Private Incentives versus Class Interests: A Theory of Optimal Institutions with An Application to Growth

Levon Barseghyan and Ani Guerdjikova*

Cornell University

lb247@cornell.edu and ag334@cornell.edu

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Abstract

We build a dynamic political economy model with a two-class society: workers and the elite. A key feature of the model is that the formation of the elite, the rate of innovation, taxes and public spending are endogenous. Differently from most of the literature on institutions and growth which emphasizes the conflict between different classes, we focus on the tension between private incentives of the members of the elite and their class interests. Our model explains the observed differences in colonization strategies by showing how the optimal choice of institutions depends on the initial conditions faced by colonizing powers. The model also creates a mapping between institutions and economic outcomes which is consistent with the observed differences in the patterns of economic growth.

Keywords: elite, class interests, institutions, optimal institutions, growth endogenous growth, patterns of growth.

JEL classification: O43, P16

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1 Introduction

Institutions governing the behavior and interaction of economic and political agents matter. As recently confirmed by the empirical studies of Hall and Jones (1999) and Acemoglu et al. (2001 and 2002) better institutions lead to better long run economic outcomes. Acemoglu et al. (2001 and 2002) show that institutional quality in former colonies can be traced back to factors that affected colonization strategy of European powers, such as settler mortality and indigenous population density. They argue that the connection between current institutions and settler mortality (population density) exists because institutional arrangements are generally persistent. This argument is supported, among others, by Engermann and Sokoloff (2000) who show that colonizers tended to introduce more efficient institutions in colonies which were technologically more developed and had better access to markets. These former colonies also tend to have better institutions and a higher rate of economic growth nowadays. This literature also documents "the reversal of fortune": some ex-colonies, that initially had very prosperous economies, nowadays tend to be poorer and less technologically advanced.

Historical evidence shows that institutions have an impact not only on the rate, but also on the source of economic growth. The most stark examples are probably Russian Empire and Soviet Union, which, at times, had tremendous rates of public investment, but low rates of productivity growth, as opposed to the market economies whose growth has been mainly driven by free enterprise and continuous innovation. However, while inferior to the growth of free market economies, during most part of the 20th century the Soviet economic growth dominated that of many developing and less developed countries, which did not have free markets.[1]

In this paper, we take a stand that institutions are persistent and study their effect on political and economic outcomes. We build a dynamic political economy model with a two-class society: workers and the elite. A key feature of the model is that the formation of the elite, the rate of innovation, taxes and public spending are endogenous. Also, differently from most of the literature

[1] Baumol (1990) provides a historical analysis of the influence of institutions on entrepreneurship and innovation. More recent examples include East Asian tigers, whose economies featured a high degree of protectionism and government intervention. Young (1995) argues that the economic growth in these countries during the 1980th and early 1990th, was mostly due to capital accumulation and (public) investment in human capital, as opposed to increases in productivity.

on institutions and growth which emphasizes the conflict between different classes, we focus on the tension between private incentives of the members of the elite and their class interests. Our model explains the observed differences in colonization strategies by showing how the optimal choice of institutions depends on the initial conditions faced by colonizing powers. The model also creates a mapping between institutions and economic outcomes which is consistent with the observed differences in the patterns of economic growth.

The point of departure for our model is the assumption that economic and political power is exclusively concentrated in the hands of the elite. The elite decides upon the rate of innovation - the number of new firms (or varieties to be produced) in the economy, and the fiscal policy - the labor tax and public investment. Workers supply labor and consume.

We use the model of Baron and Ferejohn (1989) to capture the conflict of interests between the individual members of the elite and their class. We extend this model to the case of two chambers: chamber E which decides on the rate of innovation, and chamber G which decides upon fiscal policy. In each chamber a fraction of the elite - the winning coalition - decides on the respective policy. The winning coalition in chamber E gets to extract the rents from innovation, while the costs, in terms of increased competition are borne by all members of the elite. Similarly, the winning coalition in chamber G expropriates the tax revenue net of public spending. Because taxes are distortionary, taxation imposes a cost on all members elite. Hence, the decision makers do not fully internalize the negative effect of their policy choices on the welfare of the elite as a whole. The institutional design determines the size of the winning coalition - the majority necessary to enforce a given policy in the respective chamber. These variables capture the degree of the elite's consolidation. The higher is the required majority in a given chamber, the more likely it is that the chamber will enact policies that are beneficial to the elite as a whole. The institutional design also specifies the degree of separation of power: the overlap between two winning coalitions.

Our first result concerns the relation between the degree of consolidation of the elite and patterns of economic growth when there is no separation of powers, i.e., all decisions are made by the same fraction of the elite. When this fraction is small, the incentives to innovate are high, and, thus, growth occurs mainly through high rates of innovation. However, the incentives to misappropriate fiscal revenues are also high, leading to high taxes and low public spending. When the degree of consolidation is high the opposite is true, which implies that the growth occurs mainly though public investment while innovation is suppressed.

Next, we study the relation between institutions and growth when there is substantial separation of powers, i.e. the winning collations are chosen independently. In this scenario, for a given level of fiscal consolidation of the elite, an increase in economic freedom leads to higher rates of innovation. Higher rates of innovation increase demand for labor. In addition, they also dilute the future benefits from public investment. Consequently, public investment falls. For a given level of economic freedom, an increase in fiscal consolidation naturally leads to more prudent fiscal policies, and, in particular lower taxes. This stimulates labor supply and makes innovation more attractive. Consequently, the rate of innovation increases.

These results are consistent with historical differences in patterns of economic growth. The model predicts that market economies will grow mostly via innovation, and that dictatorships (i.e., countries with a high fiscal consolidation and low economic freedom), e.g., USSR during the rule of Yoseph Stalin, will grow via high rates of public investment. The growth in market economies, in general, is substantially higher than in dictatorships. Our model also show that the worst growth rates would be observed in countries that are corrupt (i.e., have low degree of fiscal consolidation that leads to redistributive taxation and low public spending) and lack economic freedom.

The second part of the paper examines the optimal choice of institutions by the elite. Clearly, when no constraints are imposed on degree of consolidation, the elite will choose full consolidation. Then all decisions are made unanimously, which maximizes the exante payoff of the each member of the elite. However, when full consolidation with respect to economic policy is not achievable,[2] the elite will optimally reduce the level of fiscal consolidation. This happens because low fiscal consolidation leads to higher taxation and lower labor supply, which in turn lowers the incentives to innovate. In other words, by reducing fiscal consolidation the elite emposes checks and balances on those in charge of economic policy, forcing them to behave more in accord to the interests of the elite as a whole. This logic also explains an imporant dimension of the choice of optimal institutions – the decision on separation of powers. It is optimal for the elite to separate powers, i.e. to create conflict of interests among those in charge of economic and fiscal policies.

[2] We consider two different scenarios: in the first, we assume that achieving unanimity is prohibitively costly. In the second, we assume that workers have to be guaranteed a certain minimal level of life-time utility in order to prevent a revolution. We then show that this imposes an upper bound on the degree of consolidation of the elite.

Optimal institutions are not necessarily efficient, as they do not take into account the welfare of the workers. We extend our discussion of optimal institutional choice to encompass the interests of workers. We require that institutions chosen by the elite yield a minimum life time utility to the working class. Note that workers benefit both from higher rate of innovation and more prudent fiscal policies. This implies that to deliver the required life time utility to the workers, the elite either should allow for economic freedom or impose fiscal consolidation. When the required life time utility for workers is sufficiently high, the only way to deliver it is via economic freedom. When the elite is forced to deliver economic freedom, it may optimally choose to reduce fiscal consolidation for the same reasons as described above.

Our results on the optimal choice of institutions explain the dependence of institutions on initial conditions, and, in particular, the "reversal of fortunes." Consider, for example, a country that initially enjoys higher productivity. In such country the elite will optimally choose lower degree of economic freedom, as this would be enough to satisfy the demands of the workers. As a result this country will feature lower growth rates. Thus, the initial advantage productivity will disappear and the country will fall behind. Naturally, if a country initially has "extractive institutions" (Acemoglu et al., 2002), i.e., the society is geared to supress the workers demands. The required minimal life-time utility of the workers is low and the elite will choose low economic freedom.

The rest of the paper is organized as follows. In Section 2 we briefly review the related literature. In Secion 3 we present and solve our model. Section 4 describes the effect of institional design on economic and political outcomes. Section 5 discusses the optimal choice of institutions. We conclude in Section 6. All proofs are collected in the Appendix.

2 Some Historical Facts and Literature Review

In this section, we review some stylized historical facts, which provide a motivation for the structure of our model, and review the relevant literature.

2.1 Historical Facts

In most societies, economic opportunities are a function of political power. Hence, the access of new entrepreneurs to production opportunities is usually controlled by elites. This might include the necessity of specific qualification, licenses and permissions in order to start a certain business, access to credit, and / or the social and political connections necessary to establish a new enterprise. In any of these cases, the elite (implicitly or explicitly, through legal rules or informal communication) can decide on whether to grant access of new members to its resources and allow them to actively participate in the political and economic process. The Industrial Revolution provides multiple examples. In England of the 18th century, the large industrials started as small producers who needed to establish connections with the merchants operating on the markets for final goods, in order to gain independence and start a trade on their own, see Bowden (1925). In Prussia, the emergence of the coalition of "Iron and Rye" between the land aristocracy and the industrial class was necessary as a "compromise between modern industry and the feudal aristocratic groups in the country", see Gerschenkron (1943, p. 49).

It is a well documented fact that often elites not only have not actively participated in the process of implementation of new technologies, but have also vehemently opposed it, see historic references in Acemoglu and Robinson (2000). The explanation for this fact might be that, while understanding the economic gains from new technologies, the elite fears the emergence of new social classes which, by gaining economic capital might undermine its political power. An example of this is the attempt to industrialize the Russian economy by Peter I in the 18th century. The new linen- and wool-factories required skilled labor, which was sparse in Russia of that time. Hence, the manufacturers had to train their workers, recruited mainly from the peasantry, before they could actively participate in the production. At the same time, highly-skilled and trained workers had incentives to depart the factory and start a business on their own, see Daniel (1995). High incidence of such departures and lobbying against it by current producers, led to an Imperial intervention. Catherine II introduced a legislature which gave the land aristocracy hereditary rights and full control over their peasants, effectively providing full protection of the producing class from entry.

The studies of Sokoloff and Khan (1989) and Lamoreaux and Sokoloff (2005) indicate that the process of innovation indeed relies on ideas generated by outsiders to the industry. They present evidence from the US which shows that most of the inventions patented in the 19^{th} century came from independent inventors rather than from research and development activities inside the existing firms. They also demonstrate that most of the inventors not only sold patents to existing firms, but eventually established their own firms which used some of their own patents in the production.

The structure of our model is motivated by these stylized facts.

2.2 Related Literature

There is a large literature which examines the relationship between inequality and growth, such as Bertola (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994). They show that more unequal societies generate more redistribution. If redistribution is costly or diminishes incentives to produce and accumulate capital, it has a negative impact on growth. In contrast, in cases in which redistribution is productive or helps to eliminate market imperfections, it might enhance growth, see Saint-Paul and Verdier (1993), Galor and Zeira (1993) and Perotti (1993). In our model, inequality is exogenously given, but can be reduced if the elite decides to admit new members. Diminishing inequality, is equivalent to increasing the access to production possibilities and hence, promotes growth. Since political decisions are reserved to the elite, we try to identify conditions under which the elite will endogenously decide to promote equality and growth.

Our model falls into category of models with endogenous political participation, such as Ades (1996) and Gradstein and Justmann (1995). Whereas Ades (1996) assumes that to enter the elite, it is necessary to incur some fix cost, Gradstein and Justman (1995) introduce an income franchise which is exogenously fixed. Acemoglu and Robinson (1996, 2000a, 2000b, 2001) assume that the suffrage is determined by the elite. They argue that the elite faces a trade-off between preserving its political power and obtaining superior economic outcomes. They derive conditions, under which inefficient political institutions which give rise to suboptimal market structures and, hence to low economic growth, can be very persistent. If, however, the economic benefits from a reform are sufficiently large or if the elite faces the threat of a revolution, the institutional change can be initiated by the elite itself. Bourgignon and Verdier (2000) assume that the right to vote depends positively on the level of education of an individual. Allowing for transfers between the rich (who can always afford education) and the poor (who face imperfections in credit markets and therefore cannot study, unless provided with sufficient funds), they find that the rich will face the trade-off between promoting growth by educating the society and creating political competition. They demonstrate that two steady states can obtain: one, in which the education level and the rate of growth are low and one with high level of education and high level of growth.

Similarly, our model also explores the trade-off between economic growth and increased political competition. However, differently from the models cited above, we do not think of the elite as being homogenous. The trade-off between the interests of an individual member of the elite and the elite as a whole is what drives our results. In this sense, our paper is closer in spirit to the work of Lizzeri and Persico (2004) and Jack and Lagunoff (2006). Lizzeri and Persico (2004) model a society with limited suffrage. Both papers use the idea that an increase of the suffrage effectively changes the preferences of the pivotal voters. In Lizzeri and Persico (2004), the policy is determined by a simple majority vote. Hence, if the change in preferences is such that the resulting allocation benefits a majority of the members of the elite, the elite will prefer to extend the suffrage. In Jack and Lagunoff (2006), the decision in each period is made by a dictator, who also denominates his successor. While staying in power forever is always a potential choice, a dictator might decide to delegate authority if he faces commitment problems. Jack and Lagunoff (2006) demonstrate that this mechanism can be equivalently represented by an endogenous enfranchisement rule and state conditions under which the resulting enfranchisement will be monotone in time, i.e. larger and larger parts of the population will be allowed to vote.

While these models make use of the heterogeneity in the population of (potential) voters, in our model all elite members have ex-ante identical preferences. Only ex-post, when they are exogenously divided into the group of pivotal and the group of non-pivotal voters, differences in preferences emerge. Hence, adding new voters does not change the preference structure of the elite in our model. Instead, the admission of new members to the elite creates short-run economic benefits for the pivotal voters, but increases the competition for resources (labor) and political power in the long-run for the elite as a whole. This trade-off, which is not present in the rest of the literature, is the key to our result.

Persson, Roland and Tabellini (2000) examine the influence of different voting mechanisms (parliamentary versus presidential regimes) on public policy outcomes. They demonstrate that separation of powers together with a large majority required to implement a certain policy lead to more efficient outcomes, i.e. less public spending and a lower level of taxation. Our approach is similar to theirs in that we also examine the impact of the required majority and of separation of powers on economic outcomes. Our model differs in two ways: first, we consider a broader range of policies; second we assume that the legislature is controlled by the elite. The latter assumption implies that an increase in the required majority with respect to a specific issue combined with separation of powers increases the elite's welfare. Whether or not economic efficiency increases, however, depends on whether or not the interests of the elite class with respect to the specific policy issue are aligned with those of the workers. We analyze this question in detail in Sections 4.1 and 5.

Our analysis of optimal institutions is related to the literature on voting mechanisms. Barbera and Jackson (2000) provide a model of "self-stable" majority rules, defined by two parameters: majority required to make day-to-day decisions and majority required to change the constitution. A self-stable rule is defined as a rule, which would be chosen under the voting procedure it itself prescribes. They show that under certain assumptions on the voters' preferences, the rule requiring simple majority in day-to-day decisions and unanimity to change the constitution is self-stable. This provides an example of how the required majority might be determined endogenously and still differ from 1. Aghion, Alesina and Trebbi (2004) discuss how the required majority could be optimally determined under the veil of ignorance. Gersbach (2005) argues that the optimal majority should depend on the particular decision. He computes optimal flexible majority rules.

As in Aghion, Alesina and Trebbi (2004), in our model, the decision of the elite members about optimal institutions occurs under the veil of ignorance, i.e. while their specific role in the legislature has not yet been determined. Similarly to Gersbach (2005), we allow the elite to choose different majority rules governing the decision process on economic and fiscal policy. We find that without additional constraints, the optimal institutional arrangement requires unanimous decision making in both chambers. However, if the elite has to take workers' welfare into account, the optimal voting rule is flexible, in the sense of Gersbach: since workers simultaneously benefit from prudent tax policy and from a high level of public investment, the optimal required majority in chamber G is higher than the required majority in chamber E. Finally, the derived optimal voting rule is self-stable in the sense of Barbera and Jackson (2000): given the informational structure of our model, the decision about the institutional design are always made under unanimity. Hence, the elite members will never have an incentive to change the institutional design determined in the very first period.

3 The Model

3.1 Preliminaries

Our goal is to construct a model that captures the conflict between private interests of the individual members of the elite and the interests of their class as a whole and to use this model to better understand the relation between institutions and political outcomes.

We assume that economic and political power are exculsively concentrated in the hands of the elite. The access to economic power and access to political power go hand in hand, i.e., each elite member has access to both. Moreover, pursuing new economic opportunities requires political power, and, at the same time, wealth leads to political power.

We focus on two major sources of conflict between the members of the elite and their class. The first one is as follows: While the elite as a whole seeks to protect their monopoly over the means of production and over (redistributive) political process, each member of the elite wants to maximize her own welfare. This implies that while the elite as a whole opposes new entrepreneurial activity, each member is eager to initiate one, as long as she gets the rents from it. In the model below, members can initiate new entrepreneurial activity and extract a part of the resulting rents. However, such activity increases labor demand and, therefore, reduces the profits of the rest of the elite. In addition, new entrepreneurial activity delutes the political power of the existing members of the elite, as their relative wealth declines.[3] We think of the degree to which the elite can control the amount of new entrepreneurial activity as a key institutional feature. Accordingly, we paramterize it in our model.

The second conflict is over the level, the use and the distribution of fiscal revenues. The members of the elite, who find themselves in charge of fiscal policies, have an incentive to levy higher taxes, cut the spending on public investment and pocket the remaning revenues. However, this is harmful for the elite as whole, because higher taxes reduce labor supply and because lower public spending reduces future productivity and profits. We think of the degree to which the fiscal policies are capturing the interests of the elite as a whole as another key institutional variable and paramterize it in our model.

[3](FOOTNOTE: historically this is a very good and smart assumption– that's how in many cases elites have evolved!!!!!!).

These two conflicts pitch the interests of two distinct elite groups against each other - those involved with new entrepreneurial activity and those engaged in fiscal policy making. As we show below, it is this conflict that is key for understanding the forces behind a number of empirical phenomena that we seek to explain.

3.2 The Model: an Overview

There are two classes — workers and the elite. At time t workers comprise of a continuum of individuals with measure one. They supply labor and consume. The elite consists of a continuum of individuals with measure E_t. We assume that the number of workers is large relative to E_t**, i.e.** E_t **has measure** 0 **relative to the size of the working class.** The members of the elite control the means of production and the political process. We will often refer to the members of the elite as entrepreneurs.

At the beginning of each period, the elite determines: (i) the rate of innovation, e, measured as the increase in the number of operating firms, (ii) the tax on labor, τ, and (iii) the share of labor dedicated to public investment, g. After these variables are determined, workers decide on their labor supply and entrepreneurs make their production decisions.

The policy triplet (e, τ, g) is an outcome of the political process. In the beginning of the each period, one of the members of the elite is randomly chosen to control the establishment of new firms. Establishing a firm requires a new idea that some randomly chosen workers possess. Once a new firm is established, the worker who generated the idea receives a fraction $(1 - \alpha)$ of the profit earned in the first period of operation. The remaining part of these profits is distributed to the members of the elite. Some of the workers, $h(e)$, who generated the new ideas become owners of the newly created firms and joining the elite from the next period on.

The second decision of the political authority concerns the fraction of labor dedicated to public investment, g_t.[4] This investment increases the labor productivity in the next period by a factor of $n(g_t)$.

The third decision concerns the labor tax rate. Tax revenues are used in two ways: to finance public investment and make transfers to the members of the elite.

After the triplet (e_t, g_t, τ_t) is determined in the political process, the economic agents take these

[4] It can take the form of investment into education, investment into infrastructure, etc.

variables as given and choose their consumption and production plans. The wage and the price of the consumption good are determined so that market clears.

We first discuss the decisions of the entrepreneurs and households, as well as the market equilibrium, taking e_t, τ_t and g_t as given. Then, we describe the political decision-making process and solve for the political equilibrium in this economy.

3.3 The Firms

The number of entrepreneurs active in the market at time t is given by $e_t E_t$. Each firm i produces a homogenous consumption good according to the technology:

$$y_{it} = (L_t x_{it})^{\frac{1}{\lambda}},$$

where L_t is the labor producitivity, x_{it} is the amount of labor employed and $\lambda > 1$ is the parameter capturing the degree of decreasing returns to scale. The firm's maximization problem is:

$$\pi_{it} = \max_{x_{it} \in \mathbb{R}_0^+} \left\{ L_t^{\frac{1}{\lambda}} x_{it}^{\frac{1}{\lambda}} - x_{it} w_t \right\},$$

where w_t is the wage. In the optimum,

$$x_{it}^* = \left(\lambda L_t^{-\frac{1}{\lambda}} w_t\right)^{\frac{\lambda}{1-\lambda}} \text{ and } \pi_{it}^* = \left(1 - \frac{1}{\lambda}\right) (L_t x_{it}^*)^{\frac{1}{\lambda}}.$$

3.4 Workers

The (representative) household derives utility from consumption C_t and labor l_t according to the following utility function:

$$U_t(C_t, l_t) = C_t - \frac{1}{\lambda} \psi_t \frac{1}{\psi + 1} l_t^{\psi+1}$$

The time-dependent coefficient ψ_t, is assumed to grow at the rate $L_t^{\frac{1}{\lambda}} \cdot E_t^{1-\frac{1}{\lambda}}$.[5]

The household faces the following budget constraint:

$$s.t.\ C_t = (1 - \tau_t) \cdot w_t \cdot l_t$$

where w_t is the wage and τ_t is the tax level. The household's maximization problem implies that its labor supply is given by

$$l_t^* = \left[\frac{(1 - \tau_t) \cdot w_t}{\bar{\psi} \cdot \psi_t}\right]^{\frac{1}{\psi}}.$$

[5] As we show later, this is the growth rate of output.

3.5 Market Equilibrium

Recall that only the fraction $1-g_t$ of labor is used to produce consumption good. Hence, market clearing implies that

$$e_t \cdot E_t \cdot x_{it}^* = (1-g_t) \cdot l_t^*.$$

In equilibrium, labor supply, wages and profits of the entrepreneurs are given by:

$$\begin{aligned} l_t^* &= (1-\tau_t)^{\frac{1}{\psi-\frac{1}{\lambda}+1}} \left[\frac{1-g_t}{e_t}\right]^{\frac{\frac{1}{\lambda}-1}{\psi-\frac{1}{\lambda}+1}} \\ w_t^* &= \frac{1}{\lambda}\left[\frac{1-g_t}{e_t E_t} l_t^*\right]^{\frac{1}{\lambda}-1} L_t^{\frac{1}{\lambda}} \end{aligned}$$

$$\pi_{it}^* = \left(1-\frac{1}{\lambda}\right)\left[\frac{1-g_t}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}} (1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}} \left(\frac{L_t}{E_t}\right)^{\frac{1}{\lambda}}$$

3.6 Political Institutions

The decisions about (e_t, g_t, τ_t) are made by a legislature, which is fully controlled by the elite. It consists of two chambers: the first, indexed by E, controls the rate of innovation e_t; the second, indexed by G, decides on the level of public investment g_t and on the tax rate τ_t.

We assume identical structure for both chambers. Each chamber consists of N legislators, each of whom represents the interests of an equal share $\frac{E_t}{N}$ of the elite in time t. Legislator $n \in \{1...N\}$ in chamber E represents the interests of the same subset of elite members as legislator n in chamber G. Legislators maximize the utility of the elite members whom represent.

We use a version of the model by Baron and Ferejohn (1989) to describe the bargaining procedure in each of the chambers. In each period, one of the legislators in a given chamber is chosen at random to make a proposal. A proposal consists of a value for each of the variables on which the chamber has to decide (e_t for chamber E; g_t and τ_t for chamber G), as well as a transfer scheme, which specifies the fraction of the transfers received by each member of the elite (if any): in the case of chamber E, the legislator has to specify the distribution of the profits from creating new firms; in the case of chamber G, the distribution of the tax revenue.

In order for a proposal to be implemented, the proposing legislator must acquire the support of $q^i N - 1$ legislators. The parameters q^i ($i \in \{E; G\}$) determine the majority necessary to implement a decision in the two chambers E and G.

We call a subset of legislators of size $q^i N$ whose votes are decisive for the acceptance of the proposal a *winning coalition* and denote it by WC_i, $i \in \{E; G\}$. Naturally, the proposing legislator is always a member of the winning coalition. A proposal is implemented only if all members of the winning coalition vote in favor of it and it is rejected otherwise.

We assume the following information structure: the decisions in the two chambers are made simultaneously. At the time at which a legislator n in chamber $i \in \{E; G\}$ makes a proposal or votes for a proposal in chamber i, he does not know whether legislator n in the other chamber is the proposing legislator or a member of the winning coalition. The beliefs that legislator n forms will depend on the *interconnectedness* of the two legislatures, i.e. *on the relative share of legislators whose votes are decisive in both chambers.*

We capture the level of interconnectedness by a parameter c, which describes the share of legislators representing identical fractions of the elite who participate in the winning coalitions in both chambers. We consider two cases:

1. The conditional probability of being represented in the winning coalition of chamber i (or by the proposing legislator) does not depend on whether the member of the elite is represented in the winning coalition of chamber j (or by the proposing legislator). That is, $c = q^E \cdot q^G$. We call this situation the case of independent chambers.

2. Each member of the elite who is represented in the winning coalition in chamber i is also represented by the winning coalition in chamber j. That is, $c = q^E = q^G$. We call this situation the case of fully-dependent chambers.

The winning coalitions in the two chambers are chosen at random. In scenario 1, we *independently and uniformly* draw two subsets of the set $\{1...N\}$, one of size Nq^E and one of size Nq^G.[6] We call these subsets WC_E and WC_G, respectively. For each winning coalition i, the proposing legislator is drawn *independently and uniformly* out of the members of the coalition.

In scenario 2, we draw uniformly subset of Nc elements out of the set $\{1...N\}$ and the members of this set become members of both WC_E and WC_G. The proposing legislators are drawn *independently and uniformly* out of the members of each coalition.

[6] That is, each legislator has a q^E (q^G) chance to be in the winning coalition of chamber E (G), and these chances are independent of each other.

The bargaining lasts for $M \geq 2$ rounds. In scenario 1, the winning coalitions and the proposing legislators are drawn identically and independently for each round of bargaining. In scenario 2, the winning coalitions remain the same for all rounds of bargaining, but the proposing legislators are chosen independently and uniformly in each round.[7] We assume that there are no costs of bargaining. If at the end of round M no agreement is reached in one of the chambers, then a random legislator in this chamber is chosen to make a default proposal, which is then implemented without voting. The default proposals can specify arbitrary values of e_t, g_t and τ_t, but is constrained in that the transfers must be equally divided among all members of the elite.

The mechanism according to which winning coalitions are formed in our model has the following property: each member of WC_i has equal probability of being the proposing legislator or a non-proposing member in WC_j. In particular, this probability does not depend on whether the legislator is proposing or not and is identical for both chambers and across all rounds of bargaining.

We are now ready to solve for the political equilibrium, i.e. we will determine the choices of the policy variables e_t^*, g_t^* and τ_t^*.

3.7 The Payoff Functions of the Proposing Legislators

In this section, we construct the payoff functions of the proposing legislators. First note that for given values of e_t, g_t and τ_t, the total amount of profits extracted from new firms is given by:

$$\text{Profits from New Firms } = E_t \alpha \left(e_t - 1\right) \pi_t \left(e_t; g_t; \tau_t\right), \tag{3.1}$$

whereas the tax revenue to be distributed is:

$$\text{Tax Revenue} = \left(\tau_t - g_t\right) w_t \left(e_t; g_t; \tau_t\right) l_t \left(e_t; g_t; \tau_t\right). \tag{3.2}$$

The proposing legislator in WC_i will distribute transfers so that the other members of WC_i are just indifferent between voting in favor of the proposal or receiving their outside option and will keep the remaining amount to himself. As we show in the Appendix, this implies that the proposing

[7] Note that there is a difference in how the winning coalitions are formed from round to round, depending on the scenario. While in scenario 1 the independence of the winning coalitions must be preserved across bargaining periods, in scenario 2 the full dependence must be maintained. Note that we could have modelled scenario 2 as the one in which all three policy variables are chosen in the same chamber. This would have not changed any of our results that we report further in the paper.

legislator maximizes the total surplus accruing to the members of WC_i, or, equivalently, the average expected payoff per member of the elite represented in WC_i.

Consider first the payoff of those members of the elite represented in WC_E. Each member of the elite receives his own profit, $\pi_t(e_t; g_t; \tau_t)$. Since no transfers are allocated to non-members of WC_E, on average, each member of the elite represented in WC_E receives $\frac{1}{E_t q^E}$-fraction of the profits from new firms given by 3.1. Furthermore, with probability $\frac{c}{q^E}$, a member of the elite represented in WC_E is also represented in WC_G and receives a share $\frac{1}{E_t q^G}$ of the total tax revenue given by 3.2.

Note that, in period $(t+1)$, each member of the elite has equal probability of being represented in WC_E and WC_G or being a proposing legislator, independently of his role in the legislature at time t. Hence, the payoff functions of all elite members are identical from $(t+1)$ on. $v_1(E_t; L_t)$ denotes the value function of a member of the elite at the beginning of period t for given values of the state variables E_t and L_t. The average expected payoff of an elite member represented in WC_E then becomes:

$$V_t^E(e_t; g_t; \tau_t) = \underbrace{\pi_t}_{\substack{\text{individual}\\ \text{profits}}} + \underbrace{\frac{\alpha}{q^E}(e_t - 1)\pi_t}_{\substack{\text{profits from}\\ \text{new firms}}} + \underbrace{\frac{c}{q^E}\frac{1}{q^G}\frac{1}{E_t}(\tau_t - g_t)w_t l_t}_{\text{expected tax revenue}} + \underbrace{\beta v_1(E_{t+1}, L_{t+1})}_{\text{continuation value}}. \tag{3.3}$$

Analogously, the average expected payoff of an elite member represented in WC_G is given by:

$$V_t^G(e_t; g_t; \tau_t) = \underbrace{\pi_t}_{\substack{\text{individual}\\ \text{profits}}} + \underbrace{\frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1)\pi_t}_{\substack{\text{expected profits}\\ \text{from new firms}}} + \underbrace{\frac{1}{q^G}\frac{1}{E_t}(\tau_t - g_t)w_t l_t}_{\text{tax revenue}} + \underbrace{\beta v_1(E_{t+1}, L_{t+1})}_{\text{continuation value}}. \tag{3.4}$$

Denoting by e_t^* and $(g_t^*; \tau_t^*)$ the equilibrium actions of the proposing legislators in chambers E and G, we state the following:

Proposition 1 *In equilibrium, the actions of the proposing legislators in chambers E and G at time t satisfy:*

$$e_t^* = \arg\max_{e_t} V_t^E(e_t; g_t^*; \tau_t^*) \tag{3.5}$$

$$(g_t^*; \tau_t^*) = \arg\max_{g_t; \tau_t} V_t^G(e_t^*; g_t; \tau_t), \tag{3.6}$$

where V_t^E and V_t^G are defined as in 3.3 and 3.4, $E_{t+1} = h(e_t) \cdot E_t$ and $L_{t+1} = n(g_t) \cdot L_t$.

We now turn to the properties of the continuation value, $v_1(E_{t+1}; L_{t+1})$. Since at time t, each member of the elite has equal probability of being represented in WC_E and WC_G in $(t+1)$, in expectations, each member of the elite will receive a share $\frac{1}{E_{t+1}}$ of the transfers allocated in $(t+1)$.

Proposition 2 $v_1(E_{t+1}; L_{t+1})$ *can be written recursively as:*

$$v_1(E_{t+1}; L_{t+1}) = \pi_t + \alpha(e_{t+1} - 1)\pi_t + \frac{1}{E_t}(\tau_t - g_t)w_t l_t + \beta v_1(E_{t+2}; L_{t+2}). \tag{3.7}$$

3.8 The Political Equilibrium

We now make the following assumptions on the parameters α, λ and ψ, as well as on the form of the functions h and n, which guarantee that a Markov perfect equilibrium of the game exists and allow us to specify its form.

Assumption 1 *The following conditions guarantee that the goal functions of the proposing legislators satisfy concavity:*

1. $\alpha \geq q^E$; $\psi \geq \max\left\{\frac{2-\lambda}{\lambda}; 0\right\}$, *and* $\psi^2 \leq \frac{\lambda-1}{\lambda}$;

2. *The function* $h(e)$ *satisfies:*

   (i) $h'(e) > 0$

   (ii) $[h(e)]^{-\frac{1}{\lambda}}$ *is concave on* $[1; \bar{e}]$.

3. *The function* $n(g)$ *is increasing and concave on* $[0; 1]$.

Proposition 3 *(Existence) If Assumption 1 is satisfied, then there exists a Markov perfect equilibrium with the following properties:*

- $(e_t^*; g_t^*; \tau_t^*) = (e^*; g^*; \tau^*)$ *for all* $t \in \mathbb{N}$;

- *the continuation value function* $v_1(\cdot)$ *is given by*

$$v_1(E_t; L_t) = c_0 \left(\frac{L_t}{E_t}\right)^{\frac{1}{\lambda}},$$

where

$$c_0 = \left(1 - \frac{1}{\lambda}\right) \frac{\left[1 + \alpha(e^* - 1) + \frac{e^*}{\lambda-1}\frac{\tau^*-g^*}{1-g^*}\right](1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}} \left[\frac{1-g^*}{e^*}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}}{1 - \beta\left(\frac{n^*}{h^*}\right)^{\frac{1}{\lambda}}};$$

- e^* *satisfies the first-order conditions of problem 3.5:*

$$\begin{aligned}\frac{\partial V_t^E\left(e^*;g^*;\tau^*\right)}{\partial e^*} &= 0 \text{ if } e^* \in \left(1,\bar{e}\right); \\ \frac{\partial V_t^E\left(e^*;g^*;\tau^*\right)}{\partial e^*} &\leq 0 \ (\geq 0) \text{ if } e^* = 1 \ \left(e^* = \bar{e}\right);\end{aligned}$$

and g^* *and* τ^* *satisfy the first order conditions of problem 3.6:*

$$\begin{aligned}\frac{\partial V_t^G\left(e^*;g^*;\tau^*\right)}{\partial g^*} &= 0; \\ \frac{\partial V_t^G\left(e^*;g^*;\tau^*\right)}{\partial \tau^*} &= 0.\end{aligned}$$

Proposition 4 *If* v_1 *exists, then it is given by*

$$v_1\left(E_t;L_t\right) = c_0\left(\frac{L_t}{E_t}\right)^{\frac{1}{\lambda}}$$

where c_0 *is a constant.*

The uniqueness within this class of value functions depends, in general, on the concavity properties of the one-period payoff functions. For the range of parameter values that are used in this paper, uniqueness is confirmed numerically.

In the section we study the equilibrium properties of our model. In particular, we investigate the dependence of the policy variables e^*, g^* and τ^* on the institutional arrangement characterized by q^E, q^G and c.

4 Institutional Design and Equilibrium Outcomes

4.1 Institutional Design and the Conflicts of Interests

In the context of our model the effect of institutions on economic outcomes is fully captured by how the changes in parameters q^E, q^G and c alter the incentives of the proposing legislators in each chamber, i.e., how these parameters affect the payoff functions V_t^E and V_t^G and resulting equilibrium outcomes.

To start, consider the value function V_t^E in (3.3) and how it is affected by a change in the size of the winning coalition q^E. As q^E falls, i.e. the size of the winning coalition declines, the benefits from new entrepreneurial activity are shared among fewer members of the elite. In general, this implies that a decrease in q^E should lead to an increase in the rate of innovation, e^*. Whereas the cost of a higher e^*– an increase in the competition for labor and a future decline in the elite's political power[8]– are borne by every member of the elite, the benefits accrue only to those represented in the winning coalition WC^E. This difference between marginal private and marginal social benefits is captured by the factor $\frac{1}{q^E}$ in front of the term capturing the profits from new firms in the value function V_t^E. This difference diminishes as q^E gets closer to one. In other words, the parameter q^E captures the degree of conflict over the rate of innovation between the members of the elite standing to gain from new entrepreneurial activity and the elite as a whole.

Similarly, consider the value function V_t^G in (3.4) and how it is affected by a change in the size of the winning coalition q^G. As q^G falls, the tax revenues (net of public spending) are shared between fewer members of the elite. Intuitively, this means that a decrease in q^G should lead to an increase in the tax rate, τ^*, and to a decline in the public spending, g^*. Whereas the cost of a higher tax rate, a decline in labor supply, and the cost of lower public spending, lower future labor productivity, are borne by every member of the elite, the benefits are distributed only to those represented in the winning coalition WC^G. That is, the parameter q^G captures the degree of conflict over the fiscal policy between those who decide upon it and the rest of the elite.

In the next two sections we discuss in detail the effect of insitutional design on economic outcomes.

4.2 Equilibrium and Comparative Statics with Fully-Dependent Chambers

We start with the case in which the political chambers are fully dependent, i.e., $q^E = q^G = c \equiv q$. In what follows, to simplify exposition, we assume that the elite appropriates the total current profits of the new firms, i.e., $\alpha = 1$.

Recall that the proposing legislator in chamber E solves the following problem:

$$\max_{e_t} V_t^E\left(e_t; g_t; \tau_t\right) = \max_{e_t}\left\{\pi_t + \frac{1}{q}\left(e_t - 1\right)\pi_t + \frac{1}{q}\frac{1}{E_t}(\tau_t - g_t)w_t l_t + \beta v_1\left(E_{t+1}, L_{t+1}\right)\right\}; \qquad (4.1)$$

and the proposing legislator in chamber G solves

$$\max_{\tau_t, g_t} V_t^G\left(e_t; g_t; \tau_t\right) = \max_{\tau_t, g_t}\left\{\pi_t + \frac{1}{q}\left(e_t - 1\right)\pi_t + \frac{1}{q}\frac{1}{E_t}(\tau_t - g_t)w_t l_t + \beta v_1\left(E_{t+1}, L_{t+1}\right)\right\}. \qquad (4.2)$$

[8] The latter effect is due to an an increase E'.

Naturally, with fully dependent chambers, the objective functions of both legislators are identical.

The FOC for problem (4.1) is given by[9]

$$\frac{\partial V_t^E\left(e_t;g_t;\tau_t\right)}{\partial e_t}=\underbrace{\frac{\partial \pi_t}{\partial e_t}}_{(1)}+\underbrace{\frac{1}{q}\left[\pi_t+\left(e_t-1\right)\frac{\partial \pi_t}{\partial e_t}\right]}_{(2)}+\underbrace{\frac{1}{q}\frac{1}{E_t}(\tau_t-g_t)\frac{\partial\left(w_t l_t\right)}{\partial e_t}}_{(3)}+\underbrace{\beta\frac{\partial v_1}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial e_t^*}}_{(4)}=0 \quad (4.3)$$

The total effect of an increase in the rate of innovation e on $V_t^E\left(e_t;g_t;\tau_t\right)$ is given by (4.3) and can be decomposed into: (1) the decrease in the individual profits of the elite members; (2) the effect on the average rents from innovation: an increase in the number of new firms, combined with a decrease in the firms' average profits; (3) the increase in the expected tax revenue; and (4) the decline in the net present value of the elite's income due to increased future economic and political competition.

The direct effect of an increase in the majority required to make political decisions, q, on the optimal choice of the rate of innovation consists in reducing the share of total transfers accruing to each member of the winning coalition WC_E (terms (2) and (3) in (4.3)). Since, in the optimum, these transfers are increasing in e, (see the proof of Claim 1in the Appendix), for given values of τ and g, an increase in q leads to a decline in the rate of innovation e, just as our discussion in the previous section suggested.

The FOCs for problem (4.2) are given by:

$$\frac{\partial V_t^G\left(e_t;g_t;\tau_t\right)}{\partial g_t}=\underbrace{\frac{\partial \pi_t}{\partial g_t}}_{(1)}+\underbrace{\frac{1}{q}\left(e_t-1\right)\frac{\partial \pi_t}{\partial g_t}}_{(2)}+\underbrace{\frac{1}{q}\frac{1}{E_t}\frac{\partial}{\partial g_t}\left[(\tau_t-g_t)w_t l_t\right]}_{(3)}+\underbrace{\beta\frac{\partial v_1}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial g_t}}_{(4)}=0; \qquad (4.4)$$

and

$$\frac{\partial V_t^G\left(e_t;g_t;\tau_t\right)}{\partial \tau_t}=\underbrace{\frac{\partial \pi_t}{\partial \tau_t}}_{(1)}+\underbrace{\frac{1}{q}\left(e_t-1\right)\frac{\partial \pi_t}{\partial \tau_t}}_{(2)}+\underbrace{\frac{1}{q}\frac{1}{E_t}\frac{\partial}{\partial \tau_t}\left[(\tau_t-g_t)w_t l_t\right]}_{(3)}=0. \qquad (4.5)$$

The total effect of an increase in the level of public investment g on V_t^G is given by (4.4) and can be decomposed into: (1) the decrease in the individual profits of the elite members; (2) the decline in the expected rents from innovation (caused by a decrease in the profits of the new firms);

[9] We concentrate on the case when the optimal e^* s an interior solution.

(3) the decrease in the tax revenue; and (4) the increase of the net present value of the elite' income due to higher productivity.

The direct effect of an increase in q on the optimal choice of g consists in reducing the marginal cost of public investment (i.e. foregone transfers from innovation and taxation) — terms (2) and (3) in (4.4). Hence, for given values of e and τ, an increase in q leads to a higher optimal level of public investment, g.

The effects of an increase in the tax rate τ on V_t^G are summarized in (4.5): a higher τ leads to a decrease of individual profits, and, thus, also to a decline in expected rents from innovation. An increase in τ also affects the tax revenue, but for given values of e and g, has no effect on the elite's future payoffs.

The direct effect of q on tax rate can be derived from (4.5): a higher q decreases both the marginal cost of taxation in terms of foregone transfers from innovation, and the marginal benefits from taxation by reducing the fraction of transfers received by each member of the winning coalition WC_G. In the Appendix, we show that the latter effect dominates, (see the proof of Claim 2). Hence, for given e and g, an increase in the majority required for political decisions q leads to a lower optimal tax rate.

In sum, the direct effects of an increase in q are a decline in the rate of innovation, a decline in the tax rate and an increase in public investment and, hence, in the share of tax revenues used for public spending, g/τ. The discussion above neglects the fact that a change in q will, in general, affect all three policy variables simultaneously, and, hence does not capture possible equilibrium effects resulting from the interaction of e, g and τ. Our computations, however, demonstrate that for a wide range of parameter values, the responses of the equilibrium outcomes $(e^*; g^*; \tau^*)$ to changes in q are driven by the direct effects identified above.[10]

Figure 1A illustrates our finding. As q increases, innovation declines and eventually seizes. The labor tax declines. Public investment and the share of tax revenues used for it increase. Once again, the reasoning behind these results coincides with what we anticipated in section 4.1: as the elite becomes more consolidated, i.e., the proposing legislator must take into account the interests

[10] While for the case of fully dependent chambers the direct effects are sufficient to describe the comparative statics of e, g and τ, for the case of independent chambers discussed in the next section, the indirect effects will play an important role in the analysis.

of a larger fraction of the elite, the equilibrium outcomes become closer to those preferred by the elite as a whole.

The growth rate of total output in the economy is given by

$$G = \frac{E_{t+1}\left(\frac{L_t(1-g^*)}{E_{t+1}}\right)^{\frac{1}{\lambda}}}{E_t\left(\frac{L_{t-1}(1-g^*)}{E_t}\right)^{\frac{1}{\lambda}}} = h(e^*)^{1-\frac{1}{\lambda}} n(g^*)^{\frac{1}{\lambda}}.$$

Figure 1A also presents the dependence of economic growth on the insitutional variable q. This dependence is non-monotonic. In economies with lower q the higher rates of innovation act as a driving force behind economic growth while the contribution of public investment to the latter is only marginal; in economies with higher q, though the rate of innovation is zero, there is a substantial growth achieved through higher rates of public investment. It is the economices with intermediate levels of q that are the worst in terms of growth: while they are not economically free enough to have growth through entrepeneurship, the elite is not consolidated enough to put the class interests above private ones, which leads to mis-appropriation of tax revenues for private uses at the expense of the public good.[11]

4.3 Equilibrium and Comparative Statics with Independent Chambers

In this section, we turn to the case of independent chambers, $c = q^E \cdot q^G$. As before, we assume that $\alpha = 1$. We can write the maximization problem of the proposing legislator in chamber E as follows:

$$\max_{e_t} V_t^E\left(e_t; g_t; \tau_t\right) = \max_{e_t} \pi_t + \frac{1}{q^E}\left(e_t - 1\right)\pi_t + \frac{1}{E_t}(\tau_t - g_t)w_t l_t + \beta v_1\left(E_{t+1}, L_{t+1}\right), \tag{4.6}$$

while the proposing legislator in chamber G maximizes:

$$\max_{g_t;\tau_t} V_t^G\left(e_t; g_t; \tau_t\right) = \max_{g_t;\tau_t} \pi_t + \left(e_t - 1\right)\pi_t + \frac{1}{q^G}\frac{1}{E_t}(\tau_t - g_t)w_t l_t + \beta v_1\left(E_{t+1}, L_{t+1}\right). \tag{4.7}$$

Note that except for the special case, in which $q^E = q^G = c = 1$, the payoff functions of the proposing legislators are distinct. A member of the winning coalition in chamber E (G) receives

[11] This naturally resonates with well know results on the trade-offs between spending on pork-barrel transfers and spending on public goods. See, for example, Battaglini and Coate (2007).

on average a fraction $\frac{1}{Nq^E}$ $(\frac{1}{Nq^G})$ of the total transfers distributed in this chamber. His conditional probability of being a member of the winning coalition in chamber G (E) is q^G $\left(q^E\right)$, implying that the expected share of transfers he receives in coalition G (E) is $\frac{1}{N}$. This means that, in general, the proposing legislator in chamber E has higher marginal benefits from innovation and prefers a higher level of e than the average legislator in WC_G. Similarly, the proposing legislator in chamber G enjoys higher marginal benefits from taxation, and higher marginal cost of public investment, choosing a higher level of τ and a lower level of g than would be optimal from the point of view of an average legislator in WC_E.

4.3.1 Changing the Majority Required to Choose the Rate of Innovation

In this subsection, we assume that the required majority in chamber G is fixed and we compare the political and economic outcomes for different values of q^E. We can write the FOC's for problems (4.6) and (4.2) as:

$$\frac{\partial V_t^E\left(e_t;g_t;\tau_t\right)}{\partial e_t}=\frac{\partial \pi_t}{\partial e_t}+\frac{1}{q^E}\left[\pi_t+\left(e_t-1\right)\frac{\partial \pi_t}{\partial e_t}\right]+\frac{1}{E_t}(\tau_t-g_t)\frac{\partial\left(w_t l_t\right)}{\partial e_t}+\beta\frac{\partial v_1}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial e_t^*}=0, \quad (4.8)$$

for chamber E and as

$$\frac{\partial V_t^G\left(e_t;g_t;\tau_t\right)}{\partial g_t}=\frac{\partial \pi_t}{\partial g_t}+\left(e_t-1\right)\frac{\partial \pi_t}{\partial g_t}+\frac{1}{q^G}\frac{1}{E_t}\frac{\partial}{\partial g_t}\left[(\tau_t-g_t)w_t l_t\right]+\beta\frac{\partial v_1}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial g_t}=0 \qquad (4.9)$$

$$\frac{\partial V_t^G\left(e_t;g_t;\tau_t\right)}{\partial \tau_t}=\frac{\partial \pi_t}{\partial \tau_t}+\left(e_t-1\right)\frac{\partial \pi_t}{\partial \tau_t}+\frac{1}{q^G}\frac{1}{E_t}\frac{\partial}{\partial \tau_t}\left[(\tau_t-g_t)w_t l_t\right]=0, \qquad (4.10)$$

for chamber G.

The interpretation of these conditions is very similar to the case of fully dependent chambers and, hence, omitted. Note that in this scenario, the parameter of interest, q^E only appears in the payoff function V_t^E and has a negative direct effect on the choice of e. This effect is triggered by the decline in the share of total transfers from innovation received by a member of the winning coalition in chamber E.

The direct effect of q^E on the level of public investments and on the tax rate is 0. Hence, changes in q^E can only affect g and τ through changes in e. In what follows, we study the effect of an increase in the rate of innovation on the optimal choice of public investments and tax rate.

Differentiating (4.9) with respect to e_t, we obtain the total direct effect of an increase in e on the optimal choice of g (for a given τ):

$$\frac{\partial^2 V_t^G\left(e_t;g_t;\tau_t\right)}{\partial g_t \partial e_t} = \underbrace{\frac{\partial^2 \pi_t}{\partial g_t \partial e_t}}_{(1)>0} + \underbrace{\underbrace{\frac{\partial \pi_t}{\partial g_t}}_{<0} + \underbrace{\left(e_t - 1\right)\frac{\partial^2 \pi_t}{\partial g_t \partial e_t}}_{>0}}_{(2)} + \underbrace{\frac{1}{q^G}\frac{\partial^2}{\partial g_t \partial e_t}\left[(\tau_t - g_t)w_t l_t\right]}_{(3)<0} + \underbrace{\beta \frac{\partial\left(\frac{\partial v_1}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial g_t}\right)}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial e_t}}_{(4)<0}.$$

It can be decomposed into the following components: (1) the decrease in the marginal cost of public investment in terms of foregone individual profits; (2) the effect on the marginal cost of public investments in terms of foregone transfers from innovation: an increase, due to a larger number of new firms, combined with a decrease, due to the fact that innovation makes profits less sensitive to increase in public investments; (3) the increase in the marginal cost of public investment in terms of foregone tax revenue; and (4) the decrease in the marginal benefit from increased productivity in the future. In the Appendix, (see the proof of Claim 3), we show that the negative effect of e through the decrease in marginal tax revenue outweighs the positive effect of decreased marginal cost captured by (1) and (2). Hence, the total direct effect of an increase in the rate of innovation on the public investments is negative.

Finally, in the Appendix, (see the proof of Claim 4), we show that (4.10) can be rewritten as:

$$\tau^* = 1 + \frac{\lambda - 1}{\lambda\left(\psi + 1\right)}\left[q^G + \frac{1}{\lambda - 1}\right]\left(g^* - 1\right),$$

which implies that the direct effect of both q^E and e on τ is 0. However, an increase in g (triggered by a higher q^E that lowers e) leads to an increase in the tax-rate.

We conclude that an increase in q^E leads to a decline in the rate of innovation, an increase in the tax rate and an increase in public investment, and, to a decrease in the share of tax revenues used for public spending, g/τ.

The results of our computations are illustrated in Figure 1B. Innovation declines with q^E and eventually seizes at $q^E = 0.55$. Since changes in τ and g are only triggered by changes in e, for values of $q^E \geq 0.55$ (for which $e^* = 1$) τ and g are constant. The tax rate decreases with q^E, while public spending and the fraction of tax revenue spent on it increases.

Figure 1B also illustrates the effect of changes in q^E on the sources and the rate of economic growth. For low values of q^E the major source of growth in the economy is the high rate of innovation, while the fiscal authority chooses high rates of redistributional taxation and low public investment. An increase in q^E enhances growth through public investment. However, taxes increas and new entrepreneurial activity is suppressed. In this example, the increase in public investment cannot compensate for the decline in innovation: the highest rate of economic growth is achieved for low values of q^E, when the rate of innovation is at its maximum.

4.3.2 Changing the Majority Required to Choose Fiscal Policy

We now consider the case in which q^E is constant and compare institutional arrangements for different values of $q^G \in [0.5; 1]$. The first order conditions are given by (4.8), (4.9) and (4.10). Note that q^G has a direct positive effect on g and a direct negative effect on τ, but has no direct effect on e. Furthermore, we can rewrite (4.10) as

$$\tau^* = 1 - \frac{\lambda - 1}{\lambda(\psi + 1)} \left[q^G + \frac{1}{\lambda - 1} \right] (1 - g^*) . \tag{4.11}$$

Hence, the total effect of q^G on τ can be decomposed into the negative direct effect and the positive indirect effect through g. If future productivity is moderately elastic with respect to public investments, i.e. for moderate values of η, the indirect effect through g is small, and hence, the direct effect through q^G determines the changes in the tax rate.

Furthermore, for a given innovation rate, one can use expression (4.11) and the FOC with respect to g (4.9) to fully characterize the dependence of g on q^G. As expected, this relation is positive, i.e. economies with higher q^G have higher public investment. The analysis of the effect of q^G on the equilibrium values of e, g and τ becomes more complicated, when the rate of innovation changes simultaneously with the fiscal policy choices τ and g. The negative effect of q^G on τ plays a dominant role on the rate of innovation, as reduced taxation increases potential profits from new firms. That feeds into a higher rate of innovation, which in turn subdues the positive effect of q^G on g.

These effects are illustrated in Figure 1C. For the specific values of parameters used in our computations, no innovation takes place for values of $q^G \in [0.5; 0.65]$. In this range, as the elite becomes fiscally more consolidated, the tax rate declines and the amount of public investment rises. When the tax rate falls substantially, the rate of innovation starts rising, as entrepreneurship becomes more profitable. The rise in e crowds out public investment and eventually fully offsets the direct positive effect of fiscal consolidation on g. However, despite the fact that public investment starts to decline for values of q^G close to 1, the fiscal efficiency of the elite, as measured by g/τ, rises with q^G.

In sum, the effect of fiscal consolidation is an unambigous decline in the tax rate and an increase in the efficiency of the fiscal authorities. For higher levels of q^G, this leads to an increase in innovation. The effect of consolidation on public investment is, in general, non-monotonic: first, it increases and then decreases. The overall effect of fiscal consolidation on growth is positive.

4.3.3 Weak Institutions

Up to now, our discussion was concentrated on the case in which $q^G \geq 0.5$. Intuitively, these are economies with strong fiscal authorities, in which fiscal policy decisions can only be implemented if they benefit the majority of the elite members. In contrast, values of $q^G < 0.5$ describe economies with weak fiscal institutions, in which a minority of the elite can expropriate resources at the cost of the society as a whole. We now analyze the impact of changes in q^G on the policy variables for this scenario.

First note that the FOCs remain the same as before and, hence, q^G has a negative direct effect on τ and a positive direct effect on g. As before, the relationship between the equilibrium values of τ and g is described by (4.11). We conclude, therefore, that increasing the majority necessary to make fiscal policy decisions, q^G, leads to a lower tax rate τ.

Now consider the effect of q^G on the rate of innovation e. This effect is only indirect and occurs mainly through τ. For low values of q^G, the tax rate increases substantially. This implies that the main benefit from innovation is the increase in the tax revenue, a share of which (indirectly) accrues to the winning coalition in chamber E. This implies that, as q^G falls, the incentive to innovate, in order to increase the tax revenue, becomes even higher. Thus, the rate of innovation increases. In other words, the entrepreneurs innovate for a wrong reason: they do so not to increase rents from innovative activity, but to raise the rents they extract from labor via taxation[12].

Finally, the total effect from an increase in q^G on g consists of the positive direct effect, the positive effect through e and the negative effect through τ. Our computations show that the latter effect is dominated, see Figure 1D. An increase in the majority required to make fiscal policy decisions q^G, leads to a higher level of public investments, g.

[12] Note that this effect would disappear if we imposed an upper bound on the tax rate.

To sum up, an increased fiscal consolidation of the elite leads to a lower rate of innovation, lower tax rate and higher level of public investment.

The effect of q^G on growth can be immediately understood from the discussion above: as q^G increases, growth through innovations is replaced by growth through higher public investment. For very low values of q^G, the relatively slow increase in fiscal policy efficiency cannot compensate for the rapidly diminishing level of new entrepreneurial activity, and the total rate of growth declines, reaching a global minimum. As q^G increases further, the decline in the rate of innovations slows down and the increase in public investments leads to higher rates of growth. In our example, countries with values of q^G close to 0.2 perform worst in terms of economic growth.

Our results imply that for a country which has a very low value of q^G, a discrete transition to a high value of q^G is optimal. A gradual transition, however, would imply a sacrifice in terms of economic growth and, in particular, in terms of new entrepreneurial activity, before the maximal level of growth can be reached.

4.4 Separation of Powers

In this section, we discuss the impact of the parameter c on political and economic outcomes. We concentrate on the case: $q^E = q^G = q$ and consider the two scenarios of fully dependent chambers, i.e. $c = q$ and independent chambers, i.e. $c = q^2$.

If $q = 1$, the optimal policies are identical in both scenarios. Moreover, if the optimal rate of innovation is 1 in both scenarios, a comparison of the payoff functions of the proposing legislators in chamber G for the two cases immediately implies that the fiscal policies also coicncide, (see Figure 1E).

The comparison between the two scenarios becomes non-trivial, when the equilibrium value of e^* exceeds 1. Figure 1E shows that the optimal value of e^* remains 1 for all $q \in [0.5; 1]$ when chambers are independent, but starts increasing as q falls below 0.6 for the full-dependence scenario. Furthermore, the values of τ^* and g^* are consistently lower for the case of full dependence than with independent chambers.

To explain these effects, recall first the definitions of V_t^E for the two scenarios, as given in (4.1) and (4.6). The only difference in the functional forms is that the factor $\frac{1}{q}$ appears in front of the tax revenue term when the chambers are fully dependent, since the members of the winning coalition in chamber E receive a higher fraction of the tax revenue in this case. Since an increase in e has a positive effect on the tax revenue, the marginal benefits from innovation for the proposing legislator in chamber E are higher when the chambers are fully dependent. Furthermore, as q decreases the fraction of the tax revenue accruing to the members of WC_E increases for the full-dependence scenario, but remains constant for the case of independent chambers. Therefore, the marginal benefits from innovation rise faster in the case of full dependence, explaining the higher rate of innovation in this case.

The difference in the fiscal policy choices can be explained in a similar fashion. Consider the payoff functions of the proposing legislators in chamber G for the two scenarios as given in (4.2) and (4.7). This time, the functions differ with respect to the coefficients in front of the transfers for innovation. Note that an increase in both g and τ has a negative effect on transfers from innovation. Since the members of WC_G receive a larger fraction of these transfers in the case of dependent chambers, the negative effect of g and τ is stronger than for independent chambers and become even stronger as q decreases. Therefore, the level of public investment and the level of taxation are consistently lower in the full-dependence-scenario than in the case of independent chambers.

Finally, we can use the results derived above to discuss the effect of the parameter c on economic growth: the significantly higher rate of innovation in the case of fully dependent chambers combined with the lower tax rate compensates for the slightly lower level of public investment. Hence, growth is higher with fully dependent chambers. Note, however, that separation of powers leads to better outcomes for those in control of the legislature — the elite. We discuss this in more detail in the next section, when we analyze the choice of optimal institutions and the conflict of interests between the elite and the society as a whole.

5 Optimal Institutions

The analysis in Section 2 shows how empirically observed cross–country differences in the rate and patterns of economic growth can be traced back to differences in institutional design. However, the question of *why societies choose different institutional arrangements*, remains open. In this section, we try to shed some light on this issue by allowing the elite to choose political institutions.

We assume, thus, that in the initial period of the economy, the elite makes a decision about the values of q^E, q^G and c. This decision is made before the identities of the proposing legislators and the members of the winning coalitions have been determined. Hence, at this stage, all members of the elite have identical preferences w.r.t. the policy variables e, g and τ, and, thus, also w.r.t. the institutional design $\left(q^E; q^G; c\right)$ and maximize:

$$\begin{aligned} & V_0\left(q^E; q^G; c\right) \\ = & \left(1-\frac{1}{\lambda}\right)\sum_{t=0}^{\infty}\beta^t\left[e^*\left(q^E; q^G; c\right)\pi_t^*\left(q^E; q^G; c\right)+\left[\tau^*\left(q^E; q^G; c\right)-g^*\left(q^E; q^G; c\right)\right]w_t^*\left(q^E; q^G; c\right)l_t^*\left(q^E; q^G; c\right)\right] \\ = & \left(1-\frac{1}{\lambda}\right)\left(\frac{L_0}{E_0}\right)^{\frac{1}{\lambda}}\cdot\frac{e^*\left(q^E; q^G; c\right)\pi_t^*\left(q^E; q^G; c\right)+\left[\tau^*\left(q^E; q^G; c\right)-g^*\left(q^E; q^G; c\right)\right]w_t^*\left(q^E; q^G; c\right)l_t^*\left(q^E; q^G; c\right)}{1-\beta\left(\frac{n^*\left(q^E; q^G; c\right)}{h^*\left(q^E; q^G; c\right)}\right)^{\frac{1}{\lambda}}} \end{aligned} \tag{5.1}$$

If no constraints are imposed on q^E, q^G and c, the optimal choice of the elite is given by $q^E = q^G = c = 1$. The optimal institutional design maximizes the net present value of the elite's total income, thus preventing individuals from pursuing their private interests at the cost of the elite as a whole. We state this as a proposition below:

Proposition 5 V_0 *is maximized for* $q^E = q^G = c = 1$.

The proof of this proposition is straightforward and, therefore, omitted. In the next two subsections, we discuss two scenarios, which impose constraints on the choice of q^E and q^G. We analyze how the nature of these constraints affects the choice of institutional arrangements by the elite.

5.1 Optimal Institutions, When Consolidation Is Costly

Earlier on, we interpreted the value of q^E (q^G) as the degree of consolidation of the elite with respect to the economic (fiscal) policy of the country. We also showed that unanimity w.r.t. all political decisions is optimal for the elite. Such a high degree of consolidation can be, however, prohibitively costly. Decisions about fiscal policy are often made by a parliament or a ministry, which, in our view, makes consolidation less problematic. In contrast, decisions about licensing of new firms are mostly made by bureaucrats, thus making the implementation of high values of q^E relatively costly. In what follows, we concentrate on the case, in which achieving unanimity with respect to the level of innovation is costly. The analysis for the case in which the implementation of a high q^G is costly is analogous. We chose to conduct the analysis with respect to q^E for two reasons: first, we believe that in reality, consolidation with respect to economic policy will be more difficult to achieve, and second, introducing a constraint on q^E leads to some interesting, non-trivial results.

We model the cost of achieving unanimity by imposing a constraint on q^E: $q^E \leq \bar{q}^E < 1$. In what follows, we analyze the optimal choice of q^E and q^G for two possible scenarios: the case of fully dependent and the case of independent chambers.

5.1.1 Fully Dependent Chambers

Consider first the case of fully dependent chambers, $q^E = q^G = c \equiv q$. Imposing an upper bound on q^E implies that $q \leq \bar{q}^E$.

Figure 2A illustrates the elite's welfare, which is strictly increasing in q, due to the decreasing rate of innovation and the increasing level of public investment. Hence, the maximum is obtained at $q = \bar{q}^E$. The intuition behind this result is straightforward: by choosing the maximal level of q, the elite restrains as much as possible individuals in their pursue of private benefits at the cost of the elite as a whole. We now turn to the case of independent chambers and show that this conclusion is no longer valid there.

5.1.2 Independent Chambers

In the case of independent chambers, $c = q^E \cdot q^G$. Consider first the choice of q^E for a given q^G. The welfare of the elite is increasing in q^E for a given q^G (see Figure 2B), hence, in the optimum, $q^E = \bar{q}^E$.

Now consider the choice of q^G. For $q^E = \bar{q}^E = 0.5$, our analysis in Section 4.3.2 shows that e^* is increasing in q^G when q^G is close to 1, whereas g^* obtains its maximum for $q^G = 0.9$. This implies that the elite's welfare is maximized at a value of $q^G < 1$, see Figure 2C.

The intuition behind this result is as follows: when q^E is restricted from above, the elite as a whole prefers a lower value of e^* than the winning coalition in chamber E. A possible way to reduce e^* without violating the constraint is by changing q^G. A decrease in q^G implies a lower g^*, and, more importantly, a higher tax rate τ. This makes innovation less attractive and leads to a lower e^*.As long as the benefits from a lower rate of innovation excede the cost of a less efficient fiscal policy, a decrease in q^G is beneficial for the elite.

Thus, allowing the winning coalition in chamber G to more aggressively pursue its private interests makes the winning coalition in chamber E more conscious of the cost of innovation, and benefits the elite as a whole. This suggests that when consolidation is costly, the elite can profit from playing off the interests of the members of the two winning coalitions against each other. Intuitively, when the members of the winning coalition WC_E fear higher taxes imposed by WC_G, they set a lower rate of innovation. On the other hand, the members of WC_G are willing to choose a higher tax rate than would be optimal for the members of WC_E, because they receive a larger fraction of the transfers from taxation and a smaller fraction of the transfers from innovation than the members of WC_E. For this to obtain, it must be that $q^G < 1$ and that the interests of the two winning coalitions are distinct, which, in our model, occurs only for the case of independent chambers.

5.1.3 Separation of Powers and Optimal Institutions

Having analyzed the optimal choices of q^E and q^G for the two scenarios of fully dependent and independent chambers, we now consider the choice between the two scenarios. For the case in which consolidation is relatively costly ($\bar{q}^E$ is relatively low), the analysis in Section 4.4 implies that independent chambers choose lower levels of innovation, higher rates of taxation and higher levels of public investment. This increases the elite's welfare compared to the case of full dependence (see Figure 2D). Note that this analysis was conducted assuming that $q^E = q^G$. In the case of dependent chambers, this follows from the definition of the political institutions and imposes a constraint on the optimal value of q^G. In contrast, with independent chambers, q^G is unconstrained. This implies that the elite can always weakly improve its welfare by switching from the optimal institutional design with dependent chambers: $q^E = q^G = c = \bar{q}^E$ to the optimal institutional design with independent chambers with $q^E = \bar{q}^E$, $c = \bar{q}^E \cdot q^G$ and q^G being set to its optimal value. As we showed in the last subsection, this optimal institutional arrangement will not necessarily require unanimity in chamber G, i.e. $q^G < 1$ can obtain in the optimum.

The separation of powers leads to superior outcomes for the elite class, but not necessarily to the society as a whole. In the next section, we analyze the conflict of interests between the workers and the elite and model the choice of optimal institutions when workers' interests are taken into account.

5.2 Optimal Institutions when Workers Have Bargaining Power

Up to now our discussion has concentrated on the conflict of interests inside the elite In this section, we first establish and discuss the conflict of interests between the two classes: the working class and the elite. Then, we assume that workers can threaten the elite with a revolution, unless they are guaranteed a minimal level of life-time utility. We analyze the impact of the bargaining power of the workers on optimal institutions.

negotiate with the elite over the institutional design and

5.2.1 The Conflict of Interests Between the Working Class and the Elite

To understand the conflict of interests between the elite class and the workers, it is useful to compare their payoff functions. The payoff function for the elite was specified in 5.1. The payoff function for the workers is given by their life-time discounted labor income:

$$\begin{aligned} W_0 (e; g; \tau) &= \sum_{t=0}^{\infty} \beta^t (1-\tau) w_t^* l_t^* = \\ &= \frac{1}{\lambda} L_0^{\frac{1}{\lambda}} E_0^{1-\frac{1}{\lambda}} \frac{(1-\tau)^{\frac{\psi\lambda}{\psi\lambda+\lambda-1}} \left[\frac{e}{1-g}\right]^{\frac{\psi(\lambda-1)}{\psi\lambda+\lambda-1}}}{1-\beta \cdot [n(g)]^{\frac{1}{\lambda}} \cdot [h(e)]^{1-\frac{1}{\lambda}}} \end{aligned}$$

Note that W_0 is monotonically increasing in both e and g and monotonically decreasing in τ. The workers prefer to have the maximal level innovation, $e = \bar{e}$, since competition increases wages, and no redistributive taxation, $\tau = g$. The level of public investment is then chosen optimally so as to maximize W_0. In contrast, the elite as a whole would choose a much lower level of e and a tax rate which exceeds the level of public investments. Since the policy choices in our model are determined by the institutional arrangement, workers' preferences with respect to q^E, q^G and c will also differ from those of the elite as a whole.

Up to now, we assumed that this conflict of interests is latent: the workers had no influence on the choice of policy, or on the choice of institutions. We now relax this assumption, postulating that workers have to be guaranteed a minimal level of life-time utility $\bar{W}_0$ in order to prevent a revolution. A revolution is costly for the society as a whole — it destroys resources and obstructs political and economic life. For the elite, a revolution bears the risk of losing its political and economic power. We assume that the total cost of revolution for the elite is sufficiently high, so that it will always try to prevent it. For the workers, a revolution gives an opportunity to gain political and economic power. However, it also bears the risk of repressions, political anarchy and a temporary or permanent setback in economic activity. These costs and benefits are captured by $\bar{W}_0$ — the life-time utility that makes workers just indifferent between conforming to the existing regime and engaging in a revolution.

$\bar{W}_0$ can be interpreted as the bargaining power of the workers: the higher their expected benefits from a revolution, the more the institutional design chosen by the elite class will have to conform to the workers' interests. In order to prevent a revolution, the elite's choice of optimal institutional design has to satisfy:

$$\left(q^{E*}; q^{G*}; c^*\right) = \arg \max_{(q^E; q^G; c)} V_0\left(q^E; q^G; c\right) \tag{5.2}$$
$$\text{s.t. } W_0\left(q^E; q^G; c\right) \geq \bar{W}_0$$

The next subsections are devoted to a detailed analysis of this problem.

5.2.2 Fully Dependent Chambers

We start with the case of fully dependent chambers, $c = q^E = q^G \equiv q$ and consider the choice of q for a given $\bar{W}_0$. In Figure 3A we plot the welfare of the elite and of the workers for different values of $q \in [0.5; 1]$. For values of $q \in [0.5; 0.6]$, an increase in q lowers competition, which benefits the elite and makes the workers worse off. The workers' welfare reaches a minimum at $q = 0.6$. In the range $q \in [0.6; 1]$, both classes profit from an increase in q which leads to a higher level of public investments and compensates the workers for the low level of innovations. While the elite prefers to set $q = 1$, which minimizes innovations, but leads to maximally efficient fiscal policy, the workers' preferred arrangement is $q = 0.5$, the one with maximal rate of innovation.

Suppose first that $\bar{W}_0$ is relatively low, so that $\bar{W}_0 \leq W_0\left(e^*(1;1;1); g^*(1;1;1); \tau^*(1;1;1)\right)$. In this case, the constraint in problem (5.2) is not binding and $q = 1$ obtains.

As the bargaining power of the workers increases, the optimal institutional design changes in a discontinuous way. To guarantee a welfare of $\bar{W}_0$ for the workers, the elite is forced to significantly increase the level of innovation, which can only be done by choosing a sufficiently low q. Hence, in the case of fully dependent legislatures, a slight increase in the bargaining power of the workers, as captured by $\bar{W}_0$ can lead to a sharp decline in the degree of consolidation of the elite. Note that in this scenario, the workers' preferences for higher e are aligned with the private interests of the individual members of the elite. A threat of revolution, hence, encourages the elite to grant more freedom to its members, guaranteeing that the resulting policy represents the interests of the workers.

5.2.3 Independent Chambers

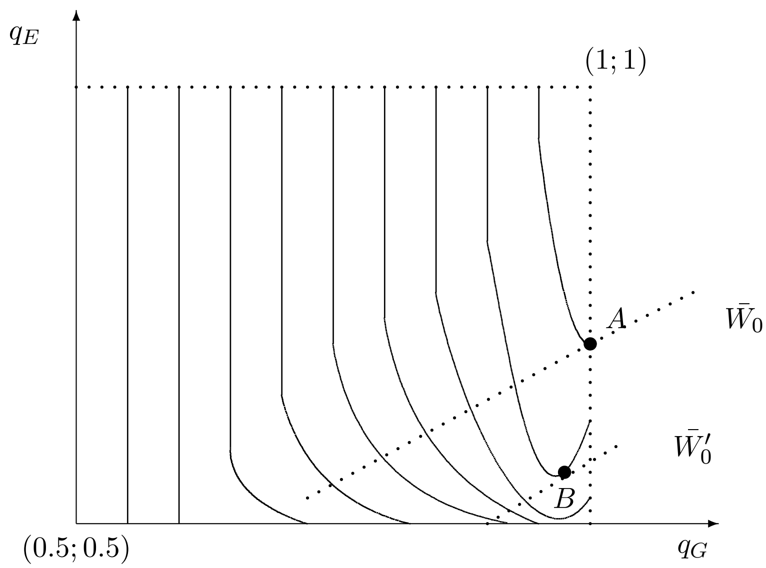
Now consider the case, in which the two chambers are independent, i.e. $c = q^E \cdot q^G$. The welfare of the elite for this scenario is plotted in Figure 3B. For a given degree of fiscal consolidation, the elite's welfare weakly decreases in q^E. As long as the level of economic freedom q^E is high, the welfare of the elite is strictly increasing with respect to q^G. For low values of q^E, the welfare of the elite is non-monotonic in q^G, in particular, it is decreasing for values of q^G close to 1. This explains the form of the elite's indifference curves, which are illustrated in Figure 4.

The welfare of the workers is plotted in Figure 3C. The welfare of the workers is (weakly) decreasing in q^E and increasing in q^G. This implies that the indifference curves of the workers are in general upward sloping: to compensate them for a decrease in economic freedom (an increase in q^E), the elite must choose a higher degree of fiscal consolidation q^G. Two such indifference curves are plotted in Figure 4.

If the workers' reservation utility is sufficiently low, i.e., $\bar{W}_0 \leq W_0\,(1;1;1)$, the constraint in problem 5.2 is not binding and the chosen institutional arrangement is $q^G = q^E = c = 1$.

For larger values of $\bar{W}_0$, the only way to satisfy the workers' demands is to induce a higher rate of innovation by reducing the degree of economic consolidation. Two cases are possible (see Figure 4). When $\bar{W}_0$ is not too high, both the elite and the workers profit from fiscal consolidation, but have conflicting preferences with respect to q^E. Hence, the optimal allocation respects the common interest, i.e., $q^G = 1$. q^E is chosen so that workers are exactly indifferent between engaging in revolution and conforming to the social order. Point A in Figure 4 illustrates the optimal choice for this case.

For high levels of $\bar{W}_0$, the workers and the elite have distinct preferences with respect to the degree of fiscal consolidation, in particular, the welfare of the elite is no longer monotone in q^G. The elite will, therefore, prefer to reduce q^G below 1. This, in turn, reduces the welfare of the workers, who have to be compensated by a further reduction in q^E. Hence, in the optimum, both q^E and q^G will be lower than 1. Point B in Figure 4 shows one such allocation.



$\bar{W}_0 < \bar{W}_0'$

Figure 4: Optimal institutions for different levels of $\bar{W}_0$

To sum up, as the bargaining power of the workers increases, we will observe a smooth transition from institutional arrangements benefitting the elite towards those preferred by the workers. At first, the elite will try to avert revolution by reducing the degree of consolidation with respect to economic policy and allowing its members to engage in entrepreneurial activities. The shape of the elite's indifference curves suggest that as the workers' outside option further improves, optimal institutions will be in the interior. The intuition behind this result goes back to Section 4.3: when forced to provide higher rates of innovation, the elite chooses a combination of institutions which on one hand encourage innovato use the conflict between the interests of the two proposing legislators to offset the negative impact of a low degree of consolidation with respect to economic policy.

Note that as the bargaining power of the workers grows, the rate of growth increases. Simultaneously, the patterns of growth change: in states with weak working class, economic growth is mainly driven by public investments. As the workers become more powerful, public investment is crowded out by an increase in innovation, which becomes the main source of growth.

5.3 Reversal of Fortune/ Initial Conditions

Our results on the optimal choice of institutions explain the dependence of institutions on initial conditions, and, in particular, the "reversal of fortunes." Consider, for example, a country that initially enjoys higher productivity due to abundace of land. In such country the elite will optimally choose lower degree of economic freedom, as this would be enough to satisfy the demands of the workers. As a result this country will feature lower growth rates. Thus, the initial advantage productivity will disappear and the country will fall behind. Naturally, if initially the society is geared to supress the workers demands, this will manisfest itself in a lower $\bar{W}_0$. In such a society the elite will optimally choose institutions detrimental to growth.

6 Conclusion

We presented a model in which institutions determine political and economic outcomes. We showed that the model generates patterns of growth which are consistent with historic evidence. We also analyzed the choice of institutions by the elite class. Our findings show that institutions will in general depend on the initial conditions. This dependence allowed us to explain the "reversal of fortune" phenomenon.

There are a number of avenues for future research. First, in the present work we assumed that the elite is small compared to the labor force. Consequently, transition from labor to the elite leaves the size of the work force unchanged. We plan to relax this assumption. We conjecture that in this case economies with good institutions will optimally choose policies that lead to outcomes that are preferred by both workers and the elite. In other words, even though the political structure of these economies would remain unchanged, the economic outcomes would converge to those chosen in countries with fair political representation. Second, we plan to extend the model to allow more elaborate participation of the working class in the political process via direct representation in the legislature. Finally, we plan to enrich the model by expanding the choice set of the legislature to other important policy variables such as trade and monetary policy.

7 Appendix

Proof of Proposition 7

Let $\tilde{S}_t^i$ be the sum of all transfers proposed by the legislator in chamber i and let $\left(\tilde{s}_{tn}^i\right)_{n\in WC^i}$ denote the transfers to the remaining members of WC_i. Obviously, the transfer to the proposing legislator in chamber i is then:

$$\tilde{s}_{tp}^i = \tilde{S}_t^i - \sum_{n\in WC^i} \tilde{s}_{tn}^i$$

We normalize by dividing the transfers by the number of elite members each legislator is representing, $\frac{E_t}{N}$ and write

$$\begin{aligned} S_t^i &= : \frac{\tilde{S}_t^i}{E_t} N \\ s_{tn}^i &= : \frac{\tilde{s}_{tn}^i}{E_t} N \end{aligned}$$

with

$$s_{tp}^i =: S_t^i - s_{tn}^i$$

The transfer schemes must satisfy the following feasibility constraints:

$$\begin{aligned} S_t^E &\leq N\alpha \left(e_t^* - 1\right) \pi_t \left(e_t^*; g_t^*; \tau_t^*\right) \\ S_t^G &\leq \frac{N}{E_t} \left(\tau_t^* - g_t^*\right) w_t \left(e_t^*; g_t^*; \tau_t^*\right) l_t \left(e_t^*; g_t^*; \tau_t^*\right) L_t \end{aligned}$$

Since the utility of the proposing legislator is strictly increasing in the size of the transfer he receives, the conditions above will hold with equality in equilibrium. Hence,

$$\begin{aligned} S_t^{*E}\left(e_t^*; g_t^*; \tau_t^*\right) &= N\alpha\left(e_t^*-1\right)\pi_t\left(e_t^*; g_t^*; \tau_t^*\right) \\ S_t^{*G}\left(e_t^*; g_t^*; \tau_t^*\right) &= \frac{N}{E_t}\left(\tau_t^*-g_t^*\right) w_t\left(e_t^*; g_t^*; \tau_t^*\right) l_t\left(e_t^*; g_t^*; \tau_t^*\right) L_t \end{aligned}$$

Note that the total amount of transfers depends on the proposals made in both chambers. Hence, a proposing legislator can only specify the share of the total transfers allocated to each of the members of WC_i. We denote these shares by:

$$\left(\left(\sigma_{tn}^i\right)_{n\in WC^i}; \sigma_{tp}^i = 1 - \sum_{n\in WC_i}\sigma_{tn}^i\right)$$

so that

$$s_{tn}^i = \sigma_{tn}^i S_t^i.$$

If both proposals are accepted, the expected payoff of a member of the elite represented by a non-proposing member of the winning coalition in chamber i is given by:

$$\begin{aligned} & V_{tn}^i\left(e_t; g_t; \tau_t; \left(\sigma_{tn}^E\right)_{n\in WC^E}; \left(\sigma_{tn}^G\right)_{n\in WC^G}\right) \\ = \; & \pi_t\left(e_t; g_t; \tau_t\right) + \sigma_{tn}^i S_t^i\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{1}{Nq^j}\sigma_{tp}^j S_t^j\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{1}{Nq^j}\sum_{n\in WC^j}\sigma_{tn}^j S_t^j\left(e_t; g_t; \tau_t\right) \\ & +\beta v_1\left(E_{t+1}\left(\cdot\right); L_{t+1}\left(\cdot\right)\right) \\ = \; & \pi_t\left(e_t; g_t; \tau_t\right) + \sigma_{tn}^i S_t^i\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{S_t^j}{Nq^j}\left(e_t; g_t; \tau_t\right) + \beta v_1\left(E_{t+1}\left(e_t; g_t\right); L_{t+1}\left(e_t; g_t\right)\right), \end{aligned}$$

whereas the utility of the proposing legislator is:

$$\begin{aligned} & V_{tp}^i\left(e_t; g_t; \tau_t; \left(\sigma_{tn}^E\right)_{n\in WC^E}; \left(\sigma_{tn}^G\right)_{n\in WC^G}\right) \\ = \; & \pi_t\left(e_t; g_t; \tau_t\right) + \sigma_{tp}^i S_t^i\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{1}{Nq^j}\sigma_{tp}^j S_t^j\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{1}{Nq^j}\sum_{n\in WC^j}\sigma_{tn}^j S_t^j\left(e_t; g_t; \tau_t\right) + \beta v_1\left(E_{t+1}\left(\cdot\right); L_{t+}\right. \\ = \; & \pi_t\left(e_t; g_t; \tau_t\right) + s_{tp}^i + \frac{c}{q^i}\frac{1}{Nq^j}\left(S_t^j\left(e_t; g_t; \tau_t\right) - \sum_{n\in WC^i}\sigma_{tn}^j S_t^j\left(e_t; g_t; \tau_t\right)\right) + \frac{c}{q^i}\frac{1}{Nq^j}\sum_{n\in WC^j}\sigma_{tn}^j S_t^j\left(e_t; g_t; \tau_t\right) \\ & +\beta v_1\left(E_{t+1}\left(\cdot\right); L_{t+1}\left(\cdot\right)\right) \end{aligned}$$

Since all non-proposing members of WC_i are symmetric, we concentrate on equilibria with the property that $\sigma_{tn}^i = \sigma_t^i$ for all non-proposing members of WC_i. We obtain:

$$
\begin{aligned}
V_{tn}^i\left(e_t; g_t; \tau_t; \sigma_t^E; \sigma_t^G\right) &= \pi_t\left(e_t; g_t; \tau_t\right) + \sigma_{tn}^i S_t^i\left(e_t; g_t; \tau_t\right) + \frac{c}{q^i}\frac{S_t^j}{Nq^j}\left(e_t; g_t; \tau_t\right) + \beta v_1\left(E_{t+1}\left(\cdot\right); L_{t+1}\left(\cdot\right)\right) \\
V_{tp}^i\left(e_t; g_t; \tau_t; \sigma_t^E; \sigma_t^G\right) &= \pi_t\left(e_t; g_t; \tau_t\right) + S_t^i\left(e_t; g_t; \tau_t\right)\left(1 - \left(q^i N - 1\right)\sigma_t^i\right) + \frac{c}{q^i}\frac{S_t^j\left(e_t; g_t; \tau_t\right)}{Nq^j} \\
&\quad + \beta v_1\left(E_{t+1}\left(\cdot\right); L_{t+1}\left(\cdot\right)\right)
\end{aligned}
$$

Consider first the proposing legislator in chamber G in subperiod m of the bargaining process. Assuming that the proposal in chamber E, $\left(e_t^*, s_{tn}^{*E}, S_t^{*E}\right)$ is accepted in the first round (we will subsequently show that this is indeed the case in equilibrium), his maximization problem is:

$$
\begin{aligned}
&\max_{\{g_m;\tau_m;\sigma_m^G\}} \pi_t\left(e_t^*; g_m; \tau_m\right) + S_t^G\left(e_t^*; g_m; \tau_m\right)\left(1 - \left(q^G N - 1\right)\sigma_m^G\right) + \frac{c}{q^G}\frac{S_t^E\left(e_t^*; g_m; \tau_m\right)}{Nq^E} \\
&+\beta v_1\left(E_{t+1}\left(e_t^*; g_m\right); L_{t+1}\left(e_t^*; g_m\right)\right)
\end{aligned}
$$

s.t.

$$
\pi_t\left(e_t^*; g_m; \tau_m\right) + s_m^G + \frac{c}{q^G}\frac{S_t^E\left(e_t^*; g_m; \tau_m\right)}{Nq^E} + \beta v_1\left(E_{t+1}\left(e_t^*; g_m\right); L_{t+1}\left(e_t^*; g_m\right)\right) \geq v_{m+1}^G\left(E_t; L_t\right)
$$

$$
\begin{aligned}
\left(q^G N - 1\right) S_t^G\left(e_t^*; g_m; \tau_m\right)\sigma_m^G &\leq \frac{N}{E_t}\left(\tau_m - g_m\right) w_t\left(e_t^*; g_m; \tau_m\right) l_t\left(e_t^*; g_m; \tau_m\right) L_t \\
\sigma_m^G &\geq 0
\end{aligned}
$$

where

$$
\begin{aligned}
v_{m+1}^G\left(E_t; L_t\right) &= \\
&= \pi_t\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right) + \frac{S_{m+1}^G\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right)}{N} + \frac{S_t^E\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right)}{N} \\
&\quad + \beta v_1\left(E_{t+1}\left(e_t^*; g_{m+1}^*\right); L_{t+1}\left(e_t^*; g_{m+1}^*\right)\right)
\end{aligned}
$$

if $c = q^E q^G$ or if $c = q^E = q^G$ and $m = M$ and

$$
\begin{aligned}
v_{m+1}^G\left(E_t; L_t\right) &= \\
&= \pi_t\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right) + \frac{S_{m+1}^G\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right)}{cN} + \frac{S_t^E\left(e_t^*; g_{m+1}^*; \tau_{m+1}^*\right)}{cN} \\
&\quad + \beta v_1\left(E_{t+1}\left(e_t^*; g_{m+1}^*\right); L_{t+1}\left(e_t^*; g_{m+1}^*\right)\right)
\end{aligned}
$$

if $c = q^E = q^G$ and $m < M$ is the continuation value of a member of WC^G in the case that the proposal in round m is rejected, but the proposal in round $m+1$ is accepted. To understand the construction of the continuation value, note that:in period $m+1$, a member of WC^G faces a probability of q^G to be a member of WC^G, in which case his expected transfer is $\frac{S^G_{m+1}\left(e^*_t;g^*_{m+1};\tau^*_{m+1}\right)}{q^G N}$. Otherwise, his transfer from chamber G is 0. Similarly, for the case of $c=q^E q^G$, without any information about the identities of the legislators in WC^E, the probability to be a member of WC^E is q^E, whereas the expected transfer is $\frac{S^E_t\left(e^*_t;g^*_{m+1};\tau^*_{m+1}\right)}{q^E N}$. Combining these gives the formula above for the case of independent chambers. For the case of full dependence, all legislators who are currently members of WC^G will also be members of WC^G in the next bargaining round and are also members of WC^E, which establishes the dependence above.

If the round M proposal is rejected, then:

$$\begin{aligned} v^G_{M+1} &= \max \pi_t\left(e^*_t;g_t;\tau_t\right)+\frac{S^G_t\left(e^*_t;g_t;\tau_t\right)}{N}+\frac{S^E_t\left(e^*_t;g_t;\tau_t\right)}{N}+\beta v_1\left(E_{t+1}\left(e^*_t;g_t\right);L_{t+1}\left(e^*_t;g_t\right)\right) \\ \text{s.t. } S^G_t &\geq 0,\ S^E_t\geq 0 \end{aligned}$$

Lemma 6 *Let* $\left(e^*_m;g^*_m;\tau^*_m;\sigma^{E*}_m;\sigma^{G*}_m\right)^M_{m=1}$ *be an equilibrium with associated value function* $v_1(E;L)$. $e^*_1=e^*_t$, $g^*_1=g^*_t$, $\tau^*_1=\tau^*_t$. *Then, for all states* E *and* L, $e^*_m(E;L)$ *and* $\left(g^*_m(E;L);\tau^*_m(E;L)\right)$ *solve the problems:*

$$\begin{aligned} & \max_{g_m;\tau_m} \pi_t\left(e^*_t;g_m;\tau_m\right)+\frac{S^G_t\left(e^*_t;g_m;\tau_m\right)}{q^G}+\frac{c}{q^G}\frac{S^E_t\left(e^*_t;g_m;\tau_m\right)}{Nq^E} \\ & +\beta v_1\left(E_{t+1}\left(e^*_t;g_m\right);L_{t+1}\left(e^*_t;g_m\right)\right) \\ s.t.\ S^G_t\left(e^*_t;g_m;\tau_m\right) &\geq 0,\ g_m\in[0;1],\ \tau_m\in[0;1] \\ & \max_{e_m}\pi_t\left(e_m;g^*_t;\tau^*_t\right)+\frac{S^E_t\left(e_m;g^*_t;\tau^*_t\right)}{q^E}+\frac{c}{q^G}\frac{S^G_t\left(e_m;g^*_t;\tau^*_t\right)}{Nq^G} \\ & +\beta v_1\left(E_{t+1}\left(e_m;g^*_t\right);L_{t+1}\left(e_m;g^*_t\right)\right) \\ s.t.\ S^E_t\left(e_m;g^*_t;\tau^*_t\right) &\geq 0,\ e_m\in[1;\bar{e}]. \end{aligned}$$

Moreover, σ^E_m *and* σ^G_m *satisfy:*

$$\begin{aligned} \sigma^E_m S^E_t\left(e_m;g^*_t;\tau^*_t\right) &= v_{m+1}\left(E_t;L_t\right)-\pi_t\left(e_m;g^*_t;\tau^*_t\right)-\frac{c}{q^G}\frac{S^G_t\left(e_m;g^*_t;\tau^*_t\right)}{Nq^G}-\beta v_1\left(E_{t+1}\left(e_m;g^*_t\right);L_{t+1}\left(e_m;g^*_t\right)\right) \\ \sigma^G_m S^G_t\left(e^*_t;g_m;\tau_m\right) &= v_{m+1}\left(E_t;L_t\right)-\pi_t\left(e^*_m;g_m;\tau_m\right)-\frac{c}{q^E}\frac{S^E_t\left(e^*_t;g_m;\tau_m\right)}{Nq^E}-\beta v_1\left(E_{t+1}\left(e^*_t;g_m\right);L_{t+1}\left(e^*_t;g_m\right)\right) \end{aligned}$$

Proof:

The proof of the Lemma closely follows the proof of Lemma A in Battaligni and Coate (2007) and is, therefore, omitted.

Proof of Proposition 3.7

Note that in period $t+1$, each member of the elite faces the following possibilities:

- His representative might be randomly chosen to be a member of the winning coalition in chamber E, but not in chamber G. This happens with probability $q^E - c$. In this case, his expected transfer is:
$$\frac{\alpha\left(e_{t+1}-1\right)}{q^E}\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)$$

- His representative might be randomly chosen to be a member of the winning coalition in chamber G, but not in chamber E. This happens with probability $q^G - c$. In this case, his expected transfer is:
$$\frac{1}{\lambda-1}\frac{e_{t+1}}{q^G}\cdot\frac{\tau_{t+1}-g_{t+1}}{1-g_{t+1}}\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)$$

- His representative might be randomly chosen to be a member of both the winning coalition in chamber G and chamber E. This happens with probability c. In this case, his expected transfer is:
$$\left[\frac{\alpha\left(e_{t+1}-1\right)}{q^E}+\frac{1}{\lambda-1}\frac{e_{t+1}}{q^G}\cdot\frac{\tau_{t+1}-g_{t+1}}{1-g_{t+1}}\right]\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)$$

- His representative might be randomly chosen to be a member of neither the winning coalition in chamber G nor in chamber E. This happens with probability $1-q^E-q^G$. In this case, his expected transfer is 0.

Since at time t, the constitution of the winning coalitions in period $(t+1)$ is not known to the legislators, they have to take into account the expected transfers obtained by the elite members they represent when making their decisions, given by:

$$\begin{aligned}
&\left(q^E-c\right)\frac{\alpha\left(e_{t+1}-1\right)}{q^E}\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)+\left(q^G-c\right)\frac{1}{\lambda-1}\frac{e_{t+1}}{q^G}\cdot\frac{\tau_{t+1}-g_{t+1}}{1-g_{t+1}}\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)\\
&+c\left[\frac{\alpha\left(e_{t+1}-1\right)}{q^E}+\frac{1}{\lambda-1}\frac{e_{t+1}}{q^G}\cdot\frac{\tau_{t+1}-g_{t+1}}{1-g_{t+1}}\right]\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)\\
=\ &\left[\alpha\left(e_{t+1}-1\right)+\frac{1}{\lambda-1}e_{t+1}\cdot\frac{\tau_{t+1}-g_{t+1}}{1-g_{t+1}}\right]\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)
\end{aligned}$$

leading to the form of v_1 specified above.■

Proof of Proposition 7

We proof the proposition in two steps. First, we assume that the continuation strategies of the legislators after period t are Markov and the proposed values of e, g and τ are independent of the state variables E and L. Fix such a continuation strategy combination from time $(t+1)$ on and consider the game between the two legislatures at time t. We demonstrate that there exist optimal strategies of the proposing legislators e_t^*, g_t^* and τ_t^*, together with optimal transfer schemes, optimal voting strategies for the non-proposing members of the winning coalitions and beliefs supporting these strategies as a perfect Bayesian equilibrium in the stage game given the continuation strategies specified above.

The second step of the proof consists of actually showing that the equilibrium strategy combination will satisfy the Markov property and will be independent of the state-variables.

We first introduce some notation: let $\left(s_{tn}^i\right)_{n\in WC^i}$ be the vector of transfers *per represented member of the elite* offered to the non-proposing members of the winning coalition in chamber i. S_t^i is the total amount of transfers *per represented member of the elite* distributed in chamber i. $\omega_{\tilde{n}}^E\left(e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right)\in\{0;1\}$ denotes the voting strategy of a member of WC_E, i.e. his decision to vote in favor of (1) or against (0) a specific proposal. Similarly, $\omega_{\tilde{n}}^G\left(g_t^*;\tau_t^*;\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)\in\{0;1\}$ is the voting strategy of the members of WC_G. $\mu_{\tilde{n}}^E\left(\cdot\mid e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right)$ denotes the beliefs of the members of WC_E about the identities of the members of WC_G and about the proposal made there. $\mu_{\tilde{n}}^G\left(\cdot\mid g_t;\tau_t;\left(s_{tn}^G\right)_{n\in WC^G};S_t^G\right)$ is defined analogously.

Step 1

Consider a continuation strategy combination after period t which satisfies the Markov property and has the property that all proposed values of e, g and τ are independent of the state variables. Recall that the profit function in $(t+1)$ has the form:

$$\pi\left(e_{t+1};g_{t+1};\tau_{t+1}\right)=\left(1-\frac{1}{\lambda}\right)\left[\frac{1-g_{t+1}}{e_{t+1}}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\left(1-\tau_{t+1}\right)^{\frac{1}{\lambda\psi-1+\lambda}}\left(\frac{L_{t+1}}{E_{t+1}}\right)^{\frac{1}{\lambda}}$$

Given our assumptions about the continuation strategy combination, the continuation value $v_1\left(E_{t+1};L_{t+1}\right)$ is thus continuous in L_{t+1} and E_{t+1}, and hence also in e_t and g_t and does not depend on τ_t.

We now show that under Assumption1, $V_E\left(e_t\right)$ $V_G\left(g_t;\tau_t\right)$ are concave functions in the respective choice variables.

First, we check whether $V_E\left(e_t\right)$ is concave in e_t. To do this, we split the function into two parts, the payoff in the current period, t and the payoff in period $t+1$. The first part of the function (up to a multiplication by a constant) is given by:

$$V_E^I(e_t) = \left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} (1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}} \left[1+\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{e_t}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right]$$

$$\begin{aligned}
\frac{\partial V_E^I}{\partial e_t} &= -\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} (1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}} \left[1+\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{e_t}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right] \\
&\quad +\left(\frac{\alpha}{q^E}+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{1}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right)\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} (1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 V_E^I}{\partial e_t^2} &= \frac{\psi}{\lambda\psi-1+\lambda}\left(\frac{\psi}{\lambda\psi-1+\lambda}+1\right)\left(\frac{1}{e_t}\right)^{\frac{\psi}{\lambda\psi-1+\lambda}+2}(1-g_t)^{\frac{\psi}{\lambda\psi-1+\lambda}}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\cdot \\
&\quad \cdot\left[1+\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{e_t}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right] \\
&\quad -\left(\frac{\alpha}{q^E}+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{1}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right)\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}} \\
&\quad -\left(\frac{\alpha}{q^E}+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{1}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right)\frac{\psi}{\lambda\psi-1+\lambda}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 V_E^I}{\partial e_t^2} &= \left(\frac{\psi}{\lambda\psi-1+\lambda}+1\right)\left(\frac{1}{e_t}\right)\left[1+\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{e_t}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right] \\
&\quad -2\left(\frac{\alpha}{q^E}+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{1}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right) \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 V_E^I}{\partial e_t^2} &= \left(\frac{\psi}{\lambda\psi-1+\lambda}-1\right)\left[\frac{\alpha}{q^E}+\frac{1}{\lambda-1}\frac{c}{q^E}\frac{1}{q^G}\cdot\frac{\tau_t^*-g_t^*}{1-g_t^*}\right]+\left(\frac{\psi}{\lambda\psi-1+\lambda}+1\right)\left(\frac{1}{e_t}\right)\left[1-\frac{\alpha}{q^E}\right] \\
&< 0
\end{aligned}$$

as long as $\psi < \lambda\psi - 1 + \lambda$, which is always satisfied for $\psi < 1$ and $\lambda > 1$ and $\alpha > q^E$, which we assume. Hence, V_E^I is concave in e_t.

The second part (up to a multiplication by a constant) is given by:

$$V_E^{II} = [h(e_t)]^{-\frac{1}{\lambda}}.$$

which is concave by our Assumption 1.

$$
\begin{aligned}
V_E^{II} &= \left(K\exp(\kappa e_t^k)\right)^{-\frac{1}{\lambda}} \\
\frac{\partial V_E^{II}}{\partial e_t} &= -\frac{1}{\lambda}K^{-\frac{1}{\lambda}}\kappa k\left(\exp(\kappa e_t^k)\right)^{-\frac{1}{\lambda}}e_t^{k-1} \\
\frac{\partial^2 V_E^{II}}{\partial e_t^2} &= \left(\frac{1}{\lambda}\right)^2 K^{-\frac{1}{\lambda}}\kappa^2 k^2\left(\exp(\kappa e_t^k)\right)^{-\frac{1}{\lambda}}e_t^{2k-2} - \frac{1}{\lambda}K^{-\frac{1}{\lambda}}\kappa k\,(k-1)\left(\exp(\kappa e_t^k)\right)^{-\frac{1}{\lambda}}e_t^{k-2} \\
&= \frac{1}{\lambda}K^{-\frac{1}{\lambda}}\kappa k\left(\exp(\kappa e_t^k)\right)^{-\frac{1}{\lambda}}e_t^{k-2}\left[\frac{1}{\lambda}\kappa k e_t^k - (k-1)\right] \\
&< 0
\end{aligned}
$$

which is equivalent to

$$e_t^k \leq \frac{\lambda\,(k-1)}{\kappa k}$$

or to

$$\bar{e}^k \leq \frac{\lambda\,(k-1)}{\kappa k}$$

Hence, if

$$\lambda > \frac{k\kappa}{k-1}\bar{e}^k$$

as in Assumption 1, the condition is satisfied and V_E^{II} is concave in e_t. Since V_E is a sum of two functions, both of which are concave in e_t, it is concave in e_t.

We now proceed to check the concavity of V_G w.r.t. τ_t and g_t. Again we divide the function into two parts (again omitting constants of multiplication).

$$
\begin{aligned}
V_G^I(g_t;\tau_t) &= \left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right] \\
V_G^{II}(g_t;\tau_t) &= n(g_t) = (Ng^n)^{\frac{1}{\lambda}}
\end{aligned}
$$

Differentiating the first part w.r.t. τ gives:

$$
\begin{aligned}
\frac{\partial V_G^I}{\partial \tau_t} = &-\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}\frac{1}{\lambda\psi-1+\lambda}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&+\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t^*}{q^G}\cdot\frac{1}{1-g_t}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 V_G^I}{\partial \tau_t^2} &= \left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} \frac{1}{\lambda\psi-1+\lambda}\left(\frac{1}{\lambda\psi-1+\lambda}-1\right)(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-2} \cdot \\
&\cdot\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}\left(e_t^*-1\right)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&-\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} \frac{1}{\lambda\psi-1+\lambda}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1}{1-g_t} \\
&-\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)} \frac{1}{\lambda\psi-1+\lambda}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1}{1-g_t}<0
\end{aligned}
$$

Hence, the function is concave in τ_t. Differentiating V_G^I w.r.t. $(1-g_t)$ gives:

$$
\begin{aligned}
\frac{\partial V_G^I}{\partial(1-g_t)} &= \frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}\left(e_t^*-1\right)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&+\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1-\tau_t}{(1-g_t)^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 V_G^I}{\partial(1-g_t)^2} &= \left(\frac{\psi}{\lambda\psi-1+\lambda}-1\right)\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t^2}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)-2}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}} \cdot \\
&\cdot\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}\left(e_t-1\right)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&+\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1-\tau_t}{(1-g_t)^2}+ \\
&+\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1-\tau_t}{(1-g_t)^2}- \\
&-\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{2(1-\tau_t)}{(1-g_t)^3} \\
&= (1-g_t)^{\frac{\psi}{\lambda\psi-1+\lambda}-2}\left(\frac{1}{e_t}\right)^{\frac{\psi}{\lambda\psi-1+\lambda}}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}[\left(1+\frac{c}{q^G}\frac{\alpha}{q^E}\left(e_t-1\right)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right)\cdot \\
&\cdot\left(\frac{\psi}{\lambda\psi-1+\lambda}-1\right)+ \\
&+2\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1-\tau_t}{(1-g_t)}-\frac{\lambda\psi-1+\lambda}{\psi(\lambda-1)}\frac{e_t}{q^G}\cdot\frac{2(1-\tau_t)}{(1-g_t)}]\frac{\psi}{\lambda\psi-1+\lambda} \\
&= (1-g_t)^{\frac{\psi}{\lambda\psi-1+\lambda}-2}\left(\frac{1}{e_t}\right)^{\frac{\psi}{\lambda\psi-1+\lambda}}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}[\left(1+\frac{c}{q^G}\frac{\alpha}{q^E}\left(e_t^*-1\right)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right)\cdot \\
&\cdot\left(\frac{\psi}{\lambda\psi-1+\lambda}-1\right) \\
&+2\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{1-\tau_t}{(1-g_t)}\frac{1-\psi-\lambda}{\psi(\lambda-1)}]\frac{\psi}{\lambda\psi-1+\lambda}<0
\end{aligned}
$$

Hence, V_G^I is concave in $(1-g_t)$.

The cross-derivative of V_G^I is given by

$$\begin{aligned}
\frac{\partial^2 V_G^I}{\partial(1-g_t)\,\partial\tau_t} &= \partial[\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right] \\
&\quad+\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1-\tau_t}{(1-g_t)^2}]/\partial\tau \\
&= -\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\cdot \\
&\quad\cdot\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&\quad+\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1}{1-g_t}- \\
&\quad-\frac{1}{\lambda\psi-1+\lambda}\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1-\tau_t}{(1-g_t)^2}- \\
&\quad-\left[\frac{(1-g_t)}{e_t}\right]^{\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}}\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1}{(1-g_t)^2} \\
&= \frac{1}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\cdot \\
&\quad\cdot[-\frac{\psi}{\lambda\psi-1+\lambda}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]+ \\
&\quad+\psi(1-\tau_t)\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1}{1-g_t}-(1-\tau_t)\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{\tau_t}{(1-g_t)}- \\
&\quad-(\lambda\psi-1+\lambda)(1-\tau_t)\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{1}{(1-g_t)}] \\
&= \frac{1}{\lambda\psi-1+\lambda}\frac{1}{e_t}\left[\frac{(1-g_t)}{e_t}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}-1}(1-\tau_t)^{\frac{1}{\lambda\psi-1+\lambda}-1}\cdot \\
&\quad\cdot[-\frac{\psi}{\lambda\psi-1+\lambda}\left[1+\frac{c}{q^G}\frac{\alpha}{q^E}(e_t-1)+\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{\tau_t-g_t}{1-g_t}\right]- \\
&\quad-(\lambda\psi+\lambda-\psi)\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{(1-\tau_t)}{(1-g_t)}]<0
\end{aligned}$$

To complete the proof of concavity of V_G^I, we have to show that:

$$\frac{\partial^2 V_G^I}{\partial\tau_t^2}\frac{\partial^2 V_G^I}{\partial(1-g_t)^2}>\left(\frac{\partial^2 V_G^I}{\partial(1-g_t)\,\partial\tau_t}\right)^2.$$

After cancelling common terms on both sides of the inequality, we get the reduced expressions:

$$\frac{\partial^2 V_G^I}{\partial \tau_t^2} \approx \left(\frac{1}{\lambda\psi - 1 + \lambda} - 1\right)\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right] - 2(1 - \tau_t)\frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{1}{1 - g_t}$$

$$\frac{\partial^2 V_G^I}{\partial (1 - g_t)^2} \approx \psi\left[\left(1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right)\left(\frac{\psi}{\lambda\psi - 1 + \lambda} - 1\right) + 2\frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{1 - \tau_t}{(1 - g_t)}\frac{1 - \psi - \lambda}{\psi(\lambda - 1)}\right]$$

$$\frac{\partial^2 V_G^I}{\partial (1 - g_t)\partial \tau_t} \approx -\frac{\psi}{\lambda\psi - 1 + \lambda}\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right] - (\lambda\psi + \lambda - \psi)\frac{1}{\lambda - 1}\frac{e_t}{q^G}\frac{(1 - \tau_t)}{(1 - g_t)}$$

or:

$$\begin{aligned}\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1 - g_t)^2} \approx\ & \psi\left[\left(\frac{1}{\lambda\psi - 1 + \lambda} - 1\right)\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right] - 2\frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{(1 - \tau_t)}{1 - g_t}\right]\cdot \\ & \cdot[\left(1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right)\left(\frac{\psi}{\lambda\psi - 1 + \lambda} - 1\right) + \\ & +2\frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{1 - \tau_t}{(1 - g_t)}\frac{1 - \psi - \lambda}{\psi(\lambda - 1)}]\end{aligned}$$

$$\begin{aligned}\left(\frac{\partial^2 V_G^I}{\partial (1 - g_t)\partial \tau_t}\right)^2 \approx\ & \left(\frac{\psi}{\lambda\psi - 1 + \lambda}\right)^2[\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right] - \\ & -(\lambda\psi + \lambda - \psi)\frac{1}{\lambda - 1}\frac{e_t}{q^G}\frac{(1 - \tau_t)}{(1 - g_t)}]^2\end{aligned}$$

We now partition each of these two expressions into three parts and show that the inequality holds for each part separately:

$$\left(\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1 - g_t)^2}\right)_I \approx \left(\frac{1}{\lambda\psi - 1 + \lambda}\right)^2 \psi[2 - \lambda\psi - \lambda](\psi + 1)(1 - \lambda)\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right]^2$$

$$\left(\left(\frac{\partial^2 V_G^I}{\partial (1 - g)\partial \tau}\right)^2\right)_I \approx \left(\frac{\psi}{\lambda\psi - 1 + \lambda}\right)^2\left[1 + \frac{c}{q^G}\frac{\alpha}{q^E}(e_t - 1) + \frac{1}{\lambda - 1}\frac{e_t}{q^G}\cdot\frac{\tau_t - g_t}{1 - g_t}\right]^2$$

$$\left(\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1 - g_t)^2}\right)_I > \left(\left(\frac{\partial^2 V_G^I}{\partial (1 - g_t)\partial \tau_t}\right)^2\right)_I$$

is satisfied if:

$$\left[2-\lambda\psi-\lambda\right]\left(1-\lambda\right)\left(1+\psi\right)>\psi$$

This can be written as:

$$\begin{aligned}
\left[2-\lambda\psi-\lambda\right]\left(1-\lambda\right)\left(1+\psi\right)-\psi &> 0\\
\psi\left[2-\lambda\psi-\lambda\right]\left(1-\lambda\right)+\left[2-\lambda\psi-\lambda\right]\left(1-\lambda\right)-\psi &> 0\\
\lambda\left(\lambda-1\right)\psi^{2}+\left(2-\lambda\right)\left(1-\lambda\right)\psi-\lambda\left(1-\lambda\right)\psi-\psi+\left(1-\lambda\right)\left(2-\lambda\right) &> 0\\
\lambda\psi^{2}+2\left(\lambda-1\right)\psi+\left(\lambda-2\right) &> 0
\end{aligned}$$

$$\begin{aligned}
\frac{D}{4} &= \left(\lambda-1\right)^{2}-\lambda\left(\lambda-2\right)=\\
&= \lambda^{2}-2\lambda+1-\lambda^{2}+2\lambda=1
\end{aligned}$$

$$\begin{aligned}
\psi_{1,2} &= \frac{-\lambda+1\pm1}{\lambda}\\
\psi_{1} &= -1\\
\psi_{2} &= \frac{2-\lambda}{\lambda}
\end{aligned}$$

Hence, for all $\psi>\frac{2-\lambda}{\lambda}$, as in Assumption 1

$$\left[2-\lambda\psi-\lambda\right]\left(1-\lambda\right)\left(1+\psi\right)>\psi$$

is satisfied.

The second parts of the functions are given by:

$$\begin{aligned}
\left(\frac{\partial^{2}V_{G}^{I}}{\partial\tau_{t}^{2}}\frac{\partial^{2}V_{G}^{I}}{\partial\left(1-g_{t}\right)^{2}}\right)_{II} \approx\ & 2\frac{1}{\lambda-1}\frac{e_{t}}{q^{G}}\cdot\frac{\left(1-\tau_{t}\right)}{1-g_{t}}\left(1+\frac{c}{q^{G}}\frac{\alpha}{q^{E}}\left(e_{t}-1\right)+\frac{1}{\lambda-1}\frac{e_{t}}{q^{G}}\cdot\frac{\tau_{t}-g_{t}}{1-g_{t}}\right)\cdot\\
&\cdot\left[\left(\frac{1}{\lambda\psi-1+\lambda}-1\right)\frac{1-\psi-\lambda}{\left(\lambda-1\right)}-\left(\frac{\psi}{\lambda\psi-1+\lambda}-1\right)\right]
\end{aligned}$$

$$\begin{aligned}
\left(\left(\frac{\partial^{2}V_{G}^{I}}{\partial\left(1-g_{t}\right)\partial\tau_{t}}\right)^{2}\right)_{II} \approx\ & -2\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^{2}\left[1+\frac{c}{q^{G}}\frac{\alpha}{q^{E}}\left(e_{t}-1\right)+\frac{1}{\lambda-1}\frac{e_{t}}{q^{G}}\cdot\frac{\tau_{t}-g_{t}}{1-g_{t}}\right]\cdot\\
&\cdot\left(\lambda\psi+\lambda-\psi\right)\frac{1}{\lambda-1}\frac{e_{t}}{q^{G}}\frac{\left(1-\tau_{t}\right)}{\left(1-g_{t}\right)}
\end{aligned}$$

$$\left(\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1-g_t)^2}\right)_{II} > \left(\left(\frac{\partial^2 V_G^I}{\partial (1-g_t)\,\partial \tau_t}\right)^2\right)_{II}$$

if

$$\begin{aligned} &\left[\left(\frac{2-\lambda\psi-\lambda}{\lambda\psi-1+\lambda}\right)\frac{1-\psi-\lambda}{(\lambda-1)} - \left(\frac{(\psi+1)(1-\lambda)}{\lambda\psi-1+\lambda}\right)\right] \\ > \quad & -\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^2(\lambda\psi+\lambda-\psi) \end{aligned}$$

Note that

$$\begin{aligned} &\left[\underbrace{\underbrace{(2-\lambda\psi-\lambda)}_{<0}\overbrace{\frac{1-\psi-\lambda}{(\lambda-1)}}^{<0}}_{>0}\underbrace{-(\psi+1)(1-\lambda)}_{>0}\right] > 0 \\ > \quad & -\underbrace{\left(\frac{\psi^2}{\lambda\psi-1+\lambda}\right)}_{>0}\underbrace{(\lambda\psi+\lambda-\psi)}_{>0} \end{aligned}$$

so that the inequality is always satisfied.

The last parts of the functions are:

$$\left(\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1-g_t)^2}\right)_{III} \approx 4\left(\frac{1}{\lambda-1}\frac{e_t}{q^G}\cdot\frac{(1-\tau_t)}{1-g_t}\right)^2\frac{-(1-\psi-\lambda)}{\psi(\lambda-1)}\psi$$

$$\left(\left(\frac{\partial^2 V_G^I}{\partial (1-g_t)\,\partial \tau_t}\right)^2\right)_{III} \approx \left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^2\left[(\lambda\psi+\lambda-\psi)\frac{1}{\lambda-1}\frac{e_t}{q^G}\frac{(1-\tau_t)}{(1-g_t)}\right]^2$$

$$\left(\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1-g_t)^2}\right)_{III} > \left(\left(\frac{\partial^2 V_G^I}{\partial (1-g_t)\,\partial \tau_t}\right)^2\right)_{III}$$

if, e.g.

$$\begin{aligned} 4\frac{-1+\psi+\lambda}{(\lambda-1)} \quad &> \quad 4 \\ &> \quad \left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^2[\lambda\psi+\lambda-\psi]^2 \end{aligned}$$

$$\begin{aligned} &\left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^2[(\lambda\psi-1-\psi)+(1+\lambda)]^2 \\ = \quad & \psi^2 + \frac{2\psi^2(1+\lambda)}{\lambda\psi-1+\lambda} + \frac{\psi^2(1+\lambda)^2}{(\lambda\psi-1+\lambda)^2} \end{aligned}$$

Note that

$$\psi^2 < 1,$$

$$\begin{aligned}
\frac{2\psi^2(1+\lambda)}{\lambda\psi - 1 + \lambda} &< 2 \\
\psi^2 + \psi^2\lambda &< \lambda\psi - 1 + \lambda \\
\psi^2 &< \frac{\lambda - 1}{\lambda}
\end{aligned}$$

which is guaranteed by Assumption 1.and

$$\begin{aligned}
\frac{\psi^2(1+\lambda)^2}{(\lambda\psi - 1 + \lambda)^2} &< 1 \\
\psi^2(1+\lambda)^2 &< (\lambda\psi - 1 + \lambda)^2 \\
\psi^2 + 2\psi^2\lambda + \psi^2\lambda^2 &< \lambda^2\psi^2 + 1 + \lambda^2 - 2\lambda - 2\lambda\psi + 2\lambda^2\psi \\
0 &< 1 + \lambda^2 - 2\lambda - 2\lambda\psi + 2\lambda^2\psi - \psi^2 - 2\psi^2\lambda \\
-\psi^2(\lambda+1) + 2\lambda\psi(\lambda-1) + (\lambda-1)^2 &> 0
\end{aligned}$$

together imply the inequality above. The last inequality is satisfied for $\psi = 0$ and for $\psi = 1$, since

$$\begin{aligned}
0 &< +\lambda^2 - 6\lambda + 2\lambda^2 \\
3\lambda^2 &> 6\lambda \\
3\lambda &> 2 \\
\lambda &> 1 > \frac{2}{3}
\end{aligned}$$

And since $\lambda + 1 > 0$, the inequality is satisfied for all $\psi \in [0;1]$.

We have shown that V_G^I satisfies

$$\begin{aligned}
\frac{\partial^2 V_G^I}{\partial \tau_t^2} &< 0 \\
\frac{\partial^2 V_G^I}{\partial (1-g_t)^2} &< 0 \\
\frac{\partial^2 V_G^I}{\partial \tau_t^2}\frac{\partial^2 V_G^I}{\partial (1-g_t)^2} &> \left(\frac{\partial^2 V_G^I}{\partial (1-g_t)\,\partial \tau_t}\right)^2
\end{aligned}$$

and is, therefore, concave.

We now turn to the second part, which is only a function of g_t. It is obvious, that $(Ng^n)^{\frac{1}{\lambda}}$ is a concave function, whenever $n < 1$.

Hence, Assumption 1 implies that the goal functions of both proposing legislators are continuous in $(e_t; g_t; \tau_t)$ and concave in their respective choice variables. Furthermore, we have $e_t \in [1; \bar{e}]$, $(g_t; \tau_t) \in [0; 1]^2$. Hence, the conditions of the Kakutani fixed point Theorem are satisfied and, therefore, there exist $(e_t^*; g_t^*; \tau_t^*)$ which simultaneously solve the maximization problems of the two proposing legislators.

Now consider the beliefs of the members of the winning coalitions. Since the proposal made in chamber j does not give its members any additional information about the identities of the members of WC_i, the beliefs of each member of WC_j assign a probability of 1 to the equilibrium proposal in chamber $i \neq j$ and the legislators hold the following beliefs about the identities of the members of WC_i:

$$\begin{aligned}
\Pr\{n \in WC_i,\, n \text{ — not proposing in } i \mid n \in WC_j,\, n \text{ — not proposing in } j\} &= \frac{c}{q^j}\frac{Nq^i - 1}{Nq^i} \\
\Pr\{n \in WC_i,\, n \text{ — not proposing in } i \mid n \in WC_j,\, n \text{ — proposing in } j\} &= \frac{c}{q^j}\frac{Nq^i - 1}{Nq^i} \\
\Pr\{n \in WC_i,\, n \text{ — proposing in } i \mid n \in WC_j,\, n \text{ — proposing in } j\} &= \frac{c}{q^j}\frac{1}{Nq^i} \\
\Pr\{n \in WC_i,\, n \text{ — proposing in } i \mid n \in WC_j,\, n \text{ — not proposing in } j\} &= \frac{c}{q^j}\frac{1}{Nq^i}
\end{aligned}$$

It is immediate that the proposing legislator in the last period of bargaining will propose:

$$\begin{aligned}
e_{tM}^* &= \arg\max_{e_t}\left[1 + \alpha(e_t - 1) + \frac{1}{\lambda - 1}c\frac{e_t}{q^G}\cdot\frac{\tau_t^* - g_t^*}{1 - g_t^*}\right]\pi_t(e_t; g_t^*; \tau_t^*) + \beta v_1(E_{t+1}, L_{t+1}) \\
(g_{tM}^*; \tau_{tM}^*) &= \arg\max_{g_t;\tau_t}\left[1 + \alpha\frac{c}{q^E}(e_t^* - 1) + \frac{1}{\lambda - 1}e_t^*\cdot\frac{\tau_t - g_t}{1 - g_t}\right]\pi_t(e_t^*; g_t; \tau_t) + \beta v_1(E_{t+1}, L_{t+1}),
\end{aligned}$$

In round $M - 1$, these decisions determine the outside options of the non-proposing members of the winning coalitions, which we denote by $\bar{V}_t^E(e_{tM}^*; g_t^*; \tau_t^*)$ and $\bar{V}_t^G(e_t^*; g_{tM}^*; \tau_{tM}^*)$.

We now claim that $(e_t^*; g_t^*; \tau_t^*)$ together with the following transfer scheme, voting strategies and system of beliefs constitute a perfect Bayesian equilibrium of the game in period t, given the continuation strategy combination specified above.

The transfer schemes $\left(\left(s_{tn}^{*E}\right)_{n\in WC^E};S_t^{*E}\right)$ and $\left(\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)$ are given by:

$$\begin{aligned} s_{tn}^E &= s_t^{*E}=\max\left\{\begin{array}{c} 0;\bar{V}_{tM}^E-\pi_t\left(e_t^*;g_t^*;\tau_t^*\right)-\frac{c}{q^E}\frac{1}{Nq^G}\left(S_t^{*G}-\sum_{n\in WC^G}s_{tn}^{*G}\right) \\ -\frac{c}{q^E}\frac{1}{Nq^G}\sum_{n\in WC^G}s_{tn}^{*G}-\beta v_1\left(h\left(e_t^*\right)E_t;n\left(g_t^*\right)L_t\right)\end{array}\right\} \quad (7.1)\\ \text{for all } n &\in WC^E \text{ and} \\ s_{tn}^G &= s_t^{*G}=\max\left\{\begin{array}{c} 0;\bar{V}_{tM}^G-\pi_t\left(e_t^*;g_t^*;\tau_t^*\right)-\frac{c}{q^G}\frac{1}{Nq^E}\left(S_t^{*E}-\sum_{n\in WC^E}s_{tn}^{E}\right) \\ -\frac{c}{q^G}\frac{1}{Nq^E}\sum_{n\in WC^E}s_{tn}^{*E}-\beta v_1\left(h\left(e_t^*\right)E_t;n\left(g_t^*\right)L_t\right)\end{array}\right\} \\ \text{for all } n &\in WC^G \end{aligned}$$

and

$$\begin{aligned} S_t^{*E} &= \alpha\left(e_t^*-1\right)\pi_t\left(e_t^*;g_t^*;\tau_t^*\right)\cdot N \qquad (7.2)\\ S_t^{*G} &= \frac{1}{\lambda-1}\left(\tau_t^*-g_t^*\right)\frac{e_t^*}{1-g_t^*}\pi_t\left(e_t^*;g_t^*;\tau_t^*\right)\cdot N \end{aligned}$$

The voting strategies of the members of the winning coalitions are given by:

$$\omega_{\tilde{n}}^E\left(e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right)=1$$

if and only if:

$$V_{t\tilde{n}}^E\left(e_t;g_t^*;\tau_t^*;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E;\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)\geq\bar{V}_{tM}^E$$

and

$$\omega_{\tilde{n}}^G\left(g_t;\tau_t;\left(s_{tn}^G\right)_{n\in WC^G};S_t^G\right)=1$$

$$V_{t\tilde{n}}^G\left(e_t^*;g_t;\tau_t;\left(s_{tn}^{*E}\right)_{n\in WC^E};S_t^{*E};\left(s_{tn}^G\right)_{n\in WC^G};S_t^G\right)\geq\bar{V}_{tM}^G$$

The beliefs of the members of the winning coalitions are given by:

$$\mu_{\tilde{n}}^{*E}\left(\tilde{n}\in WC^G,\,\tilde{n}\neq p^G;\ \left(g_t^*;\tau_t^*;\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)\mid\left(e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right),\tilde{n}\in WC^E\right)=\frac{c}{q^E}\frac{Nq^G-1}{Nq^G}$$

$$\mu_{\tilde{n}}^{*E}\left(\tilde{n}\in WC^G,\,\tilde{n}=p^G;\ \left(g_t^*;\tau_t^*;\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)\mid\left(e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right),\tilde{n}\in WC^E\right)=\frac{c}{q^E}\frac{1}{Nq^G}$$

$$\mu_{\tilde{n}}^{*E}\left(\tilde{n}\notin WC^G;\ \left(g_t^*;\tau_t^*;\left(s_{tn}^{*G}\right)_{n\in WC^G};S_t^{*G}\right)\mid\left(e_t;\left(s_{tn}^E\right)_{n\in WC^E};S_t^E\right),\tilde{n}\in WC^E\right)=1-\frac{c}{q^E}$$

$$\mu_{\tilde{n}}^{*G}\left(\tilde{n}\in WC^E,\,\tilde{n}\neq p^E;\ \left(e_t^*;\left(s_{tn}^{*E}\right)_{n\in WC^E};S_t^{*E}\right)\mid\left(g_t;\tau_t;\left(s_{tn}^G\right)_{n\in WC^G};S_t^G\right),\tilde{n}\in WC^G\right)=\frac{c}{q^E}\frac{Nq^G-1}{Nq^G}$$

$$\mu_{\tilde{n}}^{*G}\left(\tilde{n}\in WC^E,\,\tilde{n}=p^E;\ \left(e_t^*;\left(s_{tn}^{*E}\right)_{n\in WC^E};S_t^{*E}\right)\mid\left(g_t;\tau_t;\left(s_{tn}^G\right)_{n\in WC^G};S_t^G\right),\tilde{n}\in WC^G\right)=\frac{c}{q^E}\frac{1}{Nq^G}$$

$$\mu_{\tilde{n}}^{*G}\left(\tilde{n} \notin WC^{E},\ \left(e_t^{*};\left(s_{tn}^{*E}\right)_{n\in WC^{E}};S_t^{*E}\right) \mid \left(g_t;\tau_t;\left(s_{tn}^{G}\right)_{n\in WC^{G}};S_t^{G}\right),\tilde{n}\in WC^{G}\right)=1-\frac{c}{q^{E}}$$

where p^E and p^G denote respectively the proposing legislators in chamber E and G.

To understand why this result is true, first note that these actions are optimal if the bargaining has reached round $M-1$. In round $M-2$.

Proposition 7 *Let*

$$v_1\left(E_t;L_t\right)=\left[1+\alpha\left(e_t-1\right)+\frac{1}{\lambda-1}e_t\cdot\frac{\tau_t-g_t}{1-g_t}\right]\pi\left(e_t;g_t;\tau_t\right)+\beta v_1\left(E_{t+1};L_{t+1}\right)$$

denote the continuation value of a member of the elite at the beginning of period t for given values of E_t and L_t.

Then, there exists a perfect Bayes-Nash equilibrium of the bargaining game with incomplete information, with equilibrium strategies

$$\left(\begin{array}{c}\left(e_t^{*};\left(s_{tn}^{*E}\right)_{n\in WC^{E}};S_t^{*E}\right);\left(\omega_{\tilde{n}}^{*E}\left(e_t;\left(s_{tn}^{E}\right)_{n\in WC^{E}};S_t^{E}\right)\right)_{\tilde{n}\in WC^{E}};\\ \left(g_t^{*};\tau_t^{*};\left(s_{tn}^{*G}\right)_{n\in WC^{G}};S_t^{*G}\right);\left(\omega_{\tilde{n}}^{G}\left(g_t^{*};\tau_t^{*};\left(s_{tn}^{*G}\right)_{n\in WC^{G}};S_t^{*G}\right)\right)_{\tilde{n}\in WC^{G}}\end{array}\right)$$

and beliefs:

$$\begin{array}{c}\left(\mu_{\tilde{n}}^{*E}\left(\cdot \mid e_t;\left(s_{tn}^{E}\right)_{n\in WC^{E}};S_t^{E}\right)\right)_{\tilde{n}\in WC^{E}}\\ \left(\mu_{\tilde{n}}^{*G}\left(\cdot \mid g_t;\tau_t;\left(s_{tn}^{G}\right)_{n\in WC^{G}};S_t^{G}\right)\right)_{\tilde{n}\in WC^{G}}\end{array}$$

such that the equilibrium strategies of the proposing legislators satisfy:

$$e_t^{*}=\arg\max_{e_t}\left[1+\frac{\alpha}{q^{E}}\left(e_t-1\right)+\frac{1}{\lambda-1}\frac{c}{q^{E}}\frac{e_t}{q^{G}}\cdot\frac{\tau_t^{*}-g_t^{*}}{1-g_t^{*}}\right]\pi_t\left(e_t;g_t^{*};\tau_t^{*}\right)+\beta v_1\left(E_{t+1},L_{t+1}\right)$$

$$\left(g_t^{*};\tau_t^{*}\right)=\arg\max_{g_t;\tau_t}\left[1+\frac{c}{q^{G}}\frac{\alpha}{q^{E}}\left(e_t^{*}-1\right)+\frac{1}{\lambda-1}\frac{e_t^{*}}{q^{G}}\cdot\frac{\tau_t-g_t}{1-g_t}\right]\pi_t\left(e_t^{*};g_t;\tau_t\right)+\beta v_1\left(E_{t+1},L_{t+1}\right)$$

and the members of the winning coalition vote in favor of the equilibrium proposals.

We already showed that equilibrium transfers have to satisfy conditions 7.2 and 7.1. Therefore, for the specified equilibrium beliefs, the voting strategies are optimal. The equilibrium beliefs are chosen so that they are consistent with the equilibrium strategies.■

Proof of Proposition 3.

The proof is by construction. Let $v_1\left(E;L\right)=c_0\left(\frac{L}{E}\right)^{\frac{1}{\lambda}}$. v_1 is continuous in e_t and g_t and does not depend on τ_t. Furthermore, according to Assumption 1, v_1 is concave in e_t and g_t. Hence, a political equilibrium exists in each period and the equilibrium values for period t are given by the first-order conditions specified in Proposition 3. Furthermore, note that for the specific function v_1, these first-order conditions are independent of the state-variables E_t and L_t. Hence, the political equilibrium defines constant values of e^*, g^* and τ^* over time. Finally, observe that for the equilibrium values e^*, g^* and τ^*, the choice of the constant c_0 ensures that $v_1(E_t; L_t)$ is indeed the discounted value of future expected profits of the elite members.■

7.1 Comparative Statics Analysis — Fully Dependent Chambers

Claim 1 Transfers from taxation are increasing in e.

Proof of Claim 1:

The transfers from taxation are given by:

$$\frac{1}{\lambda-1}e^*\frac{\tau^*-g^*}{1-g^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}.$$

Differentiating this w.r.t. e gives

$$\begin{aligned}&\left(\frac{1}{\lambda-1}e^*\frac{\tau^*-g^*}{1-g^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\right)'_e\\ = \quad &\left(\frac{\psi+1}{\lambda\psi-1+\lambda}\right)\frac{\tau^*-g^*}{1-g^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}} > 0.\blacksquare\end{aligned}$$

Claim 2 The direct effect of q on τ is negative.

Proof of Claim 2:

The first derivative of V_t^G with respect to τ is given by:

$$\frac{1}{\lambda\psi-1+\lambda}\left[1+\frac{1}{q}(e^*-1)+\frac{1}{\lambda-1}\frac{e^*}{q}\cdot\left(1-\frac{1-\tau^*}{1-g^*}\right)\right]=\left[\frac{1}{\lambda-1}\frac{e^*}{q}\cdot\frac{1-\tau^*}{1-g^*}\right],$$

which reduces to:

$$[(\lambda-1)(q-1)+\lambda e^*]=\lambda(\psi+1)\left[e^*\cdot\frac{1-\tau^*}{1-g^*}\right]$$

Hence, the direct effect of q on τ is given by the sign of

$$\left(\frac{(\lambda-1)(q-1)}{q}\right)'_q=-(\lambda-1)\frac{1}{q^2}<0.\blacksquare$$

7.2 Comparative Statics Analysis — Independent Chambers

Claim 3 The direct effect of e on g is negative.

Proof of Claim 3:

The cross-derivative of the profit function w.r.t. e and g is given by:

$$\begin{aligned}\frac{\partial^2\pi}{\partial e\partial g} &= \frac{\partial\left[-\frac{\psi}{\lambda\psi-1+\lambda}\frac{1}{e^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\right]}{\partial g}\\
&= \left(\frac{\psi}{\lambda\psi-1+\lambda}\right)^2\frac{1}{e^*}\frac{1}{1-g}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\\
&> 0\end{aligned}$$

an increase in e makes the marginal cost of g in terms of foregone profits smaller.

The cross-derivative of transfers in G -chamber w.r.t. e and g is:

$$\begin{aligned}&\left(\frac{1}{\lambda-1}e^*\frac{\tau^*-g^*}{1-g^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\right)''_{eg}\\
=\;&\left(\frac{\psi+1}{\lambda\psi-1+\lambda}\right)\left(\frac{\tau^*-g^*}{1-g^*}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\right)'_g\\
=\;&-\left(\frac{\psi+1}{\lambda\psi-1+\lambda}\right)\frac{\psi}{\lambda\psi-1+\lambda}\frac{\tau^*-g^*}{(1-g^*)^2}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\\
&-\left(\frac{\psi+1}{\lambda\psi-1+\lambda}\right)\frac{1-\tau^*}{(1-g^*)^2}(1-\tau^*)^{\frac{1}{\lambda\psi-1+\lambda}}\left[\frac{(1-g^*)}{e}\right]^{\frac{\psi}{\lambda\psi-1+\lambda}}\\
<\;&0\end{aligned}$$

If we compare the two effects — positive and negative, we get:

$$\begin{aligned}e^*\frac{\psi^2}{\lambda\psi-1+\lambda}\frac{1}{e^*} &< (\psi+1)\frac{\psi}{\lambda\psi-1+\lambda}\frac{\tau^*-g^*}{(1-g^*)}\\
&\quad+(\psi+1)\frac{1-\tau^*}{(1-g^*)}\end{aligned}$$

$$\begin{aligned}\psi^2 &< (\psi+1)\psi+(\psi+1)^2(\lambda-1)\frac{1-\tau^*}{(1-g^*)}\\
0 &< \psi+(\psi+1)^2(\lambda-1)\frac{1-\tau^*}{(1-g^*)}\end{aligned}$$

implying that the negative effect always dominates and, hence, (together with the effect on future, below) the total effect of an increase of e on g (neglecting the effect through τ) is negative.■

Claim 4 The optimal level of τ^* is given by:

$$\tau^* = 1 + \frac{\lambda - 1}{\lambda(\psi + 1)} \left[q^G + \frac{1}{\lambda - 1} \right] (g^* - 1).$$

Proof of Claim 4:

Note that in the optimum,

$$\frac{\partial V_t^G}{\partial \tau} = -\frac{1}{\lambda\psi - 1 + \lambda} e^* \left[1 + \frac{1}{\lambda - 1} \frac{1}{q^G} \cdot \frac{\tau^* - g^*}{1 - g^*} \right] \frac{1}{1 - \tau^*} + \left[\frac{1}{\lambda - 1} \frac{e^*}{q^G} \cdot \frac{1}{1 - g^*} \right] = 0.$$

Simple algebra then leads to the expression above.■

8 References

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Figure 1A. Fully Dependent Chambers.

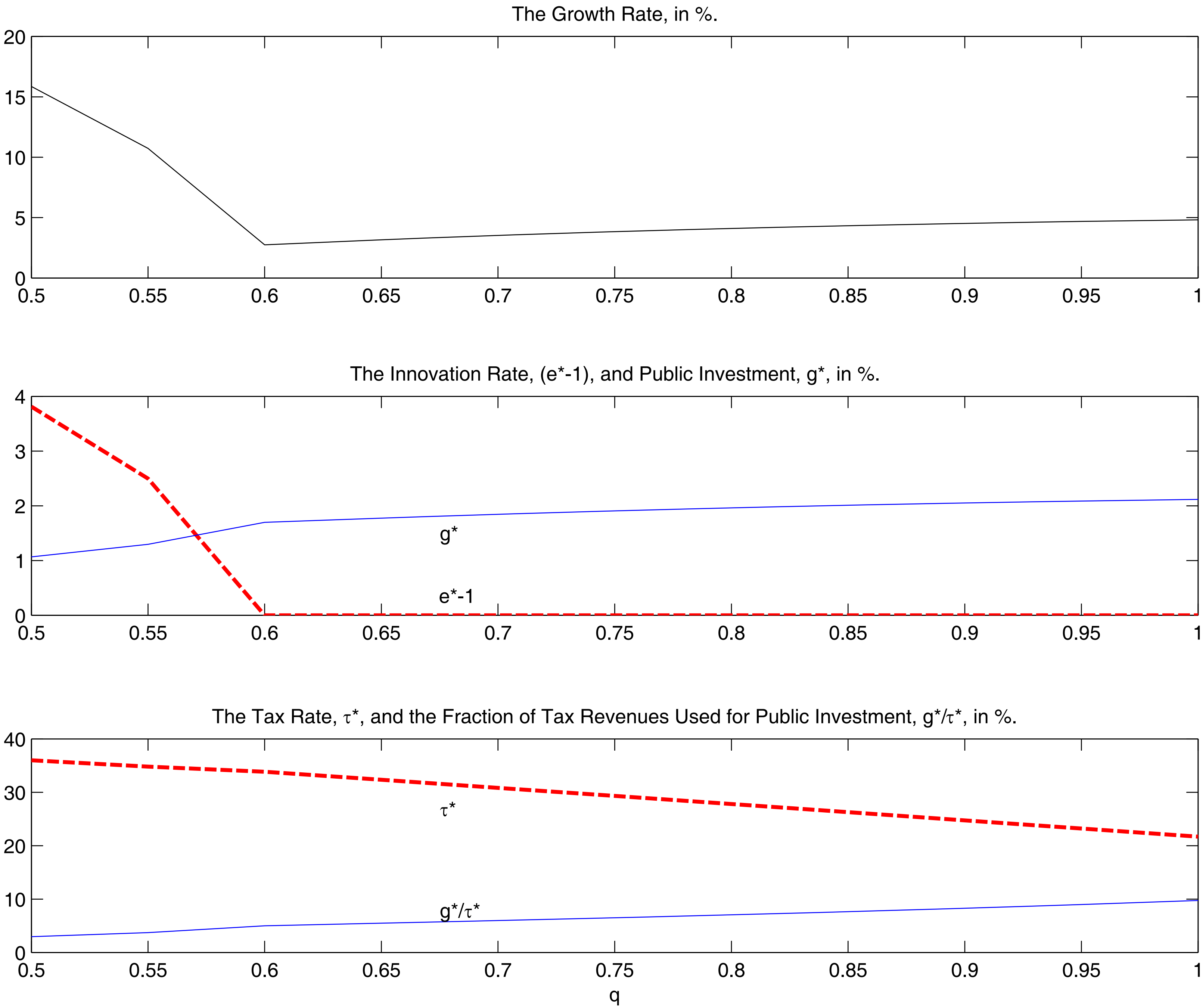


Figure 1B. Independent Chambers, the case of constant q^G.

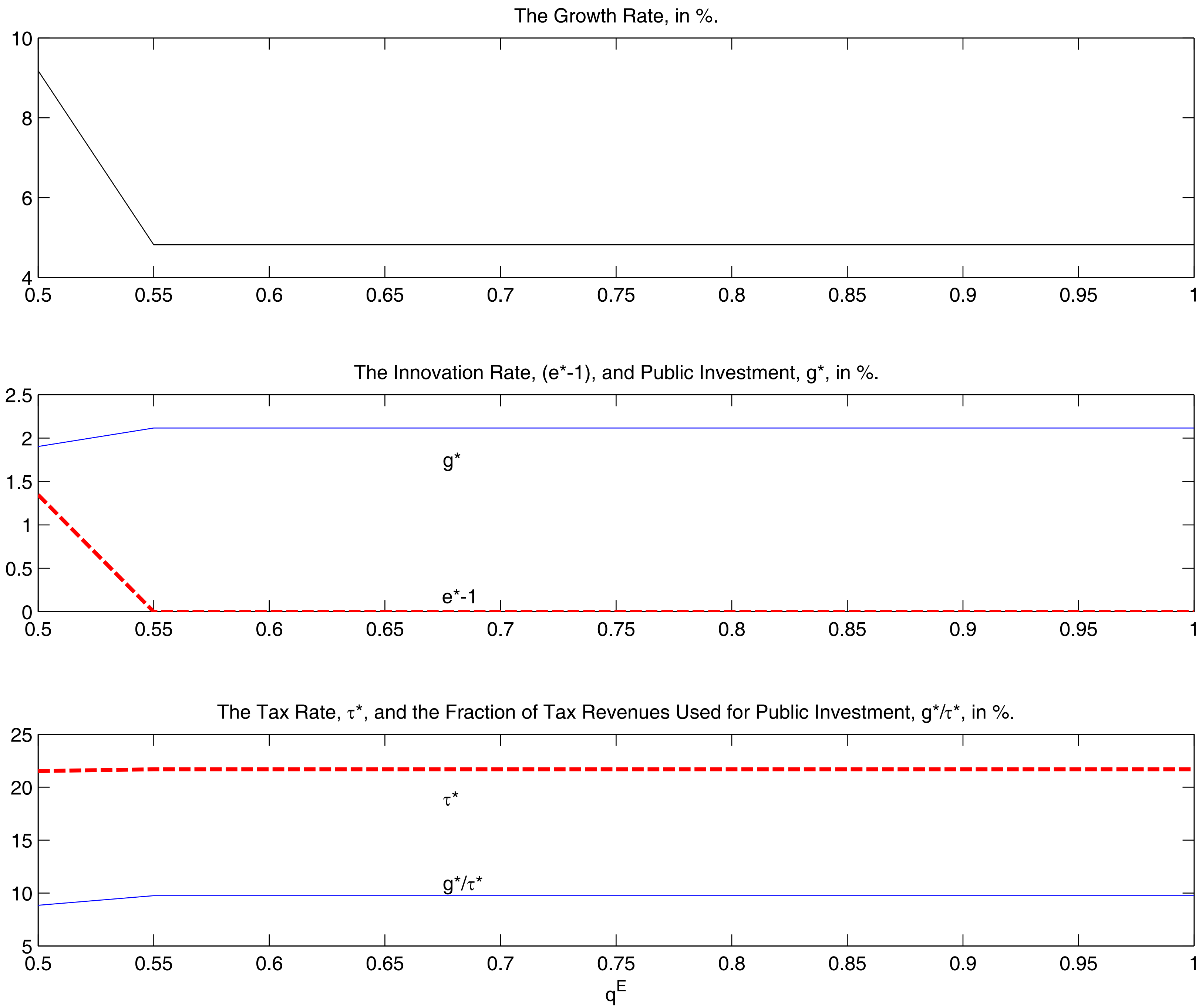


Figure 1C. Independent Chambers, the case of constant q^E.

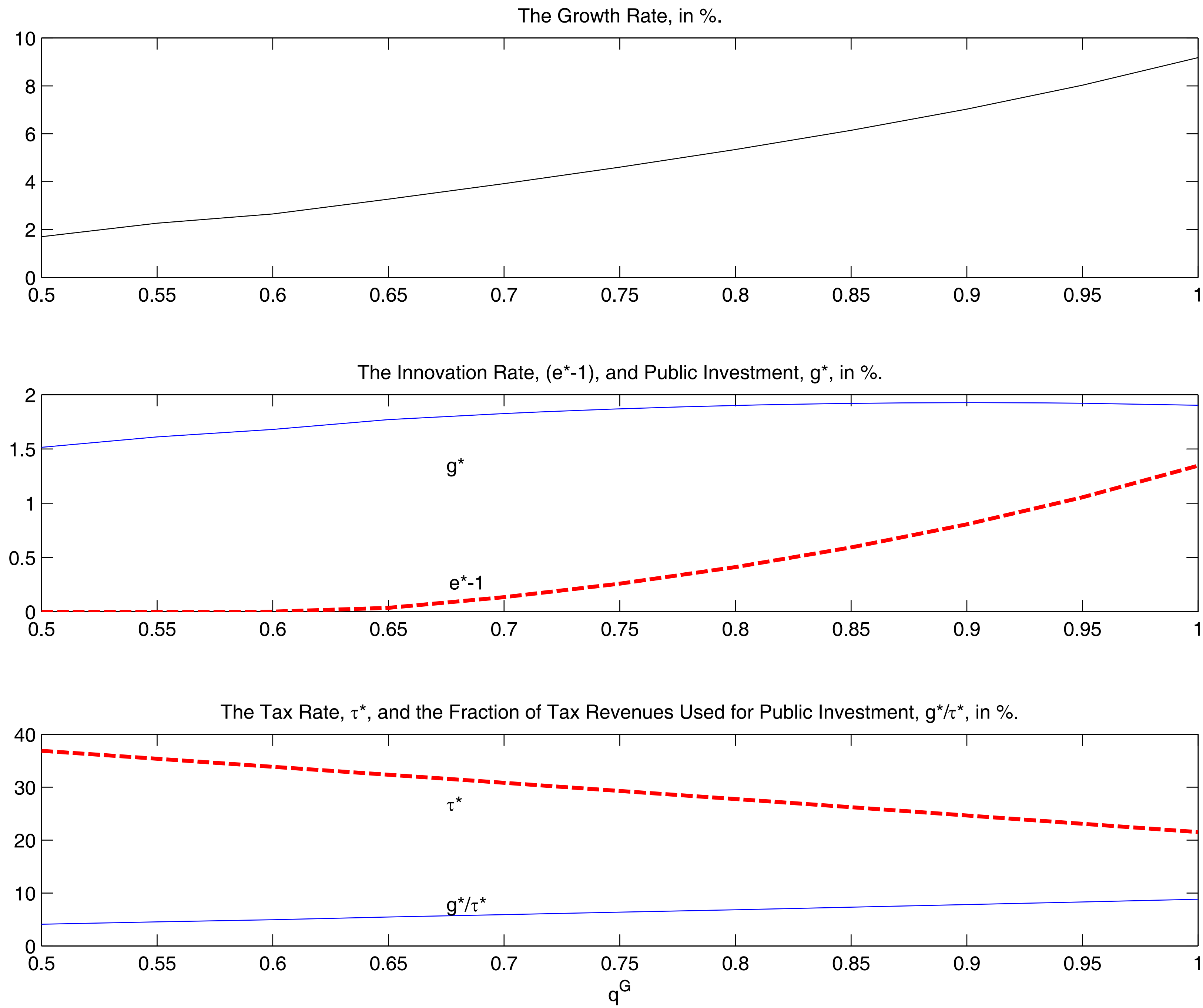


Figure 1D. Independent Chambers: the case of weak institutions.

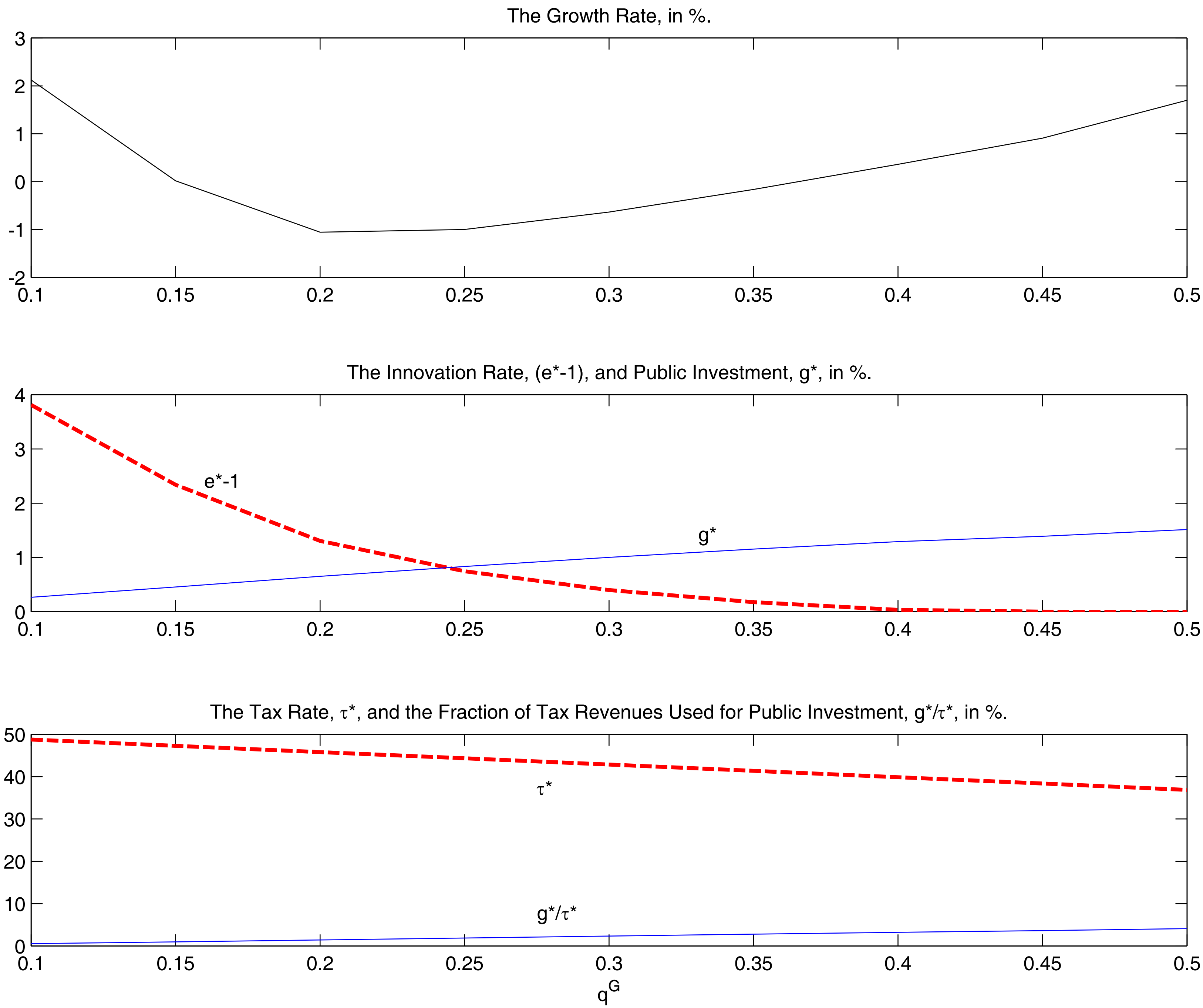
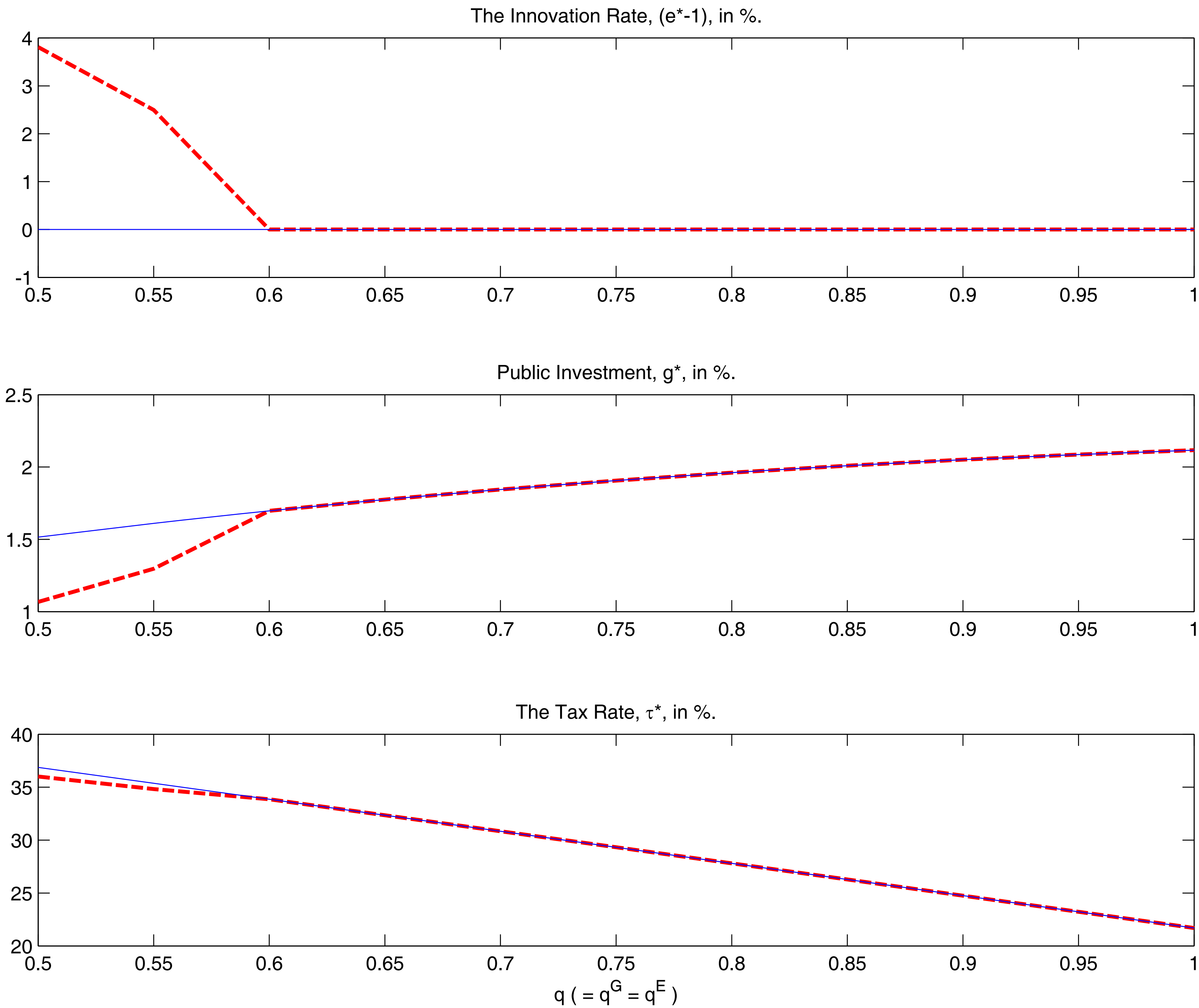


Figure 1E. Fully Dependent vs. Independent Chambers.



Figures 2A-C. Welfare of the Elite.

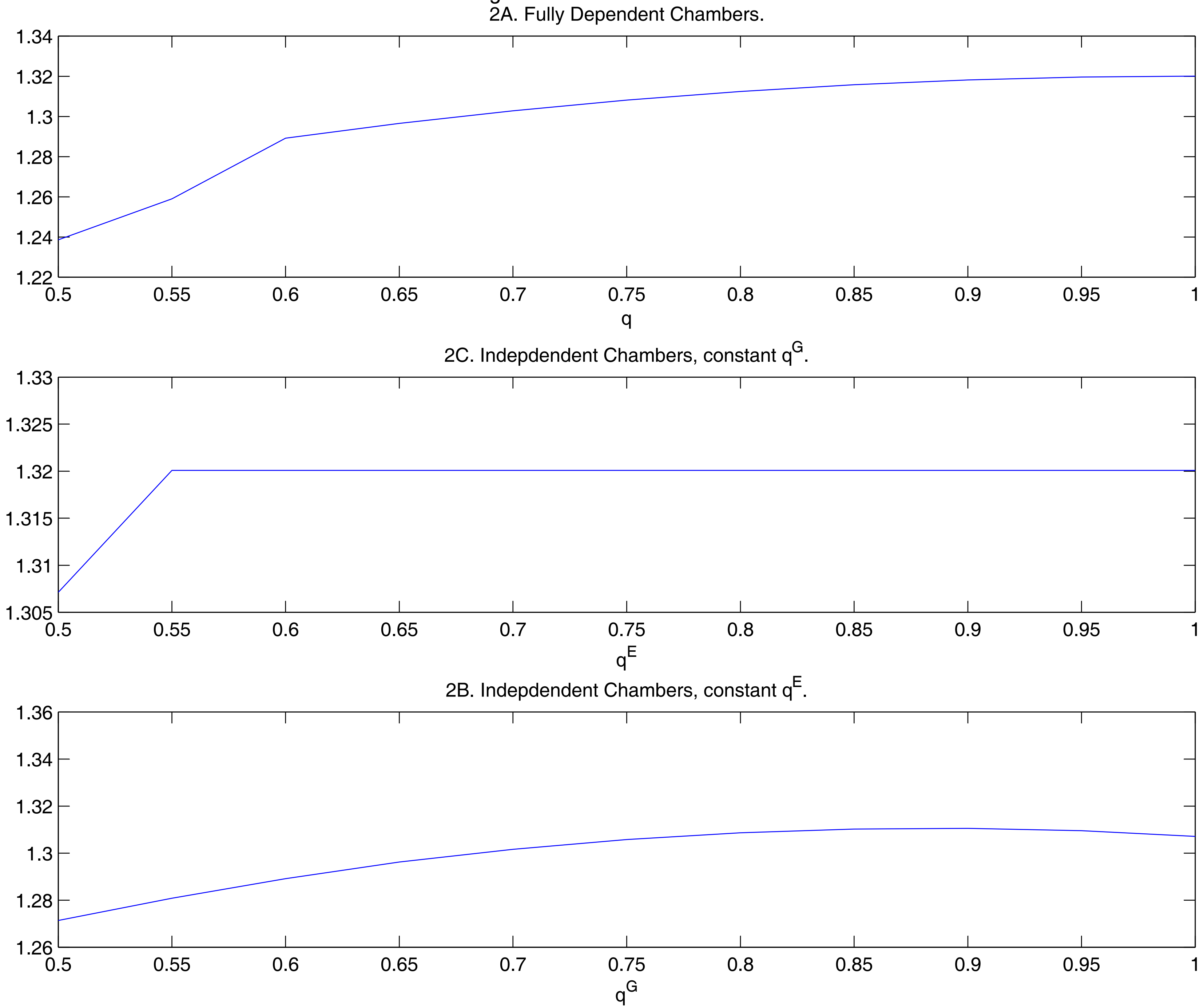


Figure 2D. Fully Dependent vs. Independent Chambers.
The Welfare of the Elite.

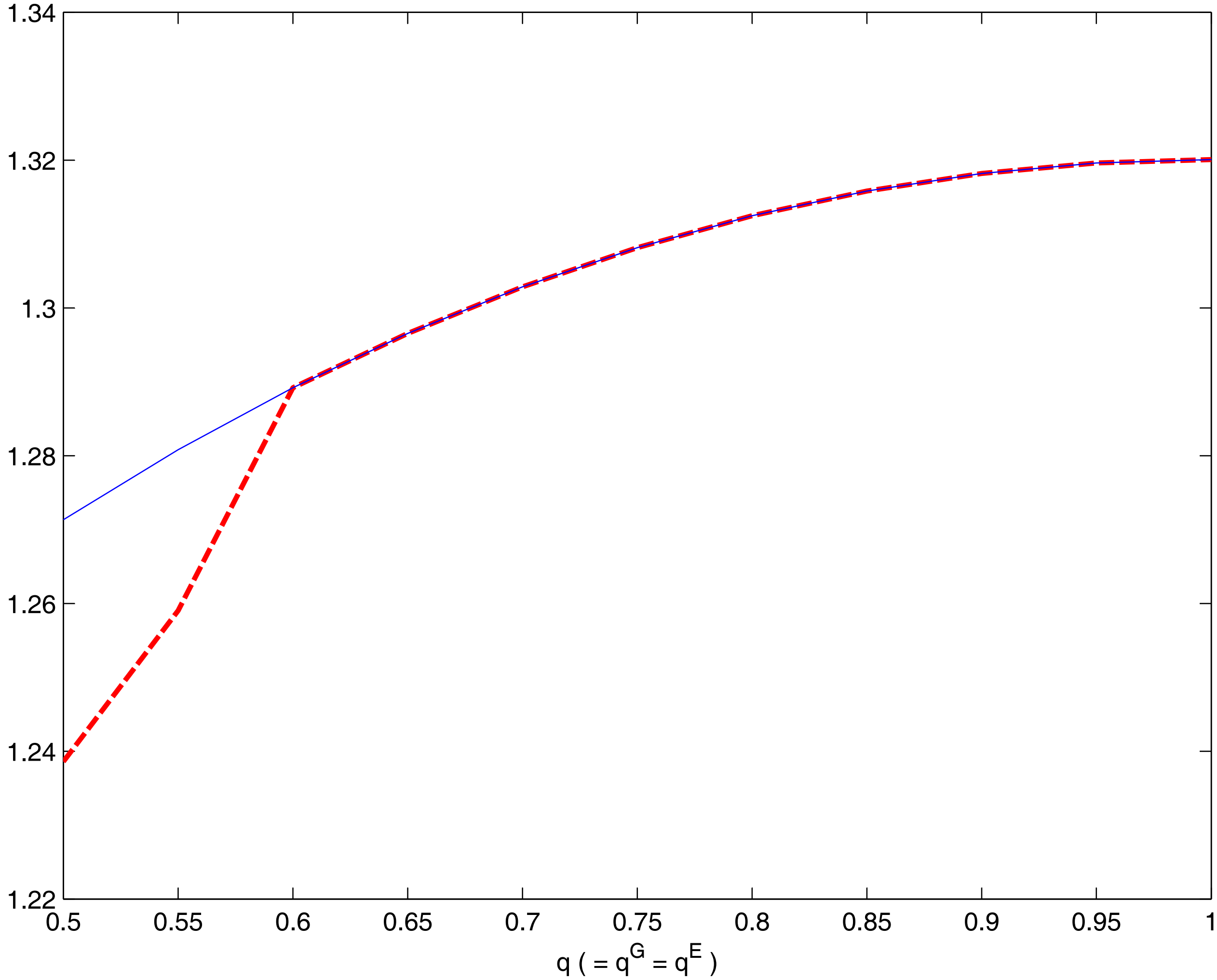


Figure 3A. Fully Dependent Chambers: Welfare of the Elite and of the Workers.

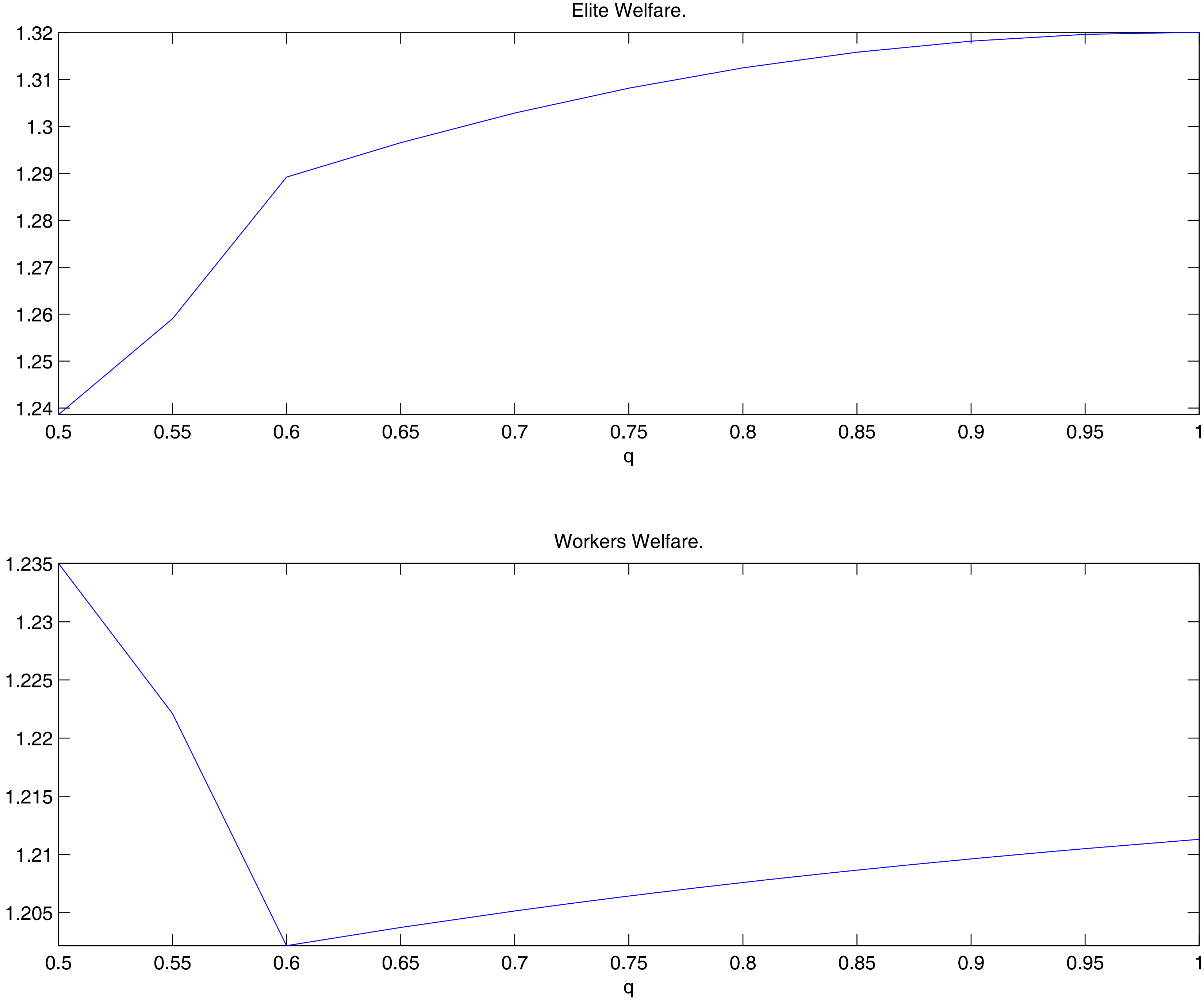


Figure 3B. Welfare of the elite.

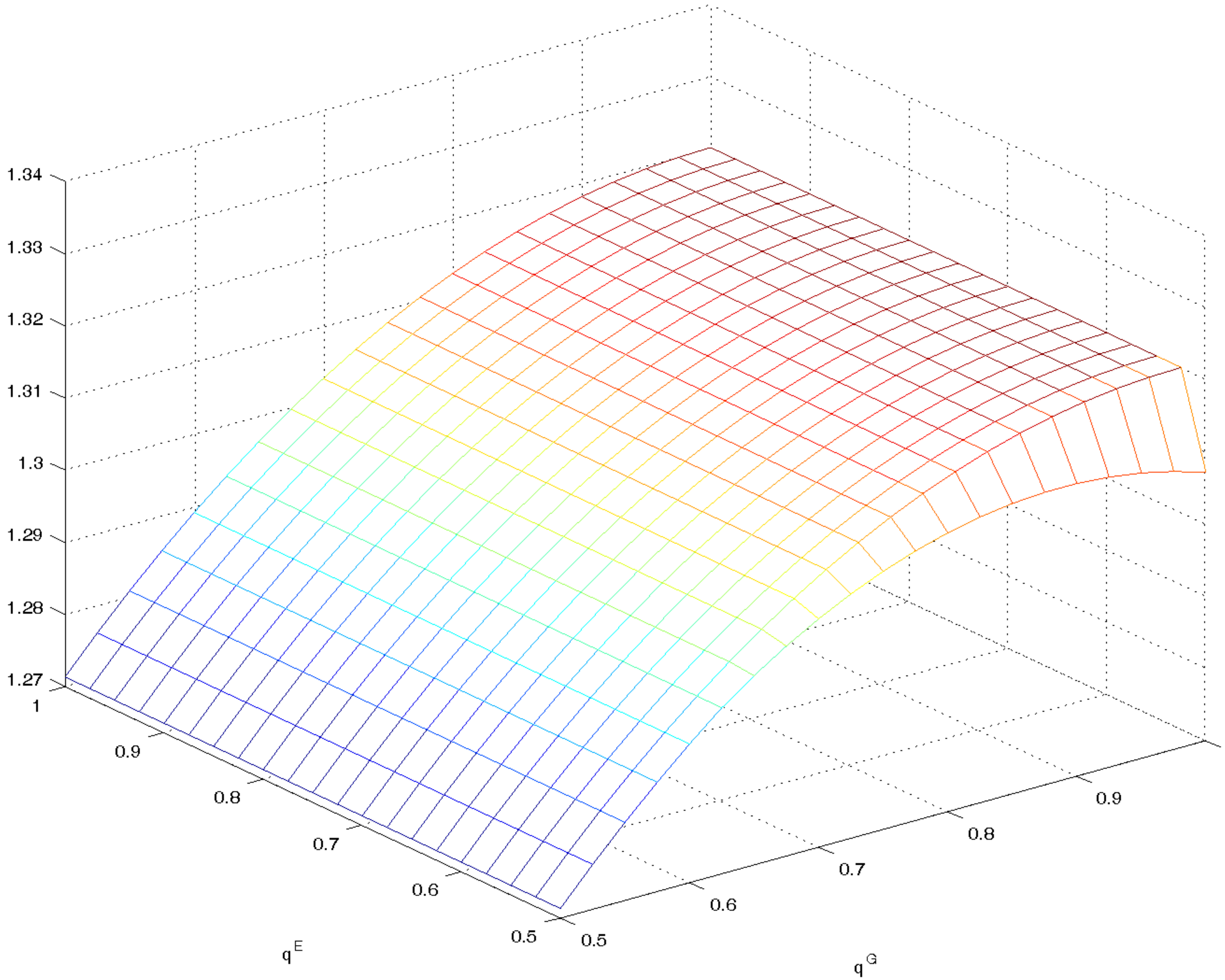


Figure 3C. Welfare of the workers.