A Subjective Model of Temporal Preferences

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Abstract

We study preferences for timing of resolution of objective uncertainty in a simple menu choice model with two stages of information arrival. We characterize two general classes of utility representations called linear hidden action representations. The representations can be interpreted as if an unobservable action is taken by the individual (or by the malevolent nature, depending on whether the preference is for early or late resolution of uncertainty) between the two periods.

We illustrate that our general representations allow for a richer class of preferences for timing of resolution of uncertainty than was possible in Kreps and Porteus (1978), and provide a unified framework for studying a variety of well-known preferences in the literature. We show that subjective versions of the class of multi-prior preferences (Gilboa and Schmeidler (1989)) and variational preferences (Maccheroni, Marinacci, and Rustichini (2006)) overlap with the class of linear hidden action preferences exhibiting a preference for late resolution of uncertainty. The costly contemplation model (Ergin and Sarver (2008a)), as well as a generalization of the Kreps and Porteus (1978) model that allows for preference for flexibility in the sense of DLR (2001) are also characterized.

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1 Introduction

This paper considers several new classes of dynamic preferences, providing representations for preferences for both early and late resolution of uncertainty. These preferences are examined in a simple menu-choice model with two-stage objective uncertainty. We axiomatize two linear hidden action representations corresponding to preferences for early and late resolution of uncertainty, and we show that, under a minimality condition, these representations are uniquely identified from the preference. In other words, preferences for early and late resolution of uncertainty can be interpreted as arising from the presence of an unobserved (hidden) action that can be taken between the resolution of the first and second period objective uncertainty.

It is well known that an individual may prefer to have uncertainty resolve at an earlier date in order to be able to condition her future actions on the realization of this uncertainty. For example, an individual may prefer to have uncertainty about her future income resolve earlier so that she can smooth her consumption across time. Suppose an individual has the possibility of receiving a promotion with a substantial salary increase several years into the future. If she is able to learn the outcome of that promotion decision now, then even if she will not actually receive the increased income until a later date, she may choose to increase her current consumption by temporarily decreasing her savings or increasing her debt. On the other hand, if she is not told the outcome of the promotion decision, then by increasing her consumption now, she risks having larger debt and hence suboptimally low consumption in the future. In this example, changing the timing of the resolution of uncertainty benefits the individual by increasing her ability to condition her choices on the outcome of that uncertainty.

Kreps and Porteus (1978) considered a broader class of dynamic preferences that allow for a preference for early (or late) resolution of uncertainty even when the individual's ability to condition her actions of this uncertainty is unchanged. For example, suppose the individual described above has no current savings and is unable to take on debt. Then, even if she learns the outcome of the promotion decision now, she is unable to increase her consumption. Even in this case, the preferences considered by Kreps and Porteus (1978) allow the individual to have a strict preference for that uncertainty to resolve earlier (or later). This additional time-preference for the resolution of uncertainty has proved quite useful in applications, in particular, in macroeconomic models of asset pricing and business cycles.

The setting for our model is a simple two-stage version of the model considered by Kreps and Porteus (1978). However, we allow for more general axioms, which permits us to model a richer set of preferences exhibiting a preference for early or late resolution of uncertainty. In particular, we relax the Strategic Rationality Axiom of Kreps and Porteus (1978) (see Axiom 7) to allow for a preference for flexibility as in Kreps (1979) and Dekel, Lipman, and Rustichini (2001, henceforth DLR). We are able to represent this general class of preferences for early and late resolution of uncertainty as if there is an unobserved (hidden) action that can be taken between the resolution of the first and second period objective uncertainty. In the case of a preference for early resolution of uncertainty, this hidden action can be thought of as an action chosen by the individual. Thus, the individual prefers to have objective uncertainty resolve in the first period so that she can choose this action optimally. In the case of a preference for late resolution of objective uncertainty, this hidden action could be though of as an action chosen by the (malevolent) nature. In this case, the individual prefers to have objective uncertainty resolve in the second period, after this action has been selected by nature, so as to mitigate natures ability to harm her.

This paper not only provides representations for a more general class of preferences for early and late resolution of uncertainty, but also provides new ways to understand and interpret the temporal preferences considered by Kreps and Porteus (1978). Our linear hidden action model is general enough to encompass the subjective versions of a number of well-known representations in the literature: the multiple priors model of Gilboa and Schmeidler (1989), the variational preferences model of Maccheroni, Marinacci, and Rustichini (2006), the costly contemplation model of Ergin and Sarver (2008a), and a version of the temporal preferences model of Kreps and Porteus (1978) extended to allow for subjective uncertainty as in DLR (2001). We identify the preference for temporal resolution of uncertainty implied by each of these representations as well as their additional behavioral implications over the linear hidden action model. The general framework in this paper provides a unification of these well-known representations and provides simple axiomatizations.

2 The Model

Let Z be a finite set of alternatives, and let $\triangle(Z)$ denote the set of all probability distributions on Z, endowed with the Euclidean metric d (generic elements $p, q, r \in \triangle(Z)$). Let $\mathcal{A}$ denote the set of all closed subsets of $\triangle(Z)$, endowed with the Hausdorff metric:

$$d_h(A,B) = \max\left\{ \max_{p\in A} \min_{q\in B} d(p,q), \max_{q\in B} \min_{p\in A} d(p,q) \right\}.$$

Elements of $\mathcal{A}$ are called menus (generic menus $A, B, C \in \mathcal{A}$). Let $\triangle(\mathcal{A})$ denote the set of all Borel probability measures on $\mathcal{A}$, endowed with the weak* topology (generic elements $P, Q, R \in \triangle(\mathcal{A})$).[1] The primitive of the model is a binary relation $\succsim$ on $\triangle(\mathcal{A})$, representing the individual's preferences over lotteries over menus.

We interpret $\succsim$ as corresponding to the individual's choices in the first period of a two-period decision problem. In the beginning of period 1, the individual chooses a lottery P over menus. Later in period 1, the uncertainty associated with the lottery P is resolved and P returns a menu A. In the unmodeled period 2, the individual chooses a lottery p out of A. Later in period 2, the lottery p resolves and returns an alternative z. We will refer to the uncertainty associated with the resolution of P as the *first-stage uncertainty*, and will refer to the uncertainty associated with the resolution of p as the *second-stage uncertainty*. Although the period 2 choice is unmodeled, it will be important for the interpretation of the representations.

Our model is a special case of Kreps and Porteus (1978) with only two periods and no consumption in period 1.[2] A lottery $P \in \triangle(\mathcal{A})$ over menus is a *temporal lottery* (Kreps and Porteus (1978)) if P returns a singleton menu with probability one. An individual facing a temporal lottery makes no choice in period 2, between the resolution of first and second stages of the uncertainty. Note that the set of temporal lotteries can be naturally associated with $\triangle(\triangle(Z))$.

For any $A, B \in \mathcal{A}$ and $\alpha \in [0,1]$, the convex combination of these two menus is defined by $\alpha A + (1-\alpha)B \equiv \{\alpha p + (1-\alpha)q : p \in A \text{ and } q \in B\}$. We let $\mathrm{co}(A)$ denote the convex hull of the menu A and let $\delta_A \in \triangle(\mathcal{A})$ denote the degenerate lottery that puts probability 1 on the menu A. Then, $\alpha\delta_A + (1-\alpha)\delta_B$ denotes the lottery that puts probability α on the menu A and probability $1-\alpha$ on the menu B. Finally, for any continuous function $V : \mathcal{A} \to \mathbb{R}$ and $P \in \triangle(\mathcal{A})$, we let $\mathbb{E}_P[V]$ denote the expected value of V under the lottery P, i.e. $\mathbb{E}_P[V] = \int_{\mathcal{A}} V(A)\, P(dA)$.

3 General Representations

We will impose the following set of axioms in all the representation results in the paper. Therefore, it will be convenient to refer to them altogether as Axiom 1.

[1]Given a metric space X, the weak* topology on the set of all finite signed Borel measures on X is the topology where a net of signed measures $\{\mu_d\}_{d \in D}$ converges to a signed measure μ if and only if $\int_X f\, \mu_d(dx) \to \int_X f\, \mu(dx)$ for every bounded continuous function $f : X \to \mathbb{R}$.

[2]The same model is also used in Epstein and Seo (2007) and in Section 4 of Epstein, Marinacci and Seo (2007).

Axiom 1

1. (Weak Order): *$\succsim$ is complete and transitive.*

2. (Continuity): *The upper and lower contour sets, $\{P \in \triangle(\mathcal{A}) : P \succsim Q\}$ and $\{P \in \triangle(\mathcal{A}) : P \precsim Q\}$, are closed in the weak* topology.*

3. (First-Stage Independence): *For any $P, Q, R \in \triangle(\mathcal{A})$ and $\alpha \in (0,1)$,*
$$P \succ Q \quad \Rightarrow \quad \alpha P + (1-\alpha)R \succ \alpha Q + (1-\alpha)R.$$

4. (L–Continuity): *There exist $A^*, A_* \in \mathcal{A}$ and $M \geq 0$ such that for every $A, B \in \mathcal{A}$ and $\alpha \in [0,1]$ with $\alpha \geq M d_h(A, B)$,*
$$(1-\alpha)\delta_A + \alpha\delta_{A^*} \succsim (1-\alpha)\delta_B + \alpha\delta_{A_*}.$$

5. (Indifference to Randomization (IR)): *For every $A \in \mathcal{A}$, $\delta_A \sim \delta_{co(A)}$.*

Axioms 1.1 and 1.2 are standard. Axiom 1.3 is the von Neumann-Morgenstern independence axiom imposed with respect to the first-stage uncertainty. Axioms 1.1–1.3 ensure that there exists a continuous function $V : \mathcal{A} \to \mathbb{R}$ such that $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$. Given Axioms 1.1–1.3, Axiom 1.4 is a technical condition implying the Lipschitz continuity of V.[3] Axiom 1.5 is introduced in DLR (2001). It is justified if the individual choosing from the menu A in period 2 can also randomly select an alternative from the menu, for example, by flipping a coin. In that case, the menus A and $\text{co}(A)$ offer the same set of options, and hence they are identical from the perspective of the individual.

The next two axioms from Kreps and Porteus (1978) will be key in our representation results.

Axiom 2 (Preference for Early Resolution of Uncertainty (PERU)) *For any $A, B \in \mathcal{A}$ and $\alpha \in (0,1)$,*
$$\alpha\delta_A + (1-\alpha)\delta_B \succsim \delta_{\alpha A + (1-\alpha)B}.$$

Axiom 3 (Preference for Late Resolution of Uncertainty (PLRU)) *For any $A, B \in \mathcal{A}$ and $\alpha \in (0,1)$,*
$$\delta_{\alpha A + (1-\alpha)B} \succsim \alpha\delta_A + (1-\alpha)\delta_B.$$

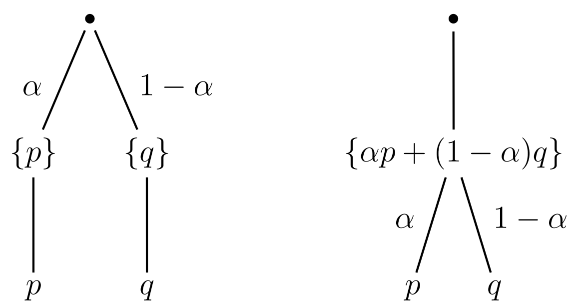

Figure 1: Timing of Resolution of Uncertainty with Temporal Lotteries

To understand Axioms 2 and 3, suppose first that $A = \{p\}$ and $B = \{q\}$ for some $p, q \in \triangle(Z)$. In this case, period 2 choice out of a menu is degenerate, and the two equations above are statements about the preference between the temporal lotteries $\alpha\delta_{\{p\}} + (1-\alpha)\delta_{\{q\}}$ and $\delta_{\alpha\{p\}+(1-\alpha)\{q\}}$. The temporal lottery $\alpha\delta_{\{p\}} + (1-\alpha)\delta_{\{q\}}$ corresponds to the first tree in Figure 1, in which the uncertainty regarding whether lottery p or q is selected resolves in period 1. The temporal lottery $\delta_{\alpha\{p\}+(1-\alpha)\{q\}}$ corresponds to the second tree in Figure 1, in which the same uncertainty resolves in period 2.[4] PERU requires a weak preference the first temporal lottery, whereas PLRU requires a weak preference for the second temporal lottery.

In the general case where A and B need not be singletons, the interpretations of PERU and PLRU are more subtle, since $\delta_{\alpha A+(1-\alpha)B}$ and $\alpha\delta_A + (1-\alpha)\delta_B$ involve non-degenerate period 2 choices. Note that in $\alpha\delta_A + (1-\alpha)\delta_B$, the uncertainty regarding whether the menu A or B is selected resolves in period 1, before the individual makes her choice out of the selected menu. On the other hand, in $\delta_{\alpha A+(1-\alpha)B}$ no uncertainty is resolved in period 1. The period 2 choice of a lottery $\alpha p + (1-\alpha)q$ from the convex combination menu $\alpha A + (1-\alpha)B$ is identical to a pair of choices $p \in A$ and $q \in B$, where after the individual chooses (p, q), p is selected with probability α and q is selected with probability $1-\alpha$. Therefore, the period 2 choice out of the menu $\alpha A + (1-\alpha)B$ can be interpreted as a complete contingent plan out of the menus A and B.

The key distinction between the two lotteries over menus is that in $\delta_{\alpha A+(1-\alpha)B}$ the period 2 choice is made prior to the resolution of the uncertainty regarding whether the choice from A or the choice from B will be implemented, whereas in $\alpha\delta_A + (1-\alpha)\delta_B$,

[3]In models with preferences over menus over lotteries, analogous L–continuity axioms can be found in and Dekel, Lipman, Rustichini, and Sarver (2007, henceforth DLRS), Sarver (2008), and Ergin and Sarver (2008a).

[4]In both temporal lotteries, the remaining uncertainty, i.e. the outcome of p conditional on p being selected and the outcome of q conditional on q being selected, is resolved in period 2.

the same uncertainty is resolved in period 1 before the individual makes a choice out of the selected menu. Therefore, PERU can be interpreted as the individual's preference to make a choice out of the menu after learning which menu is selected, whereas PLRU is the individual's preference to make her choice out of the menus before learning which menu is selected.

We will also characterize the special case of our representation where the individual has a weak preference for larger menus.

Axiom 4 (Monotonicity) *For any $A, B \in \mathcal{A}$, $A \subset B$ implies $\delta_B \succsim \delta_A$.*

Since expected-utility functions on $\triangle(Z)$ are equivalent to vectors in $\mathbb{R}^Z$, we will use the notation $u(p)$ and $u \cdot p$ interchangeably for any expected utility function $u \in \mathbb{R}^Z$. We define the set of *normalized (non-constant) expected-utility functions* on $\triangle(Z)$ to be

$$\mathcal{U} = \left\{ u \in \mathbb{R}^Z : \sum_{z \in Z} u_z = 0, \sum_{z \in Z} u_z^2 = 1 \right\}.$$

We are ready to introduce our general representations.

Definition 1 A *Maximum [Minimum] Linear Hidden Action (max-LHA [min-LHA]) representation* is a pair $(\mathcal{M}, c)$ consisting of a compact set of finite signed Borel measures $\mathcal{M}$ on $\mathcal{U}$ and a lower semi-continuous function $c : \mathcal{M} \to \mathbb{R}$ such that:

1. $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$, where $V : \mathcal{A} \to \mathbb{R}$ is defined by Equation (1) [(2)]:
$$V(A) = \max_{\mu \in \mathcal{M}} \left[\int_{\mathcal{U}} \max_{p \in A} (u \cdot p) \mu(du) - c(\mu) \right] \tag{1}$$
$$V(A) = \min_{\mu \in \mathcal{M}} \left[\int_{\mathcal{U}} \max_{p \in A} (u \cdot p) \mu(du) + c(\mu) \right]. \tag{2}$$

2. The set $\mathcal{M}$ is *minimal*: For any compact proper subset $\mathcal{M}'$ of $\mathcal{M}$, the function V' obtained by replacing $\mathcal{M}$ with $\mathcal{M}'$ in Equation (1) [(2)] is different from V.

The pair $(\mathcal{M}, c)$ is an *LHA representation* if it is a max-LHA or a min-LHA representation. An LHA representation $(\mathcal{M}, c)$ is *monotone* if all measures in $\mathcal{M}$ are positive.

Before we interpret the representations, we will first argue that after appropriately renormalizing the set of ex post utility functions, one can reinterpret the integral term in Equations (1) and (2) as an expectation. More specifically, suppose that μ is a nonnegative measure with $\lambda = \mu(\mathcal{U}) > 0$. Consider the probability measure π on $\mathcal{V} = \lambda\mathcal{U}$ which (heuristically) puts $\mu(u)/\lambda$ weight on each $v = \lambda u \in \mathcal{V}$. Then, by a simple change of variables, the above integral can be rewritten as the expectation $\int_{\mathcal{V}} \max_{p \in A}(v \cdot p)\pi(dv)$ which can be interpreted as follows. The individual anticipates that her ex post utility functions will be distributed according to π. Conditional on each realization of her ex post utility $v \in \mathcal{V}$, she will chose a lottery in A that maximizes v. She aggregates across different possible realizations of v by taking expectation with respect to π.[5] Therefore, each measure μ can be interpreted as a reduced-form representation of the individual's subjective uncertainty about her ex post (period 2) utility function over $\triangle(Z)$. We conjecture that analogous interpretations are possible if μ is a signed measure, by introducing regret and temptation to the current framework.[6]

We next interpret Equation (1). In period 1, the individual anticipates that after the first-stage uncertainty is resolved but before she makes her choice in period 2, she will be able to select an action μ from a set $\mathcal{M}$. Each action μ affects the distribution of the individual's ex post utility functions over $\triangle(Z)$, at cost $c(\mu)$. As argued in the above paragraph, the integral in Equation (1) can be interpreted as a reduced-form representation for the value of the action μ when the individual chooses from menu A, which is linear in the menu A. For each menu A, the individual maximizes the value minus cost of her action.

The interpretation of Equation (2) is dual. In period 1, the individual anticipates that after the first-stage uncertainty is resolved but before she makes her choice in period 2, the (malevolent) nature will select an action μ from a set $\mathcal{M}$. The individual anticipates the nature to chose an action which minimizes the value to the individual plus a cost term. The function c can be interpreted as being related to the pessimism attitude of the individual. For constant c, she expects the nature to chose an action that outright minimizes her utility from a menu. Different cost functions put different restrictions on the individual's perception of the malevolent nature's objective.[7]

In the above representations, both the set of available actions and and their costs are subjective in that they are part of the representation. Therefore $\mathcal{M}$ and c are

[5] It is noteworthy that any probability measure π' such that μ and π' are absolutely continuous with respect to each other can be used in the above argument. The reason is that, heuristically, in the LHA-representations, the weight $\mu(u)$ on the ex post utility function u captures both the cardinality of the ex post utility and its probability, and these two effects cannot be separated from behavior as in models with state dependent utility. See Kreps (1988) for an elaborate discussion of the state-dependence issue.

[6] See Sarver (2008), Gul and Pesendorfer (2001), and DLR (2008).

[7] This interpretation is due to Maccheroni, Marinacci, and Rustichini (2006) which we discuss in more detail in the following subsection.

not directly observable to the modeler and need to be identified from the individual's preferences. Note that in both Equations (1) and (2), it is possible to enlarge the set of actions by adding a new action μ to the set $\mathcal{M}$ at a prohibitively high cost $c(\mu)$ without affecting the equations. Therefore, in order to identify $(\mathcal{M}, c)$ from the preference, we also impose an appropriate minimality condition on the set $\mathcal{M}$.

We postpone more concrete interpretations of the set of actions and costs to the discussion of the applications of LHA-representations in the following section. We are now ready to state our general representation result.

Theorem 1 *A. The preference $\succsim$ has a max-LHA [min-LHA] representation if and only if it satisfies Axiom 1 and PERU [PLRU].*

B. The preference $\succsim$ has a monotone max-LHA [min-LHA] representation if and only if it satisfies Axiom 1, PERU [PLRU], and monotonicity.[8]

The special case of LHA representations satisfying indifference to timing of resolution of uncertainty (i.e. both PERU and PLRU) are those where $\mathcal{M}$ is a singleton. In that case, the constant cost can be dropped out from Equations (1) and (2), leading to an analogue of DLR (2001)'s additive representation where the individual is risk neutral with respect to the first-stage uncertainty.

We next give a brief intuition about Theorem 1.A. Axiom 1 guarantees the existence of a Lipschitz continuous function $V : \mathcal{A} \to \mathbb{R}$ such that $V(co(A)) = V(A)$ and $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$. In terms of this expected utility representation, it is easy to see that PERU corresponds to convexity of V and PLRU corresponds to concavity of V. The set $\mathcal{A}^c$ of convex menus can be mapped one-to-one to the set Σ of support functions, preserving the metric and the linear operations. Therefore, by using the property $V(co(A)) = V(A)$ and mimicking the construction in DLR (2001), V can be thought of as a function defined on the subset Σ of the Banach space $C(\mathcal{U})$ of continuous real-valued functions on $\mathcal{U}$. We then apply a variation of the classic duality principle that convex [concave] functions can be written as the supremum [infimum] of affine functions lying below [above] them.[9] Finally, we apply the Riesz representation theorem to write each such continuous affine function as an integral against a measure μ minus [plus] a scalar $c(\mu)$. Theorem 1.B states that additionally imposing monotonicity guarantees that all measures in the LHA representation are positive.

[8]In part B, IR can be dropped for the case of the max-LHA representation, because it is implied by weak order, continuity, PERU, and monotonicity.

[9]See Rockafellar (1970), Phelps (1993), and Ergin and Sarver (2008b) for variations of this duality result.

We show that the uniqueness of the LHA representations follows from the affine uniqueness of V and a result about the uniqueness of the dual representation of a convex function in the theory of conjugate convex functions (see Ergin and Sarver (2008b, Corollary 3.3)). A similar application of the duality and uniqueness results can be found in Ergin and Sarver (2008a).

Theorem 2 *If $(\mathcal{M}, c)$ and $(\mathcal{M}', c')$ are two max-LHA [min-LHA] representations for $\succsim$, then there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $\mathcal{M}' = \alpha\mathcal{M}$ and $c'(\alpha\mu) = \alpha c(\mu) + \beta$ for all $\mu \in \mathcal{M}$.*

4 Applications

4.1 Multiple Priors and Variational Preferences

A preference for late resolution of uncertainty could arise if an individual would like to delay the resolution of objective lotteries for hedging reasons. In this section, we formalize this intuition by showing that the monotone min-LHA model is equivalent to two representations that have natural interpretations in terms of ambiguity-aversion. The following multiple-priors representation allows for ambiguity regarding the distribution over ex post subjective states and is intuitively similar to the multiple-priors representation proposed by Gilboa and Schmeidler (1989) in the Anscombe-Aumann setting.

Definition 2 A *Subjective-State-Space Multiple-Priors (SSMP) representation* is a triple (S, U, Π) where S is a metric space, $U : \triangle(Z) \times S \to \mathbb{R}$ is a continuous and bounded state-dependent expected-utility function, and Π is a weak* compact set of Borel probability measures on S, such that $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$, where $V : \mathcal{A} \to \mathbb{R}$ is defined by

$$V(A) = \min_{\pi \in \Pi} \int_S \max_{p \in A} U(p, s)\, \pi(ds). \tag{3}$$

The next representation is similar in spirit to the variational representation considered by Maccheroni, Marinacci, and Rustichini (2006) in the Anscombe-Aumann setting.

Definition 3 A *Subjective-State-Space Variational (SSV) representation* is a quadruple (S, U, Π, c) where S is a metric space, $U : \triangle(Z) \times S \to \mathbb{R}$ is a continuous and bounded state-dependent expected-utility function, Π is a weak* compact set of Borel probability measures on S, and $c : \Pi \to \mathbb{R}$ is a weak* lower semi-continuous function, such that $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$, where $V : \mathcal{A} \to \mathbb{R}$ is defined by

$$V(A) = \min_{\pi \in \Pi} \left[\int_S \max_{p \in A} U(p, s)\, \pi(ds) + c(\pi) \right]. \tag{4}$$

The SSV representation generalizes the SSMP representation by allowing a "cost" $c(\pi)$ to be assigned to each measure π in the representation. In the Anscombe-Aumann setting, the class of variational preferences considered by Maccheroni, Marinacci, and Rustichini (2006) is strictly larger than the class of multiple-prior expected-utility preferences considered by Gilboa and Schmeidler (1989). However, we show that in the current setting, the SSMP and SSV representations are equivalent in the sense that the set of preferences that can be represented using an SSMP representation is precisely the set of preferences that can be represented using an SSV representation. The reason for this equivalence in the subjective versions of the representations is the state-dependence of the utility functions in the representations. The following Theorem formalizes this claim and, moreover, states that a preference $\succsim$ can be represented by one of these representations if and only if it has a monotone min-LHA representation.

Theorem 3 *Let $V : \mathcal{A} \to \mathbb{R}$. Then, the following are equivalent:*[10]

1. *There exists a monotone min-LHA representation such that V is given by Equation* (2).
2. *There exists an SSMP representation such that V is given by Equation* (3).
3. *There exists an SSV representation such that V is given by Equation* (4).

The following immediate corollary provides the axiomatic foundation for the SSMP and SSV representations.

Corollary 1 *A preference $\succsim$ has a SSMP representation if and only if it has a SSV representation if and only if it satisfies Axiom 1, PLRU, and monotonicity.*

[10]We conjecture that this result still holds if the topological assumptions are dropped from the definitions of the SSMP and SSV representations, so long as U (and c in the case of the SSV representation) are assumed to be bounded. The assumptions that S is a metric space, Π is compact, U is continuous, and c is lower semi-continuous are imposed solely to ensure that the minimums over $\pi \in \Pi$ exist in Equations (3) and (4). More specifically, we expect Theorem 3 to continue to hold if the definitions of the SSMP and SSV representations are altered to instead state that S is a measure space, Π is any set of probability measures on S, U is measurable and bounded, and c is bounded (and the minimums are replaced with infimums).

We next outline the intuition behind Theorem 3 for the case where in the min-LHA, SSMP, and SSV representations, the sets $\mathcal{M}$, Π, S are finite and the measures in $\mathcal{M}$ and Π have finite support. In this special case, the compactness, continuity, and boundedness properties in the representations are automatically satisfied.

First consider (1) $\Rightarrow$ (3). Fix a min-LHA representation $(\mathcal{M}, c)$, and define V by Equation (2). Take any $\mu \in \mathcal{M}$, and define a measure π_μ on the set $\mu(\mathcal{U})\mathcal{U}$ by $\pi_\mu(v) = \frac{\mu(u)}{\mu(\mathcal{U})}$ for $v = \mu(\mathcal{U})u$, $u \in \mathcal{U}$. This transformation ensures that π_μ is a probability measure. Moreover, since $\pi_\mu(v)v = \mu(u)u$ for $v = \mu(\mathcal{U})u$, for any $A \in \mathcal{A}$,

$$\int_{\mu(\mathcal{U})\mathcal{U}} \max_{p \in A}(v \cdot p)\pi_\mu(dv) = \int_{\mathcal{U}} \max_{p \in A}(u \cdot p)\mu(du).$$

Let $S = \bigcup_{\mu \in \mathcal{M}} \mu(\mathcal{U})\mathcal{U}$, and define $U : \triangle(Z) \times S \to \mathbb{R}$ by $U(p, s) = s \cdot p$. Let $\Pi = \{\pi_\mu : \mu \in \mathcal{M}\}$ and $\tilde{c}(\pi_\mu) = c(\mu)$. Then, V can be expressed in the following SSV form:

$$V(A) = \min_{\pi \in \Pi} \left[\int_S \max_{p \in A} U(p, s)\, \pi(ds) + \tilde{c}(\pi) \right].$$

The idea of this construction is identical to the interpretation we gave for the integral term in the LHA-representations: Since utility is state-dependent, the weight of the states in the linear aggregator cannot be uniquely pinned down, and integration against the positive measure μ can be reexpressed as integration against a probability measure π_μ after appropriately rescaling the state-dependent utility functions.

To see the intuition for (3) $\Rightarrow$ (2), consider an SSV representation (S, U, Π, c), and define V by Equation (4). Let $\tilde{S} = S \times \Pi$, and let $\tilde{U} : \triangle(Z) \times \tilde{S} \to \mathbb{R}$ be defined for $p \in \triangle(Z)$ and $\tilde{s} = (s, \pi)$ by $\tilde{U}(p, s, \pi) = U(p, s) + c(\pi)$. Take any probability measure $\pi \in \Pi$, and define a new measure ρ_π on $\tilde{S}$ by $\rho_\pi(s, \pi) = \pi(s)$ and $\rho_\pi(s, \pi') = 0$ for any $\pi' \neq \pi$. It is immediate that ρ_π is a probability measure. Also, for any $A \in \mathcal{A}$,

$$\begin{aligned}
\int_{\tilde{S}} \max_{p \in A} \tilde{U}(p, \tilde{s})\, \rho_\pi(d\tilde{s}) &= \int_{S \times \Pi} \left[\max_{p \in A} U(p, s) + c(\pi') \right] \rho_\pi(ds, d\pi') \\
&= \int_S \max_{p \in A} U(p, s)\, \pi(ds) + c(\pi).
\end{aligned}$$

Letting $\tilde{\Pi} = \{\rho_\pi : \pi \in \Pi\}$, we see that V can be expressed in the following SSMP form:

$$V(A) = \min_{\rho \in \tilde{\Pi}} \int_{\tilde{S}} \max_{p \in A} \tilde{U}(p, \tilde{s})\, \rho(d\tilde{s}).$$

The idea behind this argument is also a consequence of state-dependence of utility which allows any constant to be absorbed in the integral term. Above, integration against the probability measure π plus the constant $c(\pi)$ is reexpressed as integration against the probability measure ρ_π whose support is a subset of states in $S \times \{\pi\}$ where the utility functions are "shifted" by $c(\pi)$.

To see the intuition for (2) $\Rightarrow$ (1), consider an SSMP representation (S, U, Π), and define V by Equation (3). By the definition of $\mathcal{U}$, since $U(\cdot, s)$ is an expected-utility function for each $s \in S$, there exist $u(s) \in \mathcal{U}$, $\alpha(s) \geq 0$, and $\beta(s) \in \mathbb{R}$ such that

$$U(p, s) = \alpha(s)u(s) \cdot p + \beta(s), \quad \forall p \in \triangle(Z).$$

For each $\pi \in \Pi$, define the measure μ_π on $\mathcal{U}$ by $\mu_\pi(u) = \sum_{s \in S : u(s) = u} \alpha(s)\pi(s)$ for each $u \in \mathcal{U}$. Define the function $c : \Pi \to \mathbb{R}$ by $c(\pi) = \sum_{s \in S} \beta(s)\pi(s)$. Then, for any $A \in \mathcal{A}$,

$$\begin{aligned}\sum_{s \in S} \max_{p \in A} U(p, s)\, \pi(s) &= \sum_{s \in S} \max_{p \in A} [u(s) \cdot p]\, \alpha(s)\pi(s) + \sum_{s \in S} \beta(s)\pi(s) \\ &= \int_{\mathcal{U}} \max_{p \in A} (u \cdot p) \mu_\pi(du) + c(\pi).\end{aligned}$$

Let $\mathcal{M} = \{\mu_\pi : \pi \in \Pi\}$ and let $\tilde{c}(\mu) = \inf\{c(\pi) : \pi \in \Pi \text{ and } \mu_\pi = \mu\}$. Then, V can be expressed in the following min-LHA form:

$$V(A) = \min_{\mu \in \mathcal{M}} \left[\int_{\mathcal{U}} \max_{p \in A} (u \cdot p) \mu(du) + \tilde{c}(\mu) \right]. \tag{5}$$

It can also be shown that sequentially removing measures from $\mathcal{M}$ that are not strictly optimal in Equation (5) for some $A \in \mathcal{A}$ leads to a minimal set of measures $\mathcal{M}' \subset \mathcal{M}$.

Our SSMP representation bears some similarity to a representation considered by Epstein, Marinacci and Seo (2007, Theorem 1). One main distinction between our representation and theirs is that they impose a normalization on the state-dependent utility function in their representation. As the above arguments illustrate, the key to the equivalence proposed in Theorem 3 is the state-dependence of the utility functions in the SSMP and SSV representations. If a normalization as in Epstein, Marinacci and Seo (2007) were imposed on the utility functions in the SSV and SSMP representations, then the equivalence of these representations would no longer hold, as it would not be possible to "absorb" the cost function of the SSV representation into the utility function to obtain an SSMP representation. Moreover, although these representations would continue to be special cases of the monotone min-LHA representation, it would not be possible to write every monotone min-LHA representation as an SSV representation since it would not always be possible to "absorb" the magnitude of the measure into the utility function. Theorem 3 illustrates that imposing either a normalization on utility functions (as in the min-LHA representation) or a normalization that measures be probabilities (as in the SSMP and SSV representations) is not restrictive; however, imposing both normalizations simultaneously would place a non-trivial additional restriction on the representations.

4.2 Costly Contemplation

A special case of the max-LHA representation is one of subjective information acquisition/costly contemplation, where the measures in $\mathcal{M}$ can be interpreted as a reduced-form representation of information about one's tastes and the integral term $\int_{\mathcal{U}} \max_{p\in A}(u\cdot p)\mu(du)$ can be interpreted as the ex ante value of information μ given menu A. In this application, we need to additionally impose the following axiom.

Axiom 5 (Reversibility of Degenerate Decisions (RDD)) *For any $A \in \mathcal{A}$, $p, q \in \triangle(Z)$, and $\alpha \in [0,1]$,*

$$\beta\delta_{\alpha A+(1-\alpha)\{p\}} + (1-\beta)\delta_{\{q\}} \sim \beta\delta_{\alpha A+(1-\alpha)\{q\}} + (1-\beta)\delta_{\{p\}}$$

where $\beta = 1/(2-\alpha)$.

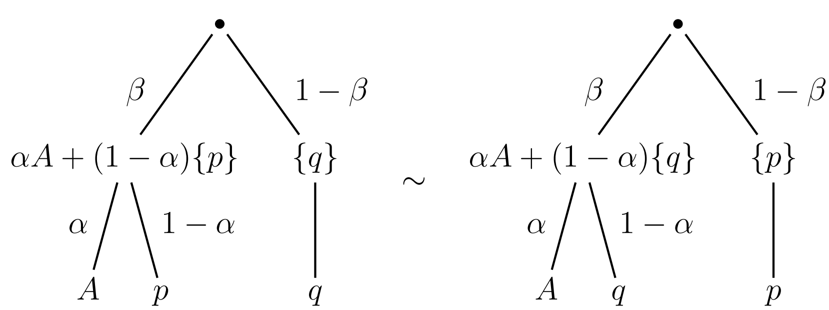

Figure 2: Reversibility of Degenerate Decisions ($\beta = 1/(2-\alpha)$)

We will call a choice out of a singleton menu as a degenerate decision. To interpret Axiom 5, consider Figure 2. The first tree represents $\beta\delta_{\alpha A+(1-\alpha)\{p\}} + (1-\beta)\delta_{\{q\}}$, where the individual makes a choice out of the menu $\alpha A + (1-\alpha)\{p\}$ with probability β, and makes a degenerate choice out of the menu $\{q\}$ with probability $1-\beta$. A choice out of the menu $\alpha A + (1-\alpha)\{p\}$ can be interpreted as a contingent plan, where initially in period 2 the individual determines a lottery out of A, and then her choice out of A is executed with probability α and the fixed lottery p is executed with the remaining $1-\alpha$ probability. Similarly, the second tree represents $\beta\delta_{\alpha A+(1-\alpha)\{q\}} + (1-\beta)\delta_{\{p\}}$ where the roles of p and q are reversed.

If one interprets the individual's behavior as one of costly contemplation/subjective information acquisition, then her optimal contemplation strategy might change as the probability α that her choice out of A is executed changes, since her return to contemplation will be higher for higher values of α. However, since the probability that her choice out of A will be executed is the same in both $\alpha A+(1-\alpha)\{p\}$ and $\alpha A+(1-\alpha)\{q\}$, it is reasonable to expect that her contemplation strategy would be the same for both contingent planning problems, although she need not be indifferent between $\delta_{\alpha A+(1-\alpha)\{p\}}$ and $\delta_{\alpha A+(1-\alpha)\{q\}}$ depending on her preference between $\delta_{\{p\}}$ and $\delta_{\{q\}}$. The RDD axiom requires the individual to be indifferent between the two trees in Figure 2 when the probabilities of the paths leading to lotteries p and q are the same, i.e. when $\beta(1-\alpha) = 1-\beta$, or equivalently, $\beta = 1/(2-\alpha)$.

Given a max-LHA representation $(\mathcal{M}, c)$, we show that RDD is equivalent to the following consistency requirement on the set of measures $\mathcal{M}$:

Definition 4 A *Reduced-Form Costly Contemplation (RFCC) representation* is a max-LHA representation $(\mathcal{M}, c)$ where the set $\mathcal{M}$ is *consistent*: For each $\mu, \nu \in \mathcal{M}$ and $p \in \triangle(Z)$,

$$\int_{\mathcal{U}} (u \cdot p)\, \mu(du) = \int_{\mathcal{U}} (u \cdot p)\, \nu(du).$$

An RFCC representation $(\mathcal{M}, c)$ is *monotone* if all the measures in $\mathcal{M}$ are positive.

We show in Ergin and Sarver (2008a) that the consistency condition above is key for the interpretation of the max-LHA representation as a subjective information acquisition problem. More specifically, V satisfies Equation (1) for some monotone RFCC representation $(\mathcal{M}, c)$ if and only if

$$V(A) = \max_{\mathcal{G} \in \mathbf{G}} \left\{ \mathbb{E}\left[\max_{p \in A} \mathbb{E}\big[U \big| \mathcal{G}\big] \cdot p \right] - c(\mathcal{G}) \right\} \tag{6}$$

where $(\Omega, \mathcal{F}, P)$ is a probability space, $U : \Omega \to \mathbb{R}^Z$ is a random vector interpreted as the individual's state-dependent utility, $\mathbf{G}$ is a collection of sub-σ-algebras of $\mathcal{F}$ representing the set of signals that the individual can acquire, and $c(\mathcal{G})$ denotes the cost of subjective signal $\mathcal{G} \in \mathbf{G}$.[11]

As a special case of the max-LHA representation, the RFCC representation satisfies PERU. However, it always satisfies indifference to timing of resolution of uncertainty when restricted to temporal lotteries, i.e. for all $p, q \in \triangle(Z)$ and $\alpha \in (0,1)$:

$$\alpha\delta_{\{p\}} + (1-\alpha)\delta_{\{q\}} \sim \delta_{\{\alpha p+(1-\alpha)q\}}.^{12}$$

Therefore, an individual with RFCC preferences never has a strict PERU unless if she has non-degenerate choices in period 2.

We next present an RFCC representation theorem as an application of Theorem 1. Since RFCC representation is a special case of the max-LHA representation, the uniqueness of the RFCC representation is immediately implied by the uniqueness of the max-LHA representation.

Theorem 4 *A. The preference $\succsim$ has a RFCC representation if and only if it satisfies Axiom 1, PERU, and RDD.*

B. The preference $\succsim$ has a monotone RFCC representation if and only if it satisfies Axiom 1, PERU, RDD, and monotonicity.

4.3 Kreps and Porteus (1978) and DLR (2001)

In this section, we will introduce a generalization of Kreps and Porteus (1978) and DLR (2001) additive representations, and characterize the intersection of this class with LHA preferences. The first axiom we consider is the standard von Neumann-Morgenstern independence axiom imposed on the second-stage uncertainty. It is satisfied by RFCC preferences, but not by SSMP and SSV preferences, since in the latter contexts the individual may benefit from hedging even when she makes no choice in period 2.

Axiom 6 (Second-Stage Independence) *For any $p, q, r \in \triangle(Z)$, and $\alpha \in (0,1)$,*

$$\delta_{\{p\}} \succ \delta_{\{q\}} \quad \Rightarrow \quad \delta_{\{\alpha p+(1-\alpha)r\}} \succ \delta_{\{\alpha q+(1-\alpha)r\}}.$$

Under weak order and continuity, the following axiom from Kreps (1979) guarantees that every menu is indifferent to its best singleton subset. Kreps and Porteus (1978)

[11] The costly contemplation representation in Equation (3) is similar to the functional form considered by Ergin (2003), whose primitive is a preference over menus taken from a finite set of alternatives.

[12] This property can also be established directly as a consequence of RDD and first-stage independence.

assume the same relationship between the individual's ranking of menus and alternatives.[13]

Axiom 7 (Strategic Rationality) *For any* $A, B \in \mathcal{A}$, $\delta_A \succsim \delta_B$ *implies* $\delta_A \sim \delta_{A\cup B}$.

Suppose there exists a continuous function $V : \mathcal{A} \to \mathbb{R}$ such that $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$. Then, the preference $\succsim$ satisfies strategic rationality if and only if

$$V(A) = \max_{p\in A} V(\{p\}), \tag{7}$$

i.e. the restriction of $\succsim$ to temporal lotteries determines the entire preference $\succsim$. If such an individual is indifferent to timing of resolution of uncertainty when choosing among temporal lotteries, then Equation (7) implies that she must always be indifferent to timing of resolution of uncertainty. This is in contrast with RFCC preferences, where the individual is indifferent to timing of resolution of uncertainty when choosing among temporal lotteries, but may exhibit a strict PERU when she faces non-degenerate choices in period 2. Although Kreps and Porteus (1978)'s setup is rich enough to distinguish between attitudes towards timing of resolution of uncertainty over temporal lotteries (without period 2 choice) and over more general lotteries over menus (with period 2 choice), the fact that they implicitly impose Axiom 7 throughout their analysis prevents them from doing so.

A second implication of strategic rationality is that it rules out a strict preference for flexibility (Kreps (1979)), i.e. situations where the union of two menus is strictly better than each menu separately: $\delta_{A\cup B} \succ \delta_A$ and $\delta_{A\cup B} \succ \delta_B$. The reason is that, by Equation (7), the individual behaves as if has she has no uncertainty in period 1 about her ex post preference ranking over $\triangle(Z)$.

The following axiom from DLR (2001) is the standard independence requirement applied to convex combinations of menus when there is no first-stage uncertainty.

Axiom 8 (Mixture Independence) *For any* $A, B, C \in \mathcal{A}$ *and* $\alpha \in (0, 1)$,

$$\delta_A \succ \delta_B \quad \Rightarrow \quad \delta_{\alpha A+(1-\alpha)C} \succ \delta_{\alpha B+(1-\alpha)C}.$$

[13] To be precise, Kreps and Porteus (1978) consider both a period 1 preference $\succsim$ over first-stage lotteries in $\triangle(\mathcal{A})$ and a period 2 preference $\succsim_2$ over second-stage lotteries in $\triangle(Z)$. It is easy to show that imposing their temporal consistency axiom (Axiom 3.1 in their paper) on this pair of preferences ($\succsim$,$\succsim_2$) implies that the period 1 preference $\succsim$ satisfies strategic rationality. Conversely, if the period 1 preference $\succsim$ satisfies strategic rationality along with continuity, then there exists *some* period 2 preference $\succsim_2$ such that the pair ($\succsim$,$\succsim_2$) satisfies their temporal consistency axiom. Moreover, in this case, the period 1 preference $\succsim$ satisfies our second-stage independence axiom if and only if this period 2 preference $\succsim_2$ satisfies the substitution axiom of Kreps and Porteus (1978, Axiom 2.3).

It is easy to see that mixture independence is stronger than second-period independence, but in the presence of Axiom 1, it is weaker than the combination of second-period independence and strategic rationality.

We next consider a class of representations that generalize the DLR (2001) additive representation where there is no objective first-stage uncertainty, and Kreps and Porteus (1978) representation where there is no subjective second-stage uncertainty. Unlike the Kreps and Porteus (1978) representation, the following class of representations are compatible with a strict preference for flexibility.

Definition 5 A *Kreps-Porteus-Dekel-Lipman-Rustichini (KPDLR) representation* is a pair (ϕ, μ) where μ is a finite signed Borel measure on $\mathcal{U}$ and $\phi : [a, b] \to \mathbb{R}$ is a Lipschitz continuous and strictly increasing function where $[a, b] = \{\int_{\mathcal{U}} \max_{p \in A} u \cdot p \, \mu(du) : A \in \mathcal{A}\}$ and $-\infty < a \leq b < +\infty$, such that $P \succsim Q$ if and only if $\mathbb{E}_P[V] \geq \mathbb{E}_Q[V]$, where $V : \mathcal{A} \to \mathbb{R}$ is defined by:

$$V(A) = \phi\Big(\int_{\mathcal{U}} \max_{p \in A} (u \cdot p) \, \mu(du) \Big). \tag{8}$$

A *monotone KPDLR representation* is a KPDLR representation (ϕ, μ) where μ is a positive measure. A *Kreps-Porteus representation* is a monotone KPDLR representation (ϕ, μ) where $\mu(\mathcal{U} \setminus \{u\}) = 0$ for some $u \in \mathbb{R}^Z$.

It is easy to see that two KPDLR representations (ϕ, μ) and (ψ, ν) induce the same preference if and only if there exists $\lambda, \alpha > 0, \beta \in \mathbb{R}$ such that $\mu = \lambda \nu$ and $\phi(t) = \alpha \psi(t/\lambda) + \beta$. Since it is possible to have $\int_{\mathcal{U}} (u \cdot p) \, \mu(du) = \lambda \int_{\mathcal{U}} (u \cdot p) \, \nu(du)$ for all $p \in \triangle(Z)$ even when $\mu \neq \lambda \nu$, this implies in particular that in KPDLR representations, the preference restricted to temporal lotteries does *not* determine the entire preference. On the other hand, in a Kreps-Porteus representation (ϕ, μ) where $\mu(\mathcal{U} \setminus \{u\}) = 0$, the V in Equation (8) can be rewritten as:

$$V(A) = \phi\Big(\max_{p \in A} v \cdot p \Big), \tag{9}$$

where $v = \mu(u) u$. Since V in Equation (9) satisfies Equation (7), in this case the preference restricted to temporal lotteries does determine the entire preference.

We are ready to state our KPDLR and Kreps-Porteus representation results. Theorem 5.B is a special case of Theorem 1 in Kreps and Porteus (1978).[14]

[14] The only difference is that Kreps and Porteus (1978) only require ϕ to be continuous. We additionally require Lipschitz continuity of ϕ, since we impose the L–continuity axiom throughout the paper.

Theorem 5 *A. The preference $\succsim$ has a KPDLR representation if and only if it satisfies Axiom 1 and mixture independence.*

B. The preference $\succsim$ has a Kreps-Porteus representation if and only if it satisfies Axiom 1, second-stage independence, and strategic rationality.

C. If the preference $\succsim$ has the KPDLR representation (ϕ, μ), then $\succsim$ satisfies PERU [PLRU] if and only if ϕ is convex [concave].

Since a Kreps-Porteus preference satisfies Axiom 7, the individual has PERU [PLRU] if and only if she has PERU [PLRU] for temporal lotteries. More generally, Theorem 5.C implies that if an individual's preference has a KPDLR representation and has the property that for every $A \in \mathcal{A}$ there exists a $p \in \triangle(Z)$ such that $\delta_A \sim \delta_{\{p\}}$, then the individual has PERU [PLRU] if and only if she has PERU [PLRU] for temporal lotteries. Therefore, under the aforementioned property, the individual must have the same attitudes towards timing of resolution of uncertainty over temporal lotteries and over more general lotteries over menus in a KPDLR representation.

The class of KPDLR preferences need not be a subset of LHA representations, but the subclass satisfying PERU or PLRU do. We next characterize the subclass of KPDLR preferences which satisfies PERU and PLRU within the class of LHA preferences.

Theorem 6 *Let $V : \mathcal{A} \to \mathbb{R}$ and let μ be a finite signed Borel measure on $\mathcal{U}$. Then, there exists a KPDLR representation (ϕ, μ) with convex [concave] ϕ such that V is given by Equation* (8) *if and only if there exists a max-LHA [min-LHA] representation $(\mathcal{M}, c)$ such that V is given by Equation* (1) *[*(2)*] and $\mathcal{M} \subset \{\lambda\mu : \lambda \in \mathbb{R}_+\}$.*

Theorem 6 suggests that if the KPDLR representation satisfies PERU or PLRU, then it is possible to rewrite the function V in Equation (8) as a LHA representation where all measures are multiples of the fixed measure μ. Consider the case of a positive measure μ, and consider again the probability measure π on $\mathcal{V} = \mu(\mathcal{U})\mathcal{U}$ that (heuristically) puts weight $\pi(v) = \mu(u)/\mu(\mathcal{U})$ on each $v = \mu(\mathcal{U})u \in \mathcal{V}$. An interpretation of the LHA representation in Theorem 6 is that all actions lead to the same distribution π over ex post utilities in $\mathcal{V}$, but each action $\lambda\mu \in \mathcal{M}$ changes the magnitude of the ex post utilities by a common scalar multiple λ.[15] In the special case of Kreps-Porteus preferences, all measures are degenerate and put their weight on the same ex post von

[15]Suppose without loss of generality that $\max\{\lambda : \lambda\mu \in \mathcal{M}\} = 1$. Then, one can also interpret each action $\lambda\mu \in \mathcal{M}$ to lead to the distribution π over $\mathcal{V}$ with probability λ, and to the ex post preference $0 \in \mathbb{R}^Z$ with probability $1 - \lambda$. Under this interpretation, the choice of action affects the probability of the 0 ex post preference, but not the conditional probability distribution π over $\mathcal{V}$.

Neumann-Morgenstern utility function over $\triangle(Z)$. In particular, the individual has no uncertainty about her ex post preference ranking over lotteries in $\triangle(Z)$, and different actions only affect the strength of her ex post preference.

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