Retrading in Competitive Equilibria with Adverse Selection

(preliminary draft)

Pamela Labadie

Department of Economics

315 Monroe

George Washington University

Washington, D.C. 20052 Phone: 202- 994-0356

Fax: 301-263-2756

Email: labadie@gwu.edu

Abstract

Competitive equilibria in adverse selection economies with private information are generally studied in models with a market structure that has risk neutral firms, market segmentation and contract exclusivity. An alternative market structure closer to the standard Arrow-Debreu framework is studied here in which risk averse agents directly trade contingent claims and markets are not segmented by type. The result is the competitive equilibrium with anonymity, which has the properties that the exclusivity of contracts cannot be enforced, agents can enter into side markets, and markets are not segmented. The allocation results in a set of endogenous transfers and subsidies. The allocations are individually incentive compatible and, because all opportunities from retrading are exhausted in a certain sense, the allocation is also multilaterally incentive compatible.

Competitive exchange economies with private information and adverse selection have been studied extensively. Prescott and Townsend [1984] determine the extent to which standard general equilibrium methods can be applied to adverse selection insurance economies. The standard market structure has risk neutral firms offering exclusive insurance contracts that are enforceable. The economy is Walrasian in that both households and firms are price takers. Exclusivity means that households cannot enter into multiple contractual agreements with insurance firms and cannot enter into subsequent risk sharing arrangements among themselves. The restriction of the market structure and the exclusivity limit the types of risk sharing arrangements that the agents can achieve in the economy. The question arises whether alternative market structures can be reasonably considered and, if so, what are the features of these structures and the associated allocations. The incentive compatible allocation studied here emerges in a market structure where exclusivity cannot be enforced, agents are allowed to enter into side markets, and risk sharing arrangements are made through a clearing house, which announces prices and balances net trades at those prices.

This is a pure exchange economy in which agents have private information about the distribution of their stochastic endowment. If an agent's type were public information, then agents of the same type could form risk sharing arrangements and achieve full consumption insurance. These risk sharing arrangements can be achieved through a standard Arrow-Debreu contingent claims market or equivalently through an insurance industry with risk-neutral firms offering a contract which is a set of contingent claims that are actuarially fair in price. The contingent claims contracts offered by risk neutral firms will have fair prices in equilibrium to eliminate arbitrage profit opportunities and the price of a claim will vary with an agent's type. When an agent's type is private information, the simple equivalence between allocations achieved through the Arrow-Debreu framework and the insurance markets with exclusive contracts is much more tenuous. The approach taken by Rothschild and Stiglitz [1976], Prescott and Townsend [1984] and Bisin and Gottardi [2006], is to preserve the market structure with risk neutral insurance firms offering exclusive contracts. Markets are segmented by type and the incentive compatibility constraints, which must hold jointly across all markets, creates a consumption externality. An alternative approach is to assume that agents trade contingent claims directly with each other through a clearing house that acts like the traditional Walrasian auctioneer. Agents are assumed to be anonymous or self-selective, which means that the prices an agent faces on contingent claims contracts do not vary type, and markets are not segmented. The net trades that an agent executes are observable and all agents face the same opportunity set in net trades. Since agents differ in terms of endowment risk, they will execute different net trades. Hence anonymity doesn't imply that the clearing house is unable to identify the net trades made by a particular agent but does imply that all agents face the identical budget constraint in net trades. Anonymous allocations are studied by Schmeidler and Vind [1972] and defined in Mas-Colell, Whinston, and Green [1995].

The competitive equilibrium with anonymity has the properties that allocations are individually incentive compatible. The price of a contingent claims contract conditional on an endowment realization is generally not actuarially fair for an agent. Hence, an agent will over-insure in some states and under-insure in others. Agents will choose net trades such that the weighted marginal rate of substitution is equalized across states, where the weight is the probability of being in that state, conditional on an agent's type. This creates a set of endogenous transfers and subsidies across states and agents. The allocation is constrained Pareto efficient and is equivalent to a central planning problem with distortionary taxes, subject to the incentive feasibility constraints.

Section 1 introduces the basic model while, in section 2, the standard framework is discussed and is based on Prescott and Townsend [1984] and Bisin and Gottardi [2006]. The anonymous allocation is introduced in section 3 and the constrained Pareto optimality is discussed in section 4. In section 5, the anonymous allocation is shown to satisfy the multi-lateral incentive compatibility constraint, specifically the requirement that side markets be in equilibrium and no coalition of agents can form a coalition to block the allocation. Concluding comments are in section 6.

1 Basic Model

The basic model is the adverse selection insurance economy of Rothschild and Stiglitz [1976], Prescott and Townsend [1984], and Bisin and Gottardi [2006].

This is a pure endowment economy with a single perishable good. There is a countable infinity of agents indexed by $i \in I = \{1, 2, 3, \ldots\}$, with φ denoting the finitely additive cumulative distribution function over I. At the beginning of time, each agent draws an η, which is private information; for reasons provided below the realization of this random variable is called the agent's type. Let $\eta \in \mathcal{N} \equiv \{\eta_1, \ldots, \eta_N\}$ be a discrete random variable and let $f(\eta)$ denote the probability over $\mathcal{N}$, where N is finite. After observing η, an agent observes the realization of his endowment.

Let $\theta \in \Theta$ denote the random endowment, where $\Theta \equiv \{\theta_1, \ldots, \theta_M\}$ such that $\theta_1 \geq 0$, θ_M is finite, and M is finite. The distribution of θ is parameterized by η so that, for a particular $\eta \in \mathcal{N}$, the distribution of θ is $g(\cdot \mid \eta)$. For example, if $\eta_2 > \eta_1$, $g(\theta \mid \eta_2)$ might be a mean-preserving spread of $g(\theta \mid \eta_1)$. The distribution over the endowment sample space Θ will differ across agents, with a fraction $f(\eta)$ facing distribution $g(\cdot \mid \eta)$. Since the realization of η determines the endowment distribution, a type η agent and type $\hat{\eta}$ agent with $\eta \neq \hat{\eta}$ are said to face different endowment distribution risk. The following assumption is standard.

Assumption 1 *For each $\eta \in \mathcal{N}$ and $\theta \in \Theta$, $g(\theta \mid \eta) > 0$.*

For any type, the probability of being in any endowment state is strictly positive.

The realization of the endowment over the population is a pair $\{\theta_i, \eta_i\}$, $i \in I$. Since there is a countable infinity of agents, the Law of Large Numbers can be used to show that the frequency of observing the pair (θ, η), $\theta \in \Theta$ and $\eta \in N$, is $g(\theta \mid \eta) f(\eta)$. Total endowment is

$$y = \sum_I \theta_i \varphi(di) = \sum_{\eta \in \mathcal{N}} \sum_{\theta \in \Theta} f(\eta) g(\theta \mid \eta) \theta, \tag{1}$$

so there is no aggregate uncertainty. The distribution of θ for the economy is a mixture

$$p(\theta) \equiv \sum_{\eta \in \mathcal{N}} f(\eta) g(\theta \mid \eta). \tag{2}$$

Denote $\theta_m(\eta)$ as the mean of the endowment for type η,

$$\theta_m(\eta) = \sum_\Theta g(\theta \mid \eta) \theta.$$

The endowment realization θ is public information and is observable by all agents.

A type-η agent has preferences

$$\sum_{\theta \in \Theta} g(\theta \mid \eta) U(c(\theta)). \tag{3}$$

The function U is continuous and twice continuously differentiable, strictly increasing and strictly concave. Also, as $c \to 0$, $U'(c) \to \infty$ (Inada condition). The expectation in (3) uses an individual agent's conditional probability of realizing θ. Since there is no aggregate uncertainty, the only risk an agent faces after realizing η is his endowment risk. The ability to diversify away that risk depends on market opportunities.

The commodity space is $\Re_+^M$. A consumption allocation for a type η agent is an M-dimensional vector $c(\eta) \equiv (c(\theta_1 \mid \eta), \ldots, c(\theta_M \mid \eta))$. A *feasible allocation* is an N-tuple $(c(1), \ldots, c(N))$ such that $c(j) \in \Re_+^M$ for $j \in \mathcal{N}$ where

$$\sum_{\eta \in N} f(\eta) \sum_{\theta \in \Theta} g(\theta \mid \eta) \theta \geq \sum_{\eta \in N} f(\eta) \sum_{\theta \in \Theta} g(\theta \mid \eta) c(\theta \mid \eta). \tag{4}$$

The set of all feasible allocations is convex and closed.

It is convenient to describe allocations in terms of net trades. Define $x : \Theta \to \Re$ by

$$x(\theta \mid \eta) = c(\theta \mid \eta) - \theta,$$

and let $x(\eta) \in \Re^M$ denote the vector of net trades for a type η agent. The set of net trades is restricted so that consumption is nonnegative, or

$$0 \leq x(\theta \mid \eta) + \theta. \tag{5}$$

Let X denote the set of net trades satisfying the nonnegativity constraint. A net trade for the economy $\hat{x}$ is an N-tuple of M dimensional vectors $\hat{x} = \hat{x}(1), \ldots, \hat{x}(N)$ such that $x(\eta) \in X$ for all $\eta \in \mathcal{N}$. A net trade for the economy $\hat{x}$ is **feasible** if $x(\eta) \in X$ for each $\eta \in \mathcal{N}$ and

$$0 \geq \sum_{\eta \in \mathcal{N}} f(\eta) \sum_{\theta \in \Theta} g(\theta \mid \eta) x(\theta \mid \eta). \tag{6}$$

Define F as the set of feasible net trades for the economy.

2 Incentive Compatibility

There is an extensive literature on optimal contracts when η is private information. In general, the optimal contract has the property of truthful revelation when individual incentive compatibility constraints are imposed along with the resource constraints. An *incentive compatible* net trade $x \in X$ satisfies the individual incentive compatibility constraint (IICC)

$$\sum_{\Theta} g(\theta \mid \eta) U(\theta + x(\theta \mid \eta)) \geq \sum_{\Theta} g(\theta \mid \eta) U(\theta + x(\theta \mid \hat{\eta})), \tag{7}$$

for $\hat{\eta}, \eta \in \mathcal{N}$ such that $\hat{\eta} \neq \eta$. When (7) is satisfied, a type η agent has no incentive to misrepresent his type. Let $\mathcal{I}$ denote the set of incentive compatible net trades and define $\mathcal{F} = F \bigcap \mathcal{I}$ as the set of feasible, incentive compatible net trades. As Prescott and Townsend have noted, the space of allocations that are feasible and incentive compatible is generally not convex.

A net trade $x \in \mathcal{F}$ is *incentive efficient* if there does not exist another feasible and incentive net trade $\tilde{x} \in \mathcal{F}$ such that

$$\sum_{\theta} g(\theta \mid \eta) U(\theta + \tilde{x}(\theta \mid \eta)) \geq \sum_{\theta} g(\theta \mid \eta) U(\theta + x(\theta \mid \eta)),$$

for all $\eta \in \mathcal{N}$, with strict inequality for at least one η.

There are many incentive feasible allocations. I will discuss two types that are most commonly studied in the literature and then introduce a third type in section (3). For convenience, I focus on the standard two agent, two state case. Assume that $\theta_2 > \theta_1$ and $g(\theta_2 \mid 2) > g(\theta_2 \mid 1)$ so that state 2 is the "good" state and type 2 agents are the "low" risk types.

An important set of allocations has the property that the high risk agents are fully insured, so their consumption lies along the 45 degree line, or certainty line, in the θ_1, θ_2 plane. Within this set, only one incentive compatibility constraint is binding, namely

$$\sum_{i=1,2} g(\theta_i \mid 1)U(\theta_i + x(\theta_i \mid 1)) = \sum_{i=1,2} g(\theta_i \mid 1)U(\theta_i + x(\theta_i \mid 2)), \tag{8}$$

so the high risk agent has no incentive to misrepresent his type. Allocations in this set are described by the net transfers from the low risk to the high risk types. If (8) holds with equality and if there are no transfers, then the consumption of the high risk is the expected value of his endowment $c(1) = \theta_m(1) = g(\theta_1 \mid 1)\theta_1 + g(\theta_2 \mid 1)\theta_2$. Low risk agents are only partially insured so that their consumption allocation satisfies

$$\sum_{i=1,2} g(\theta_i \mid 2)U(\theta_i + x(\theta_i \mid 2)) = U(\theta_m(1))$$

and

$$\theta_m(2) = g(\theta_1 \mid 2)[\theta_1 + x(\theta_1 \mid 2)] + g(\theta_2 \mid 2)[\theta_2 + x(\theta_2 \mid 2)],$$

which is a system of two equations in two unknowns $x(\theta_i \mid 2)$. This is the *separating allocation* of Rothschild and Stiglitz. This allocation is not incentive efficient because there are welfare improving transfers, which are described next.

Transfers

In the set of allocations with one binding incentive compatibility constraint, the allocations can be characterized by the transfers from low risk to high risk agents. Starting with the no-transfer allocation (Rothschild-Stiglitz), a positive transfer from the low risk to the high risk agent will generally relax the binding incentive compatibility constraint (8). The transfer shrinks the budget set of the low risk agents and expands the budget set of the high risk agents. As the consumption of a high risk agent moves out along the certainty line, increasing his utility, a low risk agent is better able to insure against endowment shocks, even with lower post-transfer expected endowment. The indifference curve of the high risk agent, who is fully insured, intersects the new budget constraint for the low risk agent at a point that is closer to the 45-degree line, so that the low risk agent has more consumption insurance. As the full insurance level of consumption for the high risk agent moves toward y, the low risk agent moves to higher indifference curve and is better off. These transfers are Pareto improving.

The maximum transfer reached when each agent has an endowment equal to y. At the maximum transfer, each type of agent has full insurance in consumption and the incentive compatibility constraints do not bind for either agent. This is the *pooling* allocation and it is Pareto optimal. The pooling allocation can be achieved only if transfers are implemented.

To summarize, two sets of feasible and incentive compatible allocations have been described: full insurance for some types with transfers and the pooling allocation. These two sets are not exhaustive and there are other examples of feasible and incentive compatible allocations. Before discussing a third set, it is useful to examine the competitive equilibria supporting the incentive compatible allocations.

Competitive Equilibria with Fair Pricing

Decentralization of adverse selection economies is problematic because the set of feasible and incentive compatible allocations is not convex. For this reason, Prescott and Townsend introduce contracts with random components as a method to ensure convexity of the space of allocations. This modification of the contract space allows them to use standard, general equilibrium analysis to study adverse selection economies. There are other approaches, such as Bisin and Gottardi, in which the set of admissible trades is restricted to lie in the space of incentive compatible net trades I. For simplicity, the discussion below follows Bisin and Gottardi in that the set of admissible trades is restricted instead of expanding the contract space to include lotteries.

While several concepts of competitive equilibria in adverse selection economies have been studied, a common feature in the literature is to assume that there are competitive risk neutral firms selling the insurance. The commodity traded is a contract, which is a set of contingent claims, and households and firms are assumed to be price takers. These insurance contracts are assumed to be exclusive: a household can purchase only one contract from a single firm. Firms are restricted to offer contracts that are incentive compatible and agents are restricted to trade only incentive compatible contracts.

Upon entering the market, an agent must announce his type, which determines the prices and the set of net trades he can execute. Although the agent is a price-taker, the net trades he chooses must be consistent with the incentive compatibility constraint, so he is restricted in both prices and quantities. The restriction on the set of admissible net trades can be thought of as a consumption externality created by the requirement that the incentive compatibility constraints must jointly hold across all markets for all types.

Risk neutral competitive firms maximize profits by offering exclusive contracts. A key feature of this market structure is that prices are "fair," so that the price paid by a type η agent, contingent on θ is $q(\theta \mid \eta)$ where

$$q(\theta \mid \eta) = g(\theta \mid \eta).$$

If a type η agent purchases a portfolio of contingent claims $x(\eta)$, $x(\eta) \in X$, the expected profit on the contract is

$$\sum_{\theta} q(\theta \mid \eta)x(\theta \mid \eta) - \sum_{\theta} g(\theta \mid \eta)x(\theta \mid \eta),$$

where the first term is the net receipts on the contract and the second term is the expected value of the net payments. Under fair pricing, the expected profit on any contract sold to an agent is zero. If prices were not fair, then firms could make arbitrarily large profits through arbitrage by selling the commodity with $q(\theta \mid \eta) > g(\theta \mid \eta)$ and buying the commodity $q(\hat{\theta} \mid \eta) < g(\hat{\theta} \mid \eta)$; see Lemma 1 of Bisin and Gottardi for a discussion. An example of an allocation achieved with fair pricing and exclusivity in contracts is the Rothschild-Stiglitz allocation: the high risk agent has full insurance and there is no cross subsidization or net transfers from the low risk to high risk agents.

Another key feature of this market structure is that the pooling allocation cannot be achieved. The pooling allocation requires the maximum net transfer from the low risk to the high risk agents. Under fair pricing without transfers, the pooling allocation is above the budget set of the high risk agents.

The fair-pricing competitive equilibrium without transfers is not incentive efficient. As already discussed, transfers from the low risk to the high risk agents will weaken the one binding incentive compatibility constraint. Consider now any incentive compatible and feasible allocation $\theta + \hat{x}(\theta \mid \eta)$ within the class of allocations with a single binding incentive compatibility constraint. For the two state-two agent case, Bisin and Gottardi show that there are transfers $\tau(\theta_1), \tau(\theta_2)$ supporting this allocation. Although these transfers are welfare improving, it is difficult to implement these transfers under the standard market structure with risk neutral firms and exclusive contracts because the market fails to take the consumption externality into account. Bisin and Gottardi introduce a market in consumption rights as a way to eliminate the consumption externality and allow the welfare improving transfers to take place. In the next section, I introduce a different market mechanism.

3 Anonymous Allocations

A key feature of the standard model just described is the existence of risk neutral insurance firms that separate markets and impose fair prices as a no-arbitrage condition. This market structure - risk neutral firms selling contingent claims contracts at fair prices - is exogenously imposed and is an arbitrary restriction on the organization of contingent claims markets. The question arises whether that market structure would arise endogenously. In this section, I will conjecture an alternate market structure that corresponds more closely to the Walrasian competitive equilibrium, in that households are price takers and can pick any set of contingent claims satisfying their budget constraint. A discussion of the issue of whether such a market structure would arise endogenously is postponed until section (5).

An alternative market structure is to assume that the risk averse households trade contingent claims contracts directly through an organized exchange or clearing house. Net trades are executed through a clearing house that announces prices and nets supply and demand for contracts as a function of the observable endowment. Markets are *non-exclusive* - agents do not announce their type to enter. All agents, regardless of type, execute state contingent net trades facing identical prices. This is an alternative restriction on the organization of contingent claims markets and, at this point, I am substituting one exogenously imposed market structure for another.

The price of a claim to one unit of the endowment conditional on realizing θ is a function $q : \Theta \to \Re_+$. The clearing house announces a price vector and then an agent submits his state contingent net trades such his budget constraint is satisfied. The market-clearing conditions are discussed later. In such an environment, agents face the identical budget constraint for net trades, specifically of the form

$$0 \geq \sum_{\theta \in \Theta} q(\theta) x(\theta \mid \eta). \tag{9}$$

Notice that the price is not a function of the agent's type and markets are not separated by type. For a type η agent, the price will, in general, not be fair so that utility maximization will result in incomplete insurance. The allocation resulting from this arrangement is an *anonymous* or *self selective* allocation. A formal definition of an anonymous allocation is discussed next.

Define B_x as the support of the distribution induced by a net trade $x \in X$. Specifically, the support is defined for any $q \in \Re_+^M$ as

$$B_x(q) = \inf\{\sum_{\theta \in \Theta} q(\theta) x(\theta) : x \in X\}.$$

For each $x \in X$, the function $B_x(\cdot)$ is a linear function of q.[1] The following definition of an anonymous allocation is based on Schmeidler and Vind [1972] and Mas-Colell, Whinston and Green [1995].

Definition 1 *A feasible allocation $c^\star = (c(1), \ldots, c(L))$ is anonymous (or self-selective or envy-free) in net trades if there is a set of net trades $B(q) \in \Re^M$, called the generalized budget set, such that, for every agent-type $\eta \in \mathcal{N}$, $x(\eta)$ maximizes*

$$\sum_{\theta \in \Theta} g(\theta \mid \eta) U(x(\theta \mid \eta) + \theta) \tag{10}$$

subject to (9) and the feasibility constraint (6).

Notice that an individual agent is restricted to net trades satisfying the nonnegativity constraint $x \in X$ and, in this sense, is a price taker because any net trade satisfying the budget constraint (9) is admissible.

[1]When the set X is convex, the support function B provides a dual description of X since X can be reconstructed from $B(q)$. In particular, $\{x \in \Re^M : q \cdot x \geq B(q) \text{for every } q\}$. The function B is homogeneous of degree one and is concave.

Anonymity or self -selection does not mean that the clearing house is unable to associate a set of net trades with a particular agent $i \in I$. For any $i \in I$, the budget constraint (9) must hold. Symmetry requires that all agents of the same type behave in the same way, so that the maximization problem is described in terms of a representative type η agent. The net trades of a specific agent are fully observable. Anonymity means that an agent doesn't need to declare his type to enter into a transaction and, whether or not he declares his type, his market opportunities in net trades are identical to those of any other agent.

Incentive Compatibility

The set of admissible net trades for an agent is restricted so that $x \in X$ and $q \in \Re^M_+$ is chosen so that the feasibility constraint is satisfied, but the opportunity set of agents is not restricted to incentive compatible net trades $\mathcal{I}$. The question arises whether the anonymous allocation is incentive compatible, specifically whether the feasible net trade x that is anonymous is an element of $\mathcal{F}$, the space of feasible incentive compatible net trades. The answer is that it is by construction. Since all net trades $x(\eta)$ for $\eta \in \mathcal{N}$ lie along the same hyperplane, or equivalently all agents face the same budget set, the net trades chosen by other agents $x^\star(\hat{\eta})$ where $\hat{\eta} \neq \eta$ are within the budget constraint faced by a type η agent. In solving his maximization problem, the type η optimally chooses from the same net trade opportunity set as any other agent. Hence, the solution to the maximization problem will satisfy (7). This introduces a third set of feasible incentive compatible allocations, namely allocations such that the net trades have a common support function B, which in this case means that agents face the same budget constraint in net trades. In general, the incentive compatibility constraints (7) may not be binding. The binding constraint is that net trades have a common support function.

Competitive Equilibria

The next step is to define the competitive equilibrium under the assumption that agents trade directly through a clearing house. The clearing house announces a price function $q : \Theta \rightarrow \Re^M_+$. Agents act as price takers and, given a price function q, maximize their objective function subject to the budget constraint (9). This results in a set of net trades as a function of the price. Prices are revised until the net trades satisfy the market clearing conditions.

A type η agent maximizes his objective function (3)) subject to the constraint (9). Admissible net trades are $x \in \Re^M$. Let $\mu(\eta)$ denote the Lagrange multiplier for the budget constraint. The first-order condition is

$$g(\theta \mid \eta) U'(\theta + x(\theta \mid \eta)) = \mu(\eta) q(\theta). \tag{11}$$

An agent will optimally choose net trades such that $x(\eta) \in X$. This follows because, even though prices are not fair, agents will not pick a net trade such that $\theta + x(\theta \mid \eta) \leq 0$ under the Inada condition. Under assumption (1), an agent has a positive probability of being in each endowment state, so that he has no incentive to short sale any state.[2] This is a critical difference between the pricing behavior with risk neutral firms and direct trade among risk averse agents. Hence prices are not required to be fair to prevent unlimited arbitrage opportunities when agents trade directly through a clearing house instead of through risk-neutral firms.

Solve (11) for the multiplier and use this equation to show, for every $\theta, \hat{\theta} \in \Theta$, each agent η selects net trades such that

$$\frac{g(\theta \mid \eta) U'(\theta + x(\theta \mid \eta))}{g(\hat{\theta} \mid \eta) U'(\hat{\theta} + x(\hat{\theta} \mid \eta))} = \frac{q(\theta)}{q(\hat{\theta})}, \tag{12}$$

subject to the budget constraint. Hence the weighted marginal rate of substitution is equalized

[2]Notice that if $g(\hat{\theta} \mid \hat{\eta}) = 0$ for some $\hat{\eta}$ and $\hat{\theta}$, that the agent, as a price taker could generate unlimited income by letting $x(\hat{\theta} \mid \hat{\eta}) \rightarrow -\infty$. Under the assumption that the probability is strictly positive and the Inada condition, agents will never optimally choose this.

across agents, where the weight is the condition probability g. For any $\theta \in \Theta$ the net trades for agents $\eta, \hat{\eta} \in \mathcal{N}$ satisfy

$$\frac{g(\theta \mid \eta)U'(\theta + x(\theta \mid \eta))}{g(\theta \mid \hat{\eta})U'(\theta + x(\theta \mid \hat{\eta}))} = \frac{\mu(\eta)}{\mu(\hat{\eta})}, \tag{13}$$

It is useful to think of the solution to the maximization problem in terms of offer curves. The offer curve for a type η can be derived as follows: start at the endowment point $\{\Theta\}$ and vary $q : \Theta \to \Re^M_+$. The net trades are adjusted so that (12) is satisfied and the offer curve traces out the points of tangency of the indifference curve and budget set as a function of the price. The level of expected utility along an offer curve is at least as large as the autarky level of expected utility $\sum_\Theta g(\theta \mid \eta)U(\theta)$. The offer curve for a type η agent intersects the certainty line at the point $\theta_m(\eta) = \sum_\theta g(\theta \mid \eta)\theta$, which occurs only when $q(\theta) = g(\theta \mid \eta)$. In an equilibrium where agents face the same price vector for net trades, it implies that the point on an agent's offer curve corresponds to a common slope determined by the price vector. Agents of different types have different offer curves, because of heterogeneity in endowment risk, but in an equilibrium, the net trades on the offer curve correspond to an indifference curve with an identical slope across types.

For the two agent-two state case, the budget constraint can be rewritten as

$$\frac{q(\theta_1)}{q(\theta_2)} = -\frac{x(\theta_2 \mid \eta)}{x(\theta_1 \mid \eta)}, \tag{14}$$

for $\eta = 1, 2$. An offer curve is a pair $x(\theta_1 \mid \eta), x(\theta_2 \mid \eta)$ satisfying

$$\frac{g(\theta_2 \mid \eta)U'(\theta_2 + x(\theta_2 \mid \eta))}{g(\theta_1 \mid \eta)U'(\theta_1 + x(\theta_1 \mid \eta))} = -\frac{x(\theta_1 \mid \eta)}{x(\theta_2 \mid \eta)}, \tag{15}$$

for $\eta = 1, 2$. The offer curve traces out the point of tangency between the indifference curve and the budget set when the slope is determined by the price function q. Since agents face the same relative prices, the net trades must satisfy

$$\frac{x(\theta_1 \mid 1)}{x(\theta_2 \mid 1)} = \frac{x(\theta_1 \mid 2)}{x(\theta_2 \mid 2)}. \tag{16}$$

The offer curve equations (15), the relative price equation (16) and the resource constraint form a system of four equations in the four unknowns $x(\theta_i \mid j)$, $i = 1, 2$ and $j = 1, 2$. This system can be solved as follows: choose a "numeraire" agent - say agent 1 - and fix his net trades at $x_{1,1}, x_{2,1}$. Using the offer curve (15) and relative price condition (16), the net trades of the type two agent can be determined as functions of the net trades of a type 1 agent. The resource constraint and the offer curve for a type 1 agent can then be used to determine the equilibrium net trades of a type 1 agent. An equilibrium allocation for this example is a set of net trades $x(\theta_i, \eta)$, $i = 1, 2$ and $\eta = 1, 2$, such that the net trades are on the offer curve (15) for each η, satisfy (16).

It is clear in the two state–two agent case that the equilibrium allocation results in transfers across the different types of agents because the insurance is not fair in the sense that, in general $q(\theta) \neq g(\theta \mid \eta)$ for $\eta \in \mathcal{N}$. For a type η who executes net trades $x(\theta \mid \eta)$, the price of the net trade is $q(\theta)$ while the expected benefit of the trade is $g(\theta \mid \eta)$. Define

$$\hat{\tau}(\theta, \eta) = q(\theta) - g(\theta \mid \eta),$$

which is the difference between the marginal cost of a state contingent net trade and the marginal benefit to a type η agent. The expected transfer for a type η agent is

$$\tau(\eta) \equiv \hat{\tau}(\theta_1, \eta) \mid \eta)x(\theta_1 \mid \eta) + \hat{\tau}(\theta_2, \eta)x(\theta_2 \mid \eta). \tag{17}$$

In general, agents shift endowment from the high state θ_2 to the low state θ_1 so that $x(\theta_1 \mid \eta) > 0$ and $x(\theta_2 \mid \eta) < 0$. For an agent with $g(\theta_1 \mid \eta) > q(\theta_1)$, the marginal cost of the contingent claim $q(\theta_1)$ is less than the expected marginal benefit $g(\theta_1 \mid \eta)$ so that the first term on the right side is negative (a subsidy). Notice that if we multiply both sides by $f(\eta)$ and then sum over all η, that

$$\sum_{\eta} \tau(\eta) = 0,$$

as expected. The transfer can also be rewritten as

$$\tau(\eta) = \theta_m(\eta) - [g(\theta_1 \mid \eta)c(\theta_1 \mid \eta) + g(\theta_2 \mid \eta)c(\theta_2 \mid \eta)]. \tag{18}$$

As a group, the low risk agents transfer $\tau(2) > 0$ to the high risk agents. The transfers occur endogenously in the market place when fair pricing is not imposed and agents trade directly with one another through the clearing house. Notice that the different types of agents generally do not have the same expected utility in equilibrium, which is a condition for incentive compatibility when there are risk neutral firms and separation of markets. Although the slope of the indifference curve at the consumption point is identical across agents, it is not a condition of equilibrium that the indifference curves of agents intersect.

For the general case $M > 2$, the market clearing conditions can be derived as follows. Multiply the budget constraint for a type η agent by $f(\eta)$ and then sum over all η to obtain

$$\sum_{\eta}\sum_{\theta} f(\eta)q(\theta)x(\theta \mid \eta) = 0.$$

Equate this equation to the feasibility constraint and rewrite to obtain

$$\begin{aligned} 0 &= \sum_{\eta}\sum_{\theta} f(\eta)[g(\theta \mid \eta) - q(\theta)]x(\theta \mid \eta) \\ &= \sum_{\eta}\sum_{\theta} f(\eta)\hat{\tau}(\theta \mid \eta)x(\theta \mid \eta) \end{aligned}$$

For each market θ, market clearing requires

$$\sum_{\eta} f(\eta)\hat{\tau}(\theta, \eta)x(\theta \mid \eta) = 0, \tag{19}$$

For each $\theta \in \Theta$ the clearing house will set the price $q(\theta)$ such that the sum of the transfers for that state is equal to zero.[3]

The clearing house acts as follows: After announcing a price function, agents submit their set of state contingent claims. If (19) is satisfied for each $\theta \in \Theta$, then the net trades are executed. If not, then the price function is changed. No trades are executed until (19) is satisfied for each $\theta \in \Theta$.

[3]This can also be interpreted as the net supply of claims equals the net demand for claims in that endowment state.

Definition 2 *A competitive equilibrium with anonymity is an N-tuple $x(\eta) \in X$ and a price function $q : \Theta \to \Re_{+}^{M}$ such that (i) $x(\eta)$ solves (3) subject to (9), given q; (ii) The net trade is feasible and incentive compatible $x \in \mathcal{F}$. (iii) Markets clear: (19) holds for each $\theta \in \Theta$.*

To solve for the equilibrium, choose a numeraire consumer - say $\eta = 1$ - and define

$$\bar{\mu}(\eta) \equiv \frac{\mu(\eta)}{\mu(1)}. \tag{20}$$

Solve (11) for the price $q(\theta)$ and observe that

$$q(\theta) = \frac{g(\theta \mid \eta) U_1(x(\theta \mid \eta) + \theta)}{\mu(\eta)} = \frac{g(\theta \mid 1) U_1(x(\theta \mid 1) + \theta)}{\mu(1)}. \tag{21}$$

Define

$$\bar{g}(\theta \mid \eta) = \frac{g(\theta \mid \eta)}{g(\theta \mid 1)}.$$

Then rewrite (21) as

$$\bar{\mu}(\eta) = \frac{\bar{g}(\theta \mid \eta) U_1(x(\theta \mid \eta) + \theta)}{U_1(x(\theta \mid 1) + \theta)}. \tag{22}$$

For a given value of μ and net trade for the numeraire agent, define the function H_η as

$$H_\eta(\mu, \theta, x) = (U_1)^{-1} \left(\frac{\mu}{\bar{g}(\theta \mid \eta)} U_1(\theta + x) \right).$$

It is straightforward to show that H_η is decreasing in μ and increasing in x. Use the function H_η to solve for $x(\theta \mid \eta)$,

$$x(\theta \mid \eta) = H_\eta(\bar{\mu}, \theta, x(\theta \mid 1)) - \theta. \tag{23}$$

Given the net trades of the numeraire agent $x(1)$, the prices are

$$q(\theta) = \frac{g(\theta \mid 1) U_1(x(\theta \mid 1) + \theta)}{\mu(1)}.$$

Substitute these functions into the budget constraint

$$0 = \sum_{\theta \in \Theta} \left[g(\theta \mid 1) U_1(x(\theta \mid 1) + \theta) H_\eta(\bar{\mu}, \theta, x(\theta \mid 1)) \right], \tag{24}$$

where $\mu(1) = 1$, without loss of generality. This is an equation in the unknown $\bar{\mu}$. The right side is strictly decreasing in $\bar{\mu}$, given $x(1)$. Let $\mu^\star(\eta)$ denote the solution. Define the operator $T_\eta : X \to \Re_+$ as

$$T_\eta(x(1)) = \mu^\star(\eta)$$

Hence, given the vector $x(1) \in X$, the operator T_η yields the value of the multiplier that solves the budget constraint for a type η agent.

The final step is to solve for $x(1)$ using the equilibrium conditions. For each $\theta \in \Theta$, the equilibrium condition is (19) which is rewritten as

$$0 = \sum_\eta f(\eta)[q(\theta) - g(\theta \mid \eta)]x(\theta \mid \eta) \tag{25}$$

Substituting for $q(\theta)$, and using the functions $H^\star_\eta$,

$$0 = \sum_\eta f(\eta)g(\theta \mid 1)\,[U_1(x(\theta \mid 1) + \theta) - \bar{g}(\theta \mid \eta)]\,[\theta - H_\eta(T_\eta(x(1)), \theta, x(\theta \mid 1))]. \tag{26}$$

This establishes a mapping $\mathcal{T} : X \to X$. The fixed point of T is $x^\star(1)$ and let $q^\star$ denote the equilibrium price function.

4 Constrained Pareto Optimality

It is useful to discuss Pareto optimal allocations when η as well as θ are observable to the central planner. If the central planner allocates resources before agents know the realization of their type, then all agents are treated identically and consume y. The competitive equilibrium supporting this allocation has a price function $\hat{q} : \Theta \times \mathcal{N} \to \Re_+$ where $\hat{q}(\theta, \eta) = f(\eta)g(\theta \mid \eta)$. No transfers are required. If η is known before the central planner makes the allocations, then there are two cases: pooled resources and separate resources. The central planner maximizes

$$\left\{ \sum_{\eta \in N} \phi(\eta) \left[\sum_{\Theta} g(\theta \mid \eta) U(\theta + x(\theta \mid \eta)) \right] \right\}, \tag{27}$$

where $\phi(\eta)$ denotes the Pareto weight attached to a type η agent, subject to the resource constraints (6). The solution has the property that marginal utility varies inversely with $\frac{\phi(\eta)}{f(\eta)}$ and consumption is invariant with respect to θ. If $\phi(\eta) = f(\eta)$, as is often assumed, then $U(x(\theta \mid \eta) + \theta) = U(y)$ and the allocations are identical for the two versions of the problem. The central planner will optimally choose to ignore the idiosyncratic income distribution risk for each type, equalizing consumption across types; this is the pooled resource allocation. The associated competitive equilibrium supporting this allocation requires lump-sum transfers, however. Any agent with θ receives a transfer $\tau^\star(\theta) = \theta - y$ and, as a group, type η agents receive

$$\tau(\eta) = f(\eta)[\theta_m(\eta) - y].$$

Define the net trade

$$x(\theta \mid \eta) = c(\theta \mid \eta) - [\theta - \tau^\star(\theta)]$$

where the mandatory state contingent transfers are incorporated. Let $q : \Theta \times \mathcal{N} \to \Re_+^M$ denote the contingent claims price function. A type η agent faces a budget constraint

$$0 \leq \sum_{\Theta} q(\theta \mid \eta)[\theta + x(\theta \mid \eta) - \tau^\star(\theta)], . \tag{28}$$

The agent maximizes his objective function (3) subject to the budget constraint (28). The simplified first-order conditions imply that prices are fair.

If the central planner treats the N types of agents as separate groups, keeping resources separate, then the central planner maximizes the expected utility of the representative type η agent (3) subject to

$$\theta_m(\eta) \geq \sum_{\Theta} g(\theta \mid \eta)c. \tag{29}$$

The solution is $c = \theta_m(\eta)$. A type η with endowment realization θ will receive a transfer equal to

$$\tau_n(\theta \mid \eta) = \theta_m(\eta) - \theta,$$

so that agents with identical realization θ receive different transfers depending on type and no resources are transferred across groups ($\sum_{\Theta} \tau_n(\theta \mid \eta) = 0$). The contingent claims market supporting this allocation is straightforward. The competitive equilibrium can be implemented without transfers under the assumption that the resources are separated by type. Clearly some agents are better off in the pooled-resources allocation, while others are worse off ($U^{\star} > \bar{U}(\eta)$ for some η), so the two versions of the problem cannot be Pareto ranked.

In both examples where η is known when the resources are allocated, the competitive equilibrium supporting the allocation required prices to vary by type $q(\eta) \in \Re_{+}^{M}$. Define the expenditure function $E : \Re^{M} \times \Re_{+} \rightarrow \Re_{+}$. The maximization of the objective function (3) subject to the budget constraint (28), is dual to an expenditure minimization problem

$$E(q(\eta), \bar{U}^{\star}) \equiv \min[\sum_{\theta} q(\theta \mid \eta) x(\theta \mid \eta)] \tag{30}$$

subject to

$$\sum_{\theta} g(\theta \mid \eta) U(x(\theta \mid \eta) + \theta - \tau^{\star}(\theta)) \geq \bar{U}, \tag{31}$$

where $\bar{U} \equiv U(y)$ and $q(\eta) = \{g(\theta_1 \mid \eta), \ldots, g(\theta_M \mid \eta)\}$. All agents achieve the same level of lifetime utility because agents are offered a net trade choice set with prices indexed by type and there is a lump-sum redistribution of wealth. Denote $B_x(\cdot \mid \eta)$ as the net trade choice set associated with the expenditure minimization problem, where

$$B_x(q(\eta) \mid \eta) = \{\sum_{\theta \in \Theta} q(\theta \mid \eta) x(\theta) : x \in X\}.$$

It is useful to summarize the four key features of the central planning problems just described:

1. The pooled resource allocation, in which agents know their type before trading, requires lump-sum transfers for the Second Welfare Theorem to apply. The central planner must be able to identify the type of an agent as well as observe his endowment realization.

2. The net trade choice set must be different for the N groups of agents to support the allocations chosen by the central planner for both the pooled resource and separate resource economies.

3. There must be exclusivity in the choice sets: a type η agent cannot choose from the net trade set $B_x(q(\hat{\eta}))$ where $\eta \neq \hat{\eta}$. Otherwise, there are arbitrage profit opportunities.

4. The utility maximization problem is dual to an expenditure minimization problem that requires a price for a unit of consumption in state θ to differ across agent types: $q(\theta \mid i) \neq q(\theta \mid j)$ for $i \neq j$ in general. Hence the fraction $m(\theta)$ of agents realization endowment θ pay different prices for consumption insurance in state θ.

If we assume that the central planner is unable to redistribute resources across agents, then the Second Welfare Theorem will generally fail to hold. If exclusivity of markets cannot be maintained, then the central planner is unable to offer different net trade sets by type and the allocations of either the pooled or separate resource economies cannot be supported by a competitive equilibrium, assuming type is known before trading starts. Exclusivity in markets may be difficult to maintain since the dual space to the utility maximization problem differs across agents. Specifically, a type η agent may find that the expenditure to implement his net trade , $x(\eta)$ is minimized for some $q(\hat{\eta})$ such that $\hat{\eta} \neq \eta$.

The anonymous allocation is based on the assumption that exclusivity in markets cannot be maintained. If agents can enter multiple markets to exploit arbitrage opportunities, then prices can not differ across types for each $\theta \in \Theta$. The utility maximization problem solved by a type η agent must be dual to the expenditure function $E(q, \cdot)$, where q is identical for all agents. Hence agents can achieve different levels of expected utility (although all agents of the same type must be identical) by choosing different bundles of net trades, subject to feasibility.

4.1 Distortionary Transfers

The anonymous competitive equilibrium results in an allocation that is feasible, incentive compatible and eliminates arbitrage opportunities when market exclusivity cannot be maintained. The inability to enforce the separation of markets is shown to create an endogenous set of transfers and subsidies. In this section, we examine how the lack of market separation is equivalent to a set of distortionary taxes. The assumption is that the central planner cannot directly allocate resources or make lump-sum transfers. Transfers can be implemented only through distortionary taxes.

Define $t(\theta, \eta)$ as a tax (subsidy) on net trades by a type η agent conditional on endowment θ. The central planner announces a tax function $t : \Theta \times \mathcal{N} \rightarrow \Re$. A type η agent announces his type which determines his tax on net trades. The expected after tax return on a net trade is $g(\theta \mid \eta)[1 - t(\theta, \eta)]$. If

$$t(\theta \mid \eta) \equiv 1 - \frac{q^{\star}(\theta)}{g(\theta \mid \eta)},$$

then the after-tax budget constraint for net trades is

$$\begin{aligned} 0 &= \sum_{\theta} g(\theta \mid \eta)[1 - \tau(\theta \mid \eta)]x(\theta \mid \eta) \\ &= \sum_{\theta} q^{\star}(\theta)x(\theta \mid \eta). \end{aligned}$$

Hence this problem can be solved as a central planning problem with distortionary taxes subject to the constraint

$$q(\theta) = g(\theta \mid \eta)[1 - t(\theta, \eta)]$$

for each $\theta \in \Theta$ and $\eta \in \mathcal{N}$. This constraint is imposed so that agents self-select into the tax schedule corresponding to their true type. This problem can be solved using the methods of Diamond and Mirrlees [1971].

5 Retrading and Side Markets

The anonymous allocation has two basic features: (i) Agents trade directly through a clearing with an auction process instead of through risk neutral firms, and (ii) Markets are not exclusive so that prices are not fair. The organization of markets supporting the anonymous allocation can be motivated in several ways.

The two main incentive compatible allocations initially described - either a separating allocation with a single binding constraint or the pooling allocation - are based on the following assumption: The central planner can ensure that agents consume their allocation. The central planner pools the resources and prevents retrading and coalition formation among agents. If we assume that the central planner, who cannot identify agents by type, also cannot prevent retrading in side markets or prevent coalition formation among agents, then the standard individual incentive compatibility constraint (IICC) in (7) may be insufficient to induce truth telling. If an agent knows that there are retrading opportunities through side markets or coalition formation, the existence of these opportunities may create an incentive for an agent may misrepresent his type to the central planner, even if the allocations offered satisfy the IICC constraints.

For example, in the standard Rothschild and Stiglitz insurance economy, the standard solution satisfying the IICC constraint results in full insurance for the high risk agent and incomplete insurance for the low risk agent. If the announcement of type to the central planner is public information, then low risk agents have an incentive to form a coalition after observing their state-contingent allocation but before observing the realization of the individual endowment state. By forming this coalition, the low risk can agree on a full risk-sharing arrangement that makes every low risk type better off in expected utility. Hence the low risk coalition can block the initial IICC allocation. But a high risk agent who knows that the low risk agents will form a risk-sharing arrangement subsequent to announcing his type will now be better off telling the central planner that he too is a low risk type. Then he can enter into the coalition and be made better off than revealing his true type and accepting the full insurance level of consumption offered to high risk agents.

The idea that subsequent retrading opportunities and potential coalition formation can change the information revealed by an agent to the central planner is discussed by Krasa [1999]. He studies private information exchange economies with allocations that cannot be improved, in that the agent would not wish to deviate by revealing further information or by retrading of goods. In his model, the revelation of private information and retrading opportunities are closely linked. A related concept, which is more appropriate for the economy studied here because it focuses on coalition formation, is multilateral incentive compatibility (MIC) developed by Hammond [1987].

Hammond argues that the appropriate constraint for an allocation is that it satisfy MIC, so that side markets are in equilibrium and there is no coalition that can block the existing allocation. There is a broader point here, which is that the assumption of IICC and restriction of subsequent coalition formation is equivalent to imposing a market structure on the economy instead of allowing the market structure to emerge endogenously. Boyd, Prescott and Smith [1988] model economic organizations as coalitions of agents instead of exogenously imposing an organizational structure and, by doing so, show that the pattern of trade and resulting allocations will be affected. The argument in this section, is this: the assumption of anonymity and the resulting allocation is coalitionally fair and satisfies MIC. Hence the market structure that is imposed under the anonymity assumption - namely that market exclusivity can't be maintained and that contingent claims prices will not differ by type - is in fact one structure that can emerge endogenously when retrading and coalition formation are allowed.

In the discussion below, only nonatomic coalitions are discussed because a finite coalition will face uncertainty in the total resources available for distribution within the coalition. Let

$\bar{\psi} = (\psi_1, \ldots \psi_N) \in \Re^L_+$ denote the measure of each announced type in a coalition. Agents form a coalition with the goal of pooling risk and exploiting gains from trade. The economy with nonpooled resources, in which market exclusivity can be enforced, can be viewed as N separate coalitions $\{\{f(1), 0, \ldots, 0\}, \ldots, \{0, \ldots, 0, f(L)\}\}$, where all members within a coalition are identical. An arrangement or a sharing rule A is mapping that specifies an allocation for a coalition as a function of its membership.

Let $x^\star(h)$, $h \in N$ denote the net transfers offered by the central planner after the agent announces his type. Agents can then enter into other risk-sharing arrangements, in the form of coalitions $\bar{\psi}$. Let $t(h) \in \Re^{M+1}$ denote the net trade in the coalition of an agent who announces he is a type h. The allocation of the agent after retrading is

$$g(\theta, x, t) = x^\star(\theta \mid h) + t(\theta \mid h).$$

An allocation $x^\star(\eta)$, $\eta \in N$ is MIC if no coalition of privately informed agents "can manipulate it by combining deception with hidden trades of exchangeable goods." [Hammond 1987 p. 399]. The following definition is based on Hammond (p. 402).

Definition 3 *An allocation $x^\star$ such that $x^\star(\eta) \in \Re^{M+1}_+$ for all $\eta \in N$ is MIC if there is no coalition $\bar{\Psi}$ that can make an exchange $t(\bar{\eta})$, $\bar{\eta} \in N$ that balance*

$$\sum_{h \in N} \psi(h) t(\theta, h) \leq 0,$$

such that every member of $\bar{\Psi}$ is better off.

As an application of [6, Theorem 3.1] in Hammond, the anonymous allocation is MIC. The allocation is also coalitionally fair. An allocation is said to be coalitionally fair if "no coalition could benefit from achieving the net trade of some other coalition" [8,p. 661]. To see this, observe that each agent faces the same generalized budget set and the same set of relative prices.

6 Conclusion

An exchange economy with adverse selection and private information is studied under the assumptions that risk averse agents trade directly through a clearing house. Markets are not segmented, contracts are not exclusive, and agents can enter into side arrangements. The result is the competitive equilibrium with anonymity. Anonymity means that all agents face the same budget set in net trades. The allocation is individual incentive compatible and multilateral incentive compatible. Since agents face the same budget constraint in net trades, but face different endowment distribution risk, agents will execute different net trades depending on type with the result that some states are under-insured while others are over-insured. The equilibrium has the property that the weighted marginal rate of substitution is equalized across states, where the weight is the probability of being in the state conditional on type.

The properties of the anonymous allocation can be intuitively explained using offer curves. For a given price vector at which a net trade can be executed, each agent determines the set of net trades that maximizes his expected utility. Starting at the autarky allocation, the offer curve traces out the points of tangency between the indifference curve and his budget set. The offer curve will cross the 45-degree line, or full insurance consumption, when the price vector equals the probability that an agent is in that state, conditional on his type. An anonymous equilibrium has the property that, for each type of agent, the equilibrium net trades correspond to a point on the offer curve will the same slope.

The results suggest that the message of Boyd, Prescott and Smith is important: exogenously imposed market structures result in allocations that are very different from those that emerge when organizations are formed as endogenous coalitions of agents.

References

[1] R. Aiyagari, D. Peled, Dominant Root Characterization of Pareto Optimality and the Existence of Optimal Equilibria in Stochastic Overlapping Generations Models, J. Econ. Theory 54(1991), 69–83.

[2] A.Bisin, P. Gottardi, Efficient Competitive Equilibria with Adverse Selection, J. Political Economy 114, no. 3(2006), 485–515.

[3] J. Boyd, E. Prescott, B. Smith, Organizations in Economic Analysis, Canadian J. Economics 21, no.3(1998), 477-91.

[4] P. Diamond, J. Mirrlees, Optimal Taxation and Public Production I: Production Efficiency, Amer. Econ. Review 61, no. 1 (1971), 8–27.

[5] P. J. Hammond, Straightforward Individual Incentive Compatibility in Large Economies, Review Econ. Stud. 46, no. (1979), 263–282.

[6] P. J. Hammond, Markets as Constraints: Multilateral Incentive Compatibility in a Continuum Economy, Review Econ. Stud. 54(1987), 399-412.

[7] J. Haubrich, Optimal Financial Structure in Exchange Economies, International Econ. Review 29, no.2(1988), 217–35.

[8] J. Jaskold-Gabszewicz, Coalitional Fairness of Allocations in Pure Exchange Economies, Econometrica 43., no.4 (1975), 661–68.

[9] S. Krasa, Unimprovable Allocations in Economies with Incomplete Information, Economic Theory 87, no. 1 (1999), 144-168.

[10] R.E. Lucas Jr., On Efficiency and Distribution, Economic Journal 102, no. 411 (1992), 233-47.

[11] A. Mas-Colell, Whinston and Green, Microeconomic Theory, Oxford University Press (1995).

[12] E. Prescott, R. Townscend, Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard, Econometrica 52, no. 1 (1984), 21-46.

[13] K. Roberts, The Theoretical Limits to Redistribution, Review Econ. Stud. 51, no. 2 (1984),177-195.

[14] M. Rothschild, J. Stiglitz, Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, Quarterly J. Economics 90 no. 4 (1976), 629–49.

[15] D. Schmeidler, K. Vind, Fair Net Trades, Econometrica 40, no. 4 (1972), 637–42.

[16] R. Wilson, The Theory of Syndicates, Econometrica 36, no.1 (1968), 119-32.