

Collateral constraints, capital specificity and the distribution of production: the role of real and financial frictions in aggregate fluctuations

Aubhik Khan

The Ohio State University

Julia K. Thomas

The Ohio State University and NBER

February 2009

ABSTRACT

We study the cyclical implications of credit market imperfections in a dynamic, stochastic general equilibrium model wherein firms face persistent shocks to both aggregate and individual productivity. In our model economy, optimal capital reallocation is distorted by two frictions. First, collateralized borrowing constraints limit the investment undertaken by small firms with relatively high productivities. Second, a quasi-specificity in firm-level capital implies investment irreversibilities that lead firms to pursue generalized (S,s) investment rules. This second friction compounds the first in implying that large and relatively unproductive firms carry a disproportionate share of the aggregate capital stock, thereby reducing endogenous aggregate total factor productivity. Moreover, because irreversibilities not only directly induce both downward and upward inertia in firm-level capital adjustment, but also tighten the borrowing limits associated with collateral constraints, they ensure that the negative consequences of a temporary tightening in financial markets are not quickly repaired. In the presence of persistent heterogeneity in both capital and total factor productivity, the effects of a financial shock can be amplified and propagated through large and long-lived disruptions to the distribution of capital that, in turn, imply large and persistent reductions in aggregate total factor productivity. Similarly, the consequences of a negative real shock can be exacerbated and prolonged in the presence of real and financial frictions. This paper seeks to measure the strength of these effects in a calibrated DSGE setting.

KEYWORDS: Financial frictions, capital adjustment frictions, irreversibilities, (S,s) policies, business cycles, total factor productivity, dynamic stochastic general equilibrium.

1 Introduction

Can a large shock to an economy's financial sector produce a large and lasting recession? Can it amplify and propagate the effects of a real shock sufficiently to transform recession into depression? In this paper, we construct a quantitative, dynamic, stochastic general equilibrium model that may better inform current and future discussions regarding the interactions of real and financial shocks in determining the size and frequency of aggregate fluctuations. In our model, firms experience persistent differences in their relative productivities, while credit market frictions interact with real frictions arising from a partial specificity in capital to yield persistent disruptions to the efficient allocation of capital across them, and thus persistent reductions in aggregate productivity. Calibrating our model to both aggregate and firm-level data, we use it as a laboratory in which to obtain quantitatively disciplined answers to the questions raised above.

In late September of 2008, Ben Bernanke urged Congress to approve a U.S. Treasury purchase of roughly \$700 billion in mortgage-backed securities in effort to revitalize various semi-frozen financial markets. In his testimony to the Joint Economic Committee, Bernanke stressed that fallout from the ongoing financial crisis would not be limited to Wall Street and the banking industry, but would have severe repercussions for economic activity overall. If not checked by a substantial policy intervention, he warned, the existing credit crisis could cause a deep and long-lived recession.

Economic data over the four months that have passed since Bernanke's remarks leave little doubt that the U.S. economy is in deep recession. In the fourth quarter of 2008, real GDP fell almost four percent at an annualized rate, the largest quarterly decline since the early 1980s. Meantime, real final sales fell five percent, and industrial production eleven percent (more than double the largest decline from the 2001 recession), while the unemployment rate has now risen to 7.5 percent. As each piece of negative news on the real sector arrives, and the difficulties in financial markets remain at the top of the business segment in nearly every newscast, the two events become increasingly difficult to disentangle. If these conditions have reawakened sleeping interest in business cycle research, they have also made vividly clear how limited are our existing macroeconomic models in their ability to answer the questions now at hand.

We develop and solve a DSGE model where firms face persistent shocks to both aggregate and individual productivity, and where optimal capital reallocation is distorted by two frictions, one financial and one real. First, collateralized borrowing constraints limit the investment undertaken

by small firms with relatively high productivities. Second, specificity in firm-level capital implies partial investment irreversibilities that lead firms to pursue generalized (S,s) rules with respect to their capital adjustments. This second friction compounds the first, further tilting the distribution of production towards large and relatively unproductive firms, and thus reducing endogenous aggregate total factor productivity. It also exacerbates the direct effects of collateral constraints by reducing the collateral value ascribed to each unit of installed capital. Moreover this added element of realism in our setting relative to existing DSGE environments may be quite important to the transmission and propagation of a financial shock.

Because specificity in capital induces both downward and upward inertia in firm-level investment activities, and because it tightens the borrowing limits implied by collateralized lending, it ensures that the negative consequences of a temporary tightening in financial markets cannot be quickly reversed. In the presence of persistent heterogeneity in both capital and total factor productivity, the effects of financial frictions are amplified and propagated through large and long-lived disruptions to the distribution of capital that, in turn, imply large and persistent reductions in aggregate total factor productivity. For example, in the presence of only a 10 percent capital irreversibility, we find that steady state output falls by nearly 2 percent when collateralized borrowing limits are introduced. This suggests the potential for large output losses in our model economy following a temporary financial shock, or following a real shock accompanied by a financial one, since the long-run output reduction in response to a permanent change in borrowing constraints fails to capture the sharp transitional reductions associated with reallocation following a temporary shock.

As suggested above, we will use our model to measure the extent to which a financial shock can spill into the real side of a plausibly calibrated economy to produce large and persistent reductions in aggregate employment and GDP on its own, as well as the extent to which it can amplify and prolong the effects of a modest-sized real shock. From the outset, understanding that investment is a small fraction of GDP, it is clear that the reductions in aggregate capital implied by a temporary reduction in available credit are unlikely to deliver sizeable or long-lived aggregate real effects. However, we also know from the disaggregated data that there is substantial heterogeneity among firms in their individual productivity levels, and there are real frictions limiting the reallocation of capital across them. As such, the transmission mechanism we explore here focuses on the economy's *effective* capital stock and endogenous total factor productivity.

Our primary question in this study is whether a temporary crisis in financial markets can generate a large and persistent drop in aggregate productivity by disrupting the distribution of capital away from that implied by firms' relative productivities, and thereby distorting the distribution of production. Of course, we are not the first to emphasize reductions in measured TFP arising from a misallocation of resources across firms. Restuccia and Rogerson (2008) show that this channel can be quite important in explaining cross-country per-capita GDP differences. However, we are to our knowledge the first to explore this channel in a quantitative DSGE setting where real frictions slow the reallocation of capital across firms, and where that reallocation is essential in determining the marginal product of the aggregate stock.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the literature most closely related to our current work. Next, in section 3, we present our model economy and define competitive equilibrium therein, thereafter, providing some analysis useful toward developing a numerical algorithm capable of its solution. Section 4 discusses how we calibrate our model using aggregate and firm-level data, as well as some specifics of our numerical method. Section 5 presents results. There, we begin by exploring our model's mechanics in the deterministic steady state, and how they compare to those in two relevant reference models - one with capital specificity and no borrowing limits, the other with collateral constraints, but perfectly reversible investment. Next, turning to dynamics, we explore the aggregate response in our economy following a persistent shock to exogenous aggregate productivity, comparing this response to each of our reference models. Finally, we consider the aggregate response to a temporary reduction in the availability of credit that coincides with a one-period rise in the death rate among firms, here contrasting the results in our model economy to those arising in the no-financial-frictions reference model to gauge the importance of interactions between the two frictions in our environment. Finally, section 6 provides brief concluding remarks.

2 Related literature

To be clear, there is a vast existing literature considering the implications of financial market imperfections. For example, Kiyotaki and Moore (1997) study a model of credit cycles and show that collateral constraints can have a role in amplifying and propagating shocks to the value of collateral. More recent studies challenge the finding, however, as one arising from an overly stylized environment. Cordoba and Ripoll (2004) argue that the effects are actually quite small in a more

plausibly calibrated model. The explanation for this may be best articulated in a short article by Kocherlakota (2000). However, a common, and likely critical, element across these papers is the abstraction from any additional source of heterogeneity across firms. One notable exception is the recent paper by Buera and Shin (2007). While Buera and Shin emphasize development concerns, their primary finding that financial frictions can have a large and persistent impact on the aggregate transition to a steady state, particularly when capital is initially misallocated, is certainly an informative one for our study. It suggests that our allowance for real capital frictions alongside the financial friction they consider may be quite relevant in magnifying and propagating business cycle fluctuations.

Elsewhere in the investment literature, various empirical and theoretical studies have together mounted a strong case that real frictions limiting the reallocation of capital are essential in explaining microeconomic investment data. (See, for instance, Cooper and Haltiwanger (2006) or Caballero and Engel (1999).) Moreover, these frictions have been shown to add persistence to an economy's aggregate response to shocks (Bertola and Caballero (1994)). Thus, the fact that the financial frictions literature has largely ignored real frictions may be costly along both empirical and theoretical margins. Of course, the same could be said of the investment literature's abstraction from financial frictions, as this abstraction may be critical in the repeated finding that nonconvex capital adjustment costs, as well as investment irreversibilities, have essentially no importance for the aggregate business cycle of a DSGE model economy (e.g., Thomas (2003) and Veracierto (2002)).

There is one existing study that does simultaneously consider real and financial frictions in a dynamic, stochastic setting. Caggese (2007) provides a careful exploration of precisely how collateralized borrowing constraints can interact with investment irreversibility to exacerbate aggregate fluctuations. However, there are two critical modeling choices there that almost surely overstate the effects of these interactions, and perhaps also alter some aspects of the channels through which they work. The first is the assumption that capital investments are entirely irreversible. The second is the abstraction from a general equilibrium environment. These are the primary dimensions along which our study is distinguished relative to his.

3 Model

In our model economy, firms face both partial capital fixity and collateralized borrowing limits, which together compound the effects of persistent differences in their total factor productivities to yield substantial heterogeneity in production. In this section, we begin our description of the economy with an initial look at the optimization problem facing each firm, then follow with a brief discussion of households and equilibrium. Next, using a simple implication of equilibrium alongside some immediate observations about firms' optimal allocation of profits across dividends and negative debt, we characterize the capital adjustment decisions of our firms as a variant of the two-sided generalized (S, s) policy that would arise in the presence of investment irreversibilities alone, absent any credit market imperfections. Finally, we use our analysis there to derive a convenient, computationally tractable algorithm with which to solve for equilibrium allocations in our model, despite the presence of three-dimensional heterogeneity in production.

3.1 Production, credit and capital adjustment

We assume a large number of firms, each producing a homogenous output using predetermined capital stock k and labor n , via an increasing and concave production function, $y = z\varepsilon F(k, n)$. Here, z represents stochastic total factor productivity common across firms, while ε is firm-specific productivity. For convenience, we assume that ε is a Markov chain, $\varepsilon \in \mathcal{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) \equiv \pi_{ij} \geq 0$, and $\sum_{j=1}^{N_\varepsilon} \pi_{ij} = 1$ for each $i = 1, \dots, N_\varepsilon$. Similarly, we assume that $z \in \{z_1, \dots, z_{N_z}\}$, where $\Pr(z' = z_m \mid z = z_l) \equiv \pi_{lm}^z \geq 0$, and $\sum_{m=1}^{N_z} \pi_{lm}^z = 1$ for each $l = 1, \dots, N_z$.

In each period, a firm is defined by its predetermined stock of capital, $k \in \mathcal{K} \subset \mathbf{R}_+$, by the level of one-period debt it incurred in the previous period, $b \in \mathcal{B} \subset \mathbf{R}$, and by its current exogenous idiosyncratic productivity level, $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$. Given this individual state, and having observed the economy's current aggregate state, the firm then takes a series of actions chosen to maximize the expected discounted value of the current and future dividends to be returned to its shareholders, the households in our economy. First, the firm chooses its current level of employment, production occurs, and its workers are paid. Next, the firm repays its existing debt and chooses its investment, i , and the level of debt (or savings) with which it will enter into the next period, b' . For each unit of debt the firm incurs for the next period, it receives q units of output that it can use toward paying current dividends or investing in its future capital. The

relative price q reflecting the interest rate at which the firm can borrow and lend is, of course, a function of the economy's aggregate state. So too is the wage rate ω that it pays its workers. However, we suppress the arguments of these equilibrium prices until we have described the model a bit further.

In contrast to the typical setting with firm-level capital adjustment frictions, and unlike a typical environment with financial frictions, these two frictions are allowed to interact with one another in our setting. Our firms' borrowing and investment decisions are necessarily inter-related, because each firm faces a collateralized borrowing constraint inside of any period. This constraint takes the form: $b' \leq \Theta k(\frac{1-\delta}{\gamma})$, where $\gamma \geq 1$ reflects the gross growth rate of aggregate output along the economy's balanced growth path and Θ represents the fraction of the firm's (growth-deflated) capital stock remaining at the end of this period that can be successfully uninstalled and returned to lenders next period in the event of default.

Two external forces together determine what fraction of its capital stock a firm can borrow against - the degree of specificity in capital and the enforceability of financial arrangements. In this early exploration of how credit market frictions interact with capital reallocation frictions in shaping aggregate productivity, we simply impose both frictions, deferring the question of their microfoundations for a future study. In particular, we assume that $\Theta = \theta_b \theta_k$, where $\theta_k \in [0, 1]$ is a parameter determining what fraction of a firm's capital stock survives when it is uninstalled and moved to another firm, and where $\theta_b \in \mathbf{R}_+$ is the fraction of uninstalled capital that creditors are confident they will actually be able to repossess should default occur.¹

If firm chooses to undertake any nonnegative level of investment, then its capital stock at the start of the next period is determined in the familiar way,

$$\gamma k' = (1 - \delta)k + i \quad \text{for } i \geq 0,$$

where $\delta \in (0, 1)$ is the rate of capital depreciation.² However, because there is some degree

¹Throughout our numerical exercises in section 5, we will assume that the degree of capital irreversibility, $1 - \theta_k$, is a fixed technological parameter. By contrast, we will allow for exogenous changes in the value of θ_b in order to consider the aggregate implications of a purely financial shock that raises or lowers the confidence of lenders. However, we assume that any such financial shock is entirely unanticipated. Inside every period, agents in our economy place zero probability weight on θ_b moving away from its usual value.

²Throughout the paper, primes indicate one-period-ahead values, and all variables measured in units of output are deflated by the level of labor-augmenting technological progress, which implies output growth at the rate $\gamma - 1$ along the balanced growth path.

of specificity in the firm's capital, the same accumulation equation does not apply when the firm undertakes negative investment. Instead, because any choice of future capital below $k(\frac{1-\delta}{\gamma})$ releases only θ_k units of capital per uninstalled unit,

$$\theta_k \gamma k' = \theta_k (1 - \delta) k + i \quad \text{for } i < 0.$$

In the analysis section to follow, we will show how the asymmetry that firms face in the cost of capital adjustment naturally gives rise to two-sided (S, s) investment decision rules. For the moment, however, we simply point out that, in contrast to a nonconvexity in the capital adjustment technology, this type of adjustment friction implies not only investment inaction among firms within their (S, s) adjustment bands, but also some inertia among firms outside of their (S, s) bands. Because there are no increasing returns in the adjustment technology, but instead a linear penalty for negative adjustments, a firm finding itself with an intolerably high capital stock (given its current productivity), will reduce its stock only to the point where it has reached the upper bound of its (S, s) inactivity range. Similarly, a firm with too little capital recognizes the penalty it will incur should it later need to shed capital, so its upward adjustment is also moderated; it invests only to the lower bound of its inactivity range.

Given the discussion above, note that, alongside the response to its current (persistent) productivity draw, a firm's current capital adjustment can also be influenced by its ability to borrow (now and in the future), with this affected by the capital (collateral) it currently holds. Note also that the firm's current investment decision can influence the level of debt that it carries into the next period. Taken together, these observations imply that we must keep track of the distinguishing features of firms in each of three dimensions: their capital, k , their debt, b , and their idiosyncratic productivity, ε . We summarize the distribution of firms over (k, b, ε) using the probability measure μ defined on the Borel algebra, $\mathcal{S}$, for the product space $\mathbf{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{E}$. The aggregate state of the economy is then described by (z, μ) , and the distribution of firms evolves over time according to a mapping, Γ , from the current aggregate state; $\mu' = \Gamma(z, \mu)$. We will define this mapping below.

Finally, because our interest is in understanding how financial constraints interact with quasi-fixity of capital in shaping the investment decisions taken by firms in our economy, we must prevent the possibility that all firms eventually grow so large that they will never again experience a binding borrowing limit. To ensure this cannot occur, we impose exogenous exit and entry in the model. In particular, we assume that the lowest value of individual productivity is $\varepsilon_1 = 0$,

that all firms have equal probability of drawing this productivity, $\pi_{i1} = \pi_1$ for all $i > 1$, and that it is an absorbing state, $\pi_{1j} = 0$ for all $j \neq 1$. In each period, those firms drawing productivity level ε_1 exit the economy and are immediately replaced by the same number of entering firms, $e \equiv \pi_1$. Each entering firm draws an initial productivity level $\varepsilon_0 \in \mathcal{E}$ from an initial distribution $H(\varepsilon_0)$, and each is endowed with an equal share of the total capital uninstalled from those exiting the economy, net of a proportional cost $(1 - \chi)$ associated with the act of replacing one firm with another.

$$k_0 = \theta_k \chi \int_{\mathbf{S}} k \mu(d[k \times b \times \varepsilon]) = \theta_k \chi K.$$

We are finally in a position to set out the optimization problem solved by each firm in our economy. Let $v(k, b, \varepsilon_i; z_l, \mu)$ represent the expected discounted value of a firm entering the period with (k, b) and drawing firm-specific productivity ε_i , when the aggregate state of the economy is (z_l, μ) . We state the dynamic optimization problem of the firm using a functional equation defined by (1) - (9) below. We first define the value of the firm as it makes a binary decision of whether to undertake upward or downward capital adjustment at the close of the period, and then identify the value associated with each option.³

$$v(k, b, \varepsilon_i; z_l, \mu) = \max \left\{ v^u(k, b, \varepsilon_i; z_l, \mu), v^d(k, b, \varepsilon_i; z_l, \mu) \right\} \quad (1)$$

Assume that $d_m(z_l, \mu)$ is the discount factor applied by firms to their next-period expected value if aggregate productivity at that time is z_m and the current aggregate state is (z_l, μ) . Taking as given the evolution of ε and z according to the transition probabilities specified above, and taking as given the evolution of the firm distribution, $\mu' = \Gamma(z, \mu)$, the firm solves each of the following optimization problems to determine whether it will undertake a nonnegative capital adjustment or a negative one. (Here forward, except where necessary for clarity, we suppress the indices for current aggregate and firm productivity.) In solving these problems, the firm selects its current employment and production, alongside the debt and capital with which it will enter into next period and its current dividends, D . These are determined as the residual of the firm's current production and borrowing after its wages, investment, and debt repayment have been covered.

³We could alternatively set out the firm's problem avoiding the binary max operator, but instead including an indicator function determining the relative price of capital as 1 in the event of $k' \geq (\frac{1-\delta}{\gamma})k$ and θ_k otherwise. Here, for sake of expositional clarity, we have opted for the less concise representation.

If it chooses to make an upward capital adjustment, the firm solves the following problem.

$$v^u(k, b, \varepsilon_i; z_l, \mu) = \max_{n, k', b', D} \left[D + \sum_{m=1}^{N_z} \pi_{lm}^z d_m(z, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{ij} v(k', b', \varepsilon_j; z_m, \mu) \right] \quad (2)$$

$$\text{subject to:} \quad k' \geq \left(\frac{1-\delta}{\gamma} \right) k \quad (3)$$

$$0 \leq D \leq z\varepsilon F(k, n) + q(z, \mu) b' - \omega(z, \mu) n - [\gamma k' - (1-\delta) k] - b \quad (4)$$

$$b' \leq \Theta \left(\frac{1-\delta}{\gamma} \right) k \quad (5)$$

The downward adjustment problem has the same objective, maximizing expected discounted dividends, and it is subject to the same non-negativity constraint on dividends and the same borrowing limit. The only difference is a $\left(\frac{1-\delta}{\gamma} \right) k$ upper (versus lower) bound on its k' choice, and a θ_k (versus unit) relative price of investment.

$$v^d(k, b, \varepsilon_i; z_l, \mu) = \max_{n, k', b', D} \left[D + \sum_{m=1}^{N_z} \pi_{lm}^z d_m(z, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{ij} v(k', b', \varepsilon_j; z_m, \mu) \right] \quad (6)$$

$$\text{subject to:} \quad k' \leq \left(\frac{1-\delta}{\gamma} \right) k \quad (7)$$

$$0 \leq D \leq z\varepsilon F(k, n) + q(z, \mu) b' - \omega(z, \mu) n - \theta_k [\gamma k' - (1-\delta) k] - b \quad (8)$$

$$b' \leq \Theta \left(\frac{1-\delta}{\gamma} \right) k \quad (9)$$

We will return to simplify the representation of the firm's problem and isolate its decision rules in section 3.4 below. For now, notice that there is no friction associated with the firm's employment choice, since the firm is allowed to pay its current wage bill after production takes place, and its capital choice for next period also has no implications for current production. Thus, irrespective of their current debt, all firms sharing in common the same (k, ε) combination select the same employment, which we will denote by $N(k, \varepsilon; z, \mu)$. The same cannot be said for the intertemporal decisions, of course, given the presence of both borrowing limits and irreversibilities. Thus, we must use $K(k, b, \varepsilon; z, \mu)$ and $B(k, b, \varepsilon; z, \mu)$ to represent the choices of next-period capital and debt, respectively, made by firms sharing in common a complete individual type (k, b, ε) .

3.2 Households

The economy is populated by a unit measure of identical households. Household wealth is held as one-period shares in firms, which we denote using the measure λ .⁴ Given the prices they receive for their current shares, $\rho_0(k, b, \varepsilon; z, \mu)$, and the real wage they receive for their labor effort, $\omega(z, \mu)$, households determine their current consumption, c , hours worked, n^h , as well as the numbers of new shares, $\lambda'(k', b', \varepsilon')$, to purchase at prices $\rho_1(k', b', \varepsilon'; z, \mu)$. The lifetime expected utility maximization problem facing each of them is listed below.

$$V^h(\lambda; z, \mu) = \max_{c, n^h, \lambda'} \left[U(c, 1 - n^h) + \beta \sum_{m=1}^{N_z} \pi_{lm}^z V^h(\lambda'; z_m, \mu') \right] \quad (10)$$

subject to

$$c + \int_{\mathbf{S}} \rho_1(k', b', \varepsilon'; z, \mu) \lambda'(d[k' \times b' \times \varepsilon']) \leq \omega(z, \mu) n^h + \int_{\mathbf{S}} \rho_0(k, b, \varepsilon; z, \mu) \lambda(d[\varepsilon \times k]).$$

Let $C^h(\lambda; z, \mu)$ describe the household choice of current consumption, and let $N^h(\lambda; z, \mu)$ be the allocation of current available to working. Finally, let $\Lambda^h(k', b', \varepsilon', \lambda; z, \mu)$ be the quantity of shares purchased in firms that will begin the next period with k' units of capital, b' units of debt, and idiosyncratic productivity ε' .

3.3 Recursive equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$\left(\omega, q, (d_j)_{j=1}^{N_z}, \rho_0, \rho_1, v, N, K, B, V^h, C^h, N^h, \Lambda^h \right),$$

that solve firm and household problems and clear the markets for assets, labor and output:

- (i) v satisfies (1) - (9), and (N, K, B) are the associated policy functions for firms.
- (ii) V^h satisfies (10), and (C^h, N^h, Λ^h) are the associated policy functions for households.
- (iii) $\Lambda^h(k', b', \varepsilon_j, \mu; z, \mu) = \mu'(k', b', \varepsilon_j)$, for each $(k', b', \varepsilon_j) \in \mathbf{S}$.
- (iv) $N^h(\mu; z, \mu) = \int_{\mathbf{S}} [N(k, \varepsilon; z, \mu)] \mu(d[k \times b \times \varepsilon])$.

⁴Households also have access to a complete set of state-contingent claims. However, as there is no heterogeneity across households, these assets are in zero net supply in equilibrium. Thus, for brevity, we do not explicitly model them.

$$\begin{aligned}
\text{(v)} \quad C^h(\mu; z, \mu) &= \int_{\mathbf{s}} \left[z \varepsilon F(k, N(\varepsilon, k; z, \mu)) - \mathcal{J}\left(K(k, b, \varepsilon; z, \mu) - \frac{(1-\delta)k}{\gamma}\right) \left(\gamma K(k, b, \varepsilon; z, \mu) \right. \right. \\
&\quad \left. \left. - (1-\delta)k \right) - \pi_1(1-\theta_k)(1-\chi)k \right] \mu(d[k \times b \times \varepsilon]), \text{ where } \mathcal{J}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ \theta_k & \text{if } x < 0 \end{cases}.
\end{aligned}$$

$$\text{(vi)} \quad \mu'(\mathcal{K}, \mathcal{B}, \varepsilon_j) = \int_{\{(k, b, \varepsilon_i) \mid X(k, b, \varepsilon_i)=1\}} \pi_{ij} \mu(d[k \times b \times \varepsilon]) + \int_{\{\varepsilon_i \mid X(k_0, 0, \varepsilon_i)=1\}} \pi_1 \pi_{ij} H(d\varepsilon_i), \text{ for all}$$

$(\mathcal{K}, \mathcal{B}, \varepsilon_j) \in \mathcal{S}$, defines Γ , where $X(k, b, \varepsilon_i) = 1$ iff $K(k, b, \varepsilon_i; z, \mu) \in \mathcal{K} \circ B(k, b, \varepsilon_i; z, \mu) \in \mathcal{B}$.

3.4 Decision rules

Using C and N to describe the market-clearing values of household consumption and hours worked satisfying conditions (iv) and (v) above, it is straightforward to show that market-clearing requires that $\omega(z, \mu) = \frac{D_2 U(C, 1-N)}{D_1 U(C, 1-N)}$, that $q(z, \mu) = \frac{\sum_{m=1}^{N_z} \pi_{lm} \beta D_1 U(C'_m, 1-N'_m)}{D_1 U(C, 1-N)}$, and that $d_j(z, \mu) = \frac{\beta D_1 U(C'_j, 1-N'_j)}{D_1 U(C, 1-N)}$. As such, we may compute equilibrium by solving a single Bellman equation that combines the firm-level profit maximization problem with these equilibrium implications of household utility maximization, effectively subsuming the implications of households' decisions into the problems faced by our firms. Defining $p(z, \mu)$ as the price firms use to value current dividends, we have the following three conditions.

$$p(z, \mu) = D_1 U(C, 1-N) \quad (11)$$

$$\omega(z, \mu) = \frac{D_2 U(C, 1-N)}{p(z, \mu)} \quad (12)$$

$$q(z, \mu) = \frac{\sum_{m=1}^{N_z} \pi_{lm}^z \beta p(z_m, \mu')}{p(z, \mu)} \quad (13)$$

A reformulation of (1) - (9) then yields an equivalent description of a firm's dynamic problem. Here, we move to express each firm's value in units of marginal utility, rather than output, a transformation that implies no change in the resulting decision rules. Suppressing the arguments of the price functions, exploiting the fact that the choice of n is independent of the choices of k' and b' , and now using the indicator function $\mathcal{J}(x)$ defined above to distinguish nonnegative from negative investment, we have:

$$V(k, b, \varepsilon_i; z_l, \mu) = \max_{n, k', b', D} \left[pD + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V(k', b', \varepsilon_j; z_m, \mu) \right] \quad (14)$$

subject to:

$$0 \leq D \leq z\varepsilon F(k, n) + qb' - \omega n - \mathcal{J}\left(k' - \frac{(1-\delta)k}{\gamma}\right)[\gamma k' - (1-\delta)k] - b, \quad (15)$$

$$b' \leq \Theta\left(\frac{1-\delta}{\gamma}\right)k. \quad (16)$$

The problem listed in equations (14) - (16) forms the basis for solving equilibrium allocations in our economy, so long as we ensure that the prices p, ω and q taken as given by our firms satisfy the restrictions in (11) - (13) above. From here, we begin to characterize the decision rules that arise from this problem.

We begin by noting that the firm chooses its labor $n = N(k, \varepsilon; z, \mu)$ to solve $z\varepsilon D_2 F(k, n) = \omega(z, \mu)$, which immediately returns its current production, $y(k, \varepsilon; z, \mu) = z\varepsilon F(k, N(k, \varepsilon; z, \mu))$. Thus, the only challenging objects to determine are the firm's choices of D, k' and b' . Turning to these, we next use a simple observation about the implications of borrowing constraints for the value a firm places on retained earnings versus current dividends. This observation will be proved in a future draft using a sequence notation with explicit multipliers on each constraint, following the same basic approach as in Caggese (2007). For now, however, it is fairly direct to see that, so long as the firm places non-zero probability weight on encountering a future state in which its borrowing constraint will bind, the shadow value of retained earnings (the discounted sequence of reductions in the multipliers on future borrowing constraints) will necessarily exceed the shadow value of current dividends, p . This means that, so long as the firm can ever hit a binding borrowing constraint in the future, it will choose to set $D = 0$. In this case, a quick look to equation 15 establishes that the firm's choice of k' will directly imply its b' , the level of debt with which it will enter into the next period. We refer to any such firm as a (potentially) *constrained* firm, and list the resulting problem that it solves after making the obvious decision to pay no dividends in the current period.

$$\begin{aligned} V^C(k, b, \varepsilon_i; z_l, \mu) &= \max_{n, k', b'} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V^C(k', b', \varepsilon_j; z_m, \mu) \quad \text{subject to:} \\ b' &= \frac{1}{q} \left[b + \mathcal{J}\left(k' - \frac{(1-\delta)k}{\gamma}\right)[\gamma k' - (1-\delta)k] - [z\varepsilon F(k, n) - \omega n] \right] \\ \text{where } D &= 0 \text{ is pre-selected.} \end{aligned} \quad (17)$$

We can also make a related observation about the value a firm will place on retained earnings versus dividends if it knows that it will never again face a binding borrowing constraint now or in

the future. In this case, the sequence of multipliers on future borrowing constraints are all zero, leaving the firm indifferent between saving its profits internally (via $b' < 0$) versus handing them over to households (via $D > 0$). Should it choose the latter, the firm faces no doubt that it will always be able to borrow the funds it requires to undertake its investments in the future, so it has no precautionary reason to hold internal funds. We can exploit the indifference (indeterminacy) this presents by choosing for the firm which means of saving and borrowing it will adopt. Here, without loss of generality, we assume that the firm returns all earnings to its shareholders and sets its debt at zero in every period here and in future, $b' = 0$. In this case, a second look to equation 15 makes clear that the choice of k' directly implies the level of dividends the firm pays this period, D . Let us refer to any such firm as an *unconstrained* firm, since it is a firm that has accumulated sufficient wealth ($k > 0$, $b < 0$) such that borrowing limits will never again curtail its investment activities, and denote its value by W .

$$W(k, b, \varepsilon_i; z_l, \mu) = \max_{k'} \left[p \left[z\varepsilon F(k, N(k, \varepsilon; z, \mu)) - \omega N(k, \varepsilon; z, \mu) - b \right. \right. \quad (18)$$

$$\left. \left. - \mathcal{J} \left(k' - \frac{(1-\delta)k}{\gamma} \right) [\gamma k' - (1-\delta)k] \right] + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} W(k', 0, \varepsilon_j; z_m, \mu) \right]$$

where $b' = 0$ is pre-selected.

Notice that, if a firm has just become unconstrained, having entered into the period with some lingering debt (savings) $b \neq 0$, its value is linearly reduced (raised) by the associated reduction (rise) in current dividends, which are valued by p . Thus, we can alternatively express the value of any unconstrained firm of type (k, b, ε_i) as $w(k, \varepsilon_i; z_l, \mu) - pb$, where $w(k, \varepsilon_i; z_l, \mu) \equiv W(k, 0, \varepsilon_i; z_l, \mu)$. This convenient representation will be used often below to represent the value of a firm that has not previously been an unconstrained type, but has become so this period.

3.4.1 (S,s) decisions among unconstrained firms

To identify the decisions made by an unconstrained firm, it may be useful to return to a less concise means of representing the problem in (18). First, note that the firm of type (k, b, ε_i) will achieve current profit flows $\pi(k, b, \varepsilon_i; z_l, \mu)$ defined below irrespective of whether it chooses to adjust its capital stock upward or downward or leave it unaltered.

$$\pi(k, b, \varepsilon_i; z_l, \mu) \equiv z\varepsilon F(k, N(k, \varepsilon; z, \mu)) - \omega N(k, \varepsilon; z, \mu) - b$$

With this summary notation in hand, we can represent an unconstrained firm's problem as follows.

$$W(k, b, \varepsilon_i; z_l, \mu) = \max\{W^u(k, b, \varepsilon_i; z_l, \mu), W^d(k, b, \varepsilon_i; z_l, \mu)\}, \text{ where:}$$

$$\begin{aligned} W^u(k, b, \varepsilon_i; z_l, \mu) &= p\pi(k, b, \varepsilon_i; z_l, \mu) + p(1 - \delta)k \\ &\quad + \max_{k'} \left[-p\gamma k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w(k', \varepsilon_j; z_m, \mu) \right] \\ \text{subject to:} \quad k' &\geq \left(\frac{1 - \delta}{\gamma} \right) k \end{aligned} \tag{19}$$

$$\begin{aligned} W^d(k, b, \varepsilon_i; z_l, \mu) &= p\pi(k, b, \varepsilon_i; z_l, \mu) + p\theta_k(1 - \delta)k \\ &\quad + \max_{k'} \left[-p\theta_k \gamma k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w(k', \varepsilon_j; z_m, \mu) \right] \\ \text{subject to :} \quad k' &\leq \left(\frac{1 - \delta}{\gamma} \right) k \end{aligned} \tag{20}$$

Quick inspection of equations 19 and 20 reveals that W^u and W^d are both strictly increasing in k . Thus, W is a strictly increasing function of the unconstrained firm's capital, as is the w function defined above.

We may characterize the capital decision rule for an unconstrained firm by reference to two target capital stocks, the upward target and the downward target, respectively, that would solve the problems in (19) and (20) were they unconstrained. First, define k_u^* as the upward target capital a firm would choose given the unit 'upward adjustment' relative price of capital:

$$k_u^*(\varepsilon_i; z_l, \mu) = \arg \max_{k'} R^u(k', \varepsilon_i; z_l, \mu), \tag{21}$$

$$\text{where } R^u(k', \varepsilon_i; z_l, \mu) \equiv -p\gamma k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w(k', \varepsilon_j; z_m, \mu).$$

Similarly, define k_d^* as the target capital a firm would choose given the 'downward adjustment' relative price of capital, θ_k :

$$k_d^*(\varepsilon_i; z_l, \mu) = \arg \max_{k'} R^d(k', \varepsilon_i; z_l, \mu), \tag{22}$$

$$\text{where } R^d(k', \varepsilon_i; z_l, \mu) \equiv -p\theta_k \gamma k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w(k', \varepsilon_j; z_m, \mu).$$

Notice that each target is independent of current capital and depends only on the aggregate state and the firm's current ε , given persistence in firms' productivities. This means that all unconstrained firms that share in common the same current productivity ε have the same upward and downward target capitals. (This fact eases the computational burden in our numerical solution of the economy.) Note also that, because $\theta_k < 1$ (and because the value function w is strictly increasing in k), the upward adjustment target necessarily lies below the downward target: $k_u^* < k_d^*$. Finally, let us refer back to the constrained problems the firm actually solves to determine its value under upward versus downward capital adjustment. Observe that, in each case, the constant purchase price of capital inside of each option implies that the firm will select a future capital as close to the target as its constraint allows. Thus, the firm's decision rule conditional on an upward adjustment is

$$k_u(\varepsilon_i; z_l, \mu) = \max \left\{ \left(\frac{1-\delta}{\gamma} \right) k, k_u^*(\varepsilon_i; z_l, \mu) \right\}, \quad (23)$$

while that conditional on a downward adjustment is

$$k_d(\varepsilon_i; z_l, \mu) = \min \left\{ \left(\frac{1-\delta}{\gamma} \right) k, k_d^*(\varepsilon_i; z_l, \mu) \right\}. \quad (24)$$

Given the conditional adjustment rules identified in (23) - (24), we know that the unconstrained firm of type (k, b, ε) will select one of three future capital levels:

$$k' \in \{k_u^*(\varepsilon_i; z_l, \mu), k_d^*(\varepsilon_i; z_l, \mu), \left(\frac{1-\delta}{\gamma} \right) k\}.$$

Which of the three it selects depends only on where its current capital lies in relation to its two targets. Recalling our observation that $k_u^* < k_d^*$, suppose first that $k \in \left[\frac{\gamma k_u^*(\varepsilon_i; z_l, \mu)}{1-\delta}, \frac{\gamma k_d^*(\varepsilon_i; z_l, \mu)}{1-\delta} \right]$. If this is the case, then $k_u(\varepsilon_i; z_l, \mu) = \left(\frac{1-\delta}{\gamma} \right) k = k_d(\varepsilon_i; z_l, \mu)$, so the firm makes no adjustment to its capital. If, instead, the firm's current capital is sufficiently low that its implied stock for next period under no adjustment lies below the upward target, $k < \frac{\gamma k_u^*(\varepsilon_i; z_l, \mu)}{1-\delta}$, we know $k_u(\varepsilon_i; z_l, \mu) = k_u^*(\varepsilon_i; z_l, \mu)$, while $k_d(\varepsilon_i; z_l, \mu) = \left(\frac{1-\delta}{\gamma} \right) k$. In this case, considering the firm's choice between k_u and k_d , we know that $\left(\frac{1-\delta}{\gamma} \right) k$ was permissible in the choice of the upward capital target, so the firm selects $k_u^*(\varepsilon_i; z_l, \mu)$. Finally, if the firm's capital is sufficiently high that its implied capital for next period under no adjustment lies above the downward target, $k > \frac{\gamma k_d^*(\varepsilon_i; z_l, \mu)}{1-\delta}$, then $k_d(\varepsilon_i; z_l, \mu) = k_d^*(\varepsilon_i; z_l, \mu)$, while $k_u(\varepsilon_i; z_l, \mu) = \left(\frac{1-\delta}{\gamma} \right) k$. Here again, since $\left(\frac{1-\delta}{\gamma} \right) k$ was in the choice set the firm selected its downward capital target, it must be at least as well off with $k_d^*(\varepsilon_i; z_l, \mu)$.

Collecting our observations from above, and given the definitions of the two targets from equations 21 and 22, we have the following (S, s) capital decision rule for an unconstrained firm.

$$K^w(k, \varepsilon_i; z_l, \mu) = \begin{cases} k_u^*(\varepsilon_i; z_l, \mu) & \text{if } k < \frac{\gamma k_u^*(\varepsilon_i; z_l, \mu)}{1-\delta} \\ \left(\frac{1-\delta}{\gamma}\right) k & \text{if } k \in \left[\frac{\gamma k_u^*(\varepsilon_i; z_l, \mu)}{1-\delta}, \frac{\gamma k_d^*(\varepsilon_i; z_l, \mu)}{1-\delta}\right] \\ k_d^*(\varepsilon_i; z_l, \mu) & \text{if } k > \frac{\gamma k_d^*(\varepsilon_i; z_l, \mu)}{1-\delta} \end{cases} \quad (25)$$

Given the capital rule, and recalling that $B^W(k, b, \varepsilon_i; z_l, \mu) = 0$, we can directly retrieve the dividends paid by this unconstrained firm, and thus retrieve the firm's value. In doing so, we suppress each target capital's dependence on (z_l, μ) to keep the equations tidy.⁵

$$\begin{aligned} D^w(k, b, \varepsilon_i; z_l, \mu) &= \pi(k, b, \varepsilon_i; z_l, \mu) \\ &+ \begin{cases} (1-\delta)k - \gamma k_u^*(\varepsilon_i) & \text{if } k < \frac{\gamma k_u^*(\varepsilon_i)}{1-\delta} \\ 0 & \text{if } k \in \left[\frac{\gamma k_u^*(\varepsilon_i)}{1-\delta}, \frac{\gamma k_d^*(\varepsilon_i)}{1-\delta}\right] \\ \theta_k[(1-\delta)k - \gamma k_d^*(\varepsilon_i)] & \text{if } k > \frac{\gamma k_d^*(\varepsilon_i)}{1-\delta} \end{cases} \end{aligned} \quad (26)$$

$$\begin{aligned} W(k, b, \varepsilon_i; z_l, \mu) &= p\pi(k, b, \varepsilon_i; z_l, \mu) \\ &+ \begin{cases} p(1-\delta)k + R^u(k_u^*(\varepsilon_i), \varepsilon_i; z_l, \mu) & \text{if } k < \frac{\gamma k_u^*(\varepsilon_i)}{1-\delta} \\ \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w\left(\left(\frac{1-\delta}{\gamma}\right) k, \varepsilon_j; z_m, \mu\right) & \text{if } k \in \left[\frac{\gamma k_u^*(\varepsilon_i)}{1-\delta}, \frac{\gamma k_d^*(\varepsilon_i)}{1-\delta}\right] \\ p\theta_k(1-\delta)k + R^d(k_d^*(\varepsilon_i), \varepsilon_i; z_l, \mu) & \text{if } k > \frac{\gamma k_d^*(\varepsilon_i)}{1-\delta} \end{cases} \end{aligned} \quad (27)$$

3.4.2 (S,s) decisions among constrained firms

Next we consider the decisions made by a firm that has not previously attained sufficient wealth to become unconstrained. It is straightforward to adopt an analogous, if slightly more cumbersome, approach to solving the problem of this potentially constrained firm as was used for unconstrained firms in the section above. The first essential step is to establish whether or not

⁵Equation (26) suggests a potential inconsistency with the restriction on our model that firms may not pay negative dividends. However, for any unconstrained firm in our economy, the Modigliani-Miller theorem applies. Thus, whenever D^w prescribes the payment of negative dividends, the firm is indifferent and instead assumes debt of $-D^w/q$.

the firm has crossed the relevant wealth threshold to become unconstrained.⁶

We can ascertain whether the firm has become unconstrained by considering the implications for current borrowing in the event that it adopts unconstrained decision rules. In particular, we need only consider whether it is feasible for the firm of type (k, b, ε_i) to adopt the capital rule $K^w(k, \varepsilon_i; z_l, \mu)$ and pay no dividends in the current period. Recall that, if a firm is in fact unconstrained (knowing it will never again face a binding borrowing constraint), then it is indifferent between paying dividends versus saving via negative debt. Thus, an alternative set of unconstrained firm decisions rules achieving equal value to those above is $k' = K^w(k, \varepsilon_i; z_l, \mu)$, $D = 0$ and $b' = -D^w(k, b, \varepsilon_i; z_l, \mu)/q$. We use these alternative rules to check whether our potentially constrained firm has indeed become an unconstrained firm, simply by considering whether adopting them would imply a level of debt allowed by the firm's current borrowing limit, $\theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k$. If the firm (k, b, ε_i) has sufficient wealth that $-D^w(k, b, \varepsilon_i; z_l, \mu)/q \leq \theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k$, then it adopts the decision rules (25) - (26), sets $b' = 0$, and achieves value $W(k, b, \varepsilon_i; z_l, \mu)$ from (27).

If the inequality above is not satisfied, then we know that the firm remains constrained, and we must treat it accordingly. Here again, we find it useful here to revert a less concise means of representing the problem in (17) as we seek to identify the decisions made by the constrained firm.

$$V^C(k, b, \varepsilon_i; z_l, \mu) = \max\{V^u(k, b, \varepsilon_i; z_l, \mu), V^d(k, b, \varepsilon_i; z_l, \mu)\}, \text{ where:}$$

$$\begin{aligned} V^u(k, b, \varepsilon_i; z_l, \mu) &= \max_{k' \in \mathbf{R}_+} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V^C(k', b'_u(k', \cdot), \varepsilon_j; z_m, \mu), \text{ with} \\ b'_u(k', \cdot) &= \frac{1}{q} \left(b + \omega N(k, \varepsilon; z, \mu) - z \varepsilon F(k, N(k, \varepsilon; z, \mu)) + [\gamma k' - (1 - \delta) k] \right) \\ \text{subject to:} \quad k' &\geq \left(\frac{1 - \delta}{\gamma} \right) k \quad \text{and} \quad b'_u(k', \cdot) \leq \theta_b \theta_k \left(\frac{1 - \delta}{\gamma} \right) k \end{aligned} \tag{28}$$

⁶ Every firm in our economy seeks to become unconstrained, since the only distinction between the value function W and the value function V^C is that the latter has a positive multiplier on at least one borrowing constraint from now forward, while the former does not. As such, W represents an upper bound for V , the original value function that we defined in equation 14.

$$\begin{aligned}
V^d(k, b, \varepsilon_i; z_l, \mu) &= \max_{k' \in \mathbf{R}_+} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V^C(k', b'_d(k', \cdot), \varepsilon_j; z_m, \mu), \text{ with} \\
b'_d(k', \cdot) &= \frac{1}{q} \left(b + \omega N(k, \varepsilon; z, \mu) - z \varepsilon F(k, N(k, \varepsilon; z, \mu)) + \theta_k [\gamma k' - (1 - \delta) k] \right) \\
\text{subject to: } &k' \leq \left(\frac{1 - \delta}{\gamma} \right) k \quad \text{and} \quad b'_d(k', \cdot) \leq \theta_b \theta_k \left(\frac{1 - \delta}{\gamma} \right) k
\end{aligned} \tag{29}$$

We may approach the solution for this firm's decision rules by first identifying two entirely unconstrained target capitals, $\hat{k}_u^*$ and $\hat{k}_d^*$, the upward and downward target that would solve (28) and (29), respectively, if the constraints listed in the final line of each problem were removed.

$$\hat{k}_u^*(k, b, \varepsilon_i; z_l, \mu) = \arg \max_{k'} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V^C(k', b'_u(k', \cdot), \varepsilon_j; z_m, \mu) \tag{30}$$

$$\hat{k}_d^*(k, b, \varepsilon_i; z_l, \mu) = \arg \max_{k'} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V^C(k', b'_d(k', \cdot), \varepsilon_j; z_m, \mu) \tag{31}$$

Unlike those we isolated for unconstrained firms in the previous section, note that these targets can depend upon the firm's entire individual state; each element therein has the potential to influence its ability to borrow in the future. Again, because the relative price of capital is higher for upward adjustments than for downward ones, it is immediate that $\hat{k}_u^* < \hat{k}_d^*$. Note also that each target is associated with a level of borrowing for next period.

$$b'_u(\hat{k}_u^*, \cdot) = \frac{1}{q} \left[[\gamma \hat{k}_u^* - (1 - \delta) k] - \pi(k, b, \varepsilon_i; z_l, \mu) \right] \tag{32}$$

$$b'_d(\hat{k}_d^*, \cdot) = \frac{1}{q} \left[\theta_k [\gamma \hat{k}_d^* - (1 - \delta) k] - \pi(k, b, \varepsilon_i; z_l, \mu) \right] \tag{33}$$

Referring back to the constrained problems the firm solves to determine its value under upward versus downward capital adjustment, observe that, in each case, the constant purchase price of capital implies that the firm will select a future capital as close to the target as its constraints allow. Before we consider the capital that will be adopted under an upward adjustment, we must first verify that the constraint set for the problem in (28) is nonempty. (Here forward, we again suppress the two target capitals' dependencies on the aggregate state to shorten some of the equations.)

Consider the lowest choice of k' permitted by the first constraint, $\left(\frac{1 - \delta}{\gamma} \right) k$. If this choice is not also permitted by the borrowing constraint, the firm cannot undertake even a trivial upward

capital adjustment. Recalling that $\pi(k, b, \varepsilon_i; z_l, \mu) \equiv z\varepsilon F(k, N(k, \varepsilon; z, \mu)) - \omega N(k, \varepsilon; z, \mu) - b$, this is the case for any firm with (k, b, ε) such that $\frac{1}{q}[b + \omega N(k, \varepsilon; z, \mu) - z\varepsilon F(k, N(k, \varepsilon; z, \mu))] > \theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k$. Thus, among any group of firms sharing a common (k, ε) , only those with $b \leq b^T(k, \varepsilon; z, \mu)$ can consider an upward adjustment, where the threshold debt b^T is:

$$b^T(k, \varepsilon; z, \mu) \equiv q\theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k + z\varepsilon F(k, N(k, \varepsilon; z, \mu)) - \omega N(k, \varepsilon; z, \mu). \quad (34)$$

Firms with $b > b^T(k, \varepsilon; z, \mu)$ do not solve (28); for them, $V^C(k, b, \varepsilon_i; z_l, \mu) = V^d(k, b, \varepsilon_i; z_l, \mu)$.

Consider the upward capital choice for a firm (k, b, ε) with $b \leq b^T(k, \varepsilon)$. Were there no borrowing limit, this would be simply $\max\left\{\left(\frac{1-\delta}{\gamma}\right) k, \hat{k}_u^*(k, b, \varepsilon_i)\right\}$. Similarly, the downward capital choice would be $\min\left\{\left(\frac{1-\delta}{\gamma}\right) k, \hat{k}_d^*(k, b, \varepsilon_i)\right\}$. Here, however, the firm may not be able to afford $\hat{k}_u^*$ ($\hat{k}_d^*$), since it may not be able to borrow the amount dictated by equation 32 (33). Define the maximum capital an upward adjusting firm can afford, and the maximum capital a downward adjusting firm can afford, as $\bar{k}^u(k, b, \varepsilon)$ and $\bar{k}^d(k, b, \varepsilon)$, respectively. These are identified as follow.

$$\begin{aligned} \bar{k}^u(k, b, \varepsilon) &\equiv \frac{1}{\gamma} \left[q\theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k + \pi(k, b, \varepsilon) + (1-\delta)k \right] \\ \bar{k}^d(k, b, \varepsilon) &\equiv \frac{1}{\gamma\theta_k} \left[q\theta_b \theta_k \left(\frac{1-\delta}{\gamma}\right) k + \pi(k, b, \varepsilon) + (1-\delta)\theta_k k \right] \end{aligned}$$

With these maximum feasible capitals in hand, we can express the constrained firm's decision rule for capital conditional on upward and downward adjustment as follow.

$$k_u^C(k, b, \varepsilon) = \max\left\{\left(\frac{1-\delta}{\gamma}\right) k, \min\{\bar{k}^u(k, b, \varepsilon), \hat{k}_u^*(k, b, \varepsilon_i)\}\right\} \quad (35)$$

$$k_d^C(k, b, \varepsilon) = \min\left\{\left(\frac{1-\delta}{\gamma}\right) k, \min\{\bar{k}^d(k, b, \varepsilon), \hat{k}_d^*(k, b, \varepsilon_i)\}\right\} \quad (36)$$

As noted above, if the firm enters the period with $b > b^T(k, \varepsilon; z, \mu)$, it has no possibility of upward adjustment. It selects $k_d^C(k, b, \varepsilon) = \min\{\bar{k}^d(k, b, \varepsilon), \hat{k}_d^*(k, b, \varepsilon_i)\}$ and the associated debt level $b'_d((k, b, \varepsilon), \cdot)$. However, if it instead enters the period with $b \leq b^T(k, \varepsilon; z, \mu)$, then it has the option of investing either to the upward capital target or to closest capital it can reach given its borrowing constraint. For any such firm, we have decision rules analogous to those derived for unconstrained firms. If $k \in \left[\frac{\gamma\hat{k}_u^*(k, b, \varepsilon_i)}{1-\delta}, \frac{\gamma\hat{k}_d^*(k, b, \varepsilon_i)}{1-\delta}\right]$, then $k_u^C(k, b, \varepsilon) = \left(\frac{1-\delta}{\gamma}\right) k = k_d^C(k, b, \varepsilon)$, so the firm makes no adjustment to its capital and its debt for next period is $b^T(k, \varepsilon; z, \mu)$. If, instead, $k < \frac{\gamma\hat{k}_u^*(k, b, \varepsilon_i)}{1-\delta}$, then $k_u^C(k, b, \varepsilon) = \min\{\bar{k}^u(k, b, \varepsilon), \hat{k}_u^*(k, b, \varepsilon_i)\}$, while $k_d^C(k, b, \varepsilon) = \left(\frac{1-\delta}{\gamma}\right) k$, so the firm selects $k' = \min\{\bar{k}^u(k, b, \varepsilon), \hat{k}_u^*(k, b, \varepsilon_i)\}$ and the associated debt $b'_u((k, b, \varepsilon), \cdot)$. Finally, if the

firm's capital is $k > \frac{\gamma \hat{k}_d^*(k, b, \varepsilon_i)}{1 - \delta}$, then $k_d^C(k, b, \varepsilon) = \hat{k}_d^*(k, b, \varepsilon_i)$, while $k_u^C(k, b, \varepsilon) = \left(\frac{1 - \delta}{\gamma}\right) k$, and the firm moves to its downward capital target, so $k' = \hat{k}_d^*(k, b, \varepsilon_i)$ and $b' = b'_d((k, b, \varepsilon), \cdot)$.

Time constraints prevent this section's completion for the moment. The next draft of this paper will finish the analysis in progress here. Thereafter, we will collect our findings from this section and use them to briefly discuss the numerical algorithm we adopt in solving our model, a variant of the method described in Khan and Thomas (2003, 2008).

4 Calibration

For this preliminary version of the paper, we provide an illustrative example in lieu of the results from a fully calibrated model economy. Here, we consider how the mechanics of our (full) model with real and financial frictions compare to those in two relevant reference models - one where there are no borrowing limits ($\theta_b \rightarrow \infty$) and one where there is no specificity in capital ($\theta_k = 1$). We use these two reference models to isolate how much the *interaction* between credit constraints and micro-level capital rigidities alters an economy's aggregate dynamics. More specifically, we compare to these reference models to explore how much this interaction may reshape the path of endogenous aggregate productivity and thus that of aggregate output.

For the baseline case of our full model, we set $\theta_k = 0.9$ and $\theta_b = 0.25$, so that both capital specificity and the limits on borrowing represent substantial frictions. It is important to note, however, that these parameters do not imply implausibly large numbers of borrowing constrained firms. In particular, given the remaining parameter selections described below, our full model gives rise to a stationary distribution of firms over (k, b, ε) wherein roughly 20 percent of firms are (potentially) financially constrained and the remaining 80 percent are unconstrained.

Aside from those governing the magnitudes of the two frictions, we select the remaining parameters in our full model to best match some selected moments drawn from the aggregate and firm-level data. Thereafter, we hold fixed these parameters, applying them also in both of our reference models. To be clear, we do not re-calibrate the reference models; thus, the average capital/output ratio, hours worked, and other important aspects of these economies are allowed to vary as each friction is eliminated.

4.1 Common parameters

Across our model economies, we assume that the representative household's period utility is the result of indivisible labor (Hansen (1985), Rogerson (1988)): $u(c, L) = \log c + \varphi L$, and the firm-level production function takes a Cobb-Douglas form, $z\varepsilon F(k, N) = z\varepsilon k^\alpha N^\nu$. We set the length of a period to correspond to one year, and we fix the mean growth rate of technological progress to imply a 1.6 percent average annual growth rate of real per capita output. The discount factor, β , is then set to imply an average real interest rate of 4 percent. Given the rate of technological progress, the depreciation rate, δ , is selected to match an average investment-to-capital ratio of roughly 10 percent, corresponding to the average value for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables. Labor's share is then set to 0.6; given this value, capital's share of output is determined by targeting an average capital-to-output ratio of roughly 2. Next, the parameter governing the preference for leisure, φ , is taken to imply an average of one-third of available time spent in market work.

In specifying our exogenous stochastic process for aggregate productivity, we begin by assuming a continuous shock following a mean zero AR(1) process in logs: $\log z' = \rho_z \log z + \eta'_z$ with $\eta'_z \sim N(0, \sigma_{\eta_z}^2)$. Next, we estimate the values of ρ_z and σ_{η_z} from Solow residuals measured using NIPA data on US real GDP and private capital, together with the total employment hours series constructed by Prescott, Ueberfeldt, and Cociuba (2005) from CPS household survey data, over the years 1959-2002. Finally, we discretize the resulting productivity process using a grid with 11 shock realizations; $N_z = 11$.

We determine idiosyncratic shocks $(\varepsilon_i)_{i=1}^{N_\varepsilon}$ and the Markov Chain determining their evolution $(\pi_{ij}^\varepsilon)_{i,j=1}^{N_\varepsilon}$ by discretizing a log-normal process, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'_\varepsilon$ using 25 values ($N_\varepsilon = 25$). In future, we will select the persistence of the idiosyncratic shock process, ρ_ε , alongside the standard deviation of its innovations $\sigma_{\eta_\varepsilon}$, to reconcile our full model with some selected moments drawn from the establishment investment distribution reported by Cooper and Haltiwanger (2006). For now, however, we simply set these parameters at 0.3 and 0.1, respectively. To our discretized idiosyncratic productivity process, we then add the realization $\varepsilon_{N_\varepsilon+1} = 0$ as an absorbing state, and we assign a common probability of moving from any ε_j to this one such that an entering firm expects to remain in the economy for roughly 15 years. Each exiting firm is replaced by a new firm endowed with roughly 20 percent of the average capital stock, no debt, and a productivity level drawn with equal probability from the $\{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$ support determined above.

5 Steady state

5.1 Full model with real and financial constraints

We begin by considering the implications of borrowing limits and irreversibilities for the typical decisions made in our economy. Figure 1 overviews the stationary distribution of firms in the baseline case of our full model, presenting three slices of the full distribution. In the top panel, we see the distribution of firms over capital and debt levels at the lowest nonzero firm-level productivity, while the middle and bottom present the counterparts at the median and highest levels of productivity.

Note that each panel of figure 1 appears to have two essentially disjoint distributions. The smooth, connected distribution where capital is relatively high (above 0.6) and debt is zero corresponds to older or wealthier firms that are unconstrained. Elsewhere, roughly 20 percent of all firms newly entering the economy each period are scattered evenly across each productivity level. These firms enter with zero debt and very low initial capital (roughly 0.30), as shown by the isolated $\mu(k, b)$ spike in each panel.

After its first date in production, each new firm begins to take on debt in effort to build up its capital. In the absence of the collateralized borrowing limits, young firms would immediately take on a large, temporary debt that would allow them to jump to the capital stock selected by unconstrained firms with the same current productivity level. Instead, however, the borrowing limits gradualize their adjustments, and we see ripples of these entering firms slowly moving into higher ranges of k and b over time. Only after a firm has accumulated sufficient collateral can it take on the one-period loan it requires to jump leftward in the distribution, joining the mass of unconstrained firms. Over time, those firms that survive long enough eventually reach a level of capital (and current productivity) such that they can no longer be affected by borrowing limits. As this occurs, the firm in question leaves the positive-debt, constrained group, jumping to the smooth unconstrained distribution. As would be expected, the mean capital among unconstrained firms rises with firm-level productivity. The same is true for constrained firms, though this is somewhat harder to see in the current figure.

Continuing our examination of low-, median-, and high-productivity slices of the (k, b, ε) space, we use figure 2 to consider which values of capital and debt allow firms to permanently escape the effects of the financial frictions in our economy. This figure illustrates, at each of the three

productivity levels, the regions of (k, b) where the indicator reflecting an unconstrained firm takes on a value of 1 (rather than 0). As one might expect, firms that have accumulated the greatest wealth via high k or large negative debt are unconstrained. Elsewhere (given ε), at any given low value of capital, firms with more debt remain constrained, while those with sufficiently low debt do not. Similarly, at any given level of debt, there is a cutoff level of capital below which firms are constrained.

As we consider how the constrained versus unconstrained firm types change as productivity rises, it is worth noting that a high value of ε has two opposing effects on a firm's likelihood of encountering binding borrowing limits. Because it increases the current level of production, it allows the firm greater retained earnings, thus reducing the reliance on borrowing for a given level of investment. However, because it also predicts high productivity in future, a high current productivity raises the firm's target capital stock for the future, thus increasing its anticipated need for funds. Consider first those firms that have large savings in the form of $b < 0$. At this debt level, as we look from lowest productivity to mean to highest productivity, notice that the cutoff capital required for a firm to be unconstrained rises. Here, given that the firm has already accumulated a large level of savings, the fact that current production is high has relatively little importance; thus, the future productivity effect of a rise in ε outweighs the current production effect. By contrast, at a very high level of debt, the current production effect associated with higher ε can be the dominant one. Thus, in the top panel of figure 2, we find considerably larger cutoff capital levels associated with the high debt region than we see in the lower panels.

Figure 3 illustrates the capital decisions taken by firms at each start-of-period capital and debt level, again conditional on firm-level productivity, by plotting the values of an indicator variable $\mathcal{J}(k, b; \varepsilon)$. This variable takes on a value of 1 if the firm undertakes positive investment to reach $\hat{k}_u^*$ (or k_u^* for unconstrained firms) and a value of 2 if the firm disinvests to reach $\hat{k}_d^*$ (or k_d^*). Elsewhere, $\mathcal{J}(k, b; \varepsilon) = 3$ reflects a firm with a positive investment limited by its borrowing constraint, while a value of 4 reflects a firm with so limited access to credit that is forced to sell some capital to pay off its current debt. Finally, $\mathcal{J}(k, b; \varepsilon) = 5$ for those firms that choose to remain inactive with respect to their capital, selecting $k' = \frac{1-\delta}{\gamma}k$.

Looking first at the highest regions of capital, we see that all firms adopt option 5 and simply allow their stocks to depreciate. Based on our previous figure, we know that the firms in this region are financially unconstrained; thus, we know that their failure to shed capital is prompted

entirely by the specificity of capital. The loss of ten percent of the value of uninstalled stock is sufficient to deter firms even with a stock at 1.8 from selling some of it, despite the fact that the upward capital target associated with the highest productivity lies below 1.5. As such, in the steady state of this economy, we see no firms selecting the second adjustment option, that of disinvesting to a downward capital target. As we look from the top panel to the bottom, we see the region of capital associated with inactivity shrink. A higher current productivity raises the expected marginal product of capital next period, thus inducing firms with ever higher current stocks to undertake positive investment.

At the floor of each panel in figure 3, we have the regions of (k, b) at which firms will invest to reach an upward target capital. As noted above, the left side of this region expands at higher levels of productivity, as firms' target capitals rise sufficiently to prompt action. By contrast, the expansion in the option 1 region to the right (and rear) as ε rises is the result of increases in current production. Consider, for example, firms that have no debt and a current capital of 0.8. Among these firms, those with relatively low current productivity (as in the top two panels of our figure) do not have sufficient current earnings to supplement the funds they can borrow toward reaching their upward target, while those with high productivity (as in the bottom panel) produce enough that the combination of their borrowing and earnings allows them to do so.

As suggested by the remarks above, the leftward, rearward regions of figure 3 (where capital is low and/or debt is high) are those where the economy's financial frictions take center stage. Adjacent to the regions of (k, b) where firms adjust to their upward target capitals, we have investments constrained by borrowing limits. In the top panel, for example, among those firms with capital around 0.5, those with zero existing debt can borrow enough to reach their upward target, while those with debt on the order of 0.1 invest to the maximum capital they can attain. Similarly, firms with a given level of debt, say 0.1, can achieve their upward targets only if their existing stock is sufficiently high. This is in part because less investment is required for a firm with a higher current stock, and it is in part because a higher existing stock represents greater collateral and thus a greater ability to borrow. Finally, at the lowest values of capital and the highest levels of debt, we have those firms that cannot borrow enough to repay their existing loans and thus are forced to sell some of their stock. This region where firms are constrained to disinvest is largest at the lowest productivity levels, since current earnings are so low there that they are of little help toward firms' efforts to repay their loans.

Through our discussion above, we have considered the logic behind the various investment decisions made over a wide range of the firm-level state space. This analysis will be useful below as we explore how our economy, in the aggregate, changes in response to a real or financial shock. However, for the purposes of our steady state discussion, a glance back to figure 1 reminds us that much of the (k, b) space we have been considering is not relevant in light of the stationary distribution of firms. Restricting attention to capital and debt combinations actually observed in the steady state, we find there are no firms being forced to sell capital to repay their loans, roughly 43 percent of firms investing to achieve their upward target capitals, 32 percent investing to a credit-constrained stock and 25 percent of firms opting to remain inactive with respect to their capital.

Finally, figure 4 illustrates the levels of output produced across the full range of capital and debt levels. As one would expect, the level of production at any given (k, b) combination rises with the level of productivity, and, examining any single current productivity, production rises with the firm's capital stock. However, the level of debt has no influence here, since we do not require our firms to pay their wage bills until after current production is done. Thus, as we consider various alternative scenarios for our real and financial frictions below, we find that the output figures corresponding to this one are essentially unchanged. Of course, this does not imply that steady state output, productivity and the distribution of production are unchanged, as each of these is influenced by the resulting stationary distribution of firms over the firm-level state space.

5.2 Reference models removing real or financial constraint

In this section, we briefly consider how the mechanics of our model are altered when we eliminate one of the two frictions therein. Figures 5 and 6 examine the case where there are no irreversibilities in investment. In this case, there is far more capital reshuffling with changes in firm-level productivity (in figure 5), and thus more dispersion in the distribution of capital across firms that are financially unconstrained. Among relatively young and unlucky firms, we continue to see the effects of the borrowing limits preventing them from joining the mass of unconstrained firms. Here again, debt and capital are slowly accumulated until such time when the firm can take one last loan to achieve sufficient capital to avoid any future effects of borrowing limits. Turning to figure 6, we see that, when we eliminate real rigidities, firms are never inactive with respect to their capital stocks. Moreover, because the collateral value of any given capital stock

is determined by what fraction of it can be successfully uninstalled and moved elsewhere in the economy in the event of default, and because this fraction has now become 1, the region of the (k, b) space where firms can invest to their target capital level has expanded relative to that seen in figure 3. In terms of the capital choices made over the stationary distribution of firms, the fraction of firms that are (potentially) constrained falls modestly with the removal of capital specificity, to roughly 18 percent, roughly 75 percent of firms invest to their target capitals each period, while the remaining 25 percent invest to a constrained stock. Finally, with the distribution of production allocated somewhat more efficiently with respect to firms' relative productivities, but collateralized borrowing limits still in place, steady state production rises by only about 0.03 percent in the aggregate.

In figures 7 and 8, we reinstate the real frictions of our full model, this time eliminating the financial ones. Specifically, by setting $\theta_b = 125$, we ensure that no firms ever face binding borrowing constraints. As a result, there are no longer two disjoint conditional distributions of firms over (k, b) , as we saw in figures 1 and 5. Instead, as soon as they have completed an initial period of production, the mass of entering firms in the spikes of figure 7 immediately borrow the necessary funds and join the distribution of incumbent firms at the start of the next period. As such, with the elimination of financial frictions, we shift the overall distribution of firms into higher average levels of capital. Nonetheless, irreversibilities continue to limit firms' willingness to alter their capital stocks in response to changes in their productivities. Thus, in figure 8, we see that (as in our full economy), there is a large region of the state space over which firms will passively allow their capital to depreciate. There is one new feature in the top panel of figure 8, however. Here, with the relaxing of credit constraints, capital no longer carries a large shadow value associated with its collateral role. Thus, we see that firms with very low productivity and high capital (at $k = 1.8$) actively choose to eliminate their excess stock. However, this region has no mass in the stationary distribution. Thus, in steady state, we see 62 percent of firms adopting their upward capital targets and 38 percent remaining inactive. Finally, when the borrowing constraint is effectively lifted from our economy, steady state output rises by 1.8 percent.

5.3 Extreme frictions

We close our examination of steady state by considering the implications of more extreme limits on borrowing or capital mobility. Figures 9 -10 present the case in which firms have no

access to credit at all. Here, firms entering the economy at the left side of each panel in figure 9 must gradually build their capital stocks over many years before they can reach the average levels of capital help by older incumbents. Correspondingly, only the $b = 0$ slices of the firm state space in figure 10 are relevant in this economy, so no firm ever actively sheds (or is forced to sell) capital. Here, roughly 28 percent of firms achieve their capital targets using retained earnings to finance their investment, while 51 percent undertake constrained investment. The shortage of capital among younger firms implied by the absence of credit, together with the partial irreversibility further limiting the efficient allocation of production across firms implies a further reduction in steady state production of roughly 0.8 percent.

Finally, returning to our baseline full model, we sharply raise the real frictions in our economy by sharply reducing θ_k while holding θ_b at its original value. We do not present the figures associated with this case, because it is an economy very similar to that discussed just above. In particular, given collateralized borrowing limits, when the fraction of capital that can effectively serve as collateral is reduced sharply via a drop in θ_k , the result is a setting where many firms (here, roughly 30 percent of all firms) are financially constrained. As a result, the steady state distribution of firms looks quite similar to that in figure 9, as are the fractions of firms undertaking each variety of capital adjustment in the steady state, though the overall output loss is somewhat larger, at 1.3 percent.

6 Dynamics

We presume that the observations of the preceding section will be useful reference as we seek to understand the aggregate behavior of actual economies facing both real and financial frictions and shocks. However, no steady state analysis can possibly be sufficient basis for answering the questions that motivate this paper. Unfortunately, while each of our reference economies have been solved, the general equilibrium solution for the dynamics of our full economy is not yet available. Thus, we must defer our dynamic results, and hence the quantitative answers we seek from a fully calibrated environment, for the next draft of this document.

6.1 Real shock in full model versus reference models

TBA

6.2 Financial shock in full model versus no-specificity reference

TBA

7 Concluding remarks

TBA

References

- [1] Bertola, G. and R. J. Caballero (1994) “Irreversibility and Aggregate Investment,” *Review of Economic Studies* 61, 223-246.
- [2] Buera, F. and Shin, Y. (2007) “Financial Frictions and the Persistence of History: A Quantitative Exploration,” manuscript, Northwestern University.
- [3] Caballero, R. J. (1999) “Aggregate Investment,” chapter 12 in M. Woodford and J. Taylor (eds.) *Handbook of Macroeconomics*, vol. IB. Amsterdam: North Holland.
- [4] Caballero, R. J and E. M. R. A. Engel (1999) “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S, s) Approach,” *Econometrica* 67, 783-826.
- [5] Caggese, A. (2007) “Financing constraints, irreversibility, and investment dynamics” *Journal of Monetary Economics* 54, 2102–2130.
- [6] Comin, D. and T. Philippon (2005) “The Rise in Firm-Level Volatility: Causes and Consequences,” NBER Working Paper No. 11388.
- [7] Cooper, R. W. and J. C. Haltiwanger (2006) “On the Nature of Capital Adjustment Costs,” *Review of Economic Studies* 73, 611-633.
- [8] Cordoba, J.C. and M. Ripoll (2004) “Credit Cycles Redux” *International Economic Review* 45, 1011-46.
- [9] Hansen, G. D. (1985) “Indivisible Labor and the Business Cycle,” *Journal of Monetary Economics* 16, 309-327.
- [10] Khan, A. and J. K. Thomas (2003) “Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?” *Journal of Monetary Economics* 50, 331-360.
- [11] Khan, A. and J. K. Thomas (2008) “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics” *Econometrica* 76, 395-436.
- [12] Kiyotaki, N. and J. Moore (1997) “Credit Cycles” *Journal of Political Economy* 105, 211-248.
- [13] Kocherlakota, N.R. (2000) “Creating Business Cycles Through Credit Constraints” *Federal Reserve Bank of Minneapolis Quarterly Review* 24, 2-10.

- [14] Krusell, P. and A.A. Smith Jr. (1997) "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics* 1, 387-422.
- [15] Modigliani, F. and M. Miller (1958). "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48, 261-297.
- [16] Prescott, E. C. (1986) "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review* 10, 9-22.
- [17] Restuccia D. and R. Rogerson (2008) "Policy distortions and aggregate productivity with heterogeneous establishments" *Review of Economic Dynamics* 11, 707-720.
- [18] Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21, 3-16.
- [19] Veracierto, M. L. (2002) "Firm-Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review* 92, 181-197.

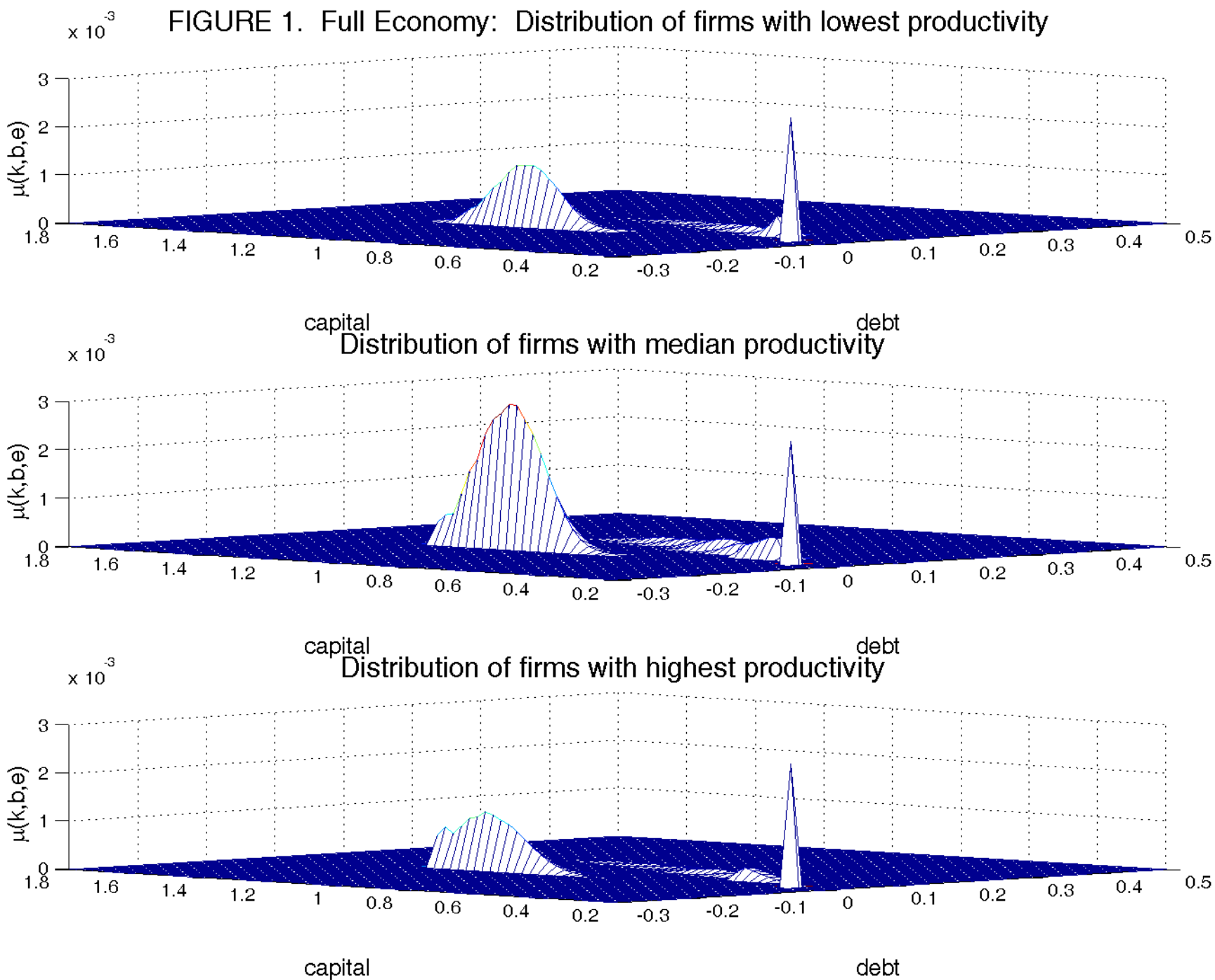
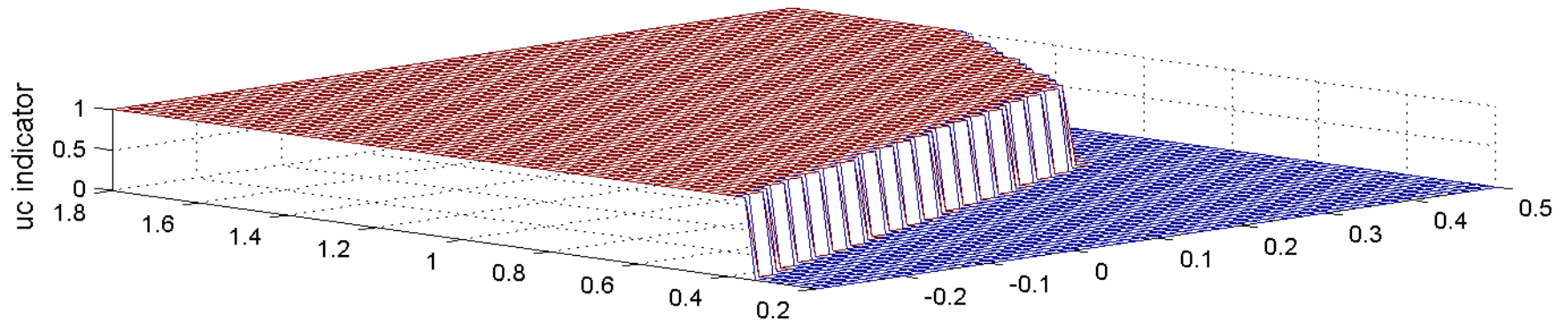
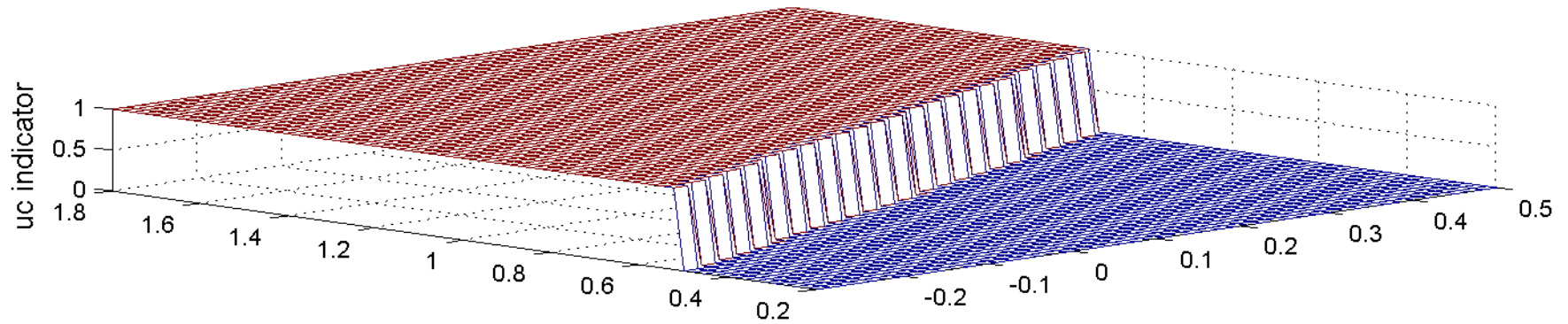


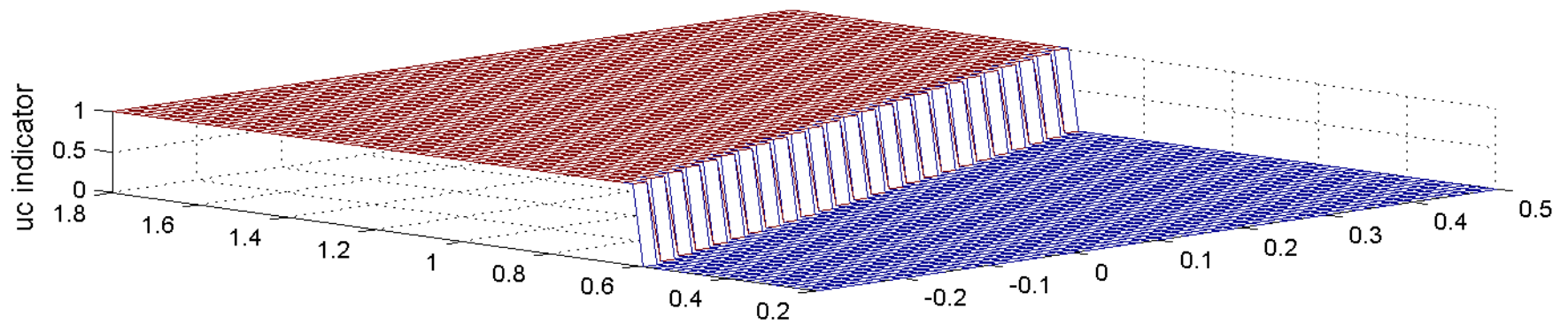
FIGURE 2. Full Economy: Firms with lowest productivity that are unconstrained



Firms with median productivity that are unconstrained



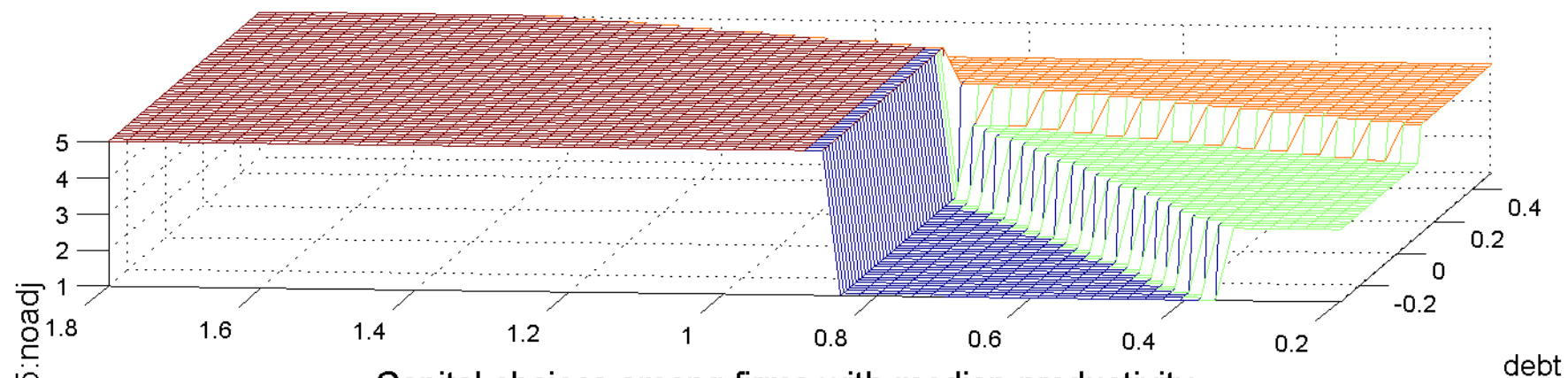
Firms with highest productivity that are unconstrained



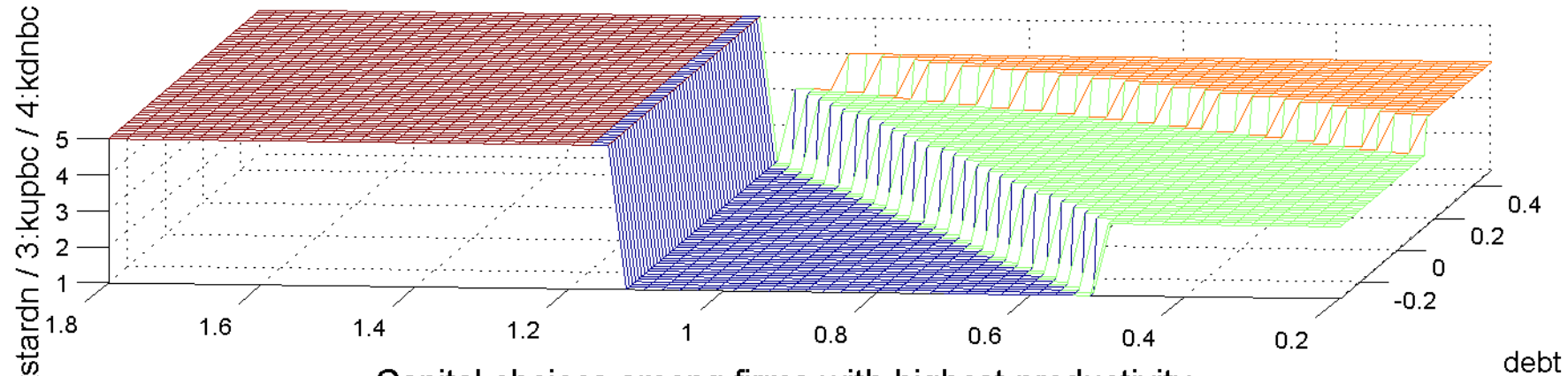
capital

debt

FIGURE 3. Full Economy: Capital choices among firms with lowest productivity



Capital choices among firms with median productivity



Capital choices among firms with highest productivity

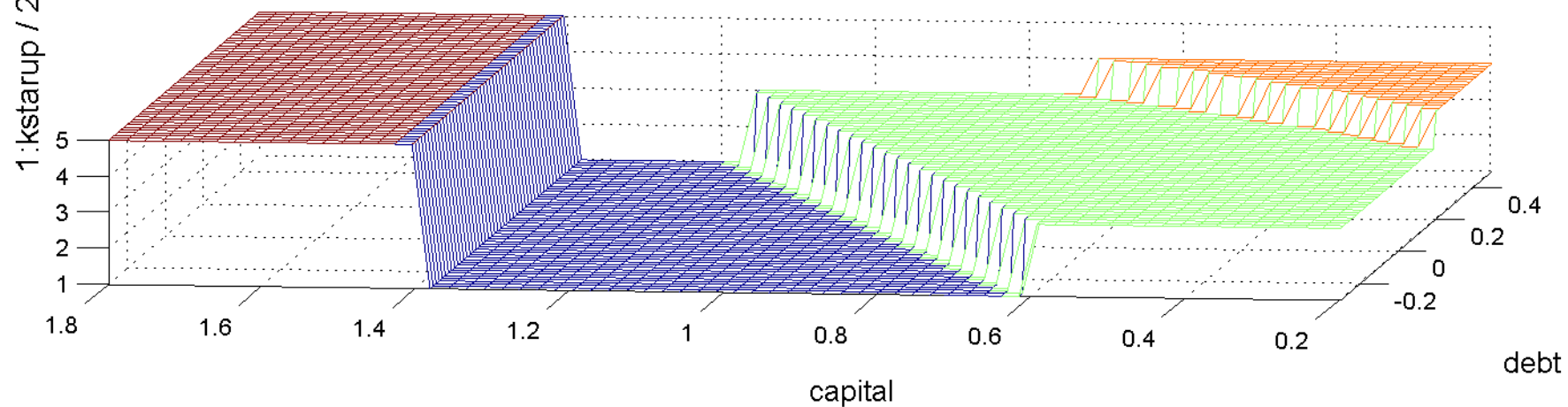
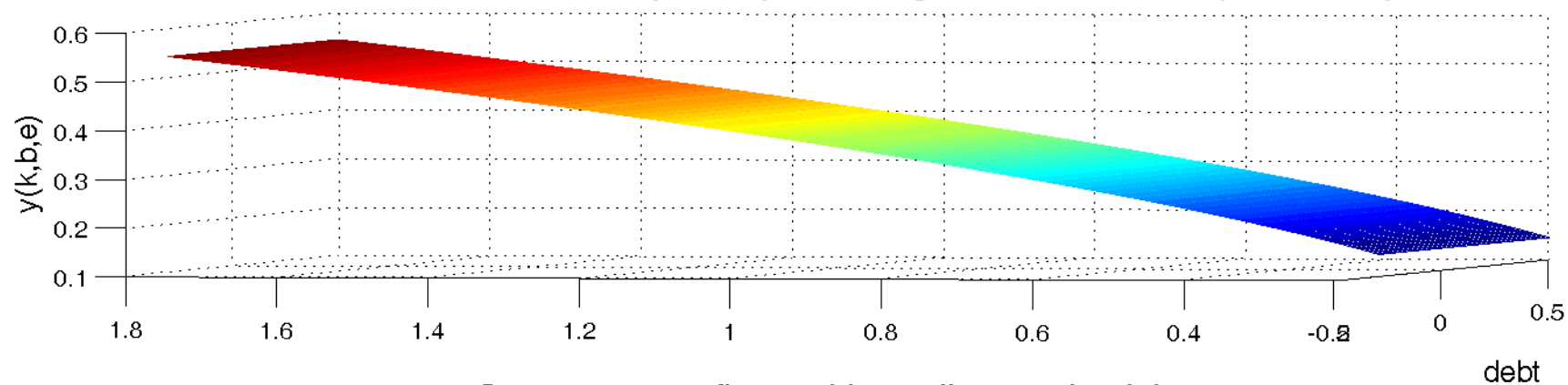
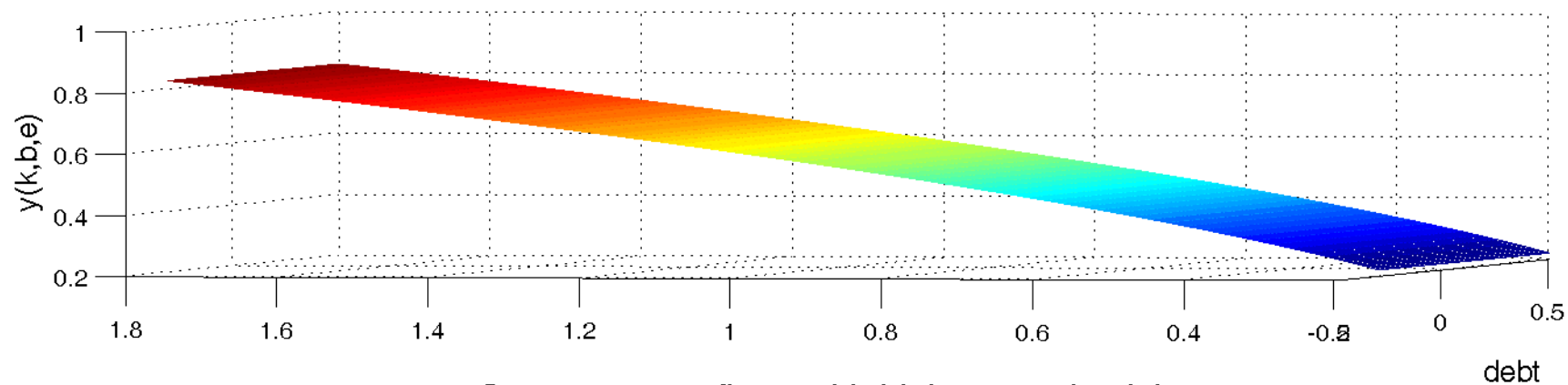


FIGURE 4. Full Economy: Output among firms with lowest productivity



Output among firms with median productivity



Output among firms with highest productivity

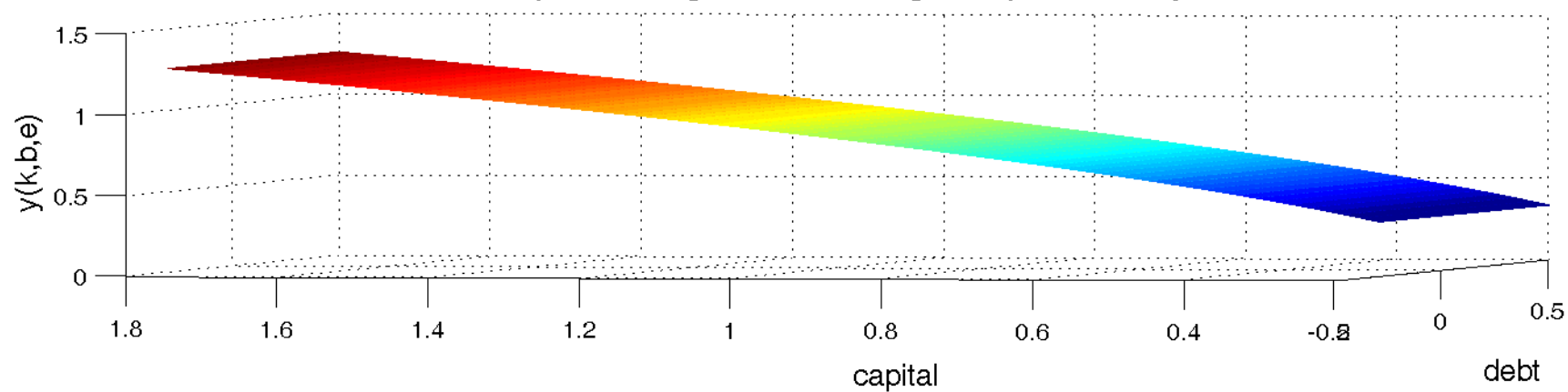


FIGURE 5. No Irreversibilities: Distribution of firms with lowest productivity

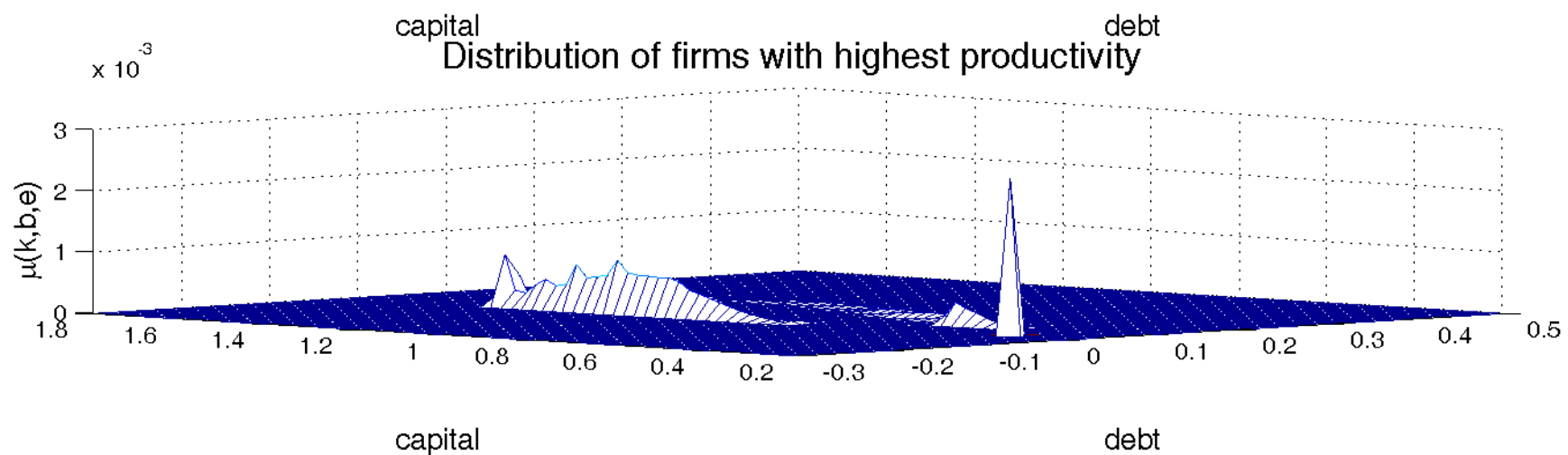
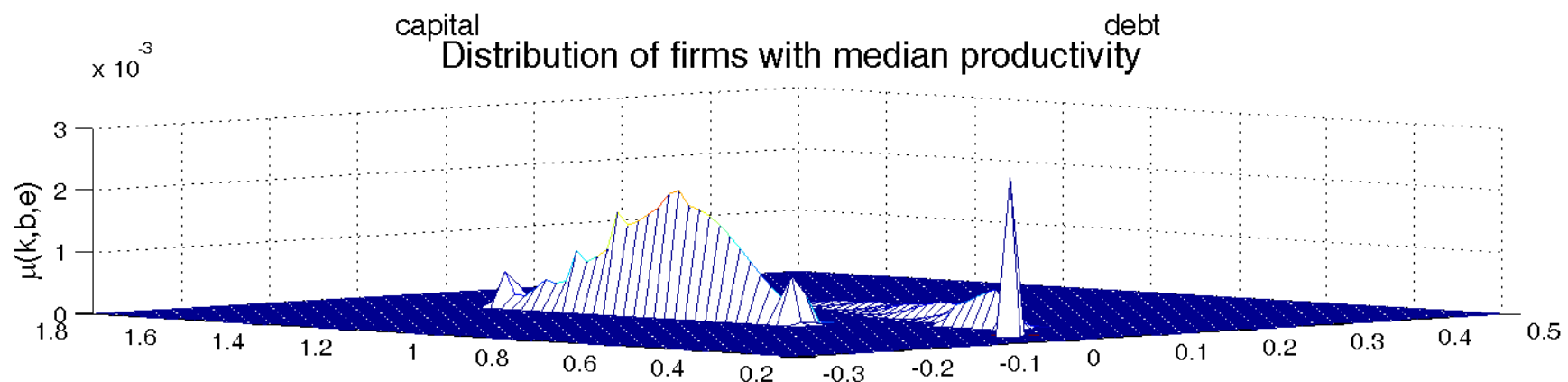
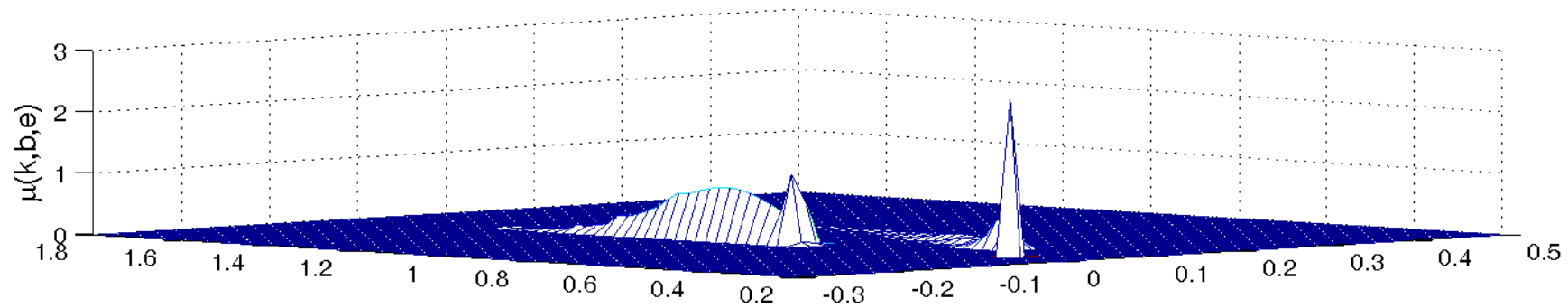
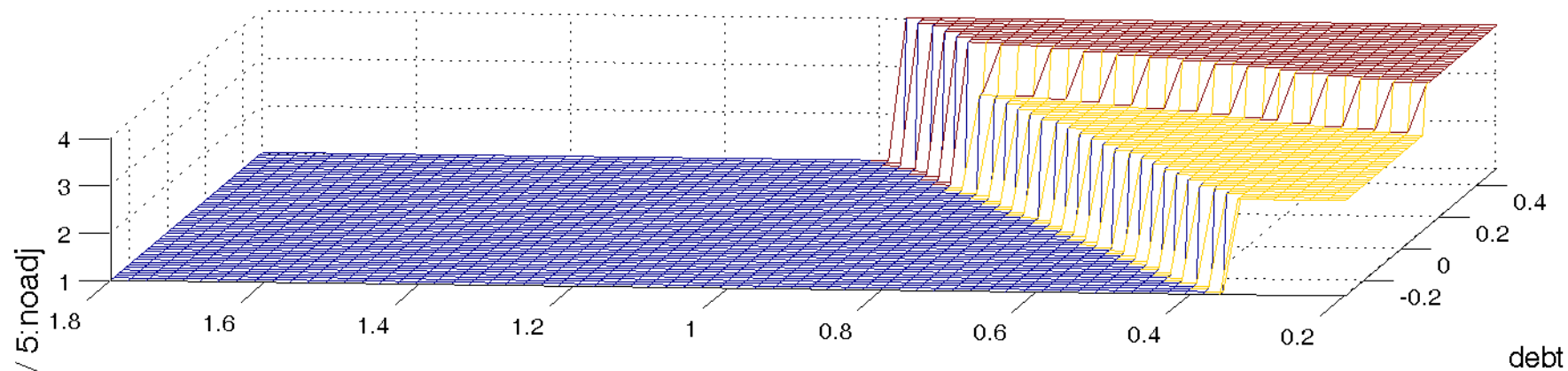
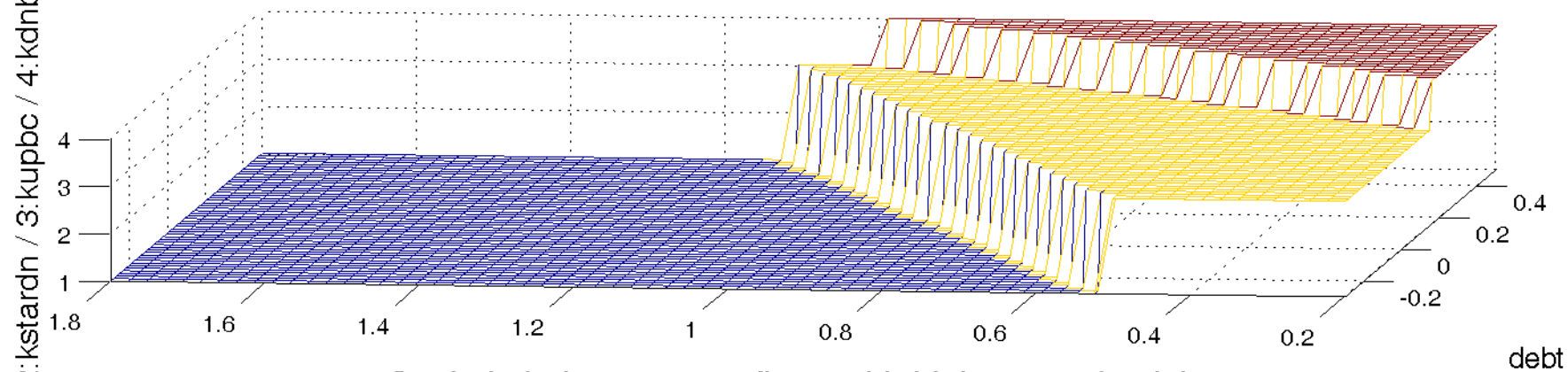


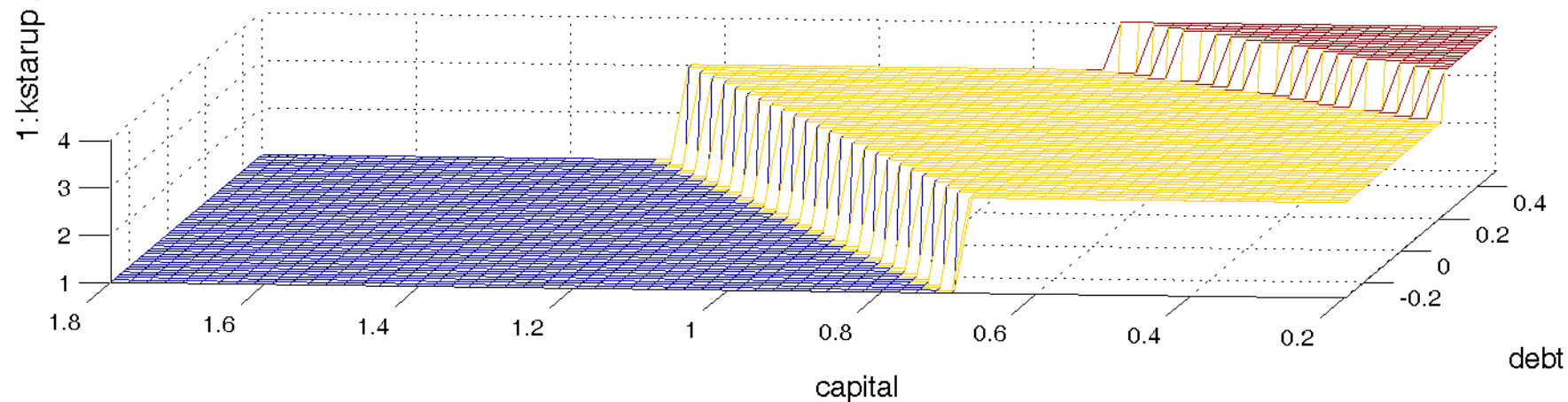
FIGURE 6. No Irreversibilities: Capital choices among firms with lowest productivity



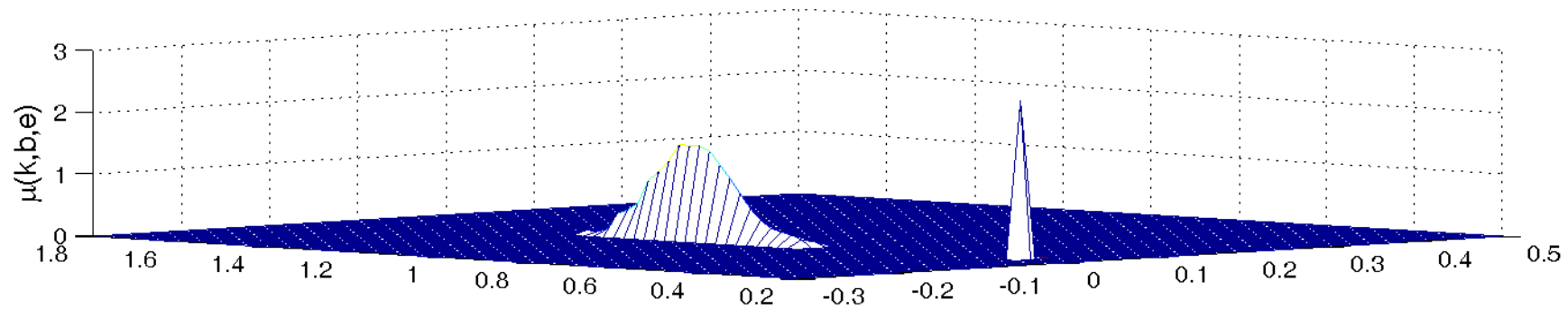
Capital choices among firms with median productivity



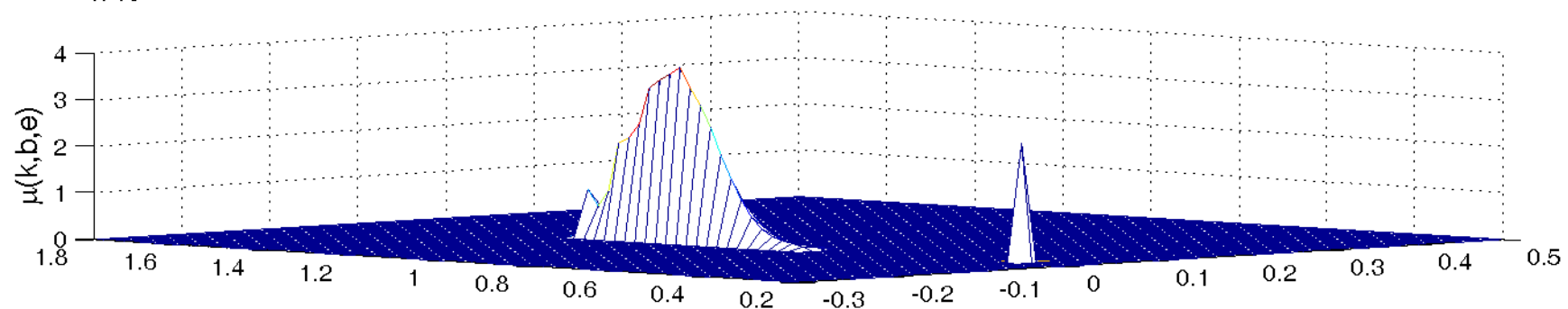
Capital choices among firms with highest productivity



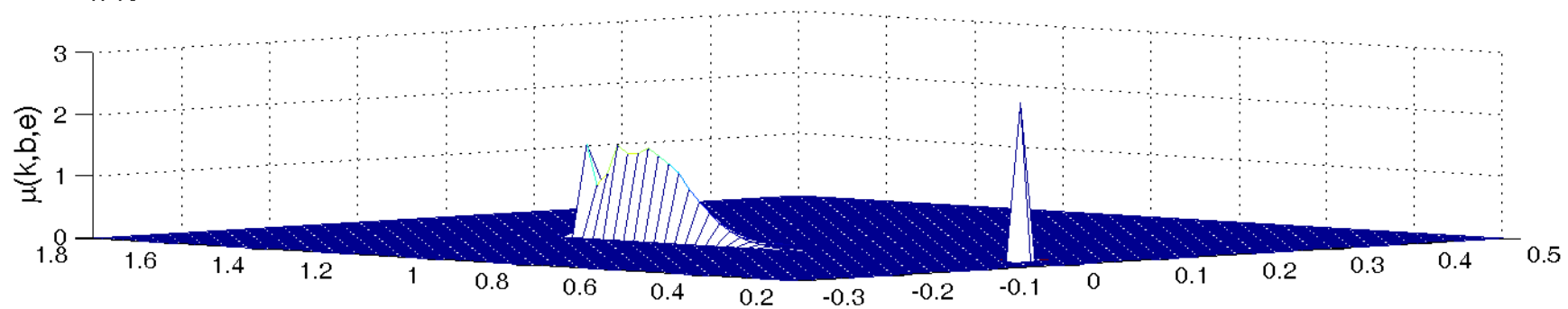
$\times 10^{-3}$ FIGURE 7. No Financial Frictions: Distribution of firms with lowest productivity



$\times 10^{-3}$ Distribution of firms with median productivity



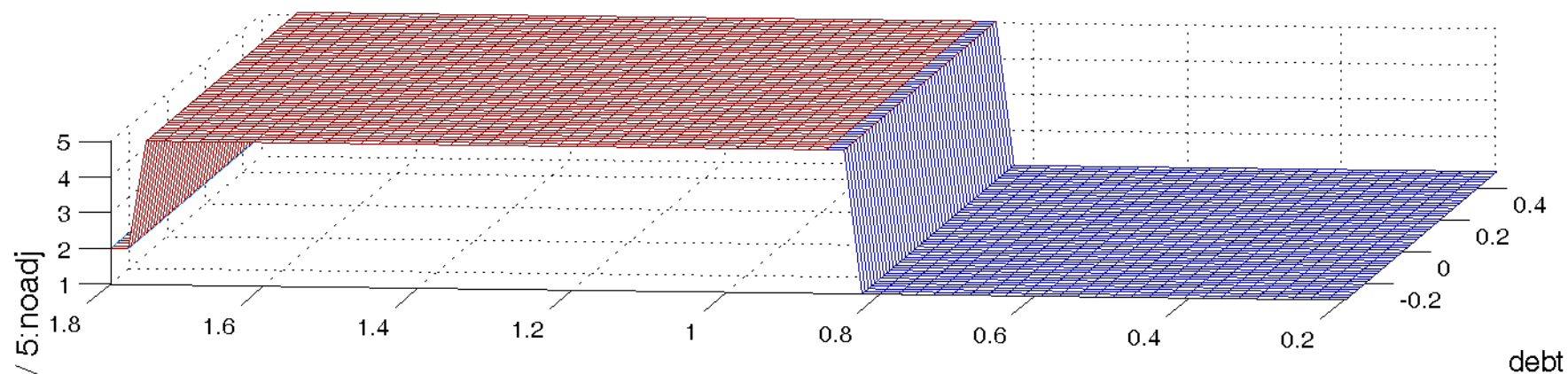
$\times 10^{-3}$ Distribution of firms with highest productivity



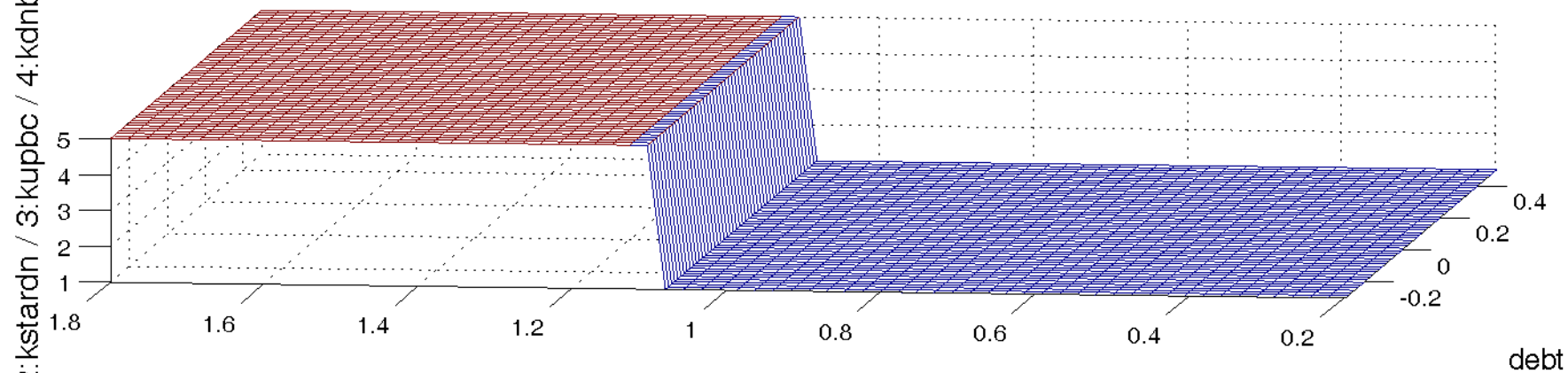
capital

debt

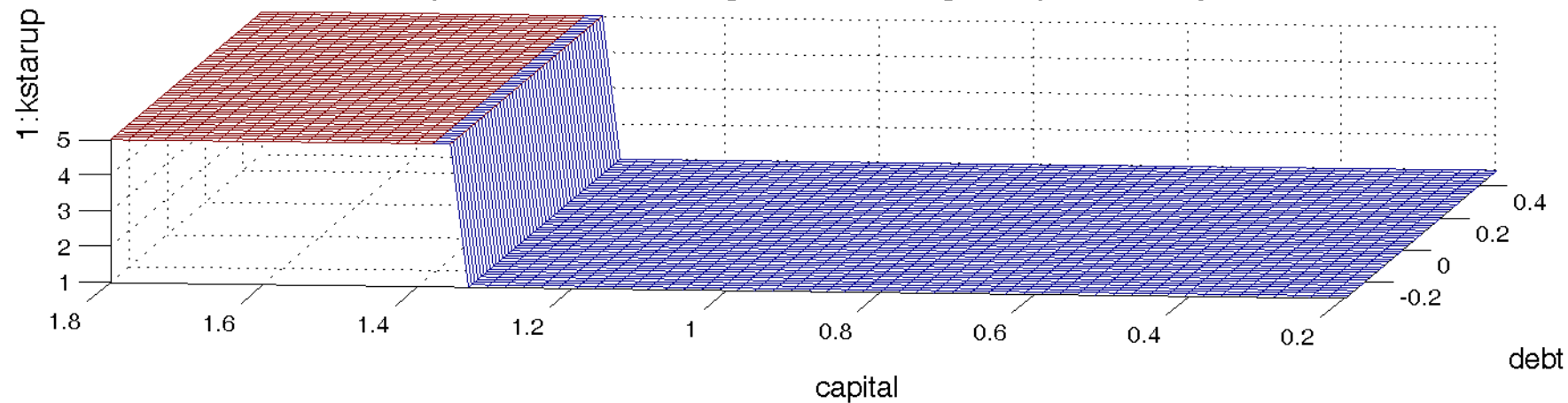
FIGURE 8. No Financial Frictions: Capital choices among firms with lowest productivity



Capital choices among firms with median productivity



Capital choices among firms with highest productivity



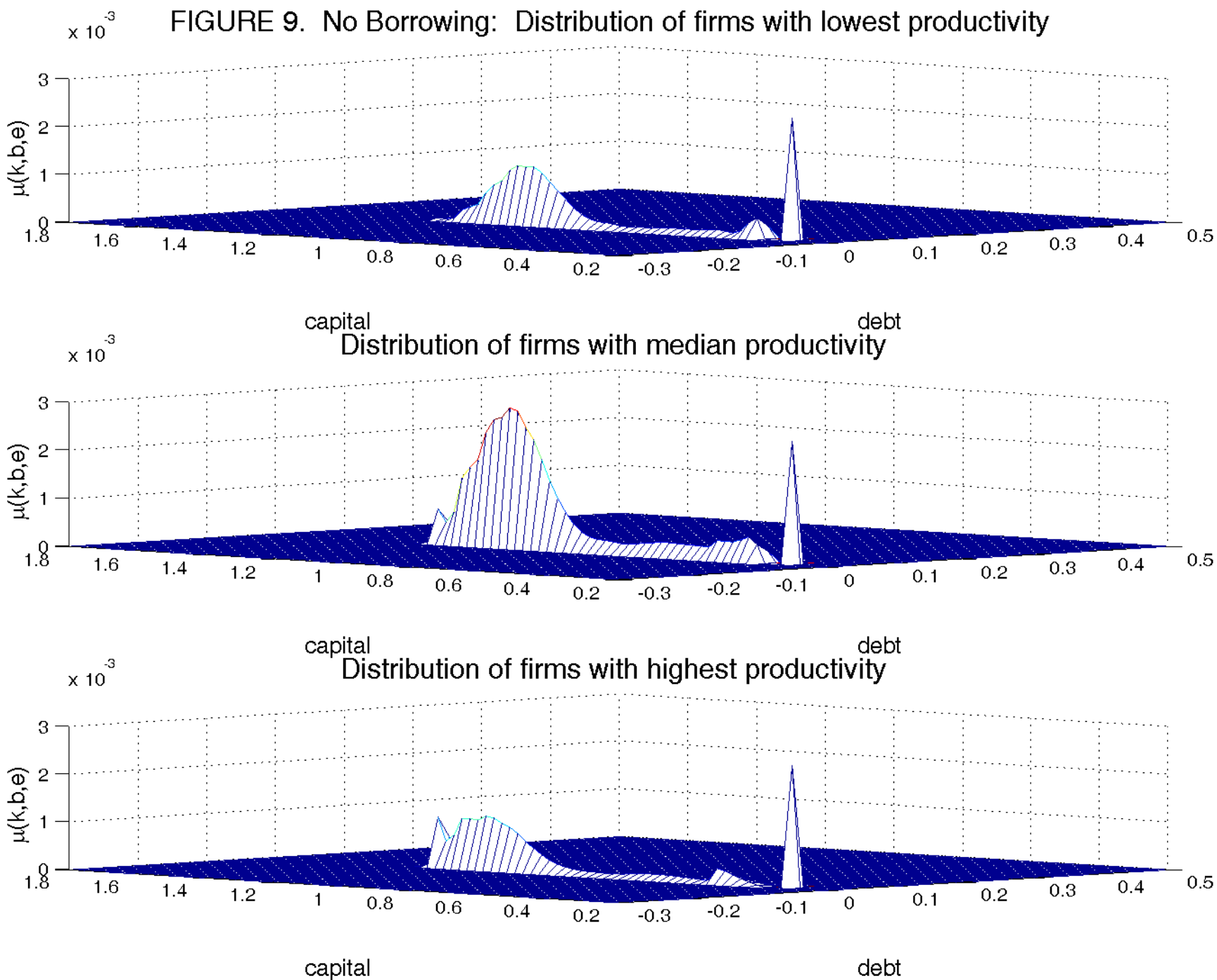
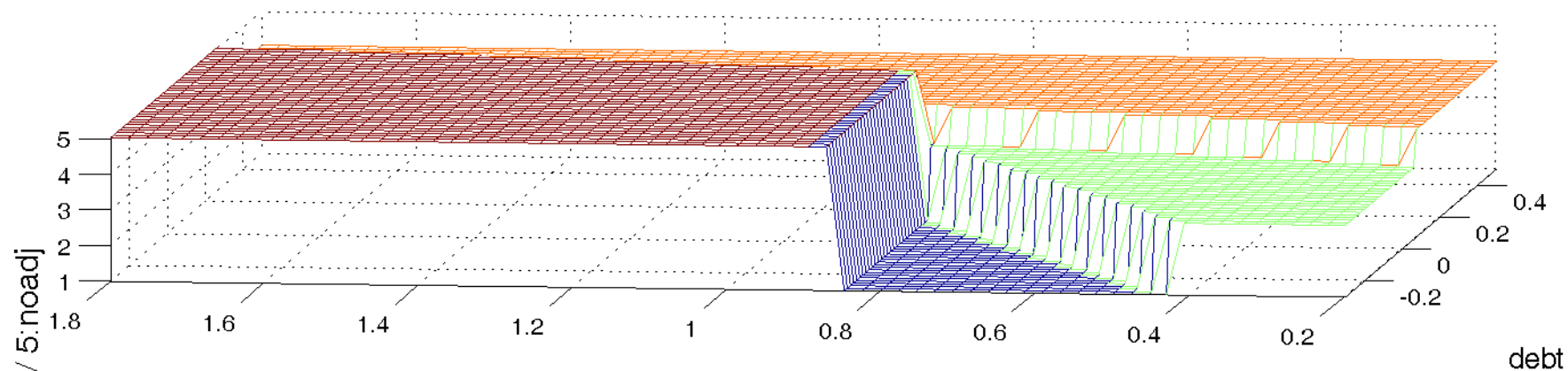
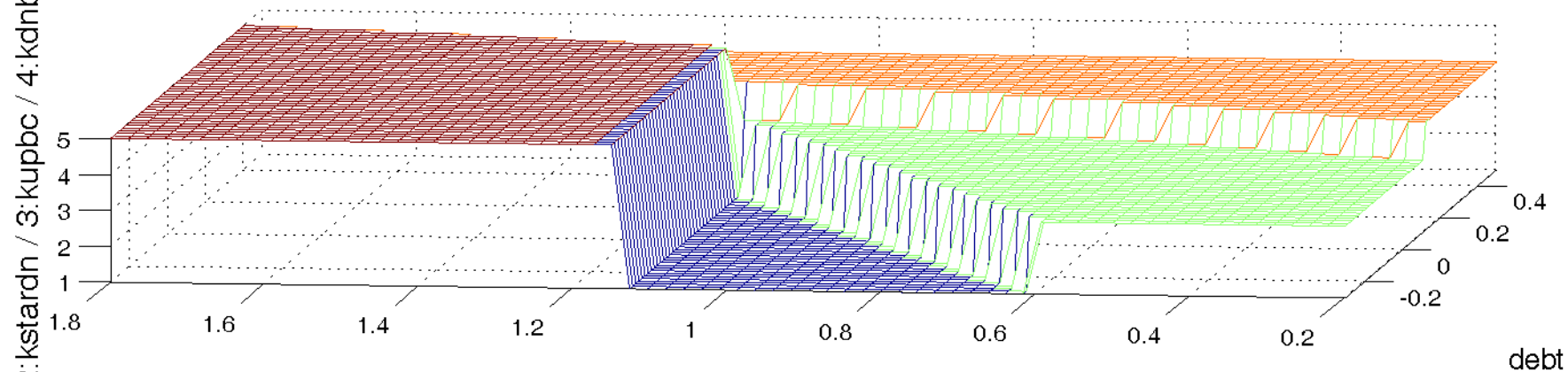


FIGURE 10. No Borrowing: Capital choices among firms with lowest productivity



Capital choices among firms with median productivity



Capital choices among firms with highest productivity

