The more we know, the less we agree: Higher-order expectations and public announcements

Péter Kondor*
Central European University

October 2008

Very preliminary

Abstract

Polarization of opinions after public announcement is widely observed, but often considered to be inconsistent with Bayesian learning. I show that this is not the case in environments where higher-order expectations play a role. I characterize informational structures where public announcement leads to polarization in all higher-order expectations, but not in first-order expectations. To illustrate the economic consequences, I modify two workhorse models of asymmetric information. I show in a Morris-Shin(1998) model that more disclosure of public information can increase the chance of successful currency attacks. I show in a dynamic Grossman-Stiglitz(1980) model that hectic trading around public announcements is consistent with common priors and Bayesian learning.

1 Introduction

Decisions are often determined by higher-order expectations (agents' opinion about other agents' opinions), rather than the agents' own opinions about the fundamentals of the economy. Public information, in such environment, aside from providing information about the uncertain fundamentals, has an important secondary role in helping each agent to guess the private information of other agents. In this paper, I focus on the economic consequences of the effect of public information on the dispersion of higher-order expectations.

*E-mail address: kondorp@ceu.hu. This paper builds on the second chapter of my PhD thesis at the LSE. I am grateful for Hyun Song Shin and Dimitri Vayanos for the guidance. I am also grateful for the help received from Amil Dasgupta, Gabrielle Demange, Bernardo Guimaraes, Gergely Ujhelyi, Jakub Steiner and seminar participants at the LSE, Chicago and Cambridge. All remaining errors are my own. I gratefully acknowledge the EU grant "Archimedes Prize" (HPAW-CT-2002-80054), and the financial support from the Central Bank of Hungary.

It is widely observed that new public information frequently polarize decision makers' opinions about a particular issue instead of decreasing their disagreement. This phenomenon was highlighted in various contexts.[1] It has been well known since Milgrom (1981) that under general conditions, Gaussian information structures with Bayesian learning are inconsistent with polarization if agents form expectations about the fundamental. The main observation of this paper is that this is not the case with higher-order expectations. When agents worry about the opinion of others about the fundamental, or about the opinion of others about the opinion of others about the fundamental, and so on to the infinite degree, then public information can increase the dispersion of these higher-order expectations. I characterize the Gaussian-Bayesian information structures where public announcement leads to polarization in all higher-order expectations and argue that many of these structures are plausible. To illustrate the economic relevance of this observation, I modify two standard workhorse models of informational economics. First, I show that in a Morris-Shin-type speculative attack model the central bank's commitment to release new information can increase the ex ante probability of speculative attacks. Second, I show that public announcement can significantly increase trading volume in a Grossman-Stiglitz type asset market.

The main result is that if the correlation among agents' private information sets is sufficiently low, a public announcement increases the dispersion of all higher-order expectations. In particular, consider T groups of agents. Each agent observes private and public signals about the value of a fundamental. Agents of the first group form expectations about the fundamental value. Agents of the t-th group, $t = 2, 3..T$, form expectations about the expectation of one of the agents of group $t-1$. All signals and the fundamental value are jointly normal and agents share the same prior.[2] As a consequence of Milgrom (1981), disagreement in the first group will decrease after the announcement under general conditions. In contrast, dispersion of expectations in any of the t-th group, $t > 1$, increases due to the public announcement if the pairwise correlation between signals of any two agents in different groups is sufficiently low.

To see the intuition, let us consider the simplest example. Suppose that the fundamental is a sum of two independent factors and a decision maker observes a noisy signal on the first

[1] Polarization was observed in political economy in the context of voters reaction to news, in finance in the context of increasing trading volume around public announcements, and in behavioral economics in relation to experiments where subjects are asked to evaluate new evidence about socially sensitive issues. See Rabin and Schrag (1998) and Dixit and Weibull (2007) for examples.

[2] Allen, Morris and Shin (2006) uses the same structure to demonstrate that the low of iterated expectations does not hold with dispersed information.

factor. In contrast, each agent in a group observes a noisy signal on the second factor. The public signal is a noisy version of the fundamental. Before the public announcement, agents in the group agree in their expectation of the expectation of the decision maker, because their private signals do not reveal any information on the decision maker's signal. However, there is disagreement after the public announcement. As the public signal is correlated to any private signals, the decision maker's signal and any other private signal cease to be independent after the observation of the public signal. Intuitively, even if the private signal is useless on its own for the forecast of the forecast of others, it helps the agent to interpret the public signal. Thus, even if each agent observes the same public signal, her inference will differ from other agents with different private signals.

To illustrate that the result has economically important consequences, I modify two standard models with learning. First, I modify the speculative currency attack model of Morris and Shin (1998). As in the original version, speculators receive private signals about the state of the economy. Based on their information, they decide whether to attack the fixed exchange rate system of the central bank. If the size of the attack is sufficiently high compared to the state of the economy, the central bank abandons the peg. As attacking is costly for speculators, they profit from an attack only if the central bank abandons the peg. The main difference between the modified version and the original one is that I do not allow the central bank to observe the state of the economy. Instead, central bank receives a signal about the state of the economy and abandons the peg if its posterior is low relative to the size of the attack. Thus, speculators have to second guess the expectation of the central bank. That is, speculators' second-order expectations matter.

I show that polarization in higher-order expectations implies that generating and disclosing more public information can destabilize the exchange rate system. In particular, if speculators have little private information on the central bank's information and the central bank decides to gather and disclose public information on the state of the economy, this increases the chance of successful currency attacks. The idea behind the result is that if public information polarizes speculators' opinion about the central banks' evaluation of the state of the economy, then it also polarizes speculators' forecast about the probability of devaluation. As speculators with sufficiently large forecast attack, polarization leads to more speculative attacks and, eventually, larger ex ante chance of devaluation.

The result highlights the interaction between different notions of transparency of central banks. In general, the central bank lacks transparency if agents in the economy "knows little" about a particular element in the central bank's private information set. Firstly, this might refer to an environment with large uncertainty, that is, where each agents' posterior about that particular element has a large variance. Secondly, this might refer to an environment with severe confusion, that is, where the variance of agent's guesses about that particular element is large. The result implies that if the central bank decreases uncertainty by disclosing more information, this might increase confusion leading to a larger chance of a currency crisis.

As a second application, I consider a Grossman-Stiglitz model of asset pricing under asymmetric information. Similarly to Allen, Morris and Shin (2006), I use a version with overlapping generation of traders. There are early traders and late comers. After trading among each other, early traders sell all their assets to late-comers. Thus, early traders are interested in the valuation of late comers, which implies that second-order expectations matter.

I show that polarization in higher-order expectations implies that public announcement can increase trading volume. This is a success given the difficulty of Grossman-Stiglitz type of models to explain the well established stylized fact of large trading volume around announcements. The idea behind the result is that polarization among early traders about the expectation of late comers implies increasing disagreement about how much they should pay for the asset. This results in hectic speculative trading around annoucements.

This paper fits in the recent flow of papers analyzing the effect of higher-order expectations in various contexts. One branch of this literature concentrates on whether imperfect information of decision makers leads to a unique equilibrium in coordination games (e.g. Morris and Shin, 1998, Goldstein and Pauzner, 2004, Angeletos and Werning, 2006, Hellwig, Mukherji and Tsyvinski, 2006). In these environments higher-order expectations matter only through the coordination motive of agents. Applications include speculative currency attacks, provision of public goods, bank runs and pricing of corporate debt[3]. Another branch concentrates on "Beauty contest" environments. That is, environments where the pay-off of agents is a weighted sum of the deviation of their actions from an optimal level and of the deviation of their actions from the average action of others (e.g. Woodford, 2002, Hellwig 2002, Angeletos and Pavan, 2007,Adam, 2003, Amato and Shin 2006). These papers focus on questions of monetary policy and social value of public information. This paper belongs to a third branch of this literature where various group of agents act sequentially and the pay-off of early actors

[3] See Morris and Shin (2001) for an overview on applications of global games for further references.

depends on the action of groups acting later. Thus, early actors has to guess the information of agents acting later (e.g. Allen, Morris and Shin, 2006, Makarov and Rytchkov, 2006, Banerjee, Kaniel and Kremer, 2008, Goldstein, Ozdenoren and Yuan, 2008, Angeletos, Lorenzoni and Pavan, 2007). Applications include financial markets, currency attacks and the interaction between stock prices and real investment. The main departure of this paper from the existing literature is that this is the first one to consider the possibility and consequences of polarization of higher-order expectations due to public announcement.

The structure of this paper is as follows. In the next section I present a statistical observation which describes conditions for polarization in higher-order expectations due to public announcements. In section 3, I show in a Morris-Shin type model that more disclosure of public information can increase the chance of successful currency attacks. In section 4, I show that in a Grossman-Stiglitz type of model, this observation generates large trading volume around announcements. Finally I conclude.

2 Increasing disagreement in higher-order expectations

In this section, I analyze the effect of public announcements to expectations of different order in a general Bayesian-Gaussian model of learning. This is a statistical exercise, where the only action of agents is to form expectations and their only objective is to make these expectations as accurate as possible. I leave the economic consequences to the rest of the paper where I will build up two economic models to analyze the implications of the presented statistical observation.

Let us consider a continuum of agents divided in T groups. In each group there is a unit mass of agents. There is a fundamental variable of interest, θ. The public information on θ is represented by the public signal y, while the the private information of agent i in group t is represented by private signals $x_{i,t}$.[4] All signals and θ are jointly normally distributed, all signals are pairwise positively correlated to θ, and, for simplicity, let us assume that the covariance

[4]For incorporating a larger number of private and public signals redifine $x_{i,t} \equiv E(\theta|S^x_{i,t})$ and $y \equiv E(\theta|S^y)$ where $S^x_{i,t}$ is the collection of all private signals of agent i in group t and S^y is the collection of all public signals. Assume that θ follows a difuse prior. Then all the results and proofs in this section are also valid for that more general case.

structure is symmetric in the sense that

$$\begin{aligned} var\left(x_{i,t}\right) &= \sigma_x^2 > 0 \\ corr(x_{1,t},\theta) &= \rho_{x,\theta} > 0 \\ corr\left(x_{i,t},y\right) &= \rho_{x,y} \geq 0 \\ corr\left(x_{i,t},x_{j,t}\right) &= \rho_{x,x} \geq 0 \\ corr\left(x_{i,t},x_{j,u}\right) &= \rho_{x,x'} \geq 0 \end{aligned}$$

for all t and $u \neq t$ and $i \neq j$. That is, each private signal have the same variance, each private signal in group 1 has the same correlation to θ, and we have a single correlation coefficient for signals within each group and another one for the correlation between signals across groups.

I am interested in the effect of the public signal y on the dispersion of expectations of different order. For this purpose, I require agents in group 1 to form expectation on θ,

$$\begin{aligned} \hat{f}_{i,1} &\equiv E\left(\theta|x_{i,1}\right) = \hat{a}_\theta x_{i,1} \\ f_{i,1} &\equiv E\left(\theta|x_{i,1},y\right) = a_\theta x_{i,1} + c_\theta y \end{aligned}$$

before and after the observing the public announcement, respectively. Throughout the paper, whenever it is important to distinguish between objects corresponding to the cases of no announcement and announcement, I distinguish the object for the case with announcement by a hat. The objects $f_{i,1}$ and $\hat{f}_{i,1}$ are first-order expectations.I require agents in group $t > 1$ to form expectations on the expectation of an agent in group $t-1$,

$$\begin{aligned} \hat{f}_{i,t} &\equiv E\left(\hat{f}_{j,t-1}|S_{i,t}\right) \\ f_{i,t} &\equiv E\left(f_{j,t-1}|S_{i,t},y\right). \end{aligned}$$

Thus, an agent in group t forms an expectation of order t.

I say that for the given information structure public announcement increases disagreement or causes polarization in the t-th expectation, if

$$\int_0^1 \left|\hat{f}_{i,t} - \overline{\hat{f}_t}\right| di < \int_0^1 \left|f_{i,t} - \overline{f_t}\right| di,$$

where $\overline{\hat{f}_t}$,$\overline{f_t}$ is the avarage forecast in the given sample before and after the announcement respectively. That is, there is polarization if the dispersion of forecasts in group t increases after the announcement.

It is a well known result that under weak conditions, there cannot be polarization in first-order expectations in a Gaussian information structure. It is a general property of distributions with the monotone likelihood ratio introduced by Milgrom (1981).[5] For completeness, in the next proposition I provide a version of the proof for the informational structure used in this paper.

Proposition 1 (Milgrom) *Let*

$$a_\theta, c_\theta \geq 0 \tag{1}$$

then the public signal does not cause polarization in the first order of expectations.

Proof. Using the Projection Theorem[6] gives

$$f_{i,1} = E\left(\theta|x_{i,1}, y\right) = E\left(\theta|x_{i,1}\right) + \frac{Cov\left(y, \theta|x_{i,1}\right)}{Var\left(y|x_{i,1}\right)}\left(y - E\left(y|x_{i,1}\right)\right)$$

which, together with the assumptions of the proposition, implies

$$a_\theta = \hat{a}_\theta - c_\theta \frac{\rho_{x,y}\sigma_y}{\sigma_x} < \hat{a}_\theta$$

for all n. The linearity of conditional expectations of jointly normal variables gives the result. ■

Conditions in (1) require that the private and the public signal remain positively related to the fundamental even conditionally on the rest of the information structure. This is an almost trivial consequence of the concept of a signal.

In the remaining of this section I show that polarization in higher-order expectations is consistent with a Gaussian information structure. Intuitively, polarization in higher-order expectations differs from polarization in first-order expectations, because of the critical role of the strength of connection between private signals of agents in different groups. The forecasts of agents in group t vary only to the extent that their forecast on the private signals of an

[5] See, for example, Dixit and Weibull (2007) for the argument.

[6] The Projection Theorem states that if $\mathbf{v}_\theta$ and $\mathbf{v}_s$ are vectors of variables which are jointly normally distributed with the vector of expected values $\boldsymbol{\mu}_\theta, \boldsymbol{\mu}_s$, respectively and covariance matrix

$$\begin{bmatrix} \Sigma_\theta & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_s \end{bmatrix},$$

then

$$\mathbf{v}_\theta|\mathbf{v}_s \tilde{} N\left(\boldsymbol{\mu}_\theta + \Sigma_{\theta,s}\Sigma_s^{-1}\left(\mathbf{v}_s - \boldsymbol{\mu}_s\right), \Sigma_\theta - \Sigma_{\theta,s}\Sigma_s^{-1}\Sigma_{s,\theta}\right)$$

(see Brunnermeier, 2001).

agent in group $t-1, x_{i,t-1}$ is sensitive to their private signals $x_{j,t}$.The next proposition shows that if the connection between private signals across groups is relatively weak, the public announcement will increase the dispersion of forecasts in all groups.

Proposition 2 *Suppose that condition in (1) hold. If and only if*

$$\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta}{\left(\hat{a}_\theta \left(1-\rho_{x,y}^2\right)+a_\theta\right)}$$

there is polarization in expectations of any order of $t>1$.

Proof. Let

$$\begin{aligned} a_x &\equiv \frac{\partial E\left(x_{t-1}|x_t,y\right)}{\partial x_t} = \frac{\sigma_{x,x'}\sigma_y^2-\sigma_{x,y}^2}{\sigma_x^2\sigma_y^2-\sigma_{x,y}^2} \\ \hat{a}_x &\equiv \frac{\partial E\left(x_{t-1}|x_t\right)}{\partial x_t} = \frac{\sigma_{x,x'}}{\sigma_x^2}. \end{aligned}$$

Note that

$$\left|\frac{\partial \hat{f}_t}{\partial x_t}\right| = \left|\left(\hat{a}_x\right)^{t-1}\frac{\partial \hat{f}_1}{\partial x_1}\right| \text{ and } \left|\frac{\partial f_t}{\partial x_t}\right| = \left|\left(a_x\right)^{t-1}\frac{\partial f_1}{\partial x_1}\right|$$

As $\frac{\partial \hat{f}_1}{\partial x_1} > \frac{\partial f_1}{\partial x_1} > 0$, $\left|\hat{a}_x\frac{\partial \hat{f}_1}{\partial x_1}\right| < \left|a_x\frac{\partial f_1}{\partial x_1}\right|$ implies $|\hat{a}_x| < |a_x|$ and $\left|\left(\hat{a}_x\right)^{t-1}\frac{\partial \hat{f}_1}{\partial x_1}\right| < \left|\left(a_x\right)^{t-1}\frac{\partial f_1}{\partial x_1}\right|$ for all t. Note also that $\left|\frac{\partial \hat{f}_t}{\partial \hat{x}_t}\right| < \left|\frac{\partial f_t}{\partial x_t}\right|$ implies polarization in the t-th order of higher-order expectations. Substituting in the definition of $\hat{a}_x$ and a_x into $\left|\hat{a}_x\frac{\partial \hat{f}_1}{\partial x_1}\right| < \left|a_x\frac{\partial f_1}{\partial x_1}\right|$ and simple algebraic manipulation gives $\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta}{\left(\frac{\sigma_\theta\rho_{\theta,x}}{\sigma_x}\left(1-\rho_{x,y}^2\right)+a_\theta\right)}$, which concludes the proof. ■

In our applications, we will be interested in second order expectations. It is easy to see that polarization of the forecasts of the second group depends only on the effect of the public announcement on the term $|a_{x_1}a_\theta|$ as

$$\begin{aligned} \int_0^1 \left|f_{i,1}-\overline{f_1}\right| di &= \left|a_{x_1}a_\theta\right|\int_0^1 \left|x_1^i-\bar{x}\right| di \qquad (2) \\ \int_0^1 \left|\hat{f}_{i,1}-\overline{\hat{f}_1}\right| di &= \left|\hat{a}_{x_1}\hat{a}_\theta^2\right|\int_0^1 \left|x_1^i-\bar{x}\right| di \end{aligned}$$

This term is the sensitivity of a second group agent's forecast on her private signal and depends crucially on the correlation between the private signals of agents in different groups. To see better the intuition why, let us consider the following extreme example where this correlation is zero.

Example 1 *Suppose θ stands for the performance of the economy and it has two components: $\theta = \theta_1 + \theta_2$, where θ_1 is the performance of the public sector and θ_2 is the performance of the private sector. Suppose that θ_1 and θ_2 are independent and have zero mean. Assume that group 1 contains only one agent, the central bank. The central bank observes a private signal on the performance of the public sector: $x_1 = \theta_1 + \varepsilon$. Group 2 contains a unit mass of speculators, each observing a private signal about the performance of the private sector, $x_2 = \theta_2 + \varepsilon_i$. We are interested in the change in the dispersion of the forecasts of group 2 after the announcement of the public signal $y = \theta + \eta$.*

Observe that before the public announcement, speculators have no private information on the private information of the central bank, so their forecast of the forecast of the central bank is

$$\hat{f}_2 = E\left(E\left(\theta|x_1\right)|x_2\right) = E_j\left(\hat{a}_\theta x_1|x_2\right) = 0.$$

Thus, there is no dispersion in the forecast of the second group at all:

$$Var\left(\hat{f}_2\right) = 0.$$

However, after the announcement of y, a speculators' forecast, f_2 changes to

$$\begin{aligned} E\left(E\left(\theta|x_1, y\right)|x_2, y\right) &= E_j\left(a_\theta x_1 + c_\theta y|x_2, y\right) = \\ &= a_\theta E_j\left(x_1|x_2, y\right) + c_\theta y = a_\theta a_{x_1} x_2 + c_\theta y, \end{aligned}$$

where $a_\theta = \frac{\partial f_1}{\partial x_1}, c_\theta = \frac{\partial f_1}{\partial y}$ and $a_{x_1} = \frac{\partial E(x_1|x_2,y)}{\partial x_2}$. It is easy to check that $a_{x_1} a_\theta \neq 0$. Thus, the dispersion of speculators' forecasts changes to

$$Var\left(f_2\right) = \left(a_{x_1} a_\theta\right)^2 \left(\sigma^2_{\theta_1} + \sigma^2_{\varepsilon_i}\right) > 0.$$

In the example, the public announcement increases the disagreement among speculators because the public announcement connects the private signals of speculators with the private signal of the central bank. Before the public announcement, a speculator with a high private signal on the performance of the private sector has the same forecast of the performance of the public sector than a speculator with a low private signal. Hence, the two speculators have the same forecast of the central bank's forecast. After the public announcement, a speculator who is pessimistic about the performance of the private sector has a higher estimation on the performance of the public sector than a speculator who is optimistic about the performance of the private sector. The reason is that the public signal gives information on the sum of the two sectors. Thus, the dispersion of the forecasts of the two speculators increases after the announcement. Proposition 1 showed that the sensitivity of first-order expectations to private signals decreases after an announcement. Proposition 2 shows that this do not have to be the case with higher-order expectations.

One might wonder why this property has not got any attention in the previous literature. There are two likely reasons. The first one is that interest in models where higher-order expectations play an important role is relatively recent. The second one is that even in such models the information structure is virtually always assumed to be the form where both private and public signals are noisy observations of the fundamental: $x_{i,t} = \theta + \varepsilon_i, y = \theta + \eta$. This structure imposes a rigid structure on the correlation structure of signals and θ. In particular,

$$cov\left(x_{i,t}, x_{j,u}\right) = cov\left(x_{i,t}, x_{j,t}\right) = cov\left(x_{i,t}, y\right) = var\left(\theta\right)$$

for any agent. It is easy to see that this structure is inconsistent with the assumptions of Proposition 2.

In this section, I highlighted a statistical property of normally distributed variables. In the next section, I argue that the statistical property highlighted in this section has important economic consequences by modifying two standard workhorse models of economics with learning.

3 Application I: Speculative currency attacks and transparency

The necessary degree of central bank's transparency has been in the forefront of monetary economics in the last decade. Transparency is commonly defined as the degree of asymmetry of information between the public and the central bank.[7] In the following application, I distinguish between two aspects of transparency. First, one can think of the lack of transparency as large uncertainty of the public about some variables in the private information set of the central bank. Second, one might think of the lack of transparency as large dispersion of opinion among the members of the public about the same variables. Although in many frameworks these two concepts are very similar, I show that the distinction is important. In particular, let

[7] The literature distinguished many different types of transparency refering to the objectives of the central bank, the information of the central bank, the followed proceeders or the future policy. In this paper, I focus on asymmetry of information about the state of the economy. See Geraats (2002) for a detailed literature review on the transparency of central banks.

us suppose that transparency is low in the first sense. I show that if the central bank decides to gather and disclose information to increase transparency in the first sense, this will reduce transparency in the second sense. Furthermore, if the public information is not sufficiently precise, the new public information destabilizes the economy by increasing the probability of speculative attacks.

I modify the currency attack model of Morris and Shin (1998). I use elements of variants of this model introduced by Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006) and Goldstein, Ozdenoren and Yuan (2008). The main point of departure from Morris and Shin (1998) is that I do not allow the central bank either to directly observe the state of the economy or to perfectly infer it from the size of the speculative attack. Instead, the central bank observes a private signal about the state and observes the size of the attack with noise. Thus, speculators have to guess the central banks' guess about the economy when they assess the chance of devaluation.[8]

The closest paper in terms of the focus of the analysis is Heinemann and Illing (2002) who stick to the original version of Morris and Shin (1998) and show that disclosing public information always decreases the chance of speculative attacks. My conclusion o$q_H(\omega)$urns this result.

3.1 The set up

There is a unit mass of risk-neutral speculators who decide whether to attack the currency regime for cost $1 > \kappa > \frac{1}{2}$. Attacking is advantageous if and only if the central bank decide to devalue the currency. Their pay-off is summarized by the following table.

	devaluation	no devaluation
attack	$1-\kappa$	$-\kappa$
do not attack	0	0

Risk-neutrality implies that a speculator attacks only if she estimates the probability of devaluation larger than κ.

Defending the exchange rate regime is costly for the central bank. If A fraction of speculators attack,the cost of defence is $\Phi^{-1}(A)$ where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function. In addition there are some noise traders who might buy or

[8] Goldstein, Ozdenoren and Yuan (2008) also assumes that the central bank has imperfect information of the fundamental. The main difference between my model and theirs is that in their model the central bank cost's function of defending the peg does not depend directly on the size of the attack.

sell the domestic currency. The cost of defending the peg from the attack of the noise-traders is $u\tilde{}N\left(0,\frac{1}{\delta}\right)$. Thus, the total cost of defending the peg is $C\equiv\Phi^{-1}(A)+u$.

The Central Bank abandons the peg if the cost,C is large relative to the central bank's estimate of the fundamentals

$$\lambda C > E\left(\theta|S_{cb}\right), \tag{3}$$

where θ is the fundamental of the economy and S_{cb} is the information set of the central bank and λ is a constant controlling for the trade-off.

The information structure is as follows. The fundamental of the economy contains three factors, $\theta=\theta_a+\theta_b+\theta_c$, where θ_a is a factor specific to the speculators information set, θ_b is a factor specific to the central bank's information set, while θ_c is a common factor. Consistently with this notation, the central bank observes a private signal $z=\theta_b+\theta_c+\varepsilon_{cb}$, and each speculator observes a private signal $x_i=\theta_a+\theta_c+\varepsilon_i$. There might be a public announcement of $y=\theta+\eta$. The cost of defence C is also public information. I assume that all factors and noise terms are i.i.d. and normally distributed:

$$\begin{aligned}\theta_a,\theta_b &\sim N\left(0,\frac{1}{\kappa}\right),\theta_c\sim N\left(0,\frac{1}{\omega}\right),\varepsilon_i\sim N\left(0,\frac{1}{\alpha}\right)\\ \varepsilon_{cb} &\sim N\left(0,\frac{1}{\upsilon}\right),\eta\sim N\left(0,\frac{1}{\beta}\right)\end{aligned}$$

Observe that the parameter ω controls the degree of uncertainty of speculators about the private information of the central bank. When ω is large, the covariance between the private information set of a speculator and the private information set of the central bank is small. Thus, speculators know little about the private information of the central bank. Our main question of interest is that whether the existence of a public signal increases or decreases the unconditional probability of a successful attack conditional on the level of uncertainty parametrized by ω.

Apart from the main point of departure of imperfect information of the central bank, all the other modifications compared to Morris and Shin (1998) are only for analytical tractability. Namely, the cost C is a particular function of A and u to keep the structure of information normally distributed.[9] I let speculators to observe C and form strategies conditional on C, so

[9]This modeling trick has become standard in the literature (e.g. Dasgupta, 2007, Angeletos, Hellwig, and Pavan, 2006 (online supplement)). However, in our particular case, this normalization has some consequences on the uniquness of equilbrium which we discuss in subsection 5.

they have to forecast only one element (the private signal) of central banks' information set.[10] None of this modifications plays a role in the mechanism of the main result of this application.

3.2 Equilibrium and existence

As it is usual in the literature, I am looking for monotone (threshold) equilibria where the central bank chooses the same action as long as her signal is below a certain threshold, z^*, and chooses the other action otherwise. Similarly, each speculator chooses the same action as long as her signal is below a certain threshold x^* and chooses the other action otherwise. Both thresholds, z^*, x^* can depend on public information only. If z^* and x^* is finite, I call the equilibrium a finite monotone equilibrium. I derive all expressions for the case when the public signal is released.

Solving for the equilibrium consists of four steps.

First, we conjecture the form of the optimal strategy and derive the information content of the cost of the attack under the conjecture. Let us conjecture that there is a threshold z^* that the central bank devalues if and only if $z < z^*$ and there is a threshold x^* that the speculators attack if and only if $x_i < x^*$. The later implies that the aggregate demand for dollars is

$$A = \Phi\left(\psi\left(x^* - \theta_s - \theta_c\right)\right) \tag{4}$$

where $\psi \equiv \sqrt{\frac{1}{\frac{1}{\omega}+\frac{1}{\kappa}+\frac{1}{\alpha}}}$ is the standard deviation of the private signal of each speculator. Using the assumption that the cost of the attack, C is $C = \Phi^{-1}(A) + u$, the cost of the attack is informationally equivalent to the cost signal, q, where

$$q \equiv x^* - \frac{1}{\psi}C = \theta_s + \theta_c - \frac{1}{\psi}u. \tag{5}$$

Second, by definition, if the central bank has a signal z^*, the central bank must be indifferent between abandoning the exchange regime or defending it. Thus, z^* must satisfy

$$\lambda C = E\left(\theta | z = z^*, q, y\right). \tag{6}$$

Third, by definition, if a speculator has a signal of x^*, she must be indifferent between attacking the exchange rate regime or not attacking. The probability of devaluation under the

[10] Letting agents to submit demand curves conditional on a contemprenous endogenous signal (typically the market price) is standard in finance as a short cut for modelling a sequential mechanism in a one-shot game. In the context of the Morris and Shin set-up, this method was followed by Hellwig, Mukherji and Tsyvinski (2006).

information set of speculator i is

$$\Pr\left(z < z^*|x_i, y, q\right) = \Phi\left(\frac{z^* - E\left(z|x_i, y, q\right)}{\sigma_{z|x_i,y,q}}\right) \tag{7}$$

where $\sigma_{z|x_i,y,q} = \sqrt{Var\left(z|x_i, y, q\right)}$. Consequently, x^* must satisfy

$$\Phi\left(\frac{z^* - E\left(z|x_i = x^*, y, q\right)}{\sigma_{z|x_i,y,q}}\right) = \kappa \tag{8}$$

Finally, substituting in the definition of q into (6) and (8) and some simple algebra, equations (6) and (8) give a system of two equations with the two unknowns z^*, x^* of the following form

$$\begin{aligned} z^* &= \frac{\lambda\psi x^* - \left(\left(b_\theta + \psi\lambda\right) q + c_\theta y\right)}{a_\theta} & (9) \\ x^* &= \frac{z^* - \Phi^{-1}\left(\kappa\right)\sigma_{z|x_i,y,q} - c_z y - b_z q}{a_z} & (10) \end{aligned}$$

where I used the notation of

$$\begin{aligned} E\left(\theta|x_i, q, y\right) &= a_\theta x_i + b_\theta q + c_\theta y \\ E\left(z|x_i, q, y\right) &= a_z x_i + b_z q + c_z y. \end{aligned}$$

Note, that both equations (9) and (10) are linear in x^* and z^*. Hence, there is a unique intercept of the two lines as long as $\frac{\lambda\psi}{a_\theta} \neq a_z$.

The derived equilibrium is consistent with our conjecture that a speculator attacks if $x < x^*$ only if expression (7) is increasing in x_i. This is true if and only if $a_z > 0$. Otherwise, we should conjecture that speculators attack if and only if $x > x^*$. In this case, (4) and (5) modify to

$$\begin{aligned} A &= \Phi\left(-\psi\left(x^* - \theta_s - \theta_c\right)\right) & (11) \\ q &\equiv x^* + \frac{1}{\psi}C = \theta_s + \theta_c + \frac{1}{\psi}u, & (12) \end{aligned}$$

respectively. Note that the difference between expressions (4)-(5) and (11)-(12) is the change of the sign before the constant ψ. Hence, the derivation of the equilibrium incorporates both cases if we substitute

$$\tilde{\psi} = \left\{ \begin{array}{ccc} \psi & if & a_z \geq 0 \\ -\psi & otherwise & \end{array} \right\}$$

instead of ψ in expressions (4), (5) and (9).

The following proposition summarizes this subsection.

Proposition 3 *As long as* $\frac{\lambda\tilde{\psi}}{a_\theta} \neq a_z$, *there is a unique finite monotone equilibrium described by*

$$\begin{aligned} x^* &= \frac{a_\theta \Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q} + (a_\theta c_z + c_\theta)\,y + \left(a_\theta b_z + \left(b_\theta + \tilde{\psi}\lambda\right)\right) q}{\lambda\tilde{\psi} - a_z a_\theta} \\ z^* &= \frac{\lambda\tilde{\psi}x^* - \left(\left(b_\theta + \tilde{\psi}\tilde{\lambda}\right) q + c_\theta y\right)}{a_\theta}. \end{aligned}$$

The central bank abandons the peg if and only if $z \leq z^*$ *and the speculators attack*

1. *if and only if* $x < x^*$ *as long as* $a_z \geq 0$ *and*

2. *if and only if* $x > x^*$ *as long as* $a_z < 0$..

3.3 Probability of successful currency attacks

I am interested in the effect of public information on the probability of a successful attack given ω, the degree of uncertainty of speculators about the information of the central bank. For a given information structure, the probability of a successful attack is the unconditional probability that the private signal of the central bank is smaller than her threshold, z^*. If the public signal is released, this is

$$\begin{aligned} \Pr(z < z^*) &= \Pr\left(z < \Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q} + a_z x^* + \left(b_\theta + \tilde{\psi}\lambda\right) q + c_z y)\right) = \\ &= \Pr\left(z - \left(\frac{a_z b_\theta + (a_z + b_z)\,\tilde{\psi}\lambda}{\lambda\tilde{\psi} - a_z a_\theta}\right) q - \left(\frac{a_z c_\theta + c_z\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right) y < \frac{\Phi^{-1}(\kappa)\,\sigma_{z|x_i,y,q}\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right) = \\ &= \Phi\left(\Phi^{-1}(\kappa)\,\frac{\lambda\psi}{\lambda\psi - |a_z|\,a_\theta}\,\frac{\sigma_{z|x_i,y,q}}{\sigma_{z-\left(\frac{a_z b_\theta + (a_z+b_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - a_z a_\theta)}\right)q - \left(\frac{a_z c_\theta + c_z\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right)y}}\right) \equiv \Phi\left(\Phi^{-1}(\kappa)\,K\right). \end{aligned}$$

where

$$K \equiv \frac{\lambda\psi}{\lambda\psi - |a_z|\,a_\theta}\,\frac{\sigma_{z|x_i,y,q}}{\sigma_{z-\left(\frac{a_z b_\theta + (a_z+b_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - a_z a_\theta)}\right)q - \left(\frac{a_z c_\theta + c_z\lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}\right)y}}$$

is a coefficient which depends only on the parameters of the model. Observe that the probability of a successful attack is increasing in K. Similarly, if there is no announcement the probability of a successful attack is

$$\Pr(z < \hat{z}^*) = \Phi\left(\Phi^{-1}(\kappa)\,\frac{\lambda\psi}{\lambda\psi - |\hat{a}_z|\,\hat{a}_\theta}\,\frac{\sigma_{z|x_i,q}}{\sigma_{z-\left(\frac{\hat{a}_z \hat{b}_\theta + (\hat{a}_z + \hat{b}_z)\tilde{\psi}\lambda}{(\lambda\tilde{\psi} - \hat{a}_z\hat{a}_\theta)}\right)q}}\right) \equiv \Phi\left(\Phi^{-1}(\kappa)\,\hat{K}\right).$$

Thus, if $K > \hat{K}$ then more public information increases the probability of a successful attack. Observe, that both K and $\hat{K}$ are products of two components. From our observation (2) it is apparent that the first term is highly related to polarization. If and only if the release of the public signal increases polarization in second-order expectations, the term, $\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}$, will increase due to the announcement. From Proposition 2, we already know that there is polarization due to the announcement if the correlation between the private signals of the central bank and speculators is small. That is, when ω, the degree of uncertainty is high. The second term is harder to interpret. It is the fraction of the precision of speculators' estimate of the central bank's signal and the standard deviation of a linear combination of z.and the public signals. The effect of an announcement on this component is ambiguous. Still, in the next proposition I show that if ω is large and the precision of the public signal is sufficiently low, the announcement will increase the probability of a successful speculative attack.

Proposition 4 *There are finite positive thresholds T_ω, T_β, that if $\omega > T_\omega$, and $\beta < T_\beta$ then*

$$K > \hat{K}$$

and the release of the public information increases the probability of a successful speculative attack.

Proof. The proof is in the appendix. ∎

This result emphasizes the interaction between two notions of transparency we discussed at the beginning of this section: uncertainty and polarization of opinions about the private information of the central bank. The Proposition shows that if transparency is low in the first sense (ω is high), then more public information can decrease transparency in the second sense. That is, if the central bank decides to generate and publish new public information, this increases polarization of opinions about the central banks' private information which increases the chance of a currency crisis.

Why polarization is connected to the probability of successful currency attacks? The intuition is based on the fact that always those speculators attack who think that the probability of devaluation is high relative to the cost of the attack κ. If there is more dispersion in speculators' assessment about the probability of devaluation, there will be more mass on the tails and more speculators with an assessment larger than κ. The form of central bank's cost function, (3), implies that the dispersion in speculators' assessment on the probability of devaluation is proportional to the dispersion in speculators' assessment on the forecast of the central bank. Thus, if the public announcement polarizes speculators second-order expectation regardless of the content of y, then the ex ante probability of a sufficiently large attack is higher in a regime with public announcement. Proposition 4 shows that this basic intuition remains valid in our equilibrium where the cost of the attack reveals information about the average signal both for the speculators and for the central bank.

3.4 Uniqueness

Compared to Morris and Shin (1998), the modification of the form of central bank's cost function changes the properties of the equilibrium regarding the issue of uniqueness versus multiplicity. The changes are two fold. First, as it is stated in proposition 3, there is a unique finite monotone equilibrium regardless of the precision of private and public signals. This is in contrast to the standard Morris and Shin (1998) model where there are multiple finite monotone equilibria if the public signal is too precise relative to the private signal. This source of multiplicity is absent in my framework. The reason is mainly technical: in the original Morris and Shin model the best response functions corresponding to equations (9)-(10) are non-linear and might intersect more than once. In my framework, there is always exactly one intersection.

Second, even if in the current framework there is always a unique finite monotone equilibrium, there is never a unique monotone equilibrium. In particular, both of the following two strategy profiles form equilibria: (1) all speculators attack regardless of their signals and the central bank devalues regardless of her signal (2) none of the speculators attack regardless of their signal and the central bank keeps the peg regardless of her signal. In Morris and Shin (1998), none of these strategy profiles forms an equilibrium. The reason is that in Morris and Shin, the cost function of the central bank maps the size of the attack to the unit interval, while in my version the cost function maps the size of the attack to the real line. Thus, in my version the cost of defending the peg might be unboundedly low or unboundedly high which validates the new equilibria.

In the rest of this section, I show that only the finite monotone equilibrium is robust to a simple perturbation of the system. Let us suppose that the cost of defending the peg from speculators is bounded from above and from below. In particular, let us suppose that the cost of defence when A proportion of speculators attack is

$$\tilde{C} = \left\{ \begin{array}{ccc} \Phi^{-1}(\xi) + u & if & \Phi^{-1}(A) \leq \Phi^{-1}(\xi) \\ \Phi^{-1}(A) + u & if & \Phi^{-1}(\xi) < \Phi^{-1}(A) < \Phi^{-1}(1-\xi) \\ \Phi^{-1}(1-\xi) + u & if & \Phi^{-1}(1-\xi) \leq \Phi^{-1}(A) \end{array} \right\} \tag{13}$$

instead of C, where ξ is a small positive constant. It is apparent that as ξ goes to zero, the system converges to the original framework. In the next proposition, I show that as long as ξ is sufficiently small, this perturbation eliminates all monotone equilibria, but the one presented in proposition 3.

Proposition 5 *Under perturbed system characterized by (13), there is a T_ξ threshold that if $\xi < T_\xi$, there is a unique monotone equilibrium. Furthermore, as $\xi \to 0$, the equilibrium converges to the one defined in Proposition 3.*

Proof. The proof is in the Appendix. ■

4 Application II: trading volume in a Grossman-Stiglitz model

In rational expectation models of asset pricing under asymmetric information has difficulties to generate the well established empirical fact that trading volume increases around public announcements.[11] To see why, let us consider the static Grossman-Stiglitz model[12] where the optimal demand function of a trader i is given by

$$d_i = \frac{E(\theta|S_i) - p}{\gamma Var(\theta|S_i)} \tag{14}$$

where θ is the true value of the asset, S_i is the information set of trader i including the equilibrium price p and γ is the absolute risk aversion of the trader. As the aggregate demand has to equal to the noisy supply, the equilibrium price is a noisy average of the best guesses of traders. Consequently, the numerator of (14) is a noisy measure of disagreement among traders. Hence, traders' aggregate position, $\int_i |d_i| di$, positively depends both on the average disagreement among traders and the precision of their estimation, $\frac{1}{Var(\theta|S_i)}$. A public announcement related to θ, will unambiguously increase the precision of traders' estimation.

[11] See Evans and Lyons (2003), Love and Payne (2003), Fleming and Remolona (1999).

[12] In the discussed form, this model was developed in steps in Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). For simplicity, I refer to the this class of models with CARA-normal framework, differential information and noisy supply as Grossman-Stiglitz type of models.

As we established in the previous section, disagreement about θ decreases after the announcement under general assumptions. Thus, the two effects work in the opposite direction and the aggregate change in position and the resulting trading volume tend to be small. In particular, in the basic Grossman-Stiglitz model where traders receive the private signal $x_i = \theta + \varepsilon_i$ and the public announcement is $y = \theta + \eta$ where $\theta, \varepsilon_i, \eta$ are normally and independently distributed with zero mean, the two effects exactly cancel out and there is no trading volume at all after the announcement. Similarly, dynamic versions of the Grossman-Stiglitz model as Brown and Jennings, (1989), Kim and Verrecchia, (1991), He and Wang (1995) do not deliver polarization of opinions either. Consequently, they have problems explaining the empirical regularities. The information structure of these models are nested in the current set up.[13]

In the rest of this section, I analyze a version of the dynamic Grossman-Stiglitz model where some traders has to liquidate their position early. This early traders are interested in the liquidation price instead of the fundamental value. As these early traders are interested in the expectation of other traders, the disagreement among them might increase due to the announcement as Proposition 2 stated. In this case, the two effects work in the same direction which leads to large trading volume around announcements.

4.1 The set-up

I modify a standard, dynamic, CARA-Normal, rational expectations model with differential information (e.g. He and Wang, 1995, Brown and Jennings, 1989). Preferences of traders are given by $U_i(W_i) = -e^{-\gamma W_i}$, where W_i is monetary wealth at the time of the exit, γ is the absolute risk-aversion parameter and in each period traders submit demand curves to an auctioneer to buy up the random supply of assets:

$$u_t \tilde{} N\left(0, \frac{1}{\delta_t^2}\right).$$

Traders base their portfolio decision on the private signal which they receive at the moment of their entry and all available public signals, i.e., past and present prices and public announcements. They update their beliefs by Bayes' Rule. Prices, p_t, are determined by market clearing in each period.

[13] In contrast, models with non-Gaussian priors and/or boundedly rational agents (Varian, 1986, Harris and Raviv, 1993, Kandel and Pearson, 1995, Evans and Lyons, 2001) can produce disagreement and volume, but at the expense of giving up some of the standard assumptions.

I introduce two non-standard assumption. First, I assume overlapping groups of traders similarly to Allen, Morris and Shin (2006). In particular, I will have three groups of traders with continuum of traders in each group, who trade in 3 periods ($t = 0, 1, 2$). Traders in the first group trade among themselves in period 0 and 1 and sell all of their remaining assets in period 2. Traders in the second group take all positions of the first group, trade among each other in period 2 and liquidate for the uncertain value of θ at the end of the period. I assume that if there is a public announcement, y, then it will be released at the beginning of period 1. Hence, I will focus on the differences in trading patterns of early traders with and without an announcement. For expositional purposes, one can imagine the set-up in the context of a 24-hour period in the USD/GBP market, where early traders are based in London and late comers are based in New York. In this context periods 0 and 1 would correspond to the trading hours in London and period 2 would correspond to the trading hours in New York.

The second departure from the standard assumptions is a more general information structure. I assume that the fundamental value of the asset – the exchange rate in this interpretation– is given by

$$\theta = \theta_s + \theta_k + \theta_w$$

where $\theta_s, \theta_k, \theta_w$ are the US factor, the UK factor and the world factor respectively. I assume that the private signal that Londoners receive contains noisy information on the UK factor and the world factor, but does not contain information on the US factor: $x_i = \theta_k + \theta_w + \varepsilon_i$, while the private signals of New Yorkers contain information on the US factor and the world factor, but not on the UK factor: $z_j = \theta_s + \theta_w + \varepsilon_j$. Hence, the world factor simply represents the common element in the information set of agents in different groups, while the US factor and the UK factor represent group-specific information. The public signal contains information on fundamental value: $y = \theta + \eta$. I assume that all factors and noise terms are i.i.d. and normally distributed:

$$\theta_k, \theta_s \sim N\left(0, \frac{1}{\kappa}\right), \theta_w \sim N\left(0, \frac{1}{\omega}\right), \varepsilon_i, \varepsilon_j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

The presented model nests the information structures of many versions of the Grossman-Stiglitz model. The next table summarizes the connections.

our parameter	*private*	*public*	*fundamental*	$p_2(\cdot)$	model
$\kappa, \delta_2 \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w$	Brown-Jennings(1989), Kim-Verrecchia(1991,1994)
$\kappa \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w + u_2$	He-Wang (1995)
$\omega \to \infty$	$\theta_k + \varepsilon_i$	$\theta_s + \eta$	$\theta_k + \theta_s$	$\approx \theta_s + u_2$	independent information sets as in Example 1

4.2 Equilibrium and existence

Finding the linear rational expectation equilibrium consists of three fairly standard steps. First, I conjecture that prices are given by the functions

$$\begin{aligned} p_2 &= a_2(\theta_s + \theta_w) + c_2 y + h_2 q_1 + g_2 q_0 - e_2 u_2 && (15)\\ p_1 &= a_1(\theta_k + \theta_w) + c_1 y + h_1 q_0 - e_1 u_1 && (16)\\ p_0 &= a_0(\theta_k + \theta_w) - e_0 u_0, && (17) \end{aligned}$$

where $c_t, b_t, e_t, f_1, f_2, g_2$ are undetermined coefficients, while q_1, q_0 are specified as follows. Prices together with past prices and the public information are informationally equivalent with the following price signals

$$\begin{aligned} q_2 &= \frac{1}{a_2}(p_2 - c_2 y - h_2 q_1 - g_2 q_0) = (\theta_s + \theta_w) - \frac{e_2}{a_2} u_2 && (18)\\ q_1 &= \frac{1}{a_1}(p_1 - c_1 y - h_1 q_0) = (\theta_k + \theta_w) - \frac{e_1}{a_1} u_1 \\ q_0 &= \frac{1}{a_0} p_0 = (\theta_k + \theta_w) - \frac{e_0}{a_0} u_0. \end{aligned}$$

I define τ_t^2 as the precision of q_t:

$$\frac{1}{\tau_t^2} \equiv \frac{e_t^2}{a_t^2 \delta_t^2} \text{ or } \frac{\tau_t}{\delta_t} = \frac{a_t}{e_t}.$$

Second, given the conjectured price functions, I derive the optimal strategies of traders. To ease on the notation, I define the following coefficients of variables in traders' information sets in different conditional expectations of New Yorkers and Londoners:

$$\begin{aligned} E(\theta | z_j, y, q_2, q_1, q_0) &= a_\theta z_j + b_\theta q_2 + c_\theta y + h_\theta q_1 + g_\theta q_0 && (19)\\ E(\theta_s + \theta_w | x_i, y, q_1, q_0) &= a_s x_i + b_s q_1 + c_s y + h_s q_0. && (20) \end{aligned}$$

From the first order conditions of the maximization problem of New Yorkers in period 2 and Londoners in period 1 are

$$d_{j,2} = \frac{E(\theta|z_j, y, q_2, q_1, q_0) - p_2}{\gamma var(\theta|z_j, y, q_2, q_1, q_0)} \tag{21}$$

$$d_{i,1} = \frac{E(p_2|x_i, y, q_1, q_0) - p_1}{\gamma var(p_2|x_i, y, q_1, q_0)}, \tag{22}$$

respectively. In period 0, Londoners maximize the following expected utility :

$$\max_{d_{i,0}} E\left(-\exp\left(-\alpha\left(p_1 - p_0\right) d_{i,0} - d_{i,1}\left(p_2 - p_1\right)\right)|x_i, q_0\right).$$

In the appendix, I show that the demand function of Londoners in period 0 has the form of

$$d_{i,0} = \frac{E(p_1|x_i, q_0) - p_0}{\gamma V_1} + \frac{E(d_{i,1}|x_i, q_0)}{\gamma V_2} \tag{23}$$

where V_1 and V_2 are functions of the variances and expectations of θ, y, q_1, q_2 conditional on the information set of Londoners in the first period. Intuitively, the first part in the nominator of expression (23) represents the short-term demand component, while the second part represents the hedging component for period-1demand.

Third, let us impose the market clearing conditions and find the undetermined coefficients which simultaneously satisfy all three market clearing conditions. From (21) and (19), the market clearing condition in period 2 is

$$D_2 = \frac{a_\theta\left(\theta_s + \theta_w\right) + b_\theta q_2 + c_\theta y + h_\theta q_1 + g_\theta q_0 - p_2}{\gamma var(\theta|z_j, y, p_2, q_1, q_0)} = u_2.$$

Using (18) gives

$$a_\theta\left(\theta_s + \theta_w\right) + b_\theta q_2 + c_\theta y + h_\theta q_1 + g_\theta q_0 - \gamma var(\theta|z_j, y, q_2, q_1, q_0)\, u_2 = a_2 q_2 + c_2 y + h_2 q_1 + g_2 q_0.$$

Setting the coefficients of random variables on the two sides of the equation to equal gives

$$c_2 = c_\theta, \quad h_2 = h_\theta, \text{ and } \quad g_2 = g_\theta \tag{24}$$

and

$$a_\theta\left(\theta_s + \theta_w\right) - \gamma var(\theta|z_j, y, q_2, q_1, q_0)\, u_2 = (a_2 - b_\theta)\, q_2.$$

Substituting in $q_2 = \theta_s + \theta_w - \frac{e_2}{a_2} u_2$ and collecting the coefficients of $\theta_s + \theta_w$ and u_2 give

$$a_2 = a_\theta + b_\theta \tag{25}$$

and

$$\frac{a_\theta}{\gamma var\left(\theta|z_j, y, q_2, q_1, q_0\right)} = \frac{a_2}{e_2}.$$

We can write the latter as

$$\frac{a_\theta}{\gamma var\left(\theta|z_j, y, q_2, q_1, q_0\right)} = \frac{\tau_2}{\delta_2}. \tag{26}$$

Note that the undetermined coefficients enter the left hand side of (26) only through the precision of the price signals, $\tau_0^2, \tau_1^2, \tau_2^2$. Thus, the market clearing condition in period 2 implies that the undetermined coefficients e_t, a_t, $t = 0, 1, 2$ has to satisfy

$$\tau_2 = F_2\left(\tau_2, \tau_1, \tau_0\right), \tag{27}$$

where $F_2(\cdot)$ is a function determined by equation (26).

Turning to the market clearing condition in period 1

$$E\left(p_2|x_i, y, q_1, q_0\right) - p_1 = \gamma var\left(p_2|x_i, y, q_1, q_0\right) u_1,$$

we get

$$a_2 E\left(\theta_s + \theta_w | x_i = \theta_k + \theta_w, y, q_1, q_0\right) + c_2 y + h_2 q_1 + g_2 q_0 - \left(a_1 q_1 + c_1 y + h_1 q_0\right) = \gamma var\left(p_2|x_i, y, q_1, q_0\right) u_1 \tag{28}$$

which implies

$$c_1 = a_2 c_s + c_2 = \left(a_\theta + b_\theta\right) c_s + c_\theta, ..h_1 = a_2 g_s + g_2 = \left(a_\theta + b_\theta\right) g_s + g_\theta \tag{29}$$

and, after substituting in $q_1 = \theta_k + \theta_w - \frac{e_1}{a_1} u_1$ and collecting the coefficients of $\theta_k + \theta_w$ and u_1,

$$a_1 = a_2\left(a_s + b_s\right) + h_2 = \left(a_\theta + b_\theta\right)\left(a_s + b_s\right) + h_\theta \tag{30}$$

and

$$\frac{\tau_1}{\delta_1} = \frac{a_2 a_s}{\gamma var\left(p_2|x_i, y, q_1, q_0\right)} = \frac{a_s}{\gamma\left(a_\theta + b_\theta\right) var\left(\theta_s + \theta_w|x_i, y, q_1, q_0\right)}. \tag{31}$$

Thus, the market clearing condition in period 1 imposes the condition that $\tau_t = 0, 1, 2$ has to satisfy the equation

$$\tau_1 = F_1\left(\tau_2, \tau_1, \tau_0\right) \tag{32}$$

where $F_1(\cdot)$ is a function determined by equation (31).

In the appendix, I show by very similar steps that the market clearing condition in period 0 imposes a third restriction on the noise term of the price signals, $\tau_t = 0, 1, 2$ which has the form of

$$\tau_0 = F_0 \left(\tau_2, \tau_1, \tau_0\right). \tag{33}$$

Thus, the existence of the equilibrium is equivalent to the existence of a fixed point of the system described by equations (27) ,(32) and 33. It is easy to see that setting $\beta = 0$ in equations (27)-(33)check that the corresponding equilibrium intensities when there is no announcement will be given by the same equations by letting the variance of the public signal to go to infinity, that is, setting $\beta = 0$. As before, when it is necessary, I distinguish between variables with and without announcement by denoting the no announcement case with a hat. The following proposition states that the equilibrium exists.

Proposition 6 *An equilibrium exists both for the announcement and the no-announcement cases.*

Proof. The proof is in the Appendix. ■

4.2.1 Trading volume

A side product of the derivation of the existence is that the demand functions turn out to have very simple forms:

$$\begin{aligned} d_{j,2} &= \frac{\tau_2}{\delta_2}(z_j - q_2) && (34) \\ d_{i,1} &= \frac{\tau_1}{\delta_1}\left(x_i - q_1\right) && (35) \\ d_{i,0} &= \frac{\tau_0}{\delta_0}\left(x_i - q_0\right). && (36) \end{aligned}$$

We get expression (34) by substituting (19), (24), (15) and (26) into (21). Similarly, substituting (20), (15), (16), (29) and (31) into (22) gives (34). I show in the appendix that (36) also holds. Expressions (34)-(36) show that traders' positions are determined as a product of the coefficient $\frac{\tau_t}{\delta_t}$ and the difference between the private information of the trader and a signal about the about the average valuation of all traders revealed by the price. Thus, $\frac{\tau_t}{\delta t}$ determines how intensively the trader uses her private information to bet against the others. I will label the fraction $\frac{\tau_t}{\delta_t}$ as the trading intensity in period t.

Using the definition of τ_t and q_t, we can rewrite expressions (34)-(36) as

$$\begin{aligned} d_{j,2} &= \frac{\tau_2}{\delta_2}\varepsilon_j + u_2 && (37)\\ d_{i,1} &= \frac{\tau_1}{\delta_1}\varepsilon_i + u_1 && (38)\\ d_{i,0} &= \frac{\tau_0}{\delta_0}\varepsilon_i + u_0. && (39) \end{aligned}$$

The expressions show that in each period, total positions consist of two parts. There is a risk-sharing part, u_t, which is purchased by each agent regardless of her information, and there is a speculative part, $\frac{\tau_t}{\delta t}\varepsilon_i$, which depends on the difference between the agent's signal and the true value of the factor, ε_i or ε_j, and the coefficient $\frac{\tau_t}{\delta t}$. So trading activity and volume will depend how these coefficients change across periods.

I am interested in the change of volume in period 1 due to the announcement. From equations (35) and (36), the amount of trading of trader i in period 1 is given by

$$v_1^i = d_{i,1} - d_{i,0} = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i - u_0 + u_1.$$

Just as total positions, the total amount of trade of individual i consists of two parts. There is an information-independent risk-sharing part, $u_1 - u_0$, and there is a speculative part $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i$. The latter is determined by the difference of trading intensities in the two periods and the private information of the trader. Aggregating across traders gives the following expression for total volume in period 1:

$$\begin{aligned} V_1^{total} &= \frac{1}{2}\int |d_{i,1} - d_{i,0}|\, di = \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i > u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi(\alpha\varepsilon_i)\, d\varepsilon_i - \\ &\quad - \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i < u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi(\alpha\varepsilon_i)\, d\varepsilon_i = \\ &= \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right| \frac{1}{\sqrt{\alpha}}\phi(T) + sgn\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)(u_1 - u_0)\frac{1}{2}(1 - 2\Phi(K)) \end{aligned}$$

with $K = \alpha\frac{u_0 - u_1}{\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}}$,where I used the result that if $\zeta \sim N(\mu, \sigma^2)$, then

$$\int_{\zeta > L} \zeta \frac{\phi\left(\frac{\zeta - \mu}{\sigma}\right)}{\Phi(\alpha)} d\zeta = E(\zeta | \zeta > L) = \mu + \sigma\lambda(\alpha)$$

with $\lambda(\alpha) = \frac{\phi(\alpha)}{1 - \Phi(\alpha)}$ and $\alpha = \frac{L - \mu}{\sigma}$.

Hence, aggregate volume depends only on the realization of $u_1 - u_0$, the precision of the private signals α, and the distance between trading intensities, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$. As the first one is unrelated to information or announcements, I focus on the speculative volume, which I define as the volume when there is no risk-sharing trade:

$$V_1 = V_1^{total}|_{u_0=u_1} = \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right| \frac{1}{\sqrt{\alpha 2\pi}}. \tag{40}$$

Hence, the effect of announcement on speculative volume, we only have to compare the change in trading intensities in the announcement case, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$, and the no-announcement case, $\left|\frac{\hat{\tau}_1}{\delta_1} - \frac{\hat{\tau}_0}{\delta_0}\right|$. I show that the outcome of this comparison depends heavily and systematically on the information structure, i.e., on the relative importance of the common factor, θ_w, and the individual factors θ_s, θ_k. The following proposition shows that as individual factors become less important and the common factor becomes more important, volume disappears. This result is in line with the observation that in a rational model with one factor trading volume around announcements is small, because the effects of increasing precision of opinions and decreasing disagreement balance out each other.

Proposition 7 *As $\delta_2, \kappa \to \infty$*

$$\frac{\hat{\tau}_2}{\delta_2} = \frac{\hat{\tau}_1}{\delta_1} = \frac{\hat{\tau}_0}{\delta_0} = \frac{\tau_2}{\delta_2} = \frac{\tau_1}{\delta_1} = \frac{\tau_0}{\delta_0} = \frac{\alpha}{\gamma},$$

hence $\hat{V}_1 = V_1 = 0$ in this limit.

Proof. The proof is in the appendix. ■

In contrast, the next proposition shows that if the importance of the common information element, θ_w, is small enough, the position of early traders is larger in absolute terms after the public announcement. Furthermore, the trading volume is also larger if there is an announcement in period 1.

Proposition 8 *If ω is large enough $D_1 = \frac{1}{2}\int \left|\frac{\tau_1}{\delta_0}\varepsilon_i\right| di > \frac{1}{2}\int \left|\frac{\hat{\tau}_1}{\delta_0}\varepsilon_i\right| di = \hat{D}_1$ and $V_1 > \hat{V}_1$ and as $\omega \to \infty$,$\frac{D_1}{\hat{D}_1} \to \infty$,$\frac{D_0}{\hat{D}_0} \to \infty$. Furthermore, $\frac{V_1}{\hat{V}_1} \to \infty$.*

Proof. The proof is in the appendix. ■

This result is based on the same intuition as the one illustrated by the Example in section 2. In the limit, when ω is very large, the importance of the world factor, θ_w, diminishes and there is no correlation between the private information of any Londoner and of any New Yorker. Traders speculate only on the value of those variables which are not public. From the

Londoners point of view in period 1, the only variable in p_2 which is not part of the public information set – apart from the noise, u_2 – is θ_s. But first period traders have no information on θ_s, they have information only on θ_k. Hence, each traders' guess on θ_s is the a priori mean, and traders do not take speculative positions against each other. But if Londoners do not trade on their private information, their private information cannot be channeled into prices. Thus, p_1 is dominated by the noisy supply. In period 0, Londoners should bet on p_1 and p_2, but they have minimal information either on p_1, as it is dominated by noise, or on p_2, because they do not know anything about θ_s. Hence, there is no speculative trade in period 0 either. It means that speculative volume, the difference between individual speculative positions in period 0 and period 1, is also very small. However, the situation changes with public announcement. Public announcement is $y \approx \theta_s + \theta_k + \eta$, if θ_w is insignificant. So Londoners will have some information on the sum of θ_s and θ_k. But together with their private information on θ_k, it gives them some information on θ_s. What is more, as they have different guesses on θ_k due to their different private signals, their guesses on θ_s will also be different. Therefore, public announcement increases disagreement. In particular, an early trader with a high private signal, will overestimate θ_k, so for a given $y \approx \theta_s + \theta_k + \eta$, will underestimate θ_s. This is why, the observation of the public signal motivates her to sell assets.[14] With the opposite logic as in the no-announcement case, there is trade in all periods and there is volume.

5 Conclusion

In this paper I characterized those Gaussian information structures under which higher-order expectations are systematically polarized by a public announcement. Furthermore, I modified two canonical models of learning to illustrate that polarization of higher-order expectations has interesting economic consequences. In particular, I showed in a Morris-Shin type of currency crisis model that more public information can increase the chance of a successful currency attack. I also showed in Grossman-Stiglitz type asset pricing model that high trading volume around public announcements is consistent with Gaussian information structure, Bayesian learning and common priors.

This analysis has a double lesson for modeling information and learning. First, the Gaussian-

[14] The fact that after the public announcement, if ω is large, traders with high private signal will sell the asset is the symptom of second-order expectations moving against first order expectations. Thus, in this case, traders have a high estimation of the fundamental value, but still they expect a low interim price. This phenomenon is not in the focus of this paper, but discussed in detail in Kondor (2004).

Bayesian framework is less restrictive than it might look. Second, the standard structure which is routinely used in many models, where the fundamental has a single factor and all signals are noisy versions of this factor, is more restrictive than it might look. In my simplest example the fundamental has two factors and some agents observe a private signal on one factor and other agents observe a private signal on the other factor. In this structure, a public announcement about the fundamental increases the dispersion of expectations of one group on the expectation of any member in the other group. In contrast, there is no polarization in higher-order expectations under the standard structure. At the same time, the structure in my example might be much more reasonable in many environments. After all, private information is typically about parts and public information is typically about aggregates. For example, if the fundamental is the state of the economy, a typical piece of private information can be the future performance, marginal cost or wage level of a given firm or sector or the political intentions of the government. While a typical pieces of public information are aggregate data releases. The paper illustrates that this richer and perhaps more realistic information structure can explain phenomena which are inconsistent with the most standard Gaussian-Bayesian structure.

References

[1] Adam, K., 2003. *Optimal monetary policy with imperfect common knowledge*, mimeo.

[2] Angeletos, George-Marios, Alessandro Pavan, 2007. Efficient use of information and social value of information, *Econometrica* 75, 1103-1142.

[3] Angeletos, George-Marios, Christian Hellwig, Alessandro Pavan, 2006. Dynamic global games of regime change: learning, multiplicity and timing of attacks, *Econometrica*, 75, 711-756.

[4] Angeletos, George-Marios, Iván Werning, 2006. Crises and Prices: Information aggregation, multiplicity and volatility, *American Economic Review*, 96, 1720-1736.

[5] Angeletos, George-Marios, Guido Lorenzoni, Alessandro Pavan, 2007. *Wall Street and Silicon Valley: A delicate interaction*, MIT, mimeo.

[6] Allen, Franklin, Stephen Morris, Hyun Song Shin, 2006. Beauty contests, bubbles and iterated expectations in asset markets, *Review of Financial Studie*s 19, 161-171.

[7] Amato, Jeffrey, Hyun Song Shin 2006. Imperfect common knowledge and the information value of prices, *Economic Theory* 27, 213-241.

[8] Banerjee, Snehal, Ron Kaniel, Ilan Kremer, 2008. Price drift as an outcome of differences in higher order beliefs, *Review of Financial Studies*, forthcoming.

[9] Brown, David P., Robert H. Jennings, 1989. On technical analysis, *Review of Financial Studies*, 2(4), 527-551.

[10] Brunnermeier, Markus K., 2001. *Asset-pricing under asymmetric information - Bubbles, crashes, technical analysis and herding*, Oxford University Press.

[11] Dasgupta, Amil 2007. Coordination and delay in global games, *Journal of Economic Theory*, 134, 195-225.

[12] Diamond, Douglas, Verrecchia, Robert E., 1981. Information aggregation in a noisy rational expectations economy, *Journal of Financial Economics*, 9, 221-235.

[13] Dixit, Avinash K., Jörgen W. Weibull, 2007. Political polarization, *PNAS*, 104, 7351-7356.

[14] Evans, Martin D. D., Richard K. Lyons, 2001. *Why order flow explains exchange rates*, Berkeley, mimeo.

[15] Evans, Martin D., Richard K. Lyons, 2003. *How is macro news transmitted to exchange rates?*, Berkeley, mimeo.

[16] Fleming, M., Remolona, E., 1999. Price formation and liquidity in the US treasury market, *Journal of Finance*, 54, 1901-1915.

[17] Geraats, Petra, 2002. Central Bank Transparency, *Economic Journal,* 112 (483), 532-565.

[18] Goldstein, Itay, Emre Ozdenoren, Kathy Yuan, 2008, Learning and strategic complementarities: Implications for speculative attacks, University of Pennsylvania, mimeo.

[19] Grossman, Sandford, Stiglitz, Joseph E., 1980. On the impossibility of informationally efficient markets, *American Economic Review*, 70, 393-408..

[20] Harris, Milt, Arthur Raviv, 1993. Differences of opinion make a horse race, *Review of Financial Studies*, 6, 473-506.

[21] He, H., Jiang Wang, 1995. Differential information and dynamic behavior of stock trading volume, *Review of Financial Studies*, 8(4), 919-972.

[22] Heinmann, Frank, Gerhard Illing, 2002. Speculative attacks: unique equilibrium and transparency, *Journal of International Economics*, 58, 429-450.

[23] Hellwig, Christian, 2002. *Public announcements, adjustment delays and the business cycle*, UCLA, mimeo.

[24] Hellwig, Christian, Arijit Mukerji, Aleh Tsyvinski, 2006. Self-fulfiling currency crises: the role of interest rates, *American Economic Review*, 96, 1769-1787.

[25] Hellwig, Martin F., 1980. On the aggregation of information in competitive markets, *Journal of Economic Theory*, 22, 477-498.

[26] Jewitt, Ian, 2004. *Information acquisition and disclosure in auctions*, Oxford, mimeo.

[27] Kandel, E., Pearson, N. D., 1995. Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy*, 103(4), 831-872.

[28] Kim, Oliver, Robert E. Verrecchia, 1991. Trading volume and price reactions to public announcements, *Journal of Accounting Research*, 29(2), 302-321.

[29] Kim, Oliver, Robert E. Verrecchia, 1994. Liquidity and volume around earning announcements, *Journal of Accounting and Economics*, 17(1-2), January, 41-67.

[30] Kondor, Peter. 2004. *Rational Trader Risk*, LSE, mimeo.

[31] Love, Ryan, Richard Payne, 2003. *Macroeconomic news, order flows, and exchange rates*, Discussion Paper 475, Financial Markets Group, London School of Economics.

[32] Makarov, Igor, Oleg Rytchkov, 2006. *Forecasting the forecasts of others: Implications for asset pricing*, MIT, mimeo.

[33] Milgrom, Paul R., 1981. Good news and bad news: Representation theorems and applications, *Bell Journal of Economics*, 12(2), 380-391.

[34] Morris, Stephen, Hyun Song Shin, 1998. Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks, *American Economic Review*, 88, 587-597.

[35] Rabin, Matthew, Joel L. Schrag, 1999. First impressions matter: A model of confirmatory bias, *Quarterly Journal of Economics*,114, 37-82.

[36] Varian, Hal R., 1989. Differences of opinion in financial markets in C. Stone (eds): *Financial risk: theory, evidence and implications, proceedings of the eleventh annual economic policy conference of the Federal Reserve Bank of St. Louis*, Kluwer, Boston pp. 3-37.

[37] Woodford, Michael, 2003. Imperfect common knowledge and the effects of monetary policy, in P. Aghion, R. Frydman, J. Stiglitz and M. Woodford (eds) *Knowledge, information and expectations in modern macroeconomics: In honor of Edmund S. Phelps*, Princeton University Press, Princeton, pp 25-58.

A Appendix: Application I

Proof of Proposition 4. Given that K is continuous in β and ω and continuously differentiable in β around $\beta = 0$ and $\omega \to \infty$, it is sufficient to show that

$$0 < \left(\lim_{\omega\to\infty} \frac{\partial K}{\partial \beta} \right) |_{\beta=0}.$$

Because of the simpler algebra, I will show instead that

$$0 < \left(\lim_{\omega\to\infty} \frac{\partial K^2}{\partial \beta} \right) |_{\beta=0}.$$

Let us define first the constants

$$E \equiv \frac{a_z b_\theta + (a_z + b_z)\,\tilde{\psi}\lambda}{\lambda\tilde{\psi} - a_z a_\theta}, H = \frac{a_z c_\theta + c_z \lambda\tilde{\psi}}{\lambda\tilde{\psi} - a_z a_\theta}.$$

Then

$$\begin{aligned}
\frac{\partial K^2}{\partial \beta} &= \frac{\partial \left(\frac{\lambda\psi}{\lambda\psi - |a_z| a_\theta}\right)^2 \frac{\sigma^2_{z|x_i,y,q}}{\sigma^2_{z-Eq-Hy}}}{\partial \beta} = \\
&= \frac{\frac{\partial \sigma^2_{z|x_i,y,q}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\partial\beta}\left(\sigma^2_{z-Eq-Hy}\right) - \frac{\partial\sigma^2_{z-Eq-Hy}}{\partial\beta}\frac{\sigma^2_{z|x_i,y,q}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\sigma^2_{z-dq-ey}}}{\left(\sigma^2_{z-Eq-Hy}\right)^2}.
\end{aligned}$$

Note that all constants, $c_\theta, a_\theta, a_z, c_z, b_z, b_\theta, \sigma^2_{z|x_i,y,q}$ are continous in ω and β and continuously differentiable in β if β is sufficiently small and ω is sufficiently large and

$$\begin{aligned}
\lim_{\omega\to\infty} c_\theta &= \frac{\beta\left(2\kappa + \psi^2\delta + \upsilon\right)}{\kappa^2 + 2\kappa\beta + \kappa\psi^2\delta + \kappa\upsilon + \beta\psi^2\delta + \beta\upsilon + \psi^2\delta\upsilon} \\
\lim_{\omega\to\infty} c_z &= \frac{\beta\left(\alpha + \kappa + \psi^2\delta\right)}{\kappa^2 + \alpha\kappa + \alpha\beta + 2\kappa\beta + \kappa\psi^2\delta + \beta\psi^2\delta} \\
\lim_{\omega\to\infty} a_\theta &= \frac{\upsilon\left(\kappa + \psi^2\delta\right)}{\kappa^2 + 2\kappa\beta + \kappa\upsilon + \kappa\psi^2\delta + \beta\upsilon + \beta\psi^2\delta + \upsilon\psi^2\delta} \\
\lim_{\omega\to\infty} a_z &= -\alpha\frac{\beta}{\kappa^2 + \alpha\kappa + \alpha\beta + 2\kappa\beta + \kappa\psi^2\delta + \beta\psi^2\delta} \\
\lim_{\omega\to\infty} b_\theta &= \psi^2\delta\frac{\kappa + \upsilon}{\kappa^2 + 2\kappa\beta + \kappa\psi^2\delta + \kappa\upsilon + \beta\psi^2\delta + \beta\upsilon + \psi^2\delta\upsilon} \\
\lim_{\omega\to\infty} b_z &= -\beta\frac{\psi^2\delta}{\kappa^2 + \alpha\kappa + \alpha\beta + 2\kappa\beta + \kappa\psi^2\delta + \beta\psi^2\delta} \\
\lim_{\omega\to\infty} \sigma^2_{z|x_i,y,q} &= \frac{\kappa^2 + \alpha\kappa + \alpha\beta + 2\kappa\beta + \alpha\nu + \kappa\psi^2\delta + \kappa\nu + \beta\psi^2\delta + \beta\nu + \delta\nu}{\nu\left(\kappa^2 + \alpha\kappa + \alpha\beta + 2\kappa\beta + \kappa\psi^2\delta + \beta\psi^2\delta\right)}.
\end{aligned}$$

Thus,

$$\lim_{\omega\to\infty} E = \lim_{\omega\to\infty} \frac{\frac{b_\theta}{a_\theta} + \frac{(a_z+b_z)}{a_z a_\theta}\tilde{\psi}\lambda}{\frac{\lambda\tilde{\psi}}{a_z a_\theta} - 1} = \frac{\frac{\psi^2\delta(\kappa+\upsilon)}{\left(\kappa+\psi^2\delta\right)\upsilon} + \frac{\left(\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta\right)}{\upsilon\left(\kappa+\psi^2\delta\right)}\frac{2\alpha+\psi^2\delta}{\alpha}\psi\lambda}{\frac{\lambda\psi}{\frac{\upsilon\left(\kappa+\psi^2\delta\right)}{\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta}\alpha\frac{\beta}{\kappa^2+\alpha\kappa+\alpha\beta+2\kappa\beta+\kappa\psi^2\delta+\beta\psi^2\delta}} - 1}$$

where the right hand side is proportional to β, so

$$\left(\lim_{\omega\to\infty}\frac{\partial\left(E^2\right)}{\partial\beta}\right)|_{\beta=0} = \left(\frac{\partial\left(\lim_{\omega\to\infty}E^2\right)}{\partial\beta}\right)|_{\beta=0} = 0.$$

Similarly,

$$\lim_{\omega\to\infty} H = \lim_{\omega\to\infty} \frac{\frac{c_\theta}{a_\theta} + \frac{c_z}{a_z}\frac{1}{a_\theta}\lambda\tilde{\psi}}{\frac{\lambda\tilde{\psi}}{a_z a_\theta} - 1} = \frac{\frac{\beta\left(2\kappa+\psi^2\delta+\upsilon\right)}{\left(\kappa+\psi^2\delta\right)\upsilon} + \frac{\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta}{\upsilon\left(\kappa+\psi^2\delta\right)}\frac{\alpha+\kappa+\psi^2\delta}{\alpha}\lambda\psi}{\frac{\lambda\psi}{\frac{\upsilon\left(\kappa+\psi^2\delta\right)}{\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta}\alpha\frac{\beta}{\kappa^2+\alpha\kappa+\alpha\beta+2\kappa\beta+\kappa\psi^2\delta+\beta\psi^2\delta}} - 1}$$

so

$$\left(\lim_{\omega\to\infty}\frac{\partial\left(H^2\right)}{\partial\beta}\right)|_{\beta=0} = \left(\frac{\partial\left(\lim_{\omega\to\infty}H^2\right)}{\partial\beta}\right)|_{\beta=0} = \left(\frac{\partial(\lim_{\omega\to\infty}EH)}{\partial\beta}\right)|_{\beta=0} = 0.$$

Consequently,

$$\begin{aligned}
&\left(\lim_{\omega\to\infty}\frac{\partial\sigma^2_{z-Eq-Hy}}{\partial\beta}\right)|_{\beta=0} = \\
&= \left(\lim_{\omega\to\infty}\frac{\partial\left(var(z)+E^2var(q)+H^2var(y)+2(EHcov(q,y)-Hcov(z,y)-Ecov(q,z))\right)}{\partial\beta}\right)|_{\beta=0} = \\
&= -2\lim_{\omega\to\infty}cov\left(z,y\right)\left(\lim_{\omega\to\infty}\frac{\partial H}{\partial\beta}\right)|_{\beta=0} - 2\lim_{\omega\to\infty}cov\left(q,z\right)\left(\lim_{\omega\to\infty}\frac{\partial E}{\partial\beta}\right)|_{\beta=0} \\
&= -2\frac{1}{\kappa}\left(\frac{\partial\lim_{\omega\to\infty}H}{\partial\beta}\right) = -2\frac{1}{\kappa}\left(\frac{\partial\lim_{\omega\to\infty}H}{\partial\beta}\right) = \\
&= -2\frac{1}{\kappa^2}.
\end{aligned}$$

where I used

$$\left(\frac{\partial\lim_{\omega\to\infty}H}{\partial\beta}\right)|_{\beta=0} = \lim_{\beta\to0}\frac{\partial\left(\frac{\frac{\beta\left(2\kappa+\psi^2\delta+\upsilon\right)}{\left(\kappa+\psi^2\delta\right)\upsilon} + \frac{\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta}{\upsilon\left(\kappa+\psi^2\delta\right)}\frac{\alpha+\kappa+\psi^2\delta}{\alpha}\lambda\psi}{\frac{\lambda\psi}{\frac{\upsilon\left(\kappa+\psi^2\delta\right)}{\kappa^2+2\kappa\beta+\kappa\upsilon+\kappa\psi^2\delta+\beta\upsilon+\beta\psi^2\delta+\upsilon\psi^2\delta}\alpha\frac{\beta}{\kappa^2+\alpha\kappa+\alpha\beta+2\kappa\beta+\kappa\psi^2\delta+\beta\psi^2\delta}}-1}\right)}{\partial\beta} = \frac{1}{\kappa}.$$

Also,

$$\lim_{\omega\to\infty}\frac{\partial\sigma^2_{z|x_i,y,q}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\partial\beta}|_{\beta=0}=$$

$$=\left(\lim_{\omega\to\infty}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|\,a_\theta}\right)^2\frac{\partial\lim_{\omega\to\infty}\sigma^2_{z|x_i,y,q}}{\partial\beta}+\lim_{\omega\to\infty}\sigma^2_{z|x_i,y,q}\frac{\partial\lim_{\omega\to\infty}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\partial\beta}\right)|_{\beta=0}=$$

$$=-\frac{1}{\kappa^2}+2\alpha\delta\frac{\psi}{\kappa^2\lambda\left(\delta^2\psi^4+\alpha\kappa+2\alpha\delta\psi^2+\kappa\delta\psi^2\right)}$$

Thus,

$$\lim_{\omega\to\infty}\frac{\partial K^2}{\partial\beta}|_{\beta=0}=\lim_{\omega\to\infty}\frac{\frac{\partial\sigma^2_{z|x_i,y,q}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\partial\beta}\left(\sigma^2_{z-Eq-Hy}\right)-\frac{\partial\sigma^2_{z-Eq-Hy}}{\partial\beta}\frac{\sigma^2_{z|x_i,y,q}\left(\frac{\lambda\psi}{\lambda\psi-|a_z|a_\theta}\right)^2}{\sigma^2_{z-dq-ey}}}{\left(\sigma^2_{z-Eq-Hy}\right)^2}|_{\beta=0}=$$

$$=\frac{-\frac{1}{\kappa^2}+2\alpha\delta\frac{\psi}{\kappa^2\lambda\left(\delta^2\psi^4+\alpha\kappa+2\alpha\delta\psi^2+\kappa\delta\psi^2\right)}+2\frac{1}{\kappa^2}}{\left(\frac{1}{\kappa}+\frac{1}{\upsilon}\right)^2}>0.$$

■

Proof of Proposition 5. Note that under (13), devaluation for any z_i is not a best response if each speculator attacks for any x_i as

$$\lim_{z_i\to\infty}E\left(\theta|z_i,y,\Phi^{-1}(\xi)+u\right)>\lambda\left(\Phi^{-1}(\xi)+u\right)$$

for any realized u. Similarly, keeping the peg for any z_i is not a best response if none of the speculator attack for any x_i Thus, all monotone equilibria must be finite monotone equilibria.

Following the derivation of the equilibrium in subsection 3, a monotone equilibrium of the perturbed system must satisfy equations

$$\begin{aligned}\lambda\tilde{C}&=E\left(\theta|z=z^*,\tilde{C},y\right) &(41)\\ \kappa&=\Pr\left(z<z^*|x_i=x^*,\tilde{C},y\right). &(42)\end{aligned}$$

Note that

$$E\left(\theta|z_i,\tilde{C},y\right)=$$

$$=E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}(\xi)>u>\tilde{C}-\Phi^{-1}(1-\xi)\right)\Pr\left(\tilde{C}-\Phi^{-1}(\xi)>u>\tilde{C}-\Phi^{-1}(1-\xi)\,|z_i,\tilde{C},y\right)+$$

$$+E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}(\xi)<u\right)\Pr\left(\tilde{C}-\Phi^{-1}(\xi)<u|z_i,\tilde{C},y\right)+$$

$$+E\left(\theta|z_i,\tilde{C},y,\tilde{C}-\Phi^{-1}(1-\xi)>u\right)\Pr\left(\tilde{C}-\Phi^{-1}(1-\xi)>u|z_i,\tilde{C},y\right)$$

and

$$\Pr\left(z < z^*|x_i, \tilde{C}, y\right) =$$
$$= \Pr\left(z < z^*|x_i, \tilde{C}, y, \tilde{C} - \Phi^{-1}(\xi) > u > \tilde{C} - \Phi^{-1}(1-\xi)\right)$$
$$\cdot \Pr\left(\tilde{C} - \Phi^{-1}(\xi) > u > \tilde{C} - \Phi^{-1}(1-\xi)|x_i, \tilde{C}, y\right) +$$
$$+ \Pr\left(z < z^*|x_i, \tilde{C}, y, \tilde{C} - \Phi^{-1}(\xi) < u\right) \Pr\left(\tilde{C} - \Phi^{-1}(\xi) < u|x_i, \tilde{C}, y\right) +$$
$$+ \Pr\left(z < z^*|x, \tilde{C}, y, \tilde{C} - \Phi^{-1}(1-\xi) > u\right) \Pr\left(\tilde{C} - \Phi^{-1}(1-\xi) > u|x_i, \tilde{C}, y\right).$$

As

$$\begin{aligned} \lim_{\xi\to 0} \Pr\left(\tilde{C} - \Phi^{-1}(\xi) < u|z_i, \tilde{C}, y\right) &= \lim_{\xi\to 0} \Pr\left(\tilde{C} - \Phi^{-1}(\xi) < u|x_i, \tilde{C}, y\right) = 0 \\ \lim_{\xi\to 0} \Pr\left(\tilde{C} - \Phi^{-1}(1-\xi) > u|z_i, \tilde{C}, y\right) &= \lim_{\xi\to 0} \Pr\left(\tilde{C} - \Phi^{-1}(1-\xi) > u|x_i, \tilde{C}, y\right) = 0 \end{aligned}$$

and

$$\begin{aligned} \lim_{\xi\to 0} E\left(\theta|z_i, \tilde{C}, y, \tilde{C} - \Phi^{-1}(\xi) > u > \tilde{C} - \Phi^{-1}(1-\xi)\right) &= E(\theta|z_i, q, y) \\ \lim_{\xi\to 0} \Pr\left(z < z^*|x_i, \tilde{C}, y, \tilde{C} - \Phi^{-1}(\xi) > u > \tilde{C} - \Phi^{-1}(1-\xi)\right) &= \Pr(z < z^*|x_i, q, y), \end{aligned}$$

if $\xi \to 0$, equations (42) and (41) converges to (8) and (6) respectively. Thus, for a sufficiently small ξ, the best response functions defined by (42) and (41) are arbitrary close to the straight lines in the (x^*, z^*) space which are given by (9) and (10). This proves the statement. ■

B Appendix: Application II

B.1 Demand in period 0

In period zero traders maximize the expected utility

$$E\left(-\exp\left(-\gamma(p_1 - p_0)d_{i,0} - \frac{E(p_2|q_1, y, x_i, q_0) - p_1}{var(p_2|q_1, y, x_i, q_0)}(p_2 - p_1)\right)|x_i, q_0\right) =$$
$$= E\left(E\left(-\exp\left(-\gamma(p_1 - p_0)d_{i,0} - \frac{E(p_2|q_1, y, x_i, q_0) - p_1}{var(p_2|q_1, y, x_i, q_0)}(p_2 - p_1)\right)|q_0, y, x_i, q_1\right)|x_i, q_0\right) =$$
$$= E\left(-\exp\left(-\gamma(p_1 - p_0)d_{i,0} - \frac{(E(p_2|q_1, y, x_i, q_0) - p_1)^2}{2var(p_2|q_1, y, x_j, q_0)}\right)|x_i, q_0\right) =$$
$$= E\left(-\exp\left(-\gamma(a_1q_1 + c_1y + h_1q_0 - a_0q_0)d_{i,0} - \frac{1}{2}\frac{a_s^2}{var(\theta_s + \theta_w|x_i, y, q_1, q_0)}(x_i - q_1)^2\right)|x_i, q_0\right)$$

where I used (28)-(31) for the last equality.

Writing the expression in the inner bracket into matrix form and using the standard result for the expectation of exponentials with quadratic forms[15] implies that

$$E\left(-\exp\left(-\gamma\left(p_1-p_0\right)d_{i,0}-\frac{\left(E\left(p_2|q_1,y,x_i,q_0\right)-p_1\right)^2}{2var\left(p_2|q_1,y,x_j,q_0\right)}\right)|x_i,q_0\right)=$$
$$=E\left(-\exp\left(-\gamma\left(a_1q_1+c_1y+h_1q_0-a_0q_0\right)d_{i,0}-\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(x_i-q_1\right)^2\right)|x_i,q_0\right)$$

wich equals to

$$E\left(-\exp\left(\begin{array}{c}-\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i^2-\gamma\left(h_1-a_0\right)q_0d_{i,0}\\ +\left(\begin{array}{cc}-\gamma c_1d_{i,0} & -\gamma a_1d_{i,0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i\end{array}\right)\left(\begin{array}{c}y\\ q_1\end{array}\right)\\ -\left(\begin{array}{cc}y & q_1\end{array}\right)\left(\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\right)\left(\begin{array}{cc}0&0\\0&1\end{array}\right)\left(\begin{array}{c}y\\ q_1\end{array}\right)\end{array}\right)|x_i,q_0\right).$$

The last expression is a monotonic function of

$$\begin{array}{c}-\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i^2-\gamma\left(h_1-a_0\right)q_0d_{i,0}+\\ +\left(\begin{array}{cc}-\gamma c_1d_{i,0} & \left(-\gamma a_1d_{i,0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i\right)\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right)+\\ \frac{1}{2}\left(\left(\begin{array}{cc}-\gamma c_1d_{i,0} & \left(-\gamma a_1d_{i,0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i\right)\end{array}\right)-2\left(\begin{array}{cc}\mu_y&\mu_q\end{array}\right)\left(\begin{array}{cc}0&0\\0&\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\gamma\end{array}\right)\right)\\ \left(\begin{array}{c}\left(\begin{array}{cc}0&0\\0&\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\end{array}\right)\\ +\left(\begin{array}{cc}\sigma^2_{y|x_i,q_0}&\sigma_{yq|x_i,q_0}\\ \sigma_{yq|x_i,q_0}&\sigma^2_{q|x_i,q_0}\end{array}\right)^{-1}\end{array}\right)^{-1}\left(\begin{array}{c}\left(\begin{array}{c}-\gamma c_1d_{i,0}\\ -\gamma a_1d_{i,0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}x_i\end{array}\right)\\ -2\left(\begin{array}{cc}0&0\\0&\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right)\end{array}\right)\\ -\left(\begin{array}{cc}\mu_y&\mu_q\end{array}\right)\left(\begin{array}{cc}0&0\\0&\frac{1}{2}\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\end{array}\right)\left(\begin{array}{c}\mu_y\\ \mu_q\end{array}\right).\end{array}$$

[15] If c is constant scalar, L is a $nx1$ constant vector, N is an nxn constant matrix and M is an $nx1$ stochastic matrix and I is an information set, then

$$E\left(-\exp\left(c+L'M-M'NM'\right)|I\right)=$$
$$-|W|^{-1/2}\left|2N+W^{-1}\right|^{-1/2}\exp\left(c+L'Q-Q'NQ+\frac{1}{2}\left(L'-2Q'N\right)\right)\left(2N+W^{-1}\right)^{-1}\left(L-2NQ\right)\Big)$$

where $Q=E\left(M|I\right)$ and $W=var\left(M|I\right)$ (see Brunnermeier, 2001, page 110).

where

$$\begin{aligned}\mu_y &= E\left(y|x_i,q_0\right)\\ \mu_q &= E\left(q_1|x_i,q_0\right).\end{aligned}$$

Maximizing the expression in the bracket gives

$$\begin{aligned}d_{i,0} &= \frac{\left((h_1-a_0)q_0+c_1\mu_y+a_1\mu_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)\left(x_i-\mu_q\right)}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}=\\ &= \frac{\left(h_1-a_0+c_1b_y+a_1b_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)-\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)b_q}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}q_0+\\ &\quad+\frac{\left(c_1a_y+a_1a_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)(1-a_q)}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}x_i.\end{aligned}$$

where I used or the definition

$$\begin{aligned}E\left(y|x_i,q_0\right) &= a_yx_i+b_yq_0\\ E\left(q_1|x_i,q_0\right) &= a_qx_i+b_qq_0.\end{aligned}$$

using the market clearing condition for period 1, substituting in $q_0=\theta_k+\theta_w-\frac{\delta_0}{\tau_0}u_0$ and collecting the coefficients of $\theta_k+\theta_w$ and u_0 implies that in equilibrium

$$\begin{aligned}&\frac{\left(h_1-a_0+c_1b_y+a_1b_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)-\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)b_q}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}=\\ &\quad=\frac{\left(c_1a_y+a_1a_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)(1-a_q)}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}\end{aligned}$$

$$\frac{\left(c_1a_y+a_1a_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy}+\sigma_q^2a_1\right)(1-a_q)}{\gamma\left(\sigma^2_{q|x_i,q_0}a_1^2+\sigma^2_{y|x_i,q_0}c_1^2+2a_1c_1\sigma_{qy|x_i,q_0}+\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)\right)}=\frac{\tau_0}{\delta_0} \tag{43}$$

must hold. Thus, in equilibrium

$$d_{i,0}=\frac{\tau_0}{\delta_0}\left(x_i-q_0\right)$$

and, using (29),

$$
\begin{aligned}
&F_0\left(\tau_0,\tau_1,\tau_2\right)\equiv\\
&\frac{\left(\left(a_\theta+b_\theta\right)\left(c_s a_y+\left(a_s+b_s\right)a_q\right)+c_\theta a_y+h_\theta a_q\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2+1\right)+\frac{a_s^2\left(\left(a_\theta+b_\theta\right)\left(c_s\sigma_{qy}+\sigma_q^2\left(a_s+b_s\right)\right)+c_\theta\sigma_{qy}+\sigma_q^2 h_\theta\right)\left(1-a_q\right)}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}}{\gamma\left(\begin{array}{c}\sigma^2_{q|x_i,q_0}\left(\left(a_\theta+b_\theta\right)\left(a_s+b_s\right)+h_\theta\right)^2+2\left(\left(a_\theta+b_\theta\right)\left(a_s+b_s\right)+h_\theta\right)\left(\left(a_\theta+b_\theta\right)c_s+c_\theta\right)\sigma_{qy|x_i,q_0}+\\ +\left(\left(a_\theta+b_\theta\right)c_s+c_\theta\right)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}-\sigma^2_{qy|x_i,q_0}\right)+\sigma^2_{y|x_i,q_0}\right)\end{array}\right)}.
\end{aligned}
$$

B.2 Expectations, variances and coefficients in the price-functions

The following coefficients are needed to analyze the equilibrium and all are obtained by the projection theorem

$$
\begin{aligned}
a_\theta &= \frac{\alpha\left(\kappa+\tau_0^2+\tau_1^2\right)\left(\kappa+\omega\right)}{B}\\
b_\theta &= \frac{\tau_2^2\left(\kappa+\tau_0^2+\tau_1^2\right)\left(\kappa+\omega\right)}{B}\\
c_\theta &= \frac{\beta\left(\kappa^2+\tau_0^2\tau_2^2+\tau_1^2\tau_2^2+\alpha\kappa+\alpha\omega+2\kappa\omega+\alpha\tau_0^2+\kappa\tau_0^2+\alpha\tau_1^2+\kappa\tau_1^2+\kappa\tau_2^2+\omega\tau_0^2+\omega\tau_1^2+\omega\tau_2^2\right)}{B}\\
h_\theta &= \frac{\tau_1^2\left(\kappa+\omega\right)\left(\alpha+\kappa+\tau_2^2\right)}{B}\\
g_\theta &= \frac{\tau_1^2\left(\kappa+\omega\right)\left(\alpha+\kappa+\tau_2^2\right)}{B}\\
var\left(\theta|z_j,q_2,q_1,q_0,y\right) &= \frac{\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa}{B}
\end{aligned}
$$

where

$$
\begin{aligned}
B \;=\;& \omega\left(\kappa^2-\tau_0^2\tau_2^2-\tau_1^2\tau_2^2-\alpha\tau_0^2-\alpha\tau_1^2\right)+\\
&+\left(\kappa+\omega\right)\left(\left(\kappa+\tau_0^2+\tau_1^2\right)\left(\alpha+\tau_2^2\right)+\left(\tau_0^2+\tau_1^2\right)\left(\alpha+\kappa+\tau_2^2\right)\right)+\\
&+\beta\left(\kappa^2+\tau_0^2\tau_2^2+\tau_1^2\tau_2^2+\alpha\kappa+\alpha\omega+2\kappa\omega+\alpha\tau_0^2+\kappa\tau_0^2+\alpha\tau_1^2+\kappa\tau_1^2+\kappa\tau_2^2+\omega\tau_0^2+\omega\tau_1^2+\omega\tau_2^2\right)
\end{aligned}
$$

and

$$
\begin{aligned}
a_s &= \frac{\alpha\left(\kappa^2-\beta\omega\right)}{\kappa^2\tau_0^2+\kappa^2\tau_1^2+\alpha\kappa^2+\kappa^2\beta+\kappa^2\omega+\kappa\beta\tau_0^2+\kappa\beta\tau_1^2+\kappa\omega\tau_0^2+\kappa\omega\tau_1^2+\beta\omega\tau_0^2+\beta\omega\tau_1^2+\alpha\kappa\beta+\alpha\kappa\omega+\alpha\beta\omega+2\kappa\beta\omega}\\
b_s &= \frac{\tau_1^2\left(\kappa^2-\beta\omega\right)}{\kappa^2\tau_0^2+\kappa^2\tau_1^2+\alpha\kappa^2+\kappa^2\beta+\kappa^2\omega+\kappa\beta\tau_0^2+\kappa\beta\tau_1^2+\kappa\omega\tau_0^2+\kappa\omega\tau_1^2+\beta\omega\tau_0^2+\beta\omega\tau_1^2+\alpha\kappa\beta+\alpha\kappa\omega+\alpha\beta\omega+2\kappa\beta\omega}\\
c_s &= \frac{\beta\left(\kappa+\omega\right)\left(\alpha+\kappa+\tau_0^2+\tau_1^2\right)}{\kappa^2\tau_0^2+\kappa^2\tau_1^2+\alpha\kappa^2+\kappa^2\beta+\kappa^2\omega+\kappa\beta\tau_0^2+\kappa\beta\tau_1^2+\kappa\omega\tau_0^2+\kappa\omega\tau_1^2+\beta\omega\tau_0^2+\beta\omega\tau_1^2+\alpha\kappa\beta+\alpha\kappa\omega+\alpha\beta\omega+2\kappa\beta\omega}\\
h_s &= \frac{\tau_0^2\left(\kappa^2-\beta\omega\right)}{\kappa^2\tau_0^2+\kappa^2\tau_1^2+\alpha\kappa^2+\kappa^2\beta+\kappa^2\omega+\kappa\beta\tau_0^2+\kappa\beta\tau_1^2+\kappa\omega\tau_0^2+\kappa\omega\tau_1^2+\beta\omega\tau_0^2+\beta\omega\tau_1^2+\alpha\kappa\beta+\alpha\kappa\omega+\alpha\beta\omega+2\kappa\beta\omega}
\end{aligned}
$$

$$\begin{aligned} a_y &= a_q = \frac{\alpha(\kappa+\omega)}{\alpha\kappa+\alpha\omega+\kappa\omega+\kappa\tau_1^2+\omega\tau_0^2} \\ b_y &= b_q = \frac{(\kappa+\omega)\tau_0^2}{\alpha\kappa+\alpha\omega+\kappa\omega+\kappa\tau_0^2+\omega\tau_0^2}. \end{aligned}$$

Furthermore,

$$\sigma^2_{\theta_s+\theta_w|x_i,q_1,q_0,y} = \frac{\tau_1^2\omega+2\tau_1^2\kappa+\omega\beta+\kappa^2+\kappa\omega+\alpha\omega+2\alpha\kappa+\tau_1^2\beta+\alpha\beta+2\tau_0^2\kappa+\tau_0^2\omega+\beta\kappa+\tau_0^2\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$$

$$\begin{aligned} \sigma^2_{y|x_i,q_0} &= \frac{2\kappa\omega\beta+\kappa^2\beta+\beta\alpha\omega+\beta\alpha\kappa+\beta\tau_0^2\omega+\beta\tau_0^2\kappa+\kappa\alpha\omega+\kappa^2\alpha+\kappa\tau_0^2\omega+\kappa^2\tau_0^2+\kappa^2\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\kappa\beta} \\ \sigma^2_{q|x_i,q_0} &= \frac{\tau_1^2\omega+\tau_1^2\kappa+\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\tau_1^2} \\ \sigma^2_{yq|x_i,q_0} &= \frac{\omega+\kappa}{\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega}. \end{aligned}$$

B.3 Proof of existence

I show the existence in three steps.

Lemma 1 *Let us fix $\tau_0 = \bar{\tau}_0$ at any arbitrary level. The system $\tau_2 = F_2(\tau_2, \tau_1, \tau_0)$, $\tau_1 = F_1(\tau_2, \tau_1, \tau_0)$ has at least one fix point, $(\tau_1^*(\bar{\tau}_0), \tau_2^*(\bar{\tau}_0))$. Additionally, $\tau_2^{\min} \leq \tau_2^* < \delta_2 \frac{1}{a}\alpha$*

where $\tau_2^{\min}$ is the single root of

$$\delta_2 \frac{1}{a}\alpha \frac{\kappa\omega+\kappa^2}{\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2} = \tau_2$$

and $\frac{(\kappa^2-\omega\beta)}{a2\kappa} < (>)\tau_1^* \leq (\geq)0$ if and only if $\kappa^2 < (\geq)\omega\beta$. Furthermore, the functions $\tau_1^*(\tau_0)$ and $\tau_2^*(\tau_0)$ are continuous.

Proof. Substituting out the coefficients of conditional expectations and variances in (27) and (32) give

$$\begin{aligned} \tau_2 &= F_2\left(\tau_2^2, \tau_1^2, \tau_0^2\right) = \\ &= \delta_2 \frac{1}{\gamma}\alpha \frac{\tau_0^2\omega+\tau_1^2\omega+\kappa\omega+\tau_0^2\kappa+\tau_1^2\kappa+\kappa^2}{\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa} \\ \tau_1 &= F_1\left(\tau_2^2, \tau_1^2, \tau_0^2\right) = \frac{\left(\kappa^2-\omega\beta\right)\delta_1\tau_2^2\alpha}{\gamma\left(\kappa\tau_0^2\tau_2^2+\kappa\omega\tau_2^2+\tau_2^2\kappa^2+\omega\tau_1^2\tau_2^2+2\alpha\omega\tau_2^2+\kappa\tau_1^2\tau_2^2+2\kappa\alpha\tau_2^2+\tau_0^2\omega\tau_2^2+\alpha\kappa^2+\kappa\omega\alpha\right)} \end{aligned}$$

Notice first, that $\tau_1 = F_1\left(\tau_2^2, \tau_1^2, \bar{\tau}_0^2\right)$ is a third degree polynom in τ_1, which is monotone increasing so it gives a single root for every τ_0 and τ_2. Similarly,$\tau_2 = F_2\left(\tau_2^2, \tau_1^2, \bar{\tau}_0^2\right)$ is also a monotone increasing third degree polynom in τ_2 which gives a single root for every τ_0 and τ_1. It is also apparent that $\tau_1^*(\bar{\tau}_0)$ and $\tau_2^*(\bar{\tau}_0)$ are continuos in τ_0.

For the existence of the fix point $(\tau_1^*(\bar{\tau}_0), \tau_2^*(\bar{\tau}_0))$, it is sufficient to show that there is a solution of

$$\tau_2 = F_2(\tau_2, \tau_1(\tau_2, \bar{\tau}_0), \bar{\tau}_0) = \frac{1}{\gamma}\alpha\delta_2 F_2^{\bar{\tau}_0}(\tau_2) =$$

where $F_2^{\bar{\tau}_0}(\cdot)$ continuously maps τ_2 to the unite interval. As

$$\lim_{\tau_2 \to 0} \frac{1}{\gamma}\alpha\delta_2 F_2^{\bar{\tau}_0}(\tau_2) = \delta_2 \frac{1}{\gamma}\alpha \frac{\kappa\omega + \kappa^2}{\tau_1^2(0,\bar{\tau}_0)\kappa + \kappa^2 + \tau_1^2(0,\bar{\tau}_0)\omega + \tau_0^2\kappa + \alpha\kappa + \tau_0^2\omega + \alpha\omega + 2\kappa\omega} > 0,$$

where $\tau_1^2(0, \bar{\tau}_0)$ is finite and

$$0 < \frac{1}{\gamma}\alpha\delta_2 F_2^{\bar{\tau}_0}(\tau_2) \leq \delta_2 \frac{1}{a}\alpha,$$

there has to be a fixed point. The rest of the lemma comes from observation. ■

Lemma 2 *The denominator of $F_0(\tau_1^2, \tau_2^2, \tau_0^2)$ is positive.*

Proof. Note that the matrix

$$Q = \left(\left(\begin{array}{cc} 0 & 0 \\ 0 & \frac{a_s^2}{\sigma^2_{\theta_s + \theta_w | x_i, y, q_1, q_0}} \end{array} \right) + \left(\begin{array}{cc} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{array} \right)^{-1} \right)^{-1}$$

is positive definite as $s > 0$. Consequently

$$0 < xQx^T$$

for all x. The lemma comes from the choice of

$$x = \left(\left(\begin{array}{cc} -ac_1 d_{i,1} & (-ab_1 d_{i,1} + sx_i) \end{array} \right) - 2 \left(\begin{array}{cc} \mu_y & \mu_q \end{array} \right) \left(\begin{array}{cc} 0 & 0 \\ 0 & \frac{1}{2}s \end{array} \right) \right)$$

as then

$$0 < xQx^T = \left(\begin{array}{c} \frac{a^2\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}{\sigma_q s + 1} d_1^2 - \\ -\frac{2ac_1\sigma_{yq}sx_i - 2\sigma_q ab_1\mu_q s - 2ac_1\sigma_{yq}\mu_q s + 2\sigma_q ab_1 sx_i}{\sigma_q s + 1} d_{i,1} \\ + \frac{s^2\sigma_q\left(x_i - \mu_q\right)^2}{\sigma_q s + 1} \end{array} \right)$$

which implies the lemma. ■

Proposition 9 *An equilibrium exists.*

Proof. Let us define $\bar{F}_0(\tau_0) \equiv F_0(\tau_2^*(\tau_0), \tau_1^*(\tau_0), \tau_0)$. I show that the expression $\tau_0 = \bar{F}_0(\tau_0)$ has at least one fix point. First, I show that $\lim_{\tau_0 \to \infty} \bar{F}_0(\tau_0) = 0$. I use the form (43). As $\tau_0 \to \infty$, τ_2^* goes to a constant, τ_1^* goes to 0 by the order of $\frac{1}{\tau_0^2}$, hence $c_1, \sigma^2_{y|x_i,q_0}, \frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,q_1,q_0,y}}$ converge to constants, $\sigma_{yq|x_i,q_0}$ and a_y, a_q goes to 0 by the order of $\frac{1}{\tau_0^2}$, $\sigma^2_{q|x_i,q_0}$ goes to infinity by the order of τ_0^4 and a_1 goes to 0 by the order of $\frac{1}{\tau_0^2}$. So the numerator of $\bar{F}_0(\tau_0)$,

$$(c_1 a_y + a_1 a_q)\left(\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\sigma_q^2 + 1\right) + \frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}\left(c_1\sigma_{qy} + \sigma_q^2 a_1\right)(1 - a_q)$$

goes to infinity by the order of τ_0^2 . The denominator,

$$\left(\sigma^2_{q|x_i,q_0}a_1^2 + \sigma^2_{y|x_i,q_0}c_1^2 + 2a_1c_1\sigma_{qy|x_i,q_0} + \frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\left(\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0} - \sigma^2_{qy|x_i,q_0}\right)\right)$$

also goes to infinity but by the order of τ_0^4 given by the term of $\frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,y,q_1,q_0}}c_1^2\sigma^2_{q|x_i,q_0}\sigma^2_{y|x_i,q_0}$. Hence, $\lim_{\tau_0 \to \infty} g_0(\tau_0) = 0$. Second, by Lemma 2 , the function$\bar{F}_0(\tau_0)$ is continuous in τ_0.and $\bar{F}_0(\tau_0) = \bar{F}_0(-\tau_0)$, as τ_0 enters in the form of τ_0^2. Hence, $F_0'(\tau_0)$ will cross the 45° line necessarily, because it is continuous and goes to 0 as $\tau_0 \to \infty$ or $\tau_0 \to -\infty$. Therefore, there will be a fixed point with $\tau_0^* \geq (<) 0$ if $F_0'(0) \geq (<) 0$. ■

B.4 Other proofs

Proof of Proposition 7. For $\frac{\tau_2}{\delta_2}, \frac{\tau_1}{\delta_1}, \frac{\hat{\tau}_2}{\delta_2}$ and $\frac{\hat{\tau}_1}{\delta_1}$, the result comes from the simple observation that the ordered limit

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f_2(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{\gamma}$$

and, after substitution of $\tau_2 = \frac{\alpha}{\gamma}$,

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f_1(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{\gamma}.$$

The result for $\frac{\tau_0}{\delta_0}$ and $\frac{\hat{\tau}_0}{\delta_0}$ can be obtained in a similar way, if we take the limit of $F_0(\tau_2, \tau_1, \tau_0)$.
■

Proof of Proposition 8. The second half of the statement implies the first half as both the aggregate holdings and the volume are continuous functions of ω. For the second half of the statement, it is sufficient to show that $\lim_{\omega \to \infty}\left|\frac{\hat{\tau}_0}{\delta_0} - \frac{\hat{\tau}_1}{\delta_1}\right| = \lim_{\omega \to \infty}\left|\frac{\hat{\tau}_0}{\delta_0}\right| = \lim_{\omega \to \infty}\left|\frac{\hat{\tau}_1}{\delta_1}\right| \to 0$ while $\left|\frac{\tau_0}{\delta_0} - \frac{\tau_1}{\delta_1}\right| \to C_1, \left|\frac{\tau_1}{\delta_1}\right| \to C_2$ and $\left|\frac{\tau_0}{\delta_0}\right| \to C_3$, where C_1, C_2 and C_3 are non zero constants.

In the no-announcement case, the equilibrium is characterized by the following equations:

$$\begin{aligned}
\hat{\tau}_2 &= F_2\left(\hat{\tau}_2, \hat{\tau}_1, \hat{\tau}_0\right)|_{\beta=0} \\
&= \frac{\delta_2 \frac{1}{a}\alpha\left(\hat{\tau}_0^2\omega+\hat{\tau}_1^2\omega+\kappa\omega+\hat{\tau}_0^2\kappa+\hat{\tau}_1^2\kappa+\kappa^2\right)}{\hat{\tau}_0^2\kappa+\hat{\tau}_0\hat{\tau}_2^2+\hat{\tau}_0^2\omega+\alpha\hat{\tau}_0^2+\alpha\hat{\tau}_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\hat{\tau}_2^2+\omega\hat{\tau}_2^2+\hat{\tau}_1^2\hat{\tau}_2^2+\hat{\tau}_1^2\omega+\hat{\tau}_1^2\kappa} \\
\hat{\tau}_1 &= F_1\left(\hat{\tau}_2, \hat{\tau}_1, \hat{\tau}_0\right)|_{\beta=0} = \\
&= \delta_1\hat{\tau}_2^2\alpha\frac{\kappa^2}{a\left(\kappa\hat{\tau}_0^2\hat{\tau}_2^2+\kappa\omega\hat{\tau}_2^2+\hat{\tau}_2^2\kappa^2+\omega\hat{\tau}_1^2\hat{\tau}_2^2+2\alpha\omega\hat{\tau}_2^2+\kappa\hat{\tau}_1^2\hat{\tau}_2^2+2\kappa\alpha\hat{\tau}_2^2+\hat{\tau}_0^2\omega\hat{\tau}_2^2+\alpha\kappa^2+\kappa\omega\alpha\right)}
\end{aligned}$$

$$\hat{\tau}_0 = F_0\left(\hat{\tau}_2, \hat{\tau}_1, \hat{\tau}_0\right)|_{\beta=0} = \delta_0\frac{a_y + \frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,q_1,q_0}}}{a_1\gamma\sigma^2_{q|x_i,q_0}}|_{\beta=0}.$$

It is apparent, that for any $\hat{\tau}_1, \hat{\tau}_0$, $\lim_{\omega\to\infty}\hat{\tau}_2$ is a finite, positive constant, and for any finite, positive $\hat{\tau}_2$ and any $\hat{\tau}_0$, $\lim_{\omega\to\infty}\hat{\tau}_1 = 0$. Consequently,

$$\lim_{\omega\to 0}\sigma^2_{q|x_i,q_0}|_{\beta=0} = \lim_{\tau_1\to 0}\sigma^2_{q|x_i,q_0}|_{\beta=0} = \infty.$$

Hence,

$$\lim_{\omega\to\infty}\delta_0\delta_0\frac{a_y + \frac{a_s^2}{\sigma^2_{\theta_s+\theta_w|x_i,q_1,q_0}}}{a_1\gamma\sigma^2_{q|x_i,q_0}}|_{\beta=0} = 0.$$

In the announcement case, our equilibrium determining equations converge to the following expressions as $\omega\to\infty$:

$$\begin{aligned}
\tau_2 &= \delta_2\frac{1}{\gamma}\alpha\frac{\left(\kappa+\tau_0^2+\tau_1^2\right)}{\left(\tau_0^2+2\kappa+\tau_2^2+\tau_1^2+\alpha\right)} \\
\tau_1 &= -\delta_1\tau_2^2\alpha\frac{\beta}{\gamma\left(\tau_1^2\tau_2^2+\kappa\tau_2^2+\tau_0^2\tau_2^2+2\alpha\tau_2^2+\kappa\alpha\right)} \\
\tau_0 &= F_0\left(\tau_0, \tau_1, \tau_2\right)|_{\omega\to\infty},
\end{aligned}$$

By the observation of the coefficients defining $F_0\left(\tau_0, \tau_1, \tau_2\right)$, it is apparent that all of them are converging to finite, non-zero constants as $\omega\to\infty$. Hence, there must be at least one equilibrium where all τ_2, τ_1, τ_0 are finite and non-zero. Thus, $\lim_{\omega\to\infty}\left|\frac{\tau_0}{\delta_0}-\frac{\tau_1}{\delta_1}\right| > 0$, except perhaps for a zero subset of parameter values. ∎