left(r^{\mathrm{B}}_{\mathrm{B},t} - r^{\mathrm{A}}_{\mathrm{B},t}\right) + \hat{\gamma}_{\mathrm{N}}\kappa_4\left(r^{\mathrm{N}}_{\mathrm{B},t} - r^{\mathrm{T}}_{\mathrm{B},t}\right).$$

Using these decompositions, the regional real exchange rate in country H can be written as

$$rer^{\mathrm{H}}_t = \lambda_{\mathrm{N}}\kappa_2\left(p^{\mathrm{N}}_{\mathrm{B},t} - p^{\mathrm{N}}_{\mathrm{A},t}\right).$$

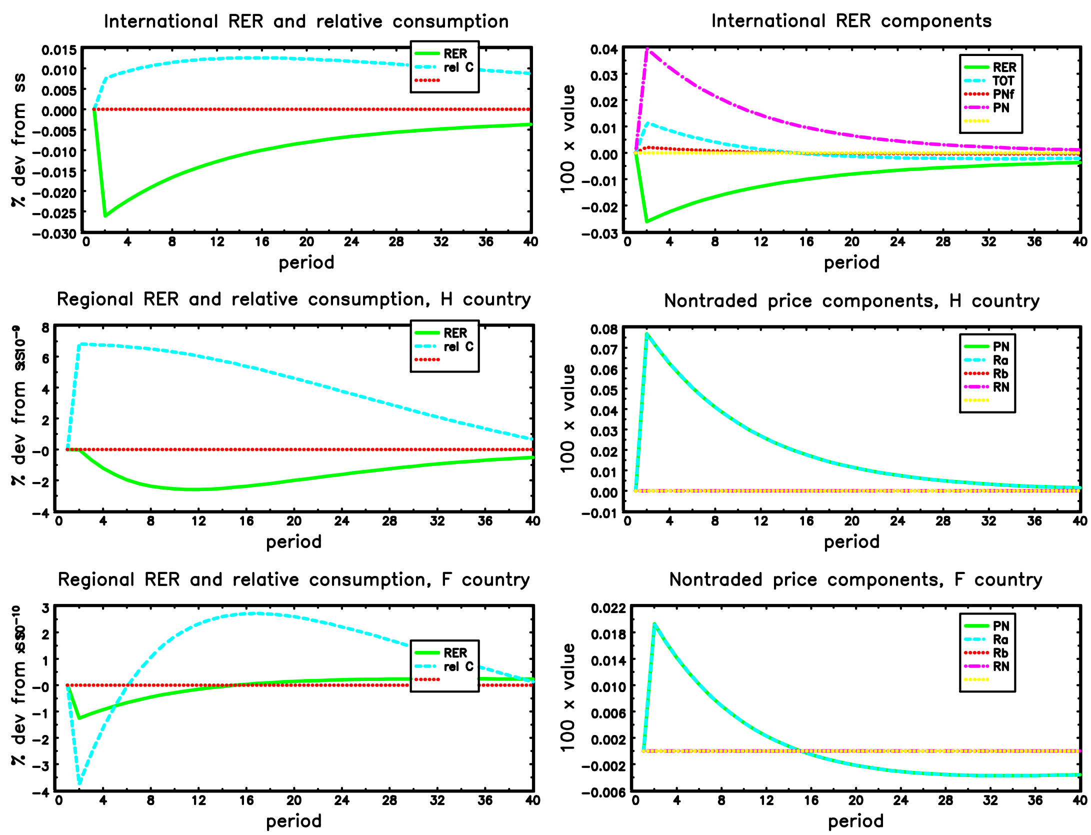
From the results in (2)-(3), the prices of regional goods in country H in deviations from the steady state are related as $r^{\mathrm{A}}_{\mathrm{A},t} = r^{\mathrm{A}}_{\mathrm{B},t} \equiv r^{\mathrm{A}}_t$ and $r^{\mathrm{B}}_{\mathrm{A},t} = r^{\mathrm{B}}_{\mathrm{B},t} \equiv r^{\mathrm{B}}_t$. Further, since regions are symmetric, we get $\hat{\gamma}_{\mathrm{B}} = \gamma_{\mathrm{A}}$, which allows to write the regional real exchange rate in country H as:

$$rer^{\mathrm{H}}_t = \lambda_{\mathrm{N}}\kappa_2\left[\left(\gamma_{\mathrm{A}} - \gamma_{\mathrm{B}}\left(\frac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}}\right)\kappa_3\left(r^{\mathrm{B}}_t - r^{\mathrm{A}}_t\right) + \hat{\gamma}_{\mathrm{N}}\kappa_4\left(r^{\mathrm{N}}_{\mathrm{B},t} - r^{\mathrm{T}}_{\mathrm{B},t}\right) - \gamma_{\mathrm{N}}\kappa_4\left(r^{\mathrm{N}}_{\mathrm{A},t} - r^{\mathrm{T}}_{\mathrm{A},t}\right)\right]. \tag{21}$$

These derivations show that the regional real exchange rate, like its international counterpart, is a composite of two components: (i) regional terms of trade, $\left(\gamma_{\mathrm{A}} - \gamma_{\mathrm{B}}\left(\frac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}}\right)\kappa_3\left(r^{\mathrm{B}}_t - r^{\mathrm{A}}_t\right)$, which depends on the degree of regional consumption home bias; and (ii) relative price of regionally-nontraded goods in regions A and B, given by $\hat{\gamma}_{\mathrm{N}}\kappa_4\left(r^{\mathrm{N}}_{\mathrm{B},t} - r^{\mathrm{T}}_{\mathrm{B},t}\right) - \gamma_{\mathrm{N}}\kappa_4\left(r^{\mathrm{N}}_{\mathrm{A},t} - r^{\mathrm{T}}_{\mathrm{A},t}\right)$. In terms of their effect on the regional RER, regionally traded and nontraded productivity shocks act very much like the internationally traded and nontraded shocks on the international RER. For instance, positive regionally nontraded shocks will tend to depreciate regional RER and increase regional consumption, implying a positive correlation between the two. Positive regionally traded shocks will worsen regional terms of trade, and effect that will be partially offset by increasing prices of regionally nontraded goods. However, contrary to internationally-traded shocks, the increase in regionally nontraded goods prices is not strong enough to appreciate the regional RER. The reason is the smaller degree of home bias in regional consumption, which results from smaller trade costs across regions than across countries. With smaller regional home bias, the terms of trade moves less in response to shocks. The resulting spillovers into the regionally nontraded sectors (through regional Balassa-Samuelson effects or wealth effects) become more symmetric across regions, leading to similar increases in prices of regionally nontraded goods. As a result, two last terms in (21) will tend to cancel out, leading to regional RER depreciation (through terms of trade effect) and an increase in regional consumption. Therefore, regional RER and relative regional consumption are always positively correlated in our model, consistent with the data.

Next, we illustrate these effects on regional real exchange rate, rer_t^{H}, and international real exchange rate, $rer_{\mathrm{A},t}^{\mathrm{I}}$ in more details using impulse response analysis. We start by analyzing the bond economy with a representative firm. The responses of international and regional real exchange rates and relative consumption, as well as their decomposition into the components derived in (19) and (21) are presented in Figures **2a** $-$ **2c**. Figure **2a** presents the effects of a 1% positive productivity shock in the H internationally-traded sector.[5]

Figure **2a**. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Int'l shock



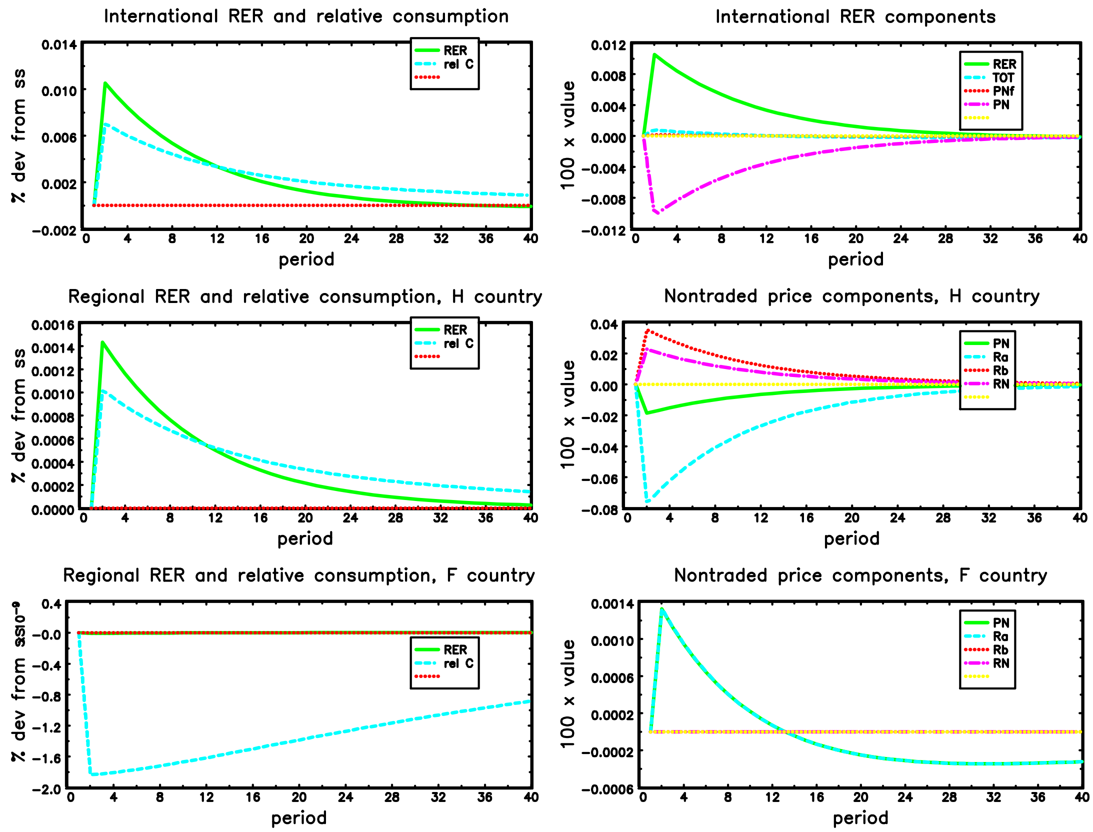
The left-hand-side panel of Figure **2a** presents the impulse responses of RER and relative consumption across countries and across regions within each country, H and F. The

[5] As before, in the model calibration we assume that the shocks to internationally-traded goods productivity experienced by the two regions within each country are perfectly positively correlated.

correlation between the two variables is clearly negative across countries. Regional RER and relative consumption are unaffected by international shocks, because both regions within the country are subject to that shock. The reason for the negative international correlation is twofold: First, there is a the strong wealth effect that is triggered by the supply shocks. Second, there is also a Balassa-Samuelson effect that results from higher wages in the sector experiencing the shock. The right-hand-side panel presents the decomposition of the international and regional RERs. In response to a positive shock to productivity of internationally-traded goods, terms of trade depreciate. Because households' preferences are such that consumption of internationally-traded goods is complementary with the consumption of internationally-nontraded goods, the demand for regional goods, both traded and nontraded, goes up. As a result, their relative prices increase, which drives P^{N} (pictured in the top right chart of Figure **2a**) up. Productivity changes in country H also spill over to country F through their effects on the international terms of trade. As the relative price of foreign internationally-traded goods goes up, households residing in the F country increase their consumption of regional goods. This drives up the relative price of these goods ($\hat{P}^{\text{N}}$ increases). This increase, however, is smaller than that in H country. While the depreciation of terms-of-trade and an increase in $\hat{P}^{\text{N}}$ tend to depreciate the international real exchange rate, and increase in P^{N} tends to appreciate it. If the supply of domestic internationally-nontraded goods is relatively inelastic, the latter effect will dominate, leading to international RER appreciation. At the same time, H country relative to F country consumption will increase, implying a negative correlation between the two variables.

Next, we look at the effects of regional shocks. Figure **2b** presents the responses of RERs and relative consumption, as well as the components of RERs to a 1% positive shock to productivity of regionally-traded goods in region A of country H.

Figure **2b**. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Reg. T shock

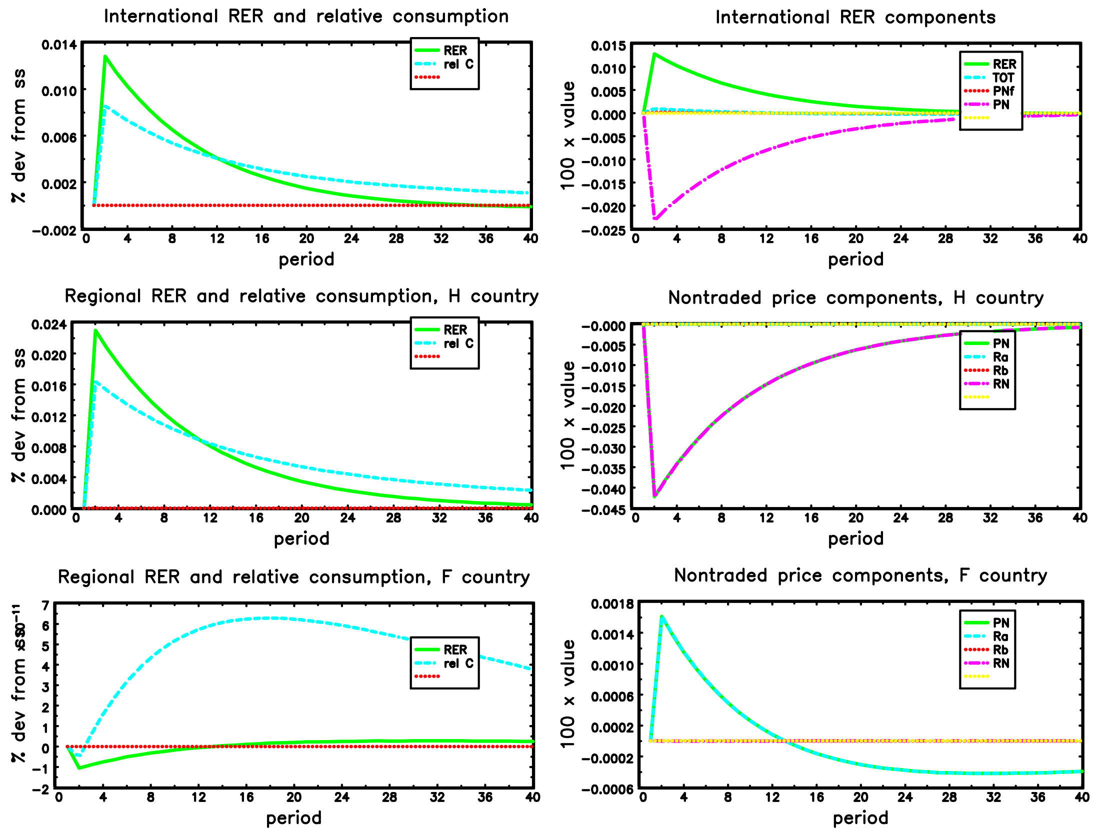


Here both international RER and relative consumption increase in response to the shock, implying a positive correlation between the two variables. The reason for such dynamics can be seen from the right-hand-side panel. In response to the shock, the international terms-of-trade and the relative price of foreign internationally-nontraded goods do not change much. The dynamics of the international RER are therefore dominated by the drop in the relative price of domestic internationally-nontraded goods, P^{N}, which leads to RER depreciation.

The regional RER follows similar dynamics following the shock. When the supply of regionally-traded goods in region A of country H increases, the price of these goods, R^{A}, drops, while the relative price of regionally-traded goods originated in region B, R^{B}, goes up. This depreciates regional terms of trade in region A. Households also demand more of regionally-nontraded goods, driving their price, R^{N} up. Overall, the fall in R^{A} dominates the other effects, and causes P^{N} to fall. In region B of country H, the effects of the shock are similar, but less pronounced, also leading to a fall in the relative price P^{N} in that region. In the presence of risk-free bonds, regions A and B are able to share regional traded shocks quite well. The regional RER, which is the sum of the regional terms of trade and the log-difference of the internationally-nontraded prices in regions B and A, therefore, increases (depreciates).

Finally, Figure **2c** presents the impulse responses of international and regional RERs and relative consumption following a 1% positive shock to productivity of the regionally-nontraded good in region A, country H.

Figure **2c**. Int'l and Regional RER and $C - \hat{C}$ with representative firm: Positive 1% Reg. N shock



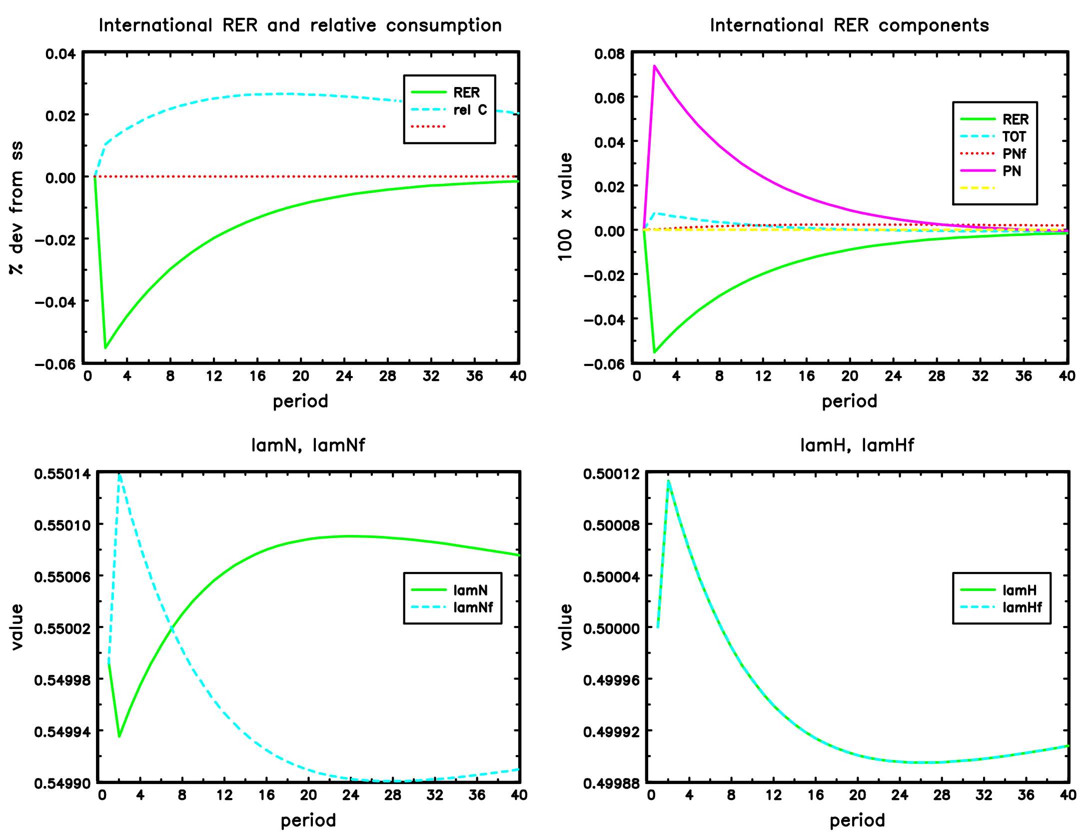
Both international and regional RERs are positively correlated with relative consumptions. An increase in the supply of regionally-nontraded goods, depresses their prices in H country, but does so more in region A than in region B. The regional RER, therefore, depreciates. The drop in the relative price of internationally-nontraded goods also leads to a depreciation of the international RER. At the same time, relative consumption rises both across countries and across regions.

To sum up, international traded shocks imply a negative correlation between international RER and relative consumption and have no effect on the corresponding correlation across regions. The shocks to productivity of internationally-nontraded goods (both regionally-traded and regionally-nontraded) generate a positive Backus-Smith correlation in the international dimension. In the regional dimension, these shocks lead to a positive regional Backus-Smith correlation. As a result, regional consumption-RER correlation is always positive in our model. The net international consumption-RER correlation depends on which shocks dominate. Under our benchmark calibration the effects of regional shocks dominate, resulting in a positive Backus-Smith correlation. This is because regions have access to an international bond, so they are able to share shocks to productivity of internationally-traded goods with the other country reasonably well. The effects of those shocks, therefore, are moderated. The shocks to regional productivity are harder to insure using international asset markets, implying that those shocks have profound effects on real exchange rates and relative consumption. However, when regions are heterogenous, they can also use international bonds to share some of the regionally-traded shocks with their countrymen in another region. This limits the effects of internationally-nontraded shocks. The effect, however, is not strong enough to overturn the Backus-Smith puzzle. Next we study the role played by endogenous non-tradability in the model.

6.3.2 Role of endogenous non-tradability

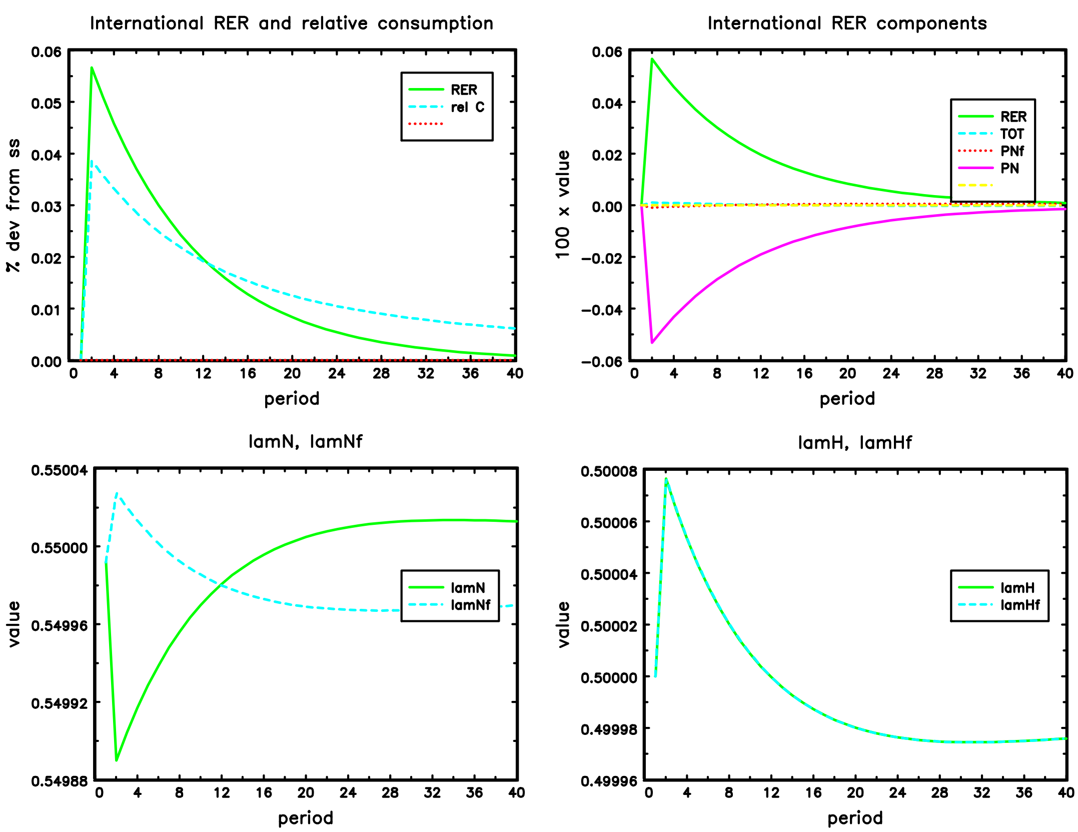
Next, consider the impulse responses arising in the model with endogenous tradability, but no regional heterogeneity. Shutting down regional heterogeneity allows us to isolate the contribution of firm heterogeneity and their export entry-exit decisions on the risk-sharing. Figures **3a** – **3b** summarize the responses.

Figure **3a**. International RER and $C - \hat{C}$ with endog. tradability: Positive 1% Int'l T shock



The top panel of Figure **3a** shows the responses of relative consumption, international RER and its components to a 1% positive productivity innovation to internationally traded sector. These responses are qualitatively similar to those in the model with no endogenous tradability presented in Figure **2a**, however, they are quantitatively larger. The amplifications is due to an endogenous change in the size of the internationally traded sector as new varieties become exported after a positive TFP shock. The bottom panel of Figure **3a** confirms this intuition. It show the impulse responses of nontraded, λ_{N} and $\hat{\lambda}_{\text{N}}$, consumption shares as well as domestic and foreign traded consumption shares in country H. Recall that λ_{N} is also equal to the threshold index i^* of the marginal firm that will export, thus controlling the size of nontraded sector in country H. Following a positive shock, more firms enter the export market in country H, while the reverse happens in country F. The expansion in the size of the H internationally traded sector leads to a larger increase in the relative price of domestic internationally-nontraded goods, P^{N}, implying a greater appreciation of the RER, as a result. Figure **3b** present impulse responses following a 1% positive productivity shock in the internationally nontraded sector.

Figure **3b**. International RER and $C - \hat{C}$ with endog. tradability: Positive 1% Int'l N shock



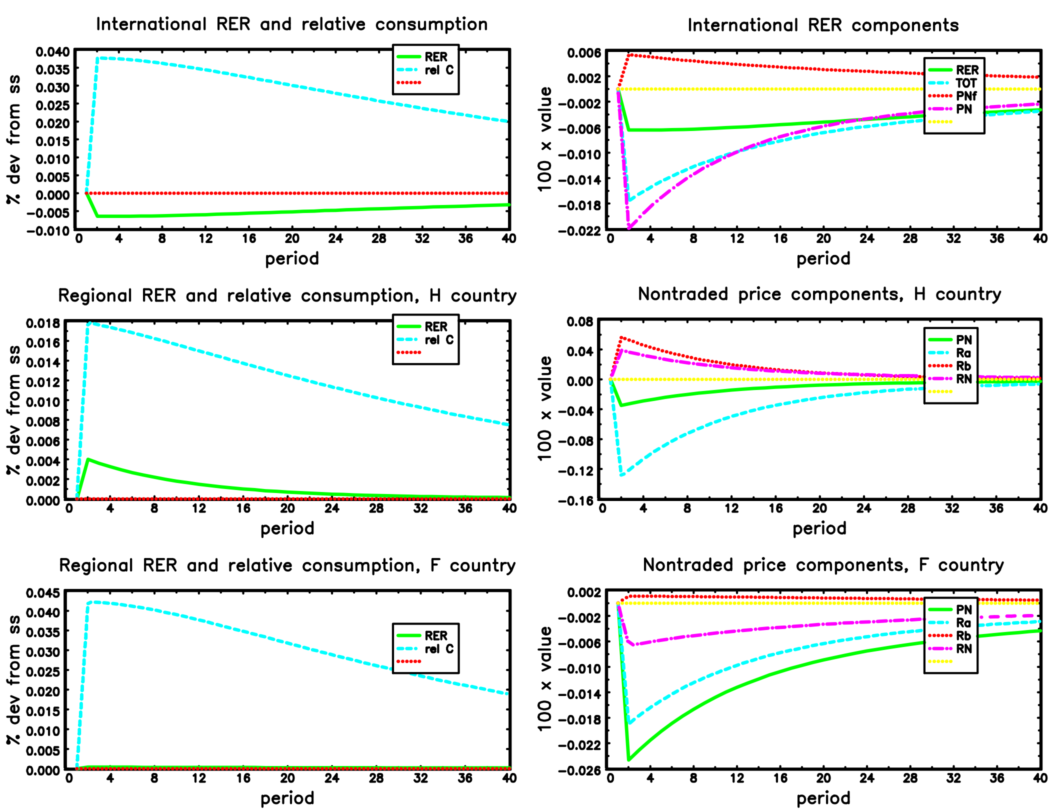
As in Figure **2c**, relative consumption in country H goes up, and RER depreciates following the shock. Much of the RER depreciation is driven by the fall in relative prices of nontraded goods in country H. In the economy with endogenous tradability, a positive shock to N sector productivity leads to a contraction in the size of internationally non-traded sector. As initially nontraded firms become more productive they enter the export market. This effect partially offsets the drop in prices of N goods in country H. As a result, RER depreciates by less, weakening the positive correlation between RER and relative consumption.

Overall, endogenous tradability enforces the effects of T shocks and counteracts the effects of N shocks on the RER, thus lowering the international Backus-Smith correlation in the model.

6.3.3 Full model

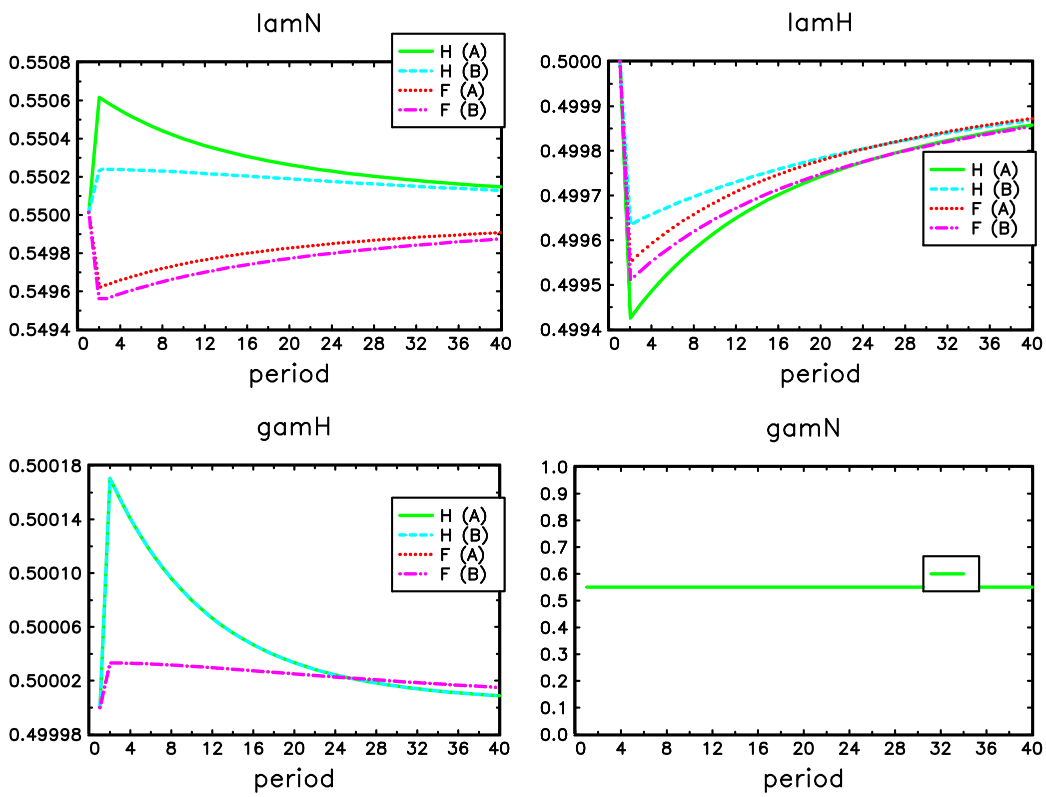
Finally, we explore the properties of the full model that allows for both within-country heterogeneity and endogenous tradability. We find that the dynamics of this model in response to internationally traded shocks and regionally-nontraded shocks are very similar to those in the two versions of the model presented above. The responses of consumption and RER, however, are quite distinct from before when we consider shocks to regionally-traded productivity. We present these responses in Figures **4** − **5** below.

Figure **4**. Int'l and Regional RER and $C - \hat{C}$ with endog. tradability: Positive 1% Reg. T shock



The main difference between Figure **2b** and Figure **4** is in the dynamics of international RER and relative consumption. In particular, with endogenous tradability, the correlation between the two variables becomes negative following the shock; that is RER appreciates, while relative consumption increases. This is in contrast to a positive correlation between the two variables in the model with a representative firm. A decomposition of the international RER in the top right figure helps us understand what drives the results. The key difference arises from the adjustment in the terms of trade. With endogenous tradability, international terms of trade improves following the shock. The improvement is large enough to offset the RER depreciation pressure coming from a fall in the relative price of nontraded goods in country H and an increase in the corresponding price in country F. The intuition for this result can be obtained from Figure **5**, which plots the impulse responses of consumption shares in the two regions (A and B) in the two countries (H and F).

Table **5**. Consumption shares: Positive 1% Reg. T shock



As the top left figure indicates, the share of internationally non-traded sector in country H increases in both regions. Since the share of regionally non-traded sector is fixed to γ_N in our calibration, all of the expansion in λ_N is due to firms' exist from the international export sector into the regional export sector (which has experienced an improvement in TFP). The internationally-traded sector thus contracts in country H. The reverse takes place in country F. With the decline in the supply of internationally-traded goods from H country, both regions in country F expand their supply of internationally traded goods ($\hat{\lambda}_{N}$ declines in both regions). These adjustments lead to a drop in relative price of F traded goods, which is the appreciation in the international terms of trade.

In summary, a combination of regionally traded shocks and endogenous tradability strengthen the negative correlation between international RER and relative consumption, thus helping us to rationalize the Backus-Smith puzzle in the international data. At the same time, the regional consumption-RER correlation remains positive, consistent with the empirical evidence.

7 Conclusion

This paper studies two well-known puzzles in the International Finance: Backus-Smith puzzle and RER volatility puzzle. First, we document the two puzzles in the international data. We complement this empirical analysis with a set of stylized facts on RER and relative consumption using regional/state level data for a large sample of countries. Second, we rationalize the two puzzles in a two-country real business cycles model, and do so in both international and intra-national dimensions. Our explanation relies on two key elements: within country heterogeneity and endogenous non-tradability. Each feature individually allows us to reduce the Backus-Smith correlation, but is not sufficient by itself. Only a combination of regional heterogeneity and endogenous tradability resolves the Backus-Smith puzzle in the international and regional dimensions, and generates realistic volatility of RER relative to consumption.

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A Appendix

A.1 Data description and sources

Canada: For Canada our data includes series for consumption and prices for 10 provinces and 3 territories over 1981-2007 period. All provincial data is obtained from CANSIM. For prices we used CANSIM Table 326-0021 "Consumer Price Index, by province"; for consumption, we used "Personal expenditure on consumer goods and services, Chained (2002) dollars" series. We converted consumption into real per capita terms by dividing the original series by CPI and population.

China: Our data for China covers consumption and prices for 31 regions over 1993-2006 period. Regional data on consumption is from China Statistical Yearbooks published by the Department of National Accounts, National Bureau of Statistics of China. We used "Household Consumption Expenditure by Region, Current Prices, 100m yuan" series. Price data was kindly made available to us by Carsten A. Holz and is "Official CPI (previous year = 100)" series. We transformed this series to obtain the index with 1995 as the base year. We also converted consumption series into real per capita terms.

Germany: Our data for Germany covers 16 bundeslander over 1995-2007 period. Consumption series are obtained from Regional Accounts database collected by Statistische Ämter des Bundes und der Länder. The series is called "Private consumption expenditure - price-settled, concatenated - for each inhabitants in Germany 1991 to 2006 after Federal states year index (2000 = 100)". Price data is from GENESIS-Tabelle: 61111-0010. The series name is "Verbraucherpreisindex für Deutschland, Verbraucherpreisindex (2005=100)".

India: Our data for India covers 15 states over 1960-2000 period. Data on consumption and CPI for India was kindly shared with us by Amartya Lahiri. Price data are from the World Bank database created by Ozler, Datt, and Ravallion (1996), for measures of price indices that control for inter-state price differentials. That dataset also includes data on consumption. Please refer to Lahiri and Yi (2009) for a detailed description of data sources.

Japan: Our data for Japan covers 47 prefectures over 1990-2005 period. All series are from "The Annual Report on Prefectural Accounts" published by ESRI. Consumption series is "Private household and nonprofit consumption, real, constant prices 2000". For prices we used "GDP Deflator: Expenditure, Constant Price 2000" series.

Spain: Our data for Spain covers 18 provinces over 1995-2004 period. All series are from Spanish National Statistics Institute. For consumption we used "Household Final Consumption Expenditure by Autonomous Communities". For prices we used "Consumer Price Index by Autonomous Communities, Spain, Annual Means" with several base years, 1992, 2001, and 2006, which we converted into a common 2001 base year.

US: Coming soon.

A.2 Derivation of consumption aggregates

In our setup households have access to three types of consumption goods: regionally-nontraded goods, goods that can be traded between regions but not internationally, and internationally-traded goods. Let's consider first the final consumption aggregate. It consists of internationally nontraded goods and a basket of internationally traded goods. The latter can be produced by local firms or imported from the foreign country. Recall that all local firms are located on $[0,1]$ interval. Firms that produce internationally nontraded goods occupy $[1,i^*]$ interval, where i^* is the threshold export index in a given region. Domestic internationally-traded goods are produced by firms located on $(i^*,1]$ interval. We append this continuum of local firms by a $[1,2-\hat{\imath}^*]$ measure of foreign firms that can export their goods to the local market. As a result, the consumption basket of households residing in region A in country H becomes

$$
\begin{aligned}
C^{\phi} &= (i^*)^{1-\phi}\left[\int_0^{i^*}\left(\tfrac{1}{i^*}\right)^{1-\rho}c_i^{\rho}di\right]^{\frac{\phi}{\rho}}+(1-i^*)^{1-\phi}\left[\left(\tfrac{1}{2-\hat{\imath}^*-i^*}\right)^{1-\rho}\left(\int_{i^*}^{1}c_i^{\rho}di+\int_1^{2-\hat{\imath}^*}\hat{c}_i^{\rho}di\right)\right]^{\frac{\phi}{\rho}}\\
&= (i^*)^{1-\phi}\left[\int_0^{i^*}\left(\tfrac{1}{i^*}\right)^{1-\rho}c_i^{\rho}di\right]^{\frac{\phi}{\rho}}+(1-i^*)^{1-\phi}\,*\\
&\quad *\left[\left(\tfrac{1}{2-\hat{\imath}^*-i^*}\right)^{1-\rho}\left((1-i^*)^{1-\rho}\int_{i^*}^{1}\left(\tfrac{1}{1-i^*}\right)^{1-\rho}c_i^{\rho}di+(1-\hat{\imath}^*)^{1-\rho}\int_1^{2-\hat{\imath}^*}\left(\tfrac{1}{1-\hat{\imath}^*}\right)^{1-\rho}\hat{c}_i^{\rho}di\right)\right]^{\frac{\phi}{\rho}}\\
&= (i^*)^{1-\phi}\left(C^{\text{N}}\right)^{\phi}+(1-i^*)^{1-\phi}\left[\left(\tfrac{1-i^*}{2-\hat{\imath}^*-i^*}\right)^{1-\rho}\left(C^{\text{H}}\right)^{\rho}+\left(\tfrac{1-\hat{\imath}^*}{2-\hat{\imath}^*-i^*}\right)^{1-\rho}\left(C^{\text{F}}\right)^{\rho}\right]^{\frac{\phi}{\rho}}\\
&= (i^*)^{1-\phi}\left(C^{\text{N}}\right)^{\phi}+(1-i^*)^{1-\phi}\left(C^{\text{T}}\right)^{\phi},
\end{aligned}
$$

where

$$
\begin{aligned}
\left(C^{\text{N}}\right)^{\rho} &= \int_0^{i^*}\left(\tfrac{1}{i^*}\right)^{1-\rho}c_i^{\rho}di,\\
\left(C^{\text{H}}\right)^{\rho} &= \int_{i^*}^{1}\left(\tfrac{1}{1-i^*}\right)^{1-\rho}c_i^{\rho}di,\\
\left(C^{\text{F}}\right)^{\rho} &= \int_1^{2-\hat{\imath}^*}\left(\tfrac{1}{1-\hat{\imath}^*}\right)^{1-\rho}\hat{c}_i^{\rho}di.
\end{aligned}
$$

From above, consumption shares are endogenously linked to the threshold export index of firms:

$$\lambda_{\text{N}}(i^*) = i^* \qquad \text{and} \qquad \lambda_{\text{T}}(i^*) = 1 - i^*,$$
$$\lambda_{\text{H}}(i^*) = \tfrac{1-i^*}{2-\hat{\imath}^*-i^*} \qquad \text{and} \qquad \lambda_{\text{F}}(i^*) = \tfrac{1-\hat{\imath}^*}{2-\hat{\imath}^*-i^*}.$$

Next, consider the internationally nontraded consumption aggregate. It consists of regionally-nontraded goods (i.e. haircuts, real estate services, etc); regionally traded goods that can be produced by the local firms or by the firms residing in another region within the same country (i.e. utility and government services, agricultural produce, many retail services, like those provided by Future Shop in Canada - a retail chain that does not operate in the US, etc). Firms residing in region A that do not export to region B and thus produce regionally nontraded goods are located on $[1, \gamma_{\text{N}}]$ interval. Firms operating in region A that produce goods that are regionally tradable occupy $[\gamma_{\text{N}}, i^*_{\text{A}}]$ interval. Appending this measure of firms with the $[i^*_{\text{A}} + i^*_{\text{B}}(1-\gamma_{\text{N}})]$ interval of firms residing in region B who supply goods to the local market. Therefore, internationally nontraded consumption aggregate in region A can be written as

$$\begin{aligned}
(i^*_{\text{A}})^{1-\alpha}(C^{\text{N}})^{\alpha} &= (\gamma_{\text{N}} i^*_{\text{A}})^{1-\alpha}\left[\int_0^{\gamma_{\text{N}} i^*_{\text{A}}} \left(\tfrac{1}{\gamma_{\text{N}} i^*_{\text{A}}}\right)^{1-\beta} c_i^{\beta}\, di\right]^{\frac{\alpha}{\beta}} + \left(i^*_{\text{A}}(1-\gamma_{\text{N}})\right)^{1-\alpha} * \\
&\quad * \left[\left(\tfrac{1}{i^*_{\text{A}}(1-\gamma_{\text{N}})+i^*_{\text{B}}(1-\gamma_{\text{N}})}\right)^{1-\beta}\left(\int_{\gamma_{\text{N}} i^*_{\text{A}}}^{i^*_{\text{A}}} c_i^{\beta}\, di + \int_{i^*_{\text{A}}}^{i^*_{\text{A}}+i^*_{\text{B}}(1-\gamma_{\text{N}})} c_{i\text{B}}^{\beta}\, di\right)\right]^{\frac{\alpha}{\beta}} \\
&= (\gamma_{\text{N}} i^*_{\text{A}})^{1-\alpha}\left[\int_0^{\gamma_{\text{N}} i^*_{\text{A}}} \left(\tfrac{1}{\gamma_{\text{N}} i^*_{\text{A}}}\right)^{1-\beta} c_i^{\beta}\, di\right]^{\frac{\alpha}{\beta}} + \left(i^*_{\text{A}}(1-\gamma_{\text{N}})\right)^{1-\alpha} * \\
&\quad * \left[\left(\left(\tfrac{i^*_{\text{A}}}{i^*_{\text{A}}+i^*_{\text{B}}}\right)^{1-\beta}\int_{\gamma_{\text{N}} i^*_{\text{A}}}^{i^*_{\text{A}}}\left(\tfrac{1}{i^*_{\text{A}}(1-\gamma_{\text{N}})}\right)^{1-\beta} c_i^{\beta}\, di + \left(\tfrac{i^*_{\text{B}}}{i^*_{\text{A}}+i^*_{\text{B}}}\right)^{1-\beta}\int_{i^*_{\text{A}}}^{i^*_{\text{A}}+i^*_{\text{B}}(1-\gamma_{\text{N}})}\left(\tfrac{1}{i^*_{\text{B}}(1-\gamma_{\text{N}})}\right)^{1-\beta} c_{i\text{B}}^{\beta}\, di\right)\right]^{\frac{\alpha}{\beta}} \\
&= (\gamma_{\text{N}} i^*_{\text{A}})^{1-\alpha}(Z^{\text{N}})^{\alpha} + \left(i^*_{\text{A}}(1-\gamma_{\text{N}})\right)^{1-\alpha}\left[\left(\tfrac{i^*_{\text{A}}}{i^*_{\text{A}}+i^*_{\text{B}}}\right)^{1-\beta}(Z^{\text{A}})^{\beta} + \left(\tfrac{i^*_{\text{B}}}{i^*_{\text{A}}+i^*_{\text{B}}}\right)^{1-\beta}(Z^{\text{B}})^{\beta}\right]^{\frac{\alpha}{\beta}} \\
&= (i^*_{\text{A}})^{1-\alpha}\left[\gamma_{\text{N}}^{1-\alpha}(Z^{\text{N}})^{\alpha} + (1-\gamma_{\text{N}})^{1-\alpha}(Z^{\text{T}})^{\alpha}\right],
\end{aligned}$$

where

$$\begin{aligned}
(Z^{\text{N}})^{\beta} &= \int_0^{\gamma_{\text{N}} i^*_{\text{A}}} \left(\tfrac{1}{\gamma_{\text{N}} i^*_{\text{A}}}\right)^{1-\beta} c_i^{\beta}\, di, \\
(Z^{\text{A}})^{\beta} &= \int_{\gamma_{\text{N}} i^*_{\text{A}}}^{i^*_{\text{A}}} \left(\tfrac{1}{i^*_{\text{A}}(1-\gamma_{\text{N}})}\right)^{1-\beta} c_i^{\beta}\, di, \\
(Z^{\text{B}})^{\beta} &= \int_{i^*_{\text{A}}}^{i^*_{\text{A}}+i^*_{\text{B}}(1-\gamma_{\text{N}})} \left(\tfrac{1}{i^*_{\text{B}}(1-\gamma_{\text{N}})}\right)^{1-\beta} c_{i\text{B}}^{\beta}\, di.
\end{aligned}$$

A.3 International and intra-national RER

In order to derive the decomposition for international RER, we log-linearize aggregate consumption price indices as follows. The log-pice of aggregate price index in region j in country H can be written as

$$\begin{aligned} p_{j,t} &= \lambda_{\text{T}} \left(\frac{P_j^{\text{T}}}{P_j}\right)^{\frac{\phi}{\phi-1}} p_t^{\text{T}} + \lambda_{\text{N}} \left(\frac{P_j^{\text{N}}}{P_j}\right)^{\frac{\phi}{\phi-1}} p_{j,t}^{\text{N}} \\ &= p_t^{\text{T}} + \lambda_{\text{N}} \left(\frac{P_j^{\text{N}}}{P_j}\right)^{\frac{\phi}{\phi-1}} \left(p_{j,t}^{\text{N}} - p_t^{\text{T}}\right), \end{aligned} \tag{A1}$$

with

$$\begin{aligned} p_t^{\text{T}} &= \lambda_{\text{H}} \left(\frac{P_j^{\text{H}}}{P_j^{\text{T}}}\right)^{\frac{\rho}{\rho-1}} p_t^{\text{H}} + \lambda_{\text{F}} \left(\frac{P_j^{\text{F}}}{P_j^{\text{T}}}\right)^{\frac{\rho}{\rho-1}} p_t^{\text{F}} \\ &= p_t^{\text{H}} + \lambda_{\text{F}} \left(\frac{P_j^{\text{F}}}{P_j^{\text{T}}}\right)^{\frac{\rho}{\rho-1}} \left(p_t^{\text{F}} - p_t^{\text{H}}\right) \end{aligned} \tag{A2}$$

and

$$p_{j,t}^{\text{N}} = \gamma_{\text{T}} \left(\frac{R_j^{\text{T}}}{P_j^{\text{N}}}\right)^{\frac{\alpha}{\alpha-1}} r_{j,t}^{\text{T}} + \gamma_{\text{N}} \left(\frac{R_j^{\text{N}}}{P_j^{\text{N}}}\right)^{\frac{\alpha}{\alpha-1}} r_{j,t}^{\text{N}}. \tag{A3}$$

Finally, the aggregate log-price of regionally traded goods in region j is

$$r_{j,t}^{\text{T}} = \gamma_{\text{A}} \left(\frac{R_j^{\text{A}}}{R_j^{\text{T}}}\right)^{\frac{\beta}{\beta-1}} r_{j,t}^{\text{A}} + \gamma_{\text{B}} \left(\frac{R_j^{\text{A}}}{R_j^{\text{T}}}\right)^{\frac{\beta}{\beta-1}} r_{j,t}^{\text{B}}. \tag{A4}$$

A similar set of conditions applies to the foreign country. For analytical tractability, we assume that international RER is a relative price of foreign to domestic consumption basket in regions A in countries H and F. In the numerical analysis of the model we use the definition of RER that relies on the average price in regions A and B as defined in (18). Now RER can be written as

$$rer_{\text{A},t}^{\text{I}} = \hat{p}_{\text{A},t} - p_{\text{A},t}.$$

Substituting in the definitions above, we get

$$rer_{\text{A},t}^{\text{I}} = \left(\lambda_{\text{H}} - \lambda_{\text{F}} \left(\frac{1}{1-\tau_I}\right)^{\frac{\rho}{\rho-1}}\right) \kappa_1 (\hat{p}_t^{\text{F}} - p_t^{\text{H}}) + \hat{\lambda}_{\text{N}} \kappa_2 \left(\hat{p}_{\text{A},t}^{\text{N}} - \hat{p}_{\text{A},t}^{\text{T}}\right) - \lambda_{\text{N}} \kappa_2 \left(p_{\text{A},t}^{\text{N}} - p_{\text{A},t}^{\text{T}}\right),$$

where lowercase letters, as before, denote the log transformations for all variables in deviations from their steady state values (e.g., $p_t^{\text{H}} \equiv \ln P_t^{\text{H}} - \ln P^{\text{H}}$, etc.); and $\kappa_1^{\frac{\rho-1}{\rho}} = \left(\hat{P}^{\text{F}}/P^{\text{T}}\right)$, $\kappa_2^{\frac{\phi-1}{\phi}} = \left(\hat{P}_{\text{A}}^{\text{N}}/\hat{P}_{\text{A}}\right) = \left(P_{\text{A}}^{\text{N}}/P_{\text{A}}\right)$. This is expression (19) in the text.

Having regional dimension in the model allows us to better understand the dynamics of nontradable goods prices. In particular, substituting equation (A4) into equation (A3), regionally nontraded log-price index can be written as:

$$p^{\mathrm{N}}_{\mathrm{A},t} = r^{\mathrm{A}}_{\mathrm{A},t} + \gamma_{\mathrm{B}} \left(\frac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(r^{\mathrm{B}}_{\mathrm{A},t} - r^{\mathrm{A}}_{\mathrm{A},t}\right) + \gamma_{\mathrm{N}}\kappa_4 \left(r^{\mathrm{N}}_{\mathrm{A},t} - r^{\mathrm{T}}_{\mathrm{A},t}\right),$$

where $\kappa_3^{\frac{\beta-1}{\beta}} = (R^{\mathrm{B}}_{\mathrm{B}}/R^{\mathrm{T}}_{\mathrm{A}}) = \left(\hat{R}^{\mathrm{B}}_{\mathrm{B}}/\hat{R}^{\mathrm{T}}_{\mathrm{A}}\right) = (R^{\mathrm{B}}_{\mathrm{B}}/R^{\mathrm{T}}_{\mathrm{B}}) = \left(\hat{R}^{\mathrm{B}}_{\mathrm{B}}/\hat{R}^{\mathrm{T}}_{\mathrm{B}}\right)$ and $\kappa_4^{\frac{\alpha-1}{\alpha}} = (R^{\mathrm{N}}_{\mathrm{A}}/P^{\mathrm{N}}_{\mathrm{A}}) = \left(\hat{R}^{\mathrm{N}}_{\mathrm{A}}/\hat{P}^{\mathrm{N}}_{\mathrm{A}}\right) = (R^{\mathrm{N}}_{\mathrm{B}}/P^{\mathrm{N}}_{\mathrm{B}}) = \left(\hat{R}^{\mathrm{N}}_{\mathrm{B}}/\hat{P}^{\mathrm{N}}_{\mathrm{B}}\right)$. This is equation (20) in the text. In region B, country H, the corresponding expression is

$$p^{\mathrm{N}}_{\mathrm{B},t} = r^{\mathrm{A}}_{\mathrm{B},t} + \hat{\gamma}_{\mathrm{B}}\kappa_3 \left(r^{\mathrm{B}}_{\mathrm{B},t} - r^{\mathrm{A}}_{\mathrm{B},t}\right) + \hat{\gamma}_{\mathrm{N}}\kappa_4 \left(r^{\mathrm{N}}_{\mathrm{B},t} - r^{\mathrm{T}}_{\mathrm{B},t}\right).$$

In country F the prices of internationally-nontraded goods can be approximated in an analogous way. For instance, in region A it is given by

$$\hat{p}^{\mathrm{N}}_{\mathrm{A},t} = \hat{r}^{\mathrm{A}}_{\mathrm{A},t} + \gamma_{\mathrm{B}} \left(\frac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}} \kappa_3 \left(\hat{r}^{\mathrm{B}}_{\mathrm{A},t} - \hat{r}^{\mathrm{A}}_{\mathrm{A},t}\right) + \gamma_{\mathrm{N}}\kappa_4 \left(\hat{r}^{\mathrm{N}}_{\mathrm{A},t} - \hat{r}^{\mathrm{T}}_{\mathrm{A},t}\right),$$

while in region B it is given by

$$p^{\mathrm{N}}_{\mathrm{B},t} = \hat{r}^{\mathrm{A}}_{\mathrm{B},t} + \hat{\gamma}_{\mathrm{B}}\kappa_3 \left(\hat{r}^{\mathrm{B}}_{\mathrm{B},t} - \hat{r}^{\mathrm{A}}_{\mathrm{B},t}\right) + \hat{\gamma}_{\mathrm{N}}\kappa_4 \left(\hat{r}^{\mathrm{N}}_{\mathrm{B},t} - \hat{r}^{\mathrm{T}}_{\mathrm{B},t}\right).$$

Using these decompositions, the regional real exchange rate in country H can be written as

$$rer^{\mathrm{H}}_t = \lambda_{\mathrm{N}}\kappa_2 \left(p^{\mathrm{N}}_{\mathrm{B},t} - p^{\mathrm{N}}_{\mathrm{A},t}\right).$$

We can simplify the derivations by recognizing that in our model $r^{\mathrm{A}}_{\mathrm{A},t} = r^{\mathrm{A}}_{\mathrm{B},t} \equiv r^{\mathrm{A}}_t$ and $r^{\mathrm{B}}_{\mathrm{A},t} = r^{\mathrm{B}}_{\mathrm{B},t} \equiv r^{\mathrm{B}}_t$. Furthermore, because regions are symmetric, we get $\hat{\gamma}_{\mathrm{B}} = \gamma_{\mathrm{A}}$. Substituting for $p^{\mathrm{N}}_{\mathrm{B},t}$ and $p^{\mathrm{N}}_{\mathrm{A},t}$ in the expression for rer^{H}_t and combining with the simplifications above allows to write rer^{H}_t as

$$rer^{\mathrm{H}}_t = \lambda_{\mathrm{N}}\kappa_2 \left[\left(\gamma_{\mathrm{A}} - \gamma_{\mathrm{B}} \left(\frac{1}{1-\tau_R}\right)^{\frac{\beta}{\beta-1}}\right) \kappa_3 \left(r^{\mathrm{B}}_t - r^{\mathrm{A}}_t\right) + \hat{\gamma}_{\mathrm{N}}\kappa_4 \left(r^{\mathrm{N}}_{\mathrm{B},t} - r^{\mathrm{T}}_{\mathrm{B},t}\right) - \gamma_{\mathrm{N}}\kappa_4 \left(r^{\mathrm{N}}_{\mathrm{A},t} - r^{\mathrm{T}}_{\mathrm{A},t}\right)\right],$$

which is equation (21) in the text.