Limited Enforcement, Firm Dynamics and Aggregate Fluctuations

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Abstract

Limitations in the punishment for default induce constraints in the credit of firms and a non-trivial dynamics for their growth. Those limitations can also shape the dynamic response of the overall economy in response to an aggregate shock. In this paper I consider a simple continuous time formulation of the optimal dynamic relationship between a banks and a firm and derive a simple and sharp characterization of the optimal contract, the implicit credit constraints (for an equivalent sequentail trading arrangment) and the implied firm dynamics. I analytically characterize the steady state firm size distribution and show how to solve the optimal contract under *any arbitrary* deterministic path of the economy-wide equilibrium prices. I use the results to study the aggregate response to interest rates and aggregate productivity fluctuations. I also study the output and welfare cost of limited commitment.

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1. Introduction

It is widely believed that financial frictions play an important role for the dynamics at various levels of aggregation. For instance, the findings of T. Dunne, M. Roberts and L.Samuelson [20, 21] , D. Evans [24, 25] and B. Hall [29] that young, small firms grow faster but die more frequently than older, larger ones is often interpreted as the result of credit constraints. At a more aggregate level some authors argue that smaller firms (M. Gertler and S. Gilchrist [27]) or firms with less access to financial markets (A. Kashyap, J. Stein and O. Lamont, and D. Wilcox [38, 36]) are more sensitive to monetary shocks than their larger or less constrained counterparts. These authors argue the financial constraints are quantitively important for the dynamics of the country as a whole.

Despite this and much other evidence, and the related strong –sometimes dogmatic– beliefs, the literature has mostly focused on very restrictive environments.[1] For the analysis of aggregate output dynamics, some authors restrict the analysis to short lived firms (Bernanke-Gertler 1989, Fisher 1996) while others consider long-lived firms but restrict contracting to be short-lived (e.g. Carlstrom-Fuerst 1997). Authors such as Cooley-Marimon-Quadrini (2002) recognize the importance of dynamic incentive problems and solve for the dynamic problems but they restrict their analysis to rather limited forms of aggregate fluctuations. The key difficulty lies in the limited validity of recursive methods since market clearing conditions makes it difficult to assume a Markov structure for the aggregate endogenous states.

In this paper I follow a different strategy. I study a simple continuous time formulation a one-sided limited commitment problem in an economy with aggregate fluctuations. I consider the optimal dynamic relationship between a banks and a firm and derive a simple and sharp characterization of the optimal contract, the implicit credit constraints (for an equivalent sequentail trading arrangment) and the implied firm dynamics. I analytically characterize the steady state firm size distribution and show how to solve the optimal contract under *any arbitrary* deterministic path of the economy-wide equilibrium prices. I use the results to study the aggregate response to interest rates and aggregate productivity fluctuations. I also study the output and welfare cost of limited commitment.

The model is a continuous time closed relative of the problem studied by Alburquerque-Hopenhayn [1]. A risk-neutral "entrepreneur" is an agent that have access to a productive

[1] A much more extensive literature on risk-sharing, consumption and assets prices has been developed on endowment economies. ADD CITES

technology but lacks the funding necessary to operate it. A "bank" is another risk-neutral agent with frictionless access to financing but who does not have a productive technology of his own. Banks can fully commit to contractual arrangements and only require that, at the time of signing the contract, they at least break even. Entrepreneurs continually face the temptation to renege and run with the physical capital of the firm. The continuous temptation faced by the entrepreneur imposes a continuum of participation constraints, one for each period the firm is active.

Instead of formulating the problem in recursive form, I look directly at the sequence formulation. I find a simple expression for the optimality of consumption in each point in time. Each unit of consumption at time t, increases the utility condition of consumption of the agent at time t as well as the utility for all $\tau < t$. The latter is relevant for all the periods in which the participation constraint binds because it relaxes the set of incentive compatible investments. The optimal level of consumption is determined by an formula that includes the integral of multipliers on the participation constraints up to time t. Manipulation of this simple condition produces an analytical characterization of the optimal contract and of the conditions for a firm to be activated in the first place. I examine the optimal contract in environments with no aggregate shocks as well as in environments in which interest rates (and wages) are moving over time.

The optimal contracts has a very simple form. It is fully characterized by the time it takes for the firm to achieve maturity. For an initial interval (possibly of measure zero) of time entrepreneurs: (i) have binding participation constraints, (ii) have zero consumption (i.e. dividends), (iii) capital utilization grows over time, (iv) if there are no aggregate fluctuations or they do not change the default value of the firm, then the firm's capital grows at a constant rate. If firms survive this initial interval, entrepreneurs achieve positive consumption and the firm. The consumption is either constant (if there are no aggregate fluctuations) or fluctuates with the aggregate economy. Consumption, investment, capital, labor, etc, have simple closed form expressions as functions of the time to maturity. Comparative statics results are straightforward to obtain.

In an environment with continuous birth and death of identical cohorts, I show that the age effects of the model induce a cross-section of firm sizes that is Pareto distribution with a shape parameter that depends on the death probability and the discount rate. This simple form of the invariant distribution provides formulas for the output and welfare cost of the lack of full enforcement in the economy.

Credit constraints arise endogenously from the optimal dynamic contract with limited commitment as in R. Albuquerque and H. Hopenhayn [1] and O. Hart and J. Moore [32]. Along the lines of F. Alvarez and U. Jermann[2], I show a debt-accumulation equivalent to the optimal dynamic contract. Imposing the solvency constraints implicitly defined by the contract, an entrepreneur trading in short lived securities can and finds it optimal to replicate the same allocations. This result is important because it formalizes the link of collateral with the level of operation of the firms and their response to aggregate fluctuations.

Even aggregate variables are moving over time, the optimal contract can be solved analytically up the time to maturity. The time it takes a firm to mature can computed as the lowest root of the participation constraint at time 0 for the bank. I explain how to solve for the dynamics of mature firms and the time to maturity in models with fluctuating interest rates. I also show how to compute it in models with labor and fluctuations in wages.

When firms are mature, the capital used is a function an interest rate that is a weighted average of the market (time varying) interest faced by the bank and the discount rate of the entrepreneur. The output, investment, and entrepreneur's consumption of the mature firms can vary over time but not with the age of the firm. On the contrary, for firms who have not achieve maturity, these variables are determined by the participation constraint. The response of new firms to aggregate fluctuations is in terms of the time they take to achieve maturity.

The rest of the paper proceeds as follows. In the next section, I describe the environment, set up the optimal contracting problem and describe its solution. In this section I also discuss the equivalent debt accumulation problem. In the ensuing section, I consider the case of time-varying interest rates. In the following section, I discuss the invariant distribution of firm sizes and characterize aggregate output, capital and consumption. Here I also discuss the welfare and output cost of limited enforcement. The fifth section considers a simple general equilibrium economy in which labor and capital are used in the production of output. In that section I solved for the optimal contract with time varying interest rates and wages. The last section concludes. An short appendix at the end contains the proofs that are not directly derived in the body of the paper.

2. The Basic Contracting Problem

Consider a standard one-sided limited commitment environment. An "entrepreneur" is an agent that has access to a productive technology but lacks funding. A "bank" is an agent with frictionless access to financing but lacks productive projects of his own. Financial frictions arise from the limited punishments faced by entrepreneurs if they renege on financial liabilities. Banks design financial arragements to compete with each other but coping with the limited commitment problem of borrowers. At the time a constract is signed, time-zero participation constraints of both, the bank and the entrepreneur must be satisfied. In addition, the arrangement must insure that a participation constraint is satisfied for the entrepreneur in each of the periods in which the project is active.

2.1. The Environment

Preferences, Endowments, Technology. Entrepreneurs and banks are risk-neutral. As of any time t, the utility of the entrepreneur is

$$U(t) = \int_t^\infty e^{-\rho(s-t)} c(s)\, ds, \tag{2.1}$$

and the payoff to the bank is

$$P(t) = \int_t^\infty e^{-\rho(s-t)} p(s)\, ds. \tag{2.2}$$

Here $c(s)$ is the consumption for entrepreneur and $p(s)$ the net flow payoff for the bank at time s. For now I assume that both parties discount the future at the same rate $\rho > 0$.

Each entrepreneur can only run one project with which he is endowed. Activating the project requires an uniform set up investment of K_I units of the single consumption/capital good. The productivity of each project is fixed and given by a project-specific term $z > 0$. Once activated, projects can, in principle, remain active forever. The flow ouput of period t of an active project is

$$y(t) = zk(t)^\nu, \tag{2.3}$$

where $0 < \nu < 1$. Installed capital $k(t)$ evolves according to

$$k(t) = k_0 e^{-\delta t} + \int_0^t e^{-\delta(s-t)} i(s)\, ds. \tag{2.4}$$

Here k_0 is the initial investment $(t = 0)$ and $i(s)$ the stream of investments up to period t. Investment is fully reversible, i.e. $i(t)$ can be positive or negative. Capital depreciates at rate $\delta > 0$.

Incentive Problems. Entrepreneurs have the option to default on their financial liabilities and run away with a fraction of the capital under their control. Specifically, an entrepreneur who is in control of $k(t)$ units of capital can obtain a level of utility

$$V_D[k(t)] = \theta_D k(t), \tag{2.5}$$

where $0 < \theta_D < \rho^{-1}$. These inequality implicitly impose that defaulting do not run away with all the capital or that, after the default, they can only use it partially or invest it at a lower rate of return. For tractability I have assumed that defaulting entrepreneurs would lose access to their productive project.

Notice that the possibility of default is always present. Contracts have to satisfy a continuum of participation constraints.

2.2. Optimal Dynamic Contracts

I first consider the design of contracts and the implied allocations in the absence of any incentive problems. Then, I consider the optimal contract under limited commitment as laid out above. I assume that banks are competitive, and therefore, they offer the best contracts subject to they break even with the costs.

2.2.1. Full Enforcement

The problem for the bank is to design streams of consumption, investment and capital $\{c(s), i(s), k(s)\}$ so as to maximize 2.1 subject to the evolution of capital 2.4 and the breaking even condition:

$$\left[\int_t^\infty e^{-\rho t}\left[zf[k(t)] - i(t) - c(t)\right]dt\right] \geq k_0 + K_I.$$

The last condition derives from the fact that, after the set up cost K_I and the initial capital k_0 the payoff for the bank is equal to the cash-flow of the firm is $p(t) = zf[k(t)] - i(t) - c(t)$, i.e. the cash flow of the firm, minus the investment and the consumption of the agent. The Lagrangian for this maximization is:

$$
\begin{aligned}
L &= \max_{\{c,k,i,k_0\}} \int_0^\infty e^{-\rho t} c(s)\, dt + \\
&+ \int_0^\infty e^{-\rho t} \left[\lambda(t)\right] \left[k_0 e^{-\delta t} + \int_0^t e^{-\delta(s-t)} i(s)\, ds - k(t)\right] dt \\
&+ \vartheta \left[\int_0^\infty e^{-\rho t} \left[z\left[k(t)\right]^v - i(t) - c(t)\right] dt - k_0 - K_I\right].
\end{aligned}
$$

where $\lambda(s)$ is the multiplier associated to the evolution of capital at time t and υ the multiplier associated with the breaking even condition of the bank.

The optimal allocation is fully characterized by the first order conditions

$$
\begin{aligned}
k(\tau) &: \lambda(\tau) = \vartheta v z \left[k(\tau)\right]^{v-1} \\
i(\tau) &: \vartheta = \int_\tau^\infty e^{-[\rho+\delta](t-\tau)} \left[\lambda(t)\right] dt \\
c(\tau) &: \vartheta = 1.
\end{aligned}
$$

These conditions must hold for any τ, implying that the optimal solution is unique. It entails a constant capital

$$
k(\tau) = k^u(z) \equiv \left[\frac{vz}{\delta+\rho}\right]^{\frac{1}{1-v}}.
$$

After $t = 0$, investments only compensates the depreciation $i(t) = i^u = \delta k^u$. It is straightforward to establish the minimum productivity required for the entrepreneur to obtain funding and activate the firm. After simple algebraic manipulations, it can be shown that:

Lemma 2.1. *The project is funded if and only if is productivity z is above a threshold z_L^u, i.e.:*

$$
z > z_L^u \equiv \frac{(\delta+\rho)^v (\rho K_I)^{1-v}}{\left[v^{\frac{v}{1-v}} - v^{\frac{1}{1-v}}\right]^{1-v}}
$$

Since both parties are risk neutral, the timing of consumption is indeterminate. The entrepreneur would be entitled to a consumption stream with a present value equal to the difference between the LHS and the RHS of the previous inequality.

Notice that ϑ implicitly defines the relative weight of the entrepreneur in the relationship. In the absence of incentive problems, this weight remains constant over time.

2.2.2. Contracts with Limited Commitment

The optimal with limited commitment entails choosing the same objects but satisfying additional restrictions. First, it has to be the case that capital must be restricted so that the entrepreneur does not run away with it. This restriction introduces a continuum of participation constraints,

$$\int_t^{\infty} e^{-\rho[s-t]} c(s)\, ds \geq \theta_D k(t) \text{ all } t.$$

I will denote by $\mu(t)$ the multiplier associated with the participation constraint at time t. Second, I need to explicitly consider the limited liability constraint that was implicit in the limited commitment problem. Consumption cannot be negative. If it could be made negative, then an additional source of funds is operative eliminating the need of external funding. I denote by $\theta(t)$ the multiplier on the restriction that the consumption at time t be non-negative. With these extra restrictions, the Lagrangian becomes:

$$\begin{aligned}
L \;=\; & \max_{\{c,k,i,k_0\}} \int_0^{\infty} e^{-\rho t} c(t)\, dt + \\
& \int_0^{\infty} e^{-\rho t} \theta(t)\, c(t)\, dt + \\
& \int_0^{\infty} e^{-\rho t} [\lambda(t)] \left[k_0 e^{-\delta t} + \int_0^t e^{-\delta(s-t)} i(s)\, ds - k(t) \right] dt + \\
& \int_0^{\infty} e^{-\rho t} \mu(t) \left[\int_t^{\infty} e^{-\rho[s-t]} c(s)\, ds - \theta_D k(t) \right] dt + \\
& \vartheta \left[\int_t^{\infty} e^{-\rho t} [z f[k(t)] - i(t) - c(t)]\, dt - k_0 - K_I \right].
\end{aligned}$$

It is not hard to see that as the previous problem, this optimization satisfies convexity. The necessary and sufficient first order conditions are:

$$[i(\tau)] \; : \; \vartheta = \int_{\tau}^{\infty} e^{-[\rho+\delta](s-\tau)} \lambda(s)\, ds, \tag{2.6}$$

$$[k(\tau)] \; : \; \lambda(\tau) = \vartheta \left[z f'[k(\tau)] - \mu(\tau) \frac{\theta_D}{\vartheta} \right] \tag{2.7}$$

$$[c(\tau)] \; : \; \vartheta = 1 + \theta(\tau) + \int_0^{\tau} \mu(s)\, ds. \tag{2.8}$$

Condition 2.6 indicates that as before, investment is chosen so as to equate the net of depreciation present value of the stream $\lambda(s)$, the shadow values of capital. With limited commitment $\lambda(s)$ subtracts from the marginal product of capital at time s the cost of increased temptation of the entrepreneur to default on the contract. The same expression is the first order condition for k_0.

The intuition for expression 2.8 is as follows. As of $t = 0$, an increment of consumption in period τ increases the cost for the bank by $e^{-\rho\tau}$ and is weighted by ϑ in the program. The benefits are that it increases the utility of the entrepreneur by a factor $e^{-\rho\tau}$ and also relaxes non-negativity, having a shadow value $\theta(\tau) e^{-\rho\tau}$. More interesting, increasing the consumption at time τ also increases $U(t)$, the utility of the entrepreneur as of any time $t < \tau$, by $e^{-\rho(\tau-t)}$. In the program, the shadow value of this increment is $e^{-\rho t}\mu(t) e^{-\rho(\tau-t)}$ and hence, it is relevant only if it relaxes the participation constraint at that time. This effect is present for all $t \in [0, \tau]$. Integrating over this interval, we obtain 2.8 after simplifying out the common factor $e^{-\rho\tau}$. It is optimal to postpone consumption as much as possible. In this way, credit constraints are relaxed at maximum speed.

The form of the optimal contract is remarkably simple. With linear preferences, entrepreneurs have zero consumption up to time t_m ("time to maturity"). After that, consumption and capital are constant. The optimal contract is characterized in the next two propositions.

Proposition 2.2. *Assume that the participation constraint of the bank can be satisfied. Then, the optimal contract is as follows: there exists a $t_m < \infty$, such that: (a) for $t \in [0, t_m]$,*

$$
\begin{aligned}
k(t) &= \begin{cases} k^u e^{-\rho(t-t_m)} & t \in [0, t_m] \\ k^u & t > t_m. \end{cases} \\
c(t) &= \begin{cases} 0 & t \in [0, t_m] \\ \rho\theta_D k^u & t > t_m. \end{cases} \\
i(t) &= \begin{cases} k^u e^{-\rho t_m} & t = 0 \\ (\delta + \rho) k^u e^{-\rho(t-t_m)} & t \in (0, t_m) \\ k^u & t \geq t_m. \end{cases}
\end{aligned}
$$

Proof. See appendix. ■

If signed, the optimal contract under limited commitment can be fully characterized by the unrestricted optimum and the value of t_m, the time needed to attain the unrestricted investment profile. I now characterize t_m and describe under which circumstance the bank finances the firm.

At the time of signing the contract $(t = 0)$ value for the entrepreneur with productivity z, $U_0(t_m, z)$, is simply

$$U_0(t_m; z) = e^{-\rho t_m}\theta_D [k^u(z)],$$

which is always decreasing in t_m. This is because the entrepreneur consumes zero until $t > t_m$ an amount $\rho\theta_D\left[k^u(z)\right]$ which is independent of t_m. For the bank, on the other hand, after integration and simplification, foresees a net present value $P_0(t_m, z)$ equal to

$$P_0(t_m; z) = e^{-\rho t_m}\left\{z\left[k^u(z)\right]^v\left[\frac{e^{\rho(1-v)t_m}-1}{\rho(1-v)}+\frac{1}{\rho}\right]-\left[k^u(z)\right]\left[(\rho+\delta)t_m+\frac{[\delta+\rho\theta_D]}{\rho}+1\right]\right\}-K_I.$$

For the bank there are two forces. On the one hand, the sooner it attains maturity (the shorter t_m), the sooner the firm will attain $k(t) = k^u(z)$ and produce the maximum profits available with productivity z. On the other hand, the sooner the firm achieves maturity, the larger is the present value of consumption and investment streams.

Assuming competitive banks, the equilibrium value of the time-to-maturity of a firm with productivity z is given the the lowest root of $P_0(t_m) = 0$. This is

$$t_m^*(z) = \min\left\{t_m \geq 0 : P_0(t_m, z) \geq 0\right\}.$$

The following proposition completes the characterization of the optimal contract.

Proposition 2.3. *For $z \in [0, \infty)$ the following is true:*

(a) There is a number $z_L > 0$ such that $\sup P_0(t, z_L) = 0$. An entrepreneur with $z < z_L$ not be activated. The threshold z_L is higher than z_L^u, the threshold under full commitment.

(b) There is a number $z_H > z_L$ such that $P_0(0, z_H) = 0$. An entrepreneur with $z > z_H$ will be activated with $t_m^*(z) = 0$, i.e. a mature firm.

(c) For $z \in (z_L, z_H)$ the equation $P_0(t_m; z) = 0$ has two strictly positive roots and $t_m^*(z)$ is the lower one. As a function of z and the parameters,

$$\frac{\partial t_m^*(z)}{\partial z} < 0, \frac{\partial t_m^*(z)}{\partial K_I} > 0, \frac{\partial t_m^*(z)}{\partial \rho} > 0, \frac{\partial t_m^*(z)}{\partial \theta_D} > 0, \frac{\partial t_m^*(z)}{\partial v} > 0.$$

Proof. *See appendix*

As with full enforcement, there is a minimum productivity required to activate a firm. The minimum z_L with limited commitment is higher than the unrestricted z_L^u since the temptation of default directly limits the capital and reduces the profits obtainable during the periods before maturity. Also, notice that with higher set up cost K_I, the bank needs to recover a longer period of repayment to recover the higher initial investment.

Contrary to models with exogenous borrowing constraints, here the more productive firms achieve maturity more quickly (lower t_m^*), and operate closer to the unrestricted optimum $k^u(z)$ even when they are constrained. This is because with higher productivity the temptation to default becomes relatively less attractive than staying in the contract, which, with a higher z means reaping sooner a higher level of consumption.

With higher ρ the entrepreneur is less patient and hence more tempted to run away with the capital under his control. The limits on the usage of capital are tighter and profits lower during the transition. Again, this will increase the time needed for the bank to recover the initial investment. A similar implication mechanism operates when θ_D is higher.

A numerical example.

2.3. Decentralization of the Dynamic Contract.

[To be added: Equivalent debt accumulation problem.]

3. Time-varying interest rates.

Consider now an economy in which the market discount rates $r(t)$ moves over time. Differences in discounting introduce a wedge in the valuation of the stream of resources by the entrepreneur and the bank. I will follow the standard assumption that in every period the bank is strictly more patient than the entrepreneur:

$$0 < r(t) < \rho.$$

Denote by $R(t)$ the compounded interest rate from 0 to t :

$$R(t) \equiv \int_0^t r(t)\, dt.$$

The Lagrangian for the optimal contract is now:

$$\begin{aligned} L \quad = \quad & \max_{\{c,k,i,k_0\}} \int_0^\infty e^{-\rho t} c(t)\, dt + \\ & \int_0^\infty e^{-R(t)} \theta(t)\, c(t)\, dt + \\ & \int_0^\infty e^{-R(t)} [\lambda(t)] \left[k_0 e^{-\delta t} + \int_0^t e^{-\delta(s-t)} i(s)\, ds - k(t) \right] dt + \\ & \int_0^\infty e^{-R(t)} \mu(t) \left[\int_t^\infty e^{-\rho[s-t]} c(s)\, ds - \theta_D k(t) \right] dt + \\ & \vartheta \left[\int_t^\infty e^{-R(t)} \left[z f[k(t)] - i(t) - c(t) \right] dt - k_0 - K_I \right]. \end{aligned}$$

The first order conditions boil down to:

$$[i(\tau)] \quad : \quad \vartheta = \int_{\tau}^{\infty} e^{-[R(s)-R(\tau)+\delta(s-\tau)]} \lambda(s)\, ds, \tag{3.1}$$

$$[k(\tau)] \quad : \quad \lambda(\tau) = \vartheta \left[z f' [k(\tau)] - \mu(\tau) \frac{\theta_D}{\vartheta} \right] \tag{3.2}$$

$$[c(\tau)] \quad : \quad \vartheta = e^{R(\tau)-\rho\tau} + \theta(\tau) + \int_0^{\tau} e^{[(R(\tau)-R(s))-\rho(\tau-s)]} \mu(s)\, ds. \tag{3.3}$$

I consider first the case of full enforcement and then the case of limited enforcement.

Full Enforcement With $\mu(t) = 0$ for all t, we can plug **??** in **??** and obtain that

$$1 = z \int_{\tau}^{\infty} e^{-[R(s)-R(\tau)+\delta(s-\tau)]} f'[k(s)]\, ds.$$

Since this holds for all τ, the derivative of the RHS with respect to τ has to be zero. Leibniz formula implies that:

$$-z f'[k(s)] + z(R'(\tau) + \delta) \int_{\tau}^{\infty} e^{-[R(s)-R(\tau)+\delta(s-\tau)]} f'[k(s)]\, ds = 0.$$

Therefore, since $R'(\tau) = r(\tau)$, and using our functional form for $f(k)$ we have that

$$k^u(\tau) = \left[\frac{vz}{\delta + r(\tau)} \right]^{\frac{1}{1-v}} \text{ all } \tau. \tag{3.4}$$

As before, we could derive the implied series of investments $i^u(t)$ and obtain the net present value of the profits $\Pi_0^u = \int_t^{\infty} e^{-R(t)} [zf[k^u(t)] - i^u(t)]\, dt - k^u(t)$. The surplus from this technology is $\Pi_0^u - K_I$ and obtain the value of the surplus.

Only if $\Pi_0^u - K_I > 0$ the contract is signed and the firm funded. With competitive banking the surplus $\Pi_0^u - K_I$ obtained by the entrepreneur. Since $r(t) < \rho$, the optimal consumption stream is to consume as soon as possible,. The consumption profile is therefore, rather striking:

$$c^u(t) = \begin{cases} \Pi_0^u - K_I & t = 0 \\ 0 & t > 0. \end{cases}$$

For all practical purposes, with full enforcement, the entrepreneur sells the technology to the bank, who, forever after, operates it at optimal scale.

Limited Enforcement When limited commitment is operative, there has to be some dates t for which the temptation to default binds and $\mu(t) > 0$. There are two opposing forces determining the behavior of the entrepreneur's consumption. On one hand, the differences in discounting make it optimal for consumption to be delivered as soon as possible, as shown in the case with full enforcement. On the other hand, after some period consumption has to be always positive if the entrepreneur is not to run-away with the capital.

For some $t_m > 0$, the non-negative constraint is no longer binding, i.e.: $\theta(\tau) = 0$ all $\tau > t_m$. For all $\tau > t_m$, $\vartheta = e^{R(\tau)-\rho\tau} + \int_0^\tau e^{[(R(\tau)-R(s))-\rho(\tau-s)]}\mu(s)\,ds$. Taking the derivative of this expression and rearranging we obtain:

$$\mu(\tau) = [\rho - r(\tau)]\,\vartheta. \tag{3.5}$$

a positive variable since since $r(\tau) < \rho$. Therefore, the difference in the discount rates implies that participation constraints remain binding forever. Plugging 3.5 in 3.2, and inserting the implied expression for $\lambda(s)$ in 3.1 renders:

$$1 = \int_\tau^\infty e^{-[R(s)-R(\tau)+\delta(s-\tau)]}\left\{zf'[k(s)] - [\rho - r(s)]\,\theta_D\right\}ds. \tag{3.6}$$

Once again, the derivative of the RHS of 3.6 with respect to τ must be equal to 0. Using Leibniz rule, and 3.6 this derivative is $-\{zf'[k(\tau)] - [\rho - r(\tau)]\,\theta_D\} + (r(\tau) + \delta) = 0$. Rearranging terms:

$$zf'[k(\tau)] = r(\tau)(1-\theta_D) + \rho\theta_D + \delta. \tag{3.7}$$

The use of capital equates the marginal product of capital with a cost of use. Here, the relevant interest rate $r(\tau)(1-\theta_D) + \rho\theta_D$. Observe that the parameter θ_D of the value of defaulting, acts as a relative weight between the entrepreneur's discount rate and the market interest rate. The relevant interest rate is higher than the market interest rate since $\rho > r(\tau)$ for all τ. The capital used under limited commitment is always below the unrestricted optimum. In order to allocating capital to the firm also entails allocating consumption to the entrepreneur for him not to default, which adds the cost $[\rho - r(\tau)]\,\theta_D$ for each unit of capital.

Interestingly, if $\theta_D = 1$, the relevant discount rate is that of the entrepreneur. Fluctuations in the market interest rate faced by the bank would have no effects on the capital used by mature firms. If $\theta_D = 0$, we are back to the case of full commitment.

With the functional form assumed for the production function f, the capital used by mature firms is:

$$k^m(\tau) = \left[\frac{zv}{r(\tau)(1-\theta_D) + \rho\theta_D + \delta}\right]^{\frac{1}{1-v}}. \tag{3.8}$$

To characterize consumption, notice that since participation constraints always bind, $U(\tau) = \int_{\tau}^{\infty} e^{-\rho[s-\tau]} c^m(s)\, ds = \theta_D k^m(\tau)$ for all periods $\tau > t_m$. Taking derivatives $\frac{\partial U(\tau)}{\partial \tau} = -c^m(\tau) + \rho U(\tau) = \theta_D \frac{\partial k^m(\tau)}{\partial \tau}$. Rearranging terms:

$$c^m(\tau) = \theta_D k^m(\tau) \left[\rho - \frac{1}{k^m(\tau)} \frac{\partial k^m(\tau)}{\partial \tau} \right].$$

In addition 3.8, with this condition we have proven the following result:

Proposition 3.1. *If $\theta_D = 1$, $k^m(\tau) = k^m$, and consumption is constant at $c^m(\tau) = \rho k^m$ and $U(\tau) = k^m$ for ever after. If $\theta_D = 0$, then we are in the unconstrained case and for $\tau > 0$, $c(\tau) = U(\tau) = 0$. If $\theta_D \in (0,1)$, consumption must follow*

$$c^m(\tau) = k^m(\tau) \left[\rho + \left(\frac{1}{1-v} \right) \frac{(1-\theta_D)}{[r(\tau)(1-\theta_D) + \rho\theta_D + \delta]} \frac{\partial r(\tau)}{\partial \tau} \right].$$

This formula generalizes the relationship between the consumption of the entrepreneur and the capital used by a mature firm. After a sustained declined in the interest rate, the use of capital is high and consumption is high.

Before achieving maturity, entrepreneur's consumption is equal to zero. As before, the utility of the agent is as of time $t \in [0, t_m]$,

$$U_0(t; t_m) = e^{-\rho(t_m - t)} \theta_D k^m(t_m).$$

The only difference with the case of $r = \rho$ is that $k^m(t_m)$ varies over time. For any $t \in [0, t_m]$, the capital used by the firm the equality of $U_0(t; t_m) =_D k^m(t_m)$. With this expression, the capital used up to time t_m is equal to

$$k(t) = e^{-\rho(t_m - t)} k^m(t_m).$$

Following the same arguments as before, we can compute the present value of the payoffs accrued by the bank as a function of t_m. Fluctuations in the interest rate impact young firms by changing the value of t_m^* at the time of signing up the contract.

4. Many Firms

To understand how the optimal contract under limited commitment shapes up the cross section distribution, in this section I consider an economy with no aggregate disturbances and no initial heterogeneity. Specifically, the interest rate is constant and equal to ρ. In each period t a new cohort of firms is born with identical productivity z. The size of each cohort is constant and equal to $\mu > 0$. Equally, in each period a fraction μ of the existing firms die. Death probabilities are independent of age, and I assume that whenever an entrepreneur dies, the capital under his control disappears. Therefore, μ can be seen as part of the discount rate of both, the bank and the entrepreneur.

The age distribution of the firms is described by the density $f_t(t) = \mu e^{-\mu t}$. As all firms have the same z, they are activated with the same time to maturity t_m.[2] Since the size of the capital is $k(t) = e^{-\rho[t_m - t]} k^u$, it is easy to verify the following proposition.

Proposition 4.1. *The size distribution is a right-truncated Pareto distribution. Specifically, for* $k \in (k_0, k^u)$, $k_0 \equiv k^u e^{-\rho t_m}$, *the density function* $f_k(k)$ *is*

$$f_k(k) = \left(\frac{\mu}{\rho k_0}\right)\left(\frac{k}{k_0}\right)^{-\left(1+\frac{\mu}{\rho}\right)};$$

and

$$\Pr[k = k^u] = e^{-\mu t_m}.$$

Having a closed form expression for the distribution of the steady state distribution enables us to derive expressions for all the aggregates, in particular those needed to compute a general equilibrium version of the economy (see below) and welfare comparisons of alternative policies or institutions.

For instance, after a few basic algebraic steps, we find that the aggregate capital K used in this economy is:

$$K \equiv \int_0^\infty k dF_k(k) = k^u \left[\frac{\rho e^{-\mu t_m} - \mu e^{-\rho t_m}}{\rho - \mu}\right].$$

Likewise, aggregate output Y is

$$Y \equiv \int_0^\infty z k^v dF_k(k) = z(k^u)^v \left[\frac{\rho v e^{-\mu t_m} - \mu e^{-v\rho t_m}}{\rho v - \mu}\right].$$

Aggregate consumption is $C \equiv Y - (\rho + \delta) K - \mu K_I$ and it is sufficient to compute welfare because agents are risk neutral. Hence, the output and welfare costs of limited commitment are easy to compute by comparing these aggregates with the economy with full enforcement, in which $K^u = k^u$ and $Y^u = z[k^u]^v$.

To illustrate the costs of limited enforcement, consider the economy parameterized as follows:

[2] To avoid trivialities, I assume that firms are initiated but with $t_m^* > 0$.

Parameter Values

$\mathbf{z}$	$\boldsymbol{\rho}$	$\boldsymbol{\mu}$	$\boldsymbol{\delta}$	$\boldsymbol{\nu}$	$\mathbf{K}_I$	$\boldsymbol{\theta}_D$
1	0.07	0.03	0.06	0.85	0.2*Π_0^U	0.75

Under this configuration, a interval of length 1 is equivalent to one year. The discount rate is equal to a risk free rate of 7% and an annual death probability of 3%. These parameters, while realistic, are aimed only to illustrate the economy and in no way I pretend to see them as the outcome of a serious calibration exercise. A serious quantitative exploration would require explicitly considering the heterogeneity in the inherent productivity z across units and also across time.

The following table displays the ratios of the economy with limited enforcement relative to an undistorted economy. It shows that the costs are significant. The temptation to default, even if it is eventually eliminated, keeps the firms from operating at the potential profitability.

Aggregate Costs of Limited Enforcement

Case	t_m	$\frac{Y}{Y^u}$	$\frac{K}{K^u}$	$\frac{C}{C^u}$
Benchmark	14.1	0.71	0.74	0.88
$\theta_D = 0.5$	8.6	0.87	0.85	0.96
$\theta_D = 1$	18.9	0.61	0.58	0.78
$K_I = 0.3 * \Pi^u$	17.2	0.66	0.62	0.81

Notice that the stronger effects are on the use of capital and on aggregate output. The loss in consumption is lower because the use of capital is costly. However, notice that a very high fraction of consumption is lost due to the inability to extract the maximum surplus from firms.

5. A Simple Small Open Economy.

I now develop a simple economy with tractable general equilibrium effects. Consider now an economy populated by two groups of agents. The economy, which we assume small and open to world financial markets, is populated by two groups of agents. On one hand, entrepreneurs use capital and labor to produce output. On the other hand, households provide labor to the entrepreneurs. Both, households and entrepreneurs have access to international financial markets, and take the interest rate as an exogenous variable. Wages are determined by the equilibrium of a competitive labor market and respond to fluctuations in international interest rates.

In this environment, households take as given interest rates $r_h(t)$ while financial intermediaries use a rate $r(t) = r_h(t) + \mu$ as the discount rate for the firms that they finance. The discount rate of the entrepreneur is still $\rho = \eta + \mu$, a pure time preference rate plus the constant death probability. The discount rate of households is υ. I maintain the assumption that $r(t) < \rho$. To avoid uninteresting trends, I will assume that

$$\lim_{t \to \infty} r_h(t) = \upsilon,$$

that is, temporary fluctuations are present, but asymptotically the discount rates are the same.

5.1. Households

There is a continuum of measure γ of infinitely lived, identical households. Each household takes the interest rates $r_h(\tau)$ and wages $w(\tau)$ as given and chooses streams of consumption $c^h(\tau)$ and of labor $n^h(\tau)$ so as to maximize the value of

$$\int_t^\infty e^{-\upsilon(\tau-t)} \left\{ \ln\left[c^h(\tau)\right] - n^h(\tau) \right\} d\tau,$$

subject to the constraint that the present value of consumption be equal to the present value of earnings.

$$\int_t^\infty e^{-R_h(t,\tau)} \left[w(\tau) n^h(\tau) - c^h(\tau) \right] d\tau = 0.$$

Here $R_h(t,\tau) \equiv \int_t^\tau r_h(s)\, ds$, is the compounded interest rates faced by households.

The quasi-linearity of preferences makes optimality conditions simple. First, consumption is equal to wages

$$c^h(\tau) = w(\tau).$$

Second, the linearity in labor pins down the behavior of wages over time:

$$w(\tau) = e^{[R(t,\tau) - \upsilon(\tau-t)]} w(t). \tag{5.1}$$

Here $w(0) = (1/\lambda)$, where λ is the shadow value of wealth for the households (the Lagrange multiplier on the budget constraint).

Fluctuations in interest rates drive proportional fluctuations in the wage rate. This property will make the ensuing analysis tractable.

5.2. Entrepreneurs.

The economic problem faced by entrepreneurs and banks is the same as before but augmented by the choice of labor $n(t)$. The instantaneous production function is

$$y = z\left(k^{\alpha} n^{1-\alpha}\right)^{v}.$$

The optimal choice of labor is intratemporal and proportional to capital:

$$n(\tau) = k(\tau)\left[\frac{zv(1-\alpha)}{w(\tau)}\right]^{\frac{1}{\alpha v}}.$$

Using this expression, the Lagrangian for the optimal contract is now:

$$\begin{aligned} L = & \max_{\{c,k,i,k_0\}} \int_0^{\infty} e^{-\rho t} c(t)\, dt + \\ & \int_0^{\infty} e^{-R(t)} \theta(t)\, c(t)\, dt + \\ & \int_0^{\infty} e^{-R(t)} [\lambda(t)] \left[k_0 e^{-\delta t} + \int_0^t e^{-\delta(s-t)} i(s)\, ds - k(t)\right] dt + \\ & \int_0^{\infty} e^{-R(t)} \mu(t) \left[\int_t^{\infty} e^{-\rho[s-t]} c(s)\, ds - \theta_D k(t)\right] dt + \\ & \vartheta \left[\int_t^{\infty} e^{-R(t)} \left[za(\tau)\, g[k(\tau)] - i(t) - c(t) - b(\tau)\, k(t)\right] dt - k_0 - K_I\right]. \end{aligned}$$

where, to save on notation, I have defined

$$g'(k) \equiv k^{v},$$

is a concentrated production function and

$$\begin{aligned} a(\tau) & \equiv [zv(1-\alpha)]^{\frac{(1-\alpha)}{\alpha}} [w(\tau)]^{\frac{\alpha-1}{\alpha}} \\ b(\tau) & \equiv w(\tau)^{\frac{\alpha v-1}{\alpha v}} [zv(1-\alpha)]^{\frac{1}{\alpha v}}, \end{aligned}$$

two strictly positive deterministic processes. The optimality conditions are the same as before, except that now the first order condition for capital is:

$$[k(\tau)] : \lambda(\tau) = \vartheta\left[zg'[k(\tau)]\, a(\tau) - b(\tau) - \frac{\mu(\tau)\,\theta_D}{\vartheta}\right]. \tag{5.2}$$

Plug 3.5 in 5.2 and the resulting expression into 3.1 to obtain

$$1=\int_{\tau}^{\infty}e^{-[R(s)-R(\tau)+\delta(s-\tau)]}\left\{zg'\left[k\left(s\right)\right]a\left(s\right)-b\left(s\right)-\left[\rho-r\left(s\right)\right]\theta_{D}\right\}ds. \tag{5.3}$$

Taking the derivative with respect to τ, $-\left\{zg'\left[k\left(\tau\right)\right]a\left(\tau\right)-b\left(\tau\right)-\left[\rho-r\left(\tau\right)\right]\theta_{D}\right\}+\left(r\left(\tau\right)+\delta\right)=0$. Rearranging terms, and using the functional form for $g\left(\cdot\right)$, the capital used by mature firms is

$$k^{m}\left(\tau\right)=\left[\frac{vza\left(\tau\right)}{b\left(\tau\right)+r\left(\tau\right)\left(1-\theta_{D}\right)+\rho\theta_{D}+\delta}\right]^{\frac{1}{1-v}}.$$

As before, the relevant interest rate is $r\left(\tau\right)\left(1-\theta_{D}\right)+\rho\theta_{D}$. With labor services required for production, increments in capital are optimally matched with labor. In addition to interest rates, fluctuations in the cost of labor also induce fluctuations in the use of capital.

As in the model without capital, consumption follows $c^{m}\left(\tau\right)=\theta_{D}k^{m}\left(\tau\right)\left[\rho-\frac{1}{k^{m}\left(\tau\right)}\frac{\partial k^{m}\left(\tau\right)}{\partial\tau}\right]$. The only difference with the solution is that now in addition to movements in $r\left(\tau\right)$ we also have movements in $a\left(\tau\right)$ and $b\left(\tau\right)$ driven by $w\left(\tau\right)$.

Before achieving maturity, entrepreneur's consumption is equal to zero. As before, the utility of the agent is as of time $t\in\left[0,t_{m}\right]$

$$U_{0}\left(t;t_{m}\right)=e^{-\rho\left(t_{m}-t\right)}\theta_{D}k^{m}\left(t_{m}\right).$$

The only difference with the case of $r=\rho$ is that $k^{m}\left(t_{m}\right)$ varies over time. For any $t\in\left[0,t_{m}\right]$, the capital used by the firm the equality of $U_{0}\left(t;t_{m}\right)=_{D}k^{m}\left(t_{m}\right)$. With this expression, the capital used up to time t_{m} is equal to

$$k\left(t\right)=e^{-\rho\left(t_{m}-t\right)}k^{m}\left(t_{m}\right).$$

Following the same arguments as before, we can compute the present value of the payoffs accrued by the bank as a function of t_{m}. Now, fluctuations in $r\left(\tau\right)$ and $w\left(\tau\right)$ affect the value of t_{m}^{*}.

5.3. Steady state.

Assume for now a steady state with $r=v+\mu$, i.e. equal discounting. With no fluctuations in interest rates, wages are constant, $w\left(\tau\right)=w_{0}$. Each household provides one unit of labor, generating an aggregate supply of labor equal to γ, the measure of households in the economy.

The cross section distribution of the firm is also Pareto. The demand for labor...[to be completed].

6. Concluding Remarks

I developed a simple model in which limited commitment restricts the operations of firms. Setting the model in continuous time and solving directly for the sequence form optimization leads to very simple and tractable characterization. Comparative statics results are straightforward. The model leads to an easy to characterize and work with invariant distribution of firms, and with it, a sharp characterization of the costs of enforcement. Also, the model is suitable to consider the optimal contract under any time behavior of interest rates and wages that affect the operation and profitability of firms.

The next step is to use the model to explore the aggregate dynamics. In the current version, all the elements are ready to study the response of the economy to fluctuations in international interest rates. In the version discussed above, wages respond to interest rates directly and the optimal operation of firms is ready to be computed. Other experiments ought to be studied extending the model to include the option of liquidating. For instance, total factor productivity innovations will increase the productivity of all firms but also the wages paid by each. The response of each firm will depend on how close they are to maturity. It will also be interesting to study the aggregate response to embedded total factor productivity innovations, those that are only available to new firms. In this case, it would be optimal to liquidate some of the existing firms. In the current framework, the firms to liquidate are the smaller, younger firms. These extensions are currently being pursued.

A. Proofs

Proof of Proposition 1

Since 2.8 holds for any τ, then, whenever $\mu(\tau)$ is positive, $\theta(\tau)$ must be positive and decreasing. Therefore, limited liability and participation constraints bind for the same periods. To see that the periods for which both bind has to be lower interval $[0,t_m]$ assume instead that they bind in two disjoint intervals $[0,t_0]$ and $[t_1,t_2]$. For the same level of utility at time 0, we could transfer the present value of consumptions for (t_0,t_1) and assign evenly it to $[t_1,t_2]$. Doing that maintains the same values for $U(t)$ and $P(t)$ (the e.d.u. of banks and entrepreneurs as of t) for all $t \in [0,t_1] \cup (t_2,\infty)$ and strictly increases the utility of the agent for $t \in [t_1,t_2]$, relaxing the constraints on $k(t) \in [t_1,t_2]$ increasing the value attainable for the bank. For $t \geq t_m$ the $\mu(t) = \theta(t) = 0$, so the unrestricted optimum is attainable and hence $k(t) = k^u$ and $i(t) = \delta k^u$. For $t > t_m$ the timing of optimal consumption is indeterminate. The minimum constant value that insures that the participation constraint does not bind at $k = k^u$ is $\rho\theta_D k^u$. For $t < t_m$ the consumption is zero. The for the entrepreneur is $U(t) = e^{-\rho(t-t_m)}\theta_D k^u$. Since participation constraints bind, the value of $k(t)$ is given by $\theta_D k(t) = U(t)$, which is the expression above. In particular, investment at $t = 0$ is the initial capital and given by $\theta_D k(t) = U(t)$. The expression for $i(t) \in (0,t_m)$ is equal to the investment needed to accumulate capital when the participation constraints are relaxed and to make up for the depreciation.■

Proof of Proposition 2

Let

$$R_0(t_m;z) \equiv e^{-\rho t_m}\left\{z\left[k^u(z)\right]^{v}\left[\frac{e^{\rho(1-v)t_m}-1}{\rho(1-v)}+\frac{1}{\rho}\right]-\left[k^u(z)\right]\left[(\rho+\delta)t_m+\frac{[\delta+\rho\theta_D]}{\rho}+1\right]\right\}$$

be the returns for the bank ex-post the initial investment K_I. $R_0(t_m;z)$ is continuous and twice differentiable in $[0,\infty)$, with $R_0(0;z) = z\left[k^u(z)\right]^{\alpha}\left[\frac{1}{\rho}\right] - k^u(z)\left[\frac{\delta+\rho\theta_D}{\rho}+1\right]$, finite and $\lim_{t_m\to\infty} R_0(t_m;z) = 0$. Direct inspection of its formula reveals that $R_0(t_m;z)$ is only positive when

$$\left[\frac{e^{\rho(1-v)t_m}-1}{\rho(1-v)}+\frac{1}{\rho}\right] > \left[vt_m+\frac{v}{\delta+\rho}\frac{[\delta+\rho\theta_D]}{\rho}+\frac{v}{\delta+\rho}\right].$$

Both LHS and RHS are increasing, but LHS is strictly convex and the the RHS linear. Hence, the crossing $\hat{t}_m$ is unique and $R_0(t_m;z)$ is only positive for $t_m > \hat{t}_m$. The only relevant segment for $t_m^*(z)$ is $\left(\hat{t}_m, \infty\right)$ because $R_0(t_m; z)$ has to be strictly positive if the bank is to recover the set up cost $K_I > 0$. By directly taking derivatives, $R_0(t_m; z)$ is log-concave with respect to t_m in $\left(\hat{t}_m, \infty\right)$, and therefore so is $P_0(t_m; z)$. Then if $P_0(t_m; z)$ ever cross zero it crosses twice as $\lim_{t_m \to \infty} R_0(t_m; z) = 0$.

On the other hand, $R_0(0; z) = z[k^u(z)]^{\alpha}\left[\frac{1}{\rho}\right] - k^u(z)\left[\frac{\delta+\rho\theta_D}{\rho} + 1\right]$ is continuous and strictly increasing in z with $R_0(0; 0) = 0$ and $\lim_{z\to\infty} R_0(0; z) = \infty$ all t_m. Letting z_H denote the unique crossing, the stated result follows. On the other hand, by the theorem of the maximum $S(z) \equiv \sup_{t_m} P_0(t_m; z)$ is a continuous strictly increasing function, with $S(0) < 0$ and $S(z_H) > 0$. Denoting z_L the crossing of $S(z)$ with 0, the stated result follows.

Finally, the sign of the derivatives in the statement of the proposition follow directly by using the implicit derivation.■

Proof of Proposition 3

Algebra to derive the consumption of mature firms:

$$
\begin{aligned}
c(\tau) &= \rho\theta_D k^m(\tau) - \theta_D \frac{\partial k^m(\tau)}{\partial \tau} \\
&= \rho\theta_D \left[\frac{zv}{r(\tau)(1-\theta_D) + \rho\theta_D + \delta}\right]^{\frac{1}{1-v}} - \theta_D [zv]^{\frac{1}{1-v}} \left[r(\tau)(1-\theta_D) + \rho\theta_D + \delta\right]^{\frac{-1}{1-v}} \\
&= \theta_D \left[\frac{zv}{r(\tau)(1-\theta_D) + \rho\theta_D + \delta}\right]^{\frac{1}{1-v}} \left[\rho + \left(\frac{1}{1-v}\right) \frac{(1-\theta_D)}{[r(\tau)(1-\theta_D) + \rho\theta_D + \delta]} \frac{\partial r(\tau)}{\partial \tau}\right] \\
c(\tau) &= k^m(\tau) \left[\rho + \left(\frac{1}{1-v}\right) \frac{(1-\theta_D)}{[r(\tau)(1-\theta_D) + \rho\theta_D + \delta]} \frac{\partial r(\tau)}{\partial \tau}\right]
\end{aligned}
$$

as claimed in the text.■

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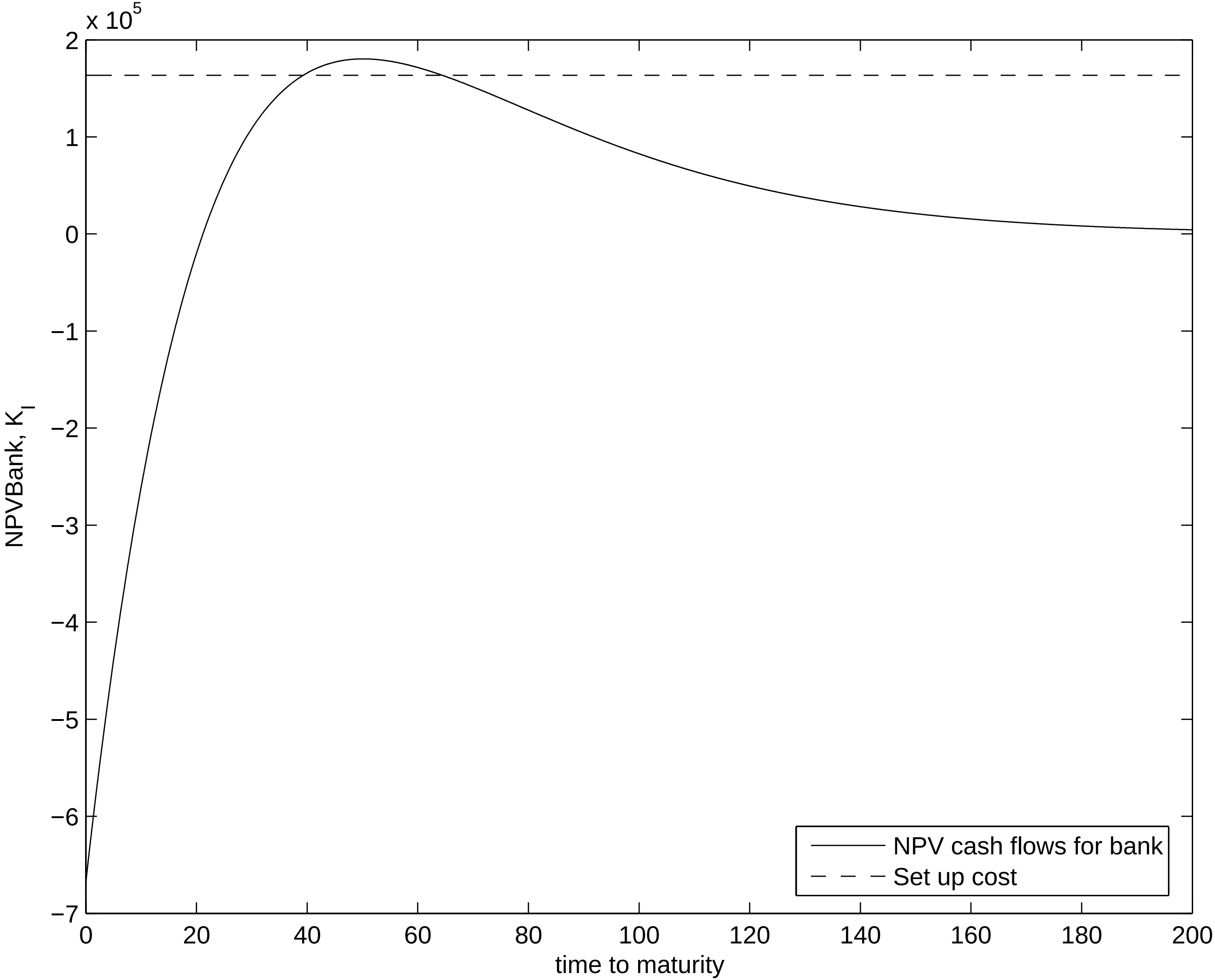
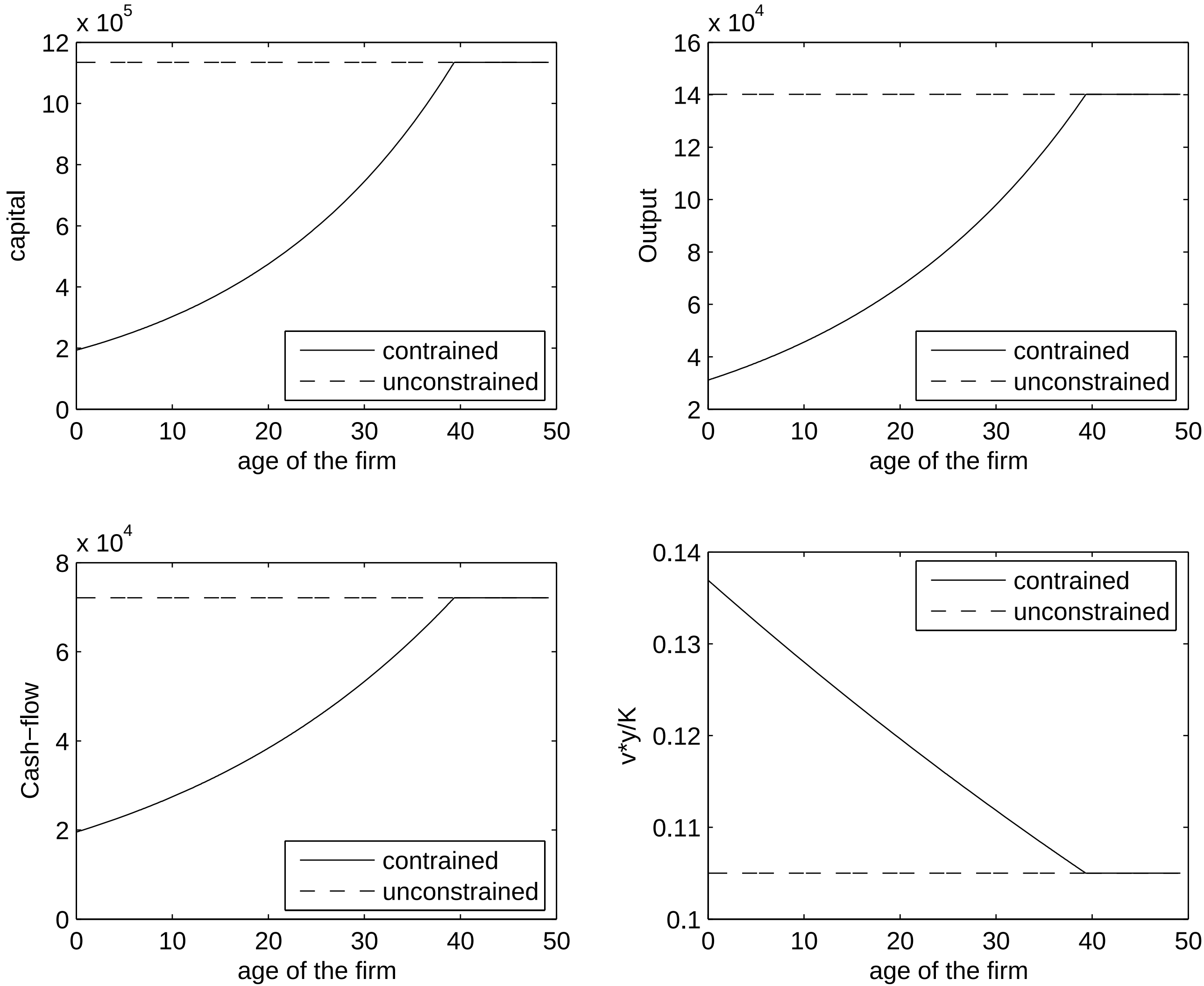
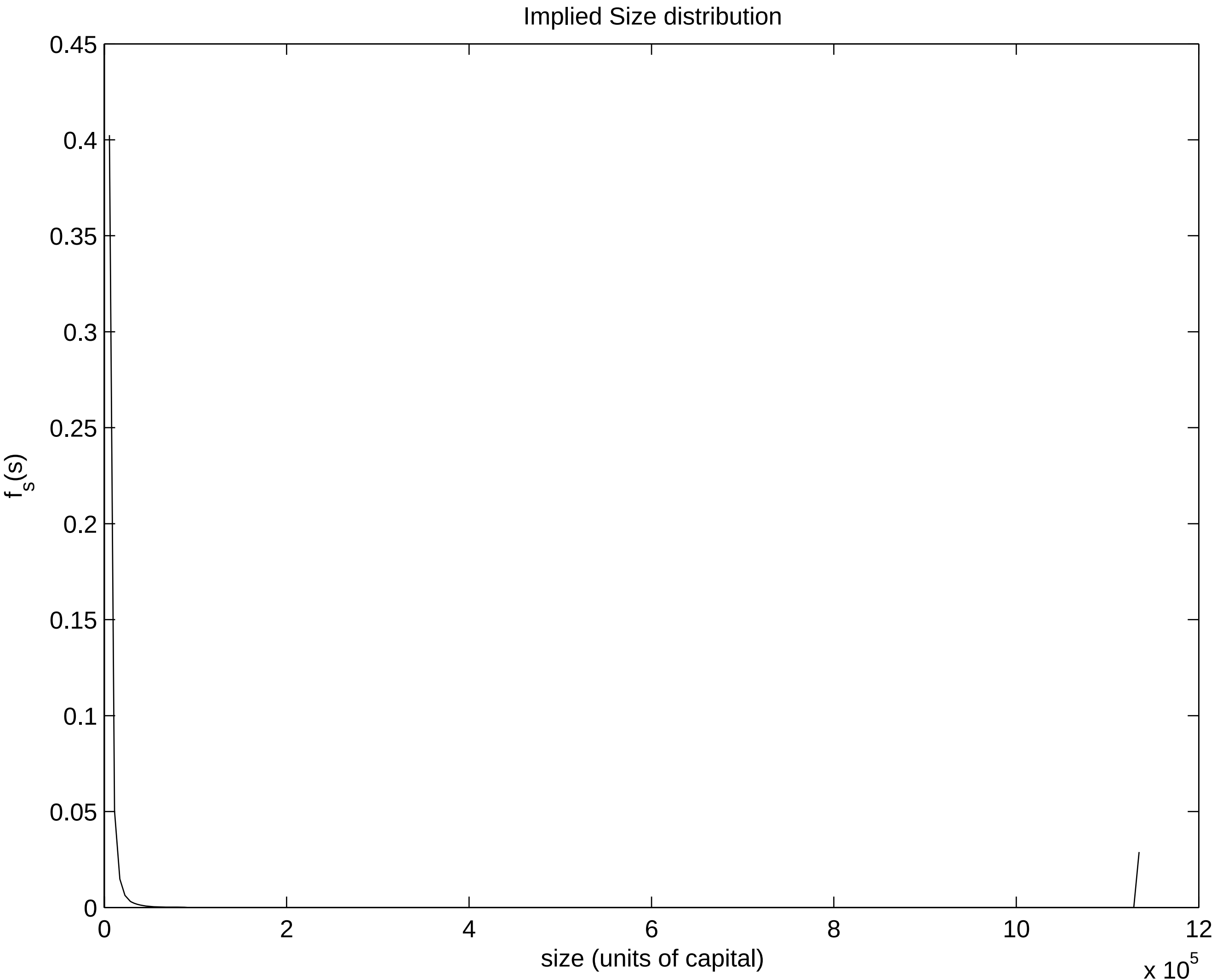


x 10^5
2
1
0
−1
−2
−3
−4
−5
−6
−7
0
20
40
60
80
100
120
140
160
180
200
NPVBank, K_I
time to maturity
NPV cash flows for bank
Set up cost


x 10^5
capital
age of the firm
contrained
unconstrained
x 10^4
Output
age of the firm
contrained
unconstrained
x 10^4
Cash−flow
age of the firm
contrained
unconstrained
v*y/K
age of the firm
contrained
unconstrained


Implied Size distribution
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$f_s(s)$
0
2
4
6
8
10
12
size (units of capital)
x 10^5