

# Assortative Matching in the Brazilian Labor Market\*

Rafael Lopes de Melo<sup>†</sup>

February 15, 2008

## 1 Introduction

The study of sorting is a classical question in economics and in particular in labor economics. The concept of sorting applies to any situation where two heterogeneous agents pair up for some reason. A classical example is the study of Becker [1] that studies the formation of pairs in the marriage market—in marriages, the heterogeneity refers to beauty, financial situation, etc. In our study, we study assortative matching between firms and workers in the labor market. The premise is that in one side there are workers with different qualifications and in the other there are firms with different levels of productivity. The study of sorting consisting in finding patterns on the ways these agents pair up. In particular, it is relevant to check whether the most skilled workers pair up with the most productive firms and vice-versa—i.e, if there is positive assortative matching.

This is an important question for two reasons. First, the type of sorting between firms and workers has strong implications for efficiency in the economy. In the seminal study of Becker [1] he shows that if there are complementarities in production then the equilibrium of the economy exhibits perfect assortative matching. i.e, the most skilled workers work for the most productive firm and the least skilled for the least productive.<sup>1</sup> This allocation, also, it the one that maximizes output in this economy. In this case, if workers and firms are not properly aligned then the economy

---

\*This paper is part of my dissertation. I am indebted to Giuseppe Moscarini for the extensive advice and support. I would like to thank Fabian Lange, Fabien Postel-Vinay, Björn Brügemann, and Theodore Papageorgiou for their valuable comments and help. Very useful feedback was also received from participants at the macroeconomics and labor/public finance seminars at Yale University, and at a seminar at Ipea-Brasilia. Confidential data collected by the Brazilian labor Ministry were used in this paper. I thank Ipea-Brasilia for granting my access to the data and for all the support given while I was doing my research there. I alone am responsible for any shortcomings in the paper.

<sup>†</sup>Department of Economics, Yale University (e-mail: rafael.melo@yale.edu).

<sup>1</sup>For this sentence we relabel pairs in a marriage as workers and firms.

might be producing less than optimal. The second reason that makes this question important is earnings inequality and mobility. Several empirical studies show that the most productive firms pay more "comparable" workers than the low productive firms. In this case, sorting can work as a mechanism of increasing inequality and mobility: the most skilled workers earn more both because they are more skilled and also because they work for the most productive firms. The opportunities for ascension in such an economy for the low skill workers are very limited because the high productivity firms don't hire them.

In our study we address the question of sorting both empirically and from a theoretical standpoint. The objective is to construct a model of the labor market that is consistent with key facts about sorting—some of them novel—, and that allows us to interpret these facts and to make inference. Our study has three main contributions.

First, we use a Brazilian matched employer-employee dataset to collect a series of moments that are relevant to the question of sorting in the labor market. This type of dataset is very useful, if not essential, to this type of question because it is necessary to obtain some measure of the worker and the firm's heterogeneity, and this is hard to obtain without following both workers and firms. There are three aspects of the data that are relevant to this question. First, we need moments to identify the distributions of heterogeneity in the economy. For that, we use both information on wage distributions across workers and also on the cross-firm wage distribution. Second, and perhaps most importantly, we need aspects of the data that refer to how the workers are allocated to firms. We begin by using the fixed effects in a wage regression as a proxy for the heterogeneities. The correlation between the fixed effects of worker and firms and the worker and his coworkers are two measures of sorting. We proceed by studying directly the allocative patterns of worker to firms. In particular, we compare the old and the new firm for workers who reallocate. Third, we need moments that give information about how efficiently the labor market allocates worker to firms. We have in mind an economy with market imperfections, and in particular search frictions. It is common practice then to use turnover data to identify this aspect of the labor market.

Second, we build a model of the labor market with two-sided heterogeneity and search frictions that has ingredients to explain the facts presented. The model is built on the framework of Shimer and Smith [2]: an economy with bilateral matches between workers and firms, complementarities in production and search frictions. In this model, when workers or firms are idle they wait for a partner, and upon appearance they decide whether they want to match or not. If there is enough complementarities in production then the equilibrium of this economy exhibits positive assortative matching, though not perfect because of the search frictions—the matching set in this economy is characterized by upward sloping matching bands. We extend the model to include on-the-job search, an important feature of labor market, that allows us to talk about endogenous separations.

We illustrate that the model has the required ingredients to explain the proposed facts, especially the ones related to sorting.

Third, we estimate the model using the proposed moments. Since the model doesn't have a closed form solution we use the simulated method of moments in order to estimate the model. Finally, with the estimated model in hand we can make counterfactual experiments to see how relevant are the consequences of sorting for inequality and efficiency. (This step is still incomplete)

The paper is divided as follows. Section 2 describes the theoretical model. Section 3 describes the data. Section 4 presents the facts. Section 5 proposes details the estimation technique.

## 2 The Model

The model shares the basic features with the model in Shimer and Smith [2]: this is a continuum time economy with heterogeneous agents, complementarities in production and search frictions. Production in this economy happens in bilateral matches. Agents meet other agents according to a stochastic process—i.e matching is not instantaneous because of search frictions—, they observe each other's types, they individually decide if they want to pair up or not, and, upon matching, they produce output and split the proceeds according to a pre-designed rule. Here, these heterogeneous agents are interpreted as firms and workers, whose distribution of types is allowed to differ. Similarly as in their model, matches are dissolved exogenously at a given rate, but we introduce a main modification to the model: on-the-job search. This has been acknowledged as an important feature of labor markets and allows us to talk about endogenous separations.

One main feature in this model is that the total stock of agents is assumed to be exogenous. In practice, we are assuming that there is a fixed stock of workers of heterogeneous skills and a fixed stock of jobs of different productivity. The equilibrium of the model prescribes the endogenous process through which these workers meet these jobs and how they match. Also, the search frictions imply that there will be unemployment and unfilled jobs in this economy. Thus, it is endogenously determined which of these workers/jobs are productive or vacant at a given point in time. Whereas the assumption of a fixed stock of workers is standard, the assumption of a fixed stock of jobs is not. In fact, other authors model vacancy creation by assuming free entry at each firm type.<sup>2</sup> We depart from this tradition in order to introduce a mechanism in the model which allows us to explain a very robust fact present in the data: the correlation between worker and firm's fixed effects in a log-linear wage regression is zero. As we show below, by

---

<sup>2</sup>For example, Gautier and Teulings [3, 4], Lentz [5].

departing from the zero-profits assumption and allowing different firms to have different vacancy values we introduce a downward distortion in the correlation between fixed effects, which allows our model to match the facts.<sup>3</sup> The implicit story we have here is that when a very productive firm— say google— opens a vacancy the expected value of that individual vacancy is higher than the expected value of a vacancy posted by an average firm, which goes against the assumption of zero-profits at each vacancy type. In the last subsection we outline a way to endogeneize vacancy creation in this environment without zero-profits.

## 2.1 The Environment

The economy has the following properties.

### Workers and Jobs

There is an atomless continuum of workers and jobs in this model, each indexed to a certain type. For each worker we assign exogenously a value  $h \in \mathbb{R}$ , which can be interpreted as his human capital, and for each job a value  $p \in \mathbb{R}$ , which can be interpreted as that firm's productivity. These exogenous random variables are respectively distributed by  $\bar{L} : \mathbb{R} \rightarrow [0, 1]$  and  $\bar{G} : \mathbb{R} \rightarrow [0, 1]$ , with density functions  $\bar{l}(h)$  and  $\bar{g}(p)$ . These types are assumed to be perfectly observable and the distributions are common knowledge. The mass of workers is normalized to 1, and the mass of jobs per worker is  $\bar{N}$ .

Workers and firms discount time at rate  $r$  and have linear preferences. i.e, firms care about expected profits and worker care about the present value of their future stream of wages.

### Production and Contact Technologies

When a worker of type  $h$  and a firm of type  $p$  match they produce a flow output of  $F(h, p)$  every instant. The production function is an essential element to our model because they have strong implications for the sorting patterns of workers. Shimer and Smith [2] provide a full characterization of the necessary and sufficient conditions for positive assortative matching in their model. In particular, they show that if the production function, the log of it's first derivative and the log of it's cross derivative are all supermodular then the economy exhibits positive assortative matching.<sup>4</sup> Although our framework is not identical to theirs— we don't assume bounded support and we introduce on-the-job search—, which precludes us to rely on their results, we use a production function that satisfy these requirements in our empirical specification:  $F(h, p) = hp$ .

---

<sup>3</sup>This insight has been previously suggested in Abowd, Kramarz and Lengermann [6].

<sup>4</sup>If a function  $f(x, y)$  is smooth then supermodularity is equivalent to  $f_{xy} > 0$ .

Unemployed workers receive a flow value  $b(h)$ , which may depend or not on their type, and vacant jobs receive 0 flow value.

This economy runs in continuous time and is assumed to be in steady state. Following the job-search literature, we use a series of Poisson processes to model the contact technology of this economy. Unemployed workers meet firms at a rate  $\lambda^W$ . Employed workers may contact other firms at rate  $\psi\lambda^W$ , where  $\psi$  represents the relative efficiency of the on-the-job search as compared to search when unemployed. On the firm side, vacancies meet workers at a rate  $\lambda^F + \phi\lambda^F$ . Note that these can be unemployed but also employed workers contact vacancies as well. Finally, matches can be exogenously destroyed at rate  $\delta$ . For our purposes, these rates are treated as exogenous. Also, we assume a random search technology, which implies that upon meeting a certain type of agent one takes a draw from the corresponding distribution—i.e, an unemployed worker takes a draw from the distribution of vacancies when finding a job.

## Wage formation

In this decentralized economy with bilateral matching the presence of search frictions creates temporary monopsony power. The flow output of this match is split every instant, with a wage going for the worker and the residual going to the firm. We follow several others and adopt the generalized Nash bargaining solution to determine wages. However, in the presence of on-the-job search things become more complicated because when an employed worker receives an offer from another firm a three-agents negotiation problem arises. We choose to adopt the solution adopted by Moscarini [7]. The proposed institution is that the worker and the firm split the procedure every instant according to the standard Nash Bargaining solution, with  $\beta$  being the bargaining power of the workers. However, when an new firm shows up the two firms engage in an ascending 1st price auction with repeated offers for the services of the worker. The firms bid in terms of a non-negative lump-sum transfer for the worker and whoever wins pays the transfer, retain the worker and resume the match with that worker splitting proceeds as before. These lump-sum transfers can be interpreted as a signing bonus if the competing firm wins the auction or a retention bonus if the incumbent firm wins.

Now, we describe how the endogenous objects of the model are determined. Before proceeding, it is useful to introduce these objects.

- Values: an employed worker of type  $h$  working for a firm of type  $p$  has value  $V^E(h, p)$ , or if this workers is unemployed he has value  $V^U(h)$ . A firm of type  $p$  has value  $J^E(h, p)$  if matched with a worker of type  $h$ , or has a value of  $J^U(p)$  if vacant. Finally, it is useful to define the surplus of a match between worker  $h$  and job  $p$  as  $S^E(h, p) \equiv [V^E(h, p) - V^U(h)] +$

$$[J^E(h, p) - J^U(h, p)].$$

- Wages,  $w(h, p)$ .
- Matching sets: these will be characterized by indicators functions. For an unemployed worker of type  $h$  who meets a vacant firm of type  $c$ , the indicator  $\alpha^U(h, p)$  assumes a value of 1 if they match or 0 otherwise. Alternatively, if worker  $h$  is currently working for firm  $p$ , but contacts firm  $p'$ , then  $\alpha^E(h, p, p')$  assumes a value 1 if the worker switches firm or 0 otherwise.
- Steady state flows: CDF of employed matches  $\Gamma(h, p)$ , with density  $\gamma(h, p)$ . Density of unemployed workers for each type,  $l(h)$ , and density of vacancies  $g(p)$ . All these distributions are stationary. Finally, the unemployment rate,  $u$ , and the number of vacancies in the economy,  $v$ , are also determined endogenously.

## 2.2 Values and Decisions

Next, we show how the values of worker and firms are determined using continuous-time Bellman equations.

An employed worker  $h$  at a firm  $p$  enjoys flow wages of  $w$ , at a rate  $\delta$  can have his match destroyed exogenously, and at a rate  $\psi\lambda^W$  meets another firm, upon which he may move or stay at the current job. This implies the following value equation

$$rV^E(h, p) = w(h, p) + \delta [V^U(h) - V^E(h, p)] + \psi\lambda^W \int_{-\infty}^{\infty} \alpha^E(h, p, p^*) [V^E(h, p^*) - V^E(h, p)] g(p^*) dp^*. \quad (1)$$

An unemployed Worker of type  $s$  enjoys flow unemployment benefits  $b(s)$  and receive job offers at rate  $\lambda^W$ :

$$rV^U(h) = b(h) + \lambda \int_{-\infty}^{\infty} \alpha^U(h, p') [V^E(h, p^*) - V^U(h)] g(p^*) dp^*. \quad (2)$$

A productive job of type  $p$ , employing a worker of type  $h$ , enjoy flow profits of  $F(h, p) - w(h, p)$ , but can be dissolved either exogenously or through on-the-job search:

$$rJ^E(h, p) = F(h, p) - w(h, p) + \left[ \delta + \psi\lambda^W \int_{-\infty}^{\infty} \alpha^E(h, p, p^*) g(p^*) dp^* \right] [J^U(p) - J^E(h, p)]. \quad (3)$$

Finally, a vacant doesn't enjoy any flow values, but can meet both employed and unemployed

workers:

$$rJ^U(p) = \lambda^F u \int_{-\infty}^{\infty} \alpha^U(h^*, p) [J^E(h^*, p) - J^U(p)] h(h^*) dh^* \\ + \varphi \lambda^F \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha^E(h^*, p^*, p) [J^E(h^*, p) - J^U(p)] \gamma(h^*, p^*) dh^* dp^*. \quad (4)$$

As we described above wages are determined by the Generalized Nash bargaining solution with repeated first price auctions. Recall that this type of mechanism involves the standard Nash bargaining solution while a match goes on, but when the worker contacts a second firm the two firms engage in an auction for the services of the worker. Let the worker's type be  $h$ , the incumbent type be  $p$  and the new firm  $p'$ . Although this game features multiple equilibria, Moscarini [05] argues that the most plausible equilibrium for this game is the one where the firm with the highest surplus wins the auction paying no lump-sum transfer and immediately starts producing with the worker splitting the proceeds as before. This is because the firm with the highest surplus can always outbid the other firm and, knowing that, the other firm doesn't even bother bidding. It is easy to see that this equilibrium is the only one that survives if we introduce an arbitrary cost of bidding. Standard results imply that

$$S^E(h, p) = \frac{[J^E(h, p) - J^U(p)]}{(1 - \beta)} = \frac{[V^E(h, p) - V^U(h)]}{\beta} \quad (5)$$

Plugging equations 1 and 3 in this formula and rearranging we arrive at the following expression for wages

$$w(h, p) = \beta [F(h, p) - rJ^U(p)] \\ + (1 - \beta) \left[ rV^U(h) - \beta \psi \lambda^v \int_{-\infty}^{\infty} \alpha^E(h, p, p') S^E(h, p') g(p') dp' \right]. \quad (6)$$

It is also useful to use these results to rewrite the surplus function in recursive form. plugging equations 1, 3 and 5 in the definition of the surplus and rearranging we attain

$$S^E(h, p) = \frac{F(h, p) - rV^U(h) - rJ^U(p) + \beta \psi \lambda^w \int_{-\infty}^{\infty} \alpha^E(h, p, p') S^E(h, p') g(p') dp'}{r + \delta + \psi \lambda^w \int_{-\infty}^{\infty} \alpha^E(h, p, p') g(p') dp'}. \quad (7)$$

The workers and job managers in this economy decide if they want to pair up or not with an agent upon meeting. Since agents are rational they decide to pair up if that increases their values.

It is easy to see from equation 5 that this is equivalent to choosing the option that provides the largest surplus. Thus, when an unemployed worker meets a vacancy the decision is given by

$$\alpha^U(h, p) = 1 [S^E(h, p) > 0] \quad (8)$$

, whereas when an employed worker meets a second firm the decision is

$$\alpha^E(h, p, p') = 1 [S^E(h, p') > S^E(h, p)] \quad (9)$$

## 2.3 Steady State Flows

In this step, we describe the equilibrium equations that jointly determine the stationary distributions  $\Gamma(h, p)$ ,  $l(h)$  and  $g(p)$ , and the idleness rates  $u$  and  $v$ . Since the economy is in steady state the flows in and out of each state have to equal.

The first equilibrium equation that we describe is the one that equates the flows in and out of employed matches of type  $(h, p)$ . Here, we show a heuristic derivation of this equation. I describe the proper derivation, starting from the CDF, in the appendix. In the outflows, we have matches that are dissolved exogenously and matches dissolved due to on-the-job search. In the inflows, we have matches formed from idle pairs and matches formed with workers retained from other matches. If  $\alpha^U(h, p) = 1$  then  $\gamma(h, p) = 0$ . Otherwise,  $\gamma(h, p)$  is determined by equating the inflows to the outflows:

$$\begin{aligned} & \delta(1-u)\gamma(h, p) + \psi\lambda^W(1-u) \left[ \int_{-\infty}^{\infty} \alpha^E(h, p, p')g(p')dc' \right] \gamma(h, p) \\ &= u\lambda^W h(h)g(p) + \psi\lambda^W(1-u)g(c) \int_{-\infty}^{\infty} \alpha^E(h, p^*, p)\gamma(h, p^*)dp^*. \end{aligned} \quad (10)$$

Second, the total amount of employed workers of type  $h$  has to equal the total number of workers of that type minus the unemployed ones. This implies that

$$\bar{l}(h) - ul(h) = (1-u) \int_{-\infty}^{\infty} \gamma(h, p^*)dp^*. \quad (11)$$

Third, the same holds for jobs of type  $p$

$$\bar{N}\bar{g}(p) - vg(p) = (\bar{N} - v) \int_{-\infty}^{\infty} \gamma(h^*, p)dh^*. \quad (12)$$

Next, the unemployment rate is determined by integrating equation 10 over the full support of  $h$

and  $p$ .

$$\delta(1-u) = u\lambda^W \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha^U(h^*, p^*) h(h^*) g(p^*) dh^* dp^*. \quad (13)$$

Finally, the last requirement for equilibrium is that the total number of employed workers has to equal the amount of filled jobs.

$$1-u = \bar{N} - v. \quad (14)$$

## 2.4 Equilibrium

The equilibrium for this economy consists of the value of the endogenous values such that the values for the workers satisfy their Bellman equations, the steady state equations are satisfied and the matching sets are in accord with rational decisions.

**Definition 1** *An equilibrium in this economy consists of values for  $\gamma(h, p)$ ,  $l(h)$ ,  $g(p)$ ,  $u$ ,  $v$ ,  $S^E(h, p)$ ,  $V^U(h)$ ,  $J^U(p)$ ,  $\alpha^U(h, p)$  and  $\alpha^E(h, p, p')$  such that equations 2, 4, 7, 8, 9, 10, 11, 12, 13 and 14 are satisfied.*

## 2.5 Proposed Algorithm

This model doesn't have a closed form solution. Thus, for practical purposes we have to solve this model computationally. For our empirical implementation we use the following algorithm to solve the model. First, we discretize the state space of workers and firms. Also, we guess an initial value for all endogenous objects. Then we take the following steps:

1. Steady state equations: given the guesses for  $\alpha^U(h, p)$  and  $\alpha^E(h, p, p')$  we iterate equations 10 to 14 until we find a fixed point. This gives a value for  $\gamma(h, p)$ ,  $l(h)$ ,  $g(p)$ ,  $u$  and  $v$ .
2. Values: given the guesses for  $\alpha^U(h, p)$  and  $\alpha^E(h, p, p')$  and the values for the stationary distributions calculated in step 1), we iterate on equations 2, 4 and 7 until finding a fixed point. This yields a value for  $S^E(h, p)$ ,  $V^U(h)$  and  $J^U(p)$ .
3. matching sets: given the inputs from steps 1) and 2), we update the matching sets using equations 8 and 9.
4. Repeat steps 1) to 3) until matching sets converge.

## 2.6 Notes on Endogeneizing Vacancy Formation

As described above, in this formulation, the total stock of jobs of each type is assumed to be exogenous and then the process through which workers are allocated to these vacancies is endogenous. We follow strictly the formulation of Shimer and Smith [2] in that respect. As we mentioned, this is because the fixed stock of jobs makes the vacant jobs have a value and, in particular, higher productivity vacancies have higher value, and this allows us to explain a very robust fact. However, a more common assumption for models that deal with firm heterogeneity is to assume that the distribution of firms in the economy is exogenous and then proceed from there on.<sup>5</sup> The next step on my project is to change the assumptions in the model so that it becomes consistent with the literature. The plan is to assume that there is an exogenous distribution of firms in the economy, then to model the vacancy posting strategy of this company. A firm would endogenously post vacancies and own a number of jobs, and to bound the size of companies we would utilize a cost structure.

## 3 The Data

### 3.1 Description of the Data and Background

In this paper, we use the labor market census RAIS (Relação Anual de Informações Sociais) as our dataset. In RAIS, an administrative dataset collected annually by the Brazilian labor ministry, all firms in the Brazilian formal sector provide information for all their workers. The dataset collects demographic information of the workers—such as age, education and sex—, provides some information about the firm/establishment—such as sector and location— and provides information about the job such as the average wage earned during that year, the wage in December, occupation, dates of admission and separation, type of contract, causes for separation, etc.

We restrict the sample for the Southeast region of Brazil—includes the states of São Paulo, Rio de Janeiro, Minas Gerais and Espírito Santo—for a number of reasons. First, because RAIS is an enormous dataset it helps to restrict the number of states, making it more manageable. Second, this region is the most important economic region of the country, producing around 50% of the country's GDP. Third, table 1 shows that the southeast region has a much smaller level of informal employment than all other regions of the country. The informality in the labor market is an important feature of the Brazilian labor market, as it is in many developing countries. Hereafter, we will not deal explicitly with this feature of the labor market, lumping it in with the non-employment

---

<sup>5</sup>For example, Bontemps, Robin and Van Den Berg [8], Postel-Vinay and Robin [9] and Lentz [5].

category. Thus, whenever we say employment we are referring to formal employment. However, it is worthwhile noting that study in Menezes, Muendler and Ramey [10], which also uses RAIS, doesn't find significant differences in their results after controlling for selectivity into the formal sector.

| Table 1 – Employed Workers in Brazil in 1999 |                  |         |               |         |
|----------------------------------------------|------------------|---------|---------------|---------|
|                                              | Southeast Region |         | Other Regions |         |
| <b>Formal Sector</b>                         | 13,064,202       | 49.32%  | 11,312,328    | 40.08%  |
| <b>Informal</b>                              | 6,028,245        | 22.76%  | 7,421,616     | 26.29%  |
| <b>Indeterminate</b>                         | 7,397,019        | 27.92%  | 9,491,019     | 33.63%  |
| <b>Total</b>                                 | 26,489,466       | 100.00% | 28,224,963    | 100.00% |
| <b>Workers in RAIS</b>                       | 15,856,851       | 59.86%  |               |         |

Source: Cacciamali e Braga (2002), e RAIS

We use 11 years of data, covering the years from 1995 to 2005. Although data from previous years is available, we choose this time-frame for two reasons. First, until 1994 Brazil suffered from hyperinflation, which causes serious measurement problems in variables such as wages, and also has structural implications for the economy, especially for the macroeconomy. Second, the data from 1995 on is more reliable.

RAIS is an enormous dataset, with more than 400 million observations, which causes serious manipulation problems. Studies that utilized analogous datasets in other countries used sampling techniques to deal with this problem. For example, in Abowd, Kramarz et al. [11, 12] the authors utilized data from 1/25 of the workers in France, and when using American data, they used data from 1/10th of the workers on the state of Washington. For the part of my study that requires longitudinal data I utilize data from 1/10 of the workers in the formal sector in the Southeast region of Brazil. We collect this subsample drawing without replacement from the pool of workers. In order not to lose information about dynamics we collect all observations from the selected workers. This procedure yields 22,133,168 observations.

In our analysis we will also consider cross-sectional aspects of the data. For this part we utilize the full sample at the year of 2000.

## 3.2 Sample Corrections

Before proceeding it is necessary to perform some sample corrections to the sample, because administrative data sets often have problems with identifiers, missing information etc. We follow the procedure outlined in Abowd, Kramarz et al. [12]. The numbers below refer to the 10%

11-year sample, but this correction is also applied to a cross-section when this is used—using all observations.

1. We start with 22,133,168 observations. Each observation corresponds to a worker-Establishment-year cell, with possible duplicates each year.
2. We remove from the sample part time and non-wage workers: weekly hours  $\geq 40$  and worker earns at least a minimum wage. This discards 3,258,299 observations.
3. Because of problems with the worker identifiers, we remove the observations of workers who show only once in the dataset and appear as working starting the year or end the year working—i.e, workers who also worked in other years. This drops 330,455 observations.
4. Next, we aggregate the information to firm level. For each worker-establishment-year, we aggregate the information from all establishments of a firm. As in Abowd et al, we use as wage the average of all the observations from the worker in that firm on that year. This drops 535,956 observations.
5. Imputations: Because of problems with the identifiers, it is common in administrative data sets for a worker to show as working in a given firm, then disappear and then showing in the same firm on the following year. As in Abowd et al, we input the middle observation whenever this happens, using the average of the previous and following wages as the measure of wages. This adds 211,062 observations.
6. Multiple jobs: As in Abowd et al, we discard the information from workers who show as working in more than three jobs in a given year. For this we remove the observations of workers who show as working more than 24 months in a given year (adding over all jobs of that year) or workers who appear in more than three firms at a given year. This discards 363,587 observations.
7. We remove from the sample workers with an age smaller than 15 or higher than 65. We also discard observations with problems in the sex, age and education identifiers. This drops 825,077 observations.

After performing these steps we are left with 17,030,856 observations (77% of initial subsample). Table 2 shows some statistics from this Sample

Table 2: Sample Statistics

| Londitudinal Sample: 10% of workers from 1995 to 2005 |         |                           |        |                    |  |
|-------------------------------------------------------|---------|---------------------------|--------|--------------------|--|
| Workers                                               |         |                           | Firms  | Firms (2+ workers) |  |
| Total number                                          | 3072908 | Total number              | 914873 | 510937             |  |
| Avg. Number of observations                           | 5.9     | Avg. # obs                | 19.8   | 33.52              |  |
| Avg. # of firms                                       | 2.2     | Avg. # workers            | 7.48   | 12.61              |  |
| Avg. # of years in sample                             | 5.33    | Avg. # of years in sample | 4.15   | 5.49               |  |
| Education                                             |         | Sector                    |        |                    |  |
| Below High School                                     | 67.10%  | Agriculture               | 0.74%  | 0.88%              |  |
| High School or College Dropout                        | 25.71%  | Commerce                  | 40.48% | 33.72%             |  |
| College Graduate                                      | 7.20%   | Manufacture               | 16.77% | 20.20%             |  |
| Male                                                  | 64.00%  | Services                  | 36.83% | 38.67%             |  |

### 3.3 Comparison With Other Datasets

Before proceeding with our inference it is useful to compare the results of common procedures applied in other administrative datasets to the results of applying these procedures to our dataset. The most common exercise applied to matched employer-employee datasets is the linear error decomposition proposed by Abowd, Kramarz et al. [11, 12]. The novel results found utilizing this methodology motivated much of the subsequent empirical and theoretical literature in this field. Thus, it is to the very least informative to compare the results of applying this methodology in our data to the results found in other datasets.

This procedure consists in estimating the following wage equation.

$$w_{it} = x_{it}\beta + \theta_i + \psi_{J(i,t)} + \varepsilon_{it}$$

,where  $w_{it}$  denotes the measure of earnings,  $x_{it}$  the vector of time-varying covariates,  $\theta$  the worker's fixed effects and  $\psi_{J(i,t)}$  the fixed effect of firm  $J$ , which employs worker  $i$  at time  $t$ . Note that  $x_{it}$  only includes time-varying variables because of the colinearity with the fixed effects.

In addition, to control for observed time-fixed characteristics such as sex and education, we consider the additional auxiliary regression

$$\theta_i = u_i\eta + \alpha_i$$

Following Abowd et al, we use experience, time dummies and region dummies  $x_{it}$ , and sex and education dummies for  $u_i$ .<sup>6</sup>

<sup>6</sup>We use the 1st-4th powers of experience as regressors. Experience is computed as potential experience for the first year and then actual experience for the subsequent years. We one dummy for each state of the southeast region.

In order to estimate this model one needs a sample with a significant amount of mobility. This is because the worker who switch firms are the ones that allow the disentangling between the fixed effects of worker and firms. Also, one other identifying assumption of this model is that the mobility patterns of workers are exogenous. It is clear that these are strong assumptions and our model will be used to make inference about that. However, the output from these regressions is still useful to summarize some aspects of the data. The most informative aspects of this regression comes from the variances and correlations between each component of this regression. Table 2 compares the standard deviations and correlations of each component of this regression for France, the US —these two from Abowd, Kramarz and Lengermann [6] —and Brazil.

The table shows that the results utilizing the Brazilian data is in accord with the results found in the US and in France. In particular, for my motivations it is worth highlighting two aspects. First, the standard deviations of  $\alpha_i$  and  $\psi_{J(i,t)}$  are, respectively 0.49/0.46, in the US 0.81/0.35 and in Brazil 0.49/0.37. Thus, in Brazil, similarly as in the other countries,  $Var(\psi_{J(i,t)})$  is relatively close to  $Var(\alpha_i)$ . This result has been interpreted as being supportive of the existence of frictions in the labor market., because it suggests that workers with similar characteristics—even unobservable—earn different wages in different firms. Second the correlation between  $\alpha_i$  and  $\psi_{J(i,t)}$  is  $-0.24$  in France,  $0.02$  in the US and  $-0.03$  in Brazil. This shows that our dataset yields the same result verified in several other datasets that this correlation is non-negative. This correlation, if taken at face value, can be interpreted as a measure of sorting in the labor market. Thus, this would suggest that although there is substantial evidence that both the heterogeneity of worker and firms are important, there is not a positive association between them.

It is also worth mentioning the results from Menezes, Muendler and Ramey [10], who also utilize RAIS. In their paper, they estimate wage regressions using a cross-section of workers, with establishment fixed effects only and compare their results with results obtained from American and French data. They conclude that

Overall, our results establish a close similarity between wage structures in a major developing country and two major industrialized countries. Key differences between the countries, in particular the high variability of Brazilian wages, emerge from the way worker characteristics are compensated and not from differences in establishment-level wage policies.

**Table 3**  
A) Abowd et al.

| Correlations of Components of Real Annual Wage Rates |        |         |          |          |         |          |         |               |
|------------------------------------------------------|--------|---------|----------|----------|---------|----------|---------|---------------|
|                                                      | StD.   | $\ln w$ | $x\beta$ | $\theta$ | $u\eta$ | $\alpha$ | $\psi$  | $\varepsilon$ |
| France                                               |        |         |          |          |         |          |         |               |
| Log real annual wage rate                            | 0.9772 | 1.0000  | 0.5377   | 0.4569   | 0.1510  | 0.4316   | 0.4287  | 0.5675        |
| Time-varying characteristics                         | 0.4087 | 0.5377  | 1.0000   | 0.0698   | -0.0469 | 0.0872   | 0.1670  | 0.0000        |
| Person effect                                        | 0.5217 | 0.4569  | 0.0698   | 1.0000   | 0.2917  | 0.9565   | -0.2225 | 0.0000        |
| Schooling and sex                                    | 0.1522 | 0.1510  | -0.0469  | 0.2917   | 1.0000  | 0.0000   | 0.0293  | 0.0000        |
| Unobservable                                         | 0.4990 | 0.4316  | 0.0872   | 0.9565   | 0.0000  | 1.0000   | -0.2415 | 0.0000        |
| Firm effect                                          | 0.4665 | 0.4287  | 0.1670   | -0.2225  | 0.0293  | -0.2415  | 1.0000  | 0.0000        |
| Residual                                             | 0.5545 | 0.5675  | 0.0000   | 0.0000   | 0.0000  | 0.0000   | 0.0000  | 1.0000        |
| US                                                   |        |         |          |          |         |          |         |               |
| Log real annual wage rate                            | 0.8941 | 1.0000  | 0.2305   | 0.4871   | 0.1733  | 0.4571   | 0.4926  | 0.4042        |
| Time-varying characteristics                         | 0.6965 | 0.2305  | 1.0000   | -0.6085  | -0.1527 | -0.5893  | 0.0635  | 0.0000        |
| Person effect                                        | 0.8434 | 0.4871  | -0.6085  | 1.0000   | 0.2748  | 0.9615   | 0.0445  | 0.0000        |
| Schooling and sex                                    | 0.2317 | 0.1733  | -0.1527  | 0.2748   | 1.0000  | 0.0000   | 0.0824  | 0.0000        |
| Unobservable                                         | 0.8110 | 0.4571  | -0.5893  | 0.9615   | 0.0000  | 1.0000   | 0.0228  | 0.0000        |
| Firm effect                                          | 0.3586 | 0.4926  | 0.0635   | 0.0445   | 0.0824  | 0.0228   | 1.0000  | 0.0000        |
| Residual                                             | 0.3614 | 0.4042  | 0.0000   | 0.0000   | 0.0000  | 0.0000   | 0.0000  | 1.0000        |

Notes: This table presents statistics based on estimating a log-wage equation with person and firm effects. See Abowd, Kramarz, Lengermann, and Roux 2003.

Sources: DADS (1976-1996) for France; LEHD (1990-1999) for the US.

## B) Our Results

| Correlations of Components of Real Annual Wage Rates – Brazil |       |         |          |          |         |          |        |               |
|---------------------------------------------------------------|-------|---------|----------|----------|---------|----------|--------|---------------|
| Brazil                                                        | STD   | $\ln w$ | $x\beta$ | $\theta$ | $u\eta$ | $\alpha$ | $\psi$ | $\varepsilon$ |
| Log Real Wage                                                 | 0.800 | 1.000   | 0.205    | 0.533    | 0.439   | 0.313    | 0.604  | 0.330         |
| Time-Varying Characteristics                                  | 0.520 | 0.205   | 1.000    | -0.551   | -0.413  | -0.366   | 0.087  | 0.000         |
| Person Effects                                                | 0.680 | 0.533   | -0.551   | 1.000    | 0.710   | 0.704    | 0.109  | 0.000         |
| schooling and sex                                             | 0.490 | 0.439   | -0.413   | 0.710    | 1.000   | 0.000    | 0.185  | 0.000         |
| Unobservable                                                  | 0.480 | 0.313   | -0.366   | 0.704    | 0.000   | 1.000    | -0.032 | 0.000         |
| Firm Effect                                                   | 0.370 | 0.604   | 0.087    | 0.109    | 0.185   | -0.032   | 1.000  | 0.000         |
| Residual                                                      | 0.320 | 0.330   | -0.041   | -0.040   | 0.036   | -0.094   | -0.016 | 1.000         |

### 3.4 Subsamples

In order to have a sample compatible with a possible realization of our theoretical model we have to make some adjustments to the sample. It is well known that observable aspects of the labor market such as region, gender and education are relevant for labor market outcomes. Since we don't explicitly model this we deal with these types of heterogeneity by restricting our sample. Thus, I select a sample of males living in the state of Sao Paulo. Moreover I stratify this sample in two levels of education: low (high school or college dropout) and high (college degree). <sup>7</sup> This combination of restrictions drops 88% of the longitudinal sample and 92% of the cross-sectional sample. <sup>8</sup> It is worth noting that each of the procedures below will be applied individually to each of these subsamples. For example, the average wage of a firm in the low education subsample will include only the low educated workers.

## 4 Moments to be Matched

There are three types of moments that are very important to the question I want to address. First, because this is an environment with both worker and firm's heterogeneity, I need moments that help identifying these two distributions. Second, sorting between workers and firms is central in my project., thus I need moments of the data that refer to the patterns of allocation of workers to firms. Third, but not less important, I need aspects of the data that give information about the degree of friction in the labor markets. Following the paradigm of the job-search theory, this is achieved by using turnover data.

### 4.1 Distributions

First, I show moments that help identify the distributions of workers and firms types. These types reflect productive capacities for these agents and in equilibrium have influence in the wages, the variable that I observe in the data. The idea is to use two distributions observable in the data to obtain this information: the distribution of wages and the distribution of "firm-wages" among workers. The concept of firm-wages is the same utilized in Postel-Vinay and Robin [9] and Christensen et al [13], and refers to the average wage of all the workers that work for a given firm. This measure helps us in our task because, in our model, these firm-wages are monotonically related to the job's types. Thus, we can use these firm-wages to obtain information about the distribution

---

<sup>7</sup>This drops the workers with less than a high school degree.

<sup>8</sup>The facts mentioned in section 4 also hold when using more representative samples, with less restrictions.

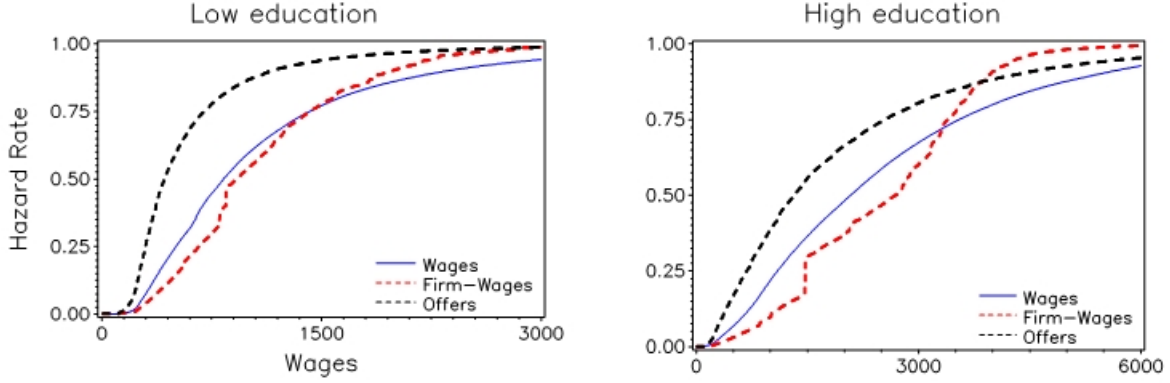
of job productivities, and then use the standard wage distribution (a composition of both firm and worker productivities) to “back out” information about the worker’s types. It is worth noting that this measure can only be computed with matched datasets, as for example RAIS.

Table 4 and Figure 1 show the distribution of wages and firm-wage among workers for both strata. It is clear that the firm-wage distribution is much more compressed than the firm-wage distribution. Also, the differences in Skewness and Kurtosis are significant and can also be explored in the identification of the distributions.

| Table 4                                                                   |         |         |      |       |        |
|---------------------------------------------------------------------------|---------|---------|------|-------|--------|
| Moments of Wage Distribution                                              |         |         |      |       |        |
|                                                                           | Mean    | Std     | Skew | Kurt. | Nobs   |
| Low Educ                                                                  | 1405.70 | 1352.87 | 3.22 | 16.81 | 309737 |
| High Educ                                                                 | 3375.18 | 2676.36 | 1.51 | 2.65  | 632361 |
| Moments of Firm-Wage Distribution (across workers)                        |         |         |      |       |        |
|                                                                           | Mean    | Std     | Skew | Kurt. | Nobs   |
| Low Educ                                                                  | 1305.65 | 766.00  | 1.48 | 5.93  | 309737 |
| High Educ                                                                 | 3252.87 | 1560.56 | 0.36 | 0.38  | 632361 |
| PS: These moments were computed using a cross section in the year of 2000 |         |         |      |       |        |

Figure 1 plots the cumulative distribution function of the wage distribution of both the wage and the firm-wage distributions. The plot confirms the patterns shown by the moments, especially that the firm-wage distribution is more compressed than the wage distribution. Additionally we also include in the graph the distribution of wages among entrants, which we label “wage offers”. This distribution is clearly stochastically dominated by the distribution of wages in the economy. This is consistent with results found in several datasets worldwide and on-the-job search, a feature present in my model, has been shown to successfully explain it.

Figure 1: Wage Distributions



## 4.2 Sorting Patterns

In this section I construct three moments that are related to sorting. The first is standard in the literature, while the other two are novel. All the moments of this section can only be computed with a matched dataset.

The standard procedure to obtain a measure of sorting between workers and firms in the availability of a matched dataset consists of a wage regression with worker and firm fixed effects. As shown above, this procedure consists in estimating the following wage regression:

$$w_{it} = x_{it}\beta + \theta_i + \psi_{J(i,t)} + \varepsilon_{it}$$

,where  $w_{it}$  denotes log-earnings,  $x_{it}$  time varying covariates such as time-dummies and experience,  $\theta_i$  are worker fixed effects and  $\psi_{J(i,t)}$  are firm fixed effects. The idea then is to use  $\theta_i$  and  $\psi_{J(i,t)}$  as a proxy for the worker and firm primitives and simply correlate them to obtain a measure of sorting. A very robust and surprising result on this literature is that this correlation is 0 or sometimes even negative.

This correlation is the first moment related to sorting that I will use in my procedure. The equilibrium of my model exhibits positive assortative matching, so it is counterintuitive how it will be able to match this moment. The feature that will allow me to gain leverage on this margin is that I allow the firms to have heterogeneous outside option. i.e, I depart from the standard assumption of zero-profits at every firm type, thus allowing the more productive firms to have higher outside

option. As we can see from equation 6 this outside option of the firm enters negatively in the wage equation, distorting this correlation between fixed effects. Because of this, in my model, the correlation between fixed effects of firm and workers is consistently smaller than the correlation between the primitives.

One other result from this model is that applying the same wage regression described above, if I correlate  $\theta_i$  with the average fixed effect of his coworkers, this correlation is not distorted downward. In fact, it is positive and fairly similar to the analogous correlation among the primitives of the model. This will be illustrated in the next section. Because of that I use as the second moment related to sorting the correlation between  $\theta_i$  and the average  $\theta_i$  among his coworkers.

Table 5 shows the results from this wage regression applied to our two subsamples.<sup>9</sup> First, the  $Corr(\theta_i, \psi_{J(i,t)})$  is slightly negative in both samples, which is not surprising and is consistent with the literature. Second,  $Corr(\theta_i, \theta_{coworkers_i})$  has the positive value of 0.3, which is much larger than any value previously found for  $Corr(\theta_i, \psi_{J(i,t)})$  in the literature. If my model's interpretation is correct, the second correlation indicates that workers and firms sort positively, and the first correlation is zero because of non-linearities introduced by outside option of firms.

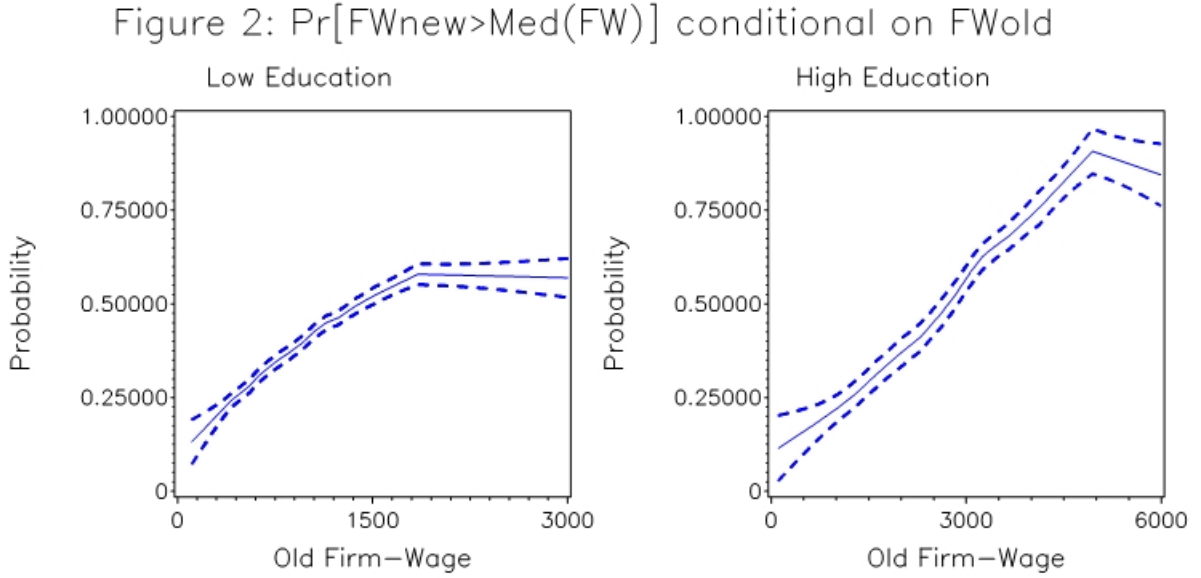
**Table 5: Correlation between elements of Wage Regression**

| Low Education Sample                                                  |      |         |          |          |                      |              |
|-----------------------------------------------------------------------|------|---------|----------|----------|----------------------|--------------|
|                                                                       | STD  | $\ln w$ | $x\beta$ | $\theta$ | $\theta_{coworkers}$ | $\psi$       |
| Log Wages                                                             | 0.78 | 1       | 0.52     | 0.49     | 0.25                 | 0.52         |
| time-varying characteristics                                          | 0.41 | 0.52    | 1        | -0.12    | 0.07                 | 0.15         |
| Worker Fixed Effects                                                  | 0.46 | 0.49    | -0.12    | 1        | <b>0.31</b>          | <b>-0.07</b> |
| Mean Coworker Effect                                                  | 0.25 | 0.25    | 0.07     | 0.31     | 1                    | 0.05         |
| Firm fixed effects                                                    | 0.39 | 0.52    | 0.15     | -0.07    | 0.05                 | 1            |
| Ps: Sample has 1,564,817 obs, with 252,604 workers and 144,056 firms. |      |         |          |          |                      |              |
| High Education Sample                                                 |      |         |          |          |                      |              |
|                                                                       | STD  | $\ln w$ | $x\beta$ | $\theta$ | $\theta_{coworkers}$ | $\psi$       |
| Log Wages                                                             | 0.89 | 1       | 0.35     | 0.63     | 0.16                 | 0.49         |
| time-varying characteristics                                          | 0.26 | 0.35    | 1        | 0.11     | 0.03                 | -0.05        |
| Worker Fixed Effects                                                  | 0.58 | 0.63    | 0.11     | 1        | <b>0.3</b>           | <b>-0.09</b> |
| Mean Coworker Effect                                                  | 0.3  | 0.16    | 0.03     | 0.3      | 1                    | -0.1         |
| Firm fixed effects                                                    | 0.53 | 0.49    | -0.05    | -0.09    | -0.1                 | 1            |
| PS Sample has 495,159 obs., with 71,616 workers and 33,516 firms      |      |         |          |          |                      |              |

Finally, I obtain the last moment related to sorting by observing directly the allocation of workers to firms. First, recall that in the model firm types are monotonically related to firm-wages. Thus, I will use these observables as a proxy for the firm types. I will look at the following event:

<sup>9</sup>Because the sample selection already controls for observables, the term  $x_{it}$  includes only experience and time-dummies.

a worker separates from a given firm, spends some time in unemployment (at least three months) and then finds another job. If there is no sorting between firms and workers, this workers would sort with random firms each time and thus there would be no association between the first and the second firm.<sup>10</sup> On the other hand, if there is positive assortative matching, workers who show in high-type firms are likely to be high-type themselves and thus are likely to end up in another high-type firm if reallocated. Let  $FW$  denote the firm-wage of the firm a given worker is employed . To assess this, I look at the following measure:  $Pr[FW_{new} > median(FW) | FW_{old}]$ .<sup>11</sup>



It is clear from Figure 2 that the relationship is positive: workers who work for a low-type firm are very likely to end up in a low-type firm when reallocated and vice-versa. All this is consistent with the positive assortative matching of my model.

Although we argue that our model with bilateral matches between workers and firms succeeds in explaining these facts we believe that the facts can be also consistent with another type of sorting: worker sorting with other workers, as in the O-ring theory. However, we believe that the implications for allocation and efficiency between these two types of economies are relatively similar and much different from an economy without sorting, which is the benchmark.

<sup>10</sup>The reason I require the worker to spend some time in unemployment before finding this new job is to avoid on-the-job search. On-the-job search could imply a positive correlation between a new and the old firm wage, regardless of the worker type.

<sup>11</sup>I could have looked simply at  $FW_{new}$  as a function of  $FW_{old}$  , but I prefer to look at this probability instead because it is invariant to monotonic transformations in  $FW$ .

### 4.3 Transitions

Finally, the last set of moments that are relevant to my question are transition moments. The idea is to use duration moments to identify the arrival rates of the stochastic processes in the model. These are three: exogenous destruction rate of jobs; arrival rate of job offers for the unemployed and arrival rate of job offers for the employed. We can use three duration moments that are directly associated with these rates. First, let  $D1 = \frac{\text{\# of workers who find a job in a given year}}{\text{\# employed workers}}$ . This moment is associated with the arrival rate of job offers.<sup>12</sup> This parameter is usually associated to the average unemployment duration. However, since we do not have unemployment information we use alternatively number of workers who start working in a given year, and to normalize this number we divide by the total number of employed workers.<sup>13</sup> Second, to identify the exogenous destruction rate we can use  $D2 = \frac{\text{\# of workers who separate in a given year}}{\text{\# employed workers}}$ . Finally, to identify the arrival rate of offers on-the-job we use  $D3 = \text{Fraction of workers who separate due to JJT}$ .<sup>14</sup> I define a job-to-job transition if the worker shows in another firm within a month after being listed as being separated from his old job. This definition is similar to the one used in Jolivet, Postel-Vinay and Robin [14]. Actually D2 and D3 are very similar to the moments these authors use to identify these same parameters but in the context of a standard job-search model.<sup>15</sup>

Table 6 brings values for these moments in our sample. The number of job-to-job transitions relative to job-to-unemployment seems very low for this data. However, it is comparable to the numbers found in Mediterranean countries such as Spain (25.4%), Italy (29.4%) and Portugal (36.9%).<sup>16</sup> Finally, to check whether the hypothesis that the State of Sao Paulo is a “closed economy” I include migration numbers in the table. Out of the workers who start employed in my sample, around 1% find a job outside of Sao Paulo in that year. The number is smaller than one if I consider jobs outside of the Southeast region.

---

<sup>12</sup>Under this Poisson specification, this random variable is binomial, with success rate depending only on  $\lambda^U$ . The total number of trials of the binomial depends on the unemployment rate, which depends on the other parameters as well.

<sup>13</sup>For future versions of the paper we plan to obtain unemployment duration from other data sources.

<sup>14</sup>The three moments depend on the three parameters. Thus, the three moments jointly identify these parameters. The separation in the text is for intuition.

<sup>15</sup>They use three years, instead of 1.

<sup>16</sup>See Jolivet, Postel-Vinay and Robin [14].

Table 6

| Summary Statistics: Worker Turnover Over 1 Year |               |                |
|-------------------------------------------------|---------------|----------------|
|                                                 | Low Education | High Education |
| Initial Sample                                  | 632361        | 309737         |
| % remain at same job                            | 80.29%        | 81.82%         |
| % job-to-nonemployment                          | 16.44%        | 13.44%         |
| % job-to-job                                    | 3.27%         | 4.74%          |
| % instantaneous job re-accessions               | 16.57%        | 26.06%         |
| (# Job entrants)/(# initially employed workers) | 20.90%        | 13.50%         |
| migration                                       |               |                |
| % find job another state                        | 1.08%         | 1.54%          |
| % find job outside SE                           | 0.61%         | 0.81%          |

Ps: Cross-Section Sample year of 2000

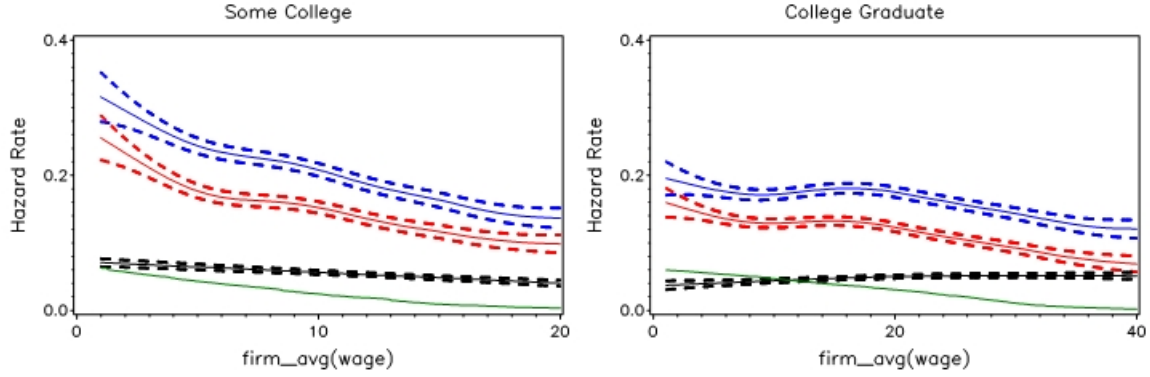
Ps2: All percentages are relative to the pool of workers employed in the beginning of the year.

#### 4.4 Other Moments

The moments outlined above are likely to be enough to estimate the parameters of the model (which aren't many). Nevertheless, there are a number of additional aspects of the data that the model can address. For example, because of the built-in heterogeneity of the model and the selection process, the durations of both unemployment and employment will be time-dependent, which is empirically accurate in most cases.

One other aspect of the data that this model can shed light on is on the relationship between separations and wages at the firm level. Christensen et al [13] looked at the relationship Firm level Separations x Average Firm Wage in the Danish data. They found a negative relationship and utilized the job-search theory interpretation to the fact: workers self-select into jobs according to wages (in specific the firm-wages). Thus, in firms with low wages there should be more separations because most offers the worker receive come from firms with higher wages, and vice-versa. This explanation depends on this negative relationship being driven by the on-the-job transitions.

In the figure below I replicate their results, but this time decomposing the separations (hazards) into job-to-job and non-JJT. The blue line indicates all separations, the red line the non-JJT separations and the black line the JJT separations. It is clear from the picture that the negative relationship between separations and firm-wages is being driven by the non-JJT separations as opposed to the JJT. In fact the relationship between job-to-job transitions and Firm wages is flat, which goes against the argument above. In fact, the green line shows what the JJT X Fwage schedule should have looked like in a standard job-search model.



My model has a mechanism that can flatten the relationship between JJT and Fwages. With the positive assortative matching of the model, the workers employed in the low-wage firms are the low-type ones. These workers are rejected by the high-type firms. Thus, it is not true anymore that these worker will leave for any firm that shows, because some of them will reject him. This difference in matching sets for the different types of workers breaks down the necessary monotonicity of the relationship between JJT and Fwages.

## 5 Estimation

## Work in Progress ##.

The objective is to estimate the parameters of the model described in Section 2 using the moments from the data described in Section 4. Since we don't have a closed form solution for this model we would use the simulated method of moments. For that, we would need to simulate both a cross-section of workers and a panel for each parameter configuration, and choose the parameters that minimize a penalty function. This penalty function would consist on the (weighted) distance between the moments from the data and the same moments, but calculated from a simulation of the model.<sup>17</sup> This procedure should be very computationally demanding in my case because of two reasons. First, it takes close to three minutes to calculate the equilibrium of the model given a set of parameters—when these converge. Second, because I have large sample sizes the simulations will also have large size.

---

<sup>17</sup>Whereas we have shown plots to illustrate the facts, we would reduce the plot to a handful of moments for this step.

## 6 Preliminary Results

We present here some preliminary results that illustrate the capacity of the model matching the proposed moments.

### 6.1 Specification

For this exercise, we choose a particular configuration of parameters in order to illustrate some features of the model that are key for our objectives. We choose the following values

- Distributions of types. First, for tractability we discretise the support of the type distributions using 100 points. We distribute the vector of probabilities of the (exogenous) worker and the job-type distributions proportionally to log-normal distributions. For the simulations we assume the feeding normal to be  $N(0, 0.5)$  for the workers and  $N(0, 0.375)$  for the firms.
- Transition parameters. First, I assume that  $\bar{N} = 1$ , which implies that  $v = u$ . Also, I normalize the transition rates the following way:  $\lambda^W = \lambda u = \lambda^F$ . Also,  $\phi \lambda^F = \psi \lambda (1 - u)$ . This reduces the transition parameters of the model to  $\lambda$ ,  $\psi$  and  $\delta$ . We choose  $\lambda = 0.75$ ,  $\psi = 0.1$  and  $\delta = 0.025$ .
- Unemployment insurance:  $b(h) = 0, \forall h$ .
- Nash Bargaining parameter: 0.35.

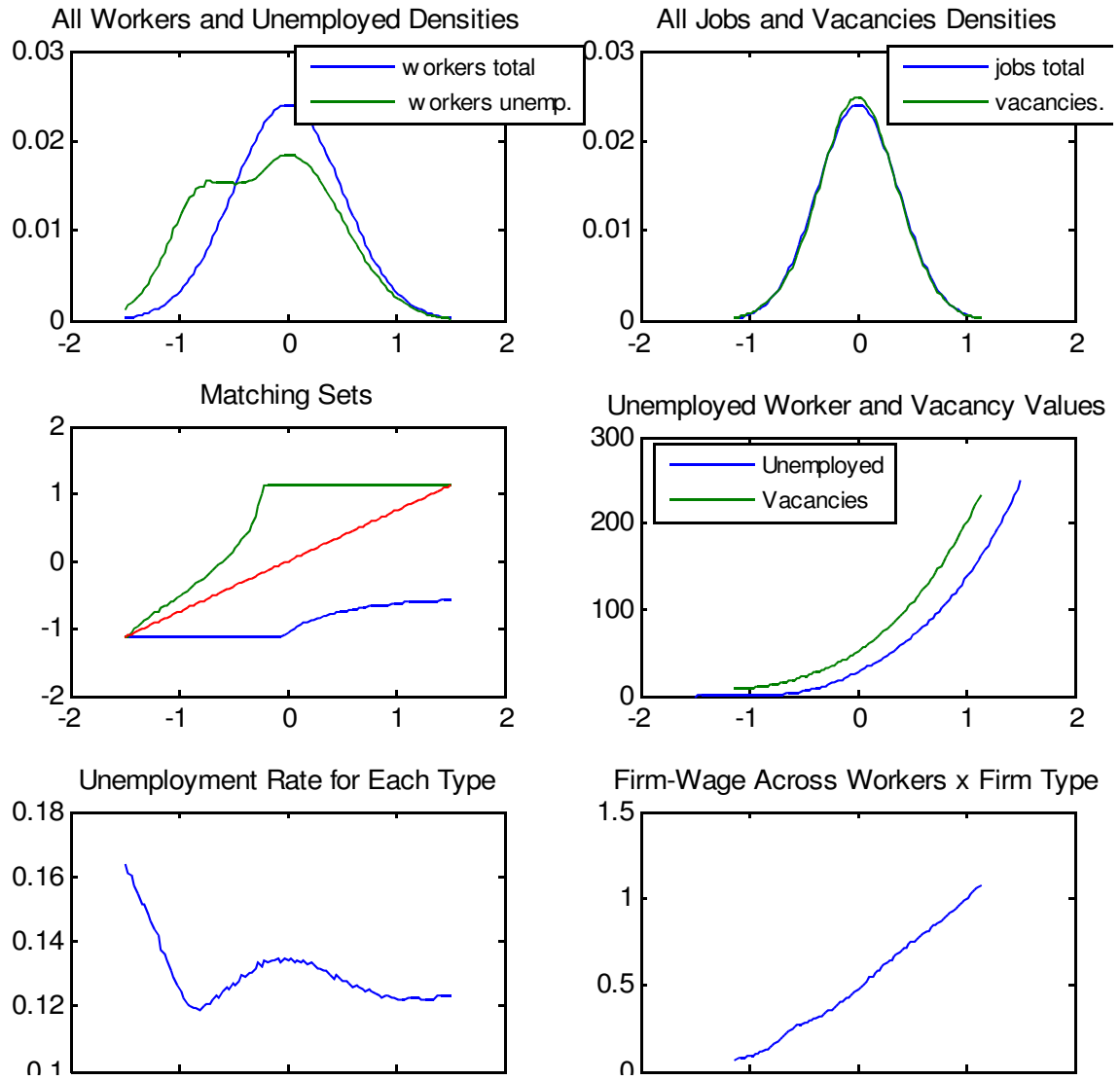
To provide a model counterpart for the data, we provide a simulation of the model. In order to do that we approximate the continuous time economy with the analogous discrete-time economy with periodicity of a week. We simulate the path of 10000 workers, whose initial labor market state is drawn from the steady state distribution, over 11 years. Also, since our model is defined in terms of jobs we also need to introduce the concept of a firm in this economy. For the results presented here, we define a firm of type  $p$  to be the owner of all the jobs of type  $p$  (employed or vacant).

### 6.2 Results

#### 6.2.1 Equilibrium

Figure 3 shows six graphs that help us characterize the equilibrium of this model.

Figure 3: Characterization of an Equilibrium



First, the two upper panels contrast the distribution of all workers/jobs in this economy with the distribution of the unemployed/vacant. In both cases the distribution of the idle types. It is worth noting that the distribution of unemployed worker types is clearly stochastically dominated by the distribution of all workers. This is a clear implication of the positive assortative matching, where the low type workers are systematically rejected by the higher type firms. On the other hand, this feature is not so prominent on the job-type distributions. This might be because on-the-job search makes the workers less picky.

The first mid panel illustrate the matching set of this economy—X axis has workers and Y axis has firms. The upward sloping bands are a direct consequence of the equilibrium with positive assortative matching: the low types are rejected by the high types. However, since the workers can search on-the-job they are less picky than the firms. The second panel of the mid row shows the outside value of workers and firms. They are clearly convex, reflecting the nature of the production technology, and the firms in general have a higher outside value than workers. This is because I chose the bargaining power of firms to be higher than the one of workers ( $\beta = 0.35$ ). The first bottom panel illustrates the unemployment rate by firm type. Following the logic of the matching set the low type workers have a much higher unemployment rate. The hump shape in the middle of the graph is a reflection of the functional forms we've chosen for the type distributions. Basically, there is a large number of middle type workers and they have to "share" the jobs with the middle types and the high types. Finally the last panel plots the firm-wage as a function of the job-type. This is clearly a positive monotonic relationship, and is useful for the empirical part of the paper, where we use the observed firm-wages to gather information on the unobserved distribution of firm-types.

### 6.2.2 Matching Sorting Moments

Next, we illustrate the model's performance in explaining the data. We focus on the "sorting moments" that we described on Section 4, because they are the most novel among the facts.

Table 7 shows a table with correlations of both the primitives of the model and the components of a linear error decomposition applied to the simulation of the model. First, it is clear from the correlations that the equilibrium of this model displays positive assortative matching: the correlation between worker types  $h$  and job types  $h$  is 0.59. Also, because workers sort positively with firms they end up sorting positively with other workers as well: the correlation between  $h$  and the average value of  $h$  for the coworkers is 0.53. Things change when we try to make inference using the results of the linear decomposition with fixed effects. In that case the correlation between the fixed effects, which would ideally capture  $\text{corr}(h, p)$ , is 0.07, which is much smaller than the

correlation between the primitives. This is because of the downward distortion introduced by the firm's outside value discussed above. However, as we anticipated this distortion does not happen when we look at the correlation between the fixed effects of the worker and his coworkers: this value is 0.61, which is fairly close to the value of  $Corr(h, p)$  and  $Corr(h, h_{other})$ . This ordering is consistent with the patterns observed in the data.

**Table 7 - Model Simulation**

| <b>A – Correlation Between Primitives</b>                    |      |         |          |             |                   |
|--------------------------------------------------------------|------|---------|----------|-------------|-------------------|
|                                                              | STD  | $\ln w$ | $h$      | $p$         | $h_{others}$      |
| Log Wages                                                    | 1.30 | 1       | 0.96     | 0.67        | 0.6               |
| h                                                            | 0.50 | 0.96    | 1        | <b>0.59</b> | <b>0.53</b>       |
| p                                                            | 0.38 | 0.67    | 0.59     | 1           | 0.77              |
| h <sub>others</sub>                                          | 0.41 | 0.6     | 0.53     | 0.77        | 1                 |
| <b>B – Correlation Between Components of Wage Regression</b> |      |         |          |             |                   |
|                                                              | STD  | $\ln w$ | $\alpha$ | $\psi$      | $\alpha_{outros}$ |
| Log Wages                                                    | 1.25 | 1       | 0.99     | 0.17        | 0.62              |
| Worker fe                                                    | 1.23 | 0.99    | 1        | <b>0.07</b> | <b>0.61</b>       |
| Fim fe                                                       | 0.12 | 0.17    | 0.07     | 1           | 0.18              |
| Coworker fe                                                  | 0.83 | 0.62    | 0.61     | 0.18        | 1                 |

Finally, we verify how does the pattern of allocation for workers who reallocate. Because of the positive assortative matching of the model, workers who go to low-wage firms keep going to these firms and vice-versa. Thus, it is not surprising that when reproducing Figure 2 using a simulation of the model we find the same patterns verified in the data: workers who show in a low-wage firm are very likely to show in a low-wage firm if relocated, and the opposite for workers who start from a high-wage firm.

Figure 4: Model simulation –  $\Pr[\text{FWnew} > \text{median}(\text{FW})]$  conditional on  $\text{Fwold}$



### 6.2.3 Counterfactual Experiment

In addition we make a simple experiment. What would happen to aggregate output and Wage inequality if we double the arrival rates of job offers, both when unemployed and on-the-job—i.e, if the search process becomes more efficient. Note that in this experiment we fix the amount of available resources but we increase the efficiency of the allocation process. The output in the economy increases 2.3% and wage inequality (measured by the STD of wages) increases almost 20%. This shows that the efficiency of the process of allocation of resources can have significant impact on real allocations. Although one may think that doubling the arrival rate of job offers is an exaggerated experiment, cross-country comparisons show sizeable differences for the estimate of these parameters. Jolivet et al [14] documents values as low as in Belgium (0.34) and Italy (0.41) , to values as high values as in the US (1.7) and Germany (0.77).

**Table 8 – Experiment: Doubling the Rates of Job Offers**

|                  | Before | After | $\Delta\%$ |
|------------------|--------|-------|------------|
| Aggregate Output | 1.31   | 1.34  | 2.29%      |
| Mean Wages       | 0.49   | 0.52  | 6.12%      |
| STD Wages        | 0.21   | 0.25  | 19.05%     |

## References

- [1] G. Becker, “A Theory of Marriage: Part I,” *The Journal of Political Economy*, vol. 81, no. 4, pp. 813–846, 1973.
- [2] R. Shimer and L. Smith, “Assortative Matching and Search,” *Econometrica*, vol. 68, no. 2, pp. 343–369, 2000.
- [3] C. Teulings and P. Gautier, “The Right Man for the Job,” *Review of Economic Studies*, vol. 71, no. 2, pp. 553–580, 2004.
- [4] P. Gautier, C. Teulings, and A. van Vuuren, *On-the-job Search and Sorting*. Centre for Economic Policy Research, 2006.
- [5] R. Lentz, “Sorting in a General Equilibrium On-the-Job Search Model,” 2007.
- [6] J. Abowd, F. Kramarz, P. Lengerman, and S. Perez-Duarte, “Are Good Workers Employed by Good Firms? A Test of a simple assortative matching model for France and the United States,” tech. rep., mimeo, 2004.
- [7] G. Moscarini, “Job Matching and the Wage Distribution,” *Econometrica*, vol. 73, no. 2, pp. 481–516, 2005.
- [8] C. Bontemps, J. Robin, and G. Berg, “An empirical equilibrium job search model with continuously distributed heterogeneity of workers’ opportunity costs of employment and firms productivities, and search on the job.” 1998.
- [9] F. Postel-Vinay and J. Robin, “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity,” *Econometrica*, vol. 70, no. 6, pp. 2295–2350, 2002.
- [10] N. Menezes-Filho, M. Muendler, and G. Ramey, *The structure of worker compensation in Brazil, with a comparison to France and the United States*. CES, 2006.
- [11] J. Abowd, F. Kramarz, and D. Margolis, “High Wage Workers and High Wage Firms,” *Econometrica*, vol. 67, no. 2, pp. 251–333, 1999.
- [12] J. Abowd, R. Creecy, and F. Kramarz, “Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data,” *Technical Paper*, vol. 6, pp. 2002–06, 2002.
- [13] B. Christensen, R. Lentz, D. Mortensen, G. Neumann, and A. Werwatz, “On the job search and wage dispersion,” *Journal of Labor Economics*, vol. 23, pp. 31–58, 2005.

- [14] G. Jolivet, F. Postel-Vinay, and J. Robin, “The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US,” 2006. CREST-INSEE mimeo. Forthcoming in H. Bunzel, BJ Christensen, GR Neumann, and JM Robin, editors, Structural Models of Wage and Employment Dynamics, Elsevier.