

Holdups and Overinvestment in Physical Capital Markets with Matching Frictions*

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Abstract

This paper considers an economy with matching frictions in the allocation of physical capital. In the absence of binding ex-ante contracts, the specificity created by this friction gives rise to a holdup problem. In partial equilibrium, firms react strategically to the holdup problem by overinvesting in order to reduce the marginal productivity of capital and thus the rental rate. In general equilibrium, overinvestment in physical capital contaminates other factor markets and, together with the matching friction, distorts the allocation of resources. Quantitative exercises in a full-blown general equilibrium model with capital and labor suggest that these inefficiencies are large, implying excessive capital accumulation and consumption losses of several percentage points relative to the social optimum.

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1 Introduction

Factor specificity and contract incompleteness are pervasive phenomena in economic transactions and considered to be a major source of inefficiency. Whenever one party expends resources that increase the value of a productive relationship relative to its outside options (i.e. specificity) and other participating parties can appropriate some of the rents from this investment (i.e. contract incompleteness), a 'fundamental transformation' or 'holdup problem' arises. In partial equilibrium, holdup problems typically reduce the incentive to invest (e.g. Simons, 1944; or Grout, 1984). In general equilibrium, markets react such as to alleviate holdup problems, resulting in underutilization of resources, missing technology adoption, and excessive destruction (e.g. Caballero and Hammour, 1998).¹

In this paper, I examine holdup problems and efficiency for the case of physical capital accumulation and show that the frictions usually associated with underinvestment – factor specificity and contract incompleteness – can lead to exactly the opposite outcome: overinvestment. At the root of this result is the assumption that the allocation of physical capital from suppliers to firms is subject to matching frictions, and that both parties need to sink resources before meeting each other. Once matched, it is thus costly for a firm to find other suppliers, respectively for a supplier to reallocate capital to another firm. In the absence of enforceable ex-ante contracts, this match specificity gives rise to a holdup problem that distorts investment decisions. In particular, under the relatively weak condition of diminishing marginal returns to capital, the firm finds it optimal to seek more capital in order to depress the rental price of capital, which under ex-post bargaining is proportional to the productivity of capital. In principle, capital suppliers would want to adopt exactly the inverse strategy and undersupply capital goods so as to boost the firm's productivity. However, as long as each capital supplier's contribution to a particular firm's capital stock is small, or, alternatively, the capital supplier has no prior information about the firm's capital stock, the capital supplier considers the firm's productivity and in turn the rental rate as exogenous. In general equilibrium, overinvestment due to the holdup problem interacts with the matching friction, leading to potentially important inefficiencies. In addition, the holdup problem in the physical cap-

¹See Caballero (2007) for an extensive review of the interplay of specificity and incomplete contracts in macroeconomics.

ital market contaminates other factor markets, which further distorts the allocation of resources. Quantitative exercises in a full-blown general equilibrium model with capital and labor suggest that these inefficiencies are large, implying excessive capital accumulation and consumption losses of several percentage points relative to the social optimum.

The overinvestment result contrasts with the conclusions of existing work with inefficiencies in capital accumulation, even though the ultimate source of inefficiencies – holdup problems in relationships with specificity – is the same. In Acemoglu and Shimer (1999), for example, firms also exploit the diminishing productivity of capital to alleviate holdup problems in the context of incomplete contracting. Capital allocation is frictionless, however, and holdup problems occur because the labor market is subject to matching frictions. Firms thus want to underinvest so as to lower labor productivity and, in turn, the bargaining set over which they negotiate the wage. Similarly, in Aruoba, Waller and Wright (2006), capital allocation is frictionless but there is imperfect matching in decentralized goods markets. Since capital reduces the cost of production and thus the bargaining set over which goods prices are negotiated, firms find it optimal to invest less.² The point here is not to question the relevance of underinvestment that arises from holdup problems in other markets. Rather, the objective is to highlight that holdup problems in the allocation of physical capital by themselves can lead to overinvestment. Loosely speaking, this is because the firm sits on the demand side rather than on the supply side of the bargaining table. Its strategic response to the holdup problem is therefore exactly the inverse. In this sense, the present paper is closely related to the analysis of intra-firm bargaining in labor markets by Stole and Zwiebel (1996a,b) that has been incorporated into a general equilibrium framework with matching frictions by Cahuc, Marque and Wasmer (2007). Holdup problems in these papers arise because of specificity in firm-worker relationships and non-binding labor contracts. Parallel to the argument made above, firms alleviate the holdup problem by increasing employment as this drives down the marginal productivity of labor and thus the bargained wage rate.

Section 2 presents the mechanism behind overinvestment in a basic setup with capital as the

²The main focus of both the paper by Acemoglu and Shimer (1999) and the paper by Aruoba, Waller and Wright (2007) is somewhat different than here, but underinvestment in physical capital due to holdup problems plays a central role in their analysis. The paper that presents an environment closest in spirit to the present one is Caballero and Hammour (1998). Their analysis is entirely static, however, which makes a comparison difficult. I return to discussing these papers in more detail at the end of Section 2.

only factor of production. A crucial ingredient in this analysis is the existence of matching frictions in physical capital markets. This assumption is rarely made in modern macroeconomics but seems easy to motivate.³ Physical capital is often highly specific to a task and difficult (if not impossible) to move from one location to another. Furthermore, both capital suppliers and firms must make important decisions about capital production and new projects long before they meet as trading partners. This is especially true for reallocations of used capital, which represent about 25% of all investments in the U.S. economy (e.g. Eisfeldt and Rampini, 2007). These technological and spatial constraints in combination with informational imperfections may be an important source of the congestion and shortages in physical capital markets that we regularly observe in the data.⁴ In the model, I formalize these frictions with a reduced-form matching function that is taken from Kurmann and Petrosky-Nadeau (2008) and that draws on the now widely employed matching approach for the labor market (e.g. Blanchard and Diamond, 1990; Mortensen and Pissarides, 1994).⁵ In order to increase the capital stock, firms must undertake projects at a cost and search for available physical capital prior to production. Likewise, capital suppliers must forgo consumption goods in order to transform them into new capital goods before trying to allocate them to firms. The probability of a capital supplier to match with a firm and vice versa is then a function of the number of projects relative to the number of new capital units.

After presenting the matching framework, Section 2 analyzes the efficiency properties of the decentralized equilibrium. I first derive the overinvestment result in partial equilibrium, and then show that in general equilibrium, ex-post Nash bargaining always implies an inefficient allocation. Firms either overinvest compared to the socially optimal level, or entry by capital suppliers is

³Modern macroeconomics has extensively analyzed the consequences of investment constraints such as time-to-build or non-linear adjustment costs (see for example the survey by Caballero, 1999). While such constraints are important ingredients to explain the distribution of investment rates across firms or the dynamics of new investment over the business cycle, the price of capital in this literature is still determined efficiently, in a perfectly competitive market. Hence, there are no holdup problems.

⁴See Kurmann and Petrosky-Nadeau (2008), for example, who document that important vacancy rates for many major capital goods (used and new) that vary inversely with the business cycle. Also note that Aruoba, Waller and Wright (2006) use a similar spatial separation argument to motivate why money is essential in a decentralized exchange economy with capital.

⁵Also see Petrongolo and Pissarides (2000) for a survey of the matching approach for labor markets, and Rogerson, Shimer and Wright (2005) for a survey on the labor search literature.

excessive, or both. Even if the bargaining power of the capital suppliers is such that the negative externality from entry for other capital suppliers cancels out the positive externality for firms – i.e. if Hosios' (1990) condition is satisfied – there is still a holdup problem that provides incentive for the firms to overinvest. The only way this inefficiency could be circumvented in the decentralized equilibrium is with binding contracts where firms commit ex-ante to rental rates and suppliers direct their capital towards more attractive firms. This extension is reminiscent of Acemoglu and Shimer (1999) who show that efficiency in a frictional labor market with holdup problems due to ex-ante investments can be achieved with wage posting and directed search. While binding ex-ante contracts may very well exist for some segments of the physical capital market, they are – as Caballero and Hammour (1998) put it – "*...much closer to a methodological benchmark than a description of actual practices*" for other segments where investment decisions must be sunk long before the actual allocation occurs. The goal here is not to argue about the existence of such contracts.⁶ Rather, the paper wants to analyze the inefficiencies that arise in the absence thereof, so as to gauge the welfare consequences of institutions that help prevent ex-post renegotiation.

Section 3 incorporates the matching friction for physical capital markets into a full-blown general equilibrium model with constant returns to scale technology in capital and labor. The model can be readily calibrated and thus allows a quantitative assessment of the welfare consequences of the holdup problem. The severity of the holdup problem in physical capital markets depends crucially on the frictions in the labor market. At one extreme, if firms can adjust labor costlessly in a Walrasian market, the holdup problem disappears. The intuition for this result is straightforward. Constant returns to scale technology implies that the marginal productivity of both capital and labor directly depend on the firm's capital-labor ratio. But with a Walrasian labor market, this implies that the capital labor ratio is proportional to the wage rate, which the firm takes as exogenous in this scenario. Hence, firm will always adjust labor in a way that makes the marginal productivity of capital and thus the rental rate independent of the firm's capital stock. At the other extreme, if labor is completely predetermined, the holdup problem for physical capital contaminates the labor market as in this case, the firm finds it optimal to further lower the marginal productivity of capital by underemploying labor relative to the social optimum.

⁶There is an extensive debate under what conditions institutions allow transaction parties to circumvent holdup problems. See for example Hart (1995), Maskin and Tirole (1999) or Hart and Moore (2004).

Section 4, finally, proceeds with a quantitative assessment of the model with capital and labor. In the case where labor is predetermined, the distortions implied by the holdup problem and the allocation friction is large, leading to losses in consumption that quickly exceed 10% relative to the social optimum when either the capital supplier's or the firm's bargaining power is too high. However, the assumption of predetermined labor seems very strong. To provide a more realistic assessment of the inefficiencies, I thus extend the model with a labor market that is subject to a standard matching friction and non-binding wage contracts. Preliminary results suggest that the holdup problems implied by each friction partly neutralize each other. Nevertheless, there remain important inefficiencies compared to the social optimum.

2 The basic model

The basic model is intentionally kept simple to convey the main intuition. Physical capital is the only factor of production and the opportunity cost of capital suppliers is kept fixed. A full model with labor as a second factor of production and an endogenous consumption-savings decision is analyzed in the next section.

2.1 Environment

The basic model is populated by a continuum of capital suppliers and a continuum of firms. All agents live forever in discrete time and discount the future at constant rate $\beta = (1 + r)^{-1}$. There is some general consumption good from which both capital suppliers and firms derive linear utility.

Capital suppliers are small in the sense that each one of them can forgo exactly one unit of consumption and convert it into one unit of physical capital, which they then try to allocate to firms. Firms, by contrast, are large in the sense that each one of them holds a productive capital stock k that is comprised of many units. The technology that turns this capital stock into consumption goods is denoted by $f(k)$, with $f(\cdot)$ strictly increasing and concave, and satisfying the usual Inada conditions.

The allocation of physical capital from suppliers to firms is subject to matching frictions as in Kurmann and Petrosky-Nadeau (2008) and occurs in two phases. In the first phase, firms open new projects at cost κ per project and search for available capital in order to transform the projects into

production. Capital suppliers, in turn, convert their consumption good into available (or liquid) capital. Then, firms with new projects and suppliers with available capital match. Let V denote the total number of new projects posted by all firms in the current period, and L the total number of liquid units of capital. Then, if suppliers do not direct their capital towards any particular (group of) firms, $\theta = \frac{V}{L}$ describes the capital market tightness, and total capital additions are governed by some matching process $m(L, V) \leq \min(L, V)$. Following most of the search literature, $m(L, V)$ is assumed constant returns to scale and satisfies $m_L > 0$, $m_{LL} < 0$, $m_V > 0$, $m_{VV} < 0$. Accordingly, each project matches with a unit of new capital with probability $p(\theta) = \frac{m(V, L)}{V}$, and each unit of new capital matches with a project with probability $q(\theta) = \frac{m(V, L)}{L}$, with $p(\theta)\theta = q(\theta)$.⁷

In the second phase, projects with matched capital become productive and, together with the existing capital stock, yield output. Unmatched capital units, in turn, remain unproductive in the hands of the capital suppliers. Finally, in the same phase, but after production has taken place, some exogenous fraction s of the capital stock separates from the firm. To distinguish capital separation from depreciation, I assume that the respective capital suppliers recover a fraction φ of the separated capital stock; i.e. φsk remains with the economy for consumption or reconversion into

⁷To provide a specific example of how matching in physical capital market operates, consider the functional form that I will employ later in the quantitative part of the paper

$$m(L, V) = \frac{LV}{(L^\chi + V^\chi)^{1/\chi}}$$

with $\chi > 1$. This formulation has been used previously by Den Haan, Ramey and Watson (2000) for the labor market and has an intuitive interpretation in the present context. In particular, suppose that the capital market is segmented into $J \equiv (L^\chi + V^\chi)^{1/\chi}$ submarkets in which matching can occur. This segmentation is easy to motivate with technological and spatial constraints. Available capital L and new projects V are assigned randomly to one of the submarkets. Once assigned, the technological and spatial constraints prevent L and V from moving to another submarket. It is thus the segmentation together with the random assignment assumption (due, for example, to information imperfections in the allocation stage) that embody the friction. A match occurs when a capital supplier and a project find themselves in the same channel. The other projects and capital suppliers remain unmatched. Under these assumptions, the probability of a capital supplier to match with a project is L/J , the probability of a project to find available capital is V/J , and the total number of matches is LV/J . As Den Haan, Ramey and Watson (2000) note, this matching function exhibits constant-returns-to-scale, is increasing in both arguments and implies probabilities $p(\theta)$ and $q(\theta)$ that are bounded between 0 and 1 (a condition that is violated by the standard Cobb-Douglas matching function).

new liquid capital units. The remainder $(1 - \varphi)sk$ is a deadweight loss incurred during separation. Caballero and Hammour (1998) interpret $(1 - \varphi)sk$ as a loss due to specificity. As I discuss at the end of this section, this specificity assumption is crucial for Caballero and Hammour's analysis, but it is not essential for the results presented here.

Given these assumptions, the evolution of the productive capital stock of a firm can be described by

$$k_{+1} = (1 - s)k + p(\theta)v, \quad (1)$$

where v denotes the number of new projects per firm.⁸ Likewise, the evolution of unmatched capital is

$$u_{+1} = (1 - q(\theta))l, \quad (2)$$

where l denotes the number of newly available capital units per firm. The temporal distinction between k_{+1} and u_{+1} on the one hand, and k , v , l and θ on the other, emphasizes the two phases: investment in new projects and liquid capital, respectively, has to be undertaken *before* production occurs. Unmatched capital, in turn, can only be reallocated the period after it was on the matching market.

2.2 Efficient allocation

An allocation in this environment is (constrained) efficient if it maximizes the discounted sum of total consumption goods subject to the technological constraint and the capital allocation friction. Consider thus a hypothetical social planner who solves the following problem

$$O(k, u) = \max_{l, \theta} [f(k) + s\varphi k + u - \kappa\theta l - l + \beta O(k_{+1}, u_{+1})]$$

subject to the equations of motion for the productive capital stock (1) and unmatched capital (2). Since all firms are identical, the social planner faces the same problem for each firm. The solution therefore implies efficiency for the entire economy.⁹

⁸The basic model thus abstracts completely from depreciation. Depreciation is introduced, however, in the full-blown model of Section 3.

⁹More precisely, the steady state solution for market tightness θ is independent of the total capital stock and total amount of new liquid capital.

The following analysis exclusively focuses on steady state equilibria, where $x = x_{+1}$ for all involved variables. I thus abstract from time subscripts. Appendix A provides an explicit derivation of the social planner's problem and proves the existence and uniqueness of a steady state equilibrium. Here, I simply focus on the characterization of that equilibrium and provide some intuition.

Proposition 1. *There exists a unique efficient steady state allocation, characterized by the solution (k^S, θ^S) to the following optimality conditions*

$$1 + \kappa\theta^S = \beta \left[q(\theta) \left(\frac{f'(k^S) + s\varphi}{1 - (1-s)\beta} \right) + (1 - q(\theta^S)) \right] \quad (3)$$

$$\kappa = q'(\theta^S)\beta \left(\frac{f'(k^S) + s\varphi}{1 - (1-s)\beta} - 1 \right) \quad (4)$$

PROOF: Appendix A.

The intuition for the two optimality condition is straightforward. The left-hand side of equation (3) is the cost of investing an additional unit of resources into new capital. This cost is comprised of the lost consumption unit plus the (negative) matching externality that an additional unit of new capital has on all other new capital units. The right-hand side denotes the marginal benefits from the additional new capital unit. With probability $q(\theta)$, it is matched with a firm and becomes productive next period. This yields an annuity of $f'(k) + s\varphi$ that has to be discounted because of both impatience and the possibility of separation. With probability $(1 - q(\theta))$, the unit remains unmatched and has to be carried forward one period, at which time it can be consumed or put again into new capital. Equation (4), in turn, describes the optimal market tightness decision. Increasing market tightness costs κ per unit of l . The marginal benefit of doing so is an increase in the matching probability $q'(\theta)$ times the net benefit from having another unit of productive capital next period.

2.3 Decentralized equilibrium with Nash bargaining

In the decentralized case with bargaining, neither the capital supplier nor the firm can commit in advance to a rental rate when deciding on the supply of the capital, and undertaking projects, respectively. Presumably, this is because of contract incompleteness: both investments are made ex-ante before the parties meet and the surplus is split ex-post according to the respective opportunity costs. In describing the different trade-offs, I start by simply describing this rental rate by $\rho = \rho(k)$.

This rate is expressed as a function of the firm's capital stock because the firm's size may be a determinant of its bargaining position and thus enters its optimization problem.

We start with the capital supplier's problem. Consider a capital supplier that is not matched with a firm at the end of period t (either because it came into the period unmatched or because it just separated from a firm). Its value of transforming a unit of resource into new capital and search for a firm is

$$W^e = -1 + q(\theta)\beta W^k + (1 - q(\theta))\beta W_{+1}^u, \quad (5)$$

where W^k and W^u denote the value of a matched and unmatched unit of capital, respectively. The former is defined by

$$W^k = \rho(k_t) + s\varphi + (1 - s)\beta W_{+1}^k, \quad (6)$$

and the latter by

$$W^u = 1. \quad (7)$$

In optimum, there is free entry; i.e. capital suppliers find it optimal to provide a unit of new capital until $W^e = 0$. Firms, in turn, enter the period with capital stock k and need to decide on how many new projects v to undertake. Their problem is described by

$$\begin{aligned} J(k) &= \max_v [f(k) - \rho(k)k - \kappa v + \beta J(k_{+1})] \\ s.t. \ k_{+1} &= (1 - s)k + p(\theta)v. \end{aligned}$$

The first-order condition is

$$\kappa = p(\theta)\beta J_k, \quad (8)$$

where, as before, I abstract from time-subscripts. The marginal value of an additional unit of capital is given by the envelope condition

$$J_k = f'(k) - \rho'(k)k - \rho(k) + (1 - s)\beta J_k. \quad (9)$$

The model is closed with a rental rate that is determined ex-post by Nash bargaining over the match surplus $J_k + W^k - W^u$. The rental rate that solves for this bargaining problem implies

$$\eta J_k = (1 - \eta)(W^k - W^u), \quad (10)$$

where η is the bargaining power of the capital supplier.

To analyze the firm's strategic investment behavior, I combine the optimal project condition in (8) with (9). After some rearrangement, this yields an expression for the firm's demand for productive capital

$$\rho(k) = f'(k) - \rho'(k)k - \frac{\kappa(r+s)}{p(\theta)}. \quad (11)$$

Without the $\rho'(k)k$ term, this equation would simply be a break-even condition, saying that the rental rate equals the firm's return on the last unit of capital minus the expected investment cost per unit in annualized terms. With the $\rho'(k)k$ term, the decision becomes more involved. The firm realizes that adding capital affects its productivity and thus the rental cost for all of its capital. To describe the impact of this addition more precisely, note that the Nash bargaining solution in (10) implies the following expression for the rental rate (see Appendix A for details)¹⁰

$$\rho(k) = \eta [f'(k) - \rho'(k)k + (1-s)\kappa\theta] + (1-\eta) [(1-\varphi)s]. \quad (12)$$

This is a non-homogenous linear differential equation of ρ in k with the following solution

$$\rho(k) = \eta \left[\int_0^1 f'(kz) z^{(1-\eta)/\eta} dz + (1-s)\kappa\theta \right] + (1-\eta)(1-\varphi)s. \quad (13)$$

The first term in brackets is the maximum amount the firm is willing to pay for an additional unit of capital. It equals a part related to the marginal product of capital, which I will discuss shortly, plus the expected project costs that are saved per unit of liquid capital in future periods because the match occurs today. The second term in brackets represents the minimum compensation the capital supplier requires to enter the rental rate relationship after the match has occurred. It equals opportunity cost of the capital supplier, which is simply the part lost to specificity in case of separation. Everything else constant, the larger the capital supplier's bargaining power (i.e. the larger η), the higher the rental rate.

Now, reconsider the first part of the firm's maximum rental rate, $\int_0^1 f'(kz) z^{(1-\eta)/\eta} dz$. It is a weighted average of the contribution of each unit of capital to the firm's productivity $f'(kz)$ weighted by $z^{(1-\eta)/\eta}$, which decreases with the distance to the margin. If $f''(\cdot) = 0$ for all k , the firm's productivity $f'(\cdot)$ is unaffected by its capital stock and the integral reduces to a constant.

¹⁰This analysis follows closely the one in Cahuc, Marque and Wasmer (2007) who implement the problem of intra-firm bargaining in a labor market with matching frictions. See the discussion at the end of this section for further details.

In this case, $\rho'(k) = 0$ and the firm's demand for capital is unaffected by its capital stock. For decreasing marginal productivity $f''(\cdot) < 0$, we have $\rho'(k) > 0$ and thus, the firm can drive down its productivity by increasing k beyond what is warranted by the break-even point. In other words, there is overinvestment, as summed up by the following proposition.

Proposition 2. *For $f''(\cdot) \leq 0$ with strict inequality over some segment of $(0, k)$, the firm overinvests relative to the social planner in order to reduce the rental rate of capital.*

PROOF: Appendix A.

The overinvestment result can be nicely illustrated when technology takes the form $f(k) = k^\alpha$. Then, equation (11) becomes

$$\rho(k) = \frac{\alpha k^{\alpha-1}}{1 - \eta(1 - \alpha)} - \frac{\kappa(r + s)}{p(\theta)}.$$

The term $\alpha k^{\alpha-1}$ is the firm's marginal productivity. The term $(1 - \eta(1 - \alpha))^{-1}$ denotes overinvestment. For $\alpha = 1$ (i.e. $f''(\cdot) = 0$), marginal productivity is constant and there is no overinvestment. For $\alpha < 1$, the firm's productivity depends on its capital stock and there is overinvestment; i.e. $(1 - \eta(1 - \alpha))^{-1} > 1$. Interestingly, this overinvestment term increases with η . Hence, a higher bargaining power by the capital supplier increases the firm's incentive to overinvest.

Proposition 1 is a partial equilibrium result because it is derived for a given level of θ . In general equilibrium, overinvestment affects the firm's optimal project decision and thus the tightness in the physical capital market. It is thus unclear under what conditions we obtain overinvestment in equilibrium and what the consequences for efficiency are. To examine these questions, I combine the capital supplier's conditions (5)-(7) with the free-entry condition and the firm's capital demand (11)

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1 - s)\beta} \right) + (1 - q(\theta)) \right]. \quad (14)$$

In turn, the solution to the Nash bargaining (10) together with (11) implies

$$\kappa = \frac{q'(\theta)}{1 - \epsilon(\theta)} (1 - \eta)\beta \left(\frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1 - s)\beta} - 1 \right), \quad (15)$$

where $1 - \epsilon(\theta) \equiv q'(\theta)\theta/q(\theta) = q'(\theta)p(\theta)$ denotes the elasticity of the capital suppliers' matching probability with respect to the market tightness. These two equations lead to the following characterization of the decentralized Nash bargaining equilibrium.

Proposition 3. *For $\eta > 0$, there exists a unique decentralized bargaining steady state equilibrium defined by (k^B, θ^B) that solves (14) and (15). This equilibrium is always inefficient. In particular,*

- *if $0 < \eta \leq \epsilon$, then $\theta^S \leq \theta^B$ and $k^S < k^B$;*
- *if $\epsilon < \eta \leq 1$, then $\theta^B < \theta^S$ and either $k^S \leq k^B$ or $k^B \leq k^S$.*

In sum, firms either overinvest ($k^S < k^B$) or entry by capital suppliers is excessive ($\theta^B < \theta^S$) or both.

PROOF: Appendix A.

To understand this proposition, consider the special case where capital suppliers' bargaining power is equal to the elasticity of the matching function $\eta = \epsilon(\theta)$. Comparing (14) and (15) with the social planner counterparts in (3) and (4), it is easy to see that the decentralized allocation would be optimal if $\rho'(k) = 0$, i.e. if there is no overinvestment. This case would be equivalent to Hosios' (1990) famous efficiency condition in decentralized search models of the labor market when productivity is exogenous. Hosios' condition trades off two externalities. On the one hand, the entry decision of a capital supplier creates a negative externality because it makes it more difficult for all other capital suppliers to find a firm. On the other hand, the same entry decision implies a positive externality for the firms because their probability of finding capital increases. When $\eta = \epsilon(\theta)$, the capital supplier's share of the rent is such that the two externalities cancel each other out – as the social planner would optimally choose.

In the current context, however, $f''(\cdot) < 0$ for there to exist an equilibrium and thus $\rho'(k) < 0$. Hence, firms have an incentive to overinvest and Hosios' condition no longer yields efficiency. Broadly speaking, the firm's strategic behavior creates an inefficiency over and beyond the allocational inefficiency that Hosios' condition resolves. Overinvestment would only be avoided if the capital supplier's bargaining power η was zero so that the rental rate is driven down to the capital supplier's opportunity cost. In this case, the firm captures all the surplus from its ex-ante investment in projects and thus invests just the right amount of capital. But then, the firm's return becomes too large, thus leading to an inefficiently large number of projects relative to the amount of available liquidity; i.e. $\lim_{\eta \rightarrow 0} \theta^B = \infty \gg \theta^S$.

In sum, Proposition 2 shows that decentralized capital markets with ex-post bargaining always lead to an inefficient allocation when the firm's capital stock influences its bargaining position and

thus the rental rate. This is because the rental rate in this decentralized market plays a dual role, one for investment by the firm and one for allocation. The two roles are fundamentally at odds with each other, thus implying inefficiency.

2.4 Decentralized equilibrium with price posting

To complete the analysis of the basic model, I consider the situation where firms post rental rates in advance in order to attract an optimal number of capital suppliers. This is similar to the setups proposed in Shimer (1996), Moen (1997) and Acemoglu and Shimer (1999) for the labor market, or Arseneau and Chugh (2008) for consumer search. As in these studies, the setup rests on two assumptions. The first is that firms can commit to a rental rate in advance, before meeting their capital suppliers. The second is that capital suppliers, after observe the posted rental rates, can direct their capital towards the firm with the best trade-off between matching probability and rental rate.¹¹ To formalize this environment, consider each firm as a submarket for capital, denoted by i . Upon observing the different rental rates postings ρ^i across submarkets, capital suppliers decide in which submarket they want to send their unit of new capital for allocation. Firms that post a higher rental rate will attract more capital suppliers; i.e. capital market tightness $\theta^i = v^i/l^i$ in submarket i is increasing in ρ^i , where v^i are the number of open projects by firm i and l^i are the number of capital suppliers that decided to send their capital to that firm. Capital suppliers thus face a trade-off between a higher rental rate if matched and a lower probability of matching $q(\theta^i)$. Vice versa, firms realize that a higher rental rate posting will attract more capital suppliers, thus increasing their matching probability $p(\theta^i)$.

As in the case with ex-post negotiation, an important assumption in this setup is that capital suppliers can allocate their capital unit to only one firm (or one submarket) at a time. If the capital unit is not matched, it cannot be moved to another firm (or submarket) in the same period. Instead, the capital supplier has to wait for the next period before that unit frees up again for either consumption or new matching. Likewise, separated capital cannot be costlessly rematched but needs to undergo the same costly allocation process than new capital.

¹¹As Acemoglu and Shimer (1999) show, the directing agent (i.e. here the capital supplier) does not need to observe the rental rate postings of all firms. Instead, all the results obtain if the directing agent only observes the postings of two random firms.

Since firms are all identical, they all make the same optimal decisions in equilibrium. I thus omit subscripts in the interest of simplification and directly work with a representative firm as before. Consider first the capital supplier's situation. Its Bellman equations are identical to the ones in (5)-(7), only that now, there is such a system for each submarket i . Combining these equations with the free-entry condition, I obtain

$$1 = q(\theta)\beta \left(\frac{\rho + s\varphi}{1 - (1-s)\beta} - 1 \right) - \beta. \quad (16)$$

This equation defines market tightness as a function of the rental rate, i.e. $\theta = \theta(\rho)$, and describes how capital supply reacts to different rental rates. The firm's problem, in turn, is also similar to the one in the decentralized model with Nash bargaining. The firm, however, now takes $\theta = v/l$ as an endogenous variable that depends on both its vacancy choice and its rental rate posting (which affects l and thus θ). The problem of the firm now consists of deciding on the number of new projects in a first stage, and then to post the rental rate so as to attract the optimal amount of liquid capital

$$J(k) = \max_{v, \rho} [f(k) - \rho k - \kappa v + \beta J(k_{+1})]$$

subject to the capital accumulation constraint $k_{+1} = (1-s)k + p(\theta)v$, and the optimal capital supply relation in (16). Note that in this formulation, the rental rate ρ no longer depends on k in the eyes of the firm, because ρ is now a choice variable.

An equilibrium in this environment must satisfy the following conditions: (i) entering capital suppliers earn zero profits; (ii) capital suppliers direct search towards the firm (or submarket) with the highest expected payoff; (iii) firm's are profit maximizing; and (iv) the equilibrium is consistent with rational expectation; i.e. rental rates and probabilities are such that capital suppliers are indifferent about where to direct capital. In other words, if a firm posted another rental rate, the trade-off between queue length and rental rate would be such that capital suppliers would not want to apply. Under these conditions, the following proposition obtains.

Proposition 4. *There exists a unique decentralized rental rate posting equilibrium that coincides with the efficient allocation (θ^S, k^S) .*

PROOF: Appendix A.

This proposition is interesting because it implies that aside from matching frictions, the root cause of the inefficiencies in the decentralized equilibrium is the absence of non-binding ex-ante

contract. If, by contrast, such contracts exist, then the firm's rental rate solves both the externality problem of the matching friction and the holdup problem.

2.5 Discussion

As mentioned in the introduction, the results of this paper relate to a number of other papers on specificity and holdups in macroeconomics. First, the overinvestment result here is closely related to the overhiring result in labor markets with non-binding contracts emphasized by Stole and Zwiebel (1996a,b), and recently incorporated into a general equilibrium framework with matching frictions in the labor market by Cahuc, Marque and Wasmer (2007). Holdup problems in these papers arise because of specificity in firm-worker relationships and non-binding labor contracts. Parallel to the argument made above, firms alleviate the holdup problem by increasing employment as this drives down the marginal productivity of labor and thus the bargained wage rate. Here, the situation is exactly inverted: labor is absent from the basic model and instead, matching frictions apply to physical capital markets. The next section will incorporate labor and then assess how frictions in both labor and capital markets interact.

The specificity embodied in productive physical capital that is implied by the matching friction is probably closest in spirit to the specificity in Caballero and Hammour's (1998) model. There, productivity is exogenous and the concept of the firm as an independent optimizing agent is absent. Hence, there is simply no role for decreasing marginal returns as a means to reduce the bargaining set and thus the rent of the other party. It is thus not surprising that they do not find overinvestment. Instead, their model introduces specificity as a deadweight loss in case the factors separate from joint production. Furthermore, their analysis is completely static. To shed more light on this last point, consider the special case of no matching friction in the basic model above. Under this scenario, the allocation of physical capital becomes immediate and the holdup problem due to non-binding contracts disappears. The only specificity left is, as in Caballero and Hammour, the deadweight loss φsk that capital suppliers incur due to separation. However, in the present *dynamic* model, it is not only the capital supplier who takes this deadweight loss into account when deciding on whether to forgo consumption for investment. The social planner equally incorporates this loss into the optimal decision problem as it affects her resource constraint. Hence, the decentralized market equilibrium yields the same solution than the efficient allocation of the social planner. Caballero

and Hammour’s static analysis ignores this last part, which is why they find underinvestment.

3 General equilibrium extension with labor

This section incorporates the capital search environment of the previous section into a full-blown dynamic general equilibrium framework with an intertemporal consumption-savings margin and labor. The first extension endogenizes the opportunity cost of capital. The second extension addresses a number of interesting questions that naturally come out of the discussion in the previous section. Under what conditions is the holdup problem in physical capital markets robust to the introduction of labor? How do holdup problems in the physical capital market seep through to the labor market? How do frictions in the capital market interact with frictions in the labor market? More generally, the extension to a full-blown general equilibrium setting allows me to conduct a variety of quantitative exercises on the effects of holdup problems, using well-known calibration techniques of the real business cycle literature.

The analysis proceeds in two steps. In the first step, labor is introduced in a frictionless market in order to address the first two questions. In a second step, the Walrasian labor market is replaced by a labor market that is subject to a standard matching friction, thus providing a quantitative answer to the third question. In particular, we know from the previous discussion that holdup problems in the labor market give rise to underinvestment when capital allocation is frictionless (e.g. Acemoglu and Shimer, 1999). So, to what extent does overinvestment due to holdup problems in the capital market cancel out underinvestment due to holdup problems in the labor market?

3.1 Model without matching frictions in the labor market

To introduce a consumption-savings margin and labor, I adopt a neoclassical growth framework with endogenous labor. Specifically, I now assume that the model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and supply of physical capital. For simplicity, there is no distinct sector for capital allocation. Instead, households directly act as capital suppliers. Dropping this assumption is straightforward but would unnecessarily complicate the model.

Firms produce output y with matched capital k and hired labor n using technology

$$y = f(n, k). \quad (17)$$

This technology is assumed constant-returns-to-scale, with $f_n, f_k > 0$ and $f_{nn}, f_{kk} < 0$. Labor is hired in a Walrasian market and thus, firms take the wage rate w as given. As before, firms need to post projects v at a cost κ and search for available capital in a frictional market described by the matching function $m(V, L)$, with $p(\theta) = m(V, L)/V$ and $q(\theta) = m(V, L)/L$. The firm's capital stock thus evolves as before with the exception that now, I allow for depreciation $0 < \delta < 1$; i.e.

$$k_{+1} = (1 - s)(1 - \delta)k + m(v, l).$$

Households, in turn, have preferences over consumption and leisure, denoted by $u(c, n)$. Each period, they have to trade off current consumption with supply of liquid capital l , and leisure with work. As before, we keep the entire analysis in terms of a representative firm. For now, the matching friction in the physical capital market is thus the only difference that distinguishes the model from the neoclassical growth benchmark.

To compute the efficient allocation, consider again a social planner who now solves the following problem

$$\begin{aligned} O(k, u) &= \max_{c, n, l, \theta} [u(c, n) + \beta O(k_{+1}, u_{+1})] \\ &\quad + \lambda [f(n, k) + \varphi(1 - \delta)sk + u - \kappa\theta l - c - l] \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + q(\theta)l \\ \text{s.t. } u_{+1} &= (1 - q(\theta))l \end{aligned}$$

where λ is the Lagrangian multiplier on the household's budget constraint. This multiplier denotes the opportunity cost, in utility terms, of capital supply relative to consumption. The analysis of the social planner's solution is very similar to the one of the basic model, except that now, it is the capital-labor ratio rather than capital itself that interacts with the matching friction in the capital market. Appendix B provides an explicit derivation of the solution and establishes the existence and uniqueness of a steady state equilibrium along the lines of the proof behind Proposition 1. The following corollary describes this equilibrium.

Corollary 5. *There exists a unique efficient steady state allocation, characterized by the solution $((k/n)^S, \theta^S)$ to the following optimality conditions*

$$-\frac{u_n(c, n)}{u_c(c, n)} = f_n(k, n) \quad (18)$$

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f_k(n, k) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} \right) + (1 - q(\theta)) \right] \quad (19)$$

$$\kappa = q'(\theta)\beta \left[\frac{f_k(n, k) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1 \right] \quad (20)$$

The first equation describes the standard efficiency condition for a Walrasian labor market. The second and third condition describe the capital market and are very similar to equations (3) and (4) of the basic model. Note in particular that λ is absent from both equations. The opportunity cost of capital therefore does not influence the optimal capital allocation.

Consider now the decentralized environment. Firms and households optimize independently over v and n , and c , l and n , respectively, taking capital market tightness θ and the wage rate w as given. Once capital is matched, the rental rate is negotiated ex-post with Nash bargaining. As before, since firms hold many units of both capital and labor, they internalize the effect of k and n on the rental rate; i.e. $\rho = \rho(k, n)$. Households, by contrast, are supposed to remain only a small contributor to each firm's capital stock. Hence, they consider ρ as exogenous when deciding on the supply of liquid capital and labor.

The household's problem is

$$\begin{aligned} V(k, u) &= \max_{c, n, l} [u(c, n) + \beta V(k_{+1}, u_{+1})] \\ &\quad + \lambda[wn + \rho(k, n)k + \varphi(1 - \delta)sk + u + d - c - l] \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + q(\theta)l \\ \text{s.t. } u_{+1} &= (1 - q(\theta))l \end{aligned}$$

where d denote firm profits transferred to households (considered exogenous). The first-order conditions combined with the envelope conditions imply

$$-\frac{u_n(c, n)}{u_c(c, n)} = w \quad (21)$$

$$1 = \beta \left[q(\theta) \frac{\rho(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} + (1 - q(\theta)) \right] \quad (22)$$

The optimality condition for the supply of liquid capital is very similar to the one in the basic model. Note, in particular, that the opportunity cost of capital supply λ drops out of this equation, as in the social planner case. The firm's problem, in turn, is

$$\begin{aligned} J(k) &= \max_{n,v} \{f(k, n) - \rho(k, n)k - wn - \kappa v + \beta J(k_{+1})\} \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + p(\theta)v. \end{aligned}$$

The first-order conditions together with the envelope conditions imply the following capital and labor demands

$$\rho(k, n) = f_k(k, n) - \rho_k(k, n)k - \frac{\kappa(r + s)}{p(\theta)} \quad (23)$$

$$w = f_n(k, n) - \rho_n(k, n)k. \quad (24)$$

The important difference to the basic model is that now, the firm not only internalizes the effect of its capital stock on the rental rate, but also the effect of its hiring decision, as both k and n affect the marginal return of capital. This has potentially important implications as I will discuss shortly. The model is closed with an ex-post Nash bargaining rule over the surplus of the marginal capital match. The solution to this bargaining game yields the rental rate $\rho(k, n)$ that satisfies $\eta J_k(k) = (1 - \eta) [V_k(k, u) - V_u(k, u)] / \lambda$. By contrast to the basic model, the part pertaining to the surplus of the household is divided by λ so as to express it in terms of goods rather than in terms of utility.

Now, consider the firm's capital demand in (23) and the labor market equilibrium defined by the combination of (21) and (24). Together with the Nash bargaining solution for the rental rate, they give rise to the following proposition.

Proposition 6. *If firms can adjust employment costlessly and immediately in a Walrasian labor market, then the rental rate $\rho(k, n)$ depends only on the firm's capital-labor ratio. In this case, $\rho_k(k, n) = 0 = \rho_n(k, n)$ and there is no holdup problem.*

If employment is predetermined, then $\rho_k(k, n) < 0$ and $\rho_n(k, n) > 0$. Firms thus find it optimal to overinvest in capital and underemploy labor relative to the social optimum in order to reduce the rental rate of capital.

PROOF: Appendix B.

The intuition for the first part of the proposition is straightforward. Constant returns to scale technology implies that the marginal productivity of both capital and labor directly depend on the firm's capital-labor ratio. But with a Walrasian labor market, this implies that the capital labor ratio is proportional to the wage rate, which the firm takes as exogenous in this scenario. Hence, firm will always adjust labor in a way that makes the marginal productivity of capital and thus the rental rate independent of the firm's capital stock.

If labor is predetermined, as stated in the second part of the proposition, the firm can no longer adjust labor immediately. In that case, both k and n are state variables that the firm considers separately in its optimal decision. As the proof in Appendix B shows, this implies $\rho_k(k, n) < 0$ and $\rho_n(k, n) > 0$. By the capital demand equation in (23), the firm thus alleviates the holdup problem in the capital market by overinvesting, identical to the basic model. But now, in addition, the firm also has employment as a second tool to influence the rental rate of capital. In particular, $\rho_n(k, n) > 0$ implies that the firm finds it optimal to keep the marginal productivity of labor above the wage rate by depressing employment relative to the social optimum. The rationale behind this decision is that a lower employment level helps reducing the marginal productivity of capital and thus the rental rate over which the firm is bargaining. The holdup problem in the capital market thus contaminates the labor market, adding another distortion with potentially important welfare effects. This result is exactly the inverse outcome of Cahuc, Marque and Wasmer's (2007) analysis of intra-firm bargaining in the labor matching model with capital. In particular, for the case of a Cobb-Douglas production function $f(k, n) = k^\alpha n^{1-\alpha}$, the capital and labor demand equations become, respectively,

$$\begin{aligned}\rho(k, n) &= \frac{1}{1 - \eta(1 - \alpha)} \alpha k^{\alpha-1} n^{1-\alpha} - \frac{\kappa(r + s)}{p(\theta)} \\ w &= \frac{(1 - \eta)}{1 - \eta(1 - \alpha)} (1 - \alpha) k^\alpha n^{-\alpha}\end{aligned}$$

Overinvestment in physical capital is captured by $(1 - \eta(1 - \alpha))^{-1} > 1$. Underemployment in the labor market is captured by $(1 - \eta)/(1 - \eta(1 - \alpha)) < 1$. More precisely, $(1 - \eta)$ captures the contamination of the labor market by the holdup problem in the capital market. This effect is mitigated partly by $(1 - \eta(1 - \alpha))^{-1}$; i.e. the effect of overinvestment in capital on labor productivity. The end result is underemployment – an effect that is increasing in the household's bargaining power η in the capital market.

In general equilibrium, the holdup distortions together with the matching friction for physical capital interact and result in a decentralized market solution that is inefficient compared to the social planner's solution. As in the basic model, this equilibrium is unique and is summarized by the following corollary.

Corollary 7. *For $\eta > 0$, there exists a unique decentralized bargaining steady state equilibrium defined by $((k/n)^B, \theta^B)$ that solves (21)-(21). If firms can adjust employment costlessly and immediately in a Walrasian market, this equilibrium is efficient if and only if $\eta = \epsilon(\theta)$. If labor is predetermined, the equilibrium is always inefficient.*

The big difference to the basic model is the existence of an efficient decentralized equilibrium for the special case when the labor market is completely frictionless and the Hosios condition holds. The frictionless labor market solves the holdup problem and Hosios' condition results in an optimal trade-off between the externalities in the matching for physical capital. This case should be interpreted more of as an extreme case intended for illustrative purposes than anything else. The labor market is obviously subject to various frictions that have been analyzed extensively. I consider such a case in what follows: matching friction in frictionless labor markets.

3.2 Model with matching frictions in the labor market

To be added.

4 Quantitative analysis

The goal of the quantitative analysis is to assess how, in a plausibly calibrated economy, changes in the bargaining power affect the decentralized equilibrium relative to the efficient allocation. I consider three cases: (i) the model without frictions in the labor market that implies no holdup problems in the capital market; (ii) the model with predetermined labor (but no other frictions in the labor market); (iii) the model with matching frictions in both the capital and the labor market. For each case, I then calibrate a baseline economy with equal bargaining power for the different parties that is consistent with salient long-run facts of the U.S. data. I then assess the distance from the efficient allocation, and show how these inefficiencies change as I vary bargaining power.

4.1 Calibration

For cases (i) and (ii), I employ the following functional forms for preferences, technology and matching

$$\begin{aligned} u(c, n) &= \log c - \phi n \\ f(k, n) &= k^\alpha n^{1-\alpha} \\ m(L, V) &= \frac{LV}{(L^\chi + V^\chi)^{1/\chi}} \end{aligned}$$

For case (iii), preferences reduce to $u(c) = \log c$, technology remains the same, and the labor matching frictions takes on the same form than the capital matching friction. The specification of both preferences and technology are standard in the neoclassical growth literature (e.g. King and Rebelo, 2000). The linearity of labor in preferences in case (i) and (ii) is not important in principle but facilitates the computation of the steady states. It can be motivated from indivisible labor as in Rogerson (1988) or Hansen (1985). The matching function is taken from Den Haan, Ramey and Watson (2000). As I discuss in the beginning of Section 2, this form has a nice intuition in the present context and, more importantly, satisfies important regularity conditions. In particular, $m(L, V)$ is constant returns to scale, and implies that both $p(\theta)$ and $q(\theta)$ are bounded between 0 and 1.

For the calibration, the standard parameters are set as in the real business cycle literature (e.g. King and Rebelo, 2000). In particular, I calibrate $\beta = 0.99$ so as to match an average annual real yield on a riskless 3-month treasury bill of 4.95%. The labor supply parameter ϕ is set such that total hours worked equal $n = 0.2$ and the consumption output ratio equals $c/y = 0.7$ in the baseline model. Furthermore, the share of capital in the production function is set to $\alpha = 1/3$, and the rate of depreciation of capital is set to $\delta = 0.025$.

For the calibration of the capital market friction, I set $s = 0.01$, which implies a steady state ratio of capital additions relative to the total capital stock of $m(v, l)/k = [1 - (1 - \delta)(1 - s)] = 0.0348$. This implies a gross investment rate of 14% on an annual basis, which is close to what Eisfeldt and Rampini (2007) report based on Compustat data (see Kurmann and Petrosky-Nadeau, 2007 for more details). I also keep $\varphi = 1$ in a first instance (no loss due to specificity).¹² With these parameters at hand, I then set the bargaining power to $\eta = 0.5$ (as discussed above) and pick κ

¹²In Kurmann and Petrosky-Nadeau (2007), the loss due to specificity is set to $\varphi = 0.95$, which implies a ratio of

and χ so that $q(\theta) = 0.5$ and $\kappa v/y = 0.01$ in the decentralized economy. Both of these values are reasonable given the evidence presented in Kurmann and Petrosky-Nadeau (2007).

4.2 Results for model without matching frictions in the labor market

Figure 1 displays capital stocks and consumption for the decentralized economy without holdups in capital markets and without labor market frictions (case (i)). All values are given relative to the efficient allocation (i.e. if consumption is at its efficient level, the relative value equals 1).

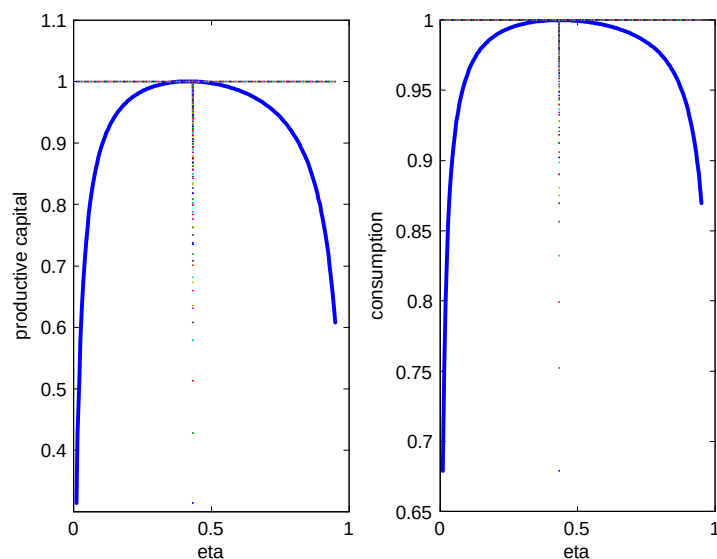


Figure 1: Capital stocks and consumption relative to the efficient allocation for the decentralized economy without holdups.

For the chosen parametrization, the elasticity of the matching function equals $\epsilon(\theta) = 0.43$, which is indicated on the graphs by the vertical line. Since in this case, there are no hold-up problems, the decentralized economy would achieve efficiency for $\eta = \epsilon(\theta)$ (Hosios' condition). For both $\eta < \epsilon(\theta)$ and $\eta > \epsilon(\theta)$, the decentralized economy implies insufficient capital accumulation, and consequently suboptimal consumption. The losses implied by departures from the Hosios condition are relatively small in the vicinity of $\eta = \epsilon(\theta)$. For example, at $\eta = 0.5$ (the baseline calibration), consumption is about 2% lower than at $\eta = \epsilon(\theta)$. However, for very small and very large values of η the losses become substantial, even in this case without holdups.

investment in new capital to total capital addition of about 25%. Leaving $\varphi = 1$ is thus reasonable, but I will explore alternative calibrations in the future.

Figure 2 displays results for case (ii); i.e. the decentralized economy with holdups in the capital market but still no matching frictions in the labor market. As discussed in Section 3.1, the holdup problem leads to an additional distortion in both the physical capital accumulation and the labor market. In particular, firms find it optimal to alleviate the holdup problem by overinvesting in physical capital and underemploying in labor so as to depress the marginal productivity of capital.

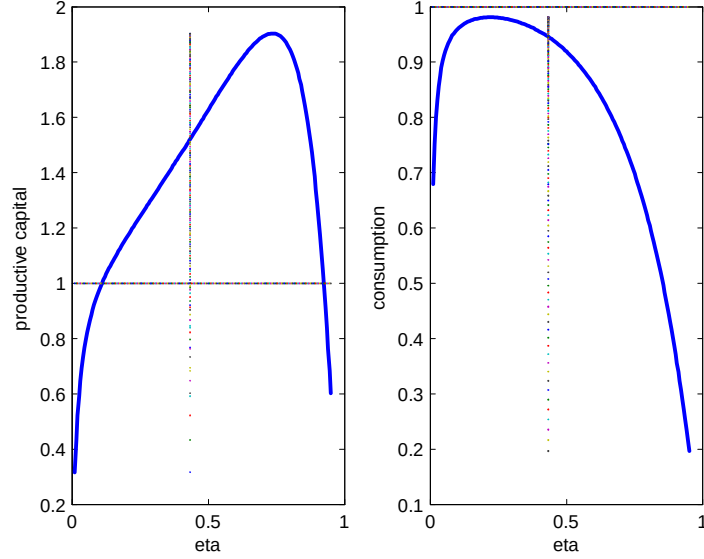


Figure 2: Capital stocks and consumption relative to the efficient allocation for the decentralized economy without holdups.

The holdup problem leads to substantial overinvestment relative to the efficient allocation for most values of η . Also, as stated in Proposition 2, the decentralized allocation is never efficient because there are now both an allocative distortion and a holdup problem that no value of η can resolve. The welfare losses are minimized for a relatively small bargaining power for the capital suppliers (around $\eta = 0.2$, close to where the productive capital stock is at its efficient level). At this point, the decentralized economy provides the best trade-off between the allocation friction and the holdup problem. As the firm's bargaining power increases, the distortions in the labor market and the capital market increase and welfare losses start mounting. For the baseline calibration $\eta = 0.5$, for example, consumption is almost 10% below its efficient level. As one moves further to the right, these losses rapidly become larger. One should not attach too much importance to these results for the moment, however, as the labor market is still frictionless.

5 Conclusion

To be added.

A Derivations for the basic model

A.1 Proof of Proposition 1

The social planner's problem is

$$\begin{aligned} O(k, u) &= \max_{l, \theta} [f(k) + s\varphi k + u - \kappa\theta l - l + \beta O(k_{+1}, u_{+1})] \\ s.t. \ k_{+1} &= (1 - s)k + q(\theta)l \\ u_{+1} &= (1 - q(\theta))l. \end{aligned}$$

The first-order conditions are

$$1 + \kappa\theta = \beta[q(\theta)O_k(k_{+1}, u_{+1}) + (1 - q(\theta))O_u(k_{+1}, u_{+1})] \quad (25)$$

$$\kappa = q'(\theta)\beta [O_k(k_{+1}, u_{+1}) - O_u(k_{+1}, u_{+1})]. \quad (26)$$

and the corresponding envelope conditions are

$$O_u(k, u) = 1 \quad (27)$$

$$O_k(k, u) = f'(k) + s\varphi + (1 - s)\beta O_k(k_{+1}, u_{+1}). \quad (28)$$

Equation (25) states that the marginal cost of an additional unit of liquid capital l_t must equal its marginal benefits. The marginal cost depends on both the resource cost (one unit of consumption less) and the negative externality that an additional unit of liquidity has on producing a match. The marginal benefit equals the marginal value of an additional unit of capital productive next period $O_k(k_{+1}, u_{+1})$ times the probability of matching, plus the marginal value of an unmatched unit times the probability of no match. Equation (26) states that the cost of increasing market tightness must equal the marginal net gain of augmenting the chances of a match.

Now, the steady state solution for (28) equals

$$\begin{aligned} O_k(k, u) &= f'(k) + s\varphi + (1 - s)\beta O_k(k, u) \\ &= \frac{f'(k) + s\varphi}{1 - (1 - s)\beta} \end{aligned}$$

Using this result in the first-order conditions (25) and (26), I obtain

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f'(k) + s\varphi}{1 - (1 - s)\beta} \right) + (1 - q(\theta)) \right] \quad (29)$$

$$\kappa = q'(\theta)\beta \left(\frac{f'(k) + s\varphi}{1 - (1 - s)\beta} - 1 \right). \quad (30)$$

These two expressions are equations (25) and (26) in the text.

To determine existence and uniqueness of the equilibrium, combine (29) and (30) to obtain, after some rearrangement

$$\theta^S = \frac{1-\beta}{\kappa} \frac{1-\epsilon(\theta)}{\epsilon(\theta)}, \quad (31)$$

where $1-\epsilon(\theta) = q'(\theta)\theta/q(\theta)$. Hence, as long as $\epsilon(\theta)$ is well behaved, there exists a unique equilibrium $\theta^S \in (0, \infty)$.¹³ Now, we take (29) and rewrite it as

$$\left[\frac{(1-\beta) + \kappa\theta^S}{\beta q(\theta^S)} + 1 \right] (1 - (1-s)\beta) = f'(k) + s\varphi. \quad (32)$$

The left-hand side is a positive constant. The right-hand side is a decreasing function in k that, given the Inada conditions, implies $\lim_{k \rightarrow 0} (f'(k) + s\varphi) = \infty$ and $\lim_{k \rightarrow \infty} (f'(k) + s\varphi) = s\varphi$. Hence, as long as $\left(\frac{\kappa}{\beta p(\theta^S)} + 1 \right) (1 - (1-s)\beta) > s\varphi$, there exists a unique equilibrium $k^S \in (0, \infty)$. This condition is not very stringent (**finish**). ■

A.2 Proof of Propositions 2 and 3

I first derive the equilibrium equations of the decentralized model with Nash bargaining. Then, I prove existence, uniqueness and (in)efficiency of the equilibrium.

Start with the capital supplier by combining conditions (5)-(7) with the free-entry condition. The result is

$$1 = q(\theta)\beta \left(\frac{\rho(k) + s\varphi}{1 - (1-s)\beta} \right) + (1 - q(\theta))\beta.$$

Likewise, on the firm side, combine (8) with (9) to obtain

$$\kappa = p(\theta)\beta \left(\frac{f'(k) - \rho'(k)k - \rho(k)}{1 - (1-s)\beta} \right).$$

Using the two equations to substitute out the rental rate $\rho(k)$, and the fact that $p(\theta)\theta = q(\theta)$ yields

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1-s)\beta} \right) + (1 - q(\theta)) \right]. \quad (33)$$

This is equation (14) in the text.

¹³To be refined. For the matching function we use in the numerical simulations, $\epsilon(\theta)$ is sufficiently well-behaved for there to be a unique equilibrium.

Next, define $S \equiv J_k + W^k - W^u$ as the surplus of a match. It equals

$$\begin{aligned} S &= \frac{f'(k) - \rho'(k)k - \rho(k)}{1 - (1-s)\beta} + \frac{\rho(k) + s\varphi}{1 - (1-s)\beta} - 1 \\ &= \frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1-s)\beta} - 1. \end{aligned}$$

Furthermore, from the Nash bargaining solution in (10), derive that $J_K = (1-\eta)S$ and $W^k - W^u = \eta S$. Hence, equation (8) can be rewritten as

$$\begin{aligned} \kappa &= p(\theta)\beta(1-\eta)S \\ &= p(\theta)(1-\eta)\beta \left[\frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1-s)\beta} - 1 \right], \end{aligned}$$

which, after applying $p(\theta) = q(\theta)/\theta = q(\theta)/\theta * q'(\theta)\theta/q(\theta) * q(\theta)/(q'(\theta)\theta) = q'(\theta)/(1 - \epsilon(\theta))$, yields

$$\kappa = \frac{q'(\theta)}{1 - \epsilon(\theta)}(1-\eta)\beta \left(\frac{f'(k) - \rho'(k)k + s\varphi}{1 - (1-s)\beta} - 1 \right). \quad (34)$$

This is equation (15) in the text.

Lastly, we use the Nash bargaining solution in (10) to derive an explicit expression for the rental rate. Using (8) and (6)-(7), we have

$$\eta \left(\frac{f'(k) - \rho'(k)k - \rho(k)}{1 - (1-s)\beta} \right) = (1-\eta) \left(\frac{\rho(k) + s\varphi}{1 - (1-s)\beta} - 1 \right)$$

which implies the following rental rate

$$\rho(k) = \eta [f'(k) - \rho'(k)k] - (1-\eta)[s\varphi - (1 - (1-s)\beta)].$$

This equation can be further modified by using the equilibrium solution, derived below, that $\theta = \frac{1-\beta}{\kappa} \frac{1-\eta}{\eta}$. Using this expression to substitute out β , we obtain after some rearrangement

$$\rho(k) = \eta [f'(k) - \rho'(k)k + (1-s)\kappa\theta] + (1-\eta)[(1-\varphi)s]. \quad (35)$$

This is equation (12) in the text.

The three equilibrium equations together imply a unique decentralized equilibrium (k^B, θ^B) . In fact, combining (33) with (34), one obtains after some rearrangement

$$\theta^B = \frac{1-\beta}{\kappa} \frac{1-\eta}{\eta}. \quad (36)$$

As long as $0 < \eta < 1$, we thus have a unique $\theta^B \in (0, \infty)$. To establish existence and uniqueness of k^B , in turn, rewrite (33) as

$$\left[\frac{(1 - \beta) + \kappa\theta}{\beta q(\theta)} + 1 \right] (1 - (1 - s)\beta) = f'(k) - \rho'(k)k + s\varphi. \quad (37)$$

The left-hand side is positive valued. To know more about the properties of the right-hand side, it is necessary to find an explicit solution for $\rho(k)$, which in turn permits establishing the properties of $\rho'(k)k$. Reconsider thus the rental rate equation in (35). This is a non-homogenous linear differential equation of ρ in k . We rewrite it as

$$\rho(k) - \eta f'(k) + \eta \rho'(k)k = \eta(1 - s)\kappa\theta - (1 - \eta)[(1 - \varphi)s].$$

With need to find a solution for the left-hand-side in terms of $\rho(K)$. Using standard tools from differential calculus, the solution for the homogenous part $\rho^h(k) = \eta f'(k) - \eta \rho'(k)k$ turns out to be

$$\rho^h(k) = k^{-\frac{1}{\eta}} \int_0^k f'(z) z^{\frac{1-\eta}{\eta}} dz + Dk^{-\frac{1}{\eta}},$$

where D is a constant from integration. For $\rho^h(k)$ being well-defined around $k = 0$, this constant needs to be $D = 0$. This expression implies that $\rho^h(k)$ is a decreasing function of k . For the sake of illustration, suppose for example that $f(k) = k^\alpha$ with $0 < \alpha \leq 1$. Hence,

$$\begin{aligned} \rho^h(k) &= \alpha k^{-\frac{1}{\eta}} \int_0^k z^{\alpha-1} z^{\frac{1-\eta}{\eta}} dz = \alpha k^{-\frac{1}{\eta}} \int_0^k z^{\frac{\eta(\alpha-1)+1-\eta}{\eta}} dz \\ &= \frac{\alpha\eta}{\eta(\alpha-1)+1} k^{-\frac{1}{\eta}} k^{\frac{\eta(\alpha-1)+1}{\eta}} \\ &= \frac{\alpha\eta}{1-\eta(1-\alpha)} k^{-(1-\alpha)} \end{aligned}$$

Hence, the rental rate equation in (35) becomes

$$\rho(k) = \eta \left[\frac{\alpha k^{\alpha-1}}{1-\eta(1-\alpha)} + (1-s)\kappa\theta \right] + (1-\eta)[(1-\varphi)s] \quad (38)$$

with $\frac{1}{1-\eta(1-\alpha)}$ denoting the overinvestment effect of the holdup problem. From this expression it is clear that $\rho(k) > 0$ and, as long as $\alpha < 1$, $\rho'(k) < 0$, $\rho'(k)k < 0$ and $\partial[\rho'(k)k]/\partial k > 0$ with $\lim_{k \rightarrow 0} \rho'(k)k = -\infty$ and $\lim_{k \rightarrow \infty} \rho'(k)k = 0$. With this, we can finally establish that the right-hand side of (37) is decreasing in k with $\lim_{k \rightarrow 0} (f'(k) - \rho'(k)k + s\varphi) = \infty$ and $\lim_{k \rightarrow \infty} (f'(k) - \rho'(k)k + s\varphi) = s\varphi$. Hence, as long as the left-hand side is larger than $s\varphi$, there exists a unique $\theta^B \in (0, \infty)$. This

last condition is not very stringent...***finish*** This proves the existence and uniqueness of a steady state equilibrium for the decentralized model with Nash bargaining.■

Now, for the efficiency, consider first $\eta = \epsilon(\theta)$. In that case, (36) coincides with (31) and thus $\theta^S = \theta^B$. By the same comparison, it is easy to see that $\eta < \epsilon$ implies $\theta^S < \theta^B$. Now, to establish the comparison of k^B with k^S , we need to compare (37) with its counterpart from the efficient allocation in (32). The left-hand side is decreasing in θ and thus smaller in the decentralized bargaining case. The right-hand side, in turn, is larger in the decentralized bargaining case for a given k . This implies that $k^B > k^S$. Finally, for the opposite case $\eta > \epsilon$, we have $\theta^S > \theta^B$. This means that for a given k , both the right-hand side and the left-hand side of (37) than its counterpart in the efficient allocation case (32). Hence, k^B can be either smaller or larger than k^S . This proves the second part of Proposition 2.■

A.3 Proof of Proposition 4

The firm's problem is

$$\begin{aligned} J(k) &= \max_{v, \rho} [f(k) - \rho k - \kappa v + \beta J(k_{+1})] \\ \text{s.t. } k_{+1} &= (1 - s)k + p(\theta)v \\ \text{s.t. } 1 &= q(\theta)\beta \left(\frac{\rho + s\varphi}{1 - (1 - s)\beta} - 1 \right) - \beta \end{aligned}$$

The first-order condition with respect to v and the associated envelope condition for k are identical to the ones in the Nash-bargaining case in (8) and (9). Together, they define the optimal entry condition

$$\kappa = p(\theta)\beta \left(\frac{f'(k) - \rho}{1 - (1 - s)\beta} \right) \quad (39)$$

where, as before, I abstract from time-subscripts. Once entered, the firm chooses the rental rate such as to maximize the value of entry in (39) subject to the constraint for market tightness. The first-order condition is

$$\begin{aligned} 0 &= p'(\theta) \frac{\partial \theta}{\partial \rho} \beta \left(\frac{f'(k) - \rho}{1 - (1 - s)\beta} \right) - p(\theta) \frac{\beta}{1 - (1 - s)\beta} \\ &= p'(\theta) \frac{\partial \theta}{\partial \rho} [f'(k) - \rho] - p(\theta) \end{aligned}$$

To find $\partial\theta/\partial\rho$, I compute the partial derivative of (16) with respect to ρ

$$0 = q'(\theta) \frac{\partial\theta}{\partial\rho} \beta \left(\frac{\rho + s\varphi}{1 - (1-s)\beta} - 1 \right) + q(\theta) \frac{\beta}{1 - (1-s)\beta}$$

Using the first of these equations to substitute out $\partial\theta/\partial\rho$, I obtain

$$\begin{aligned} 0 &= q'(\theta) p'(\theta) \frac{\partial\theta}{\partial\rho} [f'(k) - \rho] \beta \left(\frac{\rho + s\varphi}{1 - (1-s)\beta} - 1 \right) + q(\theta) p'(\theta) \frac{\beta [f'(k) - \rho]}{1 - (1-s)\beta} \\ &= q'(\theta) p(\theta) \beta \left(\frac{\rho + s\varphi}{1 - (1-s)\beta} - 1 \right) + q(\theta) p'(\theta) \frac{\beta [f'(k) - \rho]}{1 - (1-s)\beta} \\ &= (1 - \epsilon) \left(\frac{\rho + s\varphi}{1 - (1-s)\beta} - 1 \right) - \epsilon \left(\frac{f'(k) - \rho}{1 - (1-s)\beta} \right) \end{aligned}$$

Hence, we obtain an explicit solution for the rental rate

$$\rho = \epsilon f'(k) - (1 - \epsilon) [s\varphi - (1 - (1-s)\beta)]$$

Plugging this solution into (16) and (39), I obtain

$$\begin{aligned} 1 + \beta &= q(\theta) \beta \left(\frac{\epsilon f'(k) + \epsilon [s\varphi - (1 - (1-s)\beta)]}{1 - (1-s)\beta} \right) \\ &= q(\theta) \epsilon \beta \left(\frac{f'(k) + s\varphi}{1 - (1-s)\beta} - 1 \right), \end{aligned}$$

respectively,

$$\kappa = p(\theta) (1 - \epsilon) \beta \left(\frac{f'(k) + s\varphi}{1 - (1-s)\beta} - 1 \right)$$

Using similar steps than in the previous proof, it is easy to establish that this implies an efficient equilibrium (**finish**). ■

B Derivations for the general equilibrium model with labor

B.1 Efficient allocation

For the efficient allocation, the social planner solves the following problem

$$\begin{aligned} O(k, u) &= \max_{c, n, l, \theta} [u(c, n) + \beta O(k_{+1}, u_{+1})] \\ &\quad + \lambda [f(k, n) + \varphi(1 - \delta)sk + u - \kappa\theta l - c - l] \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + q(\theta)l \\ \text{s.t. } u_{+1} &= (1 - q(\theta))l \end{aligned}$$

As in the basic model, $\kappa\theta l = \kappa v$ denotes total resources expended for vacancy postings by the firm. The first-order conditions of the optimization problem are

$$u_c(c, n) = \lambda \quad (40)$$

$$-u_n(c, n) = \lambda f_n(n, k) \quad (41)$$

$$\beta[O_u(k_{+1}, u_{+1})(1 - q(\theta)) + O_k(k_{+1}, u_{+1})q(\theta)] = \lambda(1 + \kappa\theta) \quad (42)$$

$$q'(\theta)\beta[O_k(k_{+1}, u_{+1}) - O_u(k_{+1}, u_{+1})] = \lambda\kappa \quad (43)$$

and the envelope conditions are

$$O_u(k, u) = \lambda \quad (44)$$

$$O_k(k, u) = \lambda[f_k(k, n) + \varphi(1 - \delta)s] + (1 - \delta)(1 - s)\beta O_k(k_{+1}, u_{+1}). \quad (45)$$

In steady state, the combination of (42)-(45) yield two conditions that are very similar to the ones in the basic model

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f_k(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} \right) + (1 - q(\theta)) \right] \quad (46)$$

$$\kappa = q'(\theta)\beta \left[\frac{f_k(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1 \right] \quad (47)$$

Note in particular that the opportunity cost of capital λ drops out of these equations.

Given the similarity of these two equations, it is straightforward to show that the equilibrium value θ^S is determined by exactly the same expression than in the basic model

$$\theta^S = \frac{1 - \beta}{\kappa} \frac{1 - \epsilon(\theta)}{\epsilon(\theta)}.$$

Likewise, we obtain a very similar equation for the marginal productivity of capital

$$\left[\frac{(1 - \beta) + \kappa\theta^S}{\beta q(\theta^S)} + 1 \right] (1 - (1 - \delta)(1 - s)\beta) = f_k(k, n) + s(1 - \delta)\varphi.$$

This pins down the capital-labor ratio, because for any constant returns to scale production function $f(k, n)$, we know that $f_k(k, n)$ is homogenous of degree zero and thus $f_k(k, n) = f_k(\frac{k}{n}, 1)$. From here, we use the labor market equilibrium condition

$$-\frac{u_n(c, n)}{u_c(c, n)} = f_n(n, k),$$

which, together with the different constraints, determines n , c and the other equilibrium values.

To be more specific, consider the functional forms used in the different exercises

$$\begin{aligned} q(\theta) &= \frac{\theta}{(1 + \theta^\chi)^{\frac{1}{\chi}}} \\ f(k, n) &= k^\alpha n^{1-\alpha} \\ u(c, n) &= \log c - \phi n \end{aligned}$$

We thus have $1 - \epsilon(\theta) = \theta q'(\theta)/q(\theta) = 1 - \theta^l/(1 + \theta^l)$, implying

$$\theta^S = \left(\frac{1 - \beta}{\kappa} \right)^{\frac{1}{\chi+1}}$$

and

$$\frac{k}{n} = \left\{ \frac{\left(\frac{(1-\beta)+\kappa\theta^S}{\beta q(\theta^S)} + 1 \right) (1 - (1-\delta)(1-s)\beta) - s(1-\delta)\varphi}{\alpha} \right\}^{\frac{1}{\alpha-1}}.$$

Furthermore, we have

$$\phi c = (1 - \alpha) \left(\frac{k}{n} \right)^\alpha$$

which pins down consumption

$$c^S = \frac{(1 - \alpha)}{\phi} \left(\frac{k}{n} \right)^\alpha.$$

Next, we write the budget constraint as

$$1 = \frac{c}{y} + [1 + \kappa\theta] \frac{l}{y} - [\varphi(1 - \delta)s \frac{k}{y} + \frac{u}{y}]$$

The ratio l/y can be linked to k/y from the capital accumulation constraint

$$\begin{aligned} \frac{k}{y} [1 - (1 - \delta)(1 - s)] &= q(\theta) \frac{l}{y} \\ \frac{l}{y} &= \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \frac{k}{y}. \end{aligned}$$

Similarly, the ratio u/y can be linked to k/y from the evolution of unmatched capital

$$\begin{aligned} \frac{u}{y} &= (1 - q(\theta)) \frac{l}{y} \\ &= (1 - q(\theta)) \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \frac{k}{y}. \end{aligned}$$

Hence, we have

$$1 = \frac{c}{y} + [1 + \kappa\theta] \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \frac{k}{y} - \left[\varphi(1 - \delta)s + (1 - q(\theta)) \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \right] \frac{k}{y}$$

But, $k/y = (k/n)^{1-\alpha}$ and thus

$$\frac{c}{y} = 1 - [1 + \kappa\theta] \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \left(\frac{k}{n} \right)^{\alpha-1} + \left[\varphi(1 - \delta)s + (1 - q(\theta)) \left(\frac{1 - (1 - \delta)(1 - s)}{q(\theta)} \right) \right] \left(\frac{k}{n} \right)^{\alpha-1}$$

This in turn pins down the efficient equilibrium level of y and thus the level of k , n and all the other variables.

B.2 Decentralized equilibrium with Nash-bargaining

In the decentralized case, firms and households optimize independently over l and v , respectively, taking market tightness θ as given. Once matched, the rental rate is negotiated ex-post with Nash bargaining. Since firms hold many units of both capital and labor, they internalize the effect of k and n on the rental rate; i.e. $\rho = \rho(k, n)$. Households, by contrast, are supposed to be only a small contributor to each firm's capital stock. Hence, they consider ρ as exogenous when deciding on the supply of liquid capital and labor.

In the case where labor can be adjusted immediately in a Walrasian market, the firm's problem is described by

$$\begin{aligned} J(k) &= \max_{n,v} \{f(k, n) - \rho(k, n)k - wn - \kappa v + \beta J(k_{+1})\} \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + p(\theta)v. \end{aligned}$$

The first-order conditions of the optimization problem are

$$f_n(k, n) = w + \rho_n(k, n)k \tag{48}$$

$$\kappa = p(\theta)\beta J_k(k_{+1}). \tag{49}$$

and the envelope condition is

$$J_k(k) = f_k(k, n) - \rho_k(k, n)k - \rho(k, n) + (1 - \delta)(1 - s)\beta J_k(k_{+1}). \tag{50}$$

In steady state, we thus have

$$\kappa = p(\theta)\beta \left(\frac{f_k(k, n) - \rho_k(k, n)k - \rho(k, n)}{1 - (1 - \delta)(1 - s)\beta} \right), \quad (51)$$

an equation that is very similar than the one in the basic model.

The household's problem is

$$\begin{aligned} V(k, u) &= \max_{c, n, l} [u(c, n) + \beta V(k_{+1}, u_{+1})] \\ &\quad + \lambda [wn + \rho(k, n)k + \varphi(1 - \delta)sk + u + d - c - l] \\ \text{s.t. } k_{+1} &= (1 - \delta)(1 - s)k + q(\theta)l \\ \text{s.t. } u_{+1} &= (1 - q(\theta))l \end{aligned}$$

where d are firm profits transferred to households. The first-order conditions of the optimization problem are

$$u_c(c, n) = \lambda \quad (52)$$

$$-u_n(c, n) = \lambda w \quad (53)$$

$$\lambda = \beta [q(\theta)V_k(k_{+1}, u_{+1}) + (1 - q(\theta))V_u(k_{+1}, u_{+1})] \quad (54)$$

and the envelope conditions are

$$V_u(k, u) = \lambda \quad (55)$$

$$V_k(k, u) = \lambda [\rho(k, n) + \varphi(1 - \delta)s] + (1 - \delta)(1 - s)\beta V_k(k_{+1}, u_{+1}). \quad (56)$$

In steady state, (54) together with (55) and (56) yields

$$1 = \beta \left[q(\theta) \left(\frac{\rho(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} \right) + (1 - q(\theta)) \right], \quad (57)$$

very similar to the equivalent equation for the basic model. Unsurprisingly then, combining (51) with (57) to substitute out $\rho(k, n)$ yields an equilibrium condition that is very similar to the one for the basic model

$$1 + \kappa\theta = \beta \left[q(\theta) \left(\frac{f_k(k, n) - \rho_k(k, n)k + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} \right) + (1 - q(\theta)) \right]. \quad (58)$$

Next, define the surplus of a match as $S = J_k(k) + \frac{V_k(k, u) - V_u(k, u)}{\lambda}$. By contrast to the definition of the surplus in the basic model, the part pertaining to the household is divided by λ so as to

express it in terms of goods rather than in terms of utils. Continuing along the same lines than in the basic model, this surplus is defined as

$$\begin{aligned} S &= \frac{f_k(k, n) - \rho_k(k, n)k - \rho(k, n)}{1 - (1 - \delta)(1 - s)\beta} + \frac{\rho(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1 \\ &= \frac{f_k(k, n) - \rho_k(k, n)k + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1, \end{aligned}$$

the firm's entry condition becomes

$$\kappa = \frac{q'(\theta)}{1 - \epsilon(\theta)}(1 - \eta)\beta \left(\frac{f_k(k, n) - \rho_k(k, n)k + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1 \right). \quad (59)$$

Combining this equation with (58), I obtain the decentralized equilibrium solution for θ that is familiar from the basic model

$$\theta^B = \frac{1 - \beta}{\kappa} \frac{1 - \eta}{\eta}. \quad (60)$$

As long as $0 < \eta < 1$, we thus have a unique $\theta^B \in (0, \infty)$.

Finally, the Nash bargaining solution in (10) implies

$$\eta \left(\frac{f_k(k, n) - \rho_k(k, n)k - \rho(k, n)}{1 - (1 - \delta)(1 - s)\beta} \right) = (1 - \eta) \left(\frac{\rho(k, n) + \varphi(1 - \delta)s}{1 - (1 - \delta)(1 - s)\beta} - 1 \right)$$

resulting in the following rental rate

$$\rho(k, n) = \eta [f_k(k, n) - \rho_k(k, n)k] - (1 - \eta)[\varphi(1 - \delta)s - (1 - (1 - \delta)(1 - s)\beta)].$$

This equation can be further modified by using the equilibrium solution, derived below, that $\theta = \frac{1 - \beta}{\kappa} \frac{1 - \eta}{\eta}$. Substituting for β with this solution, I obtain after some rearrangement

$$\rho(k, n) = \eta [f_k(k, n) - \rho_k(k, n)k + (1 - \delta)(1 - s)\kappa\theta] + (1 - \eta)[\delta + (1 - \varphi)(1 - \delta)s]. \quad (61)$$

Here is where potential differences to the basic model come in because the rental rate is not only a function of k but also of n . In particular, the homogenous part of the differential equation for $\rho(k, n)$ is

$$\rho^h(k, n) = k^{-\frac{1}{\eta}} \int_0^k f_z(z, n) z^{\frac{1 - \eta}{\eta}} dz$$

The solution for the integral on the right-hand side depends on assumptions about the marginal productivity of capital. Consider first the case where the firm can choose labor freely in each period.

In that case, the constant-returns-to-scale assumption for production implies that $f_k(k, n)$ is a direct function of k/n and that, by virtue of the labor demand, k/n is a direct function of w . Hence, the firm takes k/n and thus $f_k(k, n)$ as exogenous. The solution for $\rho^h(k, n)$ thus simplifies to

$$\begin{aligned}\rho^h(k, n) &= k^{-\frac{1}{\eta}} f_k(k, n) \int_0^k z^{\frac{1-\eta}{\eta}} dz \\ &= f_k(k, n) K^{-\frac{1}{\eta}} \frac{K^{\frac{1}{\eta}}}{1/\eta} \\ &= \eta f_k(k, n),\end{aligned}$$

and the expression for the rental rate in (61) reduces to

$$\rho(k, n) = \eta [f_k(k, n) + (1 - \delta)(1 - s)\kappa\theta] + (1 - \eta)[\delta + (1 - \varphi)(1 - \delta)s].$$

Hence, $\rho_k(k, n) = 0 = \rho_n(k, n)$ and thus, the overinvestment motive due to the holdup problem falls away.

Now, consider the case where labor is predetermined. In that case, the firm cannot freely adjust labor so as to adjust the capital-labor ratio immediately. Hence, the marginal return to capital depends directly on k and one cannot take $f_k(k, n)$ out of the integral. To consider a concrete solution in that case, assume as before that $f(k, n) = k^\alpha n^{1-\alpha}$. We thus have

$$\begin{aligned}\rho^h(k, n) &= \alpha n^{1-\alpha} k^{-\frac{1}{\eta}} \int_0^k z^{\alpha-1} z^{\frac{1-\eta}{\eta}} dz = \alpha n^{1-\alpha} k^{-\frac{1}{\eta}} \int_0^k z^{\frac{\eta(\alpha-1)+1-\eta}{\eta}} dz \\ &= \alpha n^{1-\alpha} \frac{\eta}{\eta(\alpha-1)+1} k^{-\frac{1}{\eta}} k^{\frac{\eta(\alpha-1)+1}{\eta}} \\ &= \eta \frac{\alpha k^{\alpha-1} n^{1-\alpha}}{1 - \eta(1 - \alpha)}\end{aligned}$$

The rental rate thus becomes

$$\rho(k, n) = \eta \left[\frac{\alpha k^{\alpha-1} n^{1-\alpha}}{1 - \eta(1 - \alpha)} + (1 - s)\kappa\theta \right] + (1 - \eta)[(1 - \varphi)s] \quad (62)$$

and the demand for labor becomes

$$w = \frac{(1 - \eta)}{1 - \eta(1 - \alpha)} (1 - \alpha) k^\alpha n^{-\alpha}$$

Since $\frac{(1-\eta)}{1-\eta(1-\alpha)} < 1$, this means that firms underemploy labor relative to the competitive case so as to depress the marginal productivity of capital and thus the rental rate.

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