

A model of economic development with lumpy human capital accumulation

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Abstract

We study an economy where finite-lived households face borrowing constraints and human capital accumulation is the result of lumpy educational decisions. Households that allocate sufficient level of education to their children create skilled workers. Across skill levels, labour endowments evolve stochastically over time as a function of the skill level of the household.

There are two sectors in the economy, a manufacturing sector and an agricultural sector. In the manufacturing sector skilled and unskilled labour is used alongside capital to produce output. Skilled labour is a complement to physical capital, while unskilled labour is a substitute. In the agricultural sector, unskilled labour is the only productive input.

Educational decisions are lumpy because the accumulation of skills requires parents to allocate indivisible units of children's time to schooling. The presence of considerable income risk for unskilled households, and the existence of borrowing constraints and subsistence consumption, create a large opportunity cost of adolescent childrens' time.

In the aggregate, the model is consistent with rising rates of physical and human capital investment during the initial stages of economic development. This contrasts sharply with the predictions of the standard neoclassical growth model augmented with human capital accumulation. When borrowing constraints are sufficiently severe,

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and income risk high, initially low levels of investment increase over time. During this transition the real interest rate rises. Given any level of income per capita, the rate of growth is higher when there are less poor households.

At the household level, the model predicts very low rates of savings for poor households, and high level of intergenerational persistence in both wealth and education. Poverty is persistent across generations as poor households are unable to undertake the large investments in education that are necessary to move their children into the skilled labour force.

1 Introduction

Models of economic development typically assume that both physical and human capital accumulation involve convex choice sets. At least for physical capital, recent quantitative results provide indirect support for this assumption. The aggregate mechanics of an economy with lumpy firm-level investment is not that different from those predicted by a model where investment choices are convex (Khan and Thomas (2008)). The issue is less clear for human capital accumulation. While existing studies of the role of human capital in economic development assume linear investment technologies (Manuelli and Seshadri (2007), Erosa et al (2007)), there is evidence to suggest that this assumption of convex choices at the household level, like the assumption of convex physical capital investment at the firm level, is at odds with the data. Actual years of schooling are bunched around years associated with the completion of primary, secondary and post-secondary education. This suggests that human capital accumulation is a lumpy decision. This paper evaluates the consequences of such an assumption in a model of economic development in which households are subject to borrowing constraints.

In our economy, human capital accumulation is the result of lumpy educational decisions made by finite-lived households that face risky income streams and constraints on their borrowing. Children in each household receive at least a primary level of education. Thereafter, the household has to decide whether to invest in an additional level of education for the child, now an adolescent. If it chooses to do so, it endows its child with a secondary education.

Secondary education is necessary to become a skilled worker. Thereafter children begin their own household, working as either skilled or unskilled workers. Skilled households work in the manufacturing sector. Unskilled households work in either agriculture or in manufacturing.

Household labour endowment evolves according to a stochastic process that is a function of skills. For unskilled workers and agricultural labourers, this implies substantial income uncertainty. At the same time, educational decisions involve lumpy allocations of adolescents' time. The opportunity cost of an adolescents' time is the labour income he can bring to the household and the value of this income can be relatively high for poor households with subsistence consumption needs.

Our primary finding is that lumpy educational choices, in the face of borrowing constraints, can imply slow and gradual transition to a developed economy. When the distribution of income places many households near subsistence consumption, then industrialisation, as captured by a shift from agriculture to manufacturing, and the accumulation of skills, is far slower than that which occur in a model where households could undertake small investments in human capital. Under the assumption that skilled workers have secondary or post-secondary education, and that skills are complementary to physical capital accumulation, we find that, in the initial stages of economic development, the rate of growth rises over time as does the rate of return to physical capital. Thus relatively poor economies with low levels of physical and human capital also have low real interest rates and low rates of saving.

The relation between borrowing constraints, rising rates of growth, and a low return to physical capital investment are shared with the work of Buerra and Shin (2007). They emphasise the impact of these frictions on entrepreneurship, and the scale of production, while we study human capital accumulation. Both Banerjee and Newman (1993) and Galor and Ziera (1993) study the effects of financial frictions on occupational choice in the context of models of economic development. In contrast to these papers, our focus is on a quantitative model of economic development that seeks to replicate the slow transitions characterised by persistent poverty observed in many large poor economies.

2 The model

There are two sectors, agricultural and manufacturing. Agricultural production uses only unskilled labour. Manufacturing production uses capital as well as skilled and unskilled labour. Unskilled labour is a substitute for, and skilled labour a complement to, physical capital. Households can choose to work in the agricultural sector or the manufacturing sector.

Each adult household lives for 8 periods after spending 3 periods as a child in their parent's household. Households are identified by their wealth, their skill level, and their labour productivity. Skills are acquired through education, and there are three education levels: primary, secondary and tertiary. Households with primary education are unskilled while those with secondary or with tertiary education are skilled.

Each period households work, consume and save. A child is introduced in the second period of a household. In the following period, children receive primary education. In the fourth period of a household, they must determine the education allocated to their children, as well as the initial asset holdings of these children.

For an adult household of age $t = 1, \dots, 8$, the state vector is (a, e, l_0, l_1) where $a \in [\underline{a}, \infty)$, with $\underline{a} \leq 0$, is savings (or borrowing) and $e = u, s_0, s_1$ is the skill level. Labour productivity, $l_0 + l_1$, has two components, a permanent component, l_0 , and a stochastic component, l_1 . The permanent component is drawn from a distribution that varies with the skill-level of the household, $l_0 \in \{l_{0,1}, \dots, l_{0,N(l_0)}\}$ is $\pi^{e,0} = \{\pi_1^{e,0}, \dots, \pi_{N(l_0)}^{e,0}\}$. The stochastic component follows a Markov Chain which, again, varies with skills. Specifically, $l_1 \in \{l_{1,1}, \dots, l_{1,N(l_1)}\}$ with $\Pr \{l_1 = l_{1,i}, l'_1 = l_{1,j}\} = \pi_{ij}^e \geq 0$ with $\sum_{j=1}^{N(l_1)} \pi_{ij}^e = 1$ for each $i = 1, \dots, N(l_1)$. However this process is the same for s_0 and s_1 , which represent skilled workers with secondary and post-secondary levels of education. The initial distribution of the transitory component of human capital is $\pi^{e,1} = \{\pi_1^{e,1}, \dots, \pi_{N(l_0)}^{e,1}\}$.

2.1 Unskilled households

Household skills, for unskilled households, are completely determined by the time they form a new household. While skilled households may continue to accumulate human capital in their

first period as adults, this option is unavailable to unskilled households. In their first period, they work and save, much like their second and third periods. However in their second period they have a child and in their third period, the child receives a primary school education. In the fourth period a lumpy educational decision is made. Either the households allocates its child's time to work, or it receives a secondary school education. Thereafter the child becomes independent of the parental household. Children benefit from parental altruism, which motivates not only the desire to see children receive secondary education, but also an initial transfer of wealth from father to son. Children cannot inherit debt from their parents, hence transfers must be non-negative.

2.1.1 Unskilled households in periods 1 - 3

We describe adult households decisions using a recursive approach. In their first period, unskilled households work, in either agricultural or manufacturing, then choose how much to borrow or save and their level of consumption. Borrowing is constrained below by $\underline{a}$.

$$\begin{aligned}
V^t(a, u, l_0, l_{1,i}; S) &= \max \left(\theta^t U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^u V^{t+1}(a', u, l_0, l'_{1,j}; S') \right) \\
\text{subject to } a' + c &\leq w^u(l_0 + l_{1,i}) + (1 + r(S))a \\
a' &\geq \underline{a}, t = 1, 2, 3.
\end{aligned} \tag{1}$$

In (??), V^t is the present discounted expected lifetime value of an unskilled household in period t of their lives with state vector $(a, u, l_0, l_{1,i}; S)$. Notice that there is no sectoral specification to the unskilled household's problem. The constant average productivity of labour in the agricultural sector is denoted w^u and the ability of unskilled households to work in either agriculture or manufacturing determines the manufacturing wage at this level, for unskilled workers, independently of the aggregate state. The real interest rate on borrowing or savings, r , is however a function of this aggregate state, S , which evolves over time according to a mapping Γ that is known to all households, and $S' = \Gamma(S)$.

In their second period households have children, the consumption needs of children are represented by a time-varying coefficient θ^t . We assume that $\theta^1 = 1$ while θ^2 and θ^3 , the utility weight in periods where the household has children, exceed θ^1 .

When the household is in the third period, $t = 3$, children receive a primary school education. We assume that there is no resource cost to this education for households, and that primary school-age children provide no useful labour services.

2.1.2 Unskilled households in period 4

When $t = 4$, unskilled households must decide whether or not to allocate a secondary school education to their children. Secondary schooling is necessary for their children to become skilled workers in the manufacturing sector. Households at this stage must also determine the initial wealth of their children who will hereafter begin their own independent household. The decision to obtain secondary schooling is a discrete choice and this is the sense in which human capital accumulation is a lumpy investment. Either the adolescent's time is fully devoted to education, or to work. Human capital investment is not chosen at the margin.

Let $V^{4,0}(a, u, l_0, l_{1,i}; S)$ represent the value of being a household of type $(a, u, l_0, l_{1,i})$ that does not allocate the time of its adolescent child to school. Let $V^{4,1}(a, u, l_0, l_{1,i}; S)$ represent the value of the decision to seek secondary schooling for such a child.

$$V^4(a, u, l_0, l_{1,i}; S) = \max \{ V^{4,0}(a, u, l_0, l_{1,i}; S), V^{4,1}(a, u, l_0, l_{1,i}; S) \} \quad (2)$$

Equation 3 illustrates the value of not investing in further human capital for the child. Households with adolescent children that are not in school have a higher labour endowment due to the available time of the child. Let ω represent the relative productivity of a child. Next, let V^1 represent the value function for the child, and let $\psi \in (0, 1)$ be the weight placed by the household on its child's expected discounted lifetime utility. The initial wealth of the child, $a_0 \geq 0$, is a transfer from the parent. Finally, notice that the initial distribution of labour productivity for the child is drawn according to the initial distributions for l_0 and l_1 , which are $\pi^{u,0}$ and $\pi^{u,1}$, respectively.

$$\begin{aligned}
V^{4,0}(a, u, l_0, l_{1,i}; S) &= \max \left(\theta^4 U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^u V^{t+1}(a', u, l_0, l'_{1,j}; S') \right. \\
&\quad \left. + \beta \psi \sum_{i=1}^{N(l_0)} \pi_i^{u,0} \sum_{j=1}^{N(l_1)} \pi_j^{u,1} V^1(a_0, u, l_i, l_j; S') \right) \\
&\quad \text{subject to} \\
a' + a_0 + c &\leq w^u (1 + \omega) (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, a_0 \geq 0.
\end{aligned} \tag{3}$$

In (4) we describe the decision to enroll the child in secondary school. The cost of this is $\tau^1 \geq 0$. The decision to invest in this level of human capital implies the loss of the labour income associated with the time endowment of the adolescent. There is then both a direct cost to secondary schooling for a child, τ^0 , as well as an indirect cost of $w^u \omega (l_0 + l_{1,i})$ that is the opportunity cost of the child's time. Notice that the child's initial labour endowment is now drawn from a distribution for skilled workers.

$$\begin{aligned}
V^{4,0}(a, u, l_0, l_{1,i}; S) &= \max \left(\theta^4 U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^u V^{t+1}(a', u, l_0, l'_{1,j}; S') \right. \\
&\quad \left. + \beta \psi \sum_{i=1}^{N(l_0)} \pi_i^{s_0,0} \sum_{j=1}^{N(l_1)} \pi_j^{s_0,1} V^1(a_0, u, l_0, l_{1,i}; S') \right) \\
&\quad \text{subject to} \\
\tau^0 + a' + a_0 + c &\leq w^u (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, a_0 \geq 0.
\end{aligned} \tag{4}$$

2.1.3 Unskilled households in periods 5 – 8

After their children have become independent, in $t = 4$, the problem of an unskilled household in the four remaining periods of its life is similar to that it faced in its initial period. Each period there is labour income from work, whether as an agricultural worker or an unskilled labourer in the manufacturing sector. This is allocated between consumption and saving, except in the last period of life where there is no saving.

$$\begin{aligned}
V^t(a, u, l_0, l_{1,i}; S) &= \max \left(U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^u V^{t+1}(a', u, l_0, l'_{1,j}; S') \right) \\
\text{subject to } a' + c &\leq w^u(l_0 + l_{1,i}) + (1 + r(S))a \\
a' &\geq \underline{a}, t = 5, 6, 7.
\end{aligned} \tag{5}$$

The value in the final period is given below.

$$\begin{aligned}
V^8(a, u, l_0, l_{1,i}; S) &= U(c) \\
\text{where } c &\leq w^u(l_0 + l_{1,i}) + (1 + r(S))a
\end{aligned} \tag{6}$$

2.2 Skilled workers

The decisions facing skilled workers are analogous to those undertaken by unskilled workers with two noteworthy differences. In the first period of their adult lives they may either work or invest in further human capital. Thus an indirect return to secondary education is that it permits the household, in its first adult period, to seek further post-secondary education. Skilled households that acquire further education increase their labour productivity relative to households that do not invest in post-secondary education. Let s_1 be the labour productivity of a skilled household with three periods of education, and s_0 the productivity of a skilled household with only secondary education.

Another difference relative to unskilled households is that the manufacturing wage varies over time as a function of the aggregate state of the economy. The remaining periods of life following the same timing and decisions as faced by unskilled households and described above.

2.2.1 Skilled workers in period 1

In the first period of their lives, skilled workers have the opportunity to seek additional human capital investment which must be financed by borrowing. The cost of post-secondary school education is τ^1 . This is feasible only if they have sufficient wealth. Let $V^{1,0}(a, s_0, l_0, l_{1,i}; S)$

represent the value of being a skilled household that does not invest in further education, and let $V^{1,1}(a, s_0, l_0, l_{1,i}; S)$ be the value of a skilled household that accumulates human capital.

$$V^1(a, s_0, l_0, l_{1,i}; S) = \max \{V^{1,0}(a, s_0, l_0, l_{1,i}; S), V^{1,1}(a, s_0, l_0, l_{1,i}; S)\} \quad (7)$$

The problem facing a skilled household that does not accumulate further human capital is given below.

$$\begin{aligned} V^{1,0}(a, s_0, l_0, l_{1,i}; S) &= \max \left(U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^2(a', s_0, l_0, l'_{1,j}; S') \right) \\ \text{subject to } a' + c &\leq w(S) s_0 (l_0 + l_{1,i}) + (1 + r(S)) a \\ a' &\geq \underline{a}. \end{aligned} \quad (8)$$

Notice that the real wage for skilled households is a function of the aggregate state. The problem facing a skilled household, in its first adult period, that does pursue further education is described below.

$$\begin{aligned} V^{1,1}(a, s_0, l_0, l_{1,i}; S) &= \max \left(U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^2(a', s_1, l_0, l'_{1,j}; S') \right) \\ \text{subject to } \tau^1 + a' + c &\leq (1 + r(S)) a \\ a' &\geq \underline{a}. \end{aligned} \quad (9)$$

The cost of post-secondary education is τ^1 , and households that invest in such a level of human capital must devote the entire period to this activity. As a result, they earn no labour income.

2.2.2 Skilled households in periods 2 and 3

The decisions facing skilled households, when $t = 2, \dots, 8$, are analogous to those facing an unskilled household. Thus equation (1) applies in periods 2 and 3, subject to the labour income term $w^u(l_0 + l_{1,i})$ being replaced by $w(S) s_e(l_0 + l_{1,i})$ where $e = 1$ or 2 , and π^u replaced by π^s or π^s .

$$\begin{aligned}
V^t(a, s_e, l_0, l_{1,i}; S) &= \max \left(\theta^t U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^{t+1}(a', s_e, l_0, l'_{1,j}; S') \right) \\
\text{subject to } a' + c &\leq w(S) s_e (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, t = 1, 2, 3.
\end{aligned} \tag{10}$$

2.2.3 Skilled households in period 4

In the fourth period of the household, the decision to educate their children further is described using a version of (2) - (4), where again π_{ij}^u , for the parent, is replaced by $\pi_{ij}^{s_e}$, where $e = 0$ or 1 , and labour income is $w(S) s_e (l_0 + l_{1,i})$. The discrete choice is described by (11),

$$V^4(a, s_e, l_0, l_{1,i}; S) = \max \{ V^{4,0}(a, s_e, l_0, l_{1,i}; S), V^{4,1}(a, s_e, l_0, l_{1,i}; S) \} . \tag{11}$$

where the value of not investing in further education for the child is

$$\begin{aligned}
V^{4,0}(a, s_e, l_0, l_{1,i}; S) &= \max \left(\theta^4 U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^{t+1}(a', s_e, l_0, l'_{1,j}; S') \right. \\
&\quad \left. + \beta \psi \sum_{i=1}^{N(l_0)} \pi_i^{u,0} \sum_{j=1}^{N(l_1)} \pi_j^{u,1} V^1(a_0, u, l_i, l_j; S') \right) \\
&\text{subject to} \\
a' + a_0 + c &\leq (w(S) s_e + \omega w_u) (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, a_0 \geq 0.
\end{aligned} \tag{12}$$

The value of further education for the child is shown below.

$$\begin{aligned}
V^{4,1}(a, s_e, l_0, l_{1,i}; S) &= \max \left(\theta^4 U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^{t+1}(a', s_e, l_0, l'_{1,j}; S') \right. \\
&\quad \left. + \beta \psi \sum_{i=1}^{N(l_0)} \pi_i^{s,0} \sum_{j=1}^{N(l_1)} \pi_j^{s,1} V^1(a_0, s_1, l_0, l_{1,i}; S') \right) \\
&\text{subject to} \\
\tau^0 + a' + a_0 + c &\leq w(S) s_e (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, a_0 \geq 0.
\end{aligned} \tag{13}$$

2.2.4 Skilled households in periods 5 - 8

After children have left the household, the skilled workers problem is similar to that of unskilled workers. The value function for skilled workers in the fifth through the seventh periods of life is

$$\begin{aligned}
V^t(a, s_e, l_0, l_{1,i}; S) &= \max \left(U(c) + \beta \sum_{j=1}^{N(l_1)} \pi_{ij}^s V^{t+1}(a', s_e, l_0, l'_{1,j}; S') \right) \\
\text{subject to } a' + c &\leq w(S) s_e (l_0 + l_{1,i}) + (1 + r(S)) a \\
a' &\geq \underline{a}, t = 5, 6, 7.
\end{aligned} \tag{14}$$

In the last period, the value is given below.

$$\begin{aligned}
V^8(a, s_e, l_0, l_{1,i}; S) &= U(c) \\
\text{where } c &\leq w(S) s_e (l_0 + l_{1,i}) + (1 + r(S)) a
\end{aligned} \tag{15}$$

This completes our description of the lifetime choices of skilled households.

2.3 Production

The aggregate production function in the manufacturing sector is based on the form used by Krussel, Ohanian, Rios-Rull and Violante (1998),

$$F(u, s, k) = \left[\mu u^\sigma + (1 - \mu) (\lambda k^\rho + (1 - \lambda) s^\rho)^{\frac{\sigma}{\rho}} \right]^{\frac{1}{\sigma}}.$$

The aggregate capital stock is k , and the quality-adjusted level of skilled labour is s while u is the quality-adjusted level of unskilled labour. The elasticity of substitution between capital and skilled labour is ρ . The elasticity of substitution between unskilled labour and capital, or skilled labour, is σ .

The total stock of skilled labour is the sum of the labour endowments of skilled households weighted by their education levels, s_0 and s_1 . The total stock of unskilled labour is the sum of the labour endowments of unskilled households not working in the agricultural sector. Each period this fraction is determined by setting the number of unskilled manufacturing workers

to a level such that, given the stock of physical capital and skilled labour, the marginal product of unskilled labour in manufacturing is equal to the agricultural wage w .

Define the household state vector, now appended by the age t of the household, as $(h, t) = ((a, s, l_0, l_1), t)$ where $h_2 = s$ with $s \in \{u, s_0, s_1\}$ and $t = 1, \dots, 8$. Let $a' = A(h, t; S)$ define the decision rule for savings and $c = C(h, t; S)$ the decision rule for consumption. In addition, let $I_{s_0}(h, 4; S) = 1$ if the household sends its child to secondary school, and set this indicator function to 0 otherwise. Next, if a young skilled household chooses to acquire a university education, let $I_{s_1}(h, t; S) = 1$ where $h_2 = s_0$. We now describe the evolution of the distribution of households over time. (To be completed)

3 Results

The model is calibrated as follows. Estimates of labour earnings processes are used to determine the parameters of the labour endowment process. The aggregate production function is parameterised using Krusell et al. The educational parameters are chosen to reproduce conventional estimates of the return to human capital accumulation. Next, an initial distribution of wealth is inferred from data on the distribution of consumption in agricultural households in poor countries.

There are two sets of results. First, we contrast how the model's aggregate predictions with a benchmark model where investment in human capital may vary infinitesimally, that is when there are no discrete choices. Moreover, in this benchmark model we also abstract from borrowing constraints which serve to amplify the effect of discrete choices in the present environment. This illustrates the extent to which the approach to human capital investment taken here implies very different predictions about the path of economic development. When the distribution of income is characterised by a large number of poor households with consumption close to subsistence levels, the transition to the steady state is much slower than in the benchmark growth model. Furthermore, the rate of economic growth is non-monotonic, initially rising with the level of economic development. Investment rates initially increase with output, as do real interest rates. An economy with the same per capita income as another, but with greater inequality, in particular with more poverty, will have slower human

capital accumulation. The intergenerational movement from unskilled to skilled labour, and within skilled labour to workers with high skills, will be slower. This, in turn, will reduce the returns to physical capital accumulation, and result in slower economic development.

The second set of results reports the model's predictions for the persistence of consumption, income and skills across generations. We show that poverty is a very persistent state for poor households. (To be added.)

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