

Joint-Search Theory: New Opportunities and New Frictions*

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February 15, 2008

Abstract

Search theory routinely assumes that decisions about acceptance/rejection of job offers (hence, about labor market movements between jobs or across employment states) are made by individuals acting in isolation. In reality, the vast majority of workers are somewhat tied to their partners into couples and families, and decisions are joint. This paper studies, from a theoretical viewpoint, the joint job-search and location problem of a household formed by a couple (e.g., husband and wife) who perfectly pool income. The objective of the exercise, very much in the spirit of standard search theory, is to characterize the reservation wage behavior of the couple and compare it to the single-agent search model in order to derive the implications of partnerships for individual labor market outcomes and wage dynamics. We focus on two main cases. First, when couples are risk averse and pool income, joint-search yields new opportunities—similar to on-the-job search—relative to the single-agent search. Second, when the couple faces offers from multiple locations and a cost of living apart, joint-search features new frictions and can lead to worse outcomes than single-agent search.

*Guvnen thanks the NSF for financial support under grant SES-0649437.

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1 Introduction

In 2000, over 60% of the US population was married, the labor force participation of married women stood at 61%, and in 35% of couples, wives earned at least as much as their husbands. Put it differently, there is a large share of individuals whose labor market choices are intimately tied to those of another individual. For these couples, job search is very much a joint decision-making process.

Surprisingly, since its inception in the early 1970s, search theory has almost entirely focused on the single-agent search problem. The recent survey by Rogerson, Shimer and Wright (2005), for example, does not contain any discussion on optimal job search strategies of two-person households acting as a single decision units. This state of affairs is rather surprising given that Burdett and Mortensen (1978), in their seminal piece on “Labor Supply Under Uncertainty”, lay out a two-person search model and sketch a characterization of its solution, explicitly encouraging further work on the topic. This pioneering effort, which remained virtually unfollowed, represents the starting point of our theoretical analysis. Only very recently, a renewed interest seems to have arisen in the study of household interactions in the context of frictional labor market models (see, e.g., Dey and Flinn, 2007).

Our theoretical analysis focuses on the search problem of a couple who faces exactly the same economic environment as in the standard single-agent search problem of McCall (1970), and Mortensen (1970), without on the job search, and Burdett (1978) with on the job search. A couple is an economic unit composed by two individuals linked to each other by the assumption of perfect income pooling. There is an active and growing literature that attempts to understand the household decision making process, and emphasizes deviations from the unitary model we adopt here, e.g. Chiappori (1992). While we agree with the importance of many of those features, incorporating them into the present framework will make it harder to compare the outcomes of single-search and joint-search problems. The simple unitary model of a household adopted here is a convenient starting point, which helps to examine more transparently the role of the labor market frictions and insurance opportunities introduced by joint-search.

From a theoretical perspective, there are numerous reasons why couples would make a joint decision leading to choices different from those of a single agent. We have started from the most obvious and natural ones. First, the couple has concave preferences over pooled income. Second, the couple can receive job offers from multiple locations but faces a utility cost of living apart (here, deviation from the single-agent search problem occurs even for linear preferences).

Our theoretical analysis has the appeal of leading to two-dimensional graphs in the space of the two spouses wages (w_1, w_2) where the reservation wage policies can be easily analyzed and interpreted.

As summarized by the title of our paper, relative to single-agent search, joint search induces new opportunities and new frictions. First, the model with concave utility shows that joint-search works

similarly to on-the-job search by offering additional job opportunities to the couple to climb the wage ladder: the reservation wage of the dual-searcher couple is lower than that of the single agent, but (with CARA or DARA utility) the reservation function of the worker-searcher couple is always higher. As a result, the unemployment durations are longer in general, but the mean wage is also higher in the joint-search economy. A symbol of this property of joint-search is what we labelled the “breadwinner cycle”: spouses may alternate in quitting/searching and working according to whom receives the best job offer. In this economy, couple are always better off than singles—not just because of more effective consumption smoothing.

Second, the model with multiple locations and a cost of living apart shows that joint-search induces new frictions: the model can generate what Mincer called tied-stayers (workers who turn down a job offer in a different location that would accept as single) and tied-movers (workers who accept a job offer in the location of the partner who would turn down as single). In this economy, with risk neutrality, couple are always worse off than singles.

The set of Propositions proved in the paper formalizes the new opportunities and the new frictions in terms of comparison between reservation wage functions of the couple and reservation wage of the single agent.

The rest of the paper is organized as follows. Section 2 describes the single-agent problem which provides the benchmark of comparison throughout the paper. Section 3 develops and fully characterizes the baseline joint-search problem. Section 4 extends this baseline model in a number of directions: on-the-job search, exogenous separations, access to borrowing and saving, and symmetries in labor market characteristics between husband and wives. Section 5 studies an economy with multiple locations, and a cost of living apart for the couple. Section 6 concludes the paper. All the proofs of the lemmas and propositions in the paper are in the Appendix.

2 The single-agent search problem

To warm up, we first present the sequential job search problem of a single agent—the well-known McCall-Mortensen (McCall, 1970; Mortensen, 1970) model. This model provides a useful benchmark against which we compare the joint-search model, which we introduce in the next section. For clarity of exposition, we begin with a very stylized version of the search problem, and then consider several extensions in Section 4.

Economic Environment. Consider an economy populated with individuals who all participate in the labor force: agents are either employed or unemployed. Time is continuous and there is no aggregate uncertainty. Workers maximize the expected lifetime utility from consumption

$$E_0 \int_0^{\infty} e^{-rt} u(c(t)) dt$$

where r is the subjective rate of time preference, $c(t)$ is the instantaneous consumption flow at time t , and $u(\cdot)$ is the instantaneous utility function.

An unemployed worker is entitled to an instantaneous benefit, b , and receives wage offers, w , at rate α from an exogenous wage offer distribution, $F(w)$ with support $[0, \infty)$. The worker observes the wage offer, w , and decides whether to accept or reject it. If he accepts the offer, he becomes employed at wage w forever. If he rejects the offer, he continues to be unemployed and to receive job offers. All individuals are identical in terms of their labor market prospects, i.e., they face the same wage offer distribution and the same arrival rate of offers, α .

There is no access to financial markets, nor storage, so consumption will be equal to the wage earnings. there are no exogenous separations, and no on the job search.¹

Value functions. Denote by V and W the value functions of an unemployed and employed agent, respectively. Then, using the continuous time Bellman equations, the problem of a single worker can be written in the following flow value representation:

$$rV = u(b) + \alpha \int \max\{W(w) - V, 0\} dF(w), \quad (1)$$

$$rW(w) = u(w). \quad (2)$$

This well-known problem yields a unique reservation wage, w^* , for the unemployed such that for any wage offer above w^* , she accepts the offer and below w^* , she rejects the offer.² Furthermore, this reservation wage can be obtained as the solution to the following equation:

$$\begin{aligned} u(w^*) &= u(b) + \frac{\alpha}{r} \int_{w^*}^{\infty} (u(w) - u(w^*)) dF(w) \\ &= u(b) + \frac{\alpha}{r} \int_{w^*}^{\infty} u'(w) (1 - F(w)) dw, \end{aligned} \quad (3)$$

which equates the instantaneous utility of accepting a job offer paying the reservation wage (left hand side, LHS) to the option flow value of continuing to search in the hope of obtaining a better offer in the future (right hand side, RHS). Since the LHS is increasing in w^* whereas the RHS is a decreasing function of w^* , the above equation uniquely determines the reservation wage, w^* .

3 The joint-search problem

We now study the search problem of a couple facing the same economic environment described above. A couple is an economic unit composed by two individuals who are linked to each other by

¹ Access to financial markets, on the job search and and exogenous job separation are introduced in Section 4.

² In the equations above, and in what follows, when we abstain from specifying the upper or/and lower limits of the integral, it is implicit that they should be the upper or/and lower bound of the support of w .

the assumption that they perfectly pool income. Given the absence of storage, households simply consume their total income in each period which is the sum of the wage or benefit income of each spouse. Couples make their job acceptance/rejection/quit decisions jointly, because each spouse's search behavior affects the couple's joint welfare.

A couple can be in one of three labor market states. First, if both spouses are unemployed and searching, they are referred to as a “dual-searcher couple.” Second, if both spouses are employed (an absorbing state) we refer to them as a “dual-worker couple.” Finally, if one spouse is employed and the other one is unemployed, we refer to them as a “worker-searcher couple.” As can perhaps be anticipated, the most interesting state is the last one.

Value Functions. Let U denote the value function of a dual-searcher couple, $\Omega(w_1)$ the value function of a worker-searcher couple when the worker's wage is w_1 , and $T(w_1, w_2)$ the value function of a dual-worker couple earning wages w_1 and w_2 . The flow value in the three states becomes

$$rT(w_1, w_2) = u(w_1 + w_2), \quad (4)$$

$$rU = u(2b) + 2\alpha \int \max\{\Omega(w) - U, 0\} dF(w), \quad (5)$$

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2). \quad (6)$$

The equations determining the first two value functions (4) and (5) are straightforward analogs of their counterparts in the single-search problem. In the first case, both spouses stay employed forever, and the flow value is simply equal to the total instantaneous wage earnings of the household. In the second case, the flow value is equal to the instantaneous utility of consumption (which equals the total unemployment benefit) *plus* the expected gain in case a wage offer is received. Because both agents receive wage offers at rate α , the total offer arrival rate of a dual-searcher couple is 2α . Once a wage offer is received by either spouse, it will be accepted if it results in a gain in lifetime utility (i.e., $\Omega(w) - U > 0$), otherwise it will be rejected.

The value function of a worker-searcher couple is somewhat more involved. As can be seen in equation (6), if a couple receives a wage offer (which now arrives at rate α since only one spouse is unemployed) there are three choices facing the couple. First, the unemployed spouse can reject the offer, in which case there is no change in the value. Second, the unemployed spouse can accept the job offer and both spouses become employed, which increases the value by $T(w_1, w_2) - \Omega(w_1)$. Third, the unemployed spouse can accept the job offer *and* the employed spouse simultaneously quits his job and starts searching for a better one.

As we shall see below, this third case is the first important difference between the joint-search problem and the single-agent search problem. In the single-search problem, once an agent accepts a job offer, she will never choose to quit her job. This is because an agent strictly prefers being employed to searching at any wage offer higher than the reservation wage. Because the environment

is stationary, the agent will face the same wage offer distribution upon quitting and will have the same reservation wage. As a result, a single employed agent will never quit, even if he is given the opportunity. In contrast, in the joint-search problem, the reservation wage of each spouse depends on the income of the partner. When this income grows, for example because of a transition from unemployment to employment, also the reservation wage of the previously employed spouse may increase, which could lead to exercising the quit option. We return to this point below and discuss it in more detail.

3.1 Characterizing the couple's decisions

To better understand the optimal choices of the couple, it is instructive to treat the accept/reject decision of the unemployed spouse and the stay/quit decision of the employed spouse as two separate choices (albeit the couple makes them simultaneously). Before we begin characterizing the solution to the problem, we state the following useful lemma. We refer to Appendix A for all the proofs and derivations.

Lemma 1 *Ω is a strictly increasing function, i.e., $\Omega'(w) > 0$ for all $w \in [0, \infty)$.*

We are now ready to characterize the couple's search behavior. First, for a dual-searcher couple, the reservation wage—which is the same for both spouses by symmetry—is denoted by w^{**} , and is determined by the equation:

$$\Omega(w^{**}) = U. \quad (7)$$

Because U is a constant and Ω is a strictly increasing function (Lemma 1), w^{**} is a singleton.³

A worker-searcher couple has two decisions to make. The first decision is whether accepting the job offer to the unemployed spouse (say spouse 2) or not. The second decision, conditional on accepting, is whether the employed spouse (spouse 1) should quit his job or not. Let the current wage of the employed spouse be w_1 and denote the wage offer to the unemployed spouse by w_2 .

Let's begin by supposing that it is not optimal to exercise the quit option. The couple must decide whether to accept or reject the job offer w_2 . The offer will be accepted when $T(w_1, w_2) \geq \Omega(w_1)$. Formally, the associated reservation wage function $\phi(w_1)$ solves

$$T(w_1, \phi(w_1)) = \Omega(w_1). \quad (8)$$

³Note that no wage below w^{**} will ever be accepted by the couple, and therefore, observed in this model, which means that we can focus attention on the behavior of value functions and reservation functions for wages above w^{**} . Therefore, the statements we make below about the properties of certain function should be interpreted to apply to those functions only for $w > w^{**}$, and may or may not apply below that level.

Suppose now instead that it is optimal to exercise the quit option. Then, the job offer will be accepted when $\Omega(w_2) \geq \Omega(w_1)$ which implies the reservation rule

$$\Omega(\phi(w_1)) = \Omega(w_1). \quad (9)$$

Given the strict monotonicity of Ω , the reservation wage rule is very simple: accept the new offer and quit the old job whenever $w_2 \geq w_1$.

The worker-searcher reservation wage function $\phi(\cdot)$ is therefore piecewise, being composed of (8) and (9) in different ranges of the domain for w_1 . The kink of this piecewise function, which always lies on the 45 degree line of the (w_1, w_2) space, plays a special role in characterizing the behavior of the couple. We denote this point by $(\hat{w}, \hat{w})$.⁴ Since $T(\hat{w}, \hat{w}) = u(2\hat{w})$, $\hat{w}$ solves

$$u(2\hat{w}) = \Omega(\hat{w}). \quad (10)$$

It remains to characterize the quitting decision. If $T(w_1, w_2) \leq \Omega(w_2)$ it is optimal for the employed spouse to quit his job while the unemployed spouse accepts her job offer (that is, this choice yields higher utility than him staying at his job and the couple becoming a dual-worker couple). This inequality implies the indifference condition:

$$T(w_1, \varphi(w_1)) = \Omega(\varphi(w_1)). \quad (11)$$

Two important properties of φ should be noted. First, φ is not necessarily a function, it may be a correspondence. Second, φ is the inverse of that piece of the ϕ function defined by (8). This is easily seen. By symmetry of T , from (8) we have that $T(\phi(w_1), w_1) = \Omega(w_1)$, or $T(w_2, \phi^{-1}(w_2)) = \Omega(\phi^{-1}(w_2))$ which compared to (11) yields the desired result.

Since $\varphi = \phi^{-1}$ then φ will also cross the function ϕ on the 45 degree line at the point $\hat{w}$. Therefore, $\hat{w}$ is the wage where the unemployed spouse is indifferent between accepting and rejecting her offer *and* the employed partner is indifferent between keeping and quitting his job. To emphasize this feature, we refer to $\hat{w}$ as the “double indifference point.”

In what follows, we characterize the optimal strategy of the couple in the (w_1, w_2) space, the wage space. This means establishing the ranking between w^{**} and $\hat{w}$, especially in relation to the single-agent reservation wage w^* , and studying the function ϕ . Once we have characterized the shape of ϕ , that of ϕ^{-1} follows immediately. Overall, these different reservation rules will divide the (w_1, w_2) into four regions. One where both spouses work, one where both spouses search and the remaining two regions where spouse one (two) searches and spouse two (one) works.

⁴At this stage we have not proved that $\hat{w}$ is unique, but it will turn out that it is.

3.2 Risk-neutrality

As it will become clear, risk aversion is central to our analysis. To provide a benchmark we begin by presenting the risk-neutral case, then turn to the results with risk averse agents.

Proposition 1 (Risk neutrality) *With risk-neutral preferences, i.e., $u'' = 0$, the joint-search problem reduces to two independent single-search problems. Specifically, the value functions are:*

$$\begin{aligned} T(w_1, w_2) &= W(w_1) + W(w_2), \\ U &= 2V, \\ \Omega(w_1) &= V + W(w_1). \end{aligned}$$

*The reservation wage function $\phi(\cdot)$ of the worker-searcher couple is constant and is equal to the reservation wage value of a dual-searcher couple (regardless of the wage of the employed spouse) which, in turn, equals the reservation value in the single-search problem, i.e., $\phi(w_1) = w^{**} = w^*$.*

Figure 1 shows the relevant reservation wage functions in the (w_1, w_2) space. Without loss of generality we denote the current wage of the employed spouse (in a worker-search couple) by w_1 and display it on the horizontal axis, and denote the wage offer received by the unemployed spouse by w_2 and display it on the vertical axis. As stated in the proposition, the reservation wage function of a worker-searcher couple, $\phi(w_1)$ is simply the horizontal line at w^{**} . Similarly, the reservation wage for the quit decision is the inverse (mirror image with respect to the 45 degree line) of $\phi(w_1)$ and is shown by the vertical line at $w_1 = w^{**}$. The intersection of these two lines gives rise to four regions, in which the couple display distinct behaviors.

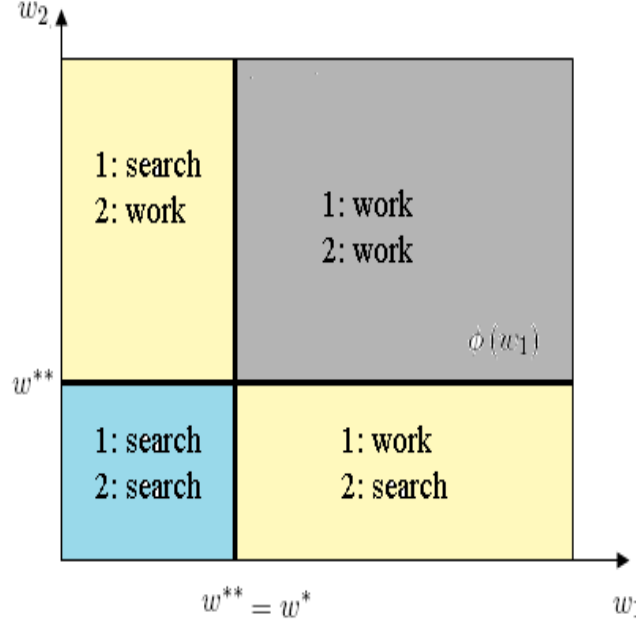
No wage below w^{**} is ever accepted in this model. Therefore, a worker-searcher couple will never be observed with a wage below w^{**} . As a result, the only wage values relevant for the employed spouse are above the $\phi(w_1)$ function. If the unemployed spouse receives a wage offer $w_2 < w^{**}$, she rejects the offer and continues to search. If she receives an offer higher than w^{**} she accepts the offer. At this point the employed partner retains his job, and the couple becomes a dual-worker couple.

For things to get interesting, risk aversion must be brought to the fore. In Section 5, we will also see that when the job-search process takes place in multiple locations and there is a cost of living separately for the couple, then even in the risk neutral case there is an important deviation from the single-agent search problem.

3.3 Risk-aversion

To introduce risk aversion into the present framework we employ preferences in the HARA (Hyperbolic Absolute Risk Aversion) class. This class encompasses several well-known utility functions as

Figure 1: **Reservation Wage Functions of a Risk-Neutral Couple.** Search behavior is identical to the single-search economy.



special cases. Formally, HARA preference are defined as the family of utility functions that have linear risk tolerance: $-u'(c)/u''(c) = a + \tau c$, where a and τ are parameters.⁵

This class can be further divided into three sub-classes depending on the sign of τ . First, when $\tau \equiv 0$, then risk tolerance (and hence absolute risk aversion) is independent of consumption level. This case corresponds to exponential utility $u(c) = -e^{-ac}/a$ and is also referred to as constant absolute risk aversion (CARA). Second, if $\tau > 0$ then absolute risk tolerance is increasing—and therefore risk aversion is decreasing with consumption—which is the decreasing absolute risk aversion (DARA) case. A well-known special case of this class is the constant relative risk aversion (CRRA) case: $u(c) = c^{1-\rho}/(1-\rho)$, which obtains when $a \equiv 0$ and $\tau = 1/\rho > 0$. Finally, if $\tau < 0$ risk aversion increases with consumption, and this class is referred to as increasing absolute risk aversion (IARA). A special case of this class is quadratic utility: $u(c) = -(a-c)^2$, which obtains when $\tau = -1$.

The results derived in this section are related to Danforth (1979) who shows that, in the presence of saving and no exogenous job separation, depending on the degree of absolute risk aversion of the utility function, the reservation wage is either increasing or decreasing in wealth.

⁵Risk tolerance is defined as the reciprocal of Pratt's measure of "absolute risk aversion", thus if risk tolerance is linear, risk aversion is hyperbolic.

3.3.1 CARA utility

We first characterize the search behavior of a couple under CARA preferences and show that it serves as the watershed for the description of search behavior under HARA preferences. The following proposition summarizes the optimal search strategy of the couple.

Proposition 2 (CARA utility) *With CARA preferences, the search behavior of a couple can be completely characterized as follows:*

- (i) *The reservation wage value of a dual-searcher couple is strictly smaller than the reservation wage of single agent: $w^{**} < w^* = \hat{w}$.*
- (ii) *The reservation wage function of a worker-searcher couple is piecewise linear in the employed spouse's wage*

$$\phi(w_1) = \begin{cases} w_1 & \text{if } w_1 \in [w^{**}, w^*) \\ w^* & \text{if } w_1 \geq w^*. \end{cases}$$

Figure 2 provides a visual summary of the contents of this proposition in the wage space. Three important remarks are in order.

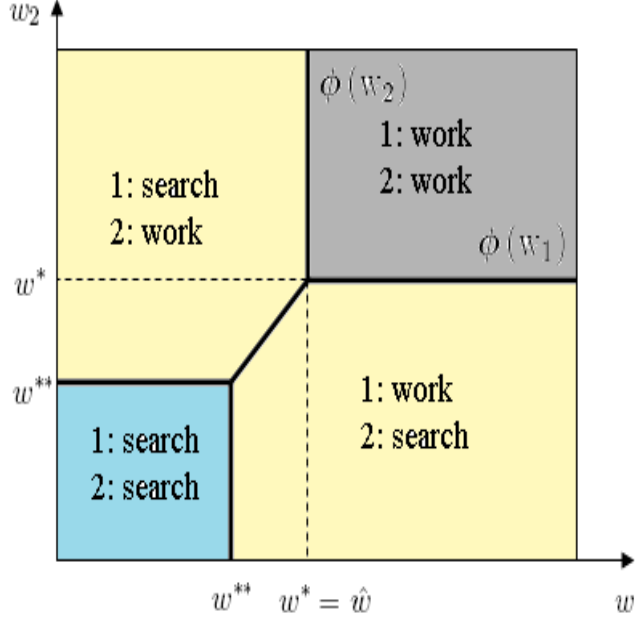
First, the dual searcher couple is less choosy than the single agent ($w^{**} < w^*$). With risk aversion, the optimal search strategy involves a trade-off between lifetime income maximization and the desire for consumption smoothing. The former force pushes up the reservation wage, the second pulls it down as risk-averse agents particularly dislike the low income state (unemployment). The dual-searcher couple can use income pooling to its advantage: it initially accepts lower wage offers (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income) which remains available to the other spouse. In contrast, when the single agent accepts his job he gives up the search option for good which induces her to be more picky at the start. Notice that joint-search plays a role similar to on-the-job search in the absence of it. We return to this point later below.

Second, for a worker-searcher couple earning a wage greater than w^* , the reservation wage function is constant and equal to w^* , the reservation wage value of the single unemployed agent. This is because with CARA utility agents' attitude towards risk does not change with the consumption (and hence wage) level. As the wage of the employed spouse increases, the couple's absolute risk aversion remains unaffected, implying a constant reservation wage for the unemployed partner.

While the appendix contains a formal proof of this result, it is instructive to sketch the argument behind the proof. To this end, first suppose that the employed spouse never quits when his wage w_1 exceeds w^* . In this case, the reservation wage function for the unemployed spouse would have to satisfy:

$$u(w_1 + \phi(w_1)) = u(w_1 + b) + \frac{\alpha}{r} \int_{\phi(w_1)}^{\infty} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] dF(w_2).$$

Figure 2: **Reservation Wage Functions with CARA Preferences.**



With exponential utility we have: $u(w_1 + w_2) = -u(w_1)u(w_2)$, which simplifies the previous condition by eliminating the dependence on w_1 :

$$u(\phi(w_1)) = u(b) + \frac{\alpha}{r} \int_{\phi(w_1)}^{\infty} (u(w_2) - u(\phi(w_1))) dF(w_2).$$

Notice that, since the dependence on the employed partner wage w_1 ceases, this condition becomes exactly the same as the one in the single-search problem (equation 3) and is thus satisfied by the constant reservation function: $\phi(w_1) = w^*$. Moreover, when ϕ is a constant function, its inverse $\phi^{-1}(w_1) = \infty$, and thus there is no wage offer w_2 that can exceed $\phi^{-1}(w_1)$ and justify quitting, which in turn justifies our conjecture that the employed spouse does not quit in the wage range $w_1 > w^*$.

Breadwinner cycle. A third remark, and a key implication of the proposition, is that the reservation wage value of a dual-searcher couple w^{**} being strictly smaller than w^* activates the region where $\phi(w_1)$ is strictly increasing, and in turn gives rise to the “breadwinner cycle.” Suppose that $w_1 \in (w^{**}, w^*)$ and the unemployed spouse receives a wage offer $w_2 > w_1 = \phi(w_1)$, where the equality only holds in the specified region (w^{**}, w^*) . Because the offer is higher than the worker-searcher couple’s reservation wage, the unemployed spouse accepts the offer and becomes employed. However, accepting this wage offer also implies $w_2 > \phi^{-1}(w_1) = w_1$ which, in turn, implies $w_1 < \phi(w_2)$. This means that the threshold for the first spouse to keep his job now exceeds

his current wage, and he will quit.

As a result, the spouses simultaneously switch roles and transition from a worker-searcher couple into another worker-searcher couple with higher wage level. This process repeats itself over and over again as long as the employed spouse's wage stays in the range (w^{**}, w^*) , although of course the identity of the employed spouse (i.e., the breadwinner) alternates. Once both spouses have in hands job offers beyond w^* , the breadwinner cycle stops and so does the search process.

3.3.2 DARA utility

As noted earlier, DARA utility is of special interest, since it encompasses the well-known and commonly used CRRA utility specification $u(c) = c^{1-\rho}/(1-\rho)$. More generally, the coefficient of absolute risk aversion with DARA preferences is $-u''(c)/u'(c) = \rho/(c + \rho a)$, which decreases with the consumption (and hence the wage) level. The following proposition characterizes the optimal search strategy for couples with DARA preferences.

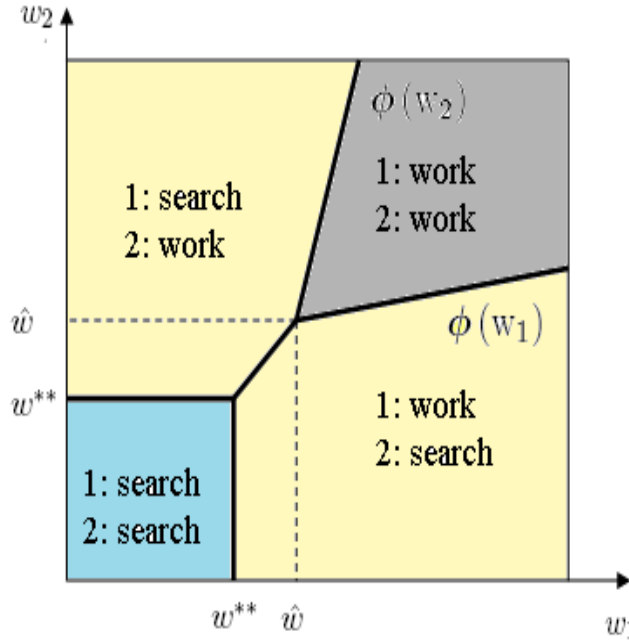
Proposition 3 (DARA utility) *With DARA preferences, the search behavior of a couple can be completely characterized as follows:*

- (i) *The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$ (with $w^* < \hat{w}$) which implies that the breadwinner cycle exists.*
- (ii) *The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly increasing with $\phi' < 1$.*

Figure 3 provides a graphical representation of the reservation wage functions associated to the DARA case. Unlike the CARA case, the reservation function of the worker-searcher couple does not feature a constant piece. It depends on the wage of the employed spouse at all wage levels. This is because with DARA utility, absolute risk aversion falls with household resources. Therefore, as the wage of the employed spouse increases, the couple becomes less concerned about smoothing consumption and becomes more picky in its job search.

The proposition also shows that the breadwinner cycle continues to exist. In contrast to the CARA case, now the breadwinner cycle is observed over a wider range of wage values of the employed spouse. This is because, as can be seen in Figure 3, ϕ is strictly increasing in w_1 , so its inverse is not a vertical line anymore but is itself an increasing function. As a result, even when $w_1 > \hat{w}$, a sufficiently high wage offer—one that exceeds $\phi^{-1}(w_1)$ —will not only be accepted by the unemployed spouse but it will also trigger the employed spouse to quit. One way to understand this result is by noting that the employed spouse will quit if his reservation wage upon quitting is higher than his current wage. If $w_2 > \phi^{-1}(w_1)$, this implies that upon quitting the job, the

Figure 3: **Reservation Wage Functions with DARA Preferences (CRRA is a Special Case).**



reservation wage for the currently employed spouse becomes $\phi(w_2) > \phi(\phi^{-1}(w_1)) = w_1$. Since this reservation wage is higher than his current wage, it is optimal for the employed spouse to quit the job. Finally, note that only if the wage offer is $w_2 \in (\phi(w_1), \phi^{-1}(w_1))$, the job offer is accepted without triggering a quit.

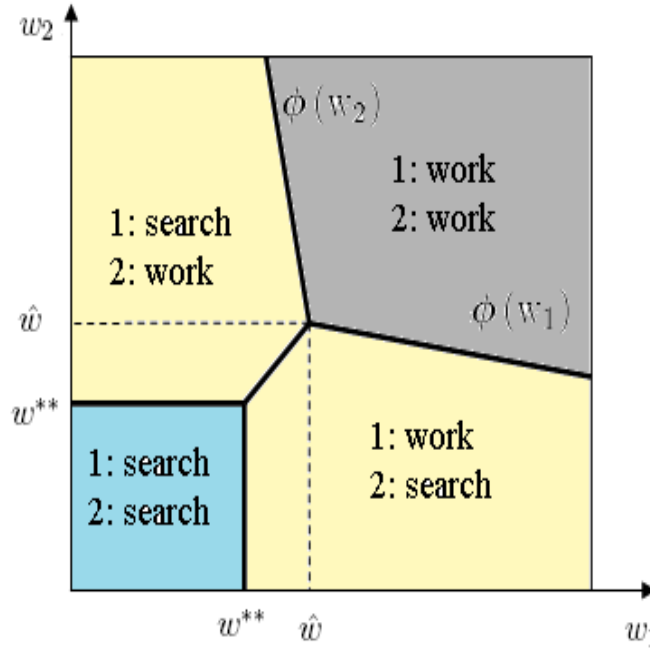
3.3.3 IARA utility

We now turn to IARA preferences, which display increasing absolute risk aversion as consumption increases. One well-known example for IARA utility is quadratic utility: $-(a - c)^2$ where $c \leq a$.

Proposition 4 (IARA utility) *With IARA preferences, the search behavior of a couple can be completely characterized as follows:*

- (i) *The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$, which implies that the breadwinner cycle exists.*
- (ii) *The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly decreasing.*

Figure 4: **Reservation Wage Functions with IARA Preferences (Quadratic Utility is a Special Case).**



The proof of the proposition is very similar to the DARA case, and is therefore omitted for brevity.⁶ Figure 4 graphically shows the IARA case.

The reservation wage function ϕ of a worker-searcher couple deviates from the CARA benchmark in the opposite direction of the DARA case. In particular, beyond wage level $\hat{w}$, the reservation function $\phi(w_1)$ is decreasing in w_1 , whereas it was increasing in the DARA case. As a result, if the unemployed spouse receives a wage offer higher than $\phi^{-1}(w_1)$, she accepts the offer, the employed stays in the job and both stay employed forever. If the wage offer instead is between $\phi(w_1)$ and $\phi^{-1}(w_1)$, then the job offer is accepted followed by a quit by the employed spouse. This behavior is the opposite of the DARA case where high wage offers resulted in quit and intermediate wages did not. Moreover, now the breadwinner cycle never happens at wage levels $w_1 > \hat{w}$. This is a direct consequence of increasing absolute risk aversion which induces a couple to become less choosy when searching as its wage level rises.

⁶The logic of the proof is as follows. Guess that at some wage w_1 the employed worker never quits, and verify the guess by using the property of IARA equivalent to (30), but with the inequality reversed. The rest of the proof is exactly as for the DARA case.

4 Extensions

The basic framework in the previous section was intended to provide the simplest possible deviation from the well known single-search problem in the direction of introducing couples jointly searching for jobs. Despite being highly stylized, this simple framework illustrated some new and potentially important mechanisms driving a couple's search behavior that are not operational in the single-agent search problem.

In this section, we enrich this basic model in three empirically relevant directions. First, we add on-the-job search. Second, we allow for exogenous job separations. Third, we allow households to access financial markets. We are able to establish analytical results in some special cases. We also simulate a calibrated version of our model to analyze the differences between a single-agent search economy and the joint-search economy in more general cases.

4.1 On-the-job search

Suppose that agents can search both off and on the job. During unemployment they draw a new wage from $F(w)$ at rate α_u while during employment they sample new job offers from the same distribution F at rate α_e . What we develop below is, essentially, a version of the Burdett (1978) wage ladder model with couples.

The flow value functions become:

$$rU = u(2b) + 2\alpha_u \int \max\{\Omega(w) - U, 0\} dF(w) \quad (12)$$

$$\begin{aligned} r\Omega(w_1) = & u(w_1 + b) + \alpha_u \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\ & + \alpha_e \int \max\{\Omega(w'_1) - \Omega(w_1), 0\} dF(w'_1), \end{aligned} \quad (13)$$

$$\begin{aligned} rT(w_1, w_2) = & u(w_1 + w_2) + \alpha_e \int \max\{T(w'_1, w_2) - T(w_1, w_2), 0\} dF(w'_1) \\ & + \alpha_e \int \max\{T(w_1, w'_2) - T(w_1, w_2), 0\} dF(w'_2) \end{aligned} \quad (14)$$

We keep denoting the reservation wage of the dual searcher couple as w^{**} , and the reservation wage of the unemployed spouse in the worker-searcher couple as $\phi(w_1)$. We now have a new reservation function, that of the employed spouse (in the dual-worker couple and in the worker-searcher couple) which we denote by $\eta(w_i)$.

It is intuitive (and can be proved easily) that under risk neutrality the joint-search problem coincides with the problem of the single agent. Below, we prove another “equivalence result” for

the special case of symmetric offer arrival rates $\alpha_u = \alpha_e$, i.e. when search is equally effective on and off the job.

Proposition 5 (On-the-job search with symmetric arrival rates) *If $\alpha_u = \alpha_e$, the joint-search problem yields the same solution as the single-agent search problem, regardless of preferences. Specifically, $w^{**} = w^* = b$, $\phi(w_1) = w^{**}$ and $\eta(w_i) = w_i$ for $i = 1, 2$.*

To understand this equivalence result, note that one way to think about joint-search is that it provides a way to climb the wage ladder for the couple even without on-the-job search: when a dual-searcher couple accepts the first job offer, it continues to receive offers, albeit at a reduced arrival rate. Therefore, one can view joint-search as “costly” version of on-the-job search. The cost comes from the fact that, absent on the job search, in order to keep the search option active, the pair must remain a worker-searcher couple, and thus it not enjoy the full wage earnings of a dual-worker couple as it would be capable of doing with on the job search. As a result, when on-the-job search is explicitly introduced and the offer arrival rate is equal across employment states, it fully neutralizes the benefits of joint-search and makes the problem equivalent to that of a single-agent.

The solution is simply that the unemployed partner should accept any offer above b and the spouse employe at w_1 any wage above its current one.

4.2 Exogenous separations

Once again, under risk neutrality it is easy to establish that the joint-search problem collapses to that of the single agent. However, when risk aversion is introduced new economic forces start playing a role. Without exogenous separation, the future wage earnings of the employed spouse are simply a deterministic income stream—constant at least as long as the searching partner remains unemployed. When a worker-searcher couple employed at w_1 sets its reservation wage $\phi(w_1)$, this wage stream acts as a risk-free asset in the household’s portfolio leading to a wealth effect whose strength depends on the degree of absolute risk aversion.

With exogenous separations, instead, the employment status becomes stochastic, which makes the future wage stream of the employed partner effectively a risky asset in the household’s portfolio.

For CARA and DARA preferences, we can prove the following result.

Proposition 6 (CARA utility with exogenous separations) *With CARA or DARA preferences and exogenous job separation, the search behavior of a couple can be completely characterized as follows:*

- (i) *The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$ (with $w^* < \hat{w}$) which implies that the breadwinner cycle exists.*

- (ii) *The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly increasing with $\phi' < 1$.*

Qualitatively, this case is similar to the DARA case without exogenous separations. When preferences are CARA, what is that twists the slope of the ϕ function from zero to positive? With exogenous separations, the job becomes a risky asset. As w_1 increases, the gap $w_1 - b$ rises and the separation risk borne by the couple goes up accordingly. Because of CARA utility, the degree of risk aversion remains constant, but since risk goes up the couple wants to optimally rebalance its portfolio towards the safe asset, which is unemployment. This choice calls for a rise in the reservation wage $\phi(w_1)$.

4.2.1 DARA utility and the “gender asymmetry puzzle”

Lentz and Tranaes (2005) document empirically, from Danish data, that while the unemployment duration of the wife (hence their reservation wage) is *increasing* in the husband’s wage, the unemployment duration of the husband is *decreasing* in the wife’s wage, a fact that they term the “gender asymmetry puzzle.”

By simulation, we can show that the extent that married women have a higher exogenous separation rate than married man, the joint-search framework is able to generate this phenomenon. Gender-specific differences in separation rates could arise due to unexpected shocks to household’s home production needs (such as childrearing, etc.) that may require the wife to quit her job (more so than the husband), or to women being overrepresented in more volatile occupations or sectors. Figure 5 plots the reservation wage functions for a couple under this assumption⁷.

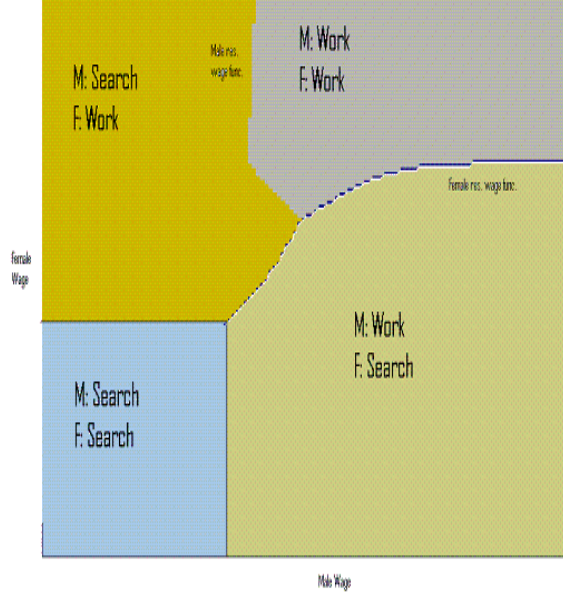
4.3 Access to borrowing and saving

With few exceptions, search models with risk-averse agents and a saving decision are typically not amenable to theoretical analysis.⁸ One such exception is when preferences are of CARA type and agents have access to a risk-free asset, an environment that has been used in some previous work to obtain analytical results (Danforth (1979), Acemoglu and Shimer (1999), Shimer and Werning (2006)). Following this tradition, we consider the CARA framework studied in Section 3.3.1 extended to borrowing and saving. Before analyzing the joint search problem, it is useful to recall here the solution to the single-agent problem.

⁷Weekly male separation rate is assumed to be 0 while female separation rate is 0.054.

⁸It is therefore not surprising that most studies of search models with risk-aversion and savings restrict attention to quantitative analyses. For examples where the decision maker is a household, see Costain (1999), Browning, Crossley and Smith (2003), Lentz (2005), Lentz and Tranaes (2005), Rendon (2006) and Lise (2006).

Figure 5: **Gender asymmetry puzzle.**



Single-agent search problem. Let a denote the asset position of the individual. Assets evolve according to the law of motion

$$\frac{da}{dt} = ra + y - c, \quad (15)$$

where r is the risk-free interest rate, y is income (equal to w during employment and b during unemployment), and c is consumption. The value functions for the employed and unemployed single agent are, respectively:

$$rW(w, a) = \max_c \{u(c) + W_a(w, a)(ra + w - c)\}, \quad (16)$$

$$rV(a) = \max_c \{u(c) + V_a(a)(ra + b - c)\} + \alpha \int \max\{W(w, a) - V(a), 0\} dF(w), \quad (17)$$

where the subscript a denotes the partial derivative with respect to wealth. These equations reflect the non-stationarity due to the change in assets over time. For example, the second term in (16) is $(dW/dt) = (dW/da) \cdot (da/dt)$. And similarly for the second term in (17).

We begin by conjecturing that $rW(w, a) = u(ra + w)$. If this is the case, then the FOC determining optimal consumption for the agent gives $u'(c) = u'(ra + w)$ which confirms the conjecture and establishes that the employed individual consumes his current wage plus the interest income on the risk free asset. Let's now guess that $rV(a) = u(ra + w^*)$. Once gain, it is easy to verify

this guess through the FOC of the unemployed agent. Substituting this solution back into equation (17) and using the CARA assumption yields

$$w^* = b + \frac{\alpha}{\rho r} \int_{w^*} [u(w - w^*) - 1] dF(w) \quad (18)$$

which shows that w^* is the reservation wage, which is independent of wealth. Therefore, the unemployed worker consumes the reservation wage plus the interest income on his wealth. This result highlights an important point: the asset position of an unemployed worker deteriorates and, in presence of a debt constraint, she may hit it. As the rest of the papers cited above which use this set up, we abstract from this possibility. The implicit assumption is that borrowing constraints are “loose” and by this we mean they do not bind along the solution for the unemployed agent.

Joint-search problem. When the couple search jointly for jobs, the asset position of the couple still evolves based on (15), but now $y = 2b$ for the dual searcher couple, $b + w_1$ for the worker-searcher couple, and $w_1 + w_2$ for the employed couple. The value functions become:

$$rT(w_1, w_2, a) = \max_c \{u(c) + T_a(w_1, w_2, a)(ra + w_1 + w_2 - c)\}, \quad (19)$$

$$rU(a) = \max_c \{u(c) + U_a(a)(ra + 2b - c)\} + \alpha \int \max\{\Omega(w, a) - U(a), 0\} dF(w) \quad (20)$$

$$\begin{aligned} r\Omega(w_1, a) &= \max_c \{u(c) + \Omega_a(w_1, a)(ra + w_1 + b - c)\} \\ &+ \alpha \int \max\{T(w_1, w_2, a) - \Omega(w_1, a), \Omega(w_2, a) - \Omega(w_1, a), 0\} dF(w_2). \end{aligned} \quad (21)$$

Solving this problem requires characterizing the optimal consumption policy for the dual-searcher couple $c_u(a)$, for the worker-searcher couple $c_\Omega(w_1, a)$, and for the dual-worker couple $c_e(w_1, w_2, a)$, as well as the reservation wage functions, now potentially a function of wealth too, which must satisfy, as usual: $\Omega(w^{**}(a), a) = U(a)$, $T(w_1, \phi(w_1, a), a) = \Omega(w_1, a)$ and $\Omega(\phi(w_1), a) = \Omega(w_1, a)$.

The following proposition characterizes the solution to this problem.

Proposition 7 (CARA utility and access to financial markets) *With CARA preferences, access to risk-free borrowing and lending, and “loose” debt constraints, the search behavior of a couple can be characterized as follows:*

- (i) *The optimal consumption policies are: $c_u(a) = ra + 2w^{**}$, $c_\Omega(w_1, a) = ra + w^{**} + w_1$ and $c_e(w_1, w_2, a) = ra + w_1 + w_2$.*
- (ii) *The reservation function ϕ of the worker-searcher couple is independent of (w_1, a) and equals w^{**} , so there is no breadwinner cycle.*

(iii) *The reservation wage w^{**} of the dual-searcher couple equals w^* , the reservation wage of the single-agent problem.*

The main message of this proposition could perhaps be anticipated by the fact that borrowing and saving effectively substitutes for the consumption smoothing provided within the household, making the latter redundant. Consequently, each spouse in the couple can implement labor market search strategies that are independent from the other spouse actions: each spouse acts as in the single-agent model.

4.4 Some illustrative simulations

In this section we wish to gain some sense about the quantitative differences in labor market outcomes between the single-search and the joint-search economy. We start from the case of CRRA utility and exogenous separations. Later we add on-the-job search. Thus the economy is characterized by the following set of parameters: $b, r, \rho, \delta, F, \alpha_u$ and α_e when we have on the job search.

We first simulate labor market histories for a large number of individuals acting as singles, compute their optimal choices and some key statistics: the reservation wage w^* , the mean wage, unemployment rate and unemployment duration. Second, we pair individuals together and we treat them as couples solving the joint-search problem in exactly the same economy (i.e., same set of parameters $\{b, r, \rho, \delta, F, \alpha_u, \alpha_e\}$).⁹

The interest of the exercise lies in comparing the key labor market statistics across economies. For example, it is not obvious whether the joint-search model would have larger or smaller unemployment rates: for the dual-searcher couples, $w^{**} < w^*$, but for the worker-searcher couple $\phi(w)$ is above w^* at least for large enough wages of the employed spouse.

Calibration. We calibrate the model to replicate salient features of the US economy. The time period in the model is set to one week of calendar time. The short duration of each period is meant to approximate the continuous time structure in the theoretical models (which, among other things, implies that the probability of both spouses receiving simultaneous offers is negligible). The coefficient of relative risk aversion ρ will vary from zero (risk-neutrality) up to eight in simulations. The weekly net interest rate, r , is set equal to 0.001, corresponding to an annual interest rate of 5.3%. Wage offers are drawn from a lognormal distribution with mean μ and standard deviation $\sigma = 0.1$ and mean $\mu = -\sigma^2/2$ so that the average wage is normalized to one. We set $\delta = 0.0054$, which corresponds to a monthly employment-unemployment (exogenous) separation rate of 0.02. The offer arrival rate α_u , is set to different values depending on the risk aversion to match unemployment

⁹To reduce the simulation variance, we use the same sequence of separation shocks and wage offers in the two economies.

Table 1: A Comparison of Single- versus Joint-Search with CRRA Preferences

	$\rho = 0$		$\rho = 2$		$\rho = 4$		$\rho = 8$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
res. wage w^*/w^{**}	1.02	1.02	0.98	0.75	0.81	0.58	0.60	0.48
res. wage $\phi(1)$	—	n/a	—	1.03	—	0.941	—	0.84
double ind. $\hat{w}$	—	1.02	—	1.02	—	0.94	—	0.82
mean wage	1.06	1.06	1.07	1.10	1.01	1.05	1.001	1.01
Mm ratio	1.04	1.04	1.09	1.47	1.23	1.81	1.67	2.10
u rate	5.5%	5.5%	5.4%	7.6%	5.4%	7.7%	5.3%	5.6%
u duration	9.9	9.9	9.7	12.6	9.8	13.3	9.6	10
dual-searcher	—	6	—	4.7	—	7.7	—	7.1
worker-searcher	—	9.8	—	14.2	—	13.6	—	9.6
job quit rate	—	0%	—	11.1%	—	5.55%	—	0.74%
EQVAR- cons.	—	0%	—	4.5%	—	14%	—	26%
EQVAR- income	—	0%	—	1.1%	—	2.8%	—	0.7%

duration and an unemployment rate of roughly 0.055.¹⁰ For the model with on the job search we set the offer arrival rate on the job, α_e , to match a monthly employment-employment transition rate of 0.02. Finally, the value of leisure b is set to 0.40, i.e., 40% of the mean of the wage offer distribution.

Table 1 reports the results of our simulation. The first two columns confirm the statement in Proposition 1 that under risk neutrality the joint-search problem reduces to the single-search problem. Let's now consider the case $\rho = 2$. The reservation wage of the dual-searcher couple is almost 25% lower than in the single-search economy. And this is reflected in the much shorter unemployment durations for the dual-searcher couples. At the same time, though, the reservation wage of the worker-searcher couples is always higher than w^* . In the second row of the table we report the reservation wage of the worker-searcher couple at the mean wage offer. Indeed, for these couples, unemployment duration is higher. Overall, this second effect dominates and the joint-search economy displays longer average unemployment duration, 12.6 weeks instead of 9.7, and considerably higher unemployment rate, 7.6% instead of 5.4%.

Comparing the mean wage tells a similar story. The job-search choosiness of the worker-searcher couples dominates the insurance motive of the dual-searcher couples and the average wage is higher in the joint-search model. The ability of the couple to climb higher up the wage ladder is reflected in the endogenous quit rate (leading to the breadwinner cycle) which is sizeable, 11.1%. Indeed, the region where the breadwinner cycle is active is rather big, as documented by the gap between w^{**} and $\hat{w}$ which is equivalent to 2.7 of the standard deviation of the wage offer.

¹⁰ As risk aversion goes up w^{**} falls, and unemployment duration decreases so to keep matching an unemployment rate of 5.5% we decrease the value of α_u . For example, for $\rho = 0$, $\alpha_u = 0.4$ and for $\rho = 8$, $\alpha_u = 0.12$.

The next six columns display how these statistics change as we increase the coefficient of relative risk aversion. As clear by looking at the first row, in the case $\rho = 0$ the difference between w^* and w^{**} is zero. As ρ goes up, both reservation wages fall. Clearly, higher risk aversion implies a stronger demand for consumption smoothing which makes the agent accept a job offer more quickly. However, the gap between w^* and w^{**} first grows but then it shrinks. Indeed, as $\rho \rightarrow \infty$, it must be true that $w^* = w^{**} = b$ so the two economies converge again. As for $\phi(1)$, it falls as risk aversion increases which means that for higher values of ρ the worker-searcher couples are less demanding which reduces unemployment. Indeed, at $\rho = 8$ the unemployment rate and the mean wage are almost the same in the two economies.

We also report a measure of frictional wage dispersion, the mean-min ratio (Mm) defined as the ratio between the mean wage and the lowest wage, i.e. the reservation wage. Hornstein, Krusell and Violante (2006) demonstrate that a large class of search models, in particular those without on the job search, when plausibly calibrated generate very little wage dispersion. The fifth row of Table 1 confirms this result. It also confirms the finding in Hornstein et al. that the *Mm* ratio increases with risk aversion. What is novel here is that the joint-search model generates more frictional inequality: the reservation wage for the dual searcher couple is lower, but the couple can climb the wage distribution faster which translates into a higher average wage. This result is consistent with the finding in Hornstein et al. that single-agent search models with on the job search fare better in terms of residual wage dispersion.

We also report two measures of welfare gains from being in a couple versus single in our economy. Recall that the jointly searching couple has two advantage: first, it can smooth consumption better, second it can get higher earnings. The first measure of welfare gain is the standard consumption-equivalent variation and embeds both advantages. The second is the change in lifetime income from being married which isolates the second aspect—the novel one.¹¹ The consumption-based measure of welfare gain is very large, not surprisingly. What is remarkable is that also the gains in terms of lifetime income can be very large, for example around 2.8% for the case $\rho = 4$. As risk aversion goes up, the welfare gains from family insurance keep increasing, but as explained above, the ones stemming from better search opportunities fade away.

Table 2 presents the results when we introduce on-the-job search into this environment. The first four columns support our theoretical results. Regardless of parameter values, if agents are risk-neutral, then on-the-job search has no additional effect and both the single-search problem and joint-search problem yield the same solution. We also proved that if the offer arrival rates are equal during employment and unemployment, then again, both economies will have the same solution.

All the qualitative difference between single-search and joint-search models that we have high-

¹¹For the sake of comparison, when we compare the consumption or the income of a married individual to her single counterpart, we take household consumption or income and divide it by two.

Table 2: Single- versus Joint-Search: CRRA Preferences and On-the-Job Search

	$\rho = 0$		$\rho = 2$		$\rho = 2$		$\rho = 4$	
	$\alpha_u = 0.2, \alpha_e = 0.03$		$\alpha_u = 0.1, \alpha_e = 0.1$		$\alpha_u = 0.11, \alpha_e = 0.02$		$\alpha_u = 0.11, \alpha_e = 0.02$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
res. wage w^*/w^{**}	0.98	0.98	0.4	0.4	0.78	0.67	0.62	0.54
res. wage $\phi(1)$	—	0.98	—	0.4	—	0.85	—	0.74
double ind. $\hat{w}$	—	0.98	—	0.4	—	0.87	—	0.8
mean wage	1.13	1.13	1.16	1.16	1.08	1.09	1.08	1.09
Mm ratio	1.15	1.15	2.90	2.90	1.38	1.63	1.74	2.02
u rate	5.4%	5.4%	5.4%	5.4%	5.3%	5.8%	5.3%	5.4%
u duration	9.8	9.8	10.5	10.5	9.7	10.6	9.6	9.8
dual-searcher	—	7	—	7.7	—	7.1	—	7
worker-searcher	—	9.4	—	9.9	—	10.2	—	9.3
EU quit rate	—	0%	—	0%	—	0.93%	—	0.19%
EE transition	0.45%	0.45%	1.03%	1.03%	0.49%	0.47%	0.51%	0.49%
EQVAR-cons.	—	0%	—	4.6%	—	4.1%	—	15%
EQVAR-income	—	0%	—	0%	—	0.2%	—	0.05%

lighted through the analysis of Table 1 remain true here. However, the differences between economies are much smaller quantitatively. As we argued in Section 4, joint-search and search on the job share many similarities so they are somewhat substitute: once on the job search is available, having a search partner is not so useful any longer to obtain higher earnings—albeit it remains obviously a great way to smooth consumption, as evident from the last two lines of the table.

5 Joint-search with multiple locations

The importance of the geographical dimension in job search is undeniable. For the single-agent search problem, accepting a job in a different market could require a relocation cost that may be high enough to induce the agent to turn down the offer. In the joint-search problem, the spatial dimension introduces a new interesting search friction. In addition to migration costs that also apply to a single agent, a couple is likely to suffer from the disutility of living apart if spouses accept jobs in different locations.

To analyze the joint-search problem with multiple locations, we modify the framework introduced in Section 2 by introducing a fixed flow cost of living separately for a couple. As we shall see below, the introduction of multiple locations leads to several important changes in the search behavior of couples compared to a single agent, even in the risk-neutral case. Furthermore, many of these changes are not favorable to couples, which serves to show that joint-search can itself create new frictions. This is in contrast to the analysis performed so far which only showed new

opportunities of joint search.¹²

To keep the analysis tractable, we first consider agents that search for jobs in two symmetric locations, and provide a theoretical characterization of the solution. In the next subsection, we examine the more general case with $L(> 2)$ locations that is more suitable for a meaningful calibration and provide some results based on numerical simulations.

5.1 Two locations

Environment. As before, we define a couple as an economic unit composed by two individuals (1,2) who are linked to each other by the assumption that income is pooled and household consumption is a public good. No storage is permitted. There are no exogenous separations.

The economy has two locations. Couples incur a flow resource cost, denoted by κ , if they live apart. Denote by i the “inside” location and by o the “outside” location. Offers arrive at rate α_i from the current location and at rate α_o from the outside location. The two locations have the same wage offer distribution F .¹³ We assume away moving costs: the point of the analysis is the comparison with the single-agent problem and such costs would be equally borne by the single agent.

A couple can be in one of four labor market states. First, if both spouses are unemployed and searching, they are referred to as a “dual-searcher couple.” Second, if both spouses are employed in the same location (in which case they will stay in their jobs forever) we refer to them as a “dual-worker couple” but if they are employed in different locations we refer to them as “separate dual-worker couple” (another absorbing state). Finally, if one spouse is employed and the other one is unemployed, we refer to them as a “worker-searcher couple.” As explained, individuals in a dual searcher couple have no advantage from living separately, so they will be in the same location.

Let $U, T(w_1, w_2), S(w_1, w_2)$ and $\Omega(w_1)$ be the value of these four states, respectively. Then,

¹²This brings to the table the issue of whether two individuals forming a couple would be better off as single and should therefore split. While the interaction between labor market frictions and changes in marital status is a fascinating question, it is beyond the scope of this paper. Here we assume that the couple has committed to stay together or, equivalently, that there is enough idiosyncratic non-monetary value in the match to justify continuing the relationship.

¹³More specifically, one should think of the two locations as offering the same job opportunities. However, being in one particular location allows to get more contacts. This set up implies that once a spouse in a dual-searcher couple finds a job in the outside location, the other spouse will follow as there is no particular advantage for job-search in remaining in the old location while the couple must pay the cost κ of living apart.

we have

$$rT(w_1, w_2) = u(w_1 + w_2) \quad (22)$$

$$rS(w_1, w_2) = u(w_1 + w_2 - \kappa) \quad (23)$$

$$rU = u(2b) + 2(\alpha_i + \alpha_o) \int \max\{\Omega(w) - U, 0\} dF(w) \quad (24)$$

$$\begin{aligned} r\Omega(w_1) = & u(w_1 + b) + \alpha_i \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\ & + \alpha_o \int \max\{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \end{aligned} \quad (25)$$

The first three value functions are easily understood and do not require explanation. The value function for a worker-searcher couple now has to account separately for inside and outside offers. If an inside offer arrives, the choice is the same as in the one-location case since no cost of living separately is incurred. If, however, an outside offer is received, the unemployed spouse may turn down the offer or may accept the job, in which case the couple has two options: either it chooses to live separately incurring cost κ , or the employed spouse quits and follows the newly employed spouse to the new location to avoid the cost.

The decision of the dual-searcher couple is entirely characterized by the reservation wage w^{**} . For the worker-searcher couple, let $\phi_i(w_1)$ and $\phi_o(w_1)$ be the reservation functions corresponding to inside and outside offers. Once again, these functions are piecewise with one piece corresponding to 45 degree line. By inspecting equation (25) it is immediate that, also as in the one location case, the correspondences $\phi_i^{-1}(w_1)$ and $\phi_o^{-1}(w_1)$ characterize the quitting decision.

Single-agent search. Before proceeding further, it is straightforward to see that the single-search problem with two locations is the same as the one-location case, except for the slight adjustment to the reservation wage to account for separate arrival rates from two locations. In the risk neutral case, we have:

$$w^* = b + \frac{\alpha_i + \alpha_o}{r} \int_{w^*} [1 - F(w)] dw. \quad (26)$$

Recall that in the one-location case, risk neutrality resulted in an equivalence between the single-search and joint-search problems. As the next proposition shows, this result does not hold in the two-location case anymore, as long as there is a positive cost κ of living apart:

Proposition 8 (Two locations) *With risk neutrality, two locations and $\kappa > 0$, the search behavior of a couple can be completely characterized as follows. There is a wage value*

$$\hat{w}_S = b + \kappa + \frac{\alpha_i}{r} \int_{\hat{w}_S - \kappa} [1 - F(w)] dw + \frac{\alpha_o}{r} \int_{\hat{w}_S} [1 - F(w)] dw$$

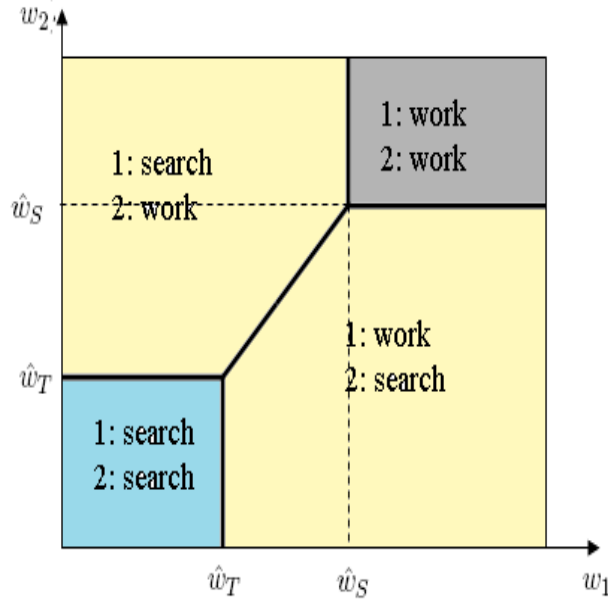


Figure 6: **Reservation Wage Functions for *Outside Offers* with Risk-Neutral Preferences and Two Locations**

and a corresponding value $\hat{w}_T = \hat{w}_S - \kappa$ such that:

- (i) The reservation wage of a dual searcher couple $w^{**} \in (\hat{w}_T, \hat{w})$ whereas $w^* \in (\hat{w}, \hat{w}_S)$. Therefore, $w^{**} < w^*$ which implies that the breadwinner cycle exists.
- (ii) For outside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}_S$, $\phi_o(w_1) = w_1$, and for $w_1 \geq \hat{w}_S$, $\phi_o(w_1) = \hat{w}_S$.
- (iii) For inside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi_i(w_1) = w_1$, for $w_1 \in [\hat{w}, \hat{w}_S)$, $\phi_i(w_1)$ is strictly decreasing and for $w_1 \geq \hat{w}_S$, $\phi_i(w_1) = \hat{w}_T$.

Figures 6 and 7 graphically show the reservation wage functions for outside offers and inside offers, respectively. As seen in these figures, the reservation wage functions for both inside and outside offers are quite different from the corresponding ones of the model with one-location (Figure 1). In particular, the reservation wage functions for both inside offers and outside offers now depend on the wage of the employed spouse at least when $w_1 \in (w^{**}, \hat{w}_S)$. This has several implications.

Consider first outside offers for a worker-searcher couple where one spouse is employed at $w_1 < \hat{w}_S$ (Figure 6). The couple will reject wage offers below w_1 , but when faced with a wage offer above w_1 , the employed worker will quit his job and follow the other spouse to the outside location. The

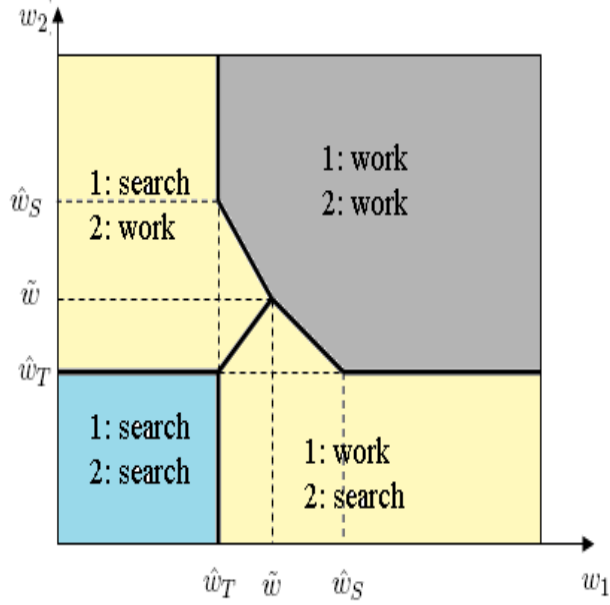


Figure 7: **Reservation Wage Functions for *Inside Offers* with Risk-Neutral Preferences and Two Locations**

cost κ is too large to justify living apart while being employed at such wages. In contrast, when $w_1 > \hat{w}_S$ if the couple receives a wage offer $w_2 > \hat{w}_S$, it will bear the cost of living separately in order to receive such high wages.

Comparing Figure 7 for inside offers to Figure 6, it is immediate that the range of wages for which inside offers are accepted by a worker-searcher couple is larger, since no cost κ has to be paid. Interestingly, the reservation function $\phi_i(w_1)$ now has three distinct pieces. For w_1 large enough, it is constant, as in the single-agent case. In the intermediate range $(\hat{w}, \hat{w}_S)$ the function is decreasing. This phenomenon is linked to the reservation function for outside offers ϕ_o which is increasing in this range: as w_1 rises the gains from search coming from outside offers are lower (it takes a very high outside wage offer w_2 to induce the employed spouse to quit), hence the reservation wage for inside offers falls.

For w_1 small enough, the reservation function $\phi_i(w_1)$ is increasing and equal to the wage of the employed spouse. In this region, the breadwinner cycle is again active, so whenever the wage offer is higher than the employed spouse's wage but smaller than $\varphi_i(w_1)$, the couple goes through the breadwinner cycle. However, if the wage offer is high enough, the potential negative impact of the outside wage offers induces the couple to become a dual-worker couple. Using the same reasoning we applied to the range $(\hat{w}, \hat{w}_S)$, the reservation wage for being a dual-worker couple decreases as w_1 increases.

Tied movers and tied stayers. In a seminal paper, Mincer (1978) has studied empirically

the job-related migration decisions of couples in the United States (during the 1960's and 70's). Following the terminology introduced by Mincer, we refer to a spouse who rejects an outside offer that she would accept when single as a “tied-stayer.” Similarly, we refer to a spouse who follows his spouse to the new destination even though her individual calculus dictates otherwise as a “tied-mover.” Using data from the 1962 BLS survey of unemployed persons, Mincer estimated that “...22 percent or two-thirds of the wives of moving families would be tied-movers, while 23 percent out of 70 percent of wives in families of stayers declared themselves to be tied-stayers (page 758).”¹⁴

As seen in figure 6, if the wage of the employed spouse is higher than $\hat{w}_S$, then the unemployed spouse rejects outside offers and stays in the current location for all wage offers less than $\hat{w}_S$. In contrast, a single agent would accept all offers w_2 above w^* , which is less than $\hat{w}_S$ by Proposition 8. Therefore, an unemployed spouse who rejects an outside wage offer $w_2 \in (w^*, \hat{w}_S)$ is formally a tied-stayer. Similarly, to see who a tied-mover is, suppose that the wage of the employed spouse, w_1 , is between w^{**} and w^* , and that the unemployed spouse receives a wage offer w_2 between w_1 and w^* . Then it is optimal for the couple to have the unemployed spouse accept the job offer and the employed spouse quit the job and both move to the other location. The unemployed spouse in this case is a tied-mover, because she would reject the offer and stay in her current location if she were single. There is a region where the employed is also a tied-mover. Suppose the wage of the employed, w_1 , is smaller than $\hat{w}_S$, and the unemployed receives an outside wage offer higher than w_1 , then the unemployed accepts the offer, the employed spouse quits the job and both move to the other location. Note that the employed spouse would not move to the other location if she were single, so the employed spouse is also as a tied-mover.

Both set of choices involve potentially large concessions by each spouse compared to the situation where he/she were single, but they are optimal from a joint decision perspective. This opens the possibility of welfare costs from being in a couple vs being single with respect to job search, an aspect of the model which we analyze quantitatively, through simulation, in the next section.

5.2 Some illustrative simulations with multiple locations

Although the two location case serves as a convenient benchmark that illustrates all the key mechanisms, it is not a natural environment for a calibrated exercise, especially when the offer arrival rates from the two locations are not the same ($\alpha_1 \neq \alpha_2$). But this is the more reasonable assumption since the outside location is more appropriately interpreted as the “rest of the world” and in

¹⁴More precisely, Mincer (1978) defines an individual to be a tied-stayer (a tied-mover) if the individual cites his/her spouses' job as the main reason for turning down (accepting) a job from a different location: Mincer wrote (page 758): “The unemployed were asked whether they would accept a job in another area comparable with the one they lost. A positive answer was given by 30 percent of the married men, 21 percent of the single women, and only 8 percent of the married women. Most people who said no cited family, home, and relatives as reasons for the reluctance to move. However, one quarter of the women singled out their husbands' job in the present area as the major deterrent factor”.

Table 3: Single- versus Joint-Search: 9 Locations and Risk Neutral Preferences

	$\kappa = 0$		$\kappa = 0.1$	$\kappa = 0.5$
	Single	Joint	Joint	Joint
res. wage w^*/w^{**}	1.02	1.02	0.998	0.997
$\hat{w}_T$	—	1.02	0.996	0.989
double ind. $\hat{w}$	—	1.02	1.014	1.013
$\hat{w}_S$	—	1.02	1.067	1.282
res. wage $\phi_i(1)$	—	n/a	1.023	1.021
mean wage	1.06	1.06	1.054	1.051
Mm ratio	1.04	1.04	1.06	1.05
unemp. rate	5.5%	5.5%	6.3%	6.6%
unemp. duration	9.9	9.9	8.7	8.7
dual-searcher	—	6.5	4.6	4.4
worker-searcher	—	9.3	11.5	12
mover	0.28%	0.55%	0.43%	0.42%
tied_mover	—	0%	43%	52%
tied_stayer	—	0%	0.12%	0.14%
job quit rate	—	0%	31.5%	37%
EQVAR-cons	—	0%	−0.8%	−1.2%

many cases could offer more job opportunities than any one home location. This “asymmetry” between the two locations cannot be captured satisfactorily with two locations, for example by setting $\alpha_2 \gg \alpha_1$, because this would imply that if one of the spouses moves to the “rest of the world,” the other spouse will have a very high probability of moving to the same location in the “rest of the world,” where the couple will reunite.

To this end, we now extend the same framework to L locations and allow on-the-job search (as well as exogenous separation later below). Consider an economy with L geographically separate symmetric labor markets. Firms in each location generate offers at flow rate ψ for employed agents and at rate ψ_u for unemployed agents. A fraction θ of both types of offers are distributed equally to the $L - 1$ outside locations and the remaining $(1 - \theta)$ is made to the local market¹⁵. The value function corresponding to this economy are described in the Appendix are a straightforward extensions of value functions in (22) – (25).

Table 3 presents the simulation results for the model with multiple locations and exogenous separation and without on-the-job search. Regarding the calibration, the number of locations, L ,

¹⁵ Assume that there are a very large number of workers in each market. Combined with the fact that the environment is stationary (i.e., no location specific shocks) this implies that we can take the number of workers in each location as constant, despite the fact that workers are free to move across locations and across employment states depending on the offers they receive.

is set to 9 representing U.S. census divisions and θ is set to $\frac{L-1}{L}$. The first two columns show that the single-search problem and joint-search problem are equivalent¹⁶. As we introduce the cost of living separately, κ , the two economies yield different results. Since κ doesn't have any effect on the single-search problem, we presented them just for the case with $\kappa = 0$. The third column shows the result of the joint-search problem when $\kappa = 0.1$, which is 10% of the mean wage. The reservation wages are in line with our theoretical results in Proposition 8: $\hat{w}_T < w^{**} < w^* < \hat{w}_S$. For the married agent the mean wage is smaller and the unemployment rate is higher than the single agent. The presence of the cost κ makes the outside offers less appealing for the couple. They reject some offers where the single would accept. So, we expect to have smaller mean wage and higher unemployment rate in the joint-search problem. The unemployment duration in the joint-search economy is smaller than the single-search economy. This may look in odds with the unemployment rates. However, decomposing the unemployment duration into the time spent as dual-searcher or worker-searcher makes it more clear. The unemployment duration as a dual-searcher is shorter while the unemployment duration as a worker-searcher is longer. Note that $w^{**} < w^*$ implies the unemployment duration for the dual-searcher to be shorter than the single agent's unemployment duration. However, the worker-searcher couple face the friction due to the outside offers. So, they have a longer unemployment duration.

The definition of the mover in our context is the household who moves geographically. So, for a dual-worker couple who live separately, if one of the spouse receives unemployment shock and becomes unemployed, she moves to her spouse's location and we even consider her as a mover. According to this definition of movers, the rate of the movers decreases from 0.55% to 0.42% as the cost of κ increases from 0.1 to 0.5. Using the definition of the tied-movers mentioned above, almost half of the movers is considered as tied-movers¹⁷. The voluntary quit rate, which is defined as the ratio of voluntary quitters to the involuntary quitters is around 1/3. Lastly, the joint-search problem creates around 1% loss compared to the single-search problem.

6 Conclusions

Our work extends naturally in two directions. First, from a theoretical viewpoint, one should explore other channels leading to joint-search decisions in the labor market. For example, complementarity/substitutability in leisure (Burdett and Mortensen, 1978), or more realistic consumption-sharing rules that deviate from full income pooling as in the collective model (e.g. Chiappori, 1992). One can also generalize the symmetry assumption we made on individuals and locations. One limit of the present framework, especially in the multiple location case where the couple may be worse-

¹⁶The moving rate is different in two economies. It's just because of the definition of the movers in this economy. If one of the spouses of a dual-worker couple who are employed in separate locations receives unemployment shock, she moves to her spouse's location to avoid the cost of living separately. This is not true in a single-search economy.

¹⁷The reported number for tied-movers is the ratio of tied-movers to total movers.

off than the single agent, is that we ignore the option to split up (see Aiyagari, Greenwood and Guner, 2000, for a quantitative model of marriage and divorce with frictional marriage market). A search-based analysis of labor and marriage market dynamics is an ambitious but necessary step forward.

Second, now that the key qualitative features of the joint-search problem have been uncovered, quantitative work can be performed more confidently and effectively. For example, the model in Section 5 generalized to asymmetric skills, locations and, perhaps, also to allow for borrowing/saving can be brought to the data and estimated structurally. the challenge is to access micro data with household level information on the detailed labor market history of both members of the couple and on their geographical movements (see Gemici, 2007, for a recent attempt of this sort on PSID data).

A Proofs

Lemma 1. Rewrite equation (6) using equation (4):

$$r\Omega(w) = u(w + b) + \alpha g(w) \quad (27)$$

where

$$g(w) \equiv \int \max \left\{ \frac{u(w + w_2)}{r}, \Omega(w_2) - \Omega(w), 0 \right\} dF(w_2).$$

We construct the proof by contradiction. Let us assume $\Omega'(w) \leq 0$. From Lemma (27), $r\Omega'(w) - u'(w + b) = \alpha g'(w)$. Then, $g'(w) < \frac{-u'(w+b)}{\alpha} < 0$. If Ω is a decreasing function, then $\frac{u(w+w_2)}{r}$ and $\Omega(w_2) - \Omega(w)$ are increasing functions of w . This means that all the terms inside the max operator of the g function are increasing, which implies that g is an increasing function, i.e., $g'(w) \geq 0$, for each w , which is a contradiction. Thus $\Omega'(w) > 0$. ■

Proposition 1. From the definition of the worker-searcher reservation wage when the quit option is not exercised, the couple has to be indifferent between both partners being employed and only one being employed. This means that ϕ has to satisfy: $\Omega(w_1) = T(w_1, \phi(w_1))$. We conjecture that the quitting option is never exercised. This allows us to disregard the second term of the max operator in (6). Using this last equality, equations (6) and (4) and the fact that workers are risk-neutral, the equation characterizing $\phi(w_1)$ becomes

$$\phi(w_1) = b + \frac{\alpha}{r} \int_{\phi(w_1)} [w_2 - \phi(w_1)] dF(w_2).$$

It is clear that $\phi(w_1)$ does not depend on w_1 , and the above equation is exactly equation (3) of the single-search problem. So, $\phi(w_1) = w^* = \hat{w}$. As a result, $\phi^{-1}(w_1) = \infty$, confirming the guess that the employed spouse never quits, since quits occur only if the wage offer w_2 exceeds $\phi^{-1}(w_1)$.

Now we will establish that $w^{**} = w^*$. Equation (7) implies that

$$r\Omega(w^{**}) = rU = 2b + \frac{2\alpha}{r} \int_{w^{**}} r\Omega'(w) [1 - F(w)] dw. \quad (28)$$

At $w_1 = w^*$, we can rewrite equation (6) in the following way

$$r\Omega(w^*) = w^* + b + \frac{\alpha}{r} \int_{w^*} r\Omega'(w) [1 - F(w)] dw. \quad (29)$$

Subtracting (28) from (29) multiplied by 2 and using the fact that $r\Omega(w^*) = 2w^*$ yields

$$r[\Omega(w^*) - \Omega(w^{**})] = \frac{2\alpha}{r} \int_{w^*}^{w^{**}} r\Omega'(w) [1 - F(w)] dw$$

Since Ω is strictly increasing, $w^* \geq w^{**}$ implies $\Omega(w^*) \geq \Omega(w^{**})$, but then the above equation in

turn implies that $w^{**} = w^*$. Thus, the quit option will never be exercised. ■

Proposition 2. It is useful by proving first part (ii). At the reservation wage for the worker-searcher couple we have $T(w_1, \phi(w_1)) = \Omega(w_1)$. Let us begin by guessing that there is a value w_1 above which the employed worker never quits his job. Therefore in this range we don't have to worry about the second argument of the max operator in (6). Using equations (6) and (4), we get

$$\begin{aligned}
u(w_1 + \phi(w_1)) - u(w_1 + b) &= \frac{\alpha}{r} \int_{\phi(w_1)} [T(w_1, w_2) - \Omega(w_1)] dF(w_2) \\
&= \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] dF(w_2) \\
-\rho u(w_1)(u(\phi(w_1)) - u(b)) &= -\rho u(w_1) \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_2) - u(\phi(w_1))] dF(w_2) \\
u(\phi(w_1)) - u(b) &= \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_2) - u(\phi(w_1))] dF(w_2)
\end{aligned}$$

where the second line uses the definition of ϕ and the third line uses the CARA assumption $u(c_1 + c_2) = -\rho u(c_1)u(c_2)$. Note that this is exactly the same equation characterizing the reservation wage of the single unemployed (equation 3). So, we can conclude that in this region $\phi(w_1) = w^*$. Moreover, $\hat{w}$ is a singleton since ϕ crosses the 45 degree line only once, so $\hat{w} = w^*$. If $w_1 \geq w^*$, the employed spouse does not quit the job, since $\phi^{-1}(w_1) = \infty$ and quits take place if $w_2 > \phi^{-1}(w_1)$ which confirms the initial guess.

From what argued above, this portion of the ϕ function holds for $w_1 \geq w^*$. Below w^* we have $\phi(w_1) = w_1$ and quits are possible as long as $w^{**} < w^*$ which is what stated in part (i) and what we prove thereafter. When the wage of the employed agent is the double indifference point $\hat{w}$, we have $r\Omega(\hat{w}) = u(2\hat{w})$ from (10). Subtracting (5) from this equation, we get

$$r[\Omega(\hat{w}) - \Omega(w^{**})] = u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}} [\Omega(w) - \Omega(w^{**})] dF(w)$$

Evaluate equation (6) at $\hat{w}$, and note that $T(\hat{w}, w) = \Omega(w)$ to arrive at

$$u(2\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] dF(w).$$

Combining these two equations results in:

$$\begin{aligned}
r[\Omega(\hat{w}) - \Omega(w^{**})] &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w) [1 - F(w)] dw \\
&= \rho[-2u(\hat{w})u(b) + u(\hat{w})u(\hat{w}) + u(b)u(b)] - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w) [1 - F(w)] dw \\
&= \rho[u(\hat{w}) - u(b)]^2 - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w) [1 - F(w)] dw,
\end{aligned}$$

where the second line again uses the CARA assumption. Suppose now, ad absurdum, that $w^{**} \geq \hat{w}$, then clearly, $LHS \leq 0$. But since obviously $\hat{w} > b$, and $2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w)(1 - F(w))dw \leq 0$, we have that $RHS > 0$, a contradiction. Thus, $w^{**} < \hat{w} = w^*$. ■

Proposition 3. We begin by part (ii). It is instructive, even though redundant, to first conjecture that there is a value w_1 above which the employed spouse never quits his job. This is equivalent to say that in this region $\phi' \leq 0$ since ϕ^{-1} would also be decreasing. Indeed, suppose that the couple draws a wage $w_2 > \phi(w_1)$. The reservation wage of the employed spouse upon quitting would be $\phi^{-1}(w_2) < w_1$, where w_1 is the current wage, which would not justify quitting. Then, the equation characterizing $\phi(w_1)$ becomes, as usual,

$$u(w_1 + \phi(w_1)) - u(w_1 + b) = \frac{\alpha}{r} \int_{\phi(w_1)}^{\infty} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] dF(w_2).$$

Consider a wage level $\tilde{w}_1 > w_1$. Then, rearranging, we get

$$\begin{aligned} 1 &= \frac{\alpha}{r} \int_{\phi(w_1)}^{\infty} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\ &\leq \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)}^{\infty} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(\tilde{w}_1))}{u(w_1 + \phi(\tilde{w}_1)) - u(w_1 + b)} \right] dF(w_2) \\ &< \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)}^{\infty} \left[\frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)} \right] dF(w_2) \\ &= 1, \end{aligned}$$

which is a contradiction. The first weak inequality comes from the fact that $\phi' \leq 0$. The second strict inequality holds because of the DARA utility assumption (Pratt, 1964, Theorem 1): if u is in the DARA class, for any $k > 0$ and m, n, p, q such that $p < q \leq m < n$, we have

$$\frac{u(n) - u(m)}{u(q) - u(p)} < \frac{u(n+k) - u(m+k)}{u(q+k) - u(p+k)}. \quad (30)$$

Here $p = w_1 + b$, $q = m = w_1 + \phi(\tilde{w}_1)$, $n = w_1 + w_2$ and $k = \tilde{w}_1 - w_1$. Thus our first conjecture is not correct.

We conclude that $\phi(w_1)$ must be strictly increasing in w_1 over this range. In this case, the employed spouse may find it optimal to quit the job if the unemployed receives a sufficiently high wage offer, i.e. whenever $w_2 > \phi^{-1}(w_1)$. This leads us to another conjecture: for any $w_1 < \hat{w}$, $\phi(w_1) = w_1$ and for $w_1 \geq \hat{w}$, $0 < \phi' < 1$. Then, the equation characterizing $\phi(w_1)$ becomes

$$\begin{aligned} 1 &= \frac{\alpha}{r} \int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\ &\quad + \frac{\alpha}{r} \int_{\phi^{-1}(w_1)}^{\infty} \left[\frac{r\Omega(w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \end{aligned}$$

Let $g(w_1, \phi, w_2) = \frac{u(w_1+w_2)-u(w_1+\phi)}{u(w_1+\phi)-u(w_1+b)}$ and $h(w_1, \phi, w_2) = \frac{r\Omega(w_2)-u(w_1+\phi)}{u(w_1+\phi)-u(w_1+b)}$ where to simplify the notation we have dropped the dependence of ϕ on its argument w_1 . Differentiating the above equation with respect to w_1 , we get

$$\begin{aligned} 0 &= \frac{\alpha}{r} \left[\int_{\phi}^{\phi^{-1}} \frac{\partial g(w_1, \phi, w_2)}{\partial w_1} dF(w_2) + \int_{\phi^{-1}} \frac{\partial h(w_1, \phi, w_2)}{\partial w_1} dF(w_2) \right] \\ &+ \frac{\alpha}{r} \left[\int_{\phi}^{\phi^{-1}} \frac{\partial g(w_1, \phi, w_2)}{\partial \phi} dF(w_2) + \int_{\phi^{-1}} \frac{\partial h(w_1, \phi, w_2)}{\partial \phi} dF(w_2) - g(w_1, \phi, \phi) \right] \frac{\partial \phi(w_1)}{\partial w_1} \\ &+ \frac{\alpha}{r} [g(w_1, \phi, \phi^{-1}) - h(w_1, \phi, \phi^{-1})] \frac{\partial \phi^{-1}(w_1)}{\partial w_1}. \end{aligned} \quad (31)$$

Note that $\frac{\partial g(w_1, \phi, w_2)}{\partial \phi} < 0$ and $\frac{\partial h(w_1, \phi, w_2)}{\partial \phi} < 0$, so the term in front of $\frac{\partial \phi(w_1)}{\partial w_1}$ in the second line is negative. By definition of $\phi^{-1}(w_1)$, $rT(w_1, \phi^{-1}(w_1)) = r\Omega(\phi^{-1}(w_1)) = u(w_1 + \phi^{-1}(w_1))$. So, $g(w_1, \phi, \phi^{-1}) = h(w_1, \phi, \phi^{-1})$ and the third line above vanishes.

In the first line we have

$$\int_{\phi}^{\phi^{-1}} \frac{\partial g(w_1, \phi, w_2)}{\partial w_1} dF(w_2) + \int_{\phi^{-1}} \frac{\partial h(w_1, \phi, w_2)}{\partial w_1} dF(w_2) \quad (32)$$

As we have done earlier, the first term is positive because of DARA property in (30). We now show that the second term is also positive.

For $w_2 > \phi^{-1}(w_1)$, the region where h is relevant, the employed spouse quits his job when the unemployed spouse accepts her offer, thus $r\Omega(w_2) = u(w_1, \phi^{-1}(w_1)) = u(\phi(w_2), w_2) > rT(w_1, w_2) = u(w_1, w_2)$. Therefore, $\phi(w_2) > w_1$. We also know that $w_2 \geq \phi(w_1) > b$. As a result, $\phi(w_2) + w_2 > w_1 + \phi(w_1) > w_1 + b$. Exploiting this ranking into (30) we have

$$\frac{u(\phi(w_2) + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} < \frac{u(\phi(w_2) + w_2 + k) - u(w_1 + \phi(w_1) + k)}{u(w_1 + \phi(w_1) + k) - u(w_1 + b + k)}$$

for $\kappa > 0$. By continuity of u , we have that, for k small enough,

$$\frac{u(\phi(w_2) + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} < \frac{u(\phi(w_2) + w_2) - u(w_1 + \phi(w_1) + k)}{u(w_1 + \phi(w_1) + k) - u(w_1 + b + k)}$$

and using the fact that $r\Omega(w_2) = u(w_2 + \phi(w_2))$ we obtain

$$h(w_1, \phi, w_2) = \frac{r\Omega(w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} < \frac{r\Omega(w_2) - u(\tilde{w}_1 + \phi(w_1))}{u(\tilde{w}_1 + \phi(w_1)) - u(\tilde{w}_1 + b)} = h(\tilde{w}_1, \phi, w_2)$$

which implies that $\frac{\partial h(w_1, \phi, w_2)}{\partial w_1} > 0$ since $\tilde{w}_1 > w_1$.

Thus, the second term in (32) is positive which implies that the whole first line in (31) is positive. Since the last line is zero and the term in front of $\frac{\partial \phi(w_1)}{\partial w_1}$ is negative, then $\frac{\partial \phi(w_1)}{\partial w_1}$ has to be positive

because the whole expression in (31) must equal zero.

We now address part (i) of the proposition. We first prove that $w^{**} < \hat{w}$. Subtracting equation (5) from equation (10) we obtain

$$r [\Omega(\hat{w}) - \Omega(w^{**})] = u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}} [\Omega(w) - \Omega(w^{**})] dF(w). \quad (33)$$

At $w_1 = \hat{w}$, we can write equation (6) as

$$r\Omega(\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] dF(w)$$

because for any wage offer $w_2 > \hat{w}$, the unemployed accepts the offer and the employed quits the job, meaning $\Omega(w_2) > T(\hat{w}, w_2)$. Multiplying the above equation by 2 and using equation (10), we arrive at

$$u(2\hat{w}) = 2u(\hat{w} + b) - u(2\hat{w}) + 2\alpha \int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] dF(w_2).$$

Substituting this expression for $u(2\hat{w})$ into the RHS of the equation (33) delivers

$$\begin{aligned} r [\Omega(\hat{w}) - \Omega(w^{**})] &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) + 2\alpha \left[\int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] dF(w) - \int_{w^{**}} [\Omega(w) - \Omega(w^{**})] dF(w) \right] \\ &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) + 2\alpha \int_{\hat{w}}^{w^{**}} \Omega'(w) [1 - F(w)] dw. \end{aligned}$$

where the second line uses integration by parts. Now, by concavity of u , $2u(\hat{w} + b) - u(2\hat{w}) - u(2b) > 0$. Suppose, ad absurdum, $w^{**} \geq \hat{w}$. Then, the RHS of the above equation is strictly positive, but the LHS is either negative or zero, which is a contradiction. Therefore, $w^{**} < \hat{w}$.

We now prove, by contradiction, that $\hat{w} > w^*$. Assume $w^* \geq \hat{w}$. Recall that equation (6) evaluated at $\hat{w}$ can be written as

$$r\Omega(\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] dF(w).$$

Since $\Omega(\hat{w}) = u(2\hat{w})$, we can rewrite the above relationship as

$$\begin{aligned} u(2\hat{w}) - u(\hat{w} + b) &= \frac{\alpha}{r} \int_{\hat{w}} [r\Omega(w) - u(2\hat{w})] dF(w) \\ &> \frac{\alpha}{r} \int_{\hat{w}} [rT(\hat{w}, w) - u(2\hat{w})] dF(w) \\ &= \int_{\hat{w}} [u(\hat{w} + w) - u(\hat{w} + \hat{w})] dF(w) \end{aligned}$$

Rearrange the above equation and, once again, use the property of DARA utility to get

$$\begin{aligned}
1 &> \frac{\alpha}{r} \int_{\hat{w}} \left[\frac{u(\hat{w} + w) - u(\hat{w} + \hat{w})}{u(\hat{w} + \hat{w}) - u(\hat{w} + b)} \right] dF(w) \\
&> \frac{\alpha}{r} \int_{\hat{w}} \left[\frac{u(w) - u(\hat{w})}{u(\hat{w}) - u(b)} \right] dF(w) \\
&\geq \frac{\alpha}{r} \int_{w^*} \left[\frac{u(w) - u(w^*)}{u(w^*) - u(b)} \right] dF(w) \\
&= 1
\end{aligned}$$

The second inequality is due to the property of DARA utility, the third weak inequality derives from the assumption $w^* \geq \hat{w}$ and from u being an increasing function. The last equality comes from the definition of reservation wage for the single agent. Since we reached a contradiction, it must be that $\hat{w} > w^*$.

Lastly, we need to prove that $\phi' < 1$. Let's assume $\phi' > 1$. This means that for $w_1 > \hat{w}$, $\phi(w_1) > \phi^{-1}(w_1) = \varphi(w_1)$. For any $w_1 > \hat{w}$, if the wage offer $w_2 > \phi(w_1)$, the unemployed accepts the offer, meaning $T(w_1, w_2) > \Omega(w_1)$, but since $w_2 > \phi(w_1) > \phi^{-1}(w_1)$, the employed quits the job at the same time, which means $\Omega(w_2) > T(w_1, w_2) > \Omega(w_1)$. With the same logic, one can see that if $w_2 \in (w_1, \phi(w_1))$, we get $\Omega(w_2) > \Omega(w_1) > T(w_1, w_2)$. If $w_2 \in (\varphi(w_1), w_1)$, we have $\Omega(w_1) > \Omega(w_2) > T(w_1, w_2)$ and if $w_2 < \varphi(w_1)$, we have $\Omega(w_1) > T(w_1, w_2) > \Omega(w_2)$. Hence, if $w_2 > w_1$, then the unemployed accepts the job and the employed quits the job, forcing the reservation wage to be w_1 . Hence $\phi(w_1) = w_1$, resulting in $\phi' = 1$, a contradiction. ■

Proposition 5. Let's conjecture that $\phi(w_1) = w^{**}$ for any value of w_1 , i.e. $T(w^{**}, w_2) = \Omega(w_2)$. This implies that the quit option is never exercised since any observed w_1 will be greater than or equal to w^{**} . So, one can disregard the second argument in the max operator in (13). Evaluating (13) at w^{**} yields

$$r\Omega(w^{**}) = u(w^{**} + b) + 2\alpha_u \int \max\{\Omega(w) - \Omega(w^{**}), 0\} dF(w) dF(w_2)$$

where we have used the fact that $\alpha_e = \alpha_u$ and the conjecture. Since $\Omega(w^{**}) = U$, comparing the above equation to (12) yields that $w^{**} = b$. We now verify our conjecture. From (14) evaluated at

$$w_2 = w^{**}$$

$$\begin{aligned}
rT(w_1, w^{**}) &= u(w_1 + b) + \alpha_e \int \max\{T(w'_1, w^{**}) - T(w_1, w^{**}), 0\} dF(w'_1) \\
&\quad + \alpha_u \int \max\{T(w_1, w'_2) - T(w_1, w^{**}), 0\} dF(w'_2) \\
&= u(w_1 + b) + \alpha_e \int \max\{\Omega(w'_1) - \Omega(w_1), 0\} dF(w'_1) \\
&\quad + \alpha_u \int \max\{T(w_1, w'_2) - \Omega(w_1), 0\} dF(w'_2) \\
&= \Omega(w_1)
\end{aligned}$$

which confirms our conjecture, since $T(w^{**}, w_2) = \Omega(w_2)$ implies that $\phi(w_2) = w^{**}$. Finally, from equation (14), it is immediate that $\eta(w_i) = w_i$ which completes the proof. ■

Proposition 6. We begin by part (ii). The value functions (4) and (6) modified to allow for exogenous separations are:

$$\begin{aligned}
(r + 2\delta)T(w_1, w_2) &= u(w_1 + w_2) - \delta[\Omega(w_1) + \Omega(w_2)] \\
r\Omega(w_1) &= u(w_1 + b) + \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) - \delta[\Omega(w_1) - U]
\end{aligned} \tag{34}$$

From the definition of reservation function ϕ for the worker-searcher couple, $T(w_1 + \phi(w_1)) = \Omega(w_1)$, we have:

$$u(w_1 + \phi(w_1)) - \delta[\Omega(w_1) - \Omega(\phi(w_1))] = r\Omega(w_1).$$

Let us assume that there is a wage value w_1 beyond which the employed worker never quits. Then, in this range $\phi(w_1)$ is a decreasing function. Using this property into (35) and substituting into the above equation, we get:

$$\begin{aligned}
u(w_1 + \phi(w_1)) &= u(w_1 + b) + \alpha \int_{\phi(w_1)} [T(w_1, w_2) - T(w_1, \phi(w_1))] dF(w_2) - \delta[\Omega(\phi(w_1)) - U] \\
&= u(w_1 + b) + \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] dF(w_2) + h(\phi(w_1))
\end{aligned} \tag{35}$$

where

$$h(x) = \frac{\alpha\delta}{r + 2\delta} \int_x [\Omega(w_2) - \Omega(x)] dF(w_2) - \delta[\Omega(x) - U]$$

with h decreasing in x . Rearrange equation (36) as:

$$1 = \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}. \tag{37}$$

Since $\phi(w_1)$ is a decreasing function of w_1 then, for any $\tilde{w}_1 > w_1$, we have:

$$0 \leq \frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \leq \frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(w_1))}{u(\tilde{w}_1 + \phi(w_1)) - u(\tilde{w}_1 + b)} \leq \frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)}$$

where the first weak inequality stems from the fact that u is CARA or DARA, and the second from the fact that ϕ is decreasing. Overall, the above condition implies the first term in equation (37) is an increasing function of w_1 .

Since h is decreasing in x , and $\phi(\tilde{w}_1) \leq \phi(w_1)$ for $\tilde{w}_1 > w_1$, we have

$$\frac{h(\phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} < \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)}.$$

And we reach the following contradiction:

$$\begin{aligned} 1 &= \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \\ &< \frac{\alpha}{r + 2\delta} \int_{\phi(w'_1)} \left[\frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)} \right] dF(w_2) + \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)} \\ &= 1 \end{aligned}$$

We conclude that $\phi(w_1)$ is strictly increasing in w_1 . Once we have established this result, similar arguments used in the proof of Proposition 3 apply here for part (i) to conclude the proof. ■

Proposition 7. We guess that $rT(w_1, w_2, a) = u(ra + w_1 + w_2)$. Then RHS of equation (19) becomes

$$\max_c \{u(c) + u'(ra + w_1 + w_2)(ra + w_1 + w_2 - c)\}.$$

The FOC implies $u'(c) = u'(ra + w_1 + w_2)$, so $c_e(a, w_1, w_2) = ra + w_1 + w_2$. If we plug this optimal consumption function back into equation (19), we arrive at $rT(w_1, w_2, a) = (ra + w_1 + w_2)$, which confirms the guess.

Similarly, let us guess that $r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1))$. Again, plugging this guess into RHS of equation (21) the FOC implies $c_\Omega(w_1, a) = ra + w_1 + \phi(w_1, a)$. Substituting this function back into (21) gives

$$\begin{aligned} r\Omega(w_1, a) &= u(ra + w_1 + \phi(w_1, a)) + u'(ra + w_1 + \phi(w_1, a))(b - \phi(w_1, a)) \\ &\quad + \frac{\alpha}{r} \int \max \{u(ra + w_1 + w_2) - u(ra + w_1 + \phi(w_1, a)), u(ra + w_2 + \phi(w_1, a)) - u(ra + w_1 + \phi(w_1, a))\} dF(w_2) \end{aligned}$$

Using the CARA property of u , we can simplify the RHS and rewrite the above equation as:

$$r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1, a)) \left[1 - \rho(b - \phi(w_1, a)) - \frac{\alpha}{r} \int \max \{u(w_2 - \phi(w_1, a)) - 1, u(w_2 - w_1) - 1, 0\} dF(w_2) \right]$$

Now, using the definition of ϕ , and the expression for $rT(w_1, \phi(w_1, a), a)$ in the above equation, we have:

$$\phi(w_1, a) = b + \frac{\alpha}{\rho r} \int [u(\max\{w_2 - \phi(w_1, a), w_2 - w_1, 0\}) - 1] dF(w_2).$$

Suppose that there is a value w_1 such that beyond that value the quitting option is never exercised. Then, in this range we can abstract from the second argument in the max operator and rewrite

$$\phi(w_1, a) = b + \frac{\alpha}{\rho r} \int_{\phi(w_1, a)} [u(w_2 - \phi(w_1, a)) - 1] dF(w_2) \quad (38)$$

which implies that ϕ is a constant function, independent of (w_1, a) . Moreover, comparing (38) to the equivalent equation for the single agent problem (18) yields that $\phi(w_1, a) = w^*$.

Finally, let's turn to U and conjecture that $rU(a) = u(ra + 2w^{**})$. Substituting this guess into equation (20) and taking the FOC leads to the optimal policy function $c_u(a) = ra + 2w^{**}$ which confirms the guess. Then, using the CARA assumption, equation (20) becomes

$$\begin{aligned} rU(a) &= u(ra + 2w^{**}) - \rho u(ra + 2w^{**})(2b - 2w^{**}) - \frac{2\alpha}{r} u(ra + 2w^{**}) \int_{w^{**}} [u(w - w^{**}) - 1] dF(w) \\ &= u(ra + 2w^{**}) \left[1 - \rho(2b - 2w^{**}) - \frac{2a}{r} \int_{w^{**}} [u(w - w^{**}) - 1] dF(w) \right]. \end{aligned}$$

and using $rU(a) = u(ra + 2w^{**})$ we arrive at:

$$w^{**} = b + \frac{a}{\rho r} \int_{w^{**}} [u(w - w^{**}) - 1] dF(w)$$

which, once again, compared to (18) implies that $w^{**} = w^*$. This concludes the proof. ■

Proposition 8. The reservation function for outside offer satisfies $S(w_1, \phi_o(w_1)) = \Omega(w_1)$. Let us first guess that there is a value w_1 above which the employed partner never exercises the quit option. In this range, from the definition of $\phi_o(w_1)$:

$$\begin{aligned} \phi_o(w_1) &= b + \kappa + \alpha_i \int_{\phi_i(w_1)} [T(w_1, w_2) - \Omega(w_1)] dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) \\ &= b + \kappa + \alpha_i \int_{\phi_i(w_1)} [S(w_1, w_2) + \kappa - \Omega(w_1)] dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) \\ &= b + \kappa + \alpha_i \int_{\phi_o(w_1) - \kappa} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) \end{aligned}$$

where the second line uses the risk neutrality assumption and a simple change of variable. Integrating by parts, we arrive at

$$\phi_o(w_1) = b + \kappa + \frac{\alpha_i}{r} \int_{\phi_o(w_1) - \kappa} [1 - F(w_2)] dw_2 + \frac{\alpha_o}{r} \int_{\phi_o(w_1)} [1 - F(w_2)] dw_2$$

which shows that $\phi_o(w_1)$ is independent of w_1 and equals the expression for $\hat{w}_S$ given in the statement of Proposition 8. This confirms the conjecture of no quitting when $w_1 > \hat{w}_S$ for outside offers. Thus, for $w \geq \hat{w}_S$, $\phi_o(w_1) = \hat{w}_S$ and for $w < \hat{w}_S$ as usual $\phi_o(w_1) = w_1$ and employed workers may quit.

We now turn to inside offers. The reservation function for inside offer satisfies $T(w_1, \phi_i(w_1)) = \Omega(w_1)$. We keep analyzing the region of w_1 above $\hat{w}_S$ where we know the employed worker does not quit upon receiving outside offers. From the definition of $\phi_i(w_1)$:

$$\begin{aligned}\phi_i(w_1) &= b + \alpha_i \int_{\phi_i(w_1)} [T(w_1, w_2) - \Omega(w_1)] dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) \\ &= b + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F(w_2)] dw_2 + \frac{\alpha_o}{r} \int_{\phi_i(w_1) + \kappa} [1 - F(w_2)] dw_2\end{aligned}$$

where the second line is derived exactly as for the outside offer case. Once again, $\phi_i(w_1)$ is independent of w_1 , which confirms the conjecture, and equals $\hat{w}_T = \hat{w}_S - \kappa$.

Let's extend our analysis of inside offers to the region where w_1 is lower than $\hat{w}_S$. Here, the reservation function ϕ_i satisfies

$$\phi_i(w_1) = b + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F(w)] dw + \frac{\alpha_o}{r} \int_{w_1}^{\hat{w}_S} \Omega'(w_2) [1 - F(w_2)] dw_2$$

since the employed worker will quit upon receiving outside offers. Clearly, $\phi_i(w_1)$ is decreasing in w_1 over this region. We conclude that for $w_1 \geq \hat{w}_S$, we have $\phi_i(w_1) = \hat{w}_T$ and in the range $[\hat{w}, \hat{w}_S]$ the function ϕ_i is decreasing, with $\hat{w}$ denoting the double indifference point, i.e. the intersection with the 45 degree line. As usual, below $\hat{w}$, $\phi_i(w_1) = w_1$.

We now want to establish the relationship between w^{**} and w^* . It is useful to recall that $\hat{w}_T < \hat{w} < \hat{w}_S$. We begin from characterizing w^{**} . The equation (25) evaluated at the point $w_1 = \hat{w}_T$ becomes

$$r\Omega(\hat{w}_T) = \hat{w}_T + b + \alpha_i \int_{\hat{w}_T} \Omega'(w) [1 - F(w)] dw. \quad (39)$$

The reservation wage of the dual-searcher couple w^{**} is characterized by the equation

$$r\Omega(w^{**}) = 2b + 2(\alpha_i + \alpha_o) \int_{w^{**}} \Omega'(w) (1 - F(w)) dw. \quad (40)$$

Now subtract equation (39) multiplied by 2 from equation (40), and get

$$r[\Omega(w^{**}) - \Omega(\hat{w}_T)] = r\Omega(\hat{w}_T) - 2\hat{w}_T + 2(\alpha_i + \alpha_o) \int_{w^{**}}^{\hat{w}_T} \Omega'(w) [1 - F(w)] dw.$$

Suppose $w^{**} \leq \hat{w}_T$, then LHS of the above equation is negative or zero. The second term of the RHS is positive. The term $r\Omega(\hat{w}_T) - 2\hat{w}_T$ is also positive because for $w_1 = \hat{w}_T$ the employed

worker would prefer to quit his job than remaining employed (more precisely, he strictly prefers it for an outside offer, he's indifferent for an inside offer). Therefore the RHS is positive which is a contradiction. So $w^{**} > \hat{w}_T$.

Similarly, consider equation (25) evaluated at $w_1 = \hat{w}$. Note that at $w_1 = \hat{w}$, for inside offers the employed spouse never exercises the quit option, while for outside offers, she does so. So, equation (25) evaluated at $w_1 = \hat{w}$ becomes

$$r\Omega(\hat{w}) = \hat{w} + b + \frac{\alpha_i}{r} \int_{\hat{w}} [1 - F(w)] dw + \frac{\alpha_o}{r} \int_{\hat{w}} r\Omega'(w) [1 - F(w)] dw.$$

Also note that since $\hat{w}$ is the double indifference point for inside offers, $r\Omega(\hat{w}) = 2\hat{w}$. Again, subtract this last equation multiplied by 2 from equation (40) to get

$$r[\Omega(w^{**}) - \Omega(\hat{w})] = \frac{2\alpha_i}{r} \left[\int_{w^{**}} r\Omega'(w) [1 - F(w)] dw - \int_{\hat{w}} [1 - F(w)] dw \right] + 2\frac{\alpha_o}{r} \int_{w^{**}}^{\hat{w}} r\Omega'(w) [1 - F(w)] dw.$$

Now, suppose $w^{**} \geq \hat{w}$. Then the LHS becomes nonnegative. The last term in the RHS is negative. From the definition of $\phi_i(w_1)$, $r\Omega(w_1) = rT(w_1, \phi_i(w_1)) = w_1 + \phi_i(w_1)$. Thus, $\phi_i'(w_1) = r\Omega'(w_1) - 1$. But since we have proved that $\phi_i'(w_1) \leq 0$ above $\hat{w}$, we have that $r\Omega'(w_1) \leq 1$. Therefore, the first term in the RHS must also be negative, which delivers a contradiction and leads to $w^{**} < \hat{w}$.

We now study w^* , the reservation wage of the single-agent search problem. Combining the equation (25) evaluated at $\hat{w}$ with the fact that $r\Omega(\hat{w}) = 2\hat{w}$, we have

$$\hat{w} = b + \frac{\alpha_i}{r} \int_{\hat{w}} [1 - F(w)] dw + \frac{\alpha_o}{r} \int_{\hat{w}} r\Omega'(w) [1 - F(w)] dw$$

Subtracting this equation from equation (26) we get

$$w^* - \hat{w} = \frac{\alpha_i}{r} \int_{w^*}^{\hat{w}} [1 - F(w)] dw + \frac{\alpha_o}{r} \left[\int_{w^*}^{\hat{w}} [1 - F(w)] dw - \int_{\hat{w}} r\Omega'(w) [1 - F(w)] dw \right]$$

Suppose, $w^* \leq \hat{w}$, then the LHS becomes non-positive, but the RHS is strictly positive since $r\Omega'(w) \leq 1$, a contradiction. Thus, $w^* > \hat{w}$.

Now we want to prove that $w^* < \hat{w}_S$. Rewrite the equation for $\hat{w}_S$ as

$$\hat{w}_S = b + \kappa + \frac{\alpha_1}{r} \int_{\hat{w}_S - \kappa} (1 - F(w)) dw + \frac{\alpha_2}{r} \int_{\hat{w}_S} (1 - F(w)) dw$$

Subtracting equation (26) from the equation defining $\hat{w}_S$, we get

$$\hat{w}_S - w^* = \kappa + \frac{\alpha_i}{r} \int_{\hat{w}_S - \kappa}^{w^*} [1 - F(w)] dw + \frac{\alpha_o}{r} \int_{\hat{w}_S}^{w^*} [1 - F(w)] dw$$

Suppose $w^* \geq \hat{w}_S$, then the LHS is non-positive. However, since $\kappa > 0$, RHS is strictly positive. Thus, $w^* < \hat{w}_S$. ■

B Additional value functions

Equations for the economy with multiple locations, exogenous separations, risk-neutral agents and on the job search.

First, consider the problem of a couple that is currently together. The arrival rate of wage offers for each spouse from the current location (in which case they can accept the job and still stay together) is $(1 - \theta)\psi$. The total arrival rate of all outside offers for each spouse is $\theta\psi$ which is obtained by multiplying the number of offers (at rate $\theta\psi/(L - 1)$ from each outside location) by the number of such locations $(L - 1)$. The equation is:

$$\begin{aligned}
rT(w_1, w_2) = & w_1 + w_2 + (1 - \theta)\psi \int \max\{T(w'_1, w_2) - T(w_1, w_2), \Omega(w'_1) - T(w_1, w_2), 0\} dF(w'_1) \\
& + (1 - \theta)\psi \int \max\{T(w_1, w'_2) - T(w_1, w_2), \Omega(w'_2) - T(w_1, w_2), 0\} dF(w'_2) \\
& + \psi\theta \int \max\{S(w'_1, w_2) - T(w_1, w_2), \Omega(w'_1) - T(w_1, w_2), 0\} dF(w'_1) \\
& + \psi\theta \int \max\{S(w_1, w'_2) - T(w_1, w_2), \Omega(w'_2) - T(w_1, w_2), 0\} dF(w'_2) \\
& + \delta[\Omega(w_1) - T(w_1, w_2)] + \delta[\Omega(w_2) - T(w_1, w_2)]
\end{aligned}$$

Notice that in all cases, an offer to one spouse can trigger a quit for the other spouse, which is taken into account in this equation. Turning to a couple whose members currently live in different locations, call A and B , the problem is somewhat different. The couple could reunite if either spouse receives an offer from the location of the other spouse. The arrival rate of job offers at location A from B (and B from A) is $\theta\psi/(L - 1)$. The arrival rate of offers that keep the couple separate is simply the total offer arrival rate minus the rate just calculated, which is $\psi(1 - \theta/(L - 1))$ for each spouse:

$$\begin{aligned}
rS(w_1, w_2) = & w_1 + w_2 - \kappa + \frac{\theta\psi}{(L-1)} \int \max \{T(w'_1, w_2) - S(w_1, w_2), \Omega(w'_1) - S(w_1, w_2), 0\} dF(w'_1) \\
& + \frac{\theta\psi}{(L-1)} \int \max \{T(w_1, w'_2) - S(w_1, w_2), \Omega(w'_2) - S(w_1, w_2), 0\} dF(w'_2) \\
& + \psi \left(1 - \frac{\theta}{L-1}\right) \int \max \{S(w'_1, w_2) - S(w_1, w_2), \Omega(w'_1) - S(w_1, w_2), 0\} dF(w'_1) \\
& + \psi \left(1 - \frac{\theta}{L-1}\right) \int \max \{S(w_1, w'_2) - S(w_1, w_2), \Omega(w'_2) - S(w_1, w_2), 0\} dF(w'_2) \\
& + \delta [\Omega(w_1) - S(w_1, w_2)] + \delta [\Omega(w_2) - S(w_1, w_2)]
\end{aligned}$$

Turning to a worker-searcher couple, their problem needs to account separately for offers received by the employed spouse and the unemployed spouse who receive offers at different rates. The unemployed spouse receives offers at rate $(1 - \theta)\psi_u$ from the current location in which case the couple faces the same options as in the one-location problem. Second, the same spouse receives outside offers at rate $\theta\psi_u$ in which case (i) the unemployed spouse can choose to accept the offer and the couple could live separately, (ii) the offer could be accepted followed by quitting by the employed spouse, or (iii) the offer could be rejected. Finally, the total offer arrival rate of the employed spouse from all locations is ψ in which case the offer can either be accepted resulting in a transition to another worker-searcher couple with a higher wage, or could be rejected. Notice that in this last case, the location of the offer does not matter since the unemployed spouse will simply follow the employed one in case the offer is accepted.

$$\begin{aligned}
r\Omega(w_1) = & w_1 + b + (1 - \theta)\psi_u \int \max \{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\
& + \theta\psi_u \int \max \{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\
& + \psi \int \max \{\Omega(w'_1) - \Omega(w_1), 0\} dF(w'_1) + \delta [U - \Omega(w_1)] \\
rU = & 2b + 2\psi_u \int \max \{\Omega(w) - U, 0\} dF(w).
\end{aligned}$$

It is easy to see that when $L = 2$ all these equations reduce to those for the two-location problem with on-the-job search.

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