

On the Individual Optimality of Economic Integration^{*}

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**Preliminary and Incomplete
Comments Welcome**

Abstract

How often, and for which countries, is it individually optimal to form an economic union? We emphasize the risk-sharing benefits on economic integration. We consider an endowment world economy model, where international financial markets are incomplete and contracts are not enforceable. A union is an arrangement that solves both the market incompleteness and the lack of enforcement problems among member countries. The union as a whole still faces these frictions when trading in the world economy. We uncover conditions on the initial income and net foreign assets of potential union members such that forming a union is welfare-improving over standing alone in the world economy. Our model predicts that economic unions (i) occur relatively infrequently, and are more likely to emerge (ii) among homogeneous countries, and (iii) among rich countries.

Keywords: Incomplete markets, endogenous borrowing constraints, risk sharing, economic integration.

JEL Codes: F15, F34, F36, F41.

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1 Introduction

How often, and for which countries, is it individually optimal to form an economic union? We emphasize a particular motivation for economic integration: improving risk sharing. An economic union is an arrangement in which its partners are better able to cope with frictions that limit risk-sharing in the world economy.

We consider an initial situation in which a continuum of individual countries are sitting in a world economy model with very limited possibilities to share idiosyncratic endowment risk. Risk sharing is limited by two frictions. First, markets are incomplete since countries may only trade a non-contingent bond. Second, international lending contracts are not legally enforceable. At any time, a country may choose to repudiate its foreign debt. The sanction for doing so is the permanent exclusion from future trade in world markets. Our world economy model is a variant of [Clarida \(1990\)](#) and [Huggett \(1993\)](#), featuring self-enforcing borrowing limits along the lines of [Kehoe and Levine \(1993\)](#), [Kocherlakota \(1996\)](#), and [Alvarez and Jermann \(2000a\)](#). Versions of this setup have been studied previously by [Zhang \(1997\)](#) and [Krueger and Perri \(2006\)](#).¹

We then consider the possibility that a pair of countries selected at random from the world economy is suddenly offered the possibility of forming an economic union. A union, by assumption, is an arrangement which solves both the market incompleteness and the lack of enforcement problems among its member countries. The union as a whole, however, still faces these frictions when trading in world markets. Since the endowment risk facing union members cannot be fully diversified away, they still have an interest in trading with the rest of the world. We focus on a centralized setting for the union's international borrowing and lending and default decisions, as if they were made by a central union government.

The key trade-off we emphasize about union formation is the following. The benefits from economic integration are two. First, forming a union improves risk-sharing among member countries. Second, a union allows for poor partners to use the rich partners' credit lines. This is a benefit for poor partners. There are also two costs of economic integration. First, borrowing limits become tighter, since defaulting on international debt becomes less costly for union partners. This is because union partners may still share risk upon default. The second cost is for rich partners. The fact that poor partners may benefit from the rich partner's credit limit generates a negative exter-

¹See [Abraham and Carceles-Poveda \(2006\)](#) and [Bai and Zhang \(2008a\)](#) for variants with capital accumulation. See also [Castro \(2005\)](#) for a variant with capital accumulation and endogenous but ad-hoc borrowing constraints.

nality: rich partners will find themselves more often borrowing constrained in a union compared to standing alone in the world economy.

There are two key ingredients in our framework. First, it generates costs as well as benefits from economic integration. Second, it generates disagreement about union formation, and the disagreement is the largest when the partners are more heterogeneous.

These two ingredients provide a potential explanation for three puzzling observations related to economic integration: (i) deep economic integration is relatively rare, and when it does take place it tends to feature (ii) relatively homogeneous partners, and (iii) relatively rich partners.

These observations are puzzling because, under a very broad set of circumstances, economic theory would imply that economic integration should happen often, particularly among heterogeneous partners. For example, this would be the case for capital market integration in the neoclassical growth model, or goods market integration in either the Heckscher-Ohlin or the Ricardian models of trade.

Instead, our framework provides a very parsimonious explanation for these three puzzling observations. Economic unions may not be formed if either the costs of economic integration are too large, or if there is disagreement among partners. Unions are unlikely to be formed among heterogeneous partners, since rich partners suffer from a negative externality imposed by poor partners. Finally, unions are also more likely to be formed among relatively rich partners, because this lowers the likelihood of either country being borrowing constrained in the future, and thus the effect of the negative externality.

This paper is related to the literature on optimal currency areas, the literature on country formation, and the literature on risk-sharing in federations. To be completed.

The paper is organized as follows. Section 2 presents the model of the world economy. Section 3 characterizes the union. Section 4 presents the results. Section 5 studies some comparative statics. Section 6 concludes. Appendices A and B describe the decentralization of the union's allocation and the numerical algorithm, respectively.

2 World economy

2.1 Model

Consider a world economy composed of a continuum of small open economies of measure one. Countries are identical ex-ante. They all have the same size, and they all share the same initial

conditions. Countries differ ex-post due to idiosyncratic endowment risk. Each period, a country receives an endowment of a non-storable consumption good. The endowment evolves over time according to a Markov chain with a finite number of states in the set Y . We denote by $y^t = \{y_s, y_{s+1}, \dots, y_t\}$ the sequence of events from the initial time period $s < 0$ up to and including period t , and by $\pi(y^t)$ the probability of such sequence. The initial event $y^s = y_s$ is given and $\pi(y^s) = 1$. We denote by $\pi(y^t|y^\tau)$ the probability of y^t conditional on y^τ where $\tau \leq t$, and by $y^\tau \leq y^t$ the sequence y^τ which is a sub-root of y^t . We assume a law of large numbers holds in the cross-section of countries, which means there is no aggregate uncertainty.

Each country is populated by a continuum of identical and infinitely lived agents. The representative agent in a country has preferences:

$$\sum_{t=s}^{\infty} \sum_{y^t \in Y^{t+1}} \beta^t \pi(y^t) u(c(y^t)), \quad (2.1)$$

where $\beta \in (0, 1)$ is the subjective discount factor. The instantaneous utility u is increasing, strictly concave, and satisfies the Inada conditions: $u(0) = 0$, $\lim_{c \rightarrow 0} u'(c) = +\infty$ and $\lim_{c \rightarrow +\infty} u'(c) = 0$.

Countries cannot completely pool their income risk on world financial markets for two reasons. First, markets are incomplete: the menu of assets is exogenously restricted to a non-contingent one-period bond. A country's resource constraint is

$$c(y^t) + b(y^t) = y_t + (1 + r)b(y^{t-1}), \quad (2.2)$$

where $b(y^t)$ is the demand for foreign bonds and r is the (time-invariant) world interest rate.

The second friction is that international lending contracts are imperfectly enforceable. At any time, a country is free to repudiate its foreign debt, the penalty being the permanent exclusion from any future trade. A country that contemplates debt repudiation faces a trade-off between current and future utility: defaulting implies higher current consumption, at a cost of lower future utility due to living in autarky. International lending contracts are self-enforcing, in the sense that borrowing countries always find the cost of repudiation larger than the benefit, and they always choose to repay. That is, allocations satisfy the following participation constraint:

$$\sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi(y^\tau|y^t) u(c(y^\tau)) \geq V_{aut}(y^t), \quad (2.3)$$

where $V_{aut}(y^t)$ is the value of entering financial autarky after the history y^t . It is the lifetime utility derived from consuming one's endowment each period from the history node y^t onwards:

$$V_{aut}(y^t) = \sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi(y^\tau|y^t) u(y_\tau).$$

The representative agent chooses contingent plans for consumption and foreign assets to maximize lifetime utility (2.1) subject to the resource constraint (2.2), the enforcement constraint (2.3) and a no-Ponzi game condition:

$$b(y^t) \geq -D, \quad (2.4)$$

where D is large enough so that the constraint never binds in equilibrium.²

2.2 Recursive competitive equilibrium

We solve for the stationary recursive competitive equilibrium with solvency constraints. It will be convenient to focus on total wealth, $\omega \equiv y + (1 + r)b$, and the current endowment, y , as state variables. The problem of each country admits the following recursive formulation (see Bai and Zhang (2008b) for a formal proof):

$$V(\omega, y) = \max_{c, b'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(\omega', y') \right\} \quad (P0)$$

subject to:

$$\begin{aligned} c + b' &= \omega \\ \omega' &\equiv y' + (1 + r)b' \\ b' &\geq \bar{b}(y). \end{aligned}$$

The state contingent borrowing constraint $\bar{b}$ is the debt level such that for every possible state next period, the country is weakly better-off by repaying:

$$\bar{b}(y) = \max_{y': \pi(y'|y) > 0} \{ b_{y'} : V(y' + (1 + r)b_{y'}, y') = V_{aut}(y') \}. \quad (2.5)$$

This constraint allows countries to borrow as much as possible while preventing them from defaulting in any possible state next period. It is state contingent only if there exist future states that cannot be reached from current state. We assume a regular transition matrix, i.e. $\pi(y'|y) > 0$ for all y, y' , so that $\bar{b}(y) = \bar{b}$ for all $y \in Y$.

The autarky value V_{aut} is the solution to the following functional equation:

$$V_{aut}(y) = u(y) + \beta \sum_{y' \in Y} \pi(y'|y) V_{aut}(y'). \quad (2.6)$$

²Note that the enforcement constraint does not prevent countries from running Ponzi schemes: an agent running a Ponzi game would never default on its debt, since this would prevent him from continuing running the scheme.

Let Ω be the set of wealth levels, $S = \Omega \times Y$ the state-space, and $\mathcal{A}_S$ the σ -Borel algebra of elements of S . We are now ready to define the stationary recursive competitive equilibrium of the world economy.

Definition. A stationary recursive competitive equilibrium is given by decision rules $c(\omega, y)$, $b'(\omega, y)$, a value function $V(\omega, y)$, a borrowing limit function $\bar{b}(y)$, an interest rate r and a distribution $\Psi(\omega, y)$ of countries over S such that:

1. Given the world interest rate r and the borrowing constraint, the decision rules solve the recursive problem (P0) and V is the associated value function.
2. The solvency constraint $\bar{b}(y)$ is not too tight, in the sense of satisfying equation (2.5).
3. The world credit market clears:

$$\int_S b'(\omega, y) d\Psi(\omega, y) = 0.$$

4. The decision rules and the transition matrix of the endowment process induce a probability distribution P over the state space, $P : S \times \mathcal{A}_S \longrightarrow [0, 1]$, where:

$$P((\omega, y); A) = \sum_{y' : (b'(\omega, y), y') \in A} \pi(y'|y)$$

is the probability of transiting from state (ω, y) to a state in the set A .

5. The distribution Ψ is stationary and consistent with P :

$$\Psi(A) = \int_S P((\omega, y); A) d\Psi(\omega, y), \text{ for all } A \in \mathcal{A}_S.$$

2.3 Parameters and computation

We calibrate the model using preference and parameter values that are standard in the literature. Preferences are isoelastic:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{2.7}$$

with a coefficient of relative risk aversion $\sigma = 1.5$. The subjective discount factor is $\beta = 0.985$. The endowment process is obtained from a first-order autoregressive process:

$$y' = \rho y + \epsilon'$$

Y			II		
y_l	y_m	y_h	0.821	0.179	0.001
0.89	1.00	1.12	0.126	0.749	0.126
			0.001	0.179	0.821

Table 1: Markov chain parameters

where $\rho = 0.90$ and ϵ' follows an i.i.d. $N(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon = 0.05$. The process is discretized into a 3-state Markov chain using Tauchen's (1986) procedure. The set of endowment values Y and the transition matrix Π are reported in Table 1.

We briefly describe our numerical algorithm, the full details of which are provided in Appendix B.1. The outer loop solves for the interest rate that clears the world bond market. For given interest rate, we solve for debt limit functions which are not too tight, using the natural borrowing limit as the initial guess. Finally, for given interest rate and debt limit functions, we solve for the decision rules that solve the system of first-order conditions for the country's problem.

3 Economic union with centralized default

We assume the existence of a central authority in the union that coordinates the international trade and default decisions. In both cases, while union partners share risk among them, access to world markets and sovereignty are exerted by a central government.³

We now describe the process of union formation in the model. We assume the world economy is in steady-state. At time $t = 0$, and without anticipating it, a pair of countries sitting in the world economy is offered the possibility of forming a union. We assume the union is still small vis-a-vis the world economy. We pick these two countries from the ergodic state-space of the world economy's stationary equilibrium. Each country is characterized by an initial state (ω_{i0}, y_{i0}) , $i = 1, 2$.

Within the union, we assume financial markets are complete. Since a union is comprised of a finite number of countries (in this case two), the endowment risk faced by its members is not fully

³An alternative decentralized setting is one in which each individual member country unilaterally decides whether to default. Jeske (2006a) provides an analysis of this situation. In this case, potentially defaulting union members presume continued indirect access to world markets via the remaining union members. This increases the chances of defaulting, and therefore tightens borrowing limits within the union. All else constant, union formation is less likely under decentralized compared to centralized default.

diversifiable within the union. The union still has an incentive to trade with the rest of the world. We assume union members still have access to world financial markets under the same conditions as before, by trading on non-contingent bonds subject to enforcement constraints. Since union members coordinate their default decisions, there is a single union-wide enforcement constraint that applies to both countries at the same time. If the union defaults, all its members are permanently excluded from world markets, but they may still share endowment risk among them.

The union's endowment is determined by the realization of two independent and identically distributed endowment processes, one for each country. We denote it compactly by a two-dimensional vector $\bar{y}_t = (y_{1t}, y_{2t}) \in Y \times Y$, where each element $y_{it} \in Y$ is country i 's endowment realization, $i = 1, 2$. With a slight abuse of notation, we also denote by π the transition probabilities for $\bar{y}$:

$$\pi(\bar{y}'|\bar{y}) = \prod_{i=1}^2 \pi(y'_i|y_i),$$

where the $\pi(y'_i|y_i)$'s are displayed in Table 1.

3.1 Planner's problem

The allocation within the union is constrained-efficient, and can be obtained by solving a benevolent planner's problem. Let λ_i be the weight the planner attaches to country i . The planner's problem is to solve for $\{c_i(\bar{y}^t)\}_{i=1,2}$ and $b(\bar{y}^t)$, for all $\bar{y}^t$, $t \geq 0$, which maximize the weighted sum of the union partners' lifetime expected utilities

$$\sum_{i=1}^2 \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t))$$

subject to the union-wide resource constraint

$$\sum_i c_i(\bar{y}^t) + b(\bar{y}^t) = \sum_i y_{it} + (1+r)b(\bar{y}^{t-1}),$$

for all $\bar{y}^t$, $t \geq 0$, to the union-wide enforcement constraint

$$\sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^{\tau}} \beta^{\tau-t} \pi(\bar{y}^{\tau}|\bar{y}^t) u(c_i(\bar{y}^{\tau})) \geq V_{aut}^U(\bar{y}^t),$$

for all $\bar{y}^t$, $t \geq 0$, where

$$V_{aut}^U(\bar{y}^t) = \max_{\{c_i(\bar{y}^{\tau})\}} \sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^{\tau}|\bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^{\tau}|\bar{y}^t) u(c_i(\bar{y}^{\tau}))$$

subject to

$$\sum_i c_i(\bar{y}^\tau) = \sum_i y_{i\tau}, \text{ for all } \bar{y}^\tau, \tau \geq t,$$

for all $\bar{y}^t, t \geq 0$, and subject also to a no-Ponzi game condition

$$b(\bar{y}^t) \geq -D, \tag{3.1}$$

for all $\bar{y}^t, t \geq 0$.

Apart from distributional issues, the planner's problem is similar to the problem of a country standing alone in the world economy, the main difference being that, because the partners' endowment processes are uncorrelated, the union faces an endowment process which is less volatile. In addition, the union partners are also able to fully share the remaining idiosyncratic country risk.

3.1.1 Reformulating the planner's problem

Under isoelastic preferences, the union planner's problem admits a simpler formulation which is very convenient. By Proposition 5 of [Jeske \(2006b\)](#), aggregate borrowing and lending is independent of distributional issues. It follows that the planner's problem may be decomposed into two steps. In the first step, the planner solves for the optimal borrowing and lending of the union assuming a single representative country facing the aggregate endowment. In the second step, the planner redistributes the optimal aggregate consumption plan obtained from the first step among the two union partners.

Formally, the step 1 problem for the planner is

$$\max_{c(\bar{y}^t), b(\bar{y}^t)} \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c(\bar{y}^t)) \tag{P1}$$

subject to the aggregate resource constraint

$$c(\bar{y}^t) + b(\bar{y}^t) = \sum_{i=1}^2 y_{it} + (1+r)b(\bar{y}^{t-1}), \tag{3.2}$$

for all $\bar{y}^t, t \geq 0$, to the enforcement constraint

$$\sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u(c(\bar{y}^\tau)) \geq V_{aut}^U(\bar{y}^t) \tag{3.3}$$

for all $\bar{y}^t, t \geq 0$, where

$$V_{aut}^U(\bar{y}^t) = \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u\left(\sum_i y_{i\tau}\right),$$

for all $\bar{y}^t$, $t \geq 0$, and to the no-Ponzi game condition (3.1).

Given the optimal plan $c(\bar{y}^t)$ from step 1, step 2 solves for the optimal distribution of aggregate consumption among the union partners. Formally, the step 2 problem is

$$\max_{\{c_i(\bar{y}^t)\}} \sum_i \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t)) \quad (\text{P2})$$

subject to

$$\sum_i c_i(\bar{y}^t) = c(\bar{y}^t),$$

for all $\bar{y}^t$, $t \geq 0$.

With isoelastic preferences, the step 2 problem admits a simple, explicit solution. It is relatively easy to show that

$$c_i(\bar{y}^t) = \alpha_i c(\bar{y}^t) \quad (3.4)$$

where $\alpha_i \equiv \lambda_i^{1/\sigma} / \sum_j \lambda_j^{1/\sigma}$, for $i = 1, 2$. That is, individual consumption is a constant fraction of aggregate consumption. The fraction is increasing in the country's welfare weight.

Similarly to Section 2.2, the step 1 planner's problem admits a recursive formulation:

$$V^U(\bar{\omega}, \bar{y}) = \max_{c, b'} \left\{ u(c) + \beta \sum_{\bar{y}'} \bar{\pi}(\bar{y}' | \bar{y}) V^U(\bar{\omega}', \bar{y}') \right\} \quad (\text{P1}')$$

subject to

$$\begin{aligned} c + b' &= \bar{\omega} \\ \bar{\omega}' &\equiv \sum_i y'_i + (1+r)b' \\ b' &\geq \bar{b}^U(\bar{y}) \end{aligned}$$

where

$$\bar{b}^U(\bar{y}) = \max_{\bar{y}': \bar{\pi}(\bar{y}' | \bar{y}) > 0} \left\{ b_{\bar{y}'} : V^U\left(\sum_i y'_i + (1+r)b_{\bar{y}'}, \bar{y}'\right) = V_{aut}^U(\bar{y}') \right\} \quad (3.5)$$

and where $V_{aut}^U(\bar{y})$ solves

$$V_{aut}^U(\bar{y}) = u\left(\sum_i y'_i\right) + \beta \sum_{\bar{y}'} \pi(\bar{y}' | \bar{y}) V_{aut}^U(\bar{y}').$$

Given (3.4), the value for country i of belonging to a union with country j is

$$V_i^U(\bar{\omega}, \bar{y}) = \alpha_i^{1-\sigma} V^U(\bar{\omega}, \bar{y}). \quad (3.6)$$

3.2 Competitive equilibrium

To perform our welfare analysis, we need to recover individual allocations from the planner's. That is, we need to compute the planner's welfare weights as a function of the initial pair of union partner states.

We use [Negishi's \(1960\)](#) iterative method to compute the welfare weights. This method exploits the first welfare theorem, which allows us to obtain the competitive equilibrium allocation as the solution to the planner's problem, for a given set of welfare weights. The goal is to obtain the unique pair of welfare weights that lead to the competitive equilibrium allocation associated with a particular set of initial states. We therefore need to consider a decentralization of the constrained efficient allocation.

We consider a competitive equilibrium with tax subsidies, in line with [Kehoe and Perri \(2004\)](#), [Wright \(2006\)](#) and [Bai and Zhang \(2008b\)](#). The decentralization works as follows. Within the union, countries trade a complete set of Arrow securities. In world credit markets, they trade freely on non-contingent bonds. However, a central government authority in the union taxes each country's income in a lump-sum fashion, and uses the proceeds to subsidize asset purchases. The government's tax and transfer policy is designed to support the constrained-efficient allocation. A subsidy is required to encourage union partners to save in those states when they would be inclined to default. Our procedure is described in more detail in [Appendix A](#).

4 Results

Our goal is to characterize which country pairs find it individually rational to form a union. The main benefit of union formation is the possibility of sharing risk with a partner. There are also costs, however. First, default becomes more attractive for union members, since they may still share risk upon default. As a result, borrowing constraints are tighter in the union. In our benchmark calibration, the borrowing limit increases from $\bar{b}_i^W = -0.109$ in the world economy, to $\bar{b}_i^U = \bar{b}^U/2 =$ in the union, on a per country basis.

Second, in asymmetric unions, poorer country members tend to borrow heavily from the rest of the world, and exhaust the whole union's borrowing limit. This creates a negative externality for richer countries, which find themselves more frequently borrowing-constrained compared to standing alone in the world economy. Although being part of an asymmetric union tends to be beneficial for poorer members, it also tends to generate losses for richer countries. Our model will

therefore produce a bias against forming asymmetric unions.

For a large set of country pairs, unions tend to generate only potential Pareto improvements. That is, one country wins and another one loses, however it would be possible to compensate losers so that both countries benefit from union formation, provided the existence of appropriate redistribution mechanisms. Our analysis assumes away such redistribution mechanisms. For one thing, richer union members would have to be compensated by poorer members, a policy which would presumably lack strong support in poor countries.

We now turn to the more detailed analysis of union formation. We compute the welfare gain for each country of forming a union with a specific partner in terms of consumption equivalents. That is, as the percentage increase in consumption, constant across time and states of nature, that leaves the country indifferent between standing alone in the world economy or forming the union.

Consider two countries sitting in the world economy at time 0, with states (ω_{i0}, y_{i0}) , $i = 1, 2$. If they form a union, the initial aggregate state is $(\bar{\omega}_0, \bar{y}_0)$, with $\bar{\omega}_0 = \omega_{10} + \omega_{20}$ and $\bar{y}_0 = (y_{10}, y_{20})$. Let $c^W(\omega_{i0}, y_{i0})$ denote the state-contingent consumption stream for country i in the world economy, from state (ω_{i0}, y_{i0}) onwards. Let $c_i^U(\bar{\omega}_0, \bar{y}_0)$ denote the state-contingent consumption stream for country i if both countries decide to form a union at time 0. Let $U(c^W(\omega_{i0}, y_{i0}))$ and $U(c_i^U(\bar{\omega}_0, \bar{y}_0))$ denote the expected lifetime utility derived from these consumption streams. Now denote by $(1 + \mu_i)c^W(\omega_{i0}, y_{i0})$ the consumption stream derived from $c_i^W(\omega_{i0}, y_{i0})$, where every state-contingent consumption level is increased by μ_i percent. The welfare gain for country i of forming a union with the other country is the μ_i that solves:

$$U((1 + \mu_i)c^W(\omega_{i0}, y_{i0})) = U(c_i^U(\bar{\omega}_0, \bar{y}_0)),$$

or, with isoelastic preferences as in (2.7),

$$\begin{aligned} \mu_i &= \left[\frac{U(c_i^U(\bar{\omega}_0, \bar{y}_0))}{U(c^W(\omega_{i0}, y_{i0}))} \right]^{\frac{1}{1-\sigma}} - 1 \\ &= \left[\frac{V_i^U(\bar{\omega}_0, \bar{y}_0)}{V(\omega_{i0}, y_{i0})} \right]^{\frac{1}{1-\sigma}} - 1, \end{aligned} \tag{4.1}$$

where the value functions have been defined in (P0) and (3.6). Notice that our welfare numbers incorporate transitional dynamics.

We next study the separate roles of wealth (foreign indebtedness) and endowment levels in union formation.

4.1 Role of wealth

Figure 1 displays the welfare gain for country 1 of forming a union, as a function of country 1 and country 2's initial wealth levels. The figure is also conditional on both countries having just received the mid-level endowment, y_m . The only asymmetry across union partners is in wealth, and different wealth levels correspond to different initial debt levels.

Several observations emerge from Figure 1. First, country 1 experiences a welfare loss for a large range of wealth levels. Second, country 1's welfare gain is always increasing in partner's wealth. Third, country 1's welfare gain is increasing in own wealth only if the partner's wealth is sufficiently low;⁴ otherwise, if the partner is rich, the welfare gain is monotonically decreasing in own wealth. Put together, the last two observation suggest the key determinant for union formation is partner's wealth: a country would like to belong to a rich club, especially if it's poor.

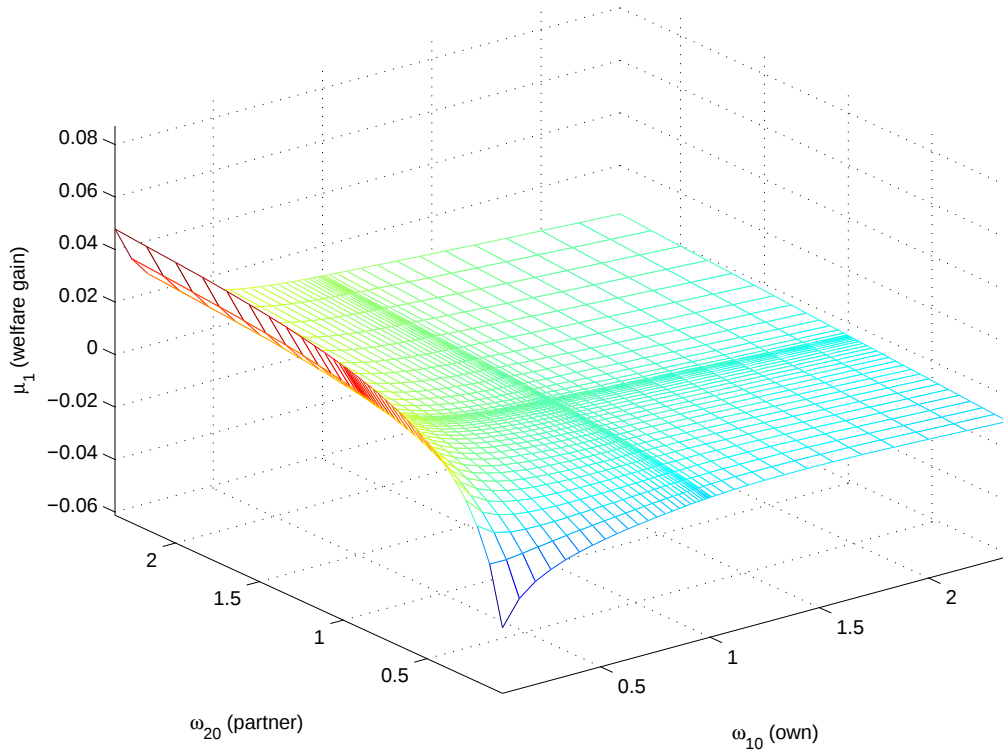


Figure 1: Welfare gain from union formation

Figure 2 displays the agreement areas, the complete set of initial country states for which both countries would experience a welfare gain, and thus agree to form a union. We focus on initial

⁴Although not apparent from the Figure 1, the welfare gain is actually non-monotonic in own wealth if partner's wealth is low enough.

net foreign debt levels, rather than wealth, and we do not display endowment pairs which, due to symmetry, are straightforward to obtain from the figure by simple relabeling of the two countries. For states above the solid lines, country 1 would improve welfare by forming a union with country 2, and similarly for country 2 for states below the dashed lines. The agreement areas are therefore represented by the light-shaded areas.

Superimposed on Figure 2 is also an area representing the ergodic space for net foreign asset positions in the world economy, $b_{10}, b_{20} \in [-0.109, 0.425]$. This is the dotted square located roughly in the middle of the figure. Notice how solving for the world equilibrium is a crucial step in our analysis of union formation. Not only it determines the world interest rate faced by the union, it also determines the relevant subset of country pairs that are faced with the option union formation.

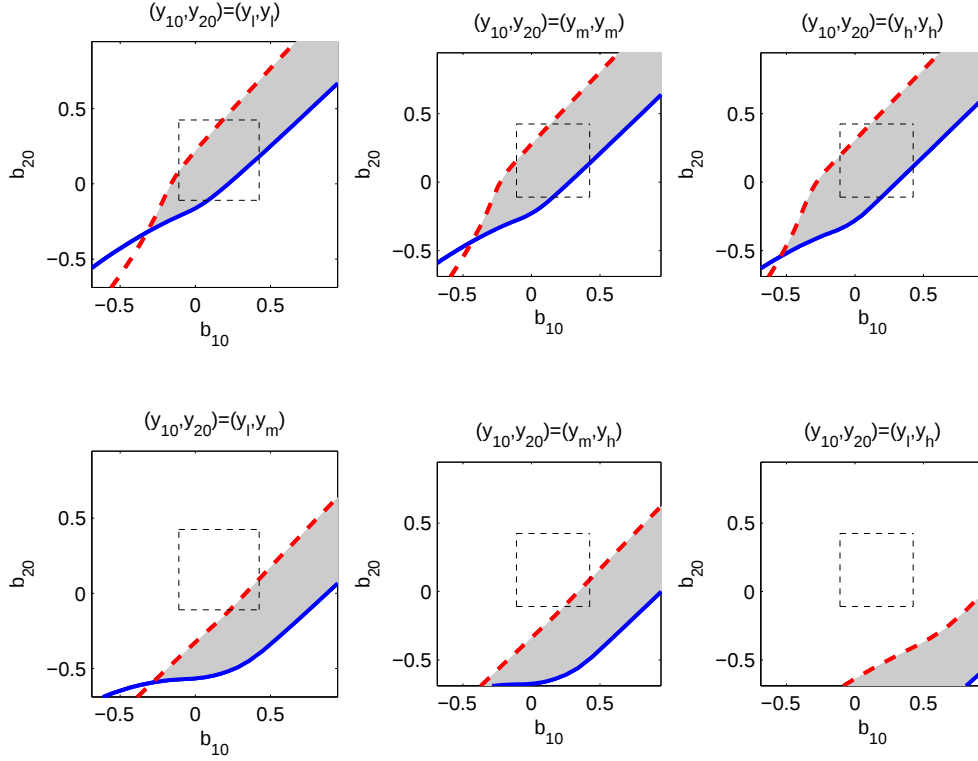


Figure 2: Agreement areas (country 1: solid, country 2: dashed)

In this subsection, we concentrate our attention on the first row of Figure 2. In this row, potential union members have identical initial endowments, but potentially different wealth levels.

The figure shows, first, that unions tend to be formed between countries sufficiently homogeneous in terms of initial wealth. Along the 45 degree line, and restricted to the ergodic space, countries always reach an agreement. The disagreement area exists when wealth levels are suffi-

ciently different from each other. Second, we also see that, in asymmetric unions for which there is disagreement, rich partners are the ones with a welfare loss. They are the ones preventing union formation. Finally, within the ergodic space, a larger union-wide level of wealth favors union formation. This is because the agreement areas get wider for larger wealth levels, particularly conditional on (y_l, y_l) . However, the effect does not appear very significant quantitatively.

4.2 Role of the endowment

We now turn to the role of the endowment. Keeping with the first row of Figure 2, we see that a larger union-wide endowment favors union formation. As we move from the left to the right panel, the agreement area fills a larger area of the ergodic space. To see this point more clearly, the left panel of Figure 3 superimposes the agreement areas for the lowest (y_l, y_l) and the highest (y_h, y_h) endowment pairs, showing that the agreement area is indeed uniformly wider under the latter.

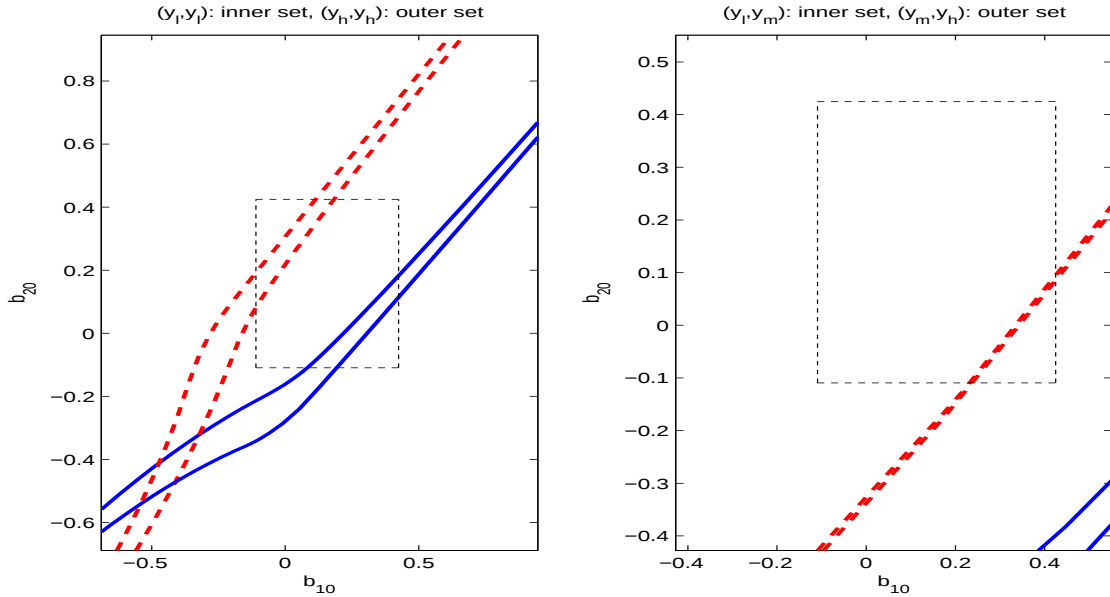


Figure 3: Agreement areas widen with larger endowments
(country 1: solid, country 2: dashed)

The three panels in the bottom row of Figure 2 are for asymmetric initial endowments. In all three cases, country 1 was relatively unlucky, having received a lower endowment shock compared to country 2. The first thing to notice now is that, for most states, an agreement cannot be reached. Although country 1 would always benefit from union formation (the ergodic space is always above the solid line), this is not the case for country 2. Only a sufficiently poor country 2 would like to

form a union with an unlucky country 1. This effect is more dramatic the more asymmetric the initial endowment levels. Indeed, the agreement area in the bottom-right panel, when endowment asymmetry is the largest, falls outside of the ergodic set. There is never an agreement.

A feature that emerges from this discussion is, once again, the importance of homogeneity for union formation. We see from the first two panels in the bottom row of Figure 2 that unions may form between a lucky and an unlucky country only if the latter is asset rich, and the former asset poor. That is, asset heterogeneity must be negatively correlated with endowment heterogeneity. Overall, the two countries must be homogenous.

The bottom row of Figure 2 also confirms our previous finding that higher union-wide endowment levels favor union formation. This can be seen by comparing the first two panels in the bottom row, and by noticing that the agreement area within the ergodic space is larger in the second. To show this point more clearly, the right panel of Figure 3 superimposes the agreement areas for the (y_l, y_m) and the (y_m, y_h) endowment pairs, showing that the agreement is indeed uniformly wider under the latter - although the effect is not very significant quantitatively.

We summarize the discussion of the previous two subsections with the following. Unions are more likely to be formed:

1. the wealthier the partners, and
2. the more homogeneous the partners,

either in terms of initial endowment or net foreign assets. Quantitatively, the most important determinant of union formation is partner homogeneity.

4.3 Quantitative results

We provide quantitative summary measures of the likelihood of union formation conditional on different regions of the state-space. More specifically, we ask: What is the probability that two randomly-picked countries from (a subset of) the world distribution of stand-alone countries agree to form a union?

To answer this question, we partition the ergodic space for net foreign asset into three subsets. The subsets are defined by two quantiles of the world distribution of b , the median and the top decile, and correspond to $B_l = [b_{\min}, b_{50}) = [-0.1094, -0.088)$, $B_m = [b_{50}, b_{90}] = [-0.088, 0.206]$ and $B_h = (b_{90}, b_{\max}] = (0.206, 0.4256]$. We computed the conditional probabilities by repeatedly

		Country 2										
		y_l			y_m			y_h				
		B_l	B_m	B_h	B_l	B_m	B_h	B_l	B_m	B_h		
Country 1	y_l	B_l	100	88.4	0	0	0	0	0	0	30.3	
		B_m	88.4	95.1	25.1	0	0	0	0	0	27.4	
		B_h	0	25.1	100	57.7	10.5	0	0	0	15.7	
	y_m	B_l	0	0	57.7	100	93.4	0	0	0	35.3	
		B_m	0	0	10.5	93.4	99.4	51.5	0	0	36.1	
		B_h	0	0	0	0	51.5	100	66.9	9.3	14.1	
	y_h	B_l	0	0	0	0	0	66.9	100	96.6	0.3	23.7
		B_m	0	0	0	0	0	9.3	96.6	100	75.9	29.0
		B_h	0	0	0	0	0	0	0.3	75.9	100	23.3
			30.3	27.4	15.7	35.3	36.1	14.1	23.7	29.0	23.3	30.8

Table 2: Probability of Union Formation (in %)

drawing two countries at random from the world economy's stationary distribution, conditional of the appropriate subset of states, and verifying whether both countries would agree to form a union.⁵ Table 2 contains the results.

The overall unconditional probability of union formation is relatively low, at about 0.3. Conditional on particular states, Table 2 confirms our previous analysis of Figure 2. The main feature is that the highest probabilities of union formation are along the diagonal. That is, partner homogeneity is the key factor. Looking at the off-diagonal elements we also see that, when endowments are heterogeneous, one needs heterogeneity in asset wealth going the opposite way for unions to be formed.

Second, asset wealth does not seem very significant for union formation. Along the diagonal, all the probabilities are either near or equal to 1.⁶ Endowment levels are relatively more important. In

⁵In practice, the world distribution of states was approximated by simulating the problem of a single country for 1 million periods. We then drew 1 million country pairs at random from this distribution.

⁶The probabilities that are less than 1 are associated with wealth levels in B_m . This is because our choice of wealth intervals. This choice implies that, conditional on endowment homogeneity, a large portion of the disagreement areas within the ergodic space fall into this asset interval.

particular, conditional on endowment homogeneity across union partners, the probabilities increase for each asset subset.

4.4 Role of the borrowing limit

As we pointed out previously, the tightening of the union's borrowing limit generates an adverse effect on richer partners in asymmetric unions. That is, a richer country standing alone in the world economy is less likely to default, and therefore faces a relatively loose borrowing limit. When a rich and a poor partner form a union, the poor partner exhausts the whole union's borrowing possibilities. This makes it more likely that the rich partner becomes credit-constrained in the union.

To better understand the role of the tightening of the borrowing limit when unions are formed, we compare the likelihood of being credit-constrained for different potential union members. We first condition on an initial (time-0) pair of country states. We then compute (i) each country's probability of being credit-constrained in the first 5 periods following union formation, starting from time 0, and (ii) each country's probability of being credit-constrained in the world economy for 5 periods, from time 0 onwards. We compute the difference between the two probabilities for each country, and report them in Table 3.

More specifically, denote by $(a_i \backslash b_j)$ a typical entry in Table 3. The a_i is the average probability that a country 1 of type i is credit-constrained following union formation with a country 2 of type j , minus the same probability if standing alone in the world economy. That is

$$a_i = \int_{\substack{s_{i0} \in (y, B)_i \\ s_{j0} \in (y, B)_j}} \frac{1}{5} \sum_{t=0}^4 \mathbf{1}\{b^{U'}(\bar{s}_t | s_{i0}, s_{j0}) = \bar{b}^U\} d\Psi(s_{i0}) d\Psi(s_{j0}) \\ - \int_{s_{i0} \in (y, B)_i} \frac{1}{5} \sum_{t=0}^4 \mathbf{1}\{b'(s_{it} | s_{i0}) = \bar{b}\} d\Psi(s_{i0}),$$

where $s_{it} = (\omega_{it}, y_{it})$ is country 1's state, $\bar{s}_{it} = (\bar{\omega}_{it}, \bar{y}_{it})$ is the union's state, and $\mathbf{1}$ is an indicator function.

The b_j element is defined analogously for a country 2 of type j . Table 3 reports only one number along the diagonal, due to symmetry. Also due to symmetry, the upper triangular elements can be obtained by switching the two countries in the appropriate lower triangular elements.

		Country 2								
		y_l			y_m			y_h		
		B_l	B_m	B_h	B_l	B_m	B_h	B_l	B_m	B_h
Country 1	B_l	0.82								$-0.31 \setminus 2.22$
	y_l B_m	$2.47 \setminus 0.76$	2.15							$1.01 \setminus 1.83$
	B_h	$3.10 \setminus -0.90$	$2.46 \setminus 0.29$	2.00						$1.51 \setminus 0.08$
	B_l	$3.01 \setminus 0.32$	$2.89 \setminus 1.91$	$0.74 \setminus 1.89$	1.88					$1.33 \setminus 1.16$
	y_m B_m	$3.88 \setminus 0.10$	$3.26 \setminus 1.20$	$0.86 \setminus 0.98$	$2.54 \setminus 1.44$	1.51				$1.96 \setminus 0.69$
	B_h	$1.80 \setminus -2.17$	$1.16 \setminus -1.15$	$0.06 \setminus 0.03$	$0.42 \setminus -0.87$	$0.15 \setminus -0.04$	0.01			$0.64 \setminus -0.81$
	B_l	$3.49 \setminus -0.48$	$3.19 \setminus 0.97$	$0.35 \setminus 0.24$	$1.65 \setminus 0.38$	$1.10 \setminus 0.92$	$0.01 \setminus 0.02$	1.03		$1.66 \setminus 0.22$
	y_h B_m	$2.56 \setminus -1.41$	$1.55 \setminus -0.72$	$0.07 \setminus 0.01$	$0.79 \setminus -0.50$	$0.26 \setminus 0.07$	$0.002 \setminus 0.002$	$0.28 \setminus 0.27$	0.019	$0.97 \setminus -0.49$
	B_h	$0.24 \setminus -3.75$	$0.10 \setminus -2.16$	$0 \setminus -0.08$	$0.01 \setminus -1.27$	$0.003 \setminus -0.18$	$0 \setminus 0$	$0 \setminus -0.01$	$0 \setminus 0$	$0 \setminus -1.38$

Table 3: Probability of being credit-constrained (union minus world economy, in % points)

The overall probability of being credit-constrained in the union is 40 percentage points higher than in the world economy (not reported in Table 3).

The rightmost column of Table 3 tells us how much more a country 1 of type i becomes constrained in a union with any type j country 2 (first element), and how much more any type j country 2 becomes constrained in a union with type i country 1 (second element).

Note that, in emphasizing the lower triangular part of the table, we are concentrating on the region where country 1 is richer than country 2, both in terms of endowment and net foreign assets.

We draw the following observations from the table:

1. Along the diagonal, countries are always more likely to become credit-constrained in the union. This is because the union's borrowing constraint is tighter.
2. The farther away from the diagonal, as country 1 becomes richer than country 2, country 1 is more likely to become credit-constrained in the union while the opposite happens with country 2.
3. The numbers are not monotonic because being richer has two effects on the probability of getting credit-constrained in the union: (i) it makes it less likely, just like it makes it less likely when you're standing alone in the world economy; (ii) makes it more likely in the sense that country 2 can use country 1's credit line. Eventually the first effect gets strong enough, and that's why, depending on the initial point, the probability may go up or down with wealth. However, this probability is always higher in the union - except when country 1 and 2 are fairly rich, in which case the probability doesn't change (it's zeros both in the union and in the world economy).

5 Experiments

We now study some experiments to help us better understand our main results.

5.1 Relaxing the union's borrowing limit

5.2 Parameter changes

5.2.1 Risk-aversion

5.2.2 Endowment risk

6 Conclusion

A Decentralization

We decentralize the planner's allocation as a competitive equilibrium with tax subsidies on saving. Our decentralization scheme is an adaptation of [Kehoe and Perri \(2004\)](#), [Wright \(2006\)](#), and [Bai and Zhang \(2008b\)](#).⁷ Within the union, countries trade a complete set of Arrow securities. In the world market, they trade freely on a riskless one-period bond. A central government authority in the union implements a tax and transfer scheme, designed to support the constrained-efficient allocation, and thus prevent default in the appropriate states.

For each country $i = 1, 2$ in the union, let $a_i(\bar{y}'; \omega_i, \bar{\omega}, \bar{y})$ denote the net stock of the Arrow security that pay in state $\bar{y}'$ tomorrow, conditional on individual wealth ω_i and the aggregate state $(\bar{\omega}, \bar{y})$, with price $q(\bar{y}'; \bar{\omega}, \bar{y})$. Let $b'_i(\omega_i, \bar{\omega}, \bar{y})$ denote the net stock of foreign bonds that earn interest r tomorrow.

Let also $\tau(\bar{\omega}, \bar{y})$ denote the subsidy rate on net asset purchases, and $T_i(\omega_i, \bar{\omega}, \bar{y})$ the lump-sum income tax faced by country i .

In a competitive equilibrium with capital controls, country i solves the following problem for every current state

$$V_i(\omega_i, \bar{\omega}, \bar{y}) = \max_{c_i, b'_i, \{a_i(\bar{y}')\}} \left\{ u(c_i) + \beta \sum_{\bar{y}'} \pi(\bar{y}' | \bar{y}) V_i(\omega'_i, \bar{\omega}, \bar{y}') \right\}$$

subject to

$$\begin{aligned} c_i + (1 - \tau(\bar{\omega}, \bar{y})) \left(b'_i + \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}) a_i(\bar{y}') \right) &= \omega_i + T_i(\omega_i, \bar{\omega}, \bar{y}) \\ \omega'_i &= y'_i + (1 + r) b'_i + a_i(\bar{y}') \end{aligned} \tag{A.1}$$

and to a perceived law of motion for aggregate wealth $\bar{\omega}$.

The government is assumed to run a balanced budget for each country separately, that is

$$\tau(\bar{\omega}, \bar{y}) \left(b'_i(\omega_i, \bar{\omega}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}) a_i(\bar{y}'; \omega_i, \bar{\omega}, \bar{y}) \right) = T_i(\omega_i, \bar{\omega}, \bar{y}) \tag{A.2}$$

for every current state and for each i .

A competitive equilibrium with tax subsidies is defined in the standard way, as (i) optimal decision rules that solve each country's problem given prices, government policy, and a perceived

⁷See [Wright \(2006\)](#) for an alternative decentralization based upon country-specific borrowing limits, along the lines of [Alvarez and Jermann \(2000b\)](#).

law of motion for aggregate wealth; (ii) a government policy that satisfies the balanced budget constraints given prices and individual decisions; (iii) Arrow security prices that clear asset markets; and (iv) consistency between the perceived law of motion for aggregate wealth and the individual decision rules.

Our goal here is to show that there exists a government tax and transfer policy that supports the constrained-efficient allocation as a competitive equilibrium. We focus on the key steps of the argument.

Consider the first-order conditions to the country's problem

$$1 - \tau(\bar{\omega}, \bar{y}) = (1 + r) \sum_{y'} \pi(\bar{y}' | \bar{y}) \frac{\beta u'(c_i(\omega'_i, \bar{\omega}', \bar{y}'))}{u'(c_i(\omega_i, \bar{\omega}, \bar{y}))} \quad (\text{A.3})$$

$$(1 - \tau(\bar{\omega}, \bar{y})) q(\bar{y}'; \bar{\omega}, \bar{y}) = \pi(\bar{y}' | \bar{y}) \frac{\beta u'(c_i(\omega'_i, \bar{\omega}', \bar{y}'))}{u'(c_i(\omega_i, \bar{\omega}, \bar{y}))}. \quad (\text{A.4})$$

Given isoelastic preferences, the last equation implies

$$\frac{c_i(\omega'_i, \bar{\omega}', \bar{y}')}{c_i(\omega_i, \bar{\omega}, \bar{y})} = \frac{c(\bar{\omega}', \bar{y}')}{c(\bar{\omega}, \bar{y})} \text{ for } i = 1, 2. \quad (\text{A.5})$$

The two Euler equations imply

$$1 = (1 + r) \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}). \quad (\text{A.6})$$

Note also that, at the optimum, we may use (A.2) to eliminate subsidies and transfers from (A.1):

$$c_i(\omega_i, \bar{\omega}, \bar{y}) + b'_i(\omega_i, \bar{\omega}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}) a_i(\bar{y}'; \omega_i, \bar{\omega}, \bar{y}) = \omega_i. \quad (\text{A.7})$$

Consider now the constrained-efficient allocation, the solution to problem (P1'). This allocation, which we denote with a star superscript, satisfies the planner's Euler equation

$$u'(c^*(\bar{\omega}, \bar{y})) - \phi^*(\bar{\omega}, \bar{y}) = \beta(1 + r) \sum_{\bar{y}'} \pi(\bar{y}' | \bar{y}) u'(c^*(\bar{\omega}', \bar{y}')). \quad (\text{A.8})$$

Using (A.5) in (A.3), and requiring that the resulting allocation be consistent with (A.8), it is easy to compute the state-contingent subsidy rates that implement the constrained-optimal allocation as

$$\tau(\bar{\omega}, \bar{y}) = \frac{\phi^*(\bar{\omega}, \bar{y})}{u'(c^*(\bar{\omega}, \bar{y}))}. \quad (\text{A.9})$$

Note that if the borrowing constraint to problem (P1') does not bind in state $(\bar{\omega}, \bar{y})$, then $\phi^*(\bar{\omega}, \bar{y}) = 0$ and so $\tau(\bar{\omega}, \bar{y}) = 0$. In this case, from (A.4) and (A.6), the domestic interest

rate equals the world interest rate. If the constraint is instead binding, then the (post-subsidy) domestic interest rate is higher than the world interest rate. This ensures that countries save in a constrained-optimal way, and that equilibrium borrowing is self-enforcing.

It is relatively straightforward to show formally that, given a constrained-efficient allocation that solves (P1') and (P2) for the appropriate set of welfare weights, one can obtain individual asset holdings from (A.7) together with the market clearing condition for Arrow securities, Arrow security prices from (A.4), and a government policy from (A.9) and (A.2) that support that allocation as a competitive equilibrium with tax subsidies.

To find the appropriate set of welfare weights, we use the method proposed by [Negishi \(1960\)](#). This method exploits the equivalence between the market and the constrained-efficient allocations.

We obtain the time-0 present value budget constraint of country i by iterating forward on the flow budget constraint (A.7). We express it as

$$C_i(\omega_{i0}, \bar{\omega}_0, \bar{y}_0) = Y_i(\bar{\omega}_0, \bar{y}_0) + (1+r)b_{i0},$$

where $C_i(\omega_{i0}, \bar{\omega}_0, \bar{y}_0)$ and $Y_i(\bar{\omega}_0, \bar{y}_0)$ are the time-0 present-values of consumption and the endowment, respectively. At time 0, the time of forming the union, $\bar{y}_0$ is the union's endowment pair, b_{i0} is country i 's net stock of foreign bonds, and $\bar{\omega}_0 = \sum_i \omega_{i0} = \sum_i y_{i0} + (1+r)\sum_i b_{i0}$ is the union's aggregate wealth.

It follows from (3.4) that we may express the present value of individual consumption as fraction of the present value of aggregate (constrained-efficient) consumption, that is $C_i(\omega_i, \bar{\omega}, \bar{y}) = \alpha_i C^*(\bar{\omega}, \bar{y})$. Replacing above allows us to recover the individual consumption share parameters as

$$\alpha_i = \frac{(1+r)b_{i0} + Y_i(\bar{\omega}_0, \bar{y}_0)}{C^*(\bar{\omega}, \bar{y})}. \quad (\text{A.10})$$

Given equilibrium Arrow security prices $q(\bar{y}'; \bar{\omega}, \bar{y})$, and optimal decision rules $c^*(\bar{\omega}, \bar{y})$ and $b^{*'}(\bar{\omega}, \bar{y})$, the C^* and Y functions solve the following functional equations

$$Y_i(\bar{\omega}, \bar{y}) = y_i + \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}) Y_i(\bar{\omega}', \bar{y}') \quad (\text{A.11})$$

$$C^*(\bar{\omega}, \bar{y}) = c^*(\bar{\omega}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{\omega}, \bar{y}) C^*(\bar{\omega}', \bar{y}') \quad (\text{A.12})$$

with

$$\bar{\omega}' = \sum_i y_i' + (1+r)b^{*'}(\bar{\omega}, \bar{y}).$$

Notice that although it is straightforward to obtain the welfare weights from the consumption share parameters, we only need to know the α_i 's in order to uncover the individual allocations.

B Numerical algorithms

B.1 World economy equilibrium

Our algorithm can be described in the following steps:

1. Solve for the autarky value function $V_{aut}(y)$ from equation (2.6).
2. Given a current guess for the equilibrium interest rate r , solve problem (P0) by iterating on the following steps:
 - (a) Consider the n^{th} iteration, with a current conjecture for the debt limit $\bar{b}_n$. For the initial conjecture, we use the natural borrowing constraint.
 - (b) Given $\bar{b}_n$, solve problem (P0) by policy function iteration. We discretize the state-space and use cubic-spline interpolation to compute decisions outside the grid.
 - i. First find the decision rules that solve the system of first-order conditions to problem (P0), ignoring the debt limit. Consider the j^{th} iteration, with a current conjecture for the consumption decision rule $c_n^j(\omega, y)$. Compute a candidate update $c_n^{j+1}(\omega, y)$ by solving

$$u'(c_n^{j+1}(\omega, y)) = \beta(1+r) \sum_{y'} \pi(y'|y) u'(c_n^j(\omega', y'))$$

with

$$\begin{aligned} \omega' &= y' + (1+r)b' \\ b' &= \omega - c_n^{j+1}(\omega, y). \end{aligned}$$

As part of the solution, we obtain $b_n'^{j+1}(\omega, y)$.

- ii. Check whether the borrowing constraint is violated. If $b_n'^{j+1}(\omega, y) < \bar{b}_n$, then update the solution as follows:

$$\begin{aligned} b_n'^{j+1}(\omega, y) &= \bar{b}_n \\ c_n^{j+1}(\omega, y) &= \omega - b_n'^{j+1}(\omega, y) \\ \phi_n^{j+1}(\omega, y) &= u'(c_n^{j+1}(\omega, y)) - \beta(1+r) \sum_{y'} \pi(y'|y) u'(c_n^{j+1}(\omega', y')), \end{aligned}$$

with

$$\omega' = y' + (1+r)b_n'^{j+1}(\omega, y).$$

If instead $b_n^{j+1}(\omega, y) \geq \bar{b}_n$, then update using the unconstrained solution, setting also $\phi_n^{j+1}(\omega, y) = 0$.

- iii. Iterate on the previous two steps until the decision rules converge. At the end, compute the value function $V_n(\omega, y)$.

(c) Given $V_n(\omega, y)$, update the debt limit as follows:

$$\bar{b}_{n+1}(y) = \max_{y': \pi(y'|y) > 0} \{b_{y'} : V_n(y' + (1+r)b_{y'}, y') = V_{aut}(y')\}.$$

(d) Iterate on steps **2b** and **2c** until the borrowing limits converge.

- 3. Check the market clearing condition by approximating the aggregate bond holding in the world economy with the total bond holding of a particular country over a very long simulation period. We discretize the state-space using a finer grid, and linearly interpolate the decision rules.
- 4. Iterate on steps **2** and **3** until we find an interest rate that approximately clears the bond market.

B.2 Union problem under centralized default

Our algorithm to solve for the union's allocation given an equilibrium world interest rate r can be described as follows:

- 1. Solve problem **(P1')** using the method described in step **2** of the algorithm of Section **B.1**. As part of the solution we obtain the union decision rule $c^*(\bar{\omega}, \bar{y})$, the multiplier function $\phi^*(\bar{\omega}, \bar{y})$, and the value function $V^U(\bar{\omega}, \bar{y})$.
- 2. Decentralize the union's constrained-efficient allocation as a competitive equilibrium with capital controls.
 - (a) Compute tax-subsidies from **(A.9)**.
 - (b) Compute pre-subsidy Arrow-security prices from **(A.4)**.
 - (c) Compute the present-value functions from **(A.11)** and **(A.12)**. In practice, we guess some arbitrary functions on a grid and then iterate on the two recursive equations until convergence. We linearly interpolate these functions when future wealth levels fall outside the grid.

- (d) Compute consumption shares from (A.10).
- (e) Compute the value function for each country from (3.6).

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