

The Bond Risk Premium and the Cross-Section of Equity Returns

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Abstract

The cross-section of returns of stock portfolios sorted along the book-to-market dimension can be understood with a one-factor model. The factor is the nominal bond risk premium, best measured as the Cochrane-Piazzesi (2005, CP) factor. This paper ties the pricing of stocks in the cross-section to the pricing of bonds of various maturities, two literatures that have been developed largely in isolation. A parsimonious stochastic discount factor model can price both the cross-section of stock and bond returns. The mean average pricing error across 5 bond and 10 book-to-market stock portfolio returns is less than 60 basis points per year. The model also replicates the dynamics of bond yields as well as the time-series predictability of stock and bond returns. Its key feature is a non-zero risk price on the state variable that governs the bond risk premium. Value stocks are riskier because their returns are high when the bond risk premium is high. Empirically, the CP factor peaks at the end of a recession, when good times lie ahead. We trace back this risk to the fundamentals: the properties of dividend growth. An equilibrium asset pricing model ties the properties of cash flows on stocks to stock and bond risk premia. The model generates the observed value spread as well as the observed sensitivities of excess returns and dividend growth to the bond risk premium. It does so because times when CP is high are times of low marginal utility growth when value stocks receive better news about future cash flows than growth stocks.

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The last twenty years have seen dramatic improvements in economists' understanding of what determines differences in yields (Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002)) and returns on bonds, as well as what determines heterogeneity in stock returns which differ by characteristics such as size and book-to-market value (Fama and French (1992)). Yet, these two literatures have developed largely separately and employ largely different asset pricing factors. This is curious from the perspective of a complete markets model because both stock and bond prices equal the expected present discounted value of future cash-flows, discounted by the same stochastic discount factor.

To be fair, there has been some work, both on the theory and on the empirical side linking stock and bond prices. On the theory side, representative-agent endowment models have been developed that have predictions for both stocks and bonds. Examples are the external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied *separately* by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) *separately* study the implications for the cross-section of equity portfolios. In more recent work, Bansal and Shaliastovich (2007) jointly studies some of the joint properties of bond yields and stock returns, although not the ones we will focus on. Other recent work that ties stock and bond markets together in a general equilibrium model is Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008), Lettau and Wachter (2007a), Campbell, Sunderam, and Viceira (2008), and Gabaix (2008).

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Cochrane and Piazzesi (2005) show that a tent-shaped function of 5 forward rates forecasts not only nominal returns on bonds, but also has some forecasting ability for future aggregate stock market returns. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Ferson and Harvey (1991) study stock and bond returns' sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios and interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Lustig, Van Nieuwerburgh, and Verdelhan (2008) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns. Finally, Baker and Wurgler (2007) show that government bonds comove most strongly with bond-

like stocks, which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. We propose that stocks and bonds are driven by a common sentiment indicator.

This paper contributes to both literatures. In Section 1, we start by showing that the bond risk premium alone, either proxied by the Cochrane-Piazzesi (henceforth CP) factor or by the slope of the yield curve, explains at least 70% of the variation in monthly, quarterly, or annual stock returns on either the 25 size and value portfolios or the 10 decile book-to-market portfolios over the period 1952-2007. The returns of value stocks have higher exposure to the CP factor than the returns of growth stocks. This essentially amounts to a one-factor model of the cross-section of stock returns, which explains as much of the cross-section as the 3-factor Fama-French or 4-factor Carhart model. Next, we show that the cross-section of bond returns can be explained by a single yield (factor). Hence, both the cross-section of stocks and the cross-section of bonds have a 1-factor structure, and in both cases, the relevant factor is a bond market factor.

This finding suggests a parsimonious unified model that can explain both the cross-section of stock and bond returns. Section 2 develops such a model of the SDF. It starts from the state of the art in term structure modeling, by writing down the four factor Cochrane and Piazzesi (2005, 2006) model. The term structure is affine, and described by three factors (level, slope, and curvature) and the CP factor. The three factors explain 99% of the cross-sectional variation in yields, while the CP factor explains much of the variation in excess bond returns. To be precise, only the level factor is priced and its price of risk varies with the CP factor. This amounts to a parsimonious model with four state variables but only two non-zero price of risk parameters. We argue that it suffices to free up only one additional price of risk in this model in order to explain the *cross-sectional variation* in stock returns: the price of *CP* risk. Finally, we introduce a fifth factor, the stock market returns, which is necessary to get the *level* of the stock returns (or the equity premium) right. This amounts to a parsimonious SDF model with five factors, three non-zero prices of risk (level, CP, and market), and only four non-zero market price of risk parameters. This model generates mean absolute pricing errors below 60 basis points per year for ten decile book-to-market portfolios, the aggregate stock market portfolio, and five bond returns on bonds of various maturities. Finally, allowing the price of level risk to vary with the CP factor allows us to capture the dynamics of bond yields.

The CP factor displays interesting time series behavior. It is typically very low at the beginning of a recession (or a period of economic turmoil such as the summer of 1998), it increases during the recession, and peaks at the end of a recession. Hence, bond risk premia increase dramatically from a very low (often negative) value to a very high value over the course of a recession. We find some predictability of the CP factor for a strategy that goes long in value stocks (highest book-to-market decile) and short in growth stocks (lowest book-to-market decile), with a positive sign. Indeed,

the best time to enter into a value minus growth strategy is at the end of a recession, when CP is high. In contrast, the same value-growth strategy generates lower returns than the average value spread when entered when CP is low. The effects are economically large: when CP is in its highest quartile of observations, the value spread is 16.6%, while it is only 2.1% when CP is in the lowest quartile of its distribution. Allowing the price of CP risk to vary with the CP factor allows the model of Section 2 to also capture the predictability of a value-minus-growth strategy by the CP factor.

We argue that this pattern can be understood in a rational model of stocks and bonds. In the absence of cash-flow risk, the higher loading of value portfolios' returns on the bond risk premium (CP factor) are puzzling at first thought. One view, advocated by Lettau and Wachter (2007b), is that growth stocks pay dividends in the far future (long duration equity) and value firms in the near future (short duration equity). In such a world, the only difference between value and growth stocks is the timing of the cash-flows. For the same reason that long-maturity bonds are more sensitive to the bond risk premium than short maturity bonds, growth stocks would have a higher risk premium than value stocks, and a *growth risk premium* would arise. Hence, cash flows on value stocks must be riskier than those on growth stocks to offset this discount rate effect. One potential explanation for the higher cash-flow risk of value firms is that they have cash-flow growth that is more cyclical than that of growth firms: expected future dividend growth on value stocks is relatively low at the start of a recession, when CP is low, and relatively high at the end of a recession, when CP is high.

This is exactly what we find in the equilibrium model we write down in Section 3. It is a long-run risk model that provides a tractable laboratory to explore the connections between stock and bond returns. Two extensions of the benchmark Bansal and Yaron (2004) or Bansal and Shaliastovich (2007) model are necessary to account for the observed sensitivity of excess stock returns and dividend growth to the CP factor on value and growth portfolios. First, expected cash flow growth rates in the economy (x) must be positively correlated with economic uncertainty (σ^2). Second, we need to allow for cross-sectional variation in expected dividend growth on different stock portfolios; not only by varying its loading with long-run risk (x), as in Bansal, Dittmar, and Lundblad (2005), but also its loading on economic uncertainty (σ^2).

The CP factor in the model has a correlation of one with economic uncertainty (σ^2); it is a perfect predictor of future bond and stock returns. We estimate a lower persistence for σ_t^2 than in the existing calibrations. This suggests that economic uncertainty behaves more like a recession variable than a low-frequency variable. Shocks to economic uncertainty carry a negative price of risk: marginal utility growth is high when uncertainty is high, so an investor is willing to pay a high price for assets with high cash-flows when uncertainty is high. In contrast, shocks to long-run risk (x) carry a positive price of risk because marginal utility growth is low when long-run growth

is high. Because of the positive correlation between σ_t^2 and x_t , a high CP_t not only implies high uncertainty but also high long-run growth. When economic uncertainty is less persistent than long-run risk, the effect of growth dominates that of uncertainty. Hence, a high CP corresponds to a period with low marginal utility growth. Stocks that have high returns when CP is high are risky. Value stocks are such stocks: they have high future expected returns when CP is high. This explains the positive CP betas of value stocks and the positive price of CP risk we find in the data. Thus, the model suggests that the underlying reason for the high risk premia on value stocks is that they have high expected future dividend growth exactly when CP is high. this occurs at the end of a recession, when good time lie ahead.

In the last Section 4 we ask *why* value stocks have cash flow growth that is more sensitive to the bond risk premium (CP factor) than growth stocks. We pursue an explanation rooted in heterogeneity in the maturity structure of corporate debt. In cross-sectional data, large mature or regulated firms (value firms) tend to have higher leverage (debt-to-equity) ratios and tend to have a higher fraction of long-term debt (Barclay and Smith 1995). Smaller, growth firms with valuable growth options tend to have less debt and a smaller fraction of long-term debt. In time series data, Baker, Greenwood, and Wurgler (2003) show that firms borrow long term when the yield spread is low, indicating predictably low excess bond returns. This connection makes the share of long-term corporate debt a good predictor of future bond returns. Moreover, their results tend to be stronger for large, old, dividend-paying firms, suggesting the strongest effect for value firms. Graham and Harvey (2001) provide direct evidence from a CFO survey that firms are trying to time the bond market. We write down a simple model which takes as given these facts on the maturity structure of debt. We show that as long as the value of a firm's assets is affected less than the value of debt, a high bond risk premium (high CP factor) reduces the value of debt and increases the value of equity. This effect is stronger for value firms because of their higher fraction of long-term debt. Such an explanation is consistent with the high value-minus-growth returns and with higher present discounted dividends in periods where CP is high.

1 Empirical Analysis

Section 1.2 shows that the bond risk premium alone, either proxied by the Cochrane-Piazzesi (henceforth CP) factor or by the slope of the yield curve, explains a substantial fraction of the cross-sectional variation in monthly, quarterly, or annual stock returns over the period 1952-2007. Our main results are for the 10 decile book-to-market portfolios. We also report results for the 25 size and value portfolios, the 10 earnings-price portfolios and the 10 dividend-price portfolios. The main finding is that the returns of value stocks have higher exposure to the CP factor or to the yield spread than the returns of growth stocks. We compare this one-factor model of the

cross-section of stock returns to the 3-factor Fama-French or 4-factor Carhart model. Sections 1.3 shows that the cross-section of bond returns can be explained by a single yield (factor). Hence, both the cross-section of stocks and the cross-section of bonds have a 1-factor structure, and in both cases, the relevant factor is a bond market factor. This suggests a unified model for pricing both stocks and bonds, which is what we provide in Section 3. But first, we briefly discuss the data and the construction of the proxies for the bond risk premium.

1.1 Data and the Cochrane-Piazzesi Factor

We use Fama-Bliss yield data for nominal bonds of maturities 1- through 5-years as well as the Fama risk-free rate on a three-month T-bill. These data are available for the sample June 1953-December 2007. We also use nominal stock returns from Kenneth French' data library on 10 book-to-market sorted portfolios, 25 size and book-to-market portfolios, 10 earnings-price portfolios, and 10 dividend-price sorted portfolios. We use monthly, quarterly, and annual data.

Following the procedure outlined in Cochrane and Piazzesi (2005), we construct one- through five-year forward rates from our monthly, quarterly, or annual nominal yield data. From these yield series, we construct one-year excess holding period returns on two- through five-year nominal bonds, in excess of the one-year nominal yield. We regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The regression coefficients display a tent-shaped function, very similar to the one reported in Cochrane and Piazzesi (2005). The variable CP_t is the fitted value of this regression. The regression R^2 is 24.1% with monthly time-series, 29.3% with quarterly, and 37.7% with annual time-series. Figure 1 plots a time-series for the Cochrane-Piazzesi factor CP_t ; each panel corresponds to a different data frequency. However, the left-hand side variables are always one-year excess returns and the right-hand side variables annual forward rates.

[Figure 1 about here.]

The yield spread is often used as another proxy for the bond risk premium. We define the yield spread as the difference between the nominal yield on a five-year T-bond and the yield on a three-month T-bill.¹ The R^2 of a regression of the equally-weighted average of the one-year excess return of bonds of maturities of two through five years on a constant and the yield spread gives an R^2 of 13.0% at monthly frequency, 14.5% with quarterly, and 18.5% with annual time-series. The correlation between the yield spread and the CP factor is 0.71 at monthly frequency, 0.63 at quarterly frequency, and also 0.63 at annual frequency. While the yield spread certainly has some

¹The results are very similar if we define the yield spread as the difference between the 5-year yield and the 1-year T-note.

predictive power for one-year ahead excess bond returns, it is a much weaker predictor than the CP factor. Hence, we will exclusively focus on CP from Section 2 onwards.

We also experimented with using the long yield (20-qtr) and the short yield (1-qtr) separately as predictors of the bond risk premium. Both are highly significant predictors of one-year excess bond returns; the long yield has a positive sign and the short yield a negative sign. The R^2 of the same bond return predictability regression is 18.4% at monthly, 20.0% at quarterly, and 24.0% at annual frequency. The combined predictive ability of the long and short yield for excess bond returns is substantially stronger than that of the yield spread, but still weaker than that of the CP factor. The CP factor is a parsimonious way to summarize information in excess bond returns. And since the results for stocks are similar, we omit them.

1.2 Pricing the Cross-Section of Stocks with a Bond Factor

For each of the portfolio returns $i \in \{1, \dots, N\}$ in the cross-section, we construct log excess returns corrected for a Jensen term: $r_{t+1}^{i,e} \equiv r_{t+1}^i - y_t^{\$}(1) + .5Var[r_{t+1}^i]$, where the lowercase letters denote logs. We use log excess returns with Jensen adjustment for ease of comparison with the models in Sections 2 and 3. All our results are quantitatively similar for gross excess returns and for gross returns. The returns are at monthly, at quarterly or at annual frequencies. We use a standard two-step OLS estimation procedure. In the first step, we run N time-series regressions of returns on the proxy for the bond risk premium, either the CP factor or the yield spread, and a constant in order to estimate the beta. In the second step we run one cross-sectional regression of the N average returns on the betas from the first step and a constant in order to estimate the market price of risk.

First Stage Betas Our benchmark set of test assets are 10 value-weighted decile portfolios sorted alongside the book-to-market dimension. The value spread, measured as $r_{t+1}^{10,e} - r_{t+1}^{1,e}$, is 5.26% per year. For comparison, the value premium for gross returns is 5.32% per year. Figure 10 shows that value firms are exposed more heavily to CP , our proxy for the bond risk premium. Each bar represents the slope coefficient (“beta”) of a time-series regression of a portfolio return on the CP factor. The data are monthly (1952.6-2007.12) in the top panel, quarterly in the middle panel (1952.II-2007.IV), and annual in the bottom panel (1953-2007). The book-to-market ratio increases reading from the left to the right. There is a clear pattern in these betas: they are much larger for value than for growth firms. At monthly and annual frequency, the CP -beta for extreme growth is negative whereas it is strong and positive at every frequency for the extreme value portfolio.

[Figure 2 about here.]

The same pattern of exposures arises when we add a size dimension to the cross-section. Figure 3 shows the contemporaneous CP betas for the 25 size and value portfolios (5-by-5 sort). The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. There is a clear pattern in these betas: they are much larger for value than for growth firms. The difference in exposure between the highest and lowest book-to-market group is more pronounced for small firms than for big firms. There is less variation along the size dimension.

[Figure 3 about here.]

The same pattern in betas also arises when we use the yield spread as an alternative proxy for the bond risk premium. For both the 10 book-to-market portfolios and the 25 size and value portfolios, we find that value stocks have a high positive loading on the yield spread, whereas growth stocks have a much lower exposure. We summarize these findings in Figure 4, which shows betas for these two sets of test assets at all three frequencies.

[Figure 4 about here.]

Second Stage Regression In a second stage, we run a cross-sectional regression of average excess stock returns on the first-stage beta and a constant in order to estimate the market price of risk $\tilde{\Lambda}$. Table 1 shows the results; the top panel is for the 10 BM portfolios, the bottom panel for the 25 size and BM portfolios. The left panel uses CP as a proxy for the bond risk premium, the right panel uses the yield spread instead. In each of the four panels, we report results at monthly (M), quarterly (Q), and annual (A) frequency. Point estimates for the price of risk $\tilde{\Lambda}$ are directly comparable across frequencies. The second-stage intercept α is expressed as a percent per year by multiplying the monthly intercept by 12 and the quarterly intercept by 4.

Using CP , we find a highly significant market price of risk estimate between 1.5 (M) and 3 (A) for the 10 BM portfolios and between 2.2 (M) and 3.3 (A) for the 25 size and BM portfolios. The second-stage regression has a high R^2 : around 75% for the 10 and around 80% for the 25 portfolios. The pricing errors are small: the mean absolute pricing error (MAPE) across the 10 portfolios is between 65 basis points per year (M) and 75 basis points (A) for the 10 BM portfolios, the root mean squared error (RMSE) between 75 (M) and 93 (A) basis points per year. For the 25 portfolios, the MAPE is between 80 and 109 basis points and the RMSE is between 1.04 and 1.44% per year. Figure 5 plots average realized excess returns against predicted excess returns, formed as $\alpha + \beta\lambda$, where α is the second-stage regression intercept. The left row is for the 10 BM, the right row for the 25FF portfolios. The sampling frequency is monthly in the top panels, quarterly in the middle panels, and annual in the bottom panels. It confirms visually that a large fraction of the cross-sectional variation in stock returns is explained by their exposure to the bond risk

premium. Combining results from the two stages, value firms (BM10) and growth firms (BM1) have a differential exposure to CP of 3.4 in the monthly data. With a market price of risk of 1.57, this translates into a value premium of 5.3%, which is essentially the entire value premium observed in the data.

[Table 1 about here.]

[Figure 5 about here.]

We repeat the analysis with the yield spread, in the right panel of Table 1. There too, we find significantly positive market prices of risk. The pricing errors are somewhat larger at monthly and quarterly frequencies, but somewhat smaller at annual frequency. The regression R^2 vary between 55% and 83%. Because exposure to the yield spread carries a positive risk price, the differential exposure of value and growth stocks leads to a value spread on value than on growth stocks. One advantage of the yield spread is that the second stage regression no longer has a significant intercept α in the monthly data. The positive α 's in all other specifications, suggest that not all cross-sectional variation is explained by the yield spread or the CP factor. In particular, there seems to be a common factor missing that increases the returns on all portfolios. We will introduce such a “market factor” in Section 2. Nevertheless, the small pricing errors throughout show that a single bond factor, linked to the bond risk premium, can explain most of the variation in the cross-section of stock returns. We consider the regression analysis of this section only preliminary evidence. Section 2 explores the links between stock returns, bond returns, and bond yields in much more detail.

Sub-sample Analysis As a first robustness check, we repeat the analysis with returns instead of excess returns. The results are very similar and are available upon request. As a second robustness check, we conduct a sub-sample analysis. We present two sets of results in Table 2 for the 10 BM portfolios (top panel) and the 25 size and BM portfolios at monthly frequency (bottom panel). The first set of results uses the CP factor constructed from the full sample (left panel); the second set of results re-estimates the CP factor over the sub-sample in question (right panel). If we start the analysis in 1963, an often used starting point for cross-sectional equity analysis, we find somewhat *stronger* results. The pattern in the betas is similar as in the full sample, but the market price of risk estimate increases and so does the second-stage R^2 . The mean absolute pricing errors drop another 4 basis points to 60 basis points per year. We also split the full sample into two sub-samples: 1952-1985 and 1986-2007. The latter period corresponds to the period after the high-inflation era, and has lower and declining bond risk premia (see Figure 1). The results remain strong in both sub-samples: pricing errors generally remain below 1.5% per year and the cross-sectional R^2 s remain high. The market price of risk estimates are lower, especially in the last

sub-sample, but they remain significantly positive in every instance. Because CP is trending down in this sub-sample, all CP betas are negative, and they are much more negative for value than for growth stocks. Given the positive price of risk (and the large intercept), that spread in exposures still explains the (positive) value premium.

[Table 2 about here.]

Comparison with Three-Factor Model As a last robustness exercise, we compare our results to the celebrated Fama-French three-factor and Carhart four-factor models. For brevity, we focus on the 25 size and BM portfolios and on monthly data. The results for quarterly and annual data are similar. The factors are the log excess return on the market including a Jensen adjustment (6.61% per year on average), the log return on SMB (1.57% per year on average), the log return on the HML (4.42% per year on average), and the log return on the UMD portfolio (9.44% per year on average). The first two columns of Table 3 repeat previously reported results for the model with only CP of the yield spread as factors. Columns 3 and 4 present the results from the three-factor (3-f) and four-factor model (4-f). The main observation is that the pricing errors are similar to, and if anything *larger* than, the pricing errors for our one-factor model reported in Column 1. Additionally, the three-factor model in Column 3 has a negative price of market risk, inconsistent with theory, and the cross-sectional intercept is very large. These issues are mitigated by including the momentum factor in Column 4.

Does the bond risk premium survive the inclusion of the three- or four-factor model's risk factors? Columns 5 through 8 show that both the CP factor and the yield spread remain significantly priced risk factors once the market, SMB, HML, and UMD are added. Adding the CP factor to the 3-factor model lowers the MAPE from .90% per year to .67% per year and increases the cross-sectional R^2 from 75.9% (Column 3) to 85.8% (Column 5). Adding the CP factor to the 4-factor model lowers the MAPE from .86% to .69% and increases the R^2 from 81.0% (Column 4) to 87.0% (Column 7). Adding the yield spread leads to even bigger gains in R^2 and bigger reductions in pricing errors. The pricing errors of the resulting 4-factor and 5-factor models in Columns 6 and 8 are .52% and .47% per year, half the size of the model without the yield spread factor. The R^2 s are 93.6% and 94.4%.

Interestingly, the addition of the CP factor or the yield spread increases the explanatory power of the SMB factor and of the HML factor. Both are robustly and highly significant. The t-stat on the SMB factor doubles and that on HML increases by 50% in many specifications. Including the bond risk premium seems to crystalize the effects of the size and value risk factors. In contrast, the momentum factor loses its significance for explaining the size and value portfolios.

[Table 3 about here.]

Quartile Results The bond risk premium proxies are clearly related to the state of the economy. Figure 8 plots the *CP* factor (top panel) and the yield spread (bottom panel) against the NBER recession dates. Both variables tend to be low at the beginning of a recession, increase substantially during a recession, and tend to peak at the end of a recession. During the average recession in the sample, *CP* increases by 224 basis points (1.27 standard deviations), typically from a negative to a positive value. Likewise, the yield spread increases by 144 basis points (1.40 standard deviations) from a negative to a positive value. The fact that the *CP* factor and the yield spread peak at the through of a recession, when good economic times are around the corner and marginal utility growth for the representative investor is presumably low, can explain the positive price of *CP* or yield spread risk we found above.

An investment strategy which is long value and short growth stocks has low future returns when entered at the start of a recession and high future returns when entered at the end of a recession. Average monthly holding period returns on the value-minus-growth strategy are only 1.5% during NBER recessions (15% of the months in the sample) but 5.8% the rest of the time. So, the fortunes of a value-growth strategy reverse over the course of a recession.

Not all periods with a high (low) *CP* factor or yield spread observations occur at the end (beginning) of recessions. However, several of these low-to-high swings in the *CP* factor are also associated with times of economic uncertainty, such as 1987, 1998 or 2003. An even starker difference between the returns on value and growth stock emerges when we condition on the top quartile of the *CP* observations or the top quartile of yield spread observations. The *CP* factor cutoffs are -0.45% (25th percentile), .58% (median), and 1.60% (75th percentile), while the yield spread cutoffs are 0.28% (25th percentile), 0.94% (median), and 1.68% (75th percentile). Figure 6 shows that the value spread is highest when *CP* is high (first row, left panel) or when the yield spread is high (first row, right panel). In that quartile the value spread is 16.9% for *CP* and 14.5% for the yield spread. As the other rows show, there is little spread in the other quartiles.

[Figure 6 about here.]

1.3 Pricing the Cross-Section of Bonds with a Bond Factor

We now show that another bond factor, the level of the term structure, has significant ability to explain the cross-section of bond returns. As a first measure of the level factor, we use the five-year Fama-Bliss bond yield. As a second measure, we use the first principal component of yields as the level factor, extracted from the cross-section of 1- through 5-year Fama-Bliss yields. We use CRSP excess bond returns on portfolios of maturities 1, 3, 5, 7, and 10 years at monthly, quarterly, or annual frequency. Average excess returns are respectively 1.08%, 1.24%, 1.53%, 1.75%, and 1.38% per year. The sample is again June 1952 until December 2007.

Figure 7 shows the level betas of excess bond returns on the 5 maturity-sorted bond portfolios. They are decreasing from short to long maturities and generally negative. Table 4 shows the corresponding market prices of risk using $y^s(20)$ (on the left) and using the first principal component (on the right). The market price of risk is estimated to be negative and significant, as predicted by theory. The pricing errors are small; the MAPE is 8 basis points and the RMSE 11 basis points per year. The explanatory power of the 20-quarter yield and the standard level factor are similar. The cross-sectional R^2 is around 75%. The negative betas combined with the negative market price of risk allow this one-factor model to capture 0.6% of the observed 0.8% spread between the excess return on the 10-year and the 1-year bonds.

Combined with the previous section, these results suggest that two term structure variables, a proxy for the bond risk premium and a level factor, can help explain both the cross-section of stock and bond returns. Section 2 will propose a model that can fit both. In addition to matching cross-sectional variation, it will also seek to match the *levels* of the risk premia on stocks and bonds. Furthermore, it will do so for a single market price of risk $\tilde{\Lambda}$, not a different one for stocks and bonds.

[Figure 7 about here.]

[Table 4 about here.]

1.4 Predictability

The previous analysis was concerned with understanding average variation across stock and bond returns. However, the covariance of CP_{t+1} or $yspr_{t+1}$ with excess returns r_{t+1}^e is the weighted average of two components: a covariance of r_{t+1}^e with innovations in the bond risk premium variable and a covariance of r_{t+1}^e with the lagged bond risk premium. In Section 2.5, we make that link explicit. The covariance of excess returns with the lagged CP factor (lagged yield spread) measures the predictability of excess stock returns by CP ($yspr$). From Cochrane and Piazzesi (2005), we know that lagged CP predicts not only future bond returns of various maturities, but that it also has some predictive ability for the aggregate stock market return. Likewise, much of the empirical literature has used the yield spread to forecast stock market returns. We add to these findings that the CP factor predicts future excess returns on value (BM10) minus growth (BM1) returns. To that end, we use monthly data and form cumulative, discounted holding period returns of horizons of 3 months up to 5 years.² The first 59 months are discarded so that all cumulative returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across holding periods.

²The discounting of future returns on each asset i is done by the constant $(\kappa_1^i)^{-1} = \exp(A_0^i)/(1 + \exp(A_0^i))$, where A_0^i is the mean log price-dividend ratio on portfolio i .

The top panel of Table 5 shows that the value-minus-growth portfolio return can be predicted by the lagged CP factor. When the CP factor increases, future (excess) value returns exceed future (excess) growth returns. This predictability is present at all horizons from 1 quarter to 5-year holding periods. The strongest effect, statistically, occurs around the 1-to-3-year holding period. In the bottom panel, we confirm that both CP has some predictive ability for future excess stock market returns on the value-weighted CRSP portfolio. The effect is significant only at horizons of 3-quarters to 2-years.

The right panel uses the yield spread as predictor instead of the CP factor. The yield spread also has positive forecasting power for value minus growth returns at horizons of 3-quarters to 2 years, but its effect is clearly weaker. The effect of the yield spread is stronger than that of CP when it comes to forecasting the market return. The stochastic discount factor model of Section 2 will match both the unconditional moments of excess returns and the predictability of the value-minus-growth strategy by the CP factor discussed here.

[Table 5 about here.]

[Figure 8 about here.]

2 A Unified Model of Stock and Bond Returns

The previous analysis suggests that the level, the cross-section, and the time-variation of expected stock returns are driven by the market factor and the CP factor, while the level, the cross-section, and the time-variation of expected bond returns are driven by the level factor and the CP factor. In this section, we develop a parsimonious stochastic-discount factor (SDF) model that captures all of these features. We show that this model is also able to match the time-series and cross-sectional properties of bond yields, thereby providing a unified pricing framework for stocks and bonds. For brevity, we focus on the 10 book-to-market sorted stock portfolios and study returns at the monthly frequency. Section 3 then develops a general equilibrium model that starts from preferences and from dividend growth rates on the stock portfolios. There, returns are generated inside the model, while here, they are taken as given.

2.1 Setup

We model the monthly dynamics of the demeaned state X as:

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}^X, \quad (1)$$

with Gaussian innovations $\varepsilon_{t+1}^X \sim \mathcal{N}(0, \Sigma_X)$. The state vector contains five variables: the CP factor, the level factor, the slope factor, the curvature factor, and aggregate stock market returns.

To construct the level, slope, and curvature factors, we regress Fama-Bliss forward rates on bonds of maturities 1-5 years (the same rates that were used in the construction of CP) on the CP factor. Level, slope, and curvature are the first three principal components of the residuals of this regression, and therefore orthogonal to CP . The first four factors have been suggested by Cochrane and Piazzesi (2006) to price the nominal term structure of interest rates. Even though we use five factors, only three of them will carry a non-zero price of risk in our preferred specification. The other factors are therefore relevant only to form expectations of the future state using (1).

The stochastic discount factor is modeled as:

$$M_{t+1} = \exp \left(-y_t^s(1) - \frac{1}{2} \Lambda_t' \Sigma_X \Lambda_t - \Lambda_t' \varepsilon_{t+1}^X \right), \quad (2)$$

with $y_t^s(1)$ the 1-month short rate and market prices of risk that are affine in the state $\Lambda_t = \Lambda_0 + \Lambda_1 X_t$. Geometric excess returns (including Jensen adjustment) on all assets are modeled as:

$$r_{t+1}^e = r_{t+1} - y_t^s(1) + \frac{1}{2} \text{var}_t[r_{t+1}] = \delta_0 + \delta_1 CP_t + \varepsilon_{t+1}^R, \quad (3)$$

with Gaussian innovations $\varepsilon_{t+1}^R \sim \mathcal{N}(0, \Sigma_R)$ and covariance matrix $\text{cov}(\varepsilon_{t+1}^R, \varepsilon_{t+1}^{X'}) = \Sigma_{XR}$.³ We focus on the ability of the CP to predict returns to replicate the predictability results in Section 1.4.⁴

The no-arbitrage condition for each asset is given by:

$$\log E_t[M_{t+1} R_{t+1}] = 0,$$

which implies:

$$E_t[r_{t+1}^e] = -\text{cov}_t(r_{t+1}^e, m_{t+1}) = \text{cov}(\varepsilon_{t+1}^R, \varepsilon_{t+1}^{X'}) \Lambda_t = \Sigma_{XR} (\Lambda_0 + \Lambda_1 X_t). \quad (4)$$

This shows that $\delta_0 = \Sigma_{XR} \Lambda_0$ and $\delta_1 = \Sigma_{XR} \Lambda_1$. Unconditional expected excess returns are computed by taking the unconditional expectation of (4):

$$E[r_{t+1}^e] = \Sigma_{XR} (\Lambda_0 + \Lambda_1 E[X_t]).$$

This section proceeds as follows. We first show that a parsimonious model is able to price the cross-section of *unconditional* expected stock and bond *returns*. Next, we demonstrate that the model is able to price the time series and cross-section of bond *yields*, and that it can match the predictability of the aggregate stock market and the return on a long growth-short value strategy.

³We will assume that the Jensen adjustment $\frac{1}{2} \text{var}_t(r_{t+1})$ is a constant. This would arise under homoscedasticity assumptions on return innovations. This adjustment is tiny in practise.

⁴It is straightforward to extend our results and to include additional predictors of either stock or bond returns.

The latter are *conditional* expected stock return moments. Finally, we connect the results from this section to those of the previous section.

Our test assets are the 10 portfolios sorted on their book-to-market ratio, the value-weighted market return from CRSP, and bond returns with maturities 1, 2, 5, 7, and 10 years from CRSP. Our estimation procedure estimates the dynamics of the state by OLS in a first step. It then finds the market price of risk coefficients Λ_0 that minimize the pricing errors on the test assets in a second step. All 16 moments are weighted equally. In a third step, we estimate the parameters in Λ_1 to fit the time series and cross-section of yields as well as the return predictability moments.

2.2 The Cross-section of Unconditional Expected Returns

In this section, we try to match the cross-section of *unconditional*, i.e. average, expected stock and bond returns. We switch off all time variation in risk prices ($\Lambda_1 = 0$) because, at this stage, we are only interested in unconditional pricing errors. We optimize over (a subset of) Λ_0 . To fix ideas, we start by pricing the 16 securities with a risk-neutral pricing kernel $M_{t+1} = \exp \{-y_t^s(1)\}$. That is, all prices of risk are zero: $\Lambda_0 = 0$. The first column of Table 6, labeled SDF1, thus recovers the risk premia we want to explain. The value spread (BM10-BM1) is 5.3% per year. Bond risk premia are not monotone in maturity and smaller than equity risk premia. The MAPE across all securities equals 6.1%.

Our first candidate SDF to explain the stock and bond risk premia is the one proposed by Cochrane and Piazzesi (2006). This is a natural candidate because their model does a great job pricing the nominal term structure of bond yields. Following Cochrane and Piazzesi (2006), we only allow the market price of level risk to be non-zero. This implies one non-zero element in Λ_0 . The second column of Table 6 shows that the best-fitting SDF, SDF2, is unable to jointly explain the cross-section of stock and bond returns. The MAPE is 3.96%. All pricing errors on the stock portfolios are large and positive, while all pricing errors on the bond portfolios are large and negative. The reason that this bond pricing model does not do better for the bond return portfolios is that the stock return risk premia command most attention because they are so much larger in magnitude. Consequently, the estimation concentrates its efforts on reducing the pricing errors of stocks. To illustrate that this bond kernel *is* able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors. The third column of Table 6 illustrates that the bond pricing errors are small in this case, but this pricing kernel SDF3 is not able to price the cross-section of expected stock returns nor the expected return on the aggregate stock market. The MAPE increases to 5.06%.

Our second candidate SDF to explain stock and bond risk premia is the canonical equity pricing model, the Capital Asset Pricing Model. The fourth column of Table 6 reports pricing errors for the CAPM. The only non-zero price of risk is the one corresponding to the aggregate stock market

return. This SDF3 model is again unable to jointly price stock and bond returns. The MAPE is 1.54%. One valuable feature is that pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the level of all expected stock returns right. However, their pattern clearly shows the value spread. Pricing errors on bond portfolios are sizeable as well and are all positive.

Having concluded that both the bond SDF and the stock SDF offer crucial ingredients to price bond returns and stock returns, but that neither is able to satisfactorily price both cross-sections, we now turn to our unified model. We allow the price of risks of the CP factor, the level factor, and the aggregate market return to be non-zero. For pricing purposes, this is a three-factor model. The three elements in Λ_0 are estimated to minimize the 16 pricing errors. As before, there is no time variation in the prices of risk: $\Lambda_1 = 0$. We call this model SDF4; the fifth column shows the results. The MAPE falls to a low 61bp. This is essentially the same pricing error we found in Section 1 for the 10 stock portfolios. Using one kernel with three price of risk parameters only, we are able to match the level and the pattern in stock and bond risk premia jointly.

The remaining issue is that there remains a strong predictable pattern in bond risk premia and that the bond risk premia are still relatively large. The level factor is the risk factor that is responsible for pricing these bond portfolios. The problem is that the covariance of the innovations to the level factor and the innovations to bond returns of various maturities goes through zero. That implies that the level factor can only “tilt” the pattern of risk premia, but cannot scale all risk premia simultaneously.

Based on this insight, we propose a *modified level* factor. This modified level factor is constructed as a weighted average of the Fama-Bliss yields of maturities one through five years, just like the standard level factor. However, the weights are additional parameters chosen to further minimize pricing errors under the restriction that they sum to one. The discipline comes from the fact that the resulting level factor is priced and needs to explain the cross-section of expected stock and bond returns. As discussed below, further discipline comes from pricing the cross-section of nominal bond yields.

With the modified level factor in hand, we construct a corresponding modified slope and curvature factors as follows. We project forward rates on the (original) CP factor and the modified level factor and collect the residuals. The modified slope and modified curvature factors are the first two principal components of these residuals. The modified level factor has a correlation of 80% with the standard level factor. The slope and the modified slope factor have a correlation of 64%, while the curvature and the modified curvature factor have a correlation of 67%.

Our final model, SDF5, contains the CP factor, the market factor, and the modified level, slope, and curvature factors as its state vector. As for SDF4, we only estimate 3 non-zero market prices of risk. This model’s pricing errors are reported in the sixth column of Table 6. The MAPE is

only 51 basis points. There hardly is a value spread in the pricing errors: The spread between the extreme portfolios is about 1% per year. Furthermore, the bond pricing errors are much lower than for SDF4, between 19 and 32 basis points. There are no more pronounced patterns in the pricing errors of either bonds or stocks. Finally, the price of CP risk and the price of market risk are estimated to be positive, while the price of (modified) level factor risk is negative. These are the signs predicted by theory.

[Table 6 about here.]

2.3 Implications for the Yield Curve

The previous section showed that a parsimonious SDF model was able to price unconditional excess returns on both stocks and bonds. An important question is how well the same SDF5 model does in pricing the term structure of nominal bond yields. After all, any SDF model has implications for both (bond) returns and (bond) yields. It is well known that three factors (level, slope, and curvature) are necessary to adequately describe bond yield dynamics, e.g., Dai and Singleton (2000, 2002). While the slope and curvature factor were unnecessary to price average returns, as shown above, we included them in the state vector X because they are necessary to adequately describe yields. They enter through the expectations formation in the state variable dynamics in equation (1). Matching yield dynamics requires only non-zero elements in Λ_1 , namely $\Lambda_{1(2,1)}$. This is the insight of (Cochrane and Piazzesi 2006), that allowing the market price of level risk to move with the CP factor suffices to match the dynamics of bond yields.

Our SDF model implies an essentially affine term structure of yields. We model the nominal short rate as

$$y_t^s(1) = \xi_0 + \xi_1' X_t.$$

Then, the SDF implies that the price of a nominal bond of maturity τ is exponentially affine in the state variables X :

$$P_t^s(\tau) = \exp \{ A^s(\tau) + B^{s'}(\tau) X_t \}.$$

By no-arbitrage, we have:

$$\begin{aligned} P_t^s(\tau) &= E_t [M_{t+1} P_{t+1}^s(\tau - 1)] \\ &= \exp \left\{ -\xi_0 - \xi_1' X_t + A^s(\tau - 1) + B^{s'}(\tau - 1) \mu_X + \right. \\ &\quad \left. B^{s'}(\tau - 1) \Gamma X_t - \Lambda_t' \Sigma B^s(\tau - 1) + \frac{1}{2} B^{s'}(\tau - 1) \Sigma_X B^s(\tau - 1) \right\}, \end{aligned}$$

which implies that $A^{\$}(\tau)$ and $B^{\$}(\tau)$ follow from the recursions:

$$\begin{aligned} A^{\$}(\tau) &= -\xi_0 + A^{\$}(\tau-1) + B^{\$'}(\tau-1)\mu_X - \Lambda'_0 \Sigma_X B^{\$}(\tau-1) + \frac{1}{2} B^{\$'}(\tau-1) \Sigma_X B^{\$}(\tau-1), \\ B^{\$'}(\tau) &= -\xi'_1 + B^{\$'}(\tau-1)\Gamma - B^{\$'}(\tau-1) \Sigma_X \Lambda_1, \end{aligned}$$

with initial conditions $A^{\$}(0) = 0$ and $B^{\$}(0) = 0_{5 \times 1}$.

We estimate the parameters in three steps. The first two steps are the same as in the unconditional analysis described above. The third step estimates ξ_0 , ξ_1 , and the first non-zero entry of Λ_1 , $\Lambda_{1(1,1)}$, to match nominal bond yields. In particular, we minimize the distance between the five annual yields from the Fama-Bliss data set (maturities 1 through 5 years) and the corresponding yields implied by the SDF:

$$\min_{\xi_0, \xi_1, \Lambda_{1(2,1)}} \left\{ \sum_{n=1}^5 \sum_{t=1}^T \left(y_t^{\$}(12 \times n) + \frac{A^{\$}(12 \times n)}{12 \times n} + \frac{1}{12 \times n} B^{\$'}(12 \times n) X_t \right)^2 \right\},$$

where the factor “12” is introduced because we have a monthly model and we price annual yields. In optimizing over Λ_1 , we modify Λ_0 to $\bar{\Lambda}_0 \equiv \Lambda_0 - \Lambda_1 E[X_t]$ to ensure that the unconditional asset pricing results from the previous section are unaffected. In related work, Adrian and Moench (2008) match bond returns first and then study the implications for yields.

The model SDF5 does a very nice job matching yields. The annualized standard deviation of the pricing errors of yields equal 18bp, 10bp, 12bp, 21bp, and 25bp for 1-5 year yields. Figure 9 plots this model’s implications for the yields of maturities one through five years. The parameter $\Lambda_{1(2,1)}$ is estimated to be negative ($\hat{\Lambda}_{1(2,1)} = -2160.2$), consistent with the results in Cochrane and Piazzesi (2006). A negative sign means that an increase in CP leads to higher bond risk premia because the price of level risk is negative and a higher CP makes it more negative.

[Figure 9 about here.]

2.4 Return Predictability

We study conditional stock return moments in this section. Our aim is to match the return predictability facts documented in Section 1.4: the predictability of aggregate stock market returns and returns on the value-minus-growth strategy by the CP factor. Matching these two moments exactly requires two more non-zero elements in Λ_1 , $\Lambda_{1(1,1)}$, and $\Lambda_{1(5,1)}$, which capture the time variation in the market price of CP risk and market risk in the CP factor. These moments can be written as $(\sigma_{XR}^{BM10} - \sigma_{XR}^{BM1})\Lambda_1 = \delta_1^{BM10-BM1}$ and $\sigma_{XR}^{Mkt}\Lambda_1 = \delta_1^{Mkt}$, where δ_1^j denotes the predictive coefficient from equation (3). We find that the price of CP risk and aggregate stock market risk both need to be increasing in the CP factor ($\hat{\Lambda}_{1(1,1)} = 2739.4$ and $\hat{\Lambda}_{1(5,1)} = 38.0$), consistent with

the results in Section 1.4. Taken together with the results from the previous section, the SDF5 model is able to not only match unconditional excess returns in the cross-section of stocks and bonds, but also bond yields, and conditional stock return moments. It does so with three priced risk factors, each of whom varies over time only with the CP factor.

2.5 Connection with Cross-sectional Regressions

The last missing piece is to tie the results of this section back to the regression analysis of Section 1. There, we estimated average returns as the product of an unconditional CP-beta and a price of CP risk $\tilde{\lambda}$

$$E[r_{t+1}^e] = \beta \tilde{\lambda} = Cov[r_{t+1}^e, CP_{t+1}] Var[CP_{t+1}]^{-1} Var[CP_{t+1}] E[\Lambda_t],$$

where Λ_t refers to the market price of risk notation of this section. Because expected excess returns only depend on CP_t according to equation (3), we can write the “contemporaneous CP beta” from Section 1 as the weighted average of the lagged betas and the innovation betas with weights w_1 and w_2 :

$$\begin{aligned}\beta &= Cov[r_{t+1}^e, CP_{t+1}] Var[CP_{t+1}]^{-1} = w_1 \beta^{Lagged} + w_2 \beta^{Innov}, \\ w_1 &= Cov[X_t, CP_t] \Gamma'_{CP} Var[CP_{t+1}]^{-1}, \\ w_2 &= Var[\varepsilon_{t+1}^{CP}] Var[CP_{t+1}]^{-1},\end{aligned}$$

where Γ_{CP} selects the first row of the companion matrix Γ , the lagged CP beta is δ_1 , and the innovation beta is defined by the regression:

$$\varepsilon_{t+1}^R = \beta^{Innov} \varepsilon_{t+1}^{CP} + \varepsilon_{t+1}^{R\perp}.$$

The lagged beta arises from a predictability regression of excess returns on the lagged CP factor, as discussed in Section 2.4. Note that when excess returns are unpredictable $\delta_1 = \beta^{Lagged} = 0$, and the contemporaneous CP beta is completely driven by the innovation beta. This innovation beta β^{Innov} is what we focused on in Section in 2.2. Note also that if CP follows an AR(1) process (uncorrelated with the other factors in X), then the weights simplify: $w_1 = \rho_{CP} = \Gamma_{(1,1)}$ and $w_2 = 1 - \rho_{CP}^2$.

Next, we derive a link between the compensation for risk in this section and in the cross-sectional exercise ($\tilde{\lambda}$). Given the expression for β just derived and given that $E[r_{t+1}^e] = \beta \tilde{\lambda}$, it follows that

$$\tilde{\lambda} = (\Lambda_{1(1,\cdot)} Cov[X_t, CP_t] \Gamma'_{CP} + I)^{-1} Var[CP_{t+1}] (\Lambda_{0(1)} + \Lambda_{1(1,\cdot)} E[CP]).$$

If excess return are not predictable, we have $\Lambda_1 = 0$ and it immediately follows that:

$$\tilde{\lambda} = Var[CP_{t+1}] \Lambda_{0(1)}. \quad (5)$$

The price of CP risk from the cross-sectional exercise, which had the interpretation of a risk premium, equals the market price of risk of innovations in CP from the SDF model pre-multiplied by the variance of the CP factor.

3 Equilibrium Model

An important question that the last section leaves open is whether the SDF of section 2 can arise in an equilibrium asset pricing model. The latter SDF admits no arbitrage opportunities so that arbitrageurs cannot earn the value premium and perfectly hedge out the risk with a position in bonds. However, it does not shed light on the fundamental determinants of the yield curve and the value premium, and hence on their deeper connection. While it did a good job simultaneously describing the cross-section of stock and bond returns in a general yet parsimonious model, it leaves open the question of whether there is a rational explanation for the observed covariances between CP , a proxy for the proxy of bond risk premium, and excess stock and bond returns. Such a model starts from preferences and technology (cash-flow growth specifications). It imposes additional discipline: it requires that the CP betas of stock returns are linked to the CP betas of bond returns and to the CP betas of dividend growth rates. We start by documenting the properties of contemporaneous CP betas for the dividend growth rates on the various stock portfolios in Section 3.1, and show that they have a distinct pattern as well. Starting in Section 3.2, we present a long-run risk model along the lines of Bansal and Yaron (2004). Our choice of the long-run risk framework is one of analytical convenience. Because this model is well-known, we only discuss the relationships that matter for our purposes. A step-by-step model derivation is relegated to the appendix.

3.1 Cash-flow Betas in the Data

For each of the ten book-to-market portfolios, we collect data on cum- and ex-dividend returns. They allow us to construct a quarterly dividend growth series for each portfolio (see Appendix A for details). Figure plots the dividend growth betas obtained from regressing dividend growth on the contemporaneous CP factor. While the pattern is not perfectly monotone, it does seem to be the case that growth firm's dividend growth has a positive correlation with contemporaneous CP and value firms' dividend growth a strong negative correlation. We find a similar pattern when we use the yield spread as proxy for the bond risk premium. Dividend growth of value stocks

is contemporaneously more negatively correlated with the yield spread than dividend growth of growth stocks. The yield spread betas of dividend growth are plotted in the right panels.

[Figure 10 about here.]

This pattern of *contemporaneous* dividend growth betas deepens the puzzle. The high (low) exposure of value (growth) stock returns to the CP factor does not seem to be coming from high (low) exposure of dividend growth to the CP factor. In fact, the opposite seems to be true. When CP is high (for example, at the end of recession), current cash-flow growth on value stocks is low, while that of growth stocks is high. The model in the following sections will reconcile these facts by appealing to the behavior of *future* dividend growth.

3.2 Model Setup

Technology We mostly adopt the consumption growth specification of Bansal and Yaron (2004):

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad (6)$$

$$x_{t+1} = \rho_x x_t + \varphi_e \sigma_t e_{t+1}, \quad (7)$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + \nu_1 (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, \quad (8)$$

$$\Delta d_{t+1}^i = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_d^i \sigma_t u_{t+1}^i \quad (9)$$

where (η_t, e_t, w_t, u_t) are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component x_t and is heteroscedastic, with conditional variance σ_t^2 . The two state variables x_t and $\sigma_t^2 - \bar{\sigma}^2$ capture time-varying growth rates and time-varying economic uncertainty. Our first major deviation is that we assume that the correlation between e_{t+1} and w_{t+1} equals $\chi \neq 0$. This creates a conditional correlation between the state variables x_t and $\sigma_t^2 - \bar{\sigma}^2$ of χ . The second major deviation is that we assume that expected dividend growth rates for portfolio i are not only a function of x_t , but also of $\sigma_t^2 - \bar{\sigma}^2$. That is, $\phi_{d,\sigma}^i \neq 0$. We introduce two other changes which are quantitatively of second-order importance. We follow Bansal and Shaliastovich (2007) and allow a correlation of ι^i between temporary consumption and dividend growth shocks η_{t+1} and u_{t+1}^i . Finally, we assume that the conditional variance of economic uncertainty moves with σ_t^2 . All other shocks at all other leads and lags are uncorrelated.

Preferences The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter α governs risk aversion and a second parameter ρ governs the willingness to substitute consumption inter-temporally. In

particular, ρ is the inverse of the inter-temporal elasticity of substitution (EIS). The appendix shows that the log SDF m has the following innovations, conditional mean, and conditional variance:

$$\begin{aligned}
m_{t+1} - E_t[m_{t+1}] &= -\alpha\sigma_t\eta_{t+1} - \frac{\alpha-\rho}{1-\rho}A_1^c\varphi_e\sigma_te_{t+1} - \frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w\sigma_tw_{t+1}, \\
E_t[m_{t+1}] &= m_0 - \rho x_t + \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c(\sigma_t^2 - \bar{\sigma}^2) \\
V_t[m_{t+1}] &= \left[\alpha^2 + \left(\frac{\alpha-\rho}{1-\rho} \right)^2 \{ (A_1^c\varphi_e)^2 + (A_2^c\sigma_w)^2 + 2\chi A_1^c\varphi_e A_2^c\sigma_w \} \right] \sigma_t^2 \\
m_0 &= \frac{1-\alpha}{1-\rho} \log \beta - \frac{\alpha-\rho}{1-\rho} [\kappa_0^c + A_0^c(1-\kappa_1^c)] - \alpha\mu_c,
\end{aligned}$$

where the log wealth-consumption ratio is given by

$$wc_t = A_0^c + A_1^c x_t + A_2^c (\sigma_t^2 - \bar{\sigma}^2),$$

and the coefficients (A_0^c, A_1^c, A_2^c) solve a system of three equations in three unknowns spelled out in the appendix. The linearization constants κ_0^c and κ_1^c are non-linear functions of the unconditional mean wealth-consumption ratio A_0^c :

$$\kappa_1^c = \frac{e^{A_0^c}}{e^{A_0^c} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log(e^{A_0^c} - 1) + \frac{e^{A_0^c}}{e^{A_0^c} - 1} A_0^c. \quad (10)$$

3.3 Expected Excess Stock Returns

This model implies expected excess log returns (including the Jensen adjustment) given by:

$$E_t[r_{t+1}^{e,i}] = \left[\alpha l^i \varphi_d^i + \frac{\alpha-\rho}{1-\rho} \{ A_1^c A_1^i \varphi_e^2 + A_2^c A_2^i \sigma_w^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w \} \right] \sigma_t^2, \quad (11)$$

where the log price-dividend ratio for portfolio i is given by

$$pd_t^i = A_0^i + A_1^i x_t + A_2^i (\sigma_t^2 - \bar{\sigma}^2),$$

and the coefficients (A_0^i, A_1^i, A_2^i) solve a (different) system of three equations in three unknowns spelled out in the appendix. There are (different) linearization constants κ_0^i and κ_1^i , given by equation (20), except with A_0^c replaced by the unconditional mean price-dividend ratio A_0^i . There is one such system of equations for each portfolio i . When the inter-temporal elasticity of substitution exceeds unity (or $\rho < 1$), an increase in long-run growth increases the price-dividend ratio ($A_1^i > 0$) and likewise for a decrease in economic uncertainty ($A_2^i < 0$).

Equation (11) shows that the equity risk premium consists of a compensation for bearing

temporary shock risk (first term), long-run risk (second term) and uncertainty risk (third term). The fourth and fifth terms arise only when there is non-zero correlation χ between the two state variables. The market price of temporary cash-flow risk is $\alpha > 0$, the market price of long-run risk is $A_1^c \varphi_e > 0$, and the market price of uncertainty risk is $A_2^c \sigma_w < 0$. These signs make sense: when long-run risk is low (economic uncertainty is high), the marginal utility growth for the representative investor is high, and she is willing to pay a high price for a security that has high dividend growth in such states of the world. Finally, we note that the only source of variation in equity risk premia over time is economic uncertainty σ_t^2 .

3.4 The Cochrane-Piazzesi Factor

Just like for the equity risk premium, the only source of time variation in bond risk premia is economic uncertainty: σ_t^2 is a *perfect* predictor of future excess bond returns. Therefore, it is natural to associate the *CP* factor with σ_t^2 . In the appendix, we make this argument rigorous and provide the details. We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as the sum of an expected part and an unexpected part.⁵ This structure gives rise to a three-factor nominal term structure model; the factors are x_t , $\sigma_t^2 - \bar{\sigma}^2$, and demeaned expected inflation $\bar{\pi}_t - \mu_\pi$. Forward rates, like yields, are affine functions of these three state variables. Hence, any three forward rates identify the three state variables exactly at each time t and a linear combination of them identifies CP_t :

$$CP_t = c_0 + c_1 f_t^{\$}(4) + c_3 f_t^{\$}(12) + c_5 f_t^{\$}(20) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2),$$

for some constants (c_1, c_3, c_5) . In order to pin down the ratio of the standard deviation of CP_t relative to that of σ_t^2 , b_{CP} , we form annual expected excess bond returns on nominal bonds of horizons 2-, 3-, 4-, and 5-years in the model. They only depend on σ_t^2 . We set the *CP* factor equal to the equally-weighted average of these four returns. This is exactly how we constructed *CP* in the data (see Section 1.1). Hence b_{CP} is not a free parameter but rather a function of the structural parameters of the model.

3.5 CP Betas

The question we want to ask of the long-run risk model is whether it is able to generate the cross-sectional variation in the exposures of excess returns of book-to-market-sorted stock portfolios to the *CP* factor. We showed in Section 1 that there was a clearly increasing pattern from growth to

⁵Unexpected inflation depends on the shocks η , e , and an additional shock ξ which is uncorrelated with all other shocks at all leads and lags. Expected inflation depends on its own lag, on the state x , and its innovations also depend on η , e , and ξ .

value. Since this is an equilibrium model, stock returns are endogenous, and only cash-flow growth can be specified exogenously. This leads us to impose the additional discipline that the model must match also the cross-sectional variation in the exposures of dividend growth rates of book-to-market-sorted stock portfolios to the CP factor. In Section 3.1, we showed dividend growth betas that were decreasing from growth (mildly positive) to value (strongly negative). Third, the CP factor is a strong forecaster of future bond returns in the data. Therefore, we would also like the model to generate the observed amount of bond return predictability by the CP factor. That is, we add lagged excess bond return betas as moments. We now provide closed-form expressions for the three sets of betas; the math is again relegated to the appendix.

The unconditional, contemporaneous, CP beta for excess stock returns is equal to:

$$\beta = \nu_1 \beta^{Lagged} + \beta^{Innov} \quad (12)$$

$$\beta^{Lagged} = \frac{\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + \frac{\alpha-\rho}{1-\rho} A_2^c(\kappa_1^c - \nu_1) + .5\Gamma(0)}{b_{CP}}, \quad (13)$$

$$\beta^{Innov} = \left(\chi A_1^i \frac{\varphi_e}{\sigma_w} + A_2^i \right) \frac{1 - \nu_1^2}{b_{CP}}, \quad (14)$$

where the expression for $\Gamma(0)$ is given in the appendix. Likewise, the contemporaneous CP beta for dividend growth rates, denoted with a superscript Δd , can be shown to equal:

$$\beta^{\Delta d} = \nu_1 \beta^{\Delta d, Lagged} = \nu_1 \left(\chi \frac{\phi_{d,x}^i}{1 - \rho_x \nu_1} \frac{\varphi_e}{\sigma_w} \frac{1 - \nu_1^2}{b_{CP}} + \frac{\phi_{d,\sigma}^i}{b_{CP}} \right). \quad (15)$$

Finally, we find the lagged CP beta for excess returns on nominal bonds of maturity τ , denoted with a superscript b for bonds:

$$\beta^{b, Lagged}(\tau) = \frac{.5\Gamma^{\$}(0) - .5\Gamma^{\$}(\tau - 1)}{b_{CP}}, \quad (16)$$

where the expression for $\Gamma^{\$}(\tau)$, for $\tau \geq 0$, is also given in the appendix.

An informative quantity is the difference between the contemporaneous return beta and the dividend growth beta. It is the difference between the CP beta of *news* about expected (discounted) *future* dividend growth rates $\Delta d_{t+1}^{i,H}$ and the CP beta of *news* about *future* stock returns:

$$\beta_{CP}^r - \beta_{CP}^{\Delta d} = \beta \left[CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] - \beta \left[CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] \quad (17)$$

In the data, the left hand side is positive for all portfolios and strongly increasing from growth to value. This is the result of increasing excess return betas and decreasing dividend growth betas. The Campbell-Shiller decomposition implies that this pattern must be due to either increasing

CP betas of news about future cash flows (first term) or decreasing CP betas of news about future discount rates (second term). The appendix provides an expression for each in terms of the structural parameters of the model.

3.6 Calibration

Our goal is to understand the patterns in CP betas and returns we documented earlier. Therefore, we refrain from a wholesale re-calibration or estimation of the long-run risk model and make minimal deviations from the literature. Table 7 compares our calibration in the first column to the calibration in Bansal and Shaliastovich (2007) (Column 2) and that of the original Bansal and Yaron (2004) (Column 3). All parameters are quarterly since we solve the model at quarterly frequency and compare it to quarterly data. This avoids complications arising from time aggregation. Our parameters are identical to those in Bansal and Shaliastovich (2007), with the following exceptions. First, we introduce heteroscedasticity in $\sigma_t^2 - \bar{\sigma}^2$. That is, we replace $\sigma_w w_{t+1}$ by $\sigma_w \sigma_t w_{t+1}$ in equation (24), and multiply our value of σ_w by $\bar{\sigma}$ to keep the unconditional variance of economic uncertainty the same. This change is inconsequential. Second, we estimate ν_1 , the autocorrelation coefficient of economic uncertainty $\sigma_t^2 - \bar{\sigma}^2$. Third, we introduce and estimate the parameter χ , which governs correlation between shocks to x_t and σ_t^2 . Fourth, we allow for a new cash-flow effect: dividend growth on each portfolio has a loading $\phi_{d,\sigma}^i$ on $\sigma_t^2 - \bar{\sigma}^2$. These loadings, along with the loadings of dividend growth on x_t , $\phi_{d,x}^i$, are estimated. We use 10 “book-to-market” portfolios and 1 “market” portfolio. Prior to the estimation, we set the unconditional mean log real dividend growth rate, μ_d^i , equal to its observed value in the data for each of the 11 portfolios. We then estimate $2 + 2 \times 11 = 24$ parameters in order to minimize the distance between model and data along the following 38 dimensions: 11 contemporaneous log excess stock return CP betas, 11 contemporaneous log real dividend growth CP betas, 11 equity risk premia, and 5 lagged bond return CP betas for maturities 1-, 2-, 5-, 7-, and 10-years. The estimation is by non-linear least squares.

[Table 7 about here.]

3.7 Benchmark Model Comparison

As a benchmark, Figure 11 shows these 38 moments for the Bansal and Shaliastovich (2007) calibration, henceforth B-S calibration. This calibration sets $\chi = 0$ and $\phi_{d,\sigma}^i = 0$. It also features a high value for ν_1 , so that economic uncertainty is very persistent, in fact more persistent than long-run growth ($\nu_1 > \rho_x$). To give this model its best chance at matching the value premium, we estimate $\phi_{d,x}^i$ in order to match risk premia on each portfolio, following Bansal, Dittmar, and Lundblad (2005), Bansal, Kiku, and Yaron (2006), and Hansen, Heaton, and Li (2008). The

bottom right panel shows that the model is flexible enough to exactly match not only the equity risk premium on the market portfolio, but also the risk premium on each of the 10 book-to-market portfolios. The model accomplishes this by having value stocks carry more long-run risk: $\phi_{d,x}^i$ is higher for value than for growth. In other words, value stocks are risky because their dividend growth is high in states of the world where long-run consumption growth x_t is high. These are states of low marginal utility growth for the representative investor. Panel A of 8 shows that $\phi_{d,x}^i$ ranges from 3.3 for BM1 to 5.3 for BM10. It is 3.5 for the market portfolio. A higher $\phi_{d,x}^i$ translates in a higher A_1^i , which increases the equity risk premium through the second term in equation (11).

[Figure 11 about here.]

However, inspection of the two top panels of Figure 11 shows that this calibration fails to generate the observed relationship between the CP factor and excess stock returns and dividend growth rates. First, equation (15) shows that when both $\phi_{d,\sigma}^i = 0$ and $\chi = 0$, the contemporaneous dividend growth betas are zero for all portfolios. Because expected dividend growth does not depend on economic uncertainty σ_t^2 , it is uncorrelated with CP (which perfectly correlates with economic uncertainty). Second, the excess stock return betas do not show the increasing pattern we found in the data, and they are at least a factor of five too big. Equation (12) helps us understand why. The innovation betas are all negative because $\chi = 0$ and $A_2^i < 0$. They become increasingly negative from growth to value: from -8 to -16. The lagged excess return betas in (13) are the sum of $\phi_{d,\sigma}^i$, $-A_2^i(\kappa_1^i - \nu_1)$, which increases from 360 to 630 from growth to value, and $\frac{\alpha-\rho}{1-\rho}A_2^c(\kappa_1^c - \nu_1) + .5\Gamma(0) = -993 + 1,046 = 53$. Given $b_{CP} = 35$, when $\phi_{d,\sigma}^i = 0$, these lagged excess stock return betas range from 11 to 19. Combining the two results in contemporaneous betas that are too high without a pronounced pattern from growth to value. Finally, the first beta on the right hand side of equation (17) is zero when $\chi = 0$ and $\phi_{d,\sigma}^i = 0$. The second term is negative: high bond risk premia (or positive innovations in CP) coincide with negative innovations in expected future stock returns, which seems counter-intuitive. In addition, there is no clear pattern in this beta difference as opposed to the strongly increasing pattern in the data.

[Table 8 about here.]

3.8 Results for Our Calibration

Our estimation sets $\chi = .2048 > 0$ and $\phi_{d,\sigma}^i < 0$ in order to match the 38 moments. It also substantially lowers the value for ν_1 from .9880 to .9296 ($\nu_1 < \rho_x$). The lower persistence we estimate in economic uncertainty facilitates the interpretation of σ_t^2 (or CP_t) as a business cycle variable. Figure 12 shows that our calibration does a good job matching the patterns and magnitudes of the CP exposures of excess stock returns, dividend growth rates, while maintaining the good fit for

the value premium. What drives these results? First, a sufficiently negative $\phi_{d,\sigma}^i$ directly generates a negative dividend growth beta. Panel B of Table 8 shows that the estimation chooses negative loadings $\phi_{d,\sigma}^i$ for all portfolios. They range from -537 for growth (BM1) and decrease monotonically to -883 for value (BM10). According to this channel, stocks, and in particular value stocks, have lower expected dividend growth when economic uncertainty, and therefore CP , is high. Second, this makes them risky because they have low cash flow growth exactly when marginal utility growth is high. Hence the value premium. This $\phi_{d,\sigma}^i$ effect reinforces the $\phi_{d,x}^i$ effect. Panel B shows that value stocks continue to have higher loadings on long run growth x_t than growth stocks, just as in Panel A. The dividend growth properties of the market portfolio resemble those of the third growth decile portfolio, consistent with the finding in Jurek and Viceira (2006) that growth stocks make up about 70% of the U.S. market capitalization.

Because χ is estimated to be .20, x_t and σ_t^2 have a conditional correlation of 20%. Positive correlation between the two state variables implies that times with higher growth are also times with more economic uncertainty. Such an economy is less risky relative to the one with $\chi = 0$ because the shocks to uncertainty (which causes prices to fall) are hedged by a higher long-term growth rate. The interest rate increases (slightly) because the demand for bonds is lower because of reduced precautionary savings motives. Because the economy is less risky, the price dividend ratios on all assets are higher and the risk premia lower. This reduction in risk is why the $\phi_{d,x}^i$ values are somewhat higher in Panel B ($\chi = 0.2$) than in Panel A ($\chi = 0$). Third, the positive χ helps to increase the excess stock return innovation betas, while the negative $\phi_{d,\sigma}^i$ helps to lower the lagged excess return betas. Both help to match the observed magnitude of the contemporaneous excess stock return betas. A positive χ suggests that economic uncertainty is highest at the end of a recession, when growth is about to resume. Such a positive correlation arises endogenously in models of information production (Van Nieuwerburgh and Veldkamp 2006). It also increases the correlation between expected returns and expected dividend growth. (Binsbergen and Koijen 2007) estimate this correlation for the aggregate stock market and conclude it is positive. In both our model and the B-S calibration, the lagged excess bond return betas are somewhat too high. This is not surprising, in the model bond returns and CP are both driven by σ_t^2 , with no offsetting effect from cash-flow growth as in the case of stocks. Finally, our calibration predicts a strongly increasing pattern in the beta difference of equation (17), just as in the data. The first beta on the right hand side is positive and increasing from growth to value. A high bond risk premium (at the end of a recession or the beginning of a boom) implies good news about future dividend growth, and more so for value stocks than for growth stocks. The second term is also positive and increasing. A high bond risk premium (at the end of a recession) coincides with a high equity risk premium, and more so on value than on growth stocks. The cash-flow effect dominates the discount rate effect.

[Figure 12 about here.]

Matching these betas does not come at the expense of matching the moments of consumption, interest rates, or inflation, for the most part because we kept these parameters at their standard values. Table 9 plots unconditional means, standard deviations (both expressed in percent per year), and quarterly autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments in our model are both close to the ones in the B-S calibration and close to the data.

[Table 9 about here.]

Discussion The higher loading of value portfolios' returns on the bond risk factor (CP) are puzzling at first thought. Long-maturity bonds are more sensitive to the bond risk factor than short maturity bonds (one-year excess returns on 5-year bonds have a higher sensitivity to CP and yield spread than one-year excess returns on 2-year bonds). In the absence of priced cash-flow risk, the only difference between various stocks is the timing (duration) of their cash flows. Lettau and Wachter (2007b) think of growth stocks as long duration equity and value stocks as short duration equity. For the same reason that long-maturity bonds are more sensitive to the bond risk premium than short maturity bonds, a *growth risk premium* would arise. In order to explain the value premium, then, future cash flows on value stocks must be riskier than those on growth stocks to offset this discount rate effect. This is exactly what we find above, based on our estimation of the CP return and dividend growth exposures. The deeper question is why future dividend growth on value and growth stocks have this covariance pattern with the bond risk premium. One potential explanation for the higher cash-flow risk of value firms is that they have cash-flow growth that is more cyclical than that of growth firms: expected future dividend growth on value stocks is relatively low at the start of a recession, when CP is low, and relatively high at the end of a recession, when CP is high. In the next section we explore a second potential explanation which starts from differences in the maturity structure of firms' debt.

4 A Maturity Structure of Debt Explanation

In the last section we ask *why* value stocks have future cash flows whose innovations are more sensitive to the bond risk premium (CP factor) than growth stocks. We pursue an explanation rooted in heterogeneity in the maturity structure of corporate debt. We take as given two important stylized facts. First, value firms (large, mature, or regulated firms) have higher leverage ratios and have a higher share of long-term debt. In contrast, growth firms (smaller firms with valuable growth options) tend to have less debt and a higher fraction of short-term debt (Barclay and Smith 1995).

Second, over time, firms issue long-term debt when the yield spread is low, indicating predictably low excess bond returns (Baker, Greenwood, and Wurgler 2003). This connection makes the share of long-term corporate debt a good predictor of future bond returns. Their results tend to be stronger for large, old, dividend-paying firms, suggesting the strongest effect for value firms.

We show that a high bond risk premium (high CP factor) reduces the value of debt and increases the value of equity. This effect is stronger for value firms because of their higher fraction of long-term debt. Such an explanation is consistent with the high value-minus-growth returns and with higher present discounted dividends in periods when CP is high. The model is simplified in that it does not attempt answer the question why value firms are more long-term debt intensive (see Barclay and Smith (1995) for the competing hypotheses and the evidence for one of them). We simply take financing choices as given. Nor does it attempt to answer why the firms' assets evolve the way we assume they do. We simply take the evolution of total cash-flow growth as given without studying the underlying investment choices that give rise to them. Obreja (2006), Garlappi and Yan (2008), and Gomes and Schmid (2008) among others, provide models that shed light on the interaction between financing and investment choices. Our contribution is to focus on how time variation in bond risk premia, based on a rich specification of the SDF, affects the value of debt and equity. We do so in the context of the long-run risk model of the previous section. In related work, Bhamra, Kuhn, and Strebulaev (2008) solve for the credit risk spread in a long-run risk model. However, they do not make the connection between value and growth stocks on the one hand and the term structure of bond yields on the other hand.

5 Conclusion

We document that differential exposure to the Cochrane-Piazzesi (CP) factor, a strong predictor of future excess bond returns and hence a proxy for the bond risk premium, can help explain why value stocks have higher average returns than growth stocks. This connection between a bond market and the cross-section of stocks leads us to develop a unified pricing model for stocks and bonds. We propose a no-arbitrage stochastic discount factor model that combines the canonical three-factor affine term structure model (level, slope, and curvature) with the CP factor, and an aggregate stock market return factor familiar from the CAPM. Only three factors need to have non-zero prices of risk to account for average returns on the book-to-market decile portfolios, the market portfolio, and maturity-sorted government bond portfolios. The price of market risk captures the common level of equity risk premia, the price of level risk captures the cross-sectional spread between long-term and short-term bond returns, and the price of CP risk captures the cross-sectional spread between value and growth stock returns. This model also captures the dynamics of bond yields and expected excess stock returns well when the market prices of risk change with the CP factor.

We explore what sources of underlying economic risk the CP factor captures in the context of the long-run risk model. We show that this equilibrium asset pricing model is able to replicate the differential exposure of excess stock returns to the CP factor of value and growth stocks, as well as the predictability facts. The CP factor is high at the end of a recession, periods that are characterized by high economic uncertainty but also good long-term growth prospects. A value-minus-growth strategy has high realized returns in periods when CP is high because value stocks receive good news about future cash flow growth.

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A Constructing Quarterly Dividend Growth

To avoid seasonality in monthly (quarterly) data, we form the dividend-price ratio as the ratio of the equally-weighted average of the current dividend and the dividends over the past 11 months (3 quarters) and the current price. We define dividend growth correspondingly so that the adjusted dividend growth rate and change in the dividend price ratio gives back the original cum-dividend return. In symbols, cum-dividend returns equal $R_{t+1} = DG_{t+1} \frac{PD_{t+1}+1}{PD_t}$, where $DG_{t+1} = D_{t+1}/D_t$ and $PD_t = P_t/D_t$. Define the ex-dividend return as R_{t+1}^{ex} and construct the dividend yield as the difference between the cum- and ex-dividend return: $DY_{t+1} = R_{t+1} - R_{t+1}^{ex}$. This leads to the quarterly dividend $D_{t+1} = DY_{t+1} P_t^{ex}$. The ex-dividend price index is updated recursively: $P_{t+1}^{ex} = P_t^{ex} (1 + R_{t+1}^{ex})$. For quarterly data, the adjusted variables, denoted with a tilde, are constructed as follows. The adjusted capital gain is defined as $\tilde{R}_{t+1}^{ex} = R_{t+1}^{ex} - (D_{t-2}/4 + D_{t-1}/4 + D_t/4 - 3D_{t+1}/4) / P_t^{ex}$. The adjusted ex-dividend price index is updated recursively: $\tilde{P}_{t+1}^{ex} = \tilde{P}_t^{ex} (1 + \tilde{R}_{t+1}^{ex})$. The adjusted dividend yield is $\tilde{DY}_{t+1} = R_{t+1} - \tilde{R}_{t+1}^{ex}$. Finally, the adjusted quarterly dividend is $\tilde{D}_{t+1} = \tilde{DY}_{t+1} \tilde{P}_t^{ex}$, dividend growth rate is $\tilde{DG}_{t+1}$, and the adjusted price-dividend ratio is $\tilde{D}P_{t+1} = \tilde{D}Y_{t+1} \tilde{P}_t^{ex} / \tilde{P}_{t+1}^{ex}$. Of course, $\tilde{P}D_t = (\tilde{D}P_t)^{-1}$. One can verify that this construction leaves gross returns unchanged: $R_{t+1} = \tilde{D}G_{t+1} \frac{\tilde{P}D_{t+1}+1}{\tilde{P}D_t}$. The construction of monthly data is similar.

B Long-Run Risk Model in Detail

B.1 Setup

Preferences The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter α governs risk aversion and a second parameter ρ governs the willingness to substitute consumption inter-temporally. In particular, ρ is the inverse of the inter-temporal elasticity of substitution (EIS). Bansal and Yaron (2004) show that the stochastic discount factor can be written as a function of consumption growth and the return to the wealth portfolio:

$$M_{t+1} = \beta^{\frac{1-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha}{1-\rho}} (R_{t+1}^c)^{\frac{\rho-\alpha}{1-\rho}}. \quad (18)$$

The return on a claim to aggregate consumption, the *total wealth return*, can be written as

$$R_{t+1}^c = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t} \frac{WC_{t+1}}{WC_t - 1}.$$

We start by using the Campbell (1991) approximation of the log total wealth return $r_t^c = \log(R_t^c)$ around the long-run average log wealth-consumption ratio $A_0^c \equiv E[w_t - c_t]$.⁶

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + wc_{t+1} - \kappa_1^c wc_t, \quad (19)$$

where we define the log *wealth-consumption* ratio wc as

$$wc_t \equiv \log \left(\frac{W_t}{C_t} \right) = w_t - c_t.$$

⁶Throughout, variables with a subscript zero denote unconditional averages.

The linearization constants κ_0^c and κ_1^c are non-linear functions of the unconditional mean wealth-consumption ratio A_0^c :

$$\kappa_1^c = \frac{e^{A_0^c}}{e^{A_0^c} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log(e^{A_0^c} - 1) + \frac{e^{A_0^c}}{e^{A_0^c} - 1} A_0^c. \quad (20)$$

Using the definition of the total wealth return, one can then show that the log SDF becomes:

$$m_{t+1} = \frac{1-\alpha}{1-\rho} [\log \beta + \kappa_0^c] - \kappa_0^c - \alpha \Delta c_{t+1} - \frac{\alpha-\rho}{1-\rho} (wc_{t+1} - \kappa_1^c wc_t) \quad (21)$$

Technology We mostly adopt the consumption growth specification of Bansal and Yaron (2004):

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad (22)$$

$$x_{t+1} = \rho_x x_t + \varphi_e \sigma_t e_{t+1}, \quad (23)$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + \nu_1(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, \quad (24)$$

where (η_t, e_t, w_t) are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component x_t and is heteroscedastic, with conditional variance σ_t^2 . The two state variables x_t and $\sigma_t^2 - \bar{\sigma}^2$ capture time-varying growth rates and time-varying economic uncertainty.

The two changes we make relative to Bansal and Yaron (2004) are that (1) the innovation in equation (24) is not $\sigma_w w_{t+1}$ but $\sigma_w \sigma_t w_{t+1}$, and (2) we assume that x_t and to $\sigma_t^2 - \bar{\sigma}^2$ have a non-zero conditional correlation χ . We engineer this through a non-zero unconditional correlation χ between the contemporaneous shocks e_{t+1} and w_{t+1} :

$$\text{Corr}[e_{t+1}, w_{t+1}] = \text{Cov}[e_{t+1}, w_{t+1}] = E[e_{t+1} w_{t+1}] = \chi.$$

All other shocks have zero correlation with each other at all leads and lags. To find the correlation between x_t and to $\sigma_t^2 - \bar{\sigma}^2$, start by working out

$$\begin{aligned} E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] &= E_t[(\rho_x x_t + \varphi_e \sigma_t e_{t+1})(\nu_1(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1})], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \rho_x x_t \sigma_w \sigma_t E_t[w_{t+1}], \\ &\quad + \varphi_e \sigma_t \nu_1 (\sigma_t^2 - \bar{\sigma}^2) E_t[e_{t+1}] + \varphi_e \sigma_t^2 \sigma_w E_t[e_{t+1} w_{t+1}], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2. \end{aligned}$$

The conditional covariance between x_{t+1} and $\sigma_{t+1}^2 - \bar{\sigma}^2$ equals:

$$\begin{aligned} \text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] - E_t[x_{t+1}] E_t[(\sigma_{t+1}^2 - \bar{\sigma}^2)], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2 - \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2), \\ &= \varphi_e \sigma_w \chi \sigma_t^2. \end{aligned}$$

It follows that the conditional correlation between x_{t+1} and $\sigma_{t+1}^2 - \bar{\sigma}^2$ is also χ :

$$\begin{aligned} \text{Corr}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{\text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{\text{Std}_t[x_{t+1}] \text{Std}_t[\sigma_{t+1}^2 - \bar{\sigma}^2]}, \\ &= \frac{\chi \varphi_e \sigma_w \sigma_t^2}{\varphi_e \sigma_w \sigma_t^2} = \chi. \end{aligned}$$

The *unconditional* variance of x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ is:

$$\begin{aligned} V[x_{t+1}] &= E[x_{t+1}^2] = \frac{\varphi_e^2 \bar{\sigma}^2}{1 - \rho_x^2}, \\ V[\sigma_{t+1}^2 - \bar{\sigma}^2] &= E[(\sigma_{t+1}^2 - \bar{\sigma}^2)^2] = \frac{\sigma_w^2 \bar{\sigma}^2}{1 - \nu_1^2}. \end{aligned}$$

The *unconditional* covariance between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ equals:

$$\text{Cov}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{1 - \rho_x \nu_1}$$

The *unconditional* correlation between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ equals:

$$\begin{aligned} \text{Corr}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{\text{Cov}[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{\text{Std}[x_{t+1}] \text{Std}[\sigma_{t+1}^2 - \bar{\sigma}^2]}, \\ &= \frac{\frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{1 - \rho_x \nu_1}}{\frac{\varphi_e \bar{\sigma}}{\sqrt{1 - \rho_x^2}} \frac{\sigma_w \bar{\sigma}}{\sqrt{1 - \nu_1^2}}} = \chi \frac{\sqrt{1 - \rho_x^2} \sqrt{1 - \nu_1^2}}{1 - \rho_x \nu_1}. \end{aligned}$$

The unconditional correlation between these two state variables is zero at all other leads and lags.

Dividend Growth We specify dividend growth processes for various assets or portfolios i , making two important modifications to the specification in Bansal and Yaron (2004):

$$\Delta d_{t+1}^i = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_d^i \sigma_t u_{t+1}^i \quad (25)$$

In Bansal and Yaron (2004), the shock u_t is assumed orthogonal to (η, e, w) and $\phi_{d,\sigma}^i = 0$. Then, correlation between consumption and dividend growth comes through the two state variables x_t and σ_t . Instead, we allow for a correlation ι^i between η_{t+1} and u_{t+1}^i . All other correlations are kept at zero. This opens up an additional channel of correlation and is a modification also made by (Bansal and Shaliastovich 2007). It turns out to be immaterial for our purposes. Much more important is the non-zero loading of dividend growth on economic uncertainty $\phi_{d,\sigma}^i \neq 0$. Its role will become clear below.

Defining returns ex-dividend and using the Campbell (1991) linearization, the log return on a claim to the dividend of portfolio i can be written as:

$$r_{t+1}^i = \Delta d_{t+1}^i + p d_{t+1}^i + \kappa_0^i - \kappa_1^i p d_t^i,$$

with coefficients

$$\kappa_1^i = \frac{e^{A_0^i}}{e^{A_0^i} - 1} > 1, \quad \text{and} \quad \kappa_0^i = -\log(e^{A_0^i} - 1) + \frac{e^{A_0^i}}{e^{A_0^i} - 1} A_0^i,$$

which depend on the long-run log price-dividend ratio A_0^i .

Inflation We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as

$$\pi_{t+1} = \bar{\pi}_t + \varphi_{\pi,\eta} \sigma_t \eta_{t+1} + \varphi_{\pi,e} \varphi_e \sigma_t e_{t+1} + \sigma_\pi \xi_{t+1}, \quad (26)$$

where expected inflation $\bar{\pi}_t = E_t[\pi_{t+1}]$ is given by

$$\bar{\pi}_{t+1} = \mu_\pi + \rho_\pi (\bar{\pi}_t - \mu_\pi) + \phi_\pi x_t + \zeta_{\pi,\eta} \sigma_t \eta_{t+1} + \zeta_{\pi,e} \varphi_e \sigma_t e_{t+1} + \sigma_z \xi_{t+1}, \quad (27)$$

The innovation ξ_t is orthogonal to all other innovations $(\eta_t, e_t, w_t, u_t^i)$. Expected inflation mean-reverts, it carries long-run risk with loading ϕ_π , and its innovations are correlated with unexpected inflation, and with the temporary and persistent component of consumption growth.

The nominal SDF is given by:

$$m_{t+1}^S = m_{t+1} - \pi_{t+1}.$$

Expected inflation (demeaned), $\bar{\pi}_t - \mu_\pi$, is a third state variable that drives the nominal yield curve. We are interested in understanding its variance and covariance with the other two state variables x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$. First, we calculate

$$\begin{aligned} Cov[x_{t+1}, \bar{\pi}_{t+1} - \mu_\pi] &= \rho_x \rho_\pi Cov[x_t, \bar{\pi}_t - \mu_\pi] + \rho_x \phi_\pi E[x_t^2] + \zeta_{\pi,e} \varphi_e^2 \bar{\sigma}^2, \\ &= \frac{\frac{\rho_x \phi_\pi}{1 - \rho_x^2} + \zeta_{\pi,e}}{1 - \rho_x \rho_\pi} \varphi_e^2 \bar{\sigma}^2. \end{aligned}$$

The covariance with economic uncertainty is

$$\begin{aligned} Cov[\sigma_{t+1}^2 - \bar{\sigma}^2, \bar{\pi}_{t+1} - \mu_\pi] &= \nu_1 \rho_\pi Cov[\sigma_t^2 - \bar{\sigma}^2, \bar{\pi}_t - \mu_\pi] + \nu_1 \phi_\pi Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] + \chi \zeta_{\pi,e} \varphi_e \sigma_w \bar{\sigma}^2, \\ &= \chi \frac{\frac{\nu_1 \phi_\pi}{1 - \rho_x \nu_1} + \zeta_{\pi,e}}{1 - \nu_1 \rho_\pi} \varphi_e \sigma_w \bar{\sigma}^2. \end{aligned}$$

We can now calculate the unconditional variance of expected inflation

$$\begin{aligned} E[(\bar{\pi}_{t+1} - \mu_\pi)^2] &= \rho_\pi^2 E[(\bar{\pi}_t - \mu_\pi)^2] + \phi_\pi^2 E[x_{t+1}^2] + \zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \zeta_{\pi,e}^2 \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2 + 2\phi_\pi \rho_\pi E[x_t, \bar{\pi}_t - \mu_\pi], \\ &= \frac{\zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \left[\frac{\phi_\pi^2}{1 - \rho_x^2} + \zeta_{\pi,e}^2 + 2\phi_\pi \rho_\pi \frac{\frac{\rho_x \phi_\pi}{1 - \rho_x^2} + \zeta_{\pi,e}}{1 - \rho_x \rho_\pi} \right] \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2}{1 - \rho_\pi^2}. \end{aligned}$$

B.2 Equity pricing

Euler Equation The starting point of the analysis is the Euler equation $E_t[M_{t+1} R_{t+1}^i] = 1$, where R_{t+1}^i denotes a gross return between dates t and $t+1$ on some asset i and M_{t+1} is the SDF. In logs:

$$E_t[m_{t+1}] + E_t[r_{t+1}^i] + \frac{1}{2} Var_t[m_{t+1}] + \frac{1}{2} Var_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0. \quad (28)$$

The same equation holds for the real risk-free rate $y_t(1)$, so that

$$y_t(1) = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}]. \quad (29)$$

The expected excess return becomes:

$$E_t[r_{t+1}^{e,i}] = E_t[r_{t+1}^i - y_t(1)] + \frac{1}{2} Var_t[r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^{e,i}], \quad (30)$$

where $r_{t+1}^{e,i}$ denotes the excess return on asset i corrected for the Jensen term.

The Consumption Claim In what follows we focus on the return on a claim to aggregate consumption, denoted r^c , where

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + w c_{t+1} - \kappa_1^c w c_t,$$

and derive the five terms in equation (28) for this asset.

Taking logs on both sides of the non-linear SDF expression in equation (18) delivers an expression of the log SDF as a function of log consumption changes and the log total wealth return

$$m_{t+1} = \frac{1-\alpha}{1-\rho} \log \beta - \frac{1-\alpha}{1-\rho} \rho \Delta c_{t+1} + \left(\frac{1-\alpha}{1-\rho} - 1 \right) r_{t+1}^c. \quad (31)$$

We conjecture that the log wealth-consumption ratio is linear in the two states x_t and $\sigma_t^2 - \bar{\sigma}^2$,

$$wc_t = A_0^c + A_1^c x_t + A_2^c (\sigma_t^2 - \bar{\sigma}^2).$$

As BY, we assume joint conditional normality of consumption growth, x , and the variance of consumption growth. We verify this conjecture from the Euler equation (28).

Using the conjecture for the wealth-consumption ratio, we compute innovations in the total wealth return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1}^c - E_t[r_{t+1}^c] &= \sigma_t \eta_{t+1} + A_1^c \varphi_e \sigma_t e_{t+1} + A_2^c \sigma_w \sigma_t w_{t+1} \\ E_t[r_{t+1}^c] &= r_0 + (1 - (\kappa_1^c - \rho_x) A_1^c) x_t - A_2^c (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2) \\ V_t[r_{t+1}^c] &= [1 + (A_1^c \varphi_e)^2 + (A_2^c)^2 \sigma_w^2 + 2 A_1^c \varphi_e A_2^c \sigma_w \chi] \sigma_t^2 \\ r_0 &= \kappa_0^c + A_0^c (1 - \kappa_1^c) + \mu_c \end{aligned}$$

Substituting in the expression for the log total wealth return r^c into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= -\alpha \sigma_t \eta_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_1^c \varphi_e \sigma_t e_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \sigma_t w_{t+1}, \\ E_t[m_{t+1}] &= m_0 + \left(-\alpha + \frac{\alpha - \rho}{1 - \rho} A_1^c (\kappa_1^c - \rho_x) \right) x_t + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c (\sigma_t^2 - \bar{\sigma}^2) \\ V_t[m_{t+1}] &= \left[\alpha^2 + \left(\frac{\alpha - \rho}{1 - \rho} \right)^2 \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \\ m_0 &= \frac{1 - \alpha}{1 - \rho} \log \beta - \frac{\alpha - \rho}{1 - \rho} [\kappa_0^c + A_0^c (1 - \kappa_1^c)] - \alpha \mu_c \end{aligned} \quad (32)$$

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$\begin{aligned} Cov_t[r_{t+1}^c, m_{t+1}] &= E_t[r_{t+1}^c - E_t[r_{t+1}^c], m_{t+1} - E_t[m_{t+1}]] \\ &= \left[-\alpha - \frac{\alpha - \rho}{1 - \rho} \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \end{aligned}$$

Using the method of undetermined coefficients and the five components of equation (28), we can solve for the

constants A_0^c , A_1^c , and A_2^c :

$$A_1^c = \frac{1 - \rho}{\kappa_1^c - \rho_x}, \quad (33)$$

$$A_2^c = .5 \frac{(1 - \rho)(1 - \alpha)}{\kappa_1^c - \nu_1} \left[1 + \frac{(A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w}{(1 - \rho)^2} \right], \quad (34)$$

$$\begin{aligned} 0 &= \frac{1 - \alpha}{1 - \rho} [\log \beta + \kappa_0^c + (1 - \kappa_1^c) A_0^c] + (1 - \alpha) \mu_c + \\ &\quad \frac{1}{2} (1 - \alpha)^2 \left[1 + \frac{(A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w}{(1 - \rho)^2} \right] \bar{\sigma}^2 \end{aligned} \quad (35)$$

The first equation is the same as in the case of no correlation between the state variables. The second and third equations are implicit functions of A_2^c and A_0^c . Because κ_0^c and κ_1^c are non-linear functions of A_0^c , this system of three equations needs to be solved simultaneously and numerically. Our computations indicate that the system has a unique solution. This verifies the conjecture that the log wealth-consumption ratio is linear in the two state variables.

The Consumption Risk Premium According to (30), the risk premium (expected excess return corrected for a Jensen term) on the consumption claim is given by

$$\begin{aligned} E_t [r_{t+1}^{c,e}] &= -Cov_t [r_{t+1}^c, m_{t+1}] \\ &= \left[\alpha + \frac{\alpha - \rho}{1 - \rho} \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \\ &= (\lambda_{m,\eta} + \lambda_{m,e} A_1^c \varphi_e + \lambda_{m,w} A_2^c \sigma_w + 2\chi \lambda_{m,e} \lambda_{m,w}) \sigma_t^2, \end{aligned} \quad (36)$$

with the market price of risk vector $\Lambda = [\lambda_{m,\eta}, \lambda_{m,e}, \lambda_{m,w}]$ given by

$$\begin{aligned} \lambda_{m,\eta} &= \alpha, \\ \lambda_{m,e} &= \frac{\alpha - \rho}{1 - \rho} A_1^c \varphi_e = \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e, \\ \lambda_{m,w} &= \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w. \end{aligned}$$

Note that the market price of temporary consumption growth shocks η_{t+1} is positive and equal to the risk aversion coefficient α , the market price of long-run consumption growth shocks e_{t+1} is positive, and the market price of economic uncertainty (innovations w_{t+1} in $\sigma^2 - \bar{\sigma}^2$) is *negative* because A_2^c will turn out to be negative.

Correlations expected consumption growth and expected total wealth return Expected total wealth returns and expected consumption growth rates are equal to:

$$E_t [r_{t+1}^c] = r_0^c + \rho x_t - A_2^c (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2), \quad (37)$$

$$E_t [\Delta c_{t+1}] = \mu_c + x_t. \quad (38)$$

The unconditional variance of these quantities is:

$$\begin{aligned} V [E_t [r_{t+1}^c]] &= \rho^2 V[x_t] + [A_2^c (\kappa_1^c - \nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] - 2\rho A_2^c (\kappa_1^c - \nu_1) Cov[x_t, \sigma_t^2 - \bar{\sigma}^2], \\ V [E_t [\Delta c_{t+1}]] &= V[x_t]. \end{aligned}$$

The unconditional covariance between expected returns and expected consumption growth rates is

$$Cov(E_t[r_{t+1}^c], E_t[\Delta c_{t+1}]) = \rho V[x] - A_2^c(\kappa_1^c - \nu_1)Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]$$

This then leads to the unconditional correlation, which is straightforward to compute.

The Dividend Claims We conjecture, as we did for the wealth-consumption ratio, that the log price dividend ratio on a stock i , where the notation i is suppressed, is linear in the two state variables:

$$pd_t^i = A_0^i + A_1^i x_t + A_2^i(\sigma_t^2 - \bar{\sigma}^2).$$

As we did for the return on the consumption claim, we compute innovations in the dividend claim return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1}^i - E_t[r_{t+1}^i] &= \varphi_d^i \sigma_t w_{t+1}^i + A_1^i \varphi_e \sigma_t e_{t+1} + A_2^i \sigma_w \sigma_t w_{t+1} \\ E_t[r_{t+1}^i] &= r_0^i + [\phi_{d,x}^i - A_1^i(\kappa_1^i - \rho_x)]x_t + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)](\sigma_t^2 - \bar{\sigma}^2) \\ V_t[r_{t+1}^i] &= [(\varphi_d^i)^2 + (A_1^i \varphi_e)^2 + (A_2^i \sigma_w)^2 + 2\chi A_1^i \varphi_e A_2^i \sigma_w] \sigma_t^2, \\ r_0^i &= \kappa_0^i + A_0^i(1 - \kappa_1^i) + \mu_d^i \end{aligned}$$

Finally, the conditional covariance between the log SDF and the log dividend claim return is

$$Cov_t[m_{t+1}, r_{t+1}^i] = - \left[\iota^i \alpha \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \right] \sigma_t^2.$$

From the Euler equation for this return $E_t[m_{t+1}] + E_t[r_{t+1}^i] + \frac{1}{2}V_t[m_{t+1}] + \frac{1}{2}V_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0$ and the method of undetermined coefficients, we can use the same procedure as above, and solve for the constants A_0^i , A_1^i , and A_2^i :

$$\begin{aligned} A_1^i &= \frac{\phi_{d,x}^i - \rho}{\kappa_1^i - \rho_x}, \\ A_2^i &= \frac{\left[\frac{\alpha - \rho}{1 - \rho} A_2^c(\kappa_1^c - \nu_1) + \phi_{d,\sigma}^i + .5H^i \right]}{\kappa_1^i - \nu_1}, \\ 0 &= m_0 + \kappa_0^i + A_0^i(1 - \kappa_1^i) + \mu_d^i + .5H^i \bar{\sigma}^2 \end{aligned}$$

where

$$\begin{aligned} H^i &= \{ \alpha^2 + (\varphi_d^i)^2 - 2\iota^i \alpha \varphi_d^i \} + \{ (A_1^i \varphi_e)^2 + (A_2^i \sigma_w)^2 + 2\chi A_1^i \varphi_e A_2^i \sigma_w \} \\ &\quad + \left(\frac{\alpha - \rho}{1 - \rho} \right)^2 \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w \} \\ &\quad - 2 \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \end{aligned} \quad (39)$$

Again, this is a non-linear system in three equations and three unknowns, which we solve numerically.

The equity risk premium on dividend claim i is:

$$E_t[r_{t+1}^{e,i}] = \left[\iota^i \alpha \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \right] \sigma_t^2. \quad (40)$$

It only varies with σ_t^2 .

Correlations expected dividend growth and expected return Expected stock returns and expected dividend growth rates on any portfolio i are equal to:

$$E_t [r_{t+1}^i] = r_0^i + \rho x_t + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)](\sigma_t^2 - \bar{\sigma}^2), \quad (41)$$

$$E_t [\Delta d_{t+1}^i] = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i(\sigma_t^2 - \bar{\sigma}^2). \quad (42)$$

The unconditional variance of these quantities is:

$$\begin{aligned} V[E_t [r_{t+1}^i]] &= \rho^2 V[x_t] + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\rho[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]Cov[x_t, \sigma_t^2 - \bar{\sigma}^2], \\ V[E_t [\Delta d_{t+1}^i]] &= (\phi_{d,x}^i)^2 V[x_t] + (\phi_{d,\sigma}^i)^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\phi_{d,\sigma}^i \phi_{d,x}^i Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

The unconditional covariance between expected stock returns and expected dividend growth rates is

$$\begin{aligned} Cov(E_t [r_{t+1}^i], E_t [\Delta d_{t+1}^i]) &= \phi_{d,x}^i \rho V[x_t] + \phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)] V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + (\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]) Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] \end{aligned}$$

This then leads to the unconditional correlation, which is straightforward to compute.

The important thing to note is that this covariance is (1) asset-specific because it depends on both $\phi_{d,x}^i$ and $\phi_{d,\sigma}^i$, and (2) it can be positive or negative depending on $\phi_{d,x}^i$ and $\phi_{d,\sigma}^i$, holding fixed χ . In the standard Bansal-Yaron model, the covariance between the state variables is zero and $\phi_{d,\sigma}^i = 0$. In that case, the covariance between expected returns and expected dividend growth is essentially equal to χ . Hence, its sign depends on the sign of χ , and it is not stock-specific. The previous section showed a high positive correlation between expected returns and expected dividend growth for growth stocks and a negative correlation for value stocks. Our model is able to generate this.

The unconditional correlation between expected and unexpected dividend growth is zero for all portfolios by the law of iterated expectations:

$$Corr(E_t [\Delta d_{t+1}^i], \Delta d_{t+1}^i - E_t [\Delta d_{t+1}^i]) = Corr(\phi_{d,x}^i x_t + \phi_{d,\sigma}^i(\sigma_t^2 - \bar{\sigma}^2), \phi_{d,\sigma}^i \sigma_t u_{t+1}) = 0.$$

Likewise, the correlation between expected and unexpected returns is zero. This is the same as in the standard BY model.

B.3 Bond Pricing

The Risk-free Rate According to equation (29), the expression for the risk-free rate is given by

$$\begin{aligned} y_t(1) &= -A(1) - B(1)x_t - C(1)(\sigma_t^2 - \bar{\sigma}^2) \\ A(1) &= m_0 + .5\Gamma(0)\bar{\sigma}^2, \\ B(1) &= -\rho, \\ C(1) &= \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma(0), \end{aligned} \quad (43)$$

where the notation $\Gamma(0)$ is defined as:

$$\Gamma(0) \equiv \alpha^2 + \left(\frac{\alpha - \rho}{1 - \rho} \right)^2 \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w \} \quad (44)$$

The Real Yield Curve The price of a τ -period real zero-coupon bond satisfies:

$$P_t(\tau) = E_t \left[e^{m_{t+1} + \log P_{t+1}(\tau-1)} \right].$$

This defines a recursion with $P_t(0) = 1$. The corresponding bond yield is $y_t(\tau) = -\log(P_t(\tau))/\tau$. It is easy to show that the LRR model gives rise to a affine real term structure (Bansal and Shaliastovich 2007):

$$y_t(\tau) = -\frac{A(\tau)}{\tau} - \frac{B(\tau)}{\tau} x_t - \frac{C(\tau)}{\tau} (\sigma_t^2 - \bar{\sigma}^2),$$

The coefficients $A(\tau)$, $B(\tau)$, and $C(\tau)$ satisfy the following recursions:

$$A(\tau+1) = m_0 + A(\tau) + .5\Gamma(\tau)\bar{\sigma}^2, \quad (45)$$

$$B(\tau+1) = \rho_x B(\tau) - \rho, \quad (46)$$

$$C(\tau+1) = \nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c + .5\Gamma(\tau) \quad (47)$$

where

$$\begin{aligned} \Gamma(\tau) \equiv & \alpha^2 + \left[B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right]^2 \varphi_e^2 + \left[C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right]^2 \sigma_w^2 \\ & + 2\chi \left[B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_e \left[C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w, \end{aligned} \quad (48)$$

and where the recursion is initialized at $A(0) = 0$, $B(0) = 0$, and $C(0) = 0$.

Proof. Guess and verify

$$\begin{aligned} P_t(\tau+1) &= E_t[\exp\{m_{t+1} + \log(P_{t+1}(\tau))\}] \\ &= E_t[\exp\{m_{t+1} + A(\tau) + B(\tau)x_{t+1} + C(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)\}] \\ &= \exp\{m_0 + A(\tau) + (B(\tau)\rho_x - \rho)x_t + \left[\nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c \right] (\sigma_t^2 - \bar{\sigma}^2)\} \times \\ & \quad E_t \left[\exp \left\{ -\alpha \sigma_t \eta_{t+1} + \left[B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_e \sigma_t e_{t+1} + \left[C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w \sigma_t w_{t+1} \right\} \right] \end{aligned}$$

Note that the conditional expectation term is equal to $\exp\{.5\Gamma(\tau)\sigma_t^2\}$, where $\Gamma(\tau)$ is defined in (48). The rest follows from recognizing that

$$\log P_t(\tau+1) = A(\tau+1) + B(\tau+1)x_t + C(\tau+1)(\sigma_t^2 - \bar{\sigma}^2)$$

and matching up coefficients. □

Real Bond Returns Define the 1-period log return on a real bond of maturity τ as:

$$r_{t+1}^b(\tau) = \tau y_t(\tau) - (\tau - 1)y_{t+1}(\tau - 1).$$

The one-period excess return subtracts the real short rate:

$$r_{t+1}^b(\tau) - y_t(1) = \tau y_t(\tau) - (\tau - 1)y_{t+1}(\tau - 1) - y_t(1).$$

In general, the excess log return on buying an τ -period real bond at time t and selling it at time $t + m$ as an $\tau - m$ period bond is:

$$\tau y_t(\tau) - (\tau - m)y_{t+m}(\tau - m) - m y_t(1).$$

The same expression holds for nominal excess bond returns, which we denote with a \$ superscript.

Innovations in one-period real bond returns are given by

$$\begin{aligned} r_{t+1}^b(\tau) - E_t[r_{t+1}^b(\tau)] &= (\tau - 1)(-y_{t+1}(\tau - 1) + E_t[y_{t+1}(\tau - 1)]), \\ &= B(\tau - 1)\varphi_e\sigma_t e_{t+1} + C(\tau - 1)\sigma_w\sigma_t w_{t+1}. \end{aligned}$$

The conditional variance of the bond return therefore equals:

$$V_t[r_{t+1}^b(\tau)] = \left\{ [B(\tau - 1)\varphi_e]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_e C(\tau - 1)\sigma_w \right\} \sigma_t^2.$$

The real bond risk premium (the expected excess return with Jensen adjustment) is:

$$\begin{aligned} E_t[r_{t+1}^{b,e}(\tau)] &= \tau y_t(\tau) - (\tau - 1)E_t[y_{t+1}(\tau - 1)] - y_t(1) + .5V_t[r_{t+1}^b(\tau)], \\ &= [.5\Gamma(0) - .5\Gamma(\tau - 1) + .5\left\{ [B(\tau - 1)\varphi_e]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_e C(\tau - 1)\sigma_w \right\}] \sigma_t^2 \\ &= \left\{ B(\tau - 1)\frac{\alpha - \rho}{\kappa_1^c - \rho_x}\varphi_e^2 + C(\tau - 1)\frac{\alpha - \rho}{1 - \rho}A_2^c\sigma_w^2 \right. \\ &\quad \left. + \chi B(\tau - 1)\varphi_e\frac{\alpha - \rho}{1 - \rho}A_2^c\sigma_w + \chi C(\tau - 1)\varphi_e\frac{\alpha - \rho}{\kappa_1^c - \rho_x}\sigma_w \right\} \sigma_t^2 \end{aligned}$$

We have verified that this risk premium equals $-Cov_t[m_{t+1}, r_{t+1}^b(\tau) - E_t[r_{t+1}^b(\tau)]]$.

The Nominal Yield Curve The price of a τ -period nominal zero-coupon bond satisfies:

$$P_t^\$(\tau) = E_t\left[e^{m_{t+1}^\$ + \log P_{t+1}^\$(\tau-1)}\right].$$

This defines a recursion with $P_t^\$(0) = 1$. The corresponding bond yield is $y_t^\$(\tau) = -\log(P_t^\$(\tau))/\tau$. Nominal bond yields are also affine in the state vector (Bansal and Shaliastovich 2007):

$$y_t^\$(\tau) = -\frac{A^\$(\tau)}{\tau} - \frac{B^\$(\tau)}{\tau}x_t - \frac{C^\$(\tau)}{\tau}(\sigma_t^2 - \bar{\sigma}^2) - \frac{D^\$(\tau)}{\tau}(\bar{\pi}_t - \mu_\pi),$$

The coefficients $A(\tau)$, $B^\$(\tau)$, and $C^\$(\tau)$ satisfy the following recursions

$$A^\$(\tau + 1) = m_0 - \mu_\pi + A^\$(\tau) + .5\left[-\sigma_\pi + D^\$(\tau)\sigma_z\right]^2 + .5\Gamma^\$(\tau)\bar{\sigma}^2, \quad (49)$$

$$B^\$(\tau + 1) = B^\$(\tau)\rho_x - \rho + D^\$(\tau)\phi_\pi, \quad (50)$$

$$C^\$(\tau + 1) = C^\$(\tau)\nu_1 + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma^\$(\tau), \quad (51)$$

$$D^\$(\tau + 1) = D^\$(\tau)\rho_\pi - 1, \quad (52)$$

initialized at $A^{\$}(0) = 0$, $B^{\$}(0) = 0$, $C^{\$}(0) = 0$, and $D^{\$}(0) = 0$, and where

$$\begin{aligned}\Gamma^{\$}(\tau) \equiv & \left[-\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta} \right]^2 + \left[B^{\$}(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e} \right]^2 \varphi_e^2 \\ & + \left[C^{\$}(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right]^2 \sigma_w^2 + 2\chi \left[B^{\$}(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e} \right] \varphi_e \left[C^{\$}(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w.\end{aligned}$$

Proof. We conjecture that the $t+1$ -price of a τ -period bond is exponentially affine in the state and solve for the coefficients in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned}P_t^{\$}(\tau+1) &= E_t[\exp\{m_{t+1}^{\$} + \log(P_{t+1}^{\$}(\tau))\}] \\ &= E_t[\exp\{m_{t+1} - \pi_{t+1} + A^{\$}(\tau) + B^{\$}(\tau)x_{t+1} + C^{\$}(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2) + D^{\$}(\tau)(\bar{\pi}_{t+1} - \mu_{\pi})\}] \\ &= \exp\{m_0 - \mu_{\pi} + A^{\$}(\tau) + [B^{\$}(\tau)\rho_x - \rho + D^{\$}(\tau)\phi_{\pi}]x_t \\ &\quad + \left[C^{\$}(\tau)\nu_1 + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c \right](\sigma_t^2 - \bar{\sigma}^2) + [D^{\$}(\tau)\rho_{\pi} - 1](\bar{\pi}_t - \mu_{\pi})\} \times \\ &\quad E_t \left[\exp\{[-\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta}] \sigma_t \eta_{t+1}\} \times \right. \\ &\quad \left. \{ + \left[-\frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + B^{\$}(\tau) + D^{\$}(\tau)\zeta_{\pi,e} \right] \varphi_e \sigma_t e_{t+1} \} \times \right. \\ &\quad \left. \{ + \left[-\frac{\alpha - \rho}{1 - \rho} A_2^c + C^{\$}(\tau) \right] \sigma_w \sigma_t w_{t+1} + [-\sigma_{\pi} + D^{\$}(\tau)\sigma_z] \xi_{t+1} \} \right]\end{aligned}$$

We use the joint standard normality of the four mutually-orthogonal shocks. Taking logs and collecting terms, we obtain a linear equation for $\log(P_t^{\$}(\tau+1))$ where the coefficients $A^{\$}(\tau+1)$ through $D^{\$}(\tau+1)$ satisfy (49)- (52). $\square$

We have verified that the one-period nominal short rate satisfies:

$$y_t^{\$(1)} = -E_t[m_{t+1}^{\$}] - .5V_t[m_{t+1}^{\$}] = -E_t[m_{t+1} - \pi_{t+1}] - .5V_t[m_{t+1} - \pi_{t+1}]$$

for the coefficients $A^{\$(1)}$ through $D^{\$(1)}$ in (49)- (52).

Inflation Risk Premium The expected excess return on a one-period nominal bond over a one-period real bond measures the inflation risk premium at the short end of the term structure. It is given by $-cov_t[m_{t+1}, \pi_{t+1}]$, where

$$-cov_t[m_{t+1}, \pi_{t+1}] = \left(\alpha\varphi_{\pi,\eta} + \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_{\pi,e} \varphi_e^2 + \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \varphi_{\pi,e} \varphi_e \right) \sigma_t^2.$$

In principle, the inflation risk premium can be either negative or positive depending on the sign of χ . However, it takes the same sign for all periods. When $\varphi_{\pi,\eta} = 0$ and $\chi = 0$, which is the case in (Bansal and Shaliastovich 2007), the inflation risk premium equals the second term.

Nominal Bond Returns Define the 1-period log return on a nominal bond of maturity τ as:

$$\begin{aligned}r_{t+1}^{b,\$}(\tau) &= \tau y_t^{\$(\tau)} - (\tau - 1)y_{t+1}^{\$(\tau - 1)}, \\ &= -A^{\$(\tau)} - B^{\$(\tau)}x_t - C^{\$(\tau)}(\sigma_t^2 - \bar{\sigma}^2) - D^{\$(\tau)}(\bar{\pi}_t - \mu_{\pi}) \\ &\quad + A^{\$(\tau - 1)} + B^{\$(\tau - 1)}x_{t+1} + C^{\$(\tau - 1)}(\sigma_{t+1}^2 - \bar{\sigma}^2) + D^{\$(\tau - 1)}(\bar{\pi}_{t+1} - \mu_{\pi})\end{aligned}$$

Expected nominal bond returns are:

$$\begin{aligned} E_t \left[r_{t+1}^{b,\$}(\tau) \right] &= A^{\$}(\tau-1) - A^{\$}(\tau) + \left[B^{\$}(\tau-1)\rho_x + D^{\$}(\tau-1)\phi_{\pi} - B^{\$}(\tau) \right] x_t \\ &\quad + \left[C^{\$}(\tau-1)\nu_1 - C^{\$}(\tau) \right] (\sigma_t^2 - \bar{\sigma}^2) + \left[D^{\$}(\tau-1)\rho_{\pi} - D^{\$}(\tau) \right] (\bar{\pi}_t - \mu_{\pi}). \end{aligned}$$

Innovation in nominal bond returns are:

$$\begin{aligned} r_{t+1}^{b,\$}(\tau) - E_t \left[r_{t+1}^{b,\$}(\tau) \right] &= (\tau-1) \left(-y_{t+1}^{\$}(\tau-1) + E_t \left[y_{t+1}^{\$}(\tau-1) \right] \right), \\ &= D^{\$}(\tau-1)\zeta_{\pi,\eta}\sigma_t\eta_{t+1} + \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e\sigma_te_{t+1} \\ &\quad + C^{\$}(\tau-1)\sigma_w\sigma_tw_{t+1} + D^{\$}(\tau-1)\sigma_z\xi_{t+1}. \end{aligned}$$

Expected excess bond returns adjusted for a Jensen term, or nominal bond risk premia, are given by:

$$\begin{aligned} E_t \left[r_{t+1}^{b,e,\$}(\tau) \right] &\equiv -Cov_t \left[m_{t+1}^{\$}, r_{t+1}^{b,e,\$}(\tau) \right] \\ &= Cov_t \left[\pi_{t+1}, r_{t+1}^{b,e,\$}(\tau) \right] - Cov_t \left[m_{t+1}, r_{t+1}^{b,e,\$}(\tau) \right] \\ &= r_0^{b,e,\$}(\tau) + F^{\$}(\tau)(\sigma_t^2 - \bar{\sigma}^2) \end{aligned}$$

where

$$\begin{aligned} r_0^{b,e,\$}(\tau) &= D^{\$}(\tau-1)\sigma_z\sigma_{\pi} + F(\tau)\bar{\sigma}^2 \\ F^{\$}(\tau) &= (\varphi_{\pi,\eta} + \alpha) D^{\$}(\tau-1)\zeta_{\pi,\eta} + \\ &\quad \left(\varphi_{\pi,e} + \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right) \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e^2 + \frac{\alpha - \rho}{1 - \rho} A_2^c C^{\$}(\tau-1)\sigma_w^2 + \\ &\quad \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e + \chi \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e C^{\$}(\tau-1)\sigma_w \end{aligned}$$

We have used the fact that:

$$\begin{aligned} Cov_t \left[\pi_{t+1}, r_{t+1}^{\$}(\tau) \right] &= \left\{ \varphi_{\pi,\eta} D^{\$}(\tau-1)\zeta_{\pi,\eta} + \varphi_{\pi,e} \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e^2 \right. \\ &\quad \left. + \chi \varphi_{\pi,e} \varphi_e C^{\$}(\tau-1)\sigma_w \right\} \sigma_t^2 + D^{\$}(\tau-1)\sigma_z\sigma_{\pi} \\ Cov_t \left[m_{t+1}, r_{t+1}^{\$}(\tau) \right] &= \left\{ -\alpha D^{\$}(\tau-1)\zeta_{\pi,\eta} - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e^2 \right. \\ &\quad \left. - \frac{\alpha - \rho}{1 - \rho} A_2^c C^{\$}(\tau-1)\sigma_w^2 - \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e} \right] \varphi_e \right. \\ &\quad \left. - \chi \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e C^{\$}(\tau-1)\sigma_w \right\} \sigma_t^2. \end{aligned}$$

In sum, nominal expected excess bond returns (nominal bond risk premia) only vary over time because economic uncertainty ($\sigma_t^2 - \bar{\sigma}^2$) varies. The loading $F(\tau)$ measures their sensitivity to this factor; it depends on the maturity of the bond.

The Cochrane-Piazzesi Factor Define the nominal forward rate for a loan between period $t + \tau$ and $t + \tau + 1$ as usual from nominal bond prices:

$$f_t^\$(\tau) = \log P_t^\$(\tau - 1) - \log P_t^\$(\tau).$$

In the model, the forward rate is

$$\begin{aligned} f_t^\$(\tau) = & \left(A^\$(\tau - 1) - A^\$(\tau) \right) + \left(B^\$(\tau - 1) - B^\$(\tau) \right) x_t + \\ & \left(C^\$(\tau - 1) - C^\$(\tau) \right) (\sigma_t^2 - \bar{\sigma}^2) + \left(D^\$(\tau - 1) - D^\$(\tau) \right) (\bar{\pi}_t - \mu_\pi) \end{aligned}$$

Given that the nominal model has three state variables, we can invert the state variables from any three forward rates, say $f_t^\$(1)$, $f_t^\$(3)$, and $f_t^\$(5)$. Put differently, one linear combination of forward rates is perfectly correlated with $(\sigma_t^2 - \bar{\sigma}^2)$. This is the linear combination of interest, because this state variable is the only source of time variation in bond and stock risk premia. Hence, this linear combination is the best predictor of future bond returns. It is perfectly correlated with expected bond returns. We call in the Cochrane-Piazzesi factor because the weights (c_1, c_3, c_5) display a tent-shaped function as in Cochrane and Piazzesi (2005):

$$CP_t = c_1 f_t^\$(1) + c_3 f_t^\$(3) + c_5 f_t^\$(5) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2). \quad (53)$$

The main text explains how b_{CP} is pinned down.

B.4 Relationship between stocks and bonds

The Equity Risk Premium and the Campbell-Shiller Decomposition Expected discounted future equity returns and dividend growth rates are given by:

$$r_t^{i,H} \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^i)^{-j} r_{t+j}^i \right] = \frac{r_0^i}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (54)$$

$$\Delta d_t^{i,H} \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^i)^{-j} \Delta d_{t+j}^i \right] = \frac{\mu_d^i}{\kappa_1^i - 1} + \frac{\phi_{d,x}^i}{\kappa_1^i - \rho_x} x_t + \frac{\phi_{d,\sigma}^i}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (55)$$

From these expressions, it is easy to see that

$$pd_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

and to compute the elements of the variance-decomposition:

$$V[pd_t^i] = Cov[pd_t^i, \Delta d_t^{i,H}] + Cov[pd_t^i, -r_t^{i,H}] = V[\Delta d_t^{i,H}] + V[r_t^{i,H}] - 2Cov[\Delta d_t^{i,H}, r_t^{i,H}].$$

We can also derive an expression for the expected future real bond yield (the risk-free rate):

$$y_t^H(1) \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^i)^{-j} y_{t+j-1}(1) \right] = \frac{-A(1)}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{-C(1)}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (56)$$

$$(57)$$

This makes it clear that the first term in the expected future stock return expression is a risk-free rate effect. It drops out once we look at expected excess returns:

$$r_t^{i,e,H} = r_t^{i,H} - y_t^H(1) = \frac{r_0^i + A(1)}{\kappa_1^i - 1} + \frac{[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{\kappa_1^i - \nu_1}(\sigma_t^2 - \bar{\sigma}^2)$$

The covariance between expected future dividend growth and expected future returns:

$$\begin{aligned} Cov(r_t^{i,H}, \Delta d_t^{i,H}) &= \frac{\phi_{d,x}^i \rho}{(\kappa_1^i - \rho_x)^2} V[x_t] + \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

It is similar to that of expected one-period ahead dividend growth and expected returns in equation (54). Indeed, if $\rho_x = \nu_1$, then it equals the latter covariance divided by $(\kappa_1^i - \rho_x)^2 = (\kappa_1^i - \nu_1)^2$. Note that when $\chi = 0$ and $\phi_{d,\sigma}^i = 0$, only the first term survives in the covariance between expected returns and expected growth rates. Because $\rho > 0$ and $\phi_{d,x}^i > 0$, it is always positive. Furthermore, this is a pure risk-free rate effect: the correlation between the expected *excess* return and expected dividend growth rate is exactly zero when $\chi = 0$ and $\phi_{d,\sigma}^i = 0$. In our model, this is no longer necessarily the case because of the additional cash-flow effect coming through $\phi_{d,\sigma}^i \neq 0$:

$$\begin{aligned} Cov(r_t^{i,e,H}, \Delta d_t^{i,H}) &= \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

Finally, for the consumption claim, we get the following expected future return, excess return, and growth expressions:

$$r_t^{c,H} = \frac{r_0^c}{\kappa_1^c - 1} + \frac{\rho}{\kappa_1^c - \rho_x} x_t - A_2^c(\sigma_t^2 - \bar{\sigma}^2) \quad (58)$$

$$\Delta c_t^H = \frac{\mu_c}{\kappa_1^c - 1} + \frac{1}{\kappa_1^c - \rho_x} x_t \quad (59)$$

$$r_t^{c,e,H} = \frac{r_0^c - A(1)}{\kappa_1^c - 1} + \frac{\frac{\alpha-1}{1-\rho} A_2^c(\kappa_1^c - \nu - 1) + .5\Gamma(0)}{(\kappa_1^c - \nu - 1)}(\sigma_t^2 - \bar{\sigma}^2) \quad (60)$$

Correlation stock returns and dividend growth with lagged CP We recall the expressions for expected returns and expected dividend growth rates on stock portfolios in equation (41)-(42). It follows that the unconditional lagged CP-betas of expected returns, $\tilde{\beta}_{CP}^r \equiv \beta(r_{t+1}^i, CP_t)$, and expected excess returns $\tilde{\beta}_{CP}^{r,e} \equiv \beta(r_{t+1}^i - y_t(1), CP_t)$ are:

$$\tilde{\beta}_{CP}^r = \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)}{b_{CP}}, \quad (61)$$

$$\tilde{\beta}_{CP}^{r,e} = \frac{\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + \frac{\alpha-\rho}{1-\rho} A_2^c(\kappa_1^c - \nu_1) + .5\Gamma(0)}{b_{CP}}, \quad (62)$$

The unconditional lagged CP-beta for expected returns may or may not be positive. The unconditional lagged CP-beta for *realized* returns is the same as for expected returns: $\beta(r_{t+1}^i, CP_t) = \beta(E_t[r_{t+1}^i], CP_t)$. The reason is that unexpected returns have a zero unconditional correlation with CP_t by the law of iterated expectations.

The unconditional correlation between *realized* dividend growth rates Δd_{t+1} and CP_t is the same as the correlation with expected growth rates because unexpected dividend growth rates have a zero correlation with CP_t because $Cov(\varphi_d^2 \sigma_t u_{t+1}, \sigma_t^2 - \bar{\sigma}^2) = 0$, again because of the law of iterated expectations.

$$\tilde{\beta}_{CP}^{\Delta d} \equiv \beta(\Delta d_{t+1}^i, CP_t) = \beta(E_t[\Delta d_{t+1}^i], CP_t) = \frac{\phi_{d,x}^i Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP} V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{\phi_{d,\sigma}^i}{b_{CP}}. \quad (63)$$

Correlation stock returns and dividend growth with contemporaneous CP It is straightforward to derive the unconditional contemporaneous CP-betas of returns $\hat{\beta}_{CP}^r \equiv \beta(r_{t+1}^i, CP_{t+1})$ and dividend growth rates $\hat{\beta}_{CP}^{\Delta d} \equiv \beta(\Delta d_{t+1}^i, CP_{t+1})$ are:

$$\hat{\beta}_{CP}^r = \nu_1 \tilde{\beta}_{CP}^r + \left(\chi A_1^i \frac{\varphi_e}{\sigma_w} + A_2^i \right) \frac{(1 - \nu_1^2)}{b_{CP}}, \quad (64)$$

$$\hat{\beta}_{CP}^{r,e} = \nu_1 \tilde{\beta}_{CP}^{r,e} + \left(\chi A_1^i \frac{\varphi_e}{\sigma_w} + A_2^i \right) \frac{(1 - \nu_1^2)}{b_{CP}}, \quad (65)$$

$$\hat{\beta}_{CP}^{\Delta d} = \nu_1 \tilde{\beta}_{CP}^{\Delta d}. \quad (66)$$

Note that the contemporaneous CP-betas for dividend growth are proportional to the lagged CP-betas, with a constant of proportionality of ν_1 . The reason is that dividend growth innovations are orthogonal to CP innovations because u_{t+1} and w_{t+1} are uncorrelated. The contemporaneous CP-beta of returns is also multiplied by ν_1 , but there is an additional term $A_2^i(1 - \nu_1^2)$ added, which arises from the fact that pd innovations and CP innovations are both driven by the shock w_{t+1} and from the fact that innovations to w_{t+1} and e_{t+1} may be correlated ($\chi \neq 0$). The contemporaneous CP beta of excess returns is the same as the one of returns.

Correlation nominal bond returns with lagged CP For comparison, we also work out the betas of nominal bond returns with respect to lagged CP, $\tilde{\beta}_{CP}^{b,\$}(\tau) \equiv \beta(r_{t+1}^{b,\$}(\tau), CP_t)$, and of excess nominal bond returns $\tilde{\beta}_{CP}^{b,e}(\tau) \equiv \beta(r_{t+1}^{b,\$}(\tau) - y_t^\$(1), CP_t) :$

$$\begin{aligned} \tilde{\beta}_{CP}^b(\tau) &= \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{1}{b_{CP}} \frac{Cov[\bar{\pi}_t - \mu_\pi, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} \\ &\quad + \frac{-\frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c - .5\Gamma^\$(\tau-1)}{b_{CP}}, \end{aligned} \quad (67)$$

$$\tilde{\beta}_{CP}^{b,e}(\tau) = \frac{.5\Gamma^\$(0) - .5\Gamma^\$(\tau-1)}{b_{CP}}, \quad (68)$$

The first equation follows because $\beta(r_{t+1}^{b,\$}(\tau), CP_t) = \beta(E_t[r_{t+1}^{b,\$}(\tau)], CP_t)$. Note that the first term is common with the beta of stock returns; it is a yield curve effect. The second term is also a yield curve effect. Both disappear if we look at excess returns.

Correlation nominal bond returns with contemporaneous CP The contemporaneous CP beta of nominal bond returns, $\hat{\beta}_{CP}^{b,\$}(\tau) \equiv \beta(r_{t+1}^{b,\$}(\tau), CP_{t+1})$, and excess returns are given by :

$$\hat{\beta}_{CP}^b(\tau) = \nu_1 \tilde{\beta}_{CP}^b(\tau) + \left(\chi [B^\$(\tau-1) + D^\$(\tau-1)\zeta_{\pi,e}] \frac{\varphi_e}{\sigma_w} + C^\$(\tau-1) \right) \frac{(1 - \nu_1^2)}{b_{CP}}. \quad (69)$$

$$\hat{\beta}_{CP}^{b,e}(\tau) = \nu_1 \tilde{\beta}_{CP}^{b,e}(\tau) + \left(\chi [B^\$(\tau-1) + D^\$(\tau-1)\zeta_{\pi,e}] \frac{\varphi_e}{\sigma_w} + C^\$(\tau-1) \right) \frac{(1 - \nu_1^2)}{b_{CP}}. \quad (70)$$

Beta decomposition Recall the definition of a return

$$r_{t+1}^i = \kappa_0^i + \Delta d_{t+1}^i + p d_{t+1}^i - \kappa_1^i p d_t^i$$

and the definition of the pd ratio:

$$p d_t^i = A_0^i + A_1^i x_t + A_2^i (\sigma_t^2 - \bar{\sigma}^2)$$

Therefore, the contemporaneous return beta is:

$$\hat{\beta}_{CP}^r - \hat{\beta}_{CP}^{\Delta d} = -\frac{A_1^i(\kappa_1^i \nu_1 - 1)}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{A_2^i(\kappa_1^i \nu_1 - 1)}{b_{CP}}$$

If $\kappa_1^i \nu_1 - 1 > 0$, which is a statement about the relative persistence of x and $\sigma_t^2 - \bar{\sigma}^2$, and $\chi < 0$, then both terms on the RHS are positive because $A_1^i > 0$ and $A_2^i < 0$. Since value stocks have higher A_1^i and more negative A_2^i , this explains the increasing pattern in the LHS from growth to value.

Another way to understand this is by using

$$p d_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

Therefore, the contemporaneous return beta is:

$$\hat{\beta}_{CP}^r - \hat{\beta}_{CP}^{\Delta d} = \beta \left[CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] - \beta \left[CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] \quad (71)$$

The first beta on the RHS measures the covariance of innovations of CP_{t+1} with *news* about future dividend growth rates. It equals:

$$\beta \left[CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] = -\frac{\phi_{d,x}^i \nu_1}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{\phi_{d,\sigma}^i \nu_1}{b_{CP}} \quad (72)$$

$$+ \left\{ \chi \frac{\phi_{d,x}^i}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{\phi_{d,\sigma}^i}{(\kappa_1^i - \nu_1)} \right\} \frac{(1 - \nu_1^2)}{b_{CP}} \quad (73)$$

The second beta on the RHS measures the covariance of innovations of CP_{t+1} with *news* about future stock returns. It equals:

$$\beta \left[CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] = \frac{-\rho \nu_1}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)] \nu_1}{b_{CP}} \quad (74)$$

$$+ \left\{ \chi \frac{\rho}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{\kappa_1^i - \nu_1} \right\} \frac{(1 - \nu_1^2)}{b_{CP}} \quad (75)$$

It is easy to verify that the difference between these two betas delivers the first expression we derived for $\hat{\beta}_{CP}^r - \hat{\beta}_{CP}^{\Delta d}$.

B.5 The Term Structure of Equity

We derive the price of dividend strips (or zero-coupon equity), the price of a claim to unit of dividends of firm i at some future date t . Recall the log-linearized stock return expression for security or portfolio i :

$$r_{t+1}^i = \kappa_0^i + \Delta d_{t+1}^i + p d_{t+1}^i - \kappa_1^i p d_t^i. \quad (76)$$

Log price-dividend ratios on dividend strips of horizon τ , $\widetilde{p d}_t^i(\tau)$, are affine in the state vector:

$$\widetilde{p d}_t^i(\tau) = A^m(\tau) + B^m(\tau)x_t + C^m(\tau)(\sigma_t^2 - \bar{\sigma}^2),$$

where the coefficients $A^m(\tau)$, $B^m(\tau)$, and $C^m(\tau)$ follow recursions

$$\begin{aligned} A^m(\tau+1) &= A^m(\tau) + m_0 + \mu_d^i + \frac{1}{2}\Gamma^m(\tau)\bar{\sigma}^2, \\ B^m(\tau+1)' &= \rho_x B^m(\tau) + \phi_{d,x}^i - \rho, \\ C^m(\tau+1)' &= \nu_1 C^m(\tau) + \phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + \frac{1}{2}\Gamma^m(\tau), \\ \Gamma^m(\tau) &= \alpha^2 + \varphi_d^2 - 2\alpha\varphi_{d\iota} + \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right]^2 \varphi_e^2 + \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right]^2 \sigma_w^2 \\ &\quad + 2\chi \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right] \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right] \varphi_e \sigma_w \end{aligned}$$

initialized at $A^m(0) = 0$, $B^m(0) = 0$, and $C^m(0) = 0$.

Proof. We conjecture that the log $t+1$ -price of a τ -period strip, scaled by the dividend in period $t+1$, is affine in the state

$$\widetilde{p d}_{t+1}^i(\tau) = A^m(\tau) + B^m(\tau)'x_{t+1} + C^m(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)$$

and solve for the coefficients $A^m(\tau+1)$, $B^m(\tau+1)$, and $C^m(\tau+1)$ in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned} \exp \widetilde{p d}_t^i(\tau+1) &= E_t \left[\exp \{ m_{t+1} + \Delta d_{t+1}^i + \widetilde{p d}_{t+1}^i(\tau) \} \right] \\ &= \exp \{ m_0 + \mu_d^i + A^m(\tau) + (\phi_{d,x}^i - \rho + B^m(\tau)\rho_x)x_t + \left[\phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + C^m(\tau)\nu_1 \right] (\sigma_t^2 - \bar{\sigma}^2) \} \times \\ &\quad E_t \left[\exp \{ -\alpha\sigma_t\eta_{t+1} + \left[B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right] \varphi_e\sigma_t e_{t+1} + \left[C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right] \sigma_w\sigma_t w_{t+1} + \varphi_d\sigma_t u_{t+1} \} \right] \end{aligned}$$

We use the log-normality of the shocks to work out the expectation. Taking logs and collecting terms, we obtain the conjectured linear expression for $\widetilde{p d}_t^i(\tau+1)$ where the recursions are as given above. $\square$

C Quarterly Calibration LRR Model

The Bansal-Yaron model is calibrated and parameterized to monthly data. Since we want to use data on quarterly consumption and dividend growth, and a quarterly series for the wealth-consumption ratio, we recast the model at quarterly frequencies. We assume that the quarterly process for consumption growth, dividend growth, the low frequency component and the variance has the exact same structure than at the monthly frequency, with mean zero, standard deviation 1 innovations, but with different parameters. This appendix explains how the monthly

parameters map into quarterly parameters. We denote all variables, shocks, and all parameters of the quarterly system with a tilde superscript. Our parameter values are listed at the end of this section, together with details on the simulation approach.

Preference Parameters Obviously, the preference parameters do not depend on the horizon ($\tilde{\alpha} = \alpha$ and $\tilde{\rho} = \rho$), except for the time discount factor $\tilde{\beta} = \beta^3$. Also, the long-run average log wealth-consumption ratio at the quarterly frequency is lower than at the monthly frequency by approximately $\log(3)$, because log of quarterly consumption is the log of three times monthly consumption.

Cash-flow Parameters We accomplish this by matching the conditional and unconditional mean and variance of log consumption and dividend growth. Log quarterly consumption growth is the sum of log consumption growth of three consecutive months. We obtain $\Delta\tilde{c}_{t+1} \equiv \Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}$

$$\Delta\tilde{c}_{t+1} = 3\mu_c + (1 + \rho_x + \rho_x^2)x_t + \sigma_t\eta_{t+1} + \sigma_{t+1}\eta_{t+2} + \sigma_{t+2}\eta_{t+3} + (1 + \rho_x)\varphi_e\sigma_te_{t+1} + \varphi_e\sigma_{t+1}e_{t+2} \quad (77)$$

Log quarterly dividend growth looks similar:

$$\Delta\tilde{d}_{t+1} = 3\mu_d + \phi(1 + \rho_x + \rho_x^2)x_t + \varphi_d\sigma_tu_{t+1} + \varphi_d\sigma_{t+1}u_{t+2} + \varphi_d\sigma_{t+2}u_{t+3} + \phi(1 + \rho_x)\varphi_e\sigma_te_{t+1} + \phi\varphi_e\sigma_{t+1}e_{t+2} \quad (78)$$

First, we rescale the long-run component in the quarterly system, so that the coefficient on it in the consumption growth equation is still 1:

$$\tilde{x}_t = (1 + \rho_x + \rho_x^2)x_t.$$

Second, we equate the unconditional mean of consumption and dividend growth :

$$\tilde{\mu} = 3\mu, \quad \tilde{\mu}_d = 3\mu_d.$$

These imply that we also match the the conditional mean of consumption growth:

$$E_t[\Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}] = 3\mu + (1 + \rho_x + \rho_x^2)x_t = \tilde{\mu} + \tilde{x}_t = E_t[\Delta\tilde{c}_{t+1}]$$

Third, we also match the conditional mean of dividend growth by setting the quarterly leverage parameter $\tilde{\phi} = \phi$.

Fourth, we match the unconditional variance of quarterly consumption growth:

$$\begin{aligned} V[\Delta\tilde{c}_{t+1}] &= (1 + \rho_x + \rho_x^2)^2 V[x_t] + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ &= (1 + \rho_x + \rho_x^2)^2 \frac{\varphi_e^2 \sigma^2}{1 - \rho_x^2} + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ &= \frac{\tilde{\varphi}_e^2 \tilde{\sigma}^2}{1 - \tilde{\rho}_x^2} + \tilde{\sigma}^2 \end{aligned}$$

The first and second equalities use the law of iterated expectations to show that

$$V[\sigma_{t+j}\eta_{t+j+1}] \equiv E[E_{t+j}\{\sigma_{t+j}^2\eta_{t+j+1}^2\}] - (E[E_{t+j}\{\sigma_{t+j}\eta_{t+j+1}\}])^2 = E[\sigma_{t+j}^2] - 0 = \sigma^2$$

and the same argument applies to terms of type $V[\sigma_{t+j}e_{t+j+1}]$. Coefficient matching on the variance of consumption

expression delivers expressions for $\tilde{\sigma}^2$ and $\tilde{\varphi}_e$:

$$\begin{aligned}\tilde{\sigma}^2 &= \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ \tilde{\varphi}_e^2 &= \varphi_e^2 \frac{(1 - \tilde{\rho}_x^2)(1 + \rho_x + \rho_x^2)^2}{1 - \rho_x^2} \frac{\sigma^2}{\tilde{\sigma}^2} \\ &= \frac{(1 - \rho_x^6)(1 + \rho_x + \rho_x^2)^2}{1 - \rho_x^2} \frac{\varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2},\end{aligned}$$

where the third equality uses the first equality. Note that we imposed $\tilde{\rho}_x = \rho_x^3$, which follows from a desire to match the persistence of the long-run cash-flow component. Recursively substituting, we find that the three-month ahead x process has the following relationship to the current value:

$$x_{t+3} = \rho_x^3 x_t + \varphi_e \sigma_{t+2} e_{t+3} + \rho_x \varphi_e \sigma_{t+1} e_{t+2} + \rho_x^2 \varphi_e \sigma_t e_{t+1}$$

which compares to the quarterly equation

$$\tilde{x}_{t+1} = \tilde{\rho}_x \tilde{x}_t + \tilde{\varphi}_e \tilde{\sigma}_t \tilde{e}_{t+1}$$

The two processes now have the same auto-correlation and unconditional variance.

Fifth, we match the unconditional variance of dividend growth. Given the assumptions we have made sofar, this pins down φ_d :

$$\tilde{\varphi}_d^2 = \frac{3\varphi_d^2 + \phi^2(1 + \rho_x)^2 \varphi_e^2 + \phi^2 \varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2}$$

Sixth, we match the autocorrelation and the unconditional variance of economic uncertainty σ_t^2 . Iterating forward, we obtain an expression that relates variance in month t to the one in month $t + 3$:

$$\sigma_{t+3}^2 - \sigma^2 = \nu_1^3 (\sigma_t^2 - \sigma^2) + \sigma_w \nu_1^2 w_{t+1} + \sigma_w \nu_1 w_{t+2} + \sigma_w w_{t+3}$$

By setting $\tilde{\nu}_1 = \nu_1^3$ and $\tilde{\sigma}_w^2 = \sigma_w^2 (1 + \nu_1^2 + \nu_1^4)$, we match the autocorrelation and variance of the quarterly equation

$$\tilde{\sigma}_{t+1}^2 - \tilde{\sigma}^2 = \tilde{\nu}_1 (\tilde{\sigma}_t^2 - \tilde{\sigma}^2) + \tilde{\sigma}_w \tilde{w}_{t+1}$$

Table 1: Contemporaneous Betas in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures (from the first-stage estimation). The intercept is in percent per year. The bond risk factor is the Cochrane-Piazzesi CP factor in the left panel and the 20-1-quarter yield spread in the right panel. The intercept is expressed as a percent per year. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the portfolios, also expressed as a percent per year. We also report the cross-sectional R^2 from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies.

10 Value Portfolios (1952-2007)						
predictor:	CP factor			Yield Spread		
	M	Q	A	M	Q	A
slope	1.57	2.57	3.00	1.46	1.48	1.41
$[t - stat]$	[5.31]	[4.56]	[5.02]	[3.15]	[4.06]	[5.03]
intercept	6.49	5.81	9.17	2.13	3.55	6.71
$[t - stat]$	[14.85]	[9.15]	[24.00]	[1.07]	[2.89]	[15.11]
MAPE	0.65	0.75	0.75	0.91	0.80	0.71
RMSE	0.75	0.85	0.93	1.07	0.92	0.93
R^2	77.9	72.2	75.9	55.4	67.3	76.0
T	666	222	55	666	222	55

25 Size- and Value Portfolios (1952-2007)						
predictor:	CP factor			Yield Spread		
	M	Q	A	M	Q	A
slope	2.20	2.81	3.30	1.31	1.28	1.70
$[t - stat]$	[10.19]	[10.26]	[7.35]	[5.43]	[7.24]	[10.43]
intercept	5.86	5.67	9.62	1.96	3.44	5.94
$[t - stat]$	[14.85]	[13.22]	[31.84]	[1.42]	[3.92]	[14.85]
MAPE	0.80	0.84	1.09	1.30	1.10	0.89
RMSE	1.04	1.04	1.44	1.62	1.35	1.10
R^2	81.9	82.1	70.1	56.2	69.5	82.5
T	666	222	55	666	222	55

Table 2: Cross-Sectional Equity Return Analysis: Subsamples

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures from the first-stage estimation. The bond risk factor is the Cochrane-Piazzesi *CP* factor. In the left panel, the *CP* factor is not re-estimated (same as in the full sample), whereas in the right panel it is re-estimated on the sub-sample in question.

10 Book-to-market Portfolios (Monthly Frequency)						
	CP from full sample			CP estimated on subsample		
	<i>1963-2007</i>	<i>1952-1985</i>	<i>1986-2007</i>	<i>1963-2007</i>	<i>1952-1985</i>	<i>1986-2007</i>
slope	2.37 [6.91]	1.67 [3.31]	0.75 [2.43]	2.96 [7.42]	1.43 [3.01]	0.70 [1.98]
intercept	4.43 [8.59]	3.75 [2.78]	11.39 [11.38]	5.40 [14.41]	3.12 [2.04]	11.58 [8.86]
MAPE	0.60	1.24	0.70	0.56	1.34	0.74
RMSE	0.73	1.47	0.76	0.68	1.54	0.81
R^2	85.6	57.7	42.4	87.3	53.1	33.0
T	540	402	264	540	402	264

25 Size- and Value Portfolios (Monthly Frequency)						
	CP from full sample			CP estimated on subsample		
	<i>1963-2007</i>	<i>1952-1985</i>	<i>1986-2007</i>	<i>1963-2007</i>	<i>1952-1985</i>	<i>1986-2007</i>
slope	2.59 [13.15]	2.09 [5.41]	1.29 [5.47]	3.14 [10.87]	1.73 [4.65]	1.42 [6.94]
intercept	4.35 [11.48]	3.44 [3.01]	12.28 [18.20]	5.88 [17.17]	3.58 [2.77]	14.66 [17.19]
MAPE	0.78	1.46	1.16	0.92	1.58	1.27
RMSE	0.94	1.83	1.70	1.10	1.98	1.47
R^2	88.3	56.0	56.6	83.7	48.4	67.7
T	540	402	264	540	402	264

Table 3: Cross-Sectional Equity Return Analysis: Comparison with 3-and 4-Factor Model

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 25 size- and value portfolios on the factor exposures from the first-stage estimation. The factors are the 3 Fama-French factors (first panel), the 4 Carhart factors (second panel), the yield spread and the 3 Fama-French factors (third panel), and the yield spread and the 4 Fama-French factors (fourth panel)

25 Size- and Value Portfolios (1952-2007)									
	<i>CP</i>	<i>yspr</i>	<i>3-f</i>	<i>4-f</i>	<i>CP+3-f</i>	<i>yspr+3-f</i>	<i>CP+4-f</i>	<i>yspr+4-f</i>	
CP	2.20				1.56		1.32		
	[10.19]				[3.77]		[2.97]		
yspr		1.31				1.35		1.25	
		[5.43]				[7.42]		[6.68]	
Mkt			-11.97	-0.35	-6.30	-1.02	-1.01	2.81	
			[-2.77]	[-0.06]	[-1.69]	[-0.38]	[-0.19]	[0.78]	
SMB			1.76	1.45	1.75	1.99	1.59	1.85	
			[3.17]	[2.79]	[4.03]	[6.77]	[3.59]	[6.21]	
HML			4.57	5.12	4.41	4.67	4.73	4.88	
			[6.72]	[7.74]	[8.24]	[13.02]	[8.20]	[13.11]	
UMD				40.31			20.06	13.83	
				[2.32]			[1.24]	[1.32]	
intercept	5.86	1.96	19.25	8.29	13.72	8.42	8.76	4.84	
	[14.85]	[1.42]	[4.28]	[1.33]	[3.58]	[3.03]	[1.65]	[1.37]	
MAPE	0.80	1.30	0.91	0.86	0.69	0.52	0.69	0.47	
RMSE	1.04	1.62	1.20	1.07	0.92	0.62	0.88	0.58	
R^2	81.9	56.2	75.9	81.0	85.8	93.6	87.0	94.4	

Table 4: Contemporaneous Betas of Bond Portfolios in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 5 CRSP bond portfolio returns on the level factor betas from a first-stage estimation. The intercept is in percent per year. The level factor is the 5-year Fama-Bliss yield in the left panel and the first principal components of the 1- through 5-year Fama-Bliss yields in the right panel. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the 25 portfolios. Both are expressed in percent per year. We also report the cross-sectional R^2 from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies.

5 CRSP Bond Portfolio Returns						
predictor: 5-Year Bond Yield factor				Level Factor		
	M	Q	A	M	Q	A
slope	-1.06	-0.89	-0.83	-2.10		
$[t - stat]$	[-3.05]	[-3.04]	[-3.16]	[-2.94]		
intercept	1.14	0.78	0.58	0.91		
$[t - stat]$	[10.43]	[5.69]	[5.21]	[5.01]		
MAPE	0.08	0.08	0.08	0.09		
RMSE	0.11	0.12	0.11	0.12		
R^2	75.7	75.5	76.9	74.3		
T	666	222	55	666	222	55

Table 5: Predictability in the Data

This table reports slope and intercept estimates from a predictability regression of cumulative (discounted) BM10-BM1 excess returns (top panel) or the cumulative discounted aggregate excess stock market return (bottom panel) on the bond risk premium. The bond risk factor is the Cochrane-Piazzesi *CP* factor in the left panel and the 20-1-quarter yield spread in the right panel. The first column denotes the holding period over which returns are cumulated. The first 59 months are discarded so that all returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across rows. The full sample is monthly data from June 1952 until December 2007.

Value-Growth Portfolio Return							
predictor:	CP factor			Yield Spread			
	slope	t-stat	R^2	slope	t-stat	R^2	
1-qtr	1.86	2.48	1.00	-0.56	-0.43	0.03	
2-qtr	1.52	2.75	1.24	-0.31	-0.32	0.02	
3-qtr	1.63	3.56	2.05	0.75	0.93	0.14	
1-yr	1.81	4.54	3.30	1.74	2.48	1.01	
2-yr	1.59	5.81	5.29	1.68	3.48	1.96	
3-yr	1.00	4.73	3.58	-0.11	-0.29	0.01	
4-yr	0.75	4.18	2.81	-0.78	-2.49	1.01	
5-yr	0.45	2.86	1.33	-0.87	-3.22	1.69	

Aggregate Stock Market Return							
predictor:	CP factor			Yield Spread			
	slope	t-stat	R^2	slope	t-stat	R^2	
1-qtr	0.50	0.69	0.08	3.98	3.20	1.67	
2-qtr	0.70	1.34	0.30	3.56	4.02	2.60	
3-qtr	0.97	2.29	0.86	3.88	5.39	4.59	
1-yr	1.16	3.18	1.64	3.59	5.78	5.24	
2-yr	0.58	2.38	0.93	1.78	4.28	2.94	
3-yr	0.31	1.67	0.46	1.44	4.61	3.39	
4-yr	0.35	2.41	0.95	1.53	6.20	5.99	
5-yr	0.48	3.71	2.22	1.60	7.42	8.35	

Table 6: Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on 10 book-to-market sorted stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year. Each column corresponds to a different SDF model, as described in the text, apart from the second and third column. In that case, we only use the set of test assets to estimate the same stochastic discount factor. The main text contains further details. The last row reports the mean absolute pricing error across all 11 securities (MAPE). Panel B reports the estimates of the prices of risk. In all specifications, we set $\Lambda_1 = 0$. The data are monthly for 1952.6 until 2007.12.

Panel A: Pricing Errors						
	SDF 1	SDF 2	SDF 2	SDF 3	SDF 4	SDF 5
	(bonds only)					
BM1	5.8888	5.3241	5.8256	-3.4608	0.0729	0.0239
BM2	6.4994	3.3423	5.8862	-2.1722	-0.6638	-0.3818
BM3	7.3279	2.7917	6.4327	-1.2192	-0.9517	-0.7492
BM4	7.2821	3.7721	6.5913	-0.8931	0.1355	0.1568
BM5	8.1376	3.7695	7.2660	0.5395	0.5638	0.6731
BM6	8.4560	3.7300	7.5011	0.7664	0.9151	0.6941
BM7	8.3899	0.9870	6.8493	1.0857	-1.2690	-1.5589
BM8	10.0592	3.5705	8.7182	2.6343	0.9732	0.8755
BM9	10.3731	3.2154	8.8965	2.6478	0.3608	0.0556
BM10	11.1598	4.8792	9.8865	2.7126	0.7499	1.0304
Market	6.8726	3.2714	6.1601	-1.6128	-0.7461	-0.7520
1-yr	1.0830	-2.3229	0.3626	0.9190	-0.6594	-0.3207
2-yr	1.2438	-3.7555	0.1878	0.9974	-0.7408	-0.3147
5-yr	1.5251	-5.8999	-0.0417	1.1422	-0.2558	-0.2039
7-yr	1.7474	-6.0792	0.0952	1.2163	0.3209	0.2558
10-yr	1.3764	-6.6745	-0.3240	0.6856	0.3625	0.1921
MAPE	6.0889	3.9616	5.0641	1.5441	0.6088	0.5149
Panel B: Estimates of Prices of Risk (Λ_0)						
	SDF 1	SDF 2	SDF 2	SDF 3	SDF 4	SDF 5
	(bonds only)					
CP factor	0	0	0	0	58.53	83.72
(Modified) Level factor	0	-67.96	-14.30	0	-21.85	-53.59
(Modified) Slope factor	0	0	0	0	0	0
(Modified) Curvature factor	0	0	0	0	0	0
Aggregate stock market return	0	0	0	4.07	3.12	3.65

Table 7: Quarterly Calibration

<i>Parameter</i>	<i>Our model</i>	<i>BS 2008</i>	<i>BY 2004</i>
Preferences			
α	8	8	10
ρ	2/3	2/3	2/3
β	.9961	.9961	.9970
Consumption Process			
μ	.0048	.0048	.0045
$\bar{\sigma}$	.0056	.0056	.0135
ρ_x	.9732	.9732	.9383
φ_e	.1617	.1617	.1264
ν_1	EST	.9880	.9615
σ_w	.1984e-5/ $\bar{\sigma}$	.1984e-5	.3932e-5
χ	EST	NA	NA
Dividend Process			
μ_d^m	.0045	.0048	.0045
φ_d^m	5.9860	5.9860	4.4960
$\phi_{d,x}^m$	EST	1.5	3
ι^m	.10	.10	NA
$\phi_{d,\sigma}^m$	EST	NA	NA
Inflation Process			
μ_π	.0096	.0096	NA
$\varphi_{\pi,\eta}$	0	0	NA
$\varphi_{\pi,e}$	-2	-2	NA
σ_π	.0061	.0061	NA
ρ_π	.5718	.5718	NA
ϕ_π	-.35	-.35	NA
$\zeta_{\pi,\eta}$	0	0	NA
$\zeta_{\pi,e}$	-1	-1	NA
σ_z	6.9282e-6	6.9282e-6	NA

Notes: This Table lists our benchmark parameter choices. All parameters are expressed in quarterly units. See Appendix C on how we go from the monthly parameters of Bansal and Shaliastovich (2007) to the quarterly values in the second column and from the monthly values of Bansal and Yaron (2004) to the quarterly values in the third column.

Table 8: Cash-Flow Parameters and the Value Premium

In Panel A, we use the benchmark calibration of Bansal and Shaliastovich (2007). In panel B, we use our own calibration. The first column reports the unconditional mean dividend growth rate. The second column reports the loading of dividend growth on the long-run risk state variable x_t . The third column reports the loading of dividend growth on the economic uncertainty state variable σ_t^2 . The fourth column reports the equity risk premium, the expected excess log stock return (including a Jensen adjustment). The last column reports the mean price-dividend ratio.

Panel A: B-S Calibration					
	μ_d^i	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	3.52	0	6.82	16.56
BM1	0.60	3.30	0	6.01	15.13
BM2	1.65	3.45	0	6.65	16.53
BM3	2.08	3.70	0	7.24	16.54
BM4	1.41	3.82	0	7.14	15.07
BM5	3.16	3.85	0	7.97	18.21
BM6	2.69	4.09	0	8.14	16.48
BM7	2.49	4.31	0	8.39	15.57
BM8	3.17	5.17	0	10.01	14.83
BM9	3.86	5.05	0	10.20	16.16
BM10	4.89	5.33	0	11.21	17.39
Panel B: Own Calibration					
	μ_d^i	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	4.42	-575.17	6.81	17.17
BM1	0.60	4.17	-536.69	6.09	15.30
BM2	1.65	4.35	-558.70	6.66	17.05
BM3	2.08	4.71	-615.94	7.32	17.11
BM4	1.41	4.74	-614.86	7.13	15.64
BM5	3.16	4.92	-655.90	8.02	19.31
BM6	2.69	5.16	-687.54	8.18	17.43
BM7	2.49	5.39	-716.69	8.43	16.49
BM8	3.17	6.28	-854.99	9.97	16.30
BM9	3.86	6.22	-851.68	10.22	17.96
BM10	4.89	6.39	-883.53	11.00	20.48

Table 9: Other Moments of Interest

This table reports unconditional means (expressed in percent per year), standard deviations (expressed in percent per year), and the quarterly first-order autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments reported for stocks are for the market portfolio, and are denoted with a m superscript. The notation follows the main text.

	Panel A: B-S Calibration			Panel B: Our Calibration		
	mean	stdev	ac(1)	mean	stdev	ac(1)
x	0.00	0.78	0.97	0.00	0.78	0.97
σ^2	0.0123	0.0026	0.99	0.0123	0.0011	0.93
$\bar{\pi} - \mu_\pi$	3.84	0.84	0.84	3.84	0.84	0.84
Δc	1.92	1.36	0.32	1.92	1.36	0.32
Δd^m	1.79	7.20	0.14	1.79	7.50	0.21
π	3.84	1.52	0.29	3.84	1.52	0.29
$y(1)$	2.19	0.54	0.97	2.34	0.52	0.97
$y^s(1)$	6.18	0.52	0.63	6.33	0.51	0.62
$y^s(20)$	6.30	0.15	0.92	6.48	0.14	0.91
$y^s(20) - y^s(1)$	0.12	0.73	0.11	0.15	0.73	0.11
$E_t \left[r_{t+1}^{b,e,s}(20) \right]$	-0.94	0.20	0.99	-0.76	0.07	0.93
$E_t \left[r_{t+1}^{b,e,s}(20) \right]$	0.21	0.04	0.99	0.26	0.02	0.93
CP	0.45	0.09	0.99	0.57	0.04	0.93
WC	67.75	5.66	0.98	74.83	4.29	0.97
PD^m	16.56	32.77	0.98	16.89	34.77	0.97
$r_{t+1}^m - y_t(1)$	5.68	15.15	0.01	5.42	17.23	0.00
$E_t[r_{t+1}^m - y_t(1)]$	5.68	1.17	0.99	5.42	0.73	0.95
$E_t[\Delta d_{t+1}^m]$	1.79	2.75	0.97	1.79	3.46	0.97
$E_t[r_{t+1}^{e,m}]$	6.82	1.42	0.99	6.90	0.60	0.93

Figure 1: Nominal Bond Risk Premia

The Cochrane-Piazzesi factor is a linear combination of the one-year nominal yield and 2- through 5-year forward rates. The data run from June 1952 until December 2007.

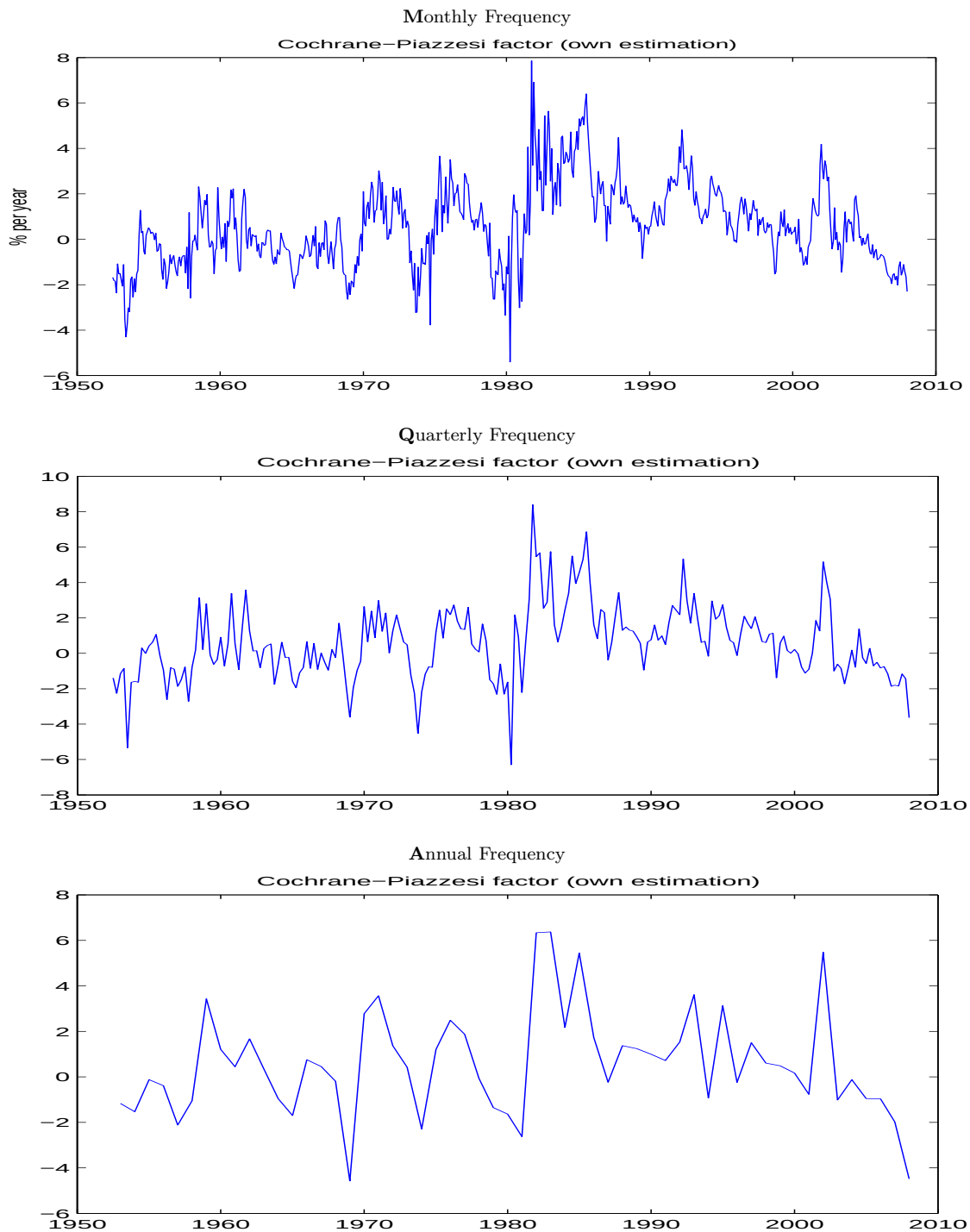


Figure 2: Exposure of 10 Book-to-Market Portfolio Excess Returns to the CP Factor in Data

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns $r_{t+1}^{i,e} \equiv r_{t+1}^i - y_t^S(1) + .5 * Var[r_{t+1}^i]$ to the contemporaneous CP_{t+1} factor. Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the CP factor. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.

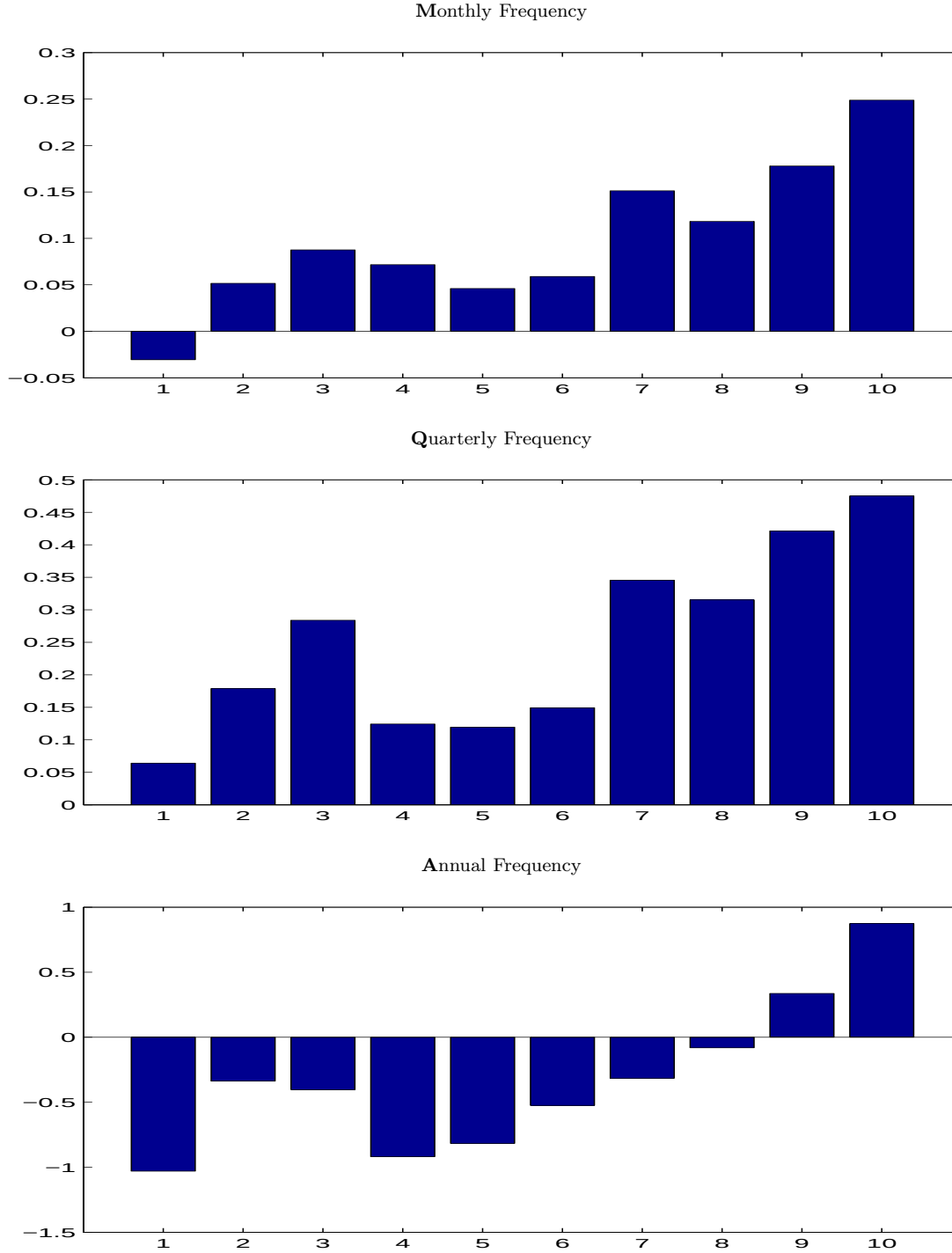


Figure 3: Exposure of 25 Size and Book-to-Market Portfolio Returns to the CP Factor in Data

The figure plots the factor exposures (betas) of the 25 size- and value portfolio returns to the CP factor. Each bar denotes the slope coefficient of a time-series regression of a portfolio return on the CP factor. The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.

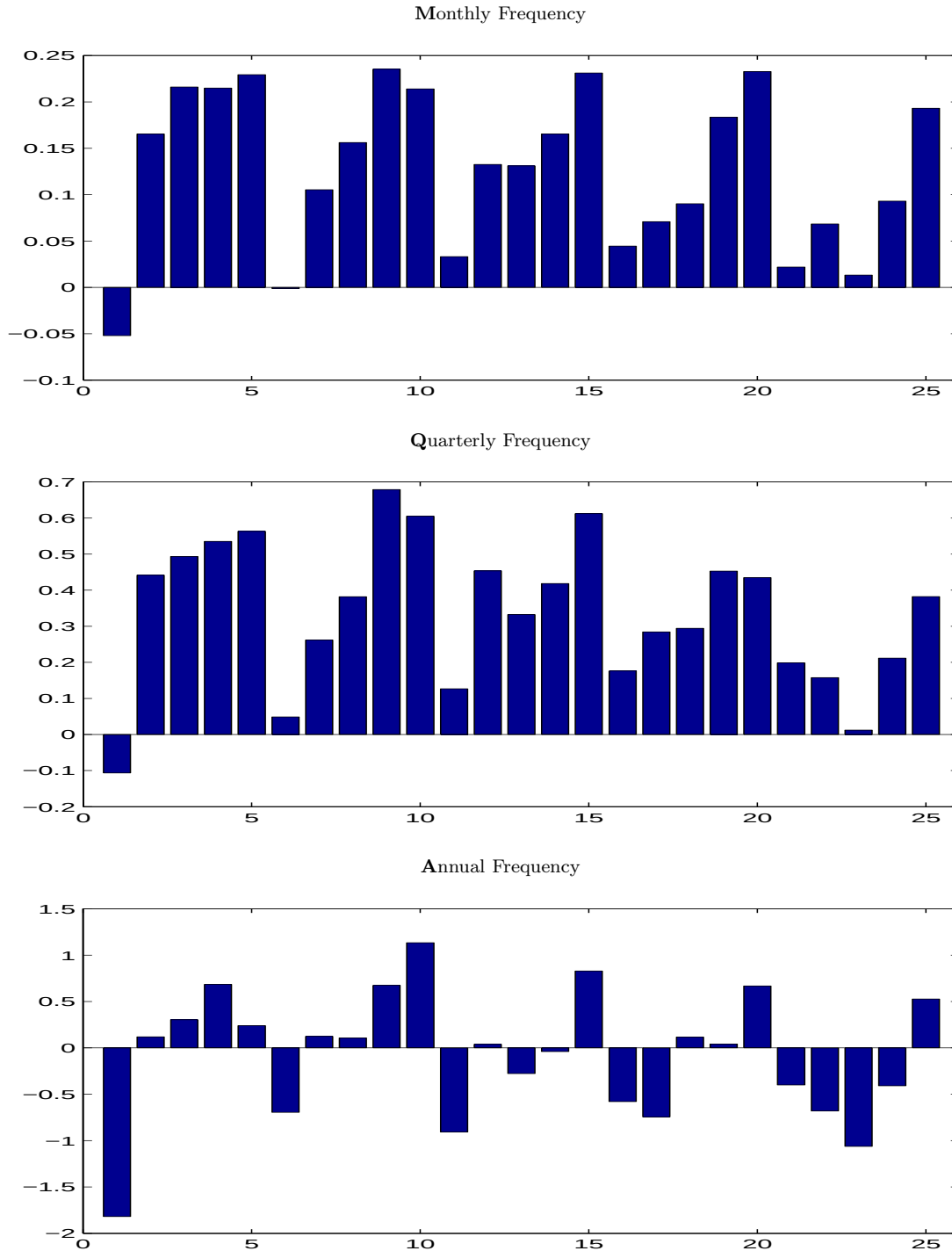


Figure 4: Exposure to the Yield Spread

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns (left panels) and the 25 size- and value portfolio returns (right panel) to the yield spread. The data are monthly for June 1952 until December 2007 (top row), quarterly (middle row), and annual (bottom row).

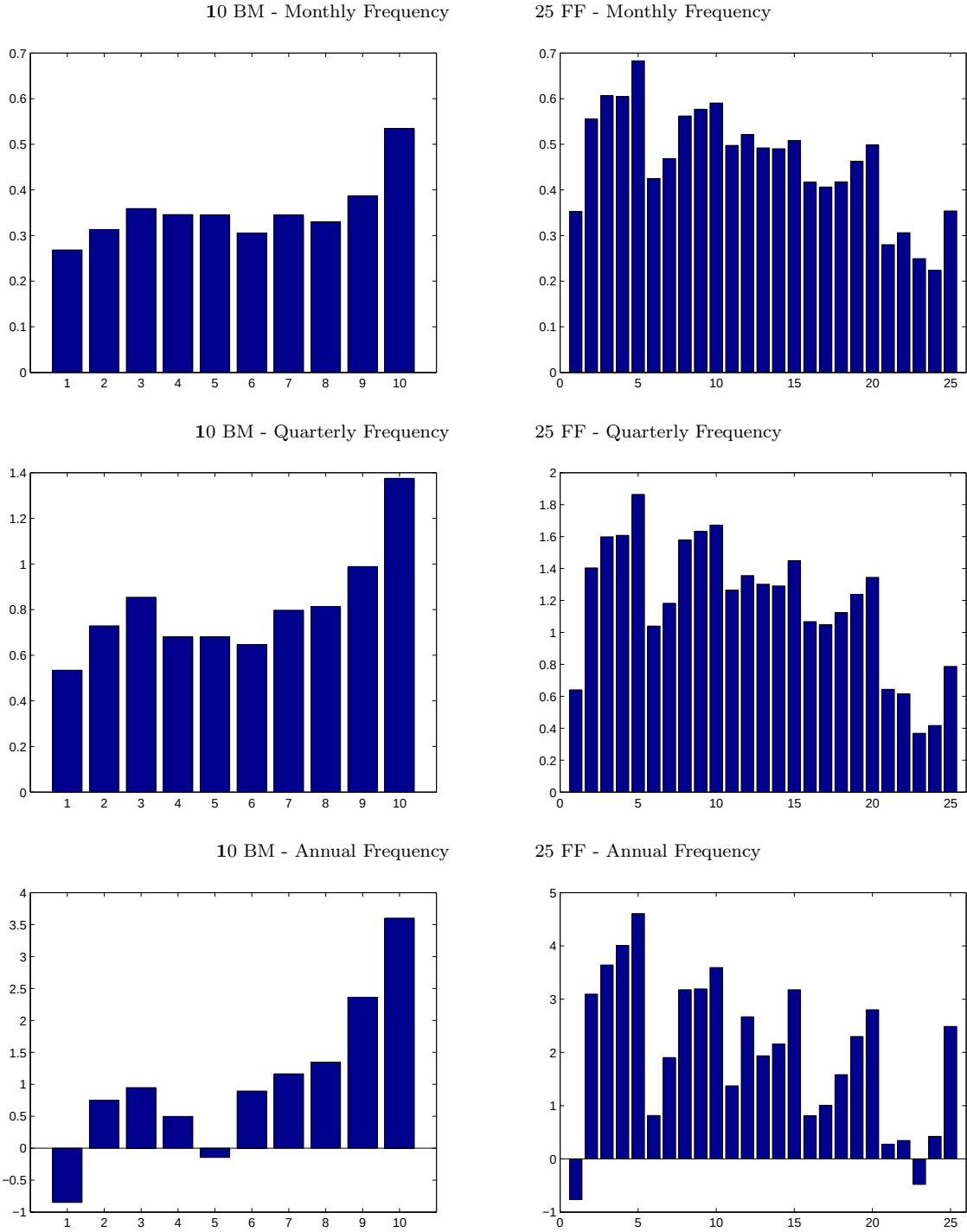


Figure 5: Average Realized vs Predicted Returns: CP Factor

Scatter diagram of average realized versus predicted excess returns. The predicted excess return is generated by the OLS estimation with the CP factor as only explanatory variable. The test assets are the 10 book-to-market portfolio returns (top panel) and the 25 size- and value portfolio returns (bottom panel). The solid line is the 45 degree line. The data run from June 1952 until December 2007.

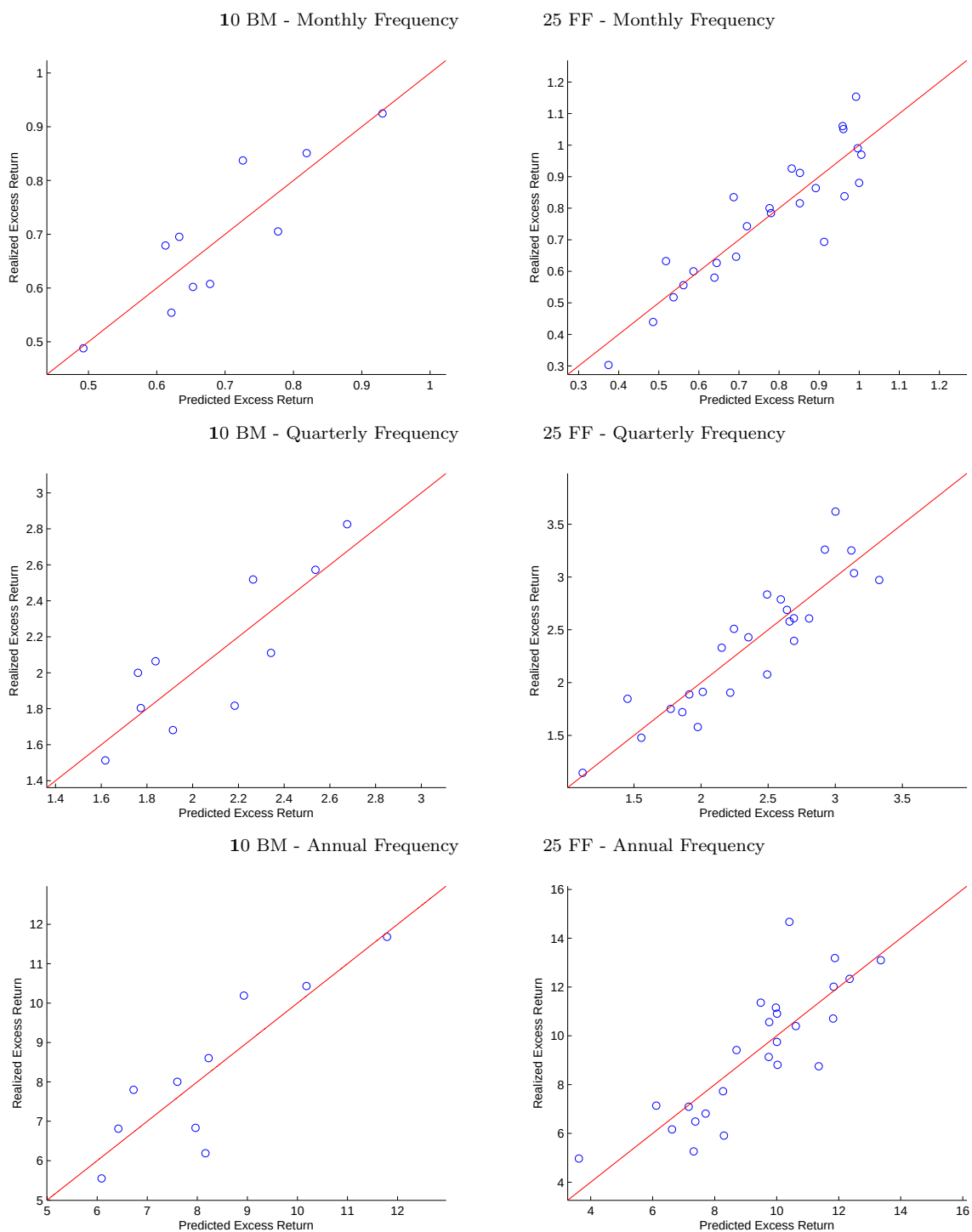


Figure 6: Value versus Growth Returns By CP Factor and Yield Spread Quartiles

The left panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by CP quartile. The right panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by yield spread quartile. The top row is for the top quartile, the second row for the 50-75 percentile, the third row for the 12-50 percentile, and the bottom row for the lowest 25 percent CP or yield spread realizations.

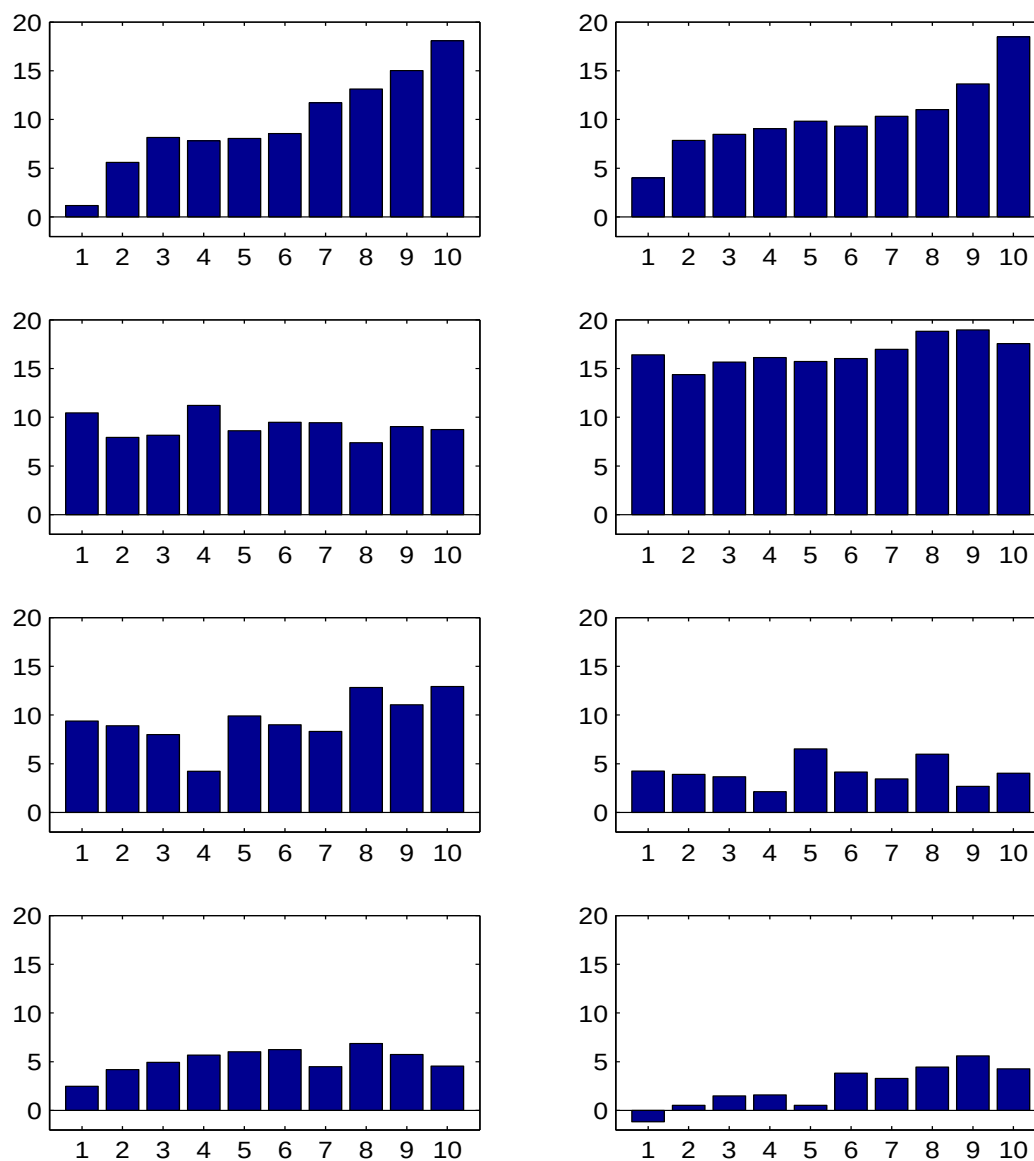


Figure 7: Exposure of 5 Bond Portfolio Excess Returns to the CP Factor in Data

The figure plots the factor exposures (betas) of five CRSP bond portfolio returns to the contemporaneous CP_{t+1} factor. Each bar denotes the slope coefficient of a time-series regression of one portfolio on the CP factor. The left bar is short-maturity bonds, the right bar for long-maturity bonds. The top panel uses monthly return data (multiplied by 12), the middle panel uses quarterly return data (multiplied by 4), and the bottom panel uses annual data. The data run from June 1952 until December 2007.

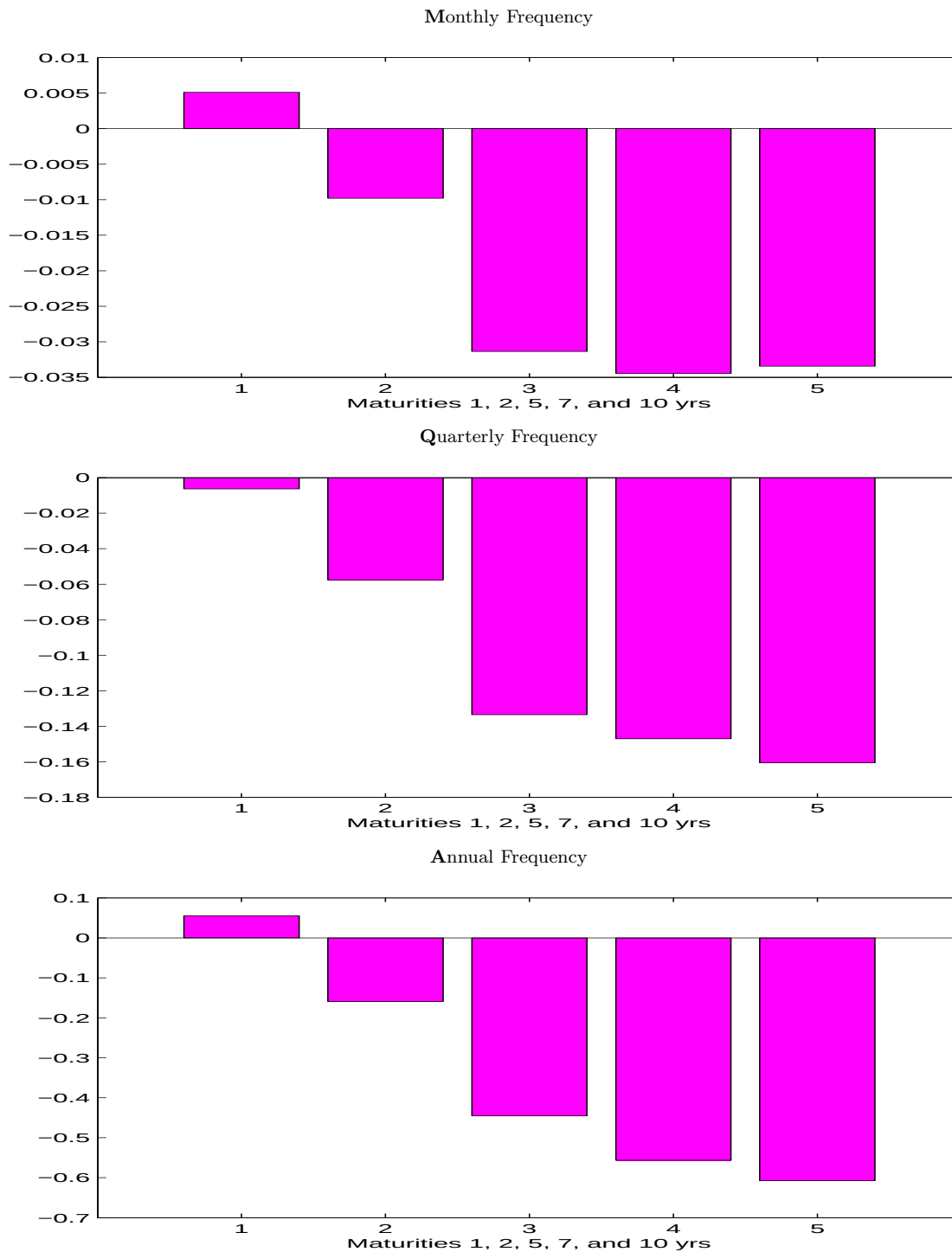


Figure 8: NBER Recessions, the CP Factor and the Yield Spread

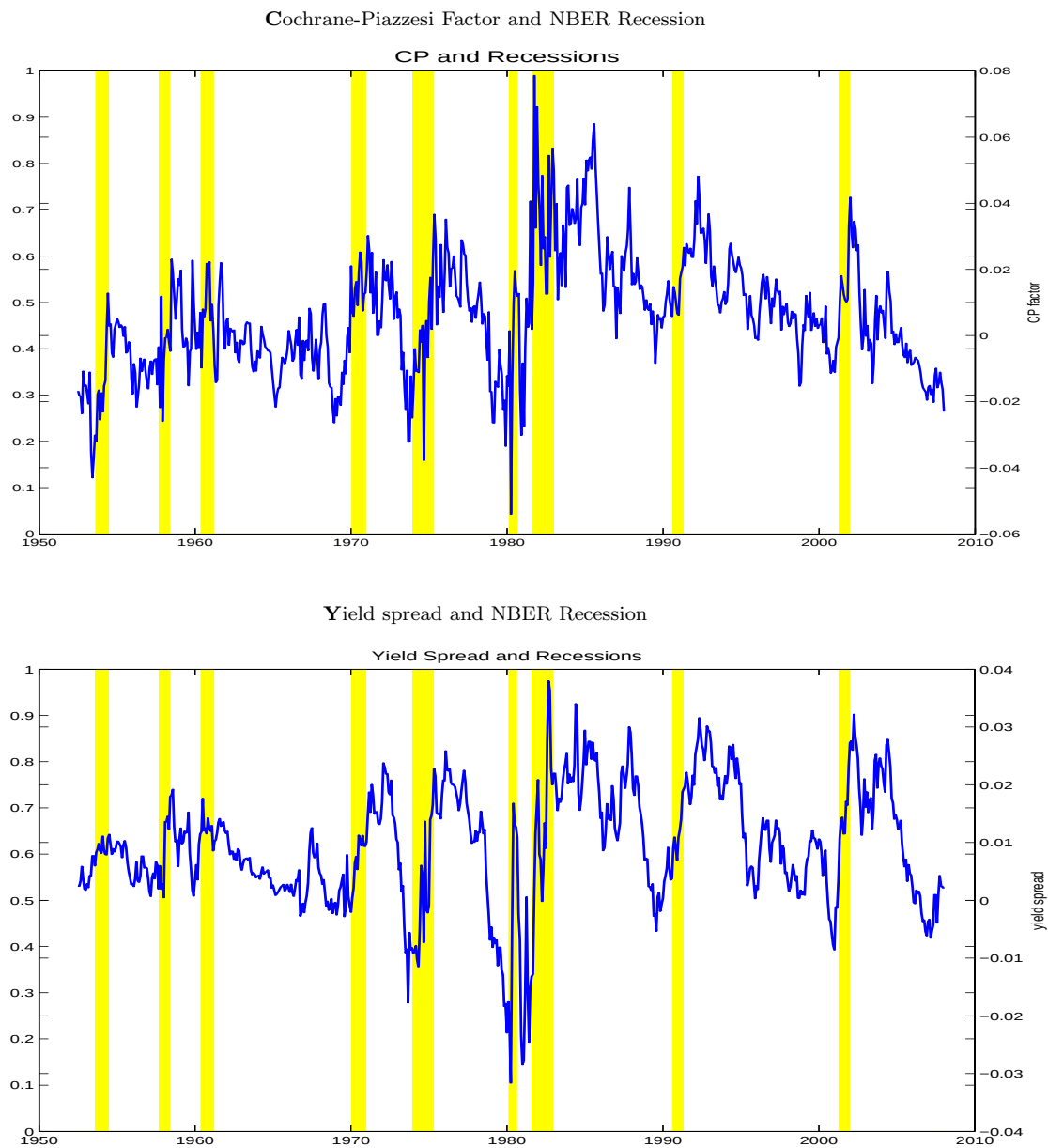


Figure 9: SDF Model-Implied Yields

This figure plots annual yields on nominal bonds of maturities 1- through 5 years as implied by stochastic discount factor model estimated in Section 2.3. The data are Fama-Bliss yields on nominal bonds of maturities 1- through 5 years.

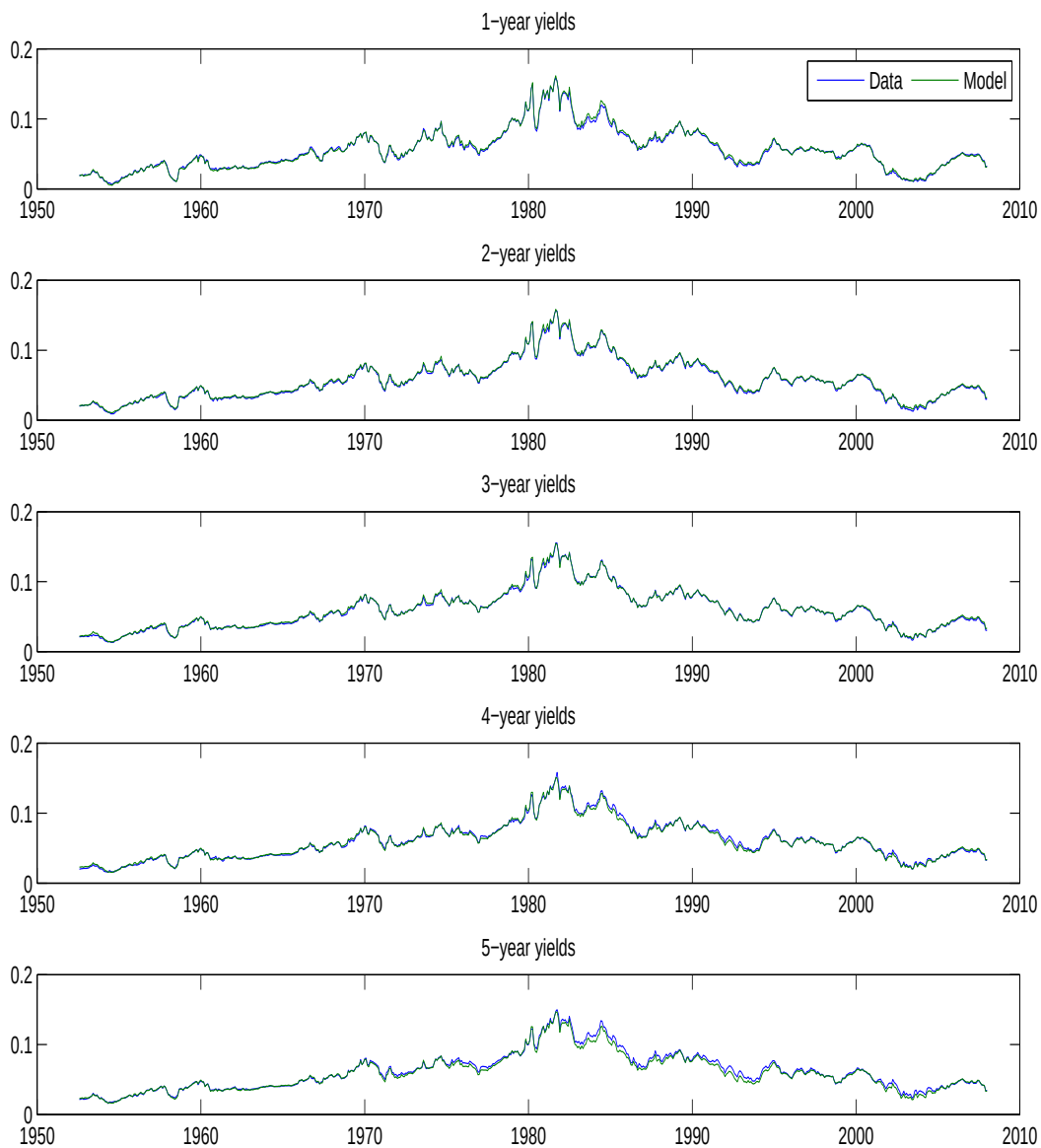


Figure 10: Exposure of 10 Book-to-Market Portfolio Dividend Growth rates to the CP and Yield Spread Factor in Data

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio dividend growth rates δd_{t+1} to the contemporaneous CP_{t+1} factor (left panels) and yield spread (right panels). Each bar denotes the slope coefficient of a time-series regression of one portfolio dividend growth rate on the CP factor or the yield spread. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2007.

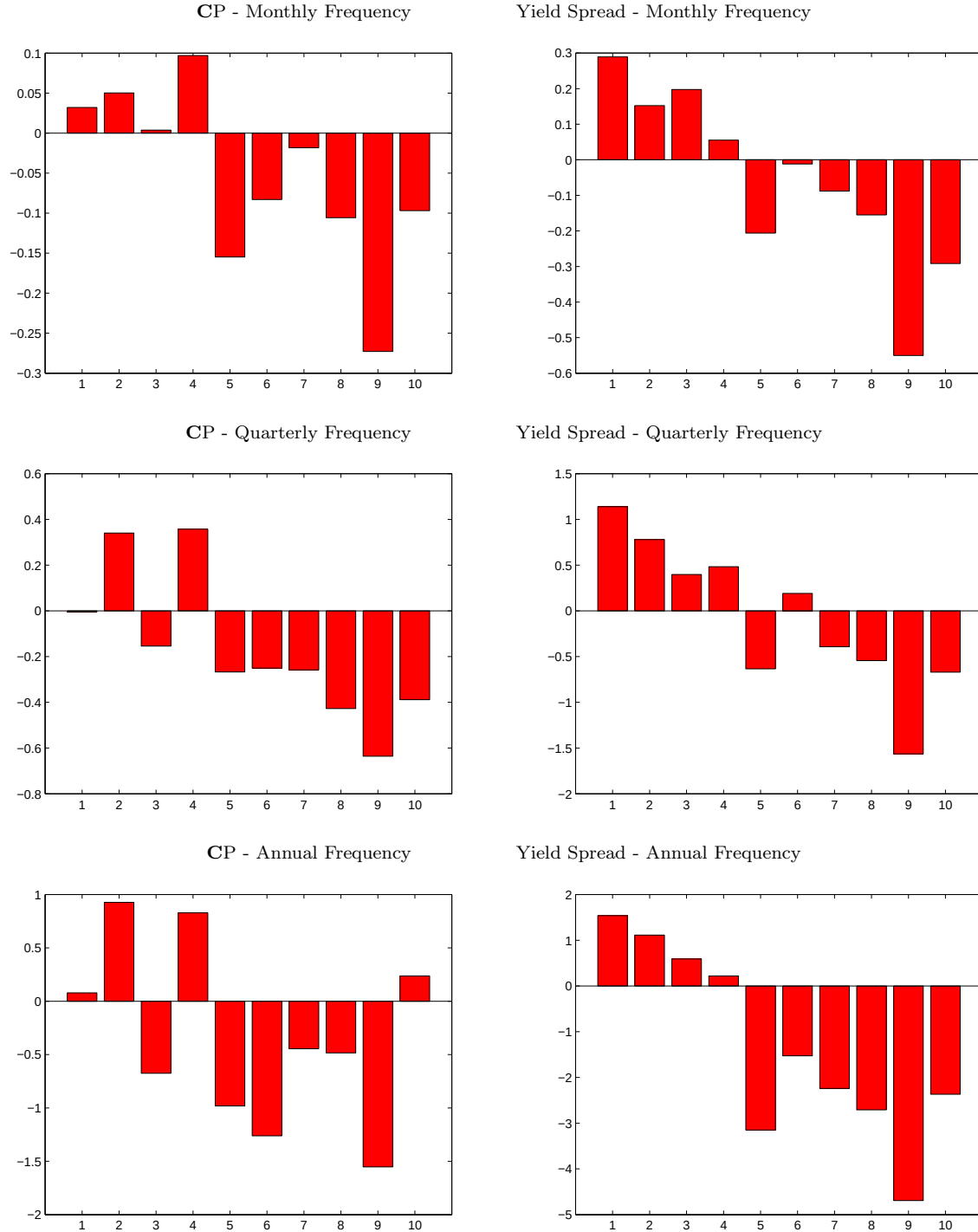


Figure 11: Risk Premia and Betas for $\chi = 0$ and $\phi_{d,\sigma}^i = 0$

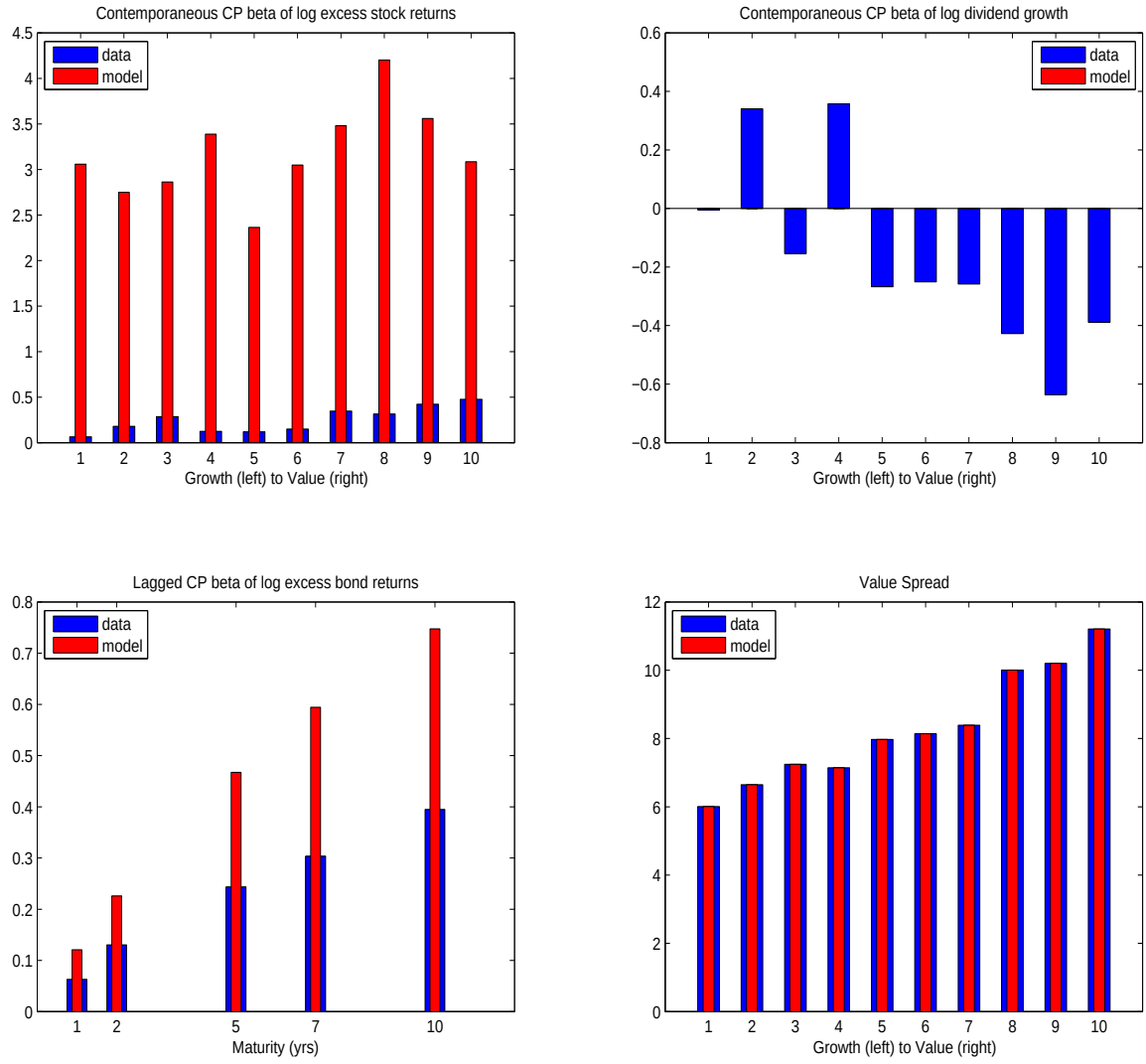


Figure 12: Risk Premia and Betas for $\chi > 0$ and $\phi_{d,\sigma}^i < 0$

