Disciplining expectations: adding survey expectations in learning models

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Abstract

Learning estimation of DSGE models is becoming a popular alternative to traditional rational expectation estimation. One of the most cited advantages of the introduction of a learning mechanism in DSGE models is that it generates the observed persistence in economic variables such as inflation and output. The evolution of the expectations obtained by learning is, however, not necessarily compatible with direct measurements of expectations such as the ones reported by surveys. I estimate a DSGE model where the expectations generated by the learning mechanism are restricted to be consistent with the dynamic of the surveys expectations. The results show that the standard learning procedure usually overestimates the persistence of the estimated model. The intuition behind these results is that the parameters and expectations estimates used in the learning procedures are designed to match as closely as possible the dynamic of current and past values of the macroeconomic variables included in the model. Therefore, including the dynamic of the surveys expectations in the objective function implies a restriction in the built-in model expectations and hence, less degree of freedoms to match the persistence observed in the actual data.

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1. Motivation

The New Keynesian model, solved and estimated under the assumption of rational expectations (RE), is the benchmark model for monetary policy in both academy and policy debates. Although theoretically appealing, its canonical version fails to match some important features of post-World War II US data such as high level of inflation persistence and the delayed and gradual response of inflation to unanticipated monetary policy shocks. Attempts to match these dimensions of the data are the introduction of habits in consumption and inflation indexation in a fully-rational setup. These procedures, however, can be seen as *ad hoc* modelling.

One alternative approach to deal with these empirical failures is to use models of expectations formation in which agents revise their forecast rule over time. This approach is referred to as (adaptive) *learning*[1] and postulates that agents do not have perfect knowledge of how the economy works. Hence, similar to applied economist or econometricians, whenever new data becomes available, the agents try to improve their knowledge about the economy by reformulating and re-estimating their models, also called *perceived law of motion* (PLM). The algorithm agents use to update their model estimations is *recursive least squares* (RLS).

The evaluation of the persistence of inflation and other macroeconomic variables implied by learning is addressed among others by Orphanides and Williams (2005a, 2005b) and Milani (2007)[2]. Both depart from the usage of RLS as the algorithm employed by agents for the estimation of the PLM and adopt instead the "constant gain" RLS. Under this algorithm the "gain" or weight of the last observations remains constant along the whole recursive estimation (in RLS the weight of the last observations tends to zero as the number of observations increases)[3]. In other words, in this case agents are aware of potential structural shifts and therefore consider that the last observations contain more relevant information of the current structure of the economy. In a calibration analysis, Orphanides and Williams (2005b) find that constant gain RLS modeling generates significantly higher persistence of inflation in comparison to the RE equilibrium[4]. Milani (2007) estimates a DGSE New Keynesian model again using constant gain RLS and shows that the parameter estimates of habit in consumption and inflation indexation stop being significant once learning is adopted. For this

[1] Under adaptive learning, agents have limited common knowledge since they estimate their own perceived laws of motion. Other approaches of learning are "eductive learning" and "rational-learning". See Evans and Honkapohja (2001) for a detailed exposition of these two types.

[2] A lot of research has been implemented to evaluate the implications of learning for the equilibrium determinacy and the design of optimal monetary policy. Examples of these type of studies are Bullard and Mitra (2002, 2004), Evans and Honkapohja (2003, 2006), Carlstrom and Fuerst (2004) and Evans and McGough (2005) among many others.

[3] This is equivalent to using weighted least squares with weights declining geometrically in time as we move backwards from the current date.

[4] In Orphanides and Williams (2005c) the authors show that under learning disturbances can give rise to significant and persistent deviations of inflation expectations from those implied by rational expectations. They refer these situations as "inflation scares" and relate them with the evidence on the excess sensitivity of yields on long-term bonds to aggregate shocks.

reason he suggests that persistence arises in the economy mainly from expectations and learning.

In the present study I analyze the effects of adding information of survey expectations on the estimated persistence under learning. As in Orphanides (2004) and Orphanides and Williams (2005a), I employ survey expectations as proxies of actual agents' expectations. Nevertheless, instead of treating survey expectations as exogenous variables, I assume that they are the outcome of a learning mechanism of expectation formation. This implies a consistency restriction between the model-implied expectations and the data of surveys expectation that, to the best of my knowledge, has not been formally adopted in any DSGE estimation.

The results provide evidence that adding survey expectations to the learning estimation, referred to as “restricted-learning”, reduces significantly the persistence of the responses of the economy to demand, supply and monetary policy. This is due to the fact that parameters and expectations estimates used under learning are designed to match as closely as possible the dynamic of current and past values of the macroeconomic variables included in the model. Therefore, adding survey expectations in the set of observable variables and restricting the built-in model expectations to be as similar as possible to the surveys implies less degrees of freedom to match the persistence observed in the actual data.

Additionally, under restricted-learning parameter estimates are better identified and closer to values obtained in studies that use microevidence and compatible with findings of studies based on exogenous survey expectations (Adam and Padula (2003)). Moreover, the time-varying dynamics of the impulse-response functions shows similarities to the empirical evidence showed by Boivin and Giannoni (2006) and Leduc et al. (2007). Specifically, the response of inflation and output to monetary policy and demand shocks has decreased since the 80s while the response of expected inflation to monetary policy shock becomes stronger.

The closest reference to the present study is Orphanides and Williams (2005a). There surveys expectations and learning are used in a reduced form New Keynesian model. Orphanides and Williamns implement a two step estimation: First they use surveys and other real-time information to estimate the coefficients of their model. Second, they calibrate the value of the constant gain that generates forecast that fits the data on survey expectations the closest possible. Keeping the estimated residuals of the model fixed, they implement counterfactual experiments to analyze what was the driving factor behind the high inflation period observed during the 70s. In contrast to their approach, this study exploits the cross equation restrictions implied by the compatibility restriction between survey expectations and the expectations generated by learning. These restrictions play an important role in the results of this study. Moreover, the full-information estimation procedure I use (Bayesian estimation) allows me to implement an impulse-response analysis and simulations of the series. These two procedures are required for the evaluation of the persistence in the dynamic of the model. Finally, I use a structural and more conventional form of the New Keynesian model which allows me to compare my parameter estimates with the ones obtained under standard learning, such as in Milani (2007), and under RE assumption.

Some studies which are related to my approach are the ones that use survey expectations as a direct measure of actual expectations in the estimation of New Keynesian models. Early references are Roberts (1998) and Rudebusch (2002). Orphanides (2004) uses survey expectations of inflation and other real-time measures to estimate a Taylor rule, while Adam and Padula (2003), Nunes (2005) and Paloviita (2004) evaluate the characteristics of the Phillips curve when surveys are added as proxy of expected inflation. Paloviita (2007) estimates a three-equation New Keynesian model using GMM and annual data of survey expectations for the Euro zone for the period 1990 – 2004. Using a TVC-VAR, Canova and Gambetti (2008) find that shocks related with survey expectations play a significant and important role in explain the volatility and persistence of the inflation and output during the whole sample. Leduc et al (2007) use series of inflation surveys expectations in a VAR estimation and find that "shocks to expectations", jointly with an accommodating monetary policy explain the persistent high inflation in the 1970's.

Finally, it is important to mention the evidence obtained from estimating medium-scale DSGE models under learning. Slobodyan and Wouters (2007) reveal that the principal difference between the estimation of a medium-scale DSGE model under RE and learning is the higher persistence of the estimated dynamic of the latter one[5]. One interesting fact of their study is that they do not find a systematic alteration of the estimated structural parameter such as the reduction in the importance of habit in consumption and indexation of inflation as in Milani (2007). One explanation for this discrepancy could be the type of habits employed in both studies (Milani's study is based on uses internal habits while Slobodyan and Wouters' approach on external habits). For this reason, one extension of the present paper is to evaluate if my results are robust when using a medium-scale model.

The rest of the paper is organized in the following way. The next section describes the methodology used in this paper in order to incorporate survey expectations information in the estimation of a DSGE model. Section three describes the standard version of the New Keynesian model employed for the analysis and section four introduces the data and priors used for the implementation of the Bayesian estimation. Section five describes the results in terms of posterior distributions, impulse-response analysis and simulated persistence. Finally, section six concludes.

2. The methodology

In this section I choose a simple model in order to illustrate the features of the methodology implemented in this study. As a first step, I represent the solution of the model obtained under learning. Subsequently, I characterize the new form that the solution acquired after imposing the compatibility restriction between built-in-model expectations and survey expectations. After discussing the theoretical features of both solutions I estimate the simple model using actual data and survey expectation of inflation. By this estimation the empirical relevance of

[5] Additionally, they find that improvements in the fit under learning depend significantly in the adoption of the initial beliefs.

the restriction implied by the use of surveys becomes clear at the same time of keeping simple the interpretation of the estimates.

A. Learning estimation of a simple model

The model consider in this section is the following:

$$P_t = \beta E_t P_{t+1} + \alpha P_{t-1} + \varepsilon_t \tag{1}$$

Under adaptive learning, which is the most common approach to modeling learning, economic agents behaves as econometricians: using their own perceived law of motion they try to improve their knowledge of the stochastic process of the economy using all the information available in each single point in time. In contrast with rational expectation agents, under this approach agents have imperfect knowledge which is reflected in their use of a perceived law of motion (PLM).

Agents are assumed to use the PLM:

$$P_t = aP_{t-1} + u_t \tag{2}$$

As it is mentioned by Evans and Honkapohja (2001), the PLM has the same functional form as the rational expectation equilibrium (REE) but possibly different coefficients since agents do not know the REE. In adaptive learning it is commonly assumed that agents uses even *recursive least squares* (RLS) or *constant-gain recursive least squares* (CG-RLS) to estimate the PLM. Asymptotically, under the first procedure we should expect to arrive to the REE while using the second one we converge to a distribution around the REE. In this study I choose CG-RLS because it is the technique chosen for the estimation New Keynesian models.

The constant gain least squares expression of the coefficient *a* in the PLM is:

$$\hat{a}_t = \frac{\sum_{i=2}^{t}(1-g)^{t-i}P_{i-1}P_i}{\sum_{i=2}^{t}(1-g)^{t-i}P_{i-2}^2}$$

g is called the *constant gain coefficient* and determines the discount factor of the relative importance of past information in the estimation of $\hat{a}_t$.The recursive form of the previous expression is:

$$R_t = R_{t-1} + g(P_{t-1}^2 - R_{t-1}) \tag{3a}$$

$$\hat{a}_t = \hat{a}_{t-1} + gR_t^{-1}P_{t-1}(P_t - \hat{a}_{t-1}P_{t-1}) \tag{3b}$$

With the estimates of *a* agents forecast $\hat{P}_{t+1} = \hat{a}_{t-1}P_t$[6]. These forecast lead to a temporary actual law of motion (ALM)

$$P_t = \frac{\alpha}{1-\beta\hat{a}_{t-1}}P_{t-1} + \frac{1}{1-\beta\hat{a}_{t-1}}\varepsilon_t \tag{4}$$

[6] In this example I use $\hat{a}_{t-1}$ instead of $\hat{a}_t$ to generate the forecast of P_{t+1} in order to avoid an endogeneity problem in the estimation of the actual law of motion. This procedure is standard in the learning literature. See for instance Carceles-Poveda and Giannitsarou (2007).

The equations that summarize the equilibrium in this economy under learning are equations (3a), (3b) and (4) for a given initial conditions R_0 and $\hat{a}_0$. The structural parameters to be estimated are α, β, σ_ε and g. Studies such as Milani (2007,2008) and Slobodyan and Wouters (2007,2008) are references of the new literature that estimate the constant gain parameter (g) jointly with the rest of structural parameters of DSGE models.

The selection of the initial conditions is an important issue in the estimation of learning (see discussion in Slobodyan and Wouters (2007) and Carceles-Poveda and Giannitsarou (2007)). I address this point in Section 5 to analyze the robustness of the main estimations presented in this study.

B. Survey expectations-implied restrictions

As I mentioned in the introduction, learning estimation arose because of some clear failures of the assumptions of rational expectations models. The use of survey expectations is one of the most common ways to test rational expectation assumptions. However, as far as I know, the information contained in these surveys is not considered formality to test how good the predictions obtained under learning estimation are.

In this study I implement a learning estimation of the structural parameters of the model imposing a compatibility condition between the expectations generated by the model and those contained in surveys (represented as $P^s_{t,t+1}$). Specifically, in the optimization process I include the minimization of the residues of the following equation:

$$P^s_{t,t+1} = \hat{a}_{t-1} P_t + u_t \tag{5}$$

It is important to notice that information is available until time t. $P^s_{t,t+1}$ represents survey expectation of P_{t+1} make at time t. The term $\hat{a}_{t-1} P_t$ is the learning forecast of P_{t+1} (remember that equation (4) is obtained from replacing $E_t P_{t+1}$ by $\hat{a}_{t-1} P_t$ in equation (1)) and u_t represents a measurement or misspecification i.i.d. error (mean-zero with standard deviation of σ_u)[7].

Under the assumption that survey expectations represent the actual expectations of the economic agent ($E_t P_{t+1} = P^s_{t,t+1}$), equations (5) and (1) can be rewritten as the following system of equations:

$$P_t = \frac{\alpha}{1-\beta\hat{a}_{t-1}} P_{t-1} + \frac{1}{1-\beta\hat{a}_{t-1}} \varepsilon_t + \frac{\beta}{1-\beta\hat{a}_{t-1}} u_t \tag{6a}$$

$$P^s_{t,t+1} = \frac{\alpha\hat{a}_{t-1}}{1-\beta\hat{a}_{t-1}} P_{t-1} + \frac{\hat{a}_{t-1}}{1-\beta\hat{a}_{t-1}} \varepsilon_t + \frac{1}{1-\beta\hat{a}_{t-1}} u_t \tag{6b}$$

In case survey expectations and learning expectations were exactly the same (so $P^s_{t,t+1} = \hat{a}_{t-1} P_t$ and $u_t = 0$), (6a) becomes the same as equation (4) and equation (6b) becomes redundant. However, in any other case the introduction of the survey-expectation

[7] Strictly speaking, the error term should have being referred as $u_{t,t+1}$ to make explicit that this error term is related with the forecast of a variable in t+1. However, to leave as simple as possible the notation I do not include this clarification in all the places where u_t appears.

compatibility condition (equation (5)) implies additional cross equations restrictions over the structural parameters of the model and the constant gain.

Another way to use survey expectations to test how different estimates are obtained under learning is to follow the procedure of Orphanides (2004). He replaces survey expectations wherever expectations appear in the model treating them as exogenous[8]. Under my procedure, however, it is not only possible to evaluate the differences in the parameter estimates but also in the dynamic of the resulting estimations. Evaluating this feature is important because it is one of the most cited advantages of learning. Therefore, by using the procedure described above we can see if the persistency obtained under learning is compatible with surveys information.

3. The model

For the estimations implemented in this study I use a similar version of the simple New Keynesian model used by Milani (2007). However, I consider two variations to the original version. The first one consists on using external habits in consumption instead of internal habits. Models such as Smets and Wouters (2007), now the main reference in DSGE estimation of New Keynesian model, use this type of habits. The second variation consist on using the difference between the (log) of the output and the (log) potential output instead of output gap. This modification allows me to use directly the information contained in the surveys of output growth. The linearized version of this model is the following:

$$Y_t = \frac{1}{1+\alpha} E_t Y_{t+1} + \frac{\alpha}{1+\alpha} Y_{t-1} - \sigma \frac{1-\alpha}{1+\alpha} [i_t - E_t \Pi_{t+1} - e_{1t}] \tag{7}$$

$$\Pi_t = \frac{\beta}{1+\beta\gamma} E_t \Pi_{t+1} + \frac{\gamma}{1+\beta\gamma} \Pi_{t-1} + \frac{\kappa}{1+\beta\gamma} (Y_t - Y_t^p) + e_{2t} \tag{8}$$

$$i_t = \phi_i i_{t-1} + (1-\phi_i)[\chi^{\pi} E_t \Pi_{t+1} + \chi^y E_t Y_{t+1} - \chi^y Y_t^p] + e_{3t} \tag{9}$$

Y_t, Y_t^p, Π_t, i_t represent the (log) real output, the (log) real potential output, (net) inflation and (net) interest rate, respectively. The parameters that describe the private sector of the economy are the inverse of the risk aversion coefficient (σ), the slope of Phillips curve (κ), the degree of habits in consumption (α), the degree of inflation indexation (γ) and the intertemporal discount factor (β). The monetary policy is described by three parameters: the degree of interest persistency (ϕ_i) and the responses of the interest rate to the inflation (χ^{π}) and output gap (χ^y). e_{1t}, e_{2t} and e_{3t} represent the IS (or demand), supply and monetary policy shocks, respectively. The first two are AR(1) processes while the later is a white noise.

[8] Orphanides (2004) uses this procedure to compare his parameter estimates of the Taylor rule for the US economy with the estimates of Clarida, Gali and Gertler (2000) based on the rational expectation assumption.

To write this specification of the model I adopt the assumption that $E_t Y_{t+1}^p = Y_t^p$. Additionally, Y_t^p is modeled separately and prior to the estimation of the complete model. The specification that fits better this variable is an AR(1) process with an error term modeled as well as an AR(1).

Given that the estimation is implemented under learning, it is necessary to specify a model of expectations formation for the agents. I follow the adaptive learning literature in assuming that agents behave as econometrician in order to learn about the relevant parameters over time. In particular, agents estimate the linear specification:

$$Z_t = a_t + b_t \pi_{t-1} + c_t Y_{t-1} \tag{10}$$

where $Z_t \equiv [\pi_t, Y_t]'$ and a_t, b_t and c_t are coefficient vectors of appropriate dimensions identified as *beliefs* in the learning literature. The previous expression represents the Perceived Law of Motion (PLM) and includes only observable variables. Milani (2008) reports not differences in the parameter estimates between using this type of PLM and one that includes non observable states of the model (e.g. e_{1t} and e_{2t}).

Agents use historical data to learn the estimate the relevant model parameters. As additional data become available in subsequent periods, they update their estimates of the coefficients (a_t, b_t and c_t) or beliefs according the constant-gain learning formula:

$$\hat{\phi}_t = \hat{\phi}_{t-1} + g \mathrm{R}_{t-1}^{-1} X_t (Z_t - X_t' \hat{\phi}_{t-1}) \tag{11a}$$

$$\mathrm{R}_t = \mathrm{R}_{t-1} + g(X_{t-1} X_{t-1}' - \mathrm{R}_{t-1}) \tag{11b}$$

where $\phi_t = (a_t', vec(b_t, c_t)')'$ describes the updating of the learning rule coefficients, and Rt the updating of the matrix of second moments of the stacked regressors $X_t \equiv \{1, \pi_t, Y_t\}_0^{t-1}$. *g* denotes the constant-gain coefficient and provides a simple way to model learning of agents concerned about future unknown structural breaks.

Using updated parameter estimates provided by (11a) and (11b) and following the expression of the PLM in (10), economic agents form expectations for t+1:

$$\hat{E}_t \begin{bmatrix} \pi_{t+1} \\ Y_{t+1} \end{bmatrix} = \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix} + \begin{bmatrix} b_{1t-1} & c_{1t-1} \\ b_{2t-1} & c_{2t-1} \end{bmatrix} \begin{bmatrix} \pi_t \\ Y_t \end{bmatrix} \tag{12}$$

DSGE estimation under learning considers not just the estimation of the structural parameters of equations (7)-(9) but also the expectations formation mechanism represented by (11a), (11b) and (12) given some initial conditions for ϕ and *R*.

When survey expectations of inflation and output are included in the DSGE estimation under learning the terms $E_t \pi_{t+1}$ and $E_t Y_{t+1}$ are replaced by $E_t \pi_{t+1}^s$ and $E_t Y_{t+1}^s$ which indicates that the survey information for both variables is directly used in the estimation and the expression (12) is modified in the following way:

$$\begin{bmatrix} E_t\pi^s_{t+1} \\ E_tY^s_{t+1} \end{bmatrix} = \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix} + \begin{bmatrix} b_{1t-1} & c_{1t-1} \\ b_{2t-1} & c_{2t-1} \end{bmatrix} \begin{bmatrix} \pi_t \\ Y_t \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix} \tag{12'}$$

Given that under this specification expectations are observable, the equations of the expectations formation (12') include residual terms (ξ_1 and ξ_2) which are assumed to be i.i.d. (with standard deviations equal to $\sigma_{\xi 1}$ and $\sigma_{\xi 2}$). By adopting this assumption this way of modeling the expectations is immune to the Sargent and Wallace's critique. The estimation considering survey expectations, therefore, uses the equations (7)-(9), (11a), (11b) and (12') and some initial conditions for ϕ and *R*.

4. Data and priors

The information that is used for the estimations presented in this study is composed by the series of real GDP, real potential GDP, implicit GDP deflactor and Federal funds rates. These series are obtained from the FREDII databank of the Federal Reserve Bank of St. Louis. Finally, information about the survey expectations of inflation and output growth are obtained from Greenbock database.

The availability of information for survey expectations defines the sample over which the estimations are implemented. This sample covers the period 1966:1 to 1995:4, in quarterly basis. The initial values of ϕ and *R* are obtained using a pre sample estimation using revised data over the period 1955-1965.

The prior distributions for each of the parameters of the structural model and the constant-gain are summarized in the following table:

Table 1: Prior distributions

Parameters	Prior distrib.	Prior mean	Prior std
β	--	0.99	--
σ	Gamma	1	0.7
κ	Normal	0.1	0.05
α	Uniform	0.5	0.29
φ	Uniform	0.5	0.29
Φi	Uniform	0.5	0.29
χ^{π}	Normal	1.5	0.5
χ^{y}	Normal	0.5	0.25
ρ_1	Uniform	0.485	0.28
ρ_2	Uniform	0.485	0.28
σ_1	InvGamma	1	0.5
σ_2	InvGamma	1	0.5
σ_3	InvGamma	1	0.5
g	Gamma	0.031	0.022
$\sigma_{\xi 1}$	InvGamma	1	0.5
$\sigma_{\xi 2}$	InvGamma	1	0.5

5. Results

The results presented in this section can be classified in three types: parameter estimates (posterior distribution statistics), impulse-response functions and simulated persistence. These results are obtained from the estimation of the model presented in section three under rational expectation assumption, learning and restricted-learning.

A. Parameter estimates: Posterior distribution analysis

Table 2 reports the mean and standard deviation of the posterior distributions for each of the parameter of the model. The most interesting features can be summarized in the following points:

a. The intertemporal elasticity of substitution (IES) has a high mean value closer to what the micro evidence indicates when the model is estimated with restricted-learning. Under RE, the mean value of the posterior distribution of this parameter is 0.088, which implies a coefficient of risk aversion higher than 10 (a value similar to what Cecchetti et al (2007) and other studies find but higher than what in average is found). Under learning this estimate increases to 0.238 which is still out of the range expected to this parameter (the risk aversion coefficient is typically consider to be between 1 and 3). However, under restricted-learning this parameter adopts a value of close to 0.5.

   It is important to note the existence of an identification problem between the IES and the standard deviation of the perturbation term of the demand shock. It can be seem that while the median of the posterior of the IES increases the value of the standard deviation of the demand shock decreases. Figure 1 shows that both parameters have problems to converge under learning (see blue and green lines that do not converge to zero) while under restricted-learning this problem disappears.

Table 2: Posterior distribution statistics

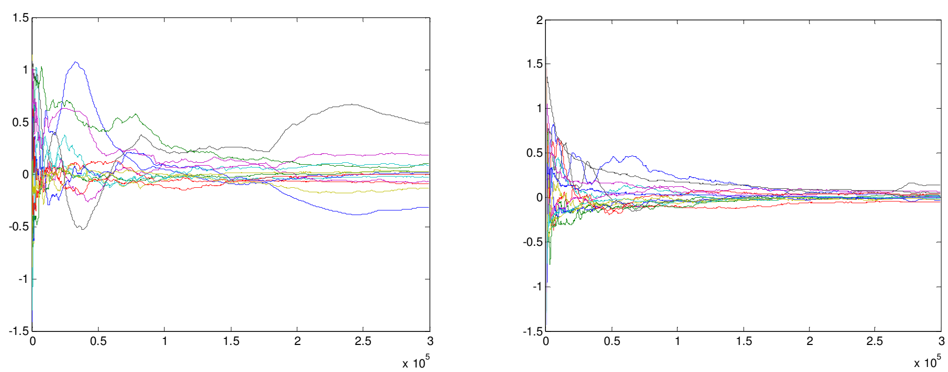
	(1) R.E.		(2) Learning		(3) Restricted learning	
Description	Mean	std	Mean	std	Mean	std
IES	0.088	0.025	0.238	0.095	0.468	0.149
PhC slope	0.014	0.016	0.028	0.031	0.145	0.065
Habits	0.772	0.046	0.760	0.048	0.778	0.028
Index. Inflat	0.032	0.029	0.024	0.024	0.961	0.038
TR: interest rate	0.860	0.023	0.871	0.028	0.844	0.026
TR: inflation	1.733	0.290	2.170	0.437	1.820	0.255
TR: output	0.784	0.204	0.733	0.194	0.844	0.185
Autoregr. dem shock	0.538	0.084	0.699	0.071	0.624	0.072
Autoregr. sup shock	0.860	0.045	0.880	0.037	0.021	0.028
S.d. dem shock	0.208	0.041	0.125	0.052	0.069	0.020
S.d. sup shock	0.002	0.001	0.008	0.001	0.011	0.001
S.d. mp shock	0.012	0.001	0.012	0.001	0.011	0.001
Gain			0.011	0.007	0.033	0.002
S.d. meas1. shock					0.012	0.001
S.d. meas2. shock					0.007	0.000
Mg. log-likelihood	1474.9		2063.1		3275.9	

b. The value of the slope of the Phillips curve hits the lower bound of zero under rational expectations and learning. This is coherent with the well-known discussion about using statistical measures of output gap in this equation. However, with restricted-learning the value of this parameter becomes significantly different from zero. This result is similar to what Adam and Padula (2003) find: when survey expectation is used in the estimation of the Phillips curve, both, output gap and marginal cost give positive values for the slope.

c. The estimate of the habits in consumption is similar for the three estimations; while the value of the indexation in inflation differs between RE/learning and the one obtained under restricted learning.

   It is important to indicate that the estimated persistence have similar foundations under RE and learning but very different under restricted-learning. In the first cases, the persistence depends on the habits and the autocorrelation of the structural shocks (specially the supply shock), while for restricted-learning the persistence is originated by the habits and the inflation indexation (almost no importance of the autocorrelation coefficients of the demand and supply shocks). Therefore, the inclusion of terms such as habit in consumption and inflation indexation is very important under restricted-learning. This result contradicts the conclusions of Milani (2007) about the irrelevance of these terms when learning is incorporated.

Figure 1: Cumsum statistics



Leaning

Restricted-learning

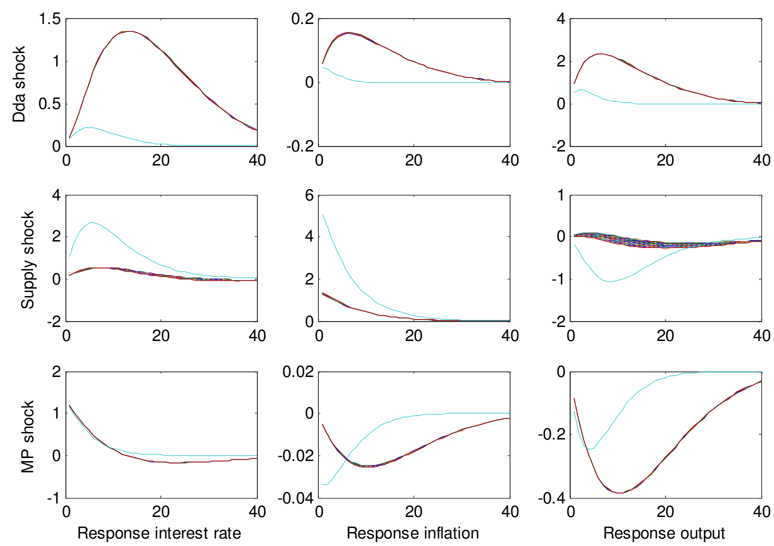
B. Impulse-response function analysis

Now I compare the Impulse-response functions (IRF) obtained under RE, learning and restricted-learning. To make this comparison possible I consider that the size of the shocks is determined by the values of the standard deviations of the perturbation terms (of the demand, supply and monetary policy shocks) obtained in the learning estimation. Additionally, it is important to mention that the IRFs calculated under learning and restricted-learning are just approximations. The correct measure of the IRFs should incorporates how the evolution of the variables, once they react to the shocks, affects the structure of the model (via the beliefs) and therefore, reinforce the response to the shocks. I follow the same procedure of Slobodyan and Wouters (2008) of estimating quasi-IRFs. This implies estimating these functions for the estimated structure at each point in time. Therefore, each graph of the IRFs for the learning and restricted-learning cases contains 144 series (which corresponds with the number of observations used in this estimation).

Figure 2 shows that under learning, the responses of inflation, output and interest rate to demand and monetary policy shocks have more inertia (takes more periods to converge to zero) than under RE. However, the response to supply shocks is weaker. The time-variability of the IRFs is almost not noticeable, except by the response of output and interest rate to supply shocks.

Figure 3 shows the IRFs for learning and restricted-learning estimations. It can be seem that in most of the cases, the IRFs fluctuates more under restricted-learning than under learning but the effects of the shocks disappear faster. Once the model´s forecasts are restricted to be compatible with surveys the persistence gained under learning is significantly reduced!

Figure 2: Impulse-response functions, Learning vs. RE



Light blue line: RE ; other(s) colour(s): Learning

Figure 3 shows as well that the time-variability of the actual law of motion is more evident under restricted-learning. This fact allows us to compare our results with what other studies find at the time of analyzing changes in the IRF using different subsamples. One important reference is Boivin and Giannoni (2006). They analyze the changes in the transmission mechanism of the monetary policy during the pre and post 1979 subsamples. In Figure 4 I plot the IRFs obtained using the structure of the model estimated for the first observation of the sample (1966:1) and the last observation of the sample (2001:4). The similarities that I find between my estimation and Boivin and Giannoni are:

i. Inflation and output responses to a monetary policy shock have become smaller in the post 1979 period. Boivin and Giannoni, using counterfactual experiments, indicate that these changes are because the monetary policy has been reacting more strongly to fluctuations in expected inflation (helping in this way to stabilize the economy in response to monetary shocks).

ii. The responses of inflation and output to a real demand shock have decreased.

iii. Output reacts stronger in the second half of the sample to supply shocks.

Figure 3: Impulse-response functions, Restricted-Learning vs. Learning

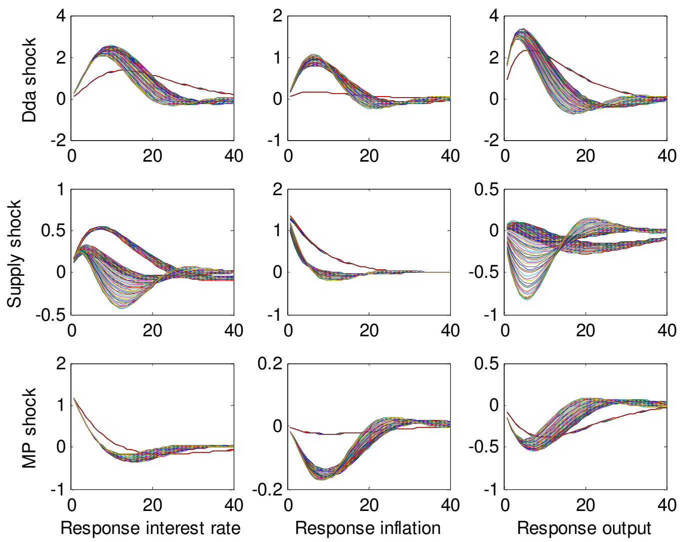
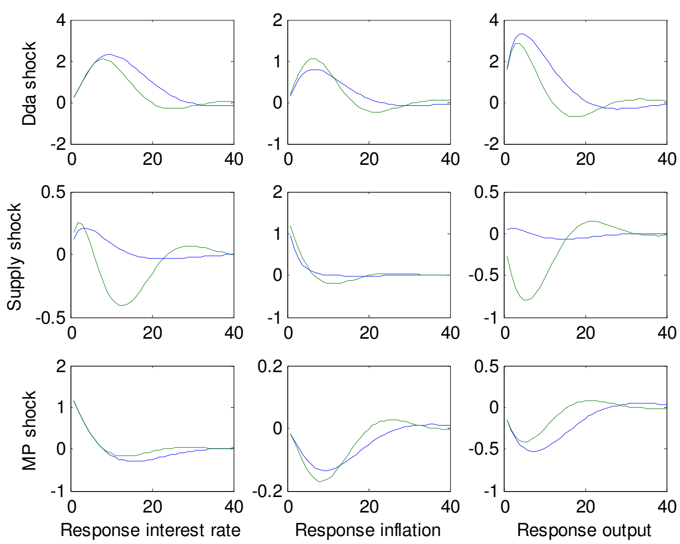
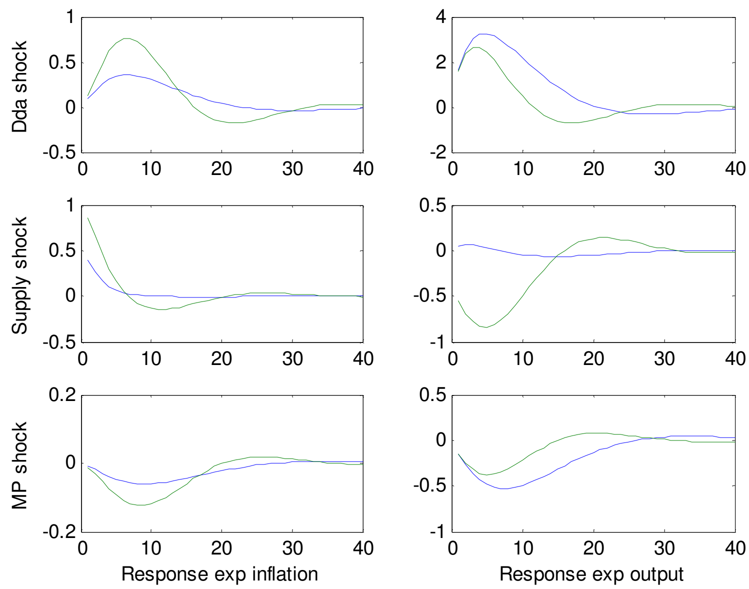


Figure 4: Impulse-response functions, Restricted-Learning in detail (1)



Blue line: 1966:1; Green line: 2001:4

Figure 5: Impulse-response functions, Restricted-Learning in detail (2)



Blue line: 1966:1; Green line: 2001:4

Figure 5 shows the same type of information that Figure 4 but this time the responses are from expected inflation and expected output. This type of evidence is not really well documented because there are few studies that include this type of variables. Leduc et al (2007) is one of the few evidences that make this type of analysis. As in Boivin and Giannoni (2006), they analyze the differences of the IRFs between the subsamples pre and post 1979. The coincidences between their results and the ones obtained in this study is that in the post 1979 subsample the response of expected inflation to monetary policy shock becomes stronger. In my estimation I find additionally that expected output responses to this shock becomes weaker. The response to a supply shock is stronger for both types of expectations at the end of the sample. In case of inflation expectations, this result is also obtained by Leduc et al (2007).

C. Estimated persistence and expected inflation

Finally, Table 3 shows a comparison of the persistence of the built-in model expected inflation obtained under RE, learning and restricted-learning. This table shows that series of expected inflation obtained under restricted-learning has closer level of persistence to the one obtained from the actual series than any of the other methods. Of course, this result is expected because the optimization process behind the estimation of the parameters includes the minimization of the residuals between the actual expectations and the estimated by the model in the case of the restricted-learning.

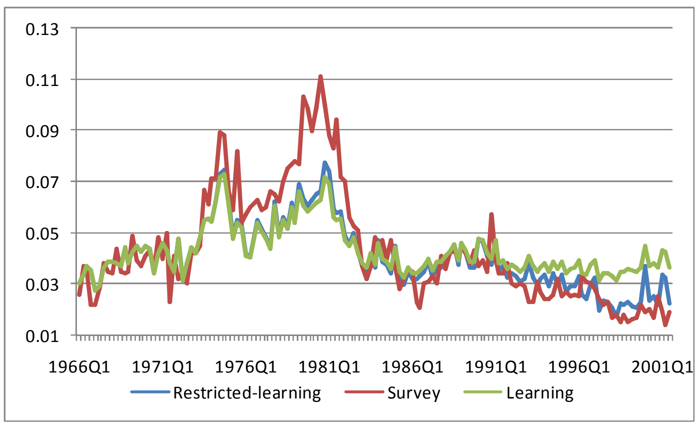
Table 3: Persistence comparison of expected inflation

Sample	Surveys	RE	Learning	Restricted Learning
1966:1 - 2001:4	0.923	0.858	0.824	0.875
	(0.033)	(0.044)	(0.047)	(0.042)
1966:1 - 1979:2	0.833	0.708	0.776	0.783
	(0.078)	(0.102)	(0.092)	(0.092)
1984:1 - 2001:4	0.789	0.507	0.409	0.700
	(0.071)	(0.104)	(0.110)	(0.088)

Note: persistency is measured as the autocorrelation coefficient.
Standard deviations are in parentesis.

Figure 2 shows the evolution of the expected inflation obtained under standard learning and restricted-learning and the actual series of survey expectation of inflation. During the first part of the sample both estimated series are similar. But after the second half of the 80s they start to differ. The series obtained under restricted-learning keeps closer to the values of the survey expectation.

Figure 2: Expected inflation



6. Conclusions

In this study I use information on survey expectations in the estimation of a New Keynesian DSGE model under adaptive learning. Survey expectations are not treated as exogenous variables but are modeled as the outcome of a learning procedure. The introduction of this compatibility restriction affects in many dimensions the results obtained under the standard learning. First, the resulting dynamics, measured by impulse-response function, exhibit less persistence. Second, the time-variability of the impulse-response functions are much clearer and are compatible with VAR evidence reported by Boivin and Giannoni (2006) and Leduc et. al. (2007). Mainly, that the response of inflation and output to monetary policy and demand shocks has decreased in the last decade while the response of expected inflation to monetary policy shock becomes stronger. Third, some identification problems seem to be resolved (e.g. the intertemporal elasticity of substitution gets values closer to what micro evidence shows and the slope of the Phillips curve becomes significantly positive). Finally, the persistence of the model now rests more in the inflation indexation than in the supply (markup) or demand shocks. This implies that under this estimation procedure this type of artifacts that usually is needed in the estimation of RE models is still needed when learning is employed (in contrast with the evidence of Milani (2007)).

The next step in this research line consist in evaluating the impact of more sophisticated but realistic ways to model the formation of expectations, such as the one proposed by Slobodyan and Wouters (2008), over the results obtained in this model. Additionally, in order to validate the methodology presented in this paper I plan to apply to different cases where learning estimates show better results than those of rational expectation. A good starting point, for instance, is the evaluation of periods of inflation scares. Orphanides and Williams (2003) discuss the advantages of learning for these cases. However, by now it is still an open question if these advantages remain once we control for the learning estimation by survey expectations.

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[TO BE COMPLETED]