

Federal Reserve Bank of Minneapolis

January 2008

Trade Liberalization, Growth, and Productivity*

(Preliminary, please do not quote)

Claustre Bajona
Ryerson University

Mark J. Gibson
Washington State University

Timothy J. Kehoe
University of Minnesota,
Federal Reserve Bank of Minneapolis,
and National Bureau of Economic Research

Kim J. Ruhl
University of Texas at Austin

ABSTRACT

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. This paper investigates the theoretical relation between trade and growth. We show that standard models — including Ricardian models, Heckscher-Ohlin models, monopolistic competition models with homogeneous firms, and monopolistic competition models with heterogeneous firms — predict that opening to trade increases welfare, not necessarily real GDP as measured in the data. In a dynamic model where trade changes the incentives to accumulate factors of production, trade liberalization may lower growth rates even as it increases welfare. To the extent that trade liberalization leads to higher rates of growth in real GDP, it must do so primarily through mechanisms outside of those analyzed in standard models.

*This paper was prepared for the conference “New Directions in International Trade Theory” at the University of Nottingham. We thank the participants and the discussant, Doug Nelson, for comments. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

How does trade liberalization affect a country's growth and productivity? How does it affect a country's social welfare? As Rodriguez and Rodrik (2001) point out, "growth and welfare are not the same thing. Trade policies can have positive effects on welfare without affecting the rate of economic growth."

There is a lively debate about the impact of trade liberalization on economic growth measured as growth in real gross domestic product (GDP). Most of this literature focuses on the empirical relation between trade and growth. The findings are mixed. Even though many studies find a connection between trade, or some other measure of openness, and growth, the literature has been subject to criticism. In particular, Rodriguez and Rodrik (2001), after a careful analysis of the methodology used in the literature, are skeptical that these studies find a connection between *trade policy* and growth. Slaughter (2001), raises the issue that this literature in general does not address the specific mechanisms through which trade may affect growth.

In this paper we investigate the theoretical relationship between trade policy and economic growth. We do so using simple versions of some of the most common international trade models, including a static Heckscher-Ohlin model, a Ricardian model with a continuum of goods, a monopolistic competition model with homogeneous firms, a monopolistic competition model with heterogeneous firms, and a dynamic Heckscher-Ohlin model. These models allow us to investigate a number of specific mechanisms by which trade liberalization is commonly thought to enhance growth or productivity: improvements in the terms of trade, increases in product variety, reallocation toward more productive firms, and increased incentive to accumulate capital.

We consider simple versions of the models for which there exist analytical solutions to the autarky and free trade equilibria. We then analyze the (extreme) case of trade liberalization by comparing autarky and free trade. Given that the objective of our paper is to directly compare the different theories with the empirical work, we measure real GDP in each of these models as real GDP is typically measured in the data, as GDP at constant prices. In each model the supply of labor is fixed, so changes in real GDP are also changes in measured labor productivity. We then contrast real GDP with a theoretical measure of real income (a measure of social welfare).

We find that in each model, trade liberalization improves social welfare. This is to be expected, but our results on real GDP may come as a surprise to many economists. In the static models, there is no general connection between trade liberalization and increases in real GDP per capita (as measured in the data) — the relationship may even be negative. Furthermore, theoretically equivalent versions of the models may have different real GDP measures. In particular, whether the traded goods are final goods or intermediate goods into the production of a final good that is used in consumption and investment (Ethier, 1979) matters in determining measured real GDP.

In a dynamic model with capital accumulation, some countries will have slower rates of growth under free trade than under autarky. Opening to trade improves welfare, but does not necessarily increase real GDP per capita or speed up growth. If openness does in fact lead to large increases in real GDP, these increases do not come from the standard mechanisms of international trade.

There is a vast empirical literature on the relationship between trade and growth. This literature typically studies the correlation between some measure of openness—for example, trade relative to GDP—and the growth of real GDP or real GDP per capita. Early papers in this line of research include Michaely (1977) and Balassa (1978). Lewer and Van den Berg (2003) present an extensive survey of this literature. They argue that most studies in this literature find a positive relationship between trade volume and growth and that they are fairly consistent on the size of this relationship. Other studies that find a positive relationship between trade openness and growth (using different techniques and openness measures) include World Bank (1987), Dollar (1992), Sachs and Warner (1995), Frankel and Romer (1999), Hall and Jones (1999), and Dollar and Kraay (2004).

Rodriguez and Rodrik (2001) question the findings of these studies. They argue that the indicators of openness used in them are either bad measures of trade barriers or are highly correlated with variables that also affect the growth rate of income. In the latter case, the studies may be attributing to trade policy the negative effects on growth of those other variables. Following this argument, Rodrik, Subramanian, and Trebbi (2002) find that openness has no significant effect on growth once institution-related variables are added in the regression analysis. Several studies using tariff rates as their specific

measures of openness have found the relationship between trade policy and growth to depend on a country's level of development. In particular, Yanikkaya (2003) and DeJong and Ripoll (2006) find a negative relationship between trade openness and growth for developing countries.

Wacziarg (2001) and Hall and Jones (1999) find that trade affects growth mainly through capital investment and productivity. A smaller set of papers study the relationship between openness to trade and productivity. Examples are Alcalá and Ciccone (2004) and Hall and Jones (1999), both of which find a significant positive relationship between trade and productivity.

Theoretical studies on the relationship between trade and growth do not offer a clear view on whether there should be a relationship between trade openness (measured as lower trade barriers) and growth in income. Models following the endogenous growth literature with increasing returns, learning-by-doing, or knowledge spillovers predict that opening to trade increases growth in the world as a whole, but may decrease growth in developing countries if they specialize in the production of goods with less potential for learning. Young (1991), Grossman and Helpman (1991), and Lucas (1988) are examples of papers in this area. By contrast, Rivera-Batiz and Romer (1991) find that trade leads to higher growth for all countries by promoting investment in research and development.

Models of trade using the Dixit-Stiglitz theory of industrial organization have typically focused on the effects of trade liberalization on welfare. Krugman (1979) shows, for instance, that trade liberalization leads to welfare increases because of increases in product variety. Ethier (1979) arrives at a similar conclusion in a model where the traded goods are intermediate inputs into the production of a final (non-traded) consumption good. The production of the final good uses a Dixit-Stiglitz type of technology, which has increasing returns to scale in product variety.

Melitz (2003) incorporates heterogeneous firms into a Krugman model and finds that trade liberalization increases a theoretical measure of productivity. When productivity is measured in the model as in the data, though, Gibson (2007) shows that trade liberalization does not, in general, increase productivity in these sorts of models. The increase is, rather, in welfare. Gibson (2007) finds that adding mechanisms to allow for technology adoption generate increases in measured productivity from trade

liberalization. Chaney (2006) also considers a simple model of heterogeneous firms, similar to the one we study here.

Standard growth models also do not have a clear prediction for the relationship between trade and growth. In particular, in dynamic Heckscher-Ohlin models — models that integrate a neoclassical growth model with a Heckscher-Ohlin model of trade — opening to trade may increase or decrease a country's growth rate of income depending on parameter values. Trade may slow down growth in the capital-scarce country even even though it raises welfare. Papers in this literature are Ventura (1997), Cuñat and Maffezzoli (2004), and Bajona and Kehoe (2006a).

2. General approach and measurement

In this paper we consider five commonly used models of trade. In each model we choose standard functional forms and, as needed, make assumptions so as to obtain analytical solutions for both the autarky equilibrium and the free trade equilibrium.¹ We analyze two versions of each model: one where the traded goods are final goods that are consumed (and invested), and another where the traded goods are intermediate inputs in to the production of a final, non-traded, good that is then consumed or invested (as in Ethier 1979). Even though these two versions of the model are theoretically equivalent, measured real GDP is, in general, different under each specification. Throughout the paper we denote autarky equilibrium objects by a superscript A and free trade equilibrium objects by a superscript T .

In each model we measure real GDP as it is measured in the data. We then contrast this with a theoretical measure of real income, or social welfare. We define this perfect real income index as $v = e^u$, where u is the utility function faced by the consumer. Gibson (2007) and Kehoe and Ruhl (2007) show that there may be substantial differences between the data-based measure of income and this theoretical measure.

We strive to measure statistics in our models the same way they are measured in the data. This allows us to directly compare the results of the model with the data. The main issue here is the measurement of real GDP. Empirical studies use real GDP as

¹ By comparing autarky and free trade we examine the extreme case of trade liberalization. In most of the models, however, it is straightforward to add *ad valorem* tariffs or iceberg transportation costs.

reported in the national income and product accounts. This is either GDP at constant prices or GDP at current prices deflated by a chain-weighted price index. We measure real GDP in each of our models as GDP at constant prices.² For instance, in each of the static models we measure real GDP as GDP at autarky prices. In the dynamic model, we measure real GDP as GDP at period-0 prices. Throughout the paper we denote GDP at current prices by *gdp* and GDP at constant prices by *GDP*.

Finally, in each model we study the effects of trade liberalization on total factor productivity, measured using real GDP. In the static models, given that capital and labor are fixed, changes in real GDP translate into changes in total factor productivity. In the dynamic Heckscher-Ohlin model this is not the case, since capital is accumulated over time.

3. Terms of trade and real GDP growth

Kehoe and Ruhl (2007) study how terms of trade affect real GDP in a small open economy model. They show that in standard models income effects due to changes in the terms of trade are not reflected in data-based measures of real GDP. Similar issues are addressed by Diewert and Morrison (1986) and Kohli (1983, 2004). In this section we study the effects of trade on real GDP in traditional trade theory models –Ricardian and Heckscher-Ohlin frameworks. In these models, trade affects income through changes in relative prices. Improvements in the terms of trade — the price of imports relative to the price of exports — lead to reallocation of resources towards goods in which a country has comparative advantage. Comparative advantage is driven by differences in technology, as in Ricardian models, or in factor endowments, as in Heckscher-Ohlin models. In this section we consider how changes in the terms of trade affect real GDP in both a Heckscher-Ohlin model and a Ricardian model with a continuum of goods. For each framework we consider two different specifications: (i) trade is in final goods, and (ii) trade is in intermediate goods. Under both specifications we obtain that trade strictly increases welfare, but never increases real GDP. More specifically, trade *lowers* real GDP in all cases except for the Ricardian model with trade in final goods where real GDP is the same under autarky and free trade. The linearity of the production possibility.

² The results of the paper should not change if a chain-weighted price index were used.

3.1. A static Heckscher-Ohlin model

Consider a world with n countries, where each country i , $i = 1, 2, \dots, n$, has measure L_i of consumers. Each consumer in country i is endowed with one unit of labor and $\bar{k}_i$ units of capital. There are two tradable goods, $j = 1, 2$, which are produced using capital and labor. The technology to produce the two tradable goods is the same across countries.

A consumer in country i derives utility from the consumption of both traded goods c_{ij} , $j = 1, 2$, with preferences given by:

$$u(c_{i1}, c_{i2}) = a_1 \log c_{i1} + a_2 \log c_{i2}, \quad (1)$$

where $a_1 + a_2 = 1$.

Good j , $j = 1, 2$, is produced by combining capital and labor according to the Cobb-Douglas production function (identical in all countries)

$$y_j = \theta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}, \quad (2)$$

where we assume that $\alpha_1 > \alpha_2$ (that is, good one is capital-intensive and good two is labor-intensive). The markets for the traded goods are perfectly competitive and producers are price takers.

The autarkic and free trade versions of this model differ in the conditions that determine feasibility in the traded goods' markets. Under autarky, both markets have to clear in each country, whereas under free trade the markets have to clear at the world level:

$$\sum_{i=1}^n L_i c_{ij} = \sum_{i=1}^n L_i y_{ij}. \quad (3)$$

Given our choice of functional forms for preferences and technologies, the model can be solved analytically. In both the autarkic and free trade equilibrium, prices and allocations can be expressed as functions of the allocation of capital per person. In what follows we list the expressions for the relevant variables for our analysis. The complete solution can be found in the appendix. To simplify the notation, let

$$A_j = a_j \alpha_j + a_2 \alpha_2 \quad (4)$$

$$A_2 = 1 - A_1 = a_1(1 - \alpha_1) + a_2(1 - \alpha_2) \quad (5)$$

$$D_j = \frac{\theta_j a_j \alpha_j^{\alpha_j} (1 - \alpha_j)^{1 - \alpha_j}}{A_1^{\alpha_j} A_2^{1 - \alpha_j}} \quad (6)$$

$$D = D_1^{a_1} D_2^{a_2} . \quad (7)$$

Autarky

The autarky prices for the traded goods, $j = 1, 2$, and the consumption and production allocations for country i , $i = 1 \dots n$, are

$$p_{ij}^A = \frac{a_j D}{D_j} \bar{k}_i^{-A_j - \alpha_j} , \quad (8)$$

$$c_{ij}^A = y_{ij}^A = D_j \bar{k}_i^{\alpha_j} \quad (9)$$

Our variables of interest are nominal and real GDP, productivity and welfare. Since we take autarky as the base year in computing real GDP, nominal and real GDP coincide in autarky:

$$gdp_i^A = GDP_i^A = p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A = D \bar{k}_i^{A_i} , \quad (10)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^A = \frac{GDP_i^A}{\bar{k}_i^{A_i}} = D . \quad (11)$$

Using our real income index to measure welfare, we obtain:

$$v_i^A = (c_{i1}^A)^{a_1} (c_{i2}^A)^{a_2} = D \bar{k}_i^{A_i} . \quad (12)$$

Free trade

In the free trade equilibrium we focus on the case where countries have similar enough factor endowments so that all countries are in the cone of diversification. That is, letting

$$\bar{k} = \frac{\sum_{i=1}^n L_i \bar{k}_i}{\sum_{i=1}^n L_i} \quad (13)$$

$$\gamma_i = \frac{\bar{k}_i}{\bar{k}} \quad (14)$$

$$\kappa_j = \left(\frac{\alpha_j}{1 - \alpha_j} \right) \frac{A_2}{A_1}, \quad (15)$$

we examine the case where $\kappa_2 \leq \gamma_i \leq \kappa_1$, $i = 1, \dots, n$. In this case, the prices and aggregate variables of the free trade equilibrium can be obtained by solving for the equilibrium of the integrated economy (a closed economy with factor endowments equal to world factor endowments) and then splitting the aggregate allocations across countries in a way that is consistent with their factor endowments. Let $\bar{k}$ be as in equation (15). Then the prices for traded good j , $j = 1, 2$, and each country i 's production and consumption patterns are given by:

$$p_j^T = \frac{a_j D}{D_j} \bar{k}^{A - \alpha_j} \quad (16)$$

$$c_{ij}^T = (A_1 \gamma_i + A_2) D_j \bar{k}^{\alpha_j} \quad (17)$$

$$y_{ij}^T = \mu_{ij} D_j \bar{k}^{\alpha_j} \quad (18)$$

where γ_i is defined in equation (16), and μ_{ij} are:

$$\mu_{i1} = \frac{A_1 \gamma_i (1 - \alpha_2) - A_2 \alpha_2}{a_1 (\alpha_1 - \alpha_2)} \quad (19)$$

$$\mu_{i2} = \frac{A_2 \alpha_1 - A_1 \gamma_i (1 - \alpha_1)}{a_2 (\alpha_1 - \alpha_2)}. \quad (20)$$

Notice that setting $\gamma_i = 1$ we obtain the same values as in autarky.

In this version of the model, nominal and real GDP do not longer coincide.

Nominal GDP in country i , is GDP measured at current prices:

$$gdp_i^T = p_1^T y_{i1}^T + p_2^T y_{i2}^T = (A_1 \gamma_i + A_2) D \bar{k}^A, \quad (21)$$

whereas real GDP is measured at autarky prices:

$$\begin{aligned} GDP_i^T &= p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T \\ &= \frac{\gamma_i^{-\alpha_1} (A_1 \gamma_i (1 - \alpha_2) - A_2 \alpha_2) + \gamma_i^{-\alpha_2} (A_2 \alpha_1 - A_1 \gamma_i (1 - \alpha_1))}{\alpha_1 - \alpha_2} D \bar{k}_i^A. \end{aligned} \quad (22)$$

Welfare under free trade is given by:

$$v_i^T = (c_{i1}^T)^{a_1} (c_{i2}^T)^{a_2} = (A_1 \gamma_i + A_2) D \bar{k}^{A_i}. \quad (23)$$

Effect of trade liberalization

Trade liberalization in this model increases the prices of the exported goods and decreases the prices of the imported goods, improving the terms of trade. This improvement in the terms of trade increases welfare, but decreases measured real GDP and productivity. The intuition for the latter is simple: given factor endowments, the autarkic production pattern in country i is the optimal production pattern for country i at the autarkic prices. Any deviation from that production pattern will lower the value of production at those prices. Since productivity is measured using real GDP, a decrease in real GDP also implies a decrease in measured productivity.

Proposition 1. In the static Heckscher-Ohlin model described above, for any country $i = 1, \dots, n$ for which $\gamma_i \neq 1$, following trade liberalization:

- (i) welfare strictly increases
- (ii) real GDP and productivity strictly decrease.

If $\gamma^i = 1$ all measures stay the same.

Proof. (i) Comparing (14) and (23) we need to show that to show that

$$(A_1 \gamma_i + A_2) D \bar{k}^{A_i} > D \bar{k}_i^{A_i}, \quad (24)$$

or equivalently, using the definition of γ_i , that

$$A_1 \gamma_i + 1 - A_1 > \gamma_i^{A_i}. \quad (25)$$

Define $f(\gamma) = A_1 \gamma + 1 - A_1$ and $g(\gamma) = \gamma^{A_i}$. The result comes from the fact that,

$$f(1) = g(1), \quad f'(1) = g'(1), \quad f'(\gamma) > g'(\gamma) \text{ if } \gamma < 1 \text{ and } f'(\gamma) < g'(\gamma) \text{ if } \gamma > 1.$$

(ii) We need to show that

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T. \quad (26)$$

Define the function

$$\begin{aligned}
\pi(p_1, p_2, k_i) &= \max \left[p_1 \theta_1 k_{i1}^{\alpha_1} \ell_{i1}^{1-\alpha_1} + p_2 \theta_2 k_{i2}^{\alpha_2} \ell_{i2}^{1-\alpha_2} \right] \\
\text{s.t. } &k_{i1} + k_{i2} \leq k_i \\
&\ell_{i1} + \ell_{i2} \leq 1 \\
&k_{ij} \geq 0, \ell_{ij} \geq 0
\end{aligned} \tag{27}$$

Since $\alpha_1 > \alpha_2$, this function is strictly concave. Notice that

$$\pi(p_{i1}^A, p_{i2}^A, \bar{k}_i) = p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A. \tag{28}$$

The free trade allocation also satisfies the feasibility constraints in (27), so

$$p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A > p_{i1}^A y_{i1}^T + p_{i2}^A y_{i2}^T, \tag{29}$$

where the strict inequality follows from the strict concavity of π . Figure 1 illustrates the proof.

The decrease in productivity follows immediately from the decrease in real GDP. ■

Traded goods as intermediate inputs

Consider a version of the previous model where individuals instead of consuming the traded goods directly, they purchase a final, non-traded good Y , which is produced using both traded goods as intermediate inputs. Assume that the consumer preferences are represented by the utility function $u(C) = \log C$, and that good Y is produced using the technology:

$$Y = c_1^{a_1} c_2^{a_2} \tag{30}$$

where c_i is the input of good i and a_i are as described above. It is straight forward to see that this model is theoretically equivalent to the model presented above, where in equilibrium $C = (w + r\bar{k}) / P$, and $P = a_1^{a_1} a_2^{a_2} p_1^{a_1} p_2^{a_2}$.

Real GDP under autarky is given by:

$$\widehat{GDP}_i^A = P_i^A C_i^A = D \bar{k}_i^A \tag{31}$$

and under free trade:

$$\widehat{GDP}_i^T = P_i^A C_i^T + \bar{p}_{i1} (y_{i1}^T - c_{i1}^T) + \bar{p}_{i2} (y_{i2}^T - c_{i2}^T) \tag{32}$$

where $\bar{p}_{ij}$ denotes the base-year price of intermediate good $j=1,2$. Here we use hats to distinguish the value of real GDP from its value in the case where final goods are traded. Notice that all the other variables are the same in both frameworks, given that they are theoretically equivalent.

At this point a question arises on which prices to use as $\bar{p}_{ij}$. For the exported good, the autarky price is used as base-year price. For the imported good, the choice is not that clear. Given that in the model domestically-produced and imported varieties of a given good are perfect substitutes, the sensible approach seems to be to consider the autarky price of the imported good as a base-year price also.

Proposition 2: Consider the version of the static Heckscher-Ohlin model with a final, non-traded, good stated above. for any country $i=1,\dots,n$ for which $\gamma_i \neq 1$, following trade liberalization real GDP and productivity strictly decrease, that is $\widehat{GDP}_i^A > \widehat{GDP}_i^T$. Furthermore, real GDP under free trade is smaller than in the framework with final traded goods. If $\gamma^i = 1$ all measures stay the same.

Proof: Real GDP under autarky is the same as in the final traded goods case, and it is equal to

$$\widehat{GDP}_i^A = P_i^A Y_i^A = p_{i1}^A y_{i1}^A + p_{i2}^A y_{i2}^A = GDP_i^A.$$

Real GDP under free trade can be written as:

$$\widehat{GDP}_i^T = (P_i^A C_i^T - p_{i1}^A c_{i1}^T - p_{j2}^A c_{i2}^T) + (p_{i1}^A y_{i1}^T + p_{j2}^A y_{i2}^T),$$

which can be expressed as a function of real GDP in the final traded goods framework as follows:

$$\widehat{GDP}_i^T = (P_i^A C_i^T - p_{i1}^A c_{i1}^T - p_{j2}^A c_{i2}^T) + GDP_i^T$$

From proposition 1, $GDP_i^A > GDP_i^T$. Therefore, to prove the proposition it is enough to show that $P_i^A C_i^T - p_{i1}^A c_{i1}^T - p_{j2}^A c_{i2}^T < 0$. For this, notice that as long as $\gamma_i \neq 1$,

$p_1^T / p_2^T \neq p_{i1}^A / p_{j2}^A$. Strict concavity and homotheticity of the production function for the

final good implies that the bundle c_{i1}^T, c_{i2}^T costs strictly more than the cheapest bundle that produces C^T at the autarky prices. Therefore, $C_i^T - p_{i1}^A x_{i1}^T - p_{j2}^A x_{i2}^T < 0$, and

$$\widehat{GDP}_i^A > GDP_i^T > \widehat{GDP}_i^T.$$

If $\gamma^i = 1$, then $p_1^T / p_2^T = p_{i1}^A / p_{j2}^A$ and both measures are equal. ■

3.2. A Ricardian model with a continuum of goods

Consider a world with two symmetric countries. In each country i , $i = 1, 2$, the representative consumer is endowed with $\bar{\ell}$ units of labor. There is a continuum of tradable goods, $z \in [0, 1]$.

The representative consumer derives utility from the consumption of all tradable goods $c_i(z)$, $z \in [0, 1]$, with preferences given by

$$u(c_i) = \int_0^1 \log c_i(z) dz \quad (33)$$

Good z is produced using only labor. The production technology to produce good z differs across countries and it is given by:

$$y_i(z) = \ell_i(z) / a_i(z), \quad (34)$$

where $a_i(z)$ is the quantity of labor required to produce one unit of good z in country i .

Let us assume that goods are ordered from lowest to highest unit labor requirements in country 1, and that countries are symmetric in terms of productivities in the sense that good z in country 1 uses the same technology in production as good $1 - z$ in country 2.

That is, let

$$a_1(z) = e^{\alpha z} \quad (35)$$

$$a_2(z) = e^{\alpha(1-z)}, \quad (36)$$

where $\alpha > 0$. The markets for the traded goods are perfectly competitive and producers are price takers.

The autarkic and free trade versions of this model differ in the conditions that determine feasibility in the traded goods' markets. Under autarky, the market for good

all goods $z \in [0,1]$ has to clear in each country, whereas under free trade, only the world market for each good has to clear:

$$c_1(z) + c_2(z) = y_1(z) + y_2(z). \quad (37)$$

We have chosen functional forms for which there is an analytical solution of the model. In what follows, we list the values of the relevant variables, as well as the values for nominal and real GDP, productivity and welfare for the model under both, autarky and free trade.

Autarky

Let us normalize $w_i = 1$. The autarkic prices for good $z \in [0,1]$ in each country and the consumption and production levels are:

$$p_1^A(z) = e^{\alpha z} \quad (38)$$

$$p_2^A(z) = e^{\alpha(1-z)}. \quad (39)$$

$$c_i^A(z) = y_i^A(z) = \frac{\bar{\ell}}{p_i^A(z)}. \quad (40)$$

In measuring real GDP, we take the autarkic prices as the base prices. Therefore, in the autarkic equilibrium, nominal and real GDP coincide, and take the value:

$$gdp_i^A = GDP_i^A = \int_0^1 p_i^A(z) y_i^A(z) dz = \bar{\ell}. \quad (41)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^A = \frac{GDP_i^A}{\bar{\ell}} = 1. \quad (42)$$

Our measure of welfare becomes:

$$v_i^A = \exp \int_0^1 \log c_i^A(z) dz = \bar{\ell} e^{-\alpha^2/2}. \quad (43)$$

Free trade

Given the symmetry imposed in the model, we normalize $w_1 = w_2 = 1$. Country 1 produces and exports goods $z \in [0, 0.5]$ and country 2 produces and exports goods $z \in (0.5, 1]$. The prices of the goods and the consumption patterns in each country are

$$p^T(z) = \begin{cases} e^{\alpha z} & z \in [0, 0.5] \\ e^{\alpha(1-z)} & z \in (0.5, 1] \end{cases}. \quad (44)$$

$$c_1^T(z) = c_2^T(z) = \frac{\bar{\ell}}{p^T(z)}. \quad (45)$$

The production patterns are as follows. For goods $z \in [0, 0.5]$, the production plans are

$$\text{Equation changed} \quad y_1^T(z) = \frac{2\bar{\ell}}{p^T(z)}, \quad y_2^T(z) = 0. \quad (46)$$

For goods $z \in (0.5, 1]$, the production plans are

$$y_1^T(z) = 0, \quad y_2^T(z) = \frac{2\bar{\ell}}{p^T(z)}. \quad (47)$$

Regarding the variables of interest in our paper notice that, in a given country, autarky and trade prices differ for the goods that the country is importing, but they are the same for the goods that the country is exporting. Overall, though, the changes in production completely offset the changes in prices, and nominal and real GDP coincide in the free trade model. In particular, GDP at current prices is:

$$gdp_i^T = \int_0^1 p^T(z) y_i^T(z) dz = \bar{\ell}, \quad (48)$$

and GDP at autarky prices is

$$GDP_i^T = \int_0^1 p_i^A(z) y_i^T(z) dz = \bar{\ell}. \quad (49)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^T = \frac{GDP_i^T}{\bar{\ell}} = 1 \quad (50)$$

Using the same measure of welfare as in the autarky model, we obtain that welfare in the free trade economy equals:

$$v_i^T = \exp \int_0^1 \log c_i^T(z) dz = \bar{\ell}. \quad (51)$$

Effect of trade liberalization

After trade liberalization, the prices of each country's imports decrease, resulting in an improvement in the terms of trade. As a result, real income increases in both

countries, but real GDP and productivity stay the same. The next proposition summarizes the results:

Proposition 3: In the Ricardian model described above, in each country $i = 1, 2$, following trade liberalization:

- (i) welfare increases
- (ii) real GDP and measured productivity do not change.

Proof: (i) We need to show $\bar{\ell}e^{-\alpha^2/2} < \bar{\ell}$, which follows directly from the fact that $e^{-\alpha^2/2} < 1$.

(ii) We prove this result for general unit labor requirement functions, mimicking the proof in proposition 1.

Define the function

$$\begin{aligned} \pi_i(p) &= \max \left[\int_0^1 p_i(z) \frac{\ell_i(z)}{a_i(z)} dz \right] \\ \text{s.t. } \int_0^1 \ell_i(z) dz &= \bar{\ell} \\ \ell_i(z) &\geq 0 \text{ for all } z \in [0, 1] \end{aligned} \quad (52)$$

Notice that, given the linearity of the production functions, under autarky $p_i^A(z) / p_i^A(z') = a_i^A(z) / a_i^A(z')$ and all bundle in the production possibility frontier cost the same. In particular, since $\{y_i^A(z)\}$ and $\{y_i^T(z)\}$ are both in the frontier, they have the same value under autarky prices, and $GDP_i^A = GDP_i^T$. ■

Traded goods as intermediate inputs

Consider a version of the previous model where individuals consume final, non-traded good Y , which is produced using both traded goods as intermediate inputs. Assume that the consumer preferences are represented by the utility function $u(C) = \log C$, and that good Y is produced using the technology:

$$Y = e^{\int_0^1 \log c(z) dz} \quad (53)$$

where $c(z)$ is the input of good z . It is straight forward to see that this model is theoretically equivalent to the model presented above, where in equilibrium $C = \bar{\ell} / P$, and $P = \exp\left(\int_0^1 \log p(z) dz\right)$.

Real GDP under autarky is given by:

$$\widehat{GDP}_i^A = P_i^A C_i^A = \bar{\ell} \quad (54)$$

and under free trade:

$$\widehat{GDP}_i^T = P_i^A C_i^T + \int_0^{1/2} p_i^A(z) (y_i^T(z) - c_i^T(z)) dz - \int_0^{1/2} \bar{p}_i(z) c_i^T(z) dz \quad (55)$$

where $\bar{p}_i(z)$ denotes the base-year price of intermediate good $j = 1, 2$.

The same issues regarding the base-year prices as in the previous section arise here. In contrast with the previous section, though, we obtain that under this specification of the model real GDP is higher under free trade than under autarky.

Proposition 4: Consider the version of the Ricardian model with a continuum of goods with a final, non-traded, good stated above. If $p_i^T(z) / p_i^T(z') \neq p_i^A(z) / p_i^A(z')$ for some $z, z' \in [0, 1]$, then $\widehat{GDP}_i^T > \widehat{GDP}_i^A$.

Proof: We also prove this result for general unit labor requirements. Following the proof of proposition 2, notice that $\widehat{GDP}_i^A = GDP_i^A$ and write real GDP under free trade as:

$$\widehat{GDP}_i^T = \left(P_i^A C_i^T - \int_0^1 p_i^A(z) c_i^T(z) dz \right) - \int_0^{1/2} p_i^A(z) y_i^T(z) dz,$$

which can be expressed as a function of real GDP in the final traded goods framework as follows:

$$\widehat{GDP}_i^T = \left(P_i^A C_i^T - \int_0^1 p_i^A(z) c_i^T(z) dz \right) - GDP_i^T$$

From proposition 2, $GDP_i^A = GDP_i^T$. Therefore, to prove the proposition it is enough to

show that $P_i^A C_i^T - \int_0^1 p_i^A(z) c_i^T(z) dz < 0$. For this, notice that as long as

$p_i^T(z) / p_i^T(z') \neq p_i^A(z) / p_i^A(z')$ for some $z, z' \in [0, 1]$, strict concavity and homotheticity

of the production function for the final good implies that the bundle $\{c_i^T(z)\}$ costs strictly more than the cheapest bundle that produces C^T at the autarky prices. Therefore,

$$P_i^A C_i^T - \int_0^1 p_i^A(z) c_i^T(z) dz < 0, \text{ and } \widehat{GDP}_i^A = GDP_i^T > \widehat{GDP}_i^T. \blacksquare$$

4. Product variety and real GDP growth

In standard monopolistic competition models with homogeneous firms, trade liberalization leads to an increase in the number of product varieties available to the consumer. This increase in product variety leads to an increase in real income, but does it lead to an increase in real GDP? We find that this depends on the nature of competition in the product market. If there is a continuum of product varieties, then real GDP does not change. If there is a finite number of product varieties, then real GDP increases. The reason is that, with Cournot (or Bertrand) competition among firms, markups over marginal cost decrease when the number of firms supplying goods to a market increases. Competition increases, and so does production. We make this point using a monopolistic competition model with a finite number of product varieties.

A monopolistic competition model with homogeneous firms

Consider a world with n countries. In each country i , $i = 1, 2, \dots, n$, the representative consumer is endowed with $\bar{\ell}_i$ units of labor. Let J_i be the number of goods available to the consumer in country i . Consumer i derives utility from the consumption of goods of each variety c_{ij} , $j = 1, 2, \dots, J_i$, and has preferences represented by

$$u(c) = (1/\rho) \log \sum_{j=1}^{J_i} c_{ij}^\rho \quad (56)$$

A firm producing good j in country i has the increasing-returns-to-scale technology

$$y_{ij} = (1/b) \max[\ell_{ij} - f, 0], \quad (57)$$

where f is the fixed cost of operating, measured in units of labor.

There is Cournot competition among firms. Taking as given the consumer's demand function and the decisions of all other firms, a firm's problem is to choose the quantity of output that maximizes its profits. There is free entry of firms, so there are no aggregate profits. Notice that, due to the existence of increasing returns to scale, different firms produce different goods.

Let N_i be the number of firms in country i . Under autarky, $J_i = N_i$ and

$$c_{ij} = y_{ij} \text{ for } j = 1, \dots, J_i. \quad (58)$$

Under free trade, $J = \sum_{i=1}^n N_i$ and if good j is produced in country i ,

$$y_{ij} = \sum_{i=1}^n c_{ij} \text{ for } j = 1, \dots, J. \quad (59)$$

Autarky

As in the previous sections, let us normalize $w_i = 1$. The objective of each firm is to maximize profits, taking as given its indirect demand function. Consumer i 's indirect demand function for good j is

$$p_{ij} = \frac{c_{ij}^{\rho-1}}{\sum_{m=1}^{J_i} c_{im}^{\rho}} \bar{\ell}_i. \quad (60)$$

Imposing symmetry across firms (which allows us to drop the subscript j), from the firm's problem we obtain that production of each good equals

$$c_i^A = y_i^A = \frac{\rho(J_i^A - 1) \bar{\ell}_i}{(J_i^A)^2 b}, \quad (61)$$

and the price of each good becomes:

$$p_i^A = \frac{b J_i^A}{\rho(J_i^A - 1)}. \quad (62)$$

Since there is free entry, firm profits must be zero in equilibrium, which determines the number of firms in each country³

Equation changed
$$J_i^A = N_i^A = \frac{(1-\rho)\bar{\ell}_i + \sqrt{(1-\rho)^2\bar{\ell}_i^2 + 4f\rho\bar{\ell}_i}}{2f}. \quad (63)$$

As in the previous models, nominal and real GDP coincide in autarky:

$$gdp_i^A = GDP_i^A = N_i^A p_i^A y_i^A = \bar{\ell}_i. \quad (64)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^A = \frac{GDP_i^A}{\bar{\ell}} = 1, \quad (65)$$

and our measure of welfare becomes

$$v_i^A = \left(J_i^A\right)^{\frac{1-\rho}{\rho}} \frac{\rho(J_i^A - 1)}{J_i^A b} \bar{\ell}_i \quad (66)$$

Free trade

We can use the above approach to solve for the integrated equilibrium of the world economy, in which the supply of labor is $\bar{\ell} = \sum_{i=1}^n \bar{\ell}_i$. By normalizing $w = 1$ we obtain production in each industry and prices as:

$$y^T = \frac{\rho(J^T - 1)\bar{\ell}}{(J^T)^2 b} \quad (67)$$

$$p^T = \frac{bJ^T}{\rho(J^T - 1)}. \quad (68)$$

The total number of firms in the world becomes:

³ Notice that the number of firms is not necessarily an integer. Alternatively, we could allow for aggregate profits and calculate N_i as the integer such that there are nonnegative profits but that, if one more firm entered, profits would be negative.

$$J^T = \frac{(1-\rho)\bar{\ell} + \sqrt{(1-\rho)^2 \bar{\ell}^2 + 4f\rho\bar{\ell}}}{2f}. \quad (69)$$

Each country's share of consumption, as well as its number of firms depends on relative income. The expressions are, respectively

$$c_i^T = \frac{\bar{\ell}_i}{\bar{\ell}} \bar{y}^T. \quad (70)$$

and

$$N_i^T = \frac{\bar{\ell}_i}{\bar{\ell}} J^T. \quad (71)$$

Notice that the equilibrium values for free trade are the same as those for autarky if $\bar{\ell}_i = \bar{\ell}$.

Regarding our variables of interest, nominal GDP in each country i becomes:

$$gdp_i^T = N_i^T p^T y^T = \bar{\ell}_i, \quad (72)$$

whereas real GDP, measured at autarky prices is:

$$GDP_i^T = N_i^T p_i^A y^T = \frac{J_i^A}{(J_i^A - 1)} \frac{(J^T - 1)}{J^T} \bar{\ell}_i. \quad (73)$$

Total factor productivity, measured using real GDP is:

$$TFP_i^T = \frac{GDP_i^T}{\bar{\ell}_i} = \frac{J_i^A}{(J_i^A - 1)} \frac{(J^T - 1)}{J^T} \quad (74)$$

Welfare becomes:

$$v_i^T = (J^T)^{\frac{1-\rho}{\rho}} \frac{\rho(J^T - 1)}{J^T b} \bar{\ell}_i. \quad (75)$$

Effect of trade liberalization

The next proposition states the effects of trade liberalization. In particular, we show that trade liberalization increases welfare in all countries and, contrary to the results in our static Heckscher-Ohlin and Ricardian models, real GDP and productivity also increase.

Proposition 5. In the model described above, assume that $\bar{\ell}_i < \bar{\ell}$. Then, in each country i , following trade liberalization:

- (i) welfare in country i increases
- (ii) real GDP and total factor productivity increase.

Proof. The proof reduces to showing that the number of varieties produced increases with trade, that is, $J^T > J_i^A$. See the appendix for details. ■

In this set up, real GDP increases after trade liberalization because markups decrease. Since $J^T > J_i^A$, there are more firms competing in each market. With Cournot competition, this lowers the markup over marginal cost:

$$\frac{J^T}{\rho(J^T - 1)} < \frac{J_i^A}{\rho(J_i^A - 1)}. \quad (76)$$

Notice that in the case where there is a continuum of product varieties, rather than a finite number, the markup over marginal cost is constant at $1/\rho$, regardless of trade policy. In this case, real GDP and productivity remain constant following trade liberalization, as in the Ricardian model with a continuum of goods.

Traded goods as intermediate inputs

Consider a version of the previous model where individuals consume final, non-traded good Y , which is produced using both traded goods as intermediate inputs. Assume that the consumer preferences are represented by the utility function $u(C) = \log C$, and that good Y is produced using the technology:

$$Y = \left(\sum_{j=1}^{J_i} c_{ij}^\rho \right)^{1/\rho} \quad (77)$$

where c_{ij} is the input of good j . For simplicity, we consider the completely symmetric case where $\bar{\ell}_i = \bar{\ell}/n$ for all i . It is straight forward to see that this model is theoretically equivalent to the model presented above, where in equilibrium $C = \bar{\ell}/P$, and

$$P = \left(\sum_{j=1}^{J_i} p_j^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho}.$$

Real GDP under autarky is given by:

$$GDP_i^A = P_i^A C_i^A = \bar{\ell} \quad (78)$$

and under free trade:

$$GDP_i^T = P_i^A C_i^T + N_i^T p_i^A (y_i^T - c_i^T) - \sum_{j \neq i} N_j^T p_j^A c_j^T = P_i^A C_i^T \quad (79)$$

where the last inequality follows from the symmetric endowments' assumption.

Proposition 6: Consider the version of the monopolistic competition model with homogeneous goods with a final, non-traded, good stated above. It is the case that $GDP_i^T > GDP_i^A$.

Proof: The proof mimics the proof of proposition 5(i).

5. Heterogeneous firms and productivity growth

Results in the previous model were obtained assuming that all firms faced the same technology. If firms are heterogeneous and there exist fixed costs of exporting, trade liberalization may lead to a reallocation of resources across firms. In a simple model, trade liberalization causes the least productive firms to exit and the most productive firms to become exporters. Intuitively, this reallocation of resources toward more productive firms should increase aggregate productivity. Surprisingly, we find that, given the way real GDP and productivity are measured, it does not. The finding here is explored further in Gibson (2007), where a positive mechanism is also provided.

A monopolistic competition model with heterogeneous firms

Consider a world with two symmetric countries, $i = 1, 2$. In each country i , the representative consumer is endowed with $\bar{\ell}$ units of labor and measure μ of potential firms (potential firms may choose not to operate). Each firm produces a differentiated good. Let Z_i be the set of goods available to consumers in country i . The consumer derives utility from the consumption of the differentiated goods, $c_i(z)$, $z \in Z_i$, with preferences give by

$$u(c_i) = (1/\rho) \log \int_{Z_i} c_i(z)^\rho dz, \quad (80)$$

where $\rho > 0$.

Firms differ in their productivity levels. Let $x(z)$ be the productivity level of the firm that produces good z in country i . This firm has the increasing-returns-to-scale technology

$$y_i(z) = \max \left[x(z) (\ell_i(z) - f_d), 0 \right], \quad (81)$$

where f_d is the fixed cost, in units of labor, of operating. If the economies are open to trade, then a firm can choose to export by paying an additional fixed cost of f_e units of labor.

Potential firms draw their productivities from a Pareto distribution $F(x) = 1 - x^{-\gamma}$, $x \geq 1$. The choice of one as the lower bound on the Pareto distribution can be thought of as a normalization. We further assume that $\gamma > \max[2, \rho/(1-\rho)]$.

Taking the consumer's demand functions as given, the firm's problem is to choose the profit-maximizing price. Each firm decides whether to operate. In the free trade environment, each firm also decides whether to export.

Autarky

We examine the case where some potential firms decide not to produce. In such case, there is a productivity cutoff $\bar{x}_d$, $\bar{x}_d > 1$, such that a firm with productivity x produces only if $x \geq \bar{x}_d$. Given that countries are symmetric, in what follows we omit the country subscripts. Let us normalize $w = 1$. The profit-maximizing prices are

$$p^A(x) = \frac{1}{\rho x}. \quad (82)$$

The productivity cutoff point is:

$$\bar{x}_d^A = \left(\frac{\mu(\gamma - \rho) f_d}{(\gamma(1 - \rho) - \rho) \bar{\ell}} \right)^{1/\gamma}, \quad (83)$$

and the demand for a good produced by a firm with productivity $x \geq \bar{x}_d^A$ is

$$c^A(x) = y^A(x) = \frac{\rho(\gamma(1-\rho)-\rho)(\bar{\ell} + \pi^A)x^{\frac{1}{1-\rho}}}{(1-\rho)\gamma\mu(\bar{x}_d^A)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}}, \quad (84)$$

Where $\pi^A = \rho\bar{\ell} / (\gamma - \rho)$.

Details on the derivation of these equations can be found in the appendix.

Nominal and real GDP in autarky become:

$$GDP^A = \mu \int_{\bar{x}_d^A}^{\infty} p^A(x) y^A(x) dF(x) = \frac{\gamma}{\gamma - \rho} \bar{\ell}. \quad (85)$$

From here we obtain the value of total factor productivity:

$$TFP^A = \frac{\gamma}{\gamma - \rho} \quad (86)$$

Welfare, measured using our real income index becomes:

$$v^A = \left(\mu \int_{\bar{x}_d^A}^{\infty} c^A(x)^\rho dF(x) \right)^{1/\rho} = \frac{\gamma}{(\gamma - \rho)P^A} \bar{\ell}, \quad (87)$$

where P^A is an aggregate price index the expression for which is given in the appendix (equation (140)).

Free trade

We again examine the case in which not all firms choose to produce. That is, firm z produces if $x(z) \geq \bar{x}_d$, $\bar{x}_d > 1$. With free trade, each firm faces an additional decision: whether to pay the fixed cost f_e to export, with $f_e > f_d$. There is a cutoff $\bar{x}_e$, $\bar{x}_e > \bar{x}_d$, such that firm z exports if $x(z) \geq \bar{x}_e$.

Since the countries are symmetric, we normalize $w_1 = w_2 = 1$. The profit-maximizing prices are

$$p^T(x) = \frac{1}{\rho x}. \quad (88)$$

By solving the model (details can be found in the appendix), we obtain the values of the productivity cutoff point to be:

$$\bar{x}_d^T = \left(\frac{\mu(\gamma - \rho) f_d \left(1 + (f_e / f_d)^{\frac{\rho - \gamma(1 - \rho)}{\rho}} \right)}{(\gamma(1 - \rho) - \rho) \bar{\ell}} \right)^{1/\gamma} \quad (89)$$

and the minimum productivity level for a firm to export is:

$$\bar{x}_e^T = \left(\frac{f_e}{f_d} \right)^{\frac{1 - \rho}{\rho}} \left(\frac{\mu(\gamma - \rho) f_d \left(1 + (f_e / f_d)^{\frac{\rho - \gamma(1 - \rho)}{\rho}} \right)}{(\gamma(1 - \rho) - \rho) \bar{\ell}} \right)^{1/\gamma} \quad (90)$$

The demand in a country for a good produced by a firm with productivity $x \geq \bar{x}_d^T$ is:

$$\begin{aligned} c^T(x) &= p^T(x)^{\frac{-1}{1 - \rho}} (P^T)^{\frac{\rho}{1 - \rho}} (\bar{\ell} + \pi^T) \\ &= \frac{\rho(\gamma(1 - \rho) - \rho) (\bar{\ell} + \pi^T) x^{\frac{1}{1 - \rho}}}{(1 - \rho) \gamma \mu \left((\bar{x}_d^T)^{\frac{\rho - \gamma(1 - \rho)}{1 - \rho}} + (\bar{x}_e^T)^{\frac{\rho - \gamma(1 - \rho)}{1 - \rho}} \right)}, \end{aligned} \quad (91)$$

where $\pi^T = \rho \bar{\ell} / (\gamma - \rho)$.

Therefore, production in each firm with productivity $x \geq \bar{x}_d^T$ becomes:

$$y^T(x) = \begin{cases} c^T(x) & \bar{x}_d^T \leq x < \bar{x}_e^T \\ 2c^T(x) & x \geq \bar{x}_e^T \end{cases}. \quad (92)$$

Using these expressions we obtain nominal GDP as

$$gdp^T = \mu \int_{\bar{x}_d^T}^{\infty} p^T(x) y^T(x) dF(x) = \frac{\gamma}{\gamma - \rho} \bar{\ell}, \quad (93)$$

Notice that real GDP, measured using autarky prices coincides with the nominal GDP:

$$GDP^T = \mu \int_{\bar{x}_d^T}^{\infty} p^A(x) y^T(x) dF(x) = \frac{\gamma}{\gamma - \rho} \bar{\ell}, \quad (94)$$

and, therefore, so does total factor productivity.

Our measure of welfare becomes:

$$v^T = \frac{\gamma}{(\gamma - \rho)P^T} \bar{\ell}, \quad (95)$$

where P^T is an aggregate price index the expression for which is given in the appendix (equation (142)).

Effect of trade liberalization

The next proposition summarizes the effect of trade liberalization in the economy. In particular, it states that trade liberalization increases the minimum productivity level at which firms operate. Real income increases, but real GDP and productivity stay the same.

Proposition 7. Consider a version of the model stated above. In each country after trade liberalization:

- (i) The productivity cutoff for operating strictly increases
- (ii) Real income increases
- (iii) Real GDP and total factor productivity stay the same

Proof. (i) Compare (82) and (88).

(ii) It follows from the fact that $P^T < P^A$ (details in the appendix).

(iii) Compare (85) and (94). ■

Notice that the effect of reallocation across firms — the exit of the least productive firms and the movement of resources toward the most productive firms which start exporting — increases welfare, not real GDP. The intuition is the following...**ADD SOME INTUITION HERE.**

Traded goods as intermediate inputs

Consider a version of the previous model where individuals consume final, non-traded good Y , which is produced using both traded goods as intermediate inputs. Assume that

the consumer preferences are represented by the utility function $u(C) = \log C$, and that good Y is produced using the technology:

$$Y = \left(\int_{Z_i} c(z)^\rho dz \right)^{1/\rho} \quad (96)$$

where $c(z)$ is the input of good z . It is straight forward to see that this model is theoretically equivalent to the model presented above, where in equilibrium $C = \bar{\ell} / P$, and $P = \left(\int_{Z_i} p(z)^{\rho/(\rho-1)} dz \right)^{(\rho-1)/\rho}$.

Real GDP under autarky is given by:

$$GDP_i^A = P_i^A C_i^A = \bar{\ell} \quad (97)$$

and under free trade, using the symmetry assumptions,

$$GDP_i^T = P^A C^T + \mu \int_{\bar{x}_e}^{\infty} p^A(x) (y^T(x) - c^T(x)) dx - \mu \int_{\bar{x}_e}^{\infty} p^A(x) c^T(x) dx = P^A C^T \quad (98)$$

Proposition 8: Consider the version of the monopolistic competition model with heterogeneous firms with a final, non-traded, good stated above. It is the case that $GDP_i^T > GDP_i^A$.

Proof: It is enough to show that $P^T < P^A$. Using the expressions for $\bar{x}_d^A$ and $\bar{x}_d^T, \bar{x}_e^T$, the inequality is equivalent to showing that

$$(1+b)^{1/\gamma} > \frac{1}{1+b} \quad (99)$$

where $b = (f_e / f_d)^{(\rho - \gamma(1-\rho))/(1-\rho)} > 0$, which is trivial, given that $\gamma > 0$. ■

6. Trade liberalization and growth

The models that we have presented are static and, therefore, can only study static effects of trade liberalization. By changing the incentives to accumulate capital, trade liberalization may also affect a country's growth rate. In particular, under free trade capital scarce countries may choose to concentrate in the production of labor intensive goods and, thus, accumulate capital at a slower rate than in autarky. In this section we analyze the effect of trade liberalization on real GDP growth under the framework of a

dynamic Heckscher-Ohlin model with endogenous capital accumulation like the one studied in Bajona and Kehoe (2006).

A dynamic Heckscher-Ohlin model

Consider a world with n countries, where in each country i , $i = 1, 2, \dots, n$, there is measure L_i of infinitely-lived consumers. Each consumer in country i is endowed with one unit of labor and $\bar{k}_{i0}$ units of capital. There are two tradable goods, $j = 1, 2$ which are produced using capital and labor. The technology to produce the two tradable goods is the same across countries.

A consumer in country i derives utility from the consumption of both traded goods in each period of his life, and chooses consumption and investment allocations $\{c_{ijt}, x_{ijt}, k_{it}\}$, $j = 1, 2$, $t = 0, 1, \dots$, to maximize lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t (a_1 \log c_{i1t} + a_2 \log c_{i2t}), \quad (100)$$

where $a_1 + a_2 = 1$, subject to the budget constraints

$$p_{i1t} (c_{i1t} + x_{i1t}) + p_{i2t} (c_{i2t} + x_{i2t}) = w_{it} + r_{it} k_{it} \quad (101)$$

and the laws of motion of capital

$$k_{it+1} = (1 - \delta) k_{it} + x_{i1t}^a x_{i2t}^{1-a}, \quad (102)$$

for $t = 0, 1, 2, \dots$, given $k_{i0} = \bar{k}_{i0}$. Here p_{ijt} is the price of good j , w_{it} is the wage rate, and r_{it} is the rental rate of capital.

In this section we want to compare real GDP in a closed and a free trade environment in such a way that the base year prices are the same in both environments. For this purpose, we consider two scenarios. In both scenarios all countries are in autarky in the first period (period 0) and they believe that they will continue in autarky forever. In the first scenario (the *autarky* scenario), the countries continue in autarky forever. In the second scenario (the *free trade* scenario), at the beginning of the second period (period 1) and before any production or consumption decisions are made, all economies are allowed to trade freely with each other. This (quite unrealistic) experiment ensures

that period-0 prices are the same under both scenarios and, therefore, real GDP is measured using the same prices in both scenarios.

The production of the traded goods, as well as the period feasibility conditions under autarky and free trade follow exactly the description of the static model in section 3.1 and we do not repeat them here. In what follows we use the same notational conventions used in the static Heckscher-Ohlin model.

The model described above has an analytical solution under the assumption of complete depreciation, $\delta = 1$. Notice that in our specification of the model, the traded goods are combined in the same way in consumption and investment. This assumption greatly simplifies the solution of the dynamic model. In particular, given k_{it} , the equilibrium prices and production patterns of the dynamic model for period t can be solved by solving a static Heckscher-Ohlin model with initial capital per person k_{it} in each country i . Values for consumption and investment in each period are solved by using the intertemporal consumer's problem. See Bajona and Kehoe (2006) for details.

Autarky

Let us normalize prices so that the price of a unit of investment is equal to one in each period. The autarky prices for the traded goods and the consumption, investment, and production allocations for country i , $i = 1 \dots n$, are

$$p_{ijt}^A = \frac{a_j D}{D_j} (k_{it}^A)^{A_1 - \alpha_j} \quad (103)$$

$$y_{ijt}^A = D_j (k_{it}^A)^{\alpha_j} \quad (104)$$

$$c_{ijt}^A = (1 - \beta A_1) D_j (k_{it}^A)^{\alpha_j} \quad (105)$$

$$x_{ijt}^A = \beta A_1 D_j (k_{it}^A)^{\alpha_j} \quad (106)$$

where

$$k_{it}^A = \beta A_1 D (k_{i,t-1}^A)^{A_1} = (\beta A_1 D)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_{i0}^{A_1^t}, \quad (107)$$

and D , D_j and A_j , $j = 1, 2$ are as described in section 3.1.

Our variables of interest are nominal and real GDP, productivity and welfare. GDP measured at current prices is equal to:

$$gdp_{it}^A = p_{i1t}^A y_{i1t}^A + p_{i2t}^A y_{i2t}^A = D \left(k_{it}^A \right)^{A_1}. \quad (108)$$

Notice that GDP at current prices is equal to $\left(y_{i1t}^A \right)^{a_1} \left(y_{i2t}^A \right)^{a_2}$. This is a direct result of our assumption that the traded goods are combined using the same technology in consumption and investment.

We measure real GDP by using the period-0 as the base year. Its value is

$$\begin{aligned} GDP_{it}^A &= p_{i10}^A y_{i1t}^A + p_{i20}^A y_{i2t}^A \\ &= \left(a_1 \left(\frac{k_{it}^A}{k_{i0}^A} \right)^{a_1} + a_2 \left(\frac{k_{it}^A}{k_{i0}^A} \right)^{a_2} \right) D \bar{k}_{i0}^{A_1}. \end{aligned} \quad (109)$$

Notice that replacing (107) in the expressions of nominal and real GDP, we can get a formula for real GDP as a function only of capital per worker and parameters.

Lifetime welfare is defined, as in the other models as $v_i^A = e^u$, where u is the utility function in (100) evaluated at the autarkic equilibrium. Using the expressions in (105) and (107) we obtain:

$$v_i^A = M \bar{k}_{i0}^{A_1} k_{i1}^{\beta A_1 / (1 - \beta A_1)}, \quad (110)$$

where $M = \left[(1 - \beta A_1) D \right]^{1/(1-\beta)} (\beta A_1 D)^{\beta^2 A_1 / [(1-\beta)(1-\beta A_1)]}$.

Free trade

Following the same strategy as in the static version of the model, we assume that the initial factor endowments are such that factor prices are equalized after the economies open to trade in the first period. Bajona and Kehoe (2006b) show that, under our assumptions on preferences and technologies, factor prices equalize in all subsequent periods. Therefore, the model after the first period can be solved by calculating the equilibrium of the integrated economy — an economy with initial endowments equal to the world endowments — and then splitting production, consumption, and investment across countries in each period. If all countries are in the cone of diversification in all periods, it can be shown that for all $t = 1, 2, \dots$

$$k_{it}^T = \gamma_i k_t, \quad (111)$$

where $\gamma_i = k_{i1}^A / k_1^T$ and $k_t = \sum_{i=1}^n L_i k_{it}^T / \sum_{i=1}^n L_i$ for $t = 0, 1, 2, \dots$. Here k_{i1}^A is the autarky value of capital per worker in the first period. Notice that, since the economies are in autarky in period 0 and they believe that autarky will prevail forever, in general it is the case that $\bar{k}_{i0} \neq \gamma_i k_0$.

The expressions for the relevant variables for our analysis, prices, consumption, production, and investment patterns, coincide with the autarky variables in period zero: $p_{ijt}^T = p_{ij0}^A$, $c_{ijt}^T = c_{ij0}^A$, $y_{ijt}^T = y_{ij0}^A$, and $k_{i1}^T = k_{i1}^A$. For $t = 1, 2, \dots$ the values of these variables become (a complete solution is described in the appendix)

$$p_{jt}^T = \frac{a_j D}{D_j} (k_t)^{A_j - \alpha_j} \quad (112)$$

$$c_{ijt}^T = ((1 - \beta) A \gamma_i + A_2) D_j (k_t)^{\alpha_j} \quad (113)$$

$$y_{ijt}^T = \mu_{ij} D_j (k_t)^{\alpha_j}, \quad (114)$$

where k_t , $t = 2, 3, \dots$ can be expressed as a function of the world's average level of capital per person in period 1:

$$k_t = \beta A_1 D k_{t-1}^{A_1} = (\beta A_1 D)^{\frac{1-A_1^{t-1}}{1-A_1}} k_1^{A_1^{t-1}}. \quad (115)$$

Our variables of interest are nominal and real GDP, productivity and welfare. Given that the economy is in autarky and the opening to trade is a surprising event, the value of these variables in period 0 in the free trade scenario coincide with their values in the autarky scenario. In what follows we specify the values of these variables for the periods where the economy is open, that is, for $t = 1, 2, \dots$

GDP at current prices is:

$$\begin{aligned} gdp_{it}^T &= p_{it}^T y_{i1t}^T + p_{2t}^T y_{i2t}^T \\ &= (A_1 \gamma_i + A_2) D k_t^{A_1} \end{aligned} \quad (116)$$

Following the methodology used in the autarky scenario, we measure real GDP using period-0 prices. The value at period 0 coincides with real GDP in autarky, since prices and output at period 0 are the same in both scenarios. The value at $t = 1, 2, \dots$, becomes:

$$\begin{aligned}
GDP_{it}^T &= p_{i10}^A y_{i1t}^T + p_{i20}^A y_{i2t}^T \\
&= \left(a_1 \mu_{i1} \left(\frac{k_t}{\bar{k}_{i0}} \right)^{\alpha_1} + a_2 \mu_{i2} \left(\frac{k_t}{\bar{k}_{i0}} \right)^{\alpha_2} \right) D \bar{k}_{i0}^{A_1},
\end{aligned} \tag{117}$$

where the expressions for μ_{ij} are given in (19) and (20).

Lifetime welfare in the free trade scenario becomes:

$$v_i^T = (\eta_i)^{\beta/(1-\beta)} M \bar{k}_{i0}^{A_1} k_1^{\beta A_1/(1-\beta A_1)}, \tag{118}$$

where $\eta_i = (A_1(1-\beta)\gamma_i + A_2)/(1-\beta A_1)$.

Effect of trade liberalization

We begin with the analysis of lifetime welfare and discuss real GDP later. The next proposition shows that opening to trade cannot decrease lifetime welfare.

Proposition 5. In the dynamic Heckscher-Ohlin framework described above, $v_i^T \geq v_i^A$, with strict inequality if $\gamma_i \neq 1$.

Proof. Using the expressions in (110) and (118), we need to show that

$$\eta_i^{1/(1-\beta)} \geq \left(\frac{k_1}{k_{i1}^T} \right)^{A_1/(1-\beta A_1)}$$

with strict inequality unless $\gamma_i = 1$. This is equivalent to show that

$$\frac{A_1(1-\beta)}{1-\beta A_1} \gamma_i + \frac{A_2}{1-\beta A_1} \geq \gamma_i^{A_1(1-\beta)/(1-\beta A_1)}, \tag{119}$$

which was proven in proposition 1. ■

What happens to real GDP following trade liberalization? We can infer from the static model that, if a country is initially in autarky, then trade liberalization initially causes a decrease, or at least a decrease in the growth rate of, real GDP in that country. Direct comparisons between the expressions in (109) and (117) are not straight forward. Instead, we provide an illustrative numerical example. There are two countries, and

country 1 is relatively capital-rich. We set $L_1 = L_2 = 1$, $\beta = 0.96$, $a_1 = a_2 = 0.5$, $\theta_1 = \theta_2 = 1$, $\alpha_1 = 0.6$, $\alpha_2 = 0.4$, $\bar{k}_{10} = 0.05$, and $\bar{k}_{20} = 0.03$.

Figures 2 and 3 compare real GDP indices in each country under both regimes. We observe that the capital abundant country grows faster under free trade than under autarky, whereas the capital-poor country does the opposite. Therefore, in this example opening to trade slows down growth in the poor country (compared to autarky). In figure 4 we compare the growth rates of the capital-poor and capital-abundant countries. We observe, as traditional growth theory predicts, that the capital-poor country grows faster. This results changes under the free trade scenario, represented in figure 5. In the first period, the poor country grows faster, but its growth rate slows down substantially after all countries unexpectedly open to trade in period 1. The growth rate of the capital-poor country remains below the growth rate of the rich country in the subsequent period (figure 6 plots the corresponding growth rates).

7. Conclusion

To the extent that trade liberalization leads to higher productivity or higher rates of growth in real GDP (as measured in the data), it does so through mechanisms that are, for the most part, different from those analyzed in standard models. In this paper we have analyzed the predictions of several trade models, each emphasizing a different mechanism by which trade liberalization is commonly thought to enhance growth and productivity. We find that in most cases trade liberalization does not lead to an increase in measured GDP (nor measured productivity), and it may actually lead a lower value for real GDP. Using a dynamic Heckscher-Ohlin model we show that opening to trade can substantially slow down measured real GDP growth in capital-poor countries. In all our examples, though, trade liberalization increases social welfare. Our results pose the more fundamental question about whether measured real GDP growth and productivity are the correct measures to use when analyzing the effects of trade liberalization. Determining the relation between trade liberalization and growth is not just a challenge for empirical research but also for theoretical research.

References

- Alcalá, F., and A. Ciccone (2004), "Trade and Productivity," *Quarterly Journal of Economic*, 119: 613-46.
- Bajona, C., and T. J. Kehoe (2006a), "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," Federal Reserve Bank of Minneapolis Staff Report.
- Bajona, C., and T. J. Kehoe (2006b), "Demographics in Dynamic Heckscher-Ohlin Models: Overlapping Generations versus Infinitely Lived Consumers," Federal Reserve Bank of Minneapolis Staff Report.
- Balassa, B. (1978), "Exports and Economic Growth: Further Evidence," *Journal of Development Economics* 5: 181-189.
- Chaney, T. (2006), "Distorted Gravity: Heterogeneous Firms, Market Structure, and the Geography of International Trade," University of Chicago.
- Cuñat A., and M. Maffezzoli (2004), "Neoclassical Growth and Commodity Trade," *Review of Economic Dynamics* 7: 707-736.
- DeJong, D. N. and M. Ripoll (2006), "Tariffs and growth: comparing relationships among the rich and poor," *Review of Economics and Statistics* 88: 625-40.
- Diewert, W. E. and C. J. Morrison (1986), "Adjusting output and productivity indexes from changes in the terms of trade," *Economic Journal* 96: 659-79.
- Dollar, D. (1992) "Outward-oriented developing economies really do grow more rapidly: evidence from 95 LDCs, 1976-1985," *Economic Development and Cultural Change* 40: 523-544.
- Dollar, D. and A. Kraay (2004), "Trade, growth, and poverty," *The Economic Journal* 114: 22-49.
- Frankel, J. A., and D. Romer (1999), "Does Trade Cause Growth?" *American Economic Review*, 89, 379-399.
- Gibson, M. J. (2007), "Trade Liberalization, Reallocation, and Productivity," University of Minnesota.
- Grossman, G. and E. Helpman (1991), *Innovation and growth in the global economy*. MIT Press, Cambridge, Massachusetts.

- Hall, R. and C. Jones (1999), "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics* 114: 83-116.
- Kehoe, T. J., and K. J. Ruhl (2007), "Are Shocks to the Terms of Trade Shocks to Productivity?" Federal Reserve Bank of Minneapolis Staff Report.
- Kohli, U. (1983), "Technology and the demand for imports," *Southern Economic Journal* 50: 137-150.
- Kohli, U. (2004), "Real GDP, real domestic income, and terms of trade changes." *Journal of International Economics* 62: 83-106.
- Krugman P. (1979), "Increasing returns, monopolistic competition, and international trade," *Journal of International Economics* 9: 469-79.
- Lewer, J. J. and H. Van den Berg (2003), "How large is international trade's effect on economic growth?" *Journal of Economic Surveys* 17: 363-396
- Lucas, R. E. Jr. (1988), "On the mechanics of economic development," *Journal of Monetary Economics* 22:3-42.
- Melitz, M. J. (2003), "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity," *Econometrica* 71, 1695-1725.
- Michaely M. (1977), "Exports and growth: an empirical investigation." *Journal of Development Economics* 4: 49-53.
- Rivera-Batiz, L. A. and P. Romer (1991), "Economic integration and endogenous growth." *Quarterly Journal of Economics* 106: 531-56.
- Rodriguez, F. and D. Rodrik (2001), "Trade policy and economic growth: a skeptics guide to the cross-national evidence," in Bernanke, B., and K. S. Rogoffs (eds) *NBER Macroeconomics Annual 2000*, MIT Press, Cambridge, Massachusetts.
- Rodrik D., A. Subramanian, and F. Trebbi (2002) "Institutions rule: the primacy of institutions over geography and integration in economic development," *NBER Working Paper* 9305.
- Sachs, J., and A. Warner (1995), "Economic Reform and the Process of Global Integration," *Brookings Papers on Economic Activity* 1: 1-118.
- Slaughter M. J. (2001), "Trade liberalization and per capita income convergence: a difference-in-differences analysis," *Journal of International Economics* 55: 203-28.

- Ventura (1997) "Growth and interdependence," *Quarterly Journal of Economics* 112: 57-84.
- Wacziarg, R. (2001), "Measuring the dynamic gains from trade," *World Bank Economic Review* 15: 393-429.
- World Bank (1987), *The World Bank World Development Report*. The World Bank, New York.
- Yanikkaya, H. (2003), "Trade openness and economic growth: a cross-country empirical investigation," *Journal of Development Economics* 72: 57-89.
- Young, A. (1991), "Learning by doing and the dynamic effects of international trade," *Quarterly Journal of Economics* 106: 369-405.

APPENDIX

1. Solution of the Static Heckscher-Ohlin model

In all the following equations indices are: $i = 1, \dots, n$ and $j = 1, 2$.

1.1 Autarky

Prices:

$$p_{ij}^A = \frac{a_j D}{D_j} \bar{k}_i^{A-\alpha_j} \quad (120)$$

$$r_i^A = A_1 D \bar{k}_i^{A-1} \quad (121)$$

$$w_i^A = A_2 D \bar{k}_i^A. \quad (122)$$

Allocations:

$$c_{ij}^A = y_{ij}^A = D_j \bar{k}_i^{\alpha_j} \quad (123)$$

$$k_{ij}^A = \frac{a_j \alpha_j}{A_1} \bar{k}_i \quad (124)$$

$$\ell_{ij}^A = \frac{a_j (1 - \alpha_j)}{A_2} \quad (125)$$

1.2 Free trade

Define $\bar{k} = \left(\sum_{i=1}^n L_i \bar{k}_i \right) / \sum_{i=1}^n L_i$, and let any generic variable λ be defined as the autarkic equilibrium of an economy (call it the integrated economy) with initial capital per worker equal to the world average $\bar{k}$. Then equilibrium prices in the trade economy coincide with equilibrium prices of the integrated economy, and allocations in each country are:

$$c_{ij}^T = \eta_i c_j^T \quad (126)$$

$$y_{ij}^T = \mu_{ij} y_j^T \quad (127)$$

$$k_{ij}^T = \mu_{ij} k_j^T \quad (128)$$

$$\ell_{ij}^T = \mu_{ij} \ell_j^T \quad (129)$$

where μ_{ij} are defined in (19) and (20), and $\eta_i = A_1 \gamma_i + A_2$.

2. Proof of proposition 5

(i) We need to show that

$$\left(J^T\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J^T-1\right)}{J^T b} \bar{\ell}_i > \left(J_i^A\right)^{\frac{1-\rho}{\rho}} \frac{\rho\left(J_i^A-1\right)}{J_i^A b} \bar{\ell}_i, \quad (130)$$

or equivalently that $J^T > J_i^A$, where

$$J^T = \frac{(1-\rho) \bar{\ell} + \sqrt{(1-\rho)^2 \bar{\ell}^2 + 4 f \rho \bar{\ell}}}{2 f} \quad (131)$$

and

$$J_i^A = N_i^A = \frac{(1-\rho) \bar{\ell}_i + \sqrt{(1-\rho)^2 \bar{\ell}_i^2 + 4 f \rho \bar{\ell}_i}}{2 f}. \quad (132)$$

The inequality derives from the fact that $\bar{\ell} = \sum_{i=1}^n \bar{\ell}_i$ and $\bar{\ell}_i > 0$ for all i .

(ii) We need to show that

$$\frac{J_i^A}{\left(J_i^A-1\right)} \frac{\left(J^T-1\right)}{J^T} \bar{\ell}_i > \bar{\ell}_i, \quad (133)$$

which also follows from $J^T > J_i^A$. ■

3. Solution of the monopolistic competition model with heterogeneous firms.

3.1. Autarky

The profit-maximizing prices are

$$p^A(x) = \frac{1}{\rho x}. \quad (134)$$

The aggregate price index is

$$\begin{aligned} P^A &= \left(\mu \int_{\bar{x}_d^A}^{\infty} p^A(x)^{\frac{-\rho}{1-\rho}} dF(x) \right)^{\frac{-(1-\rho)}{\rho}} \\ &= \left(\frac{\gamma(1-\rho) - \rho}{\mu \rho^{\frac{\rho}{1-\rho}} (1-\rho) \gamma \left(\bar{x}_d^A\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}} \right)^{\frac{1-\rho}{\rho}}. \end{aligned} \quad (135)$$

From here, we obtain the demand for a good produced by a firm with productivity x :

$$\begin{aligned} c^A(x) &= y^A(x) = p^A(x)^{\frac{-1}{1-\rho}} \left(P^A\right)^{\frac{\rho}{1-\rho}} \left(\bar{\ell} + \pi^A\right) \\ &= \frac{\rho(\gamma(1-\rho) - \rho) \left(\bar{\ell} + \pi^A\right) x^{\frac{1}{1-\rho}}}{\mu(1-\rho) \gamma \left(\bar{x}_d^A\right)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}}. \end{aligned} \quad (136)$$

A firm with productivity $\bar{x}_d^A$ must make zero profits in equilibrium, so

$$p^A(\bar{x}_d^A)c^A(\bar{x}_d^A) - \frac{c^A(\bar{x}_d^A)}{\bar{x}_d^A} - f_d = 0. \quad (137)$$

Combining the last two equations and operating, we obtain the cutoff point:

$$\begin{aligned} \bar{x}_d^A &= \left(\frac{\mu\gamma f_d}{(\gamma(1-\rho)-\rho)(\bar{\ell} + \pi^A)} \right)^{1/\gamma}, \\ &= \left(\frac{\mu(\gamma-\rho) f_d}{(\gamma(1-\rho)-\rho)\bar{\ell}} \right)^{1/\gamma}, \end{aligned} \quad (138)$$

Where the last equality uses the fact that

$$\begin{aligned} \pi^A &= \mu \int_{\bar{x}_d^A}^{\infty} \left(p^A(x)c^A(x) - \frac{c^A(x)}{x} - f_d \right) dF(x) \\ &= \frac{\rho\bar{\ell}}{\gamma-\rho} \end{aligned} \quad (139)$$

Finally, using these values we obtain an equation for the aggregate price index as a function of the parameters of the model:

$$P^A = \left(\frac{\gamma(1-\rho)-\rho}{\mu\rho^{\frac{\rho}{1-\rho}}(1-\rho)\gamma \left(\frac{\mu(\gamma-\rho) f_d}{(\gamma(1-\rho)-\rho)\bar{\ell}} \right)^{\frac{\rho-\gamma(1-\rho)}{\gamma(1-\rho)}}} \right)^{\frac{1-\rho}{\rho}}. \quad (140)$$

3.2. Free trade

The profit-maximizing prices are

$$p^T(x) = \frac{1}{\rho x}. \quad (141)$$

The aggregate price index is

$$\begin{aligned}
P^T &= \left(\mu \int_{\bar{x}_d^T}^{\infty} p^T(x) \frac{-\rho}{1-\rho} dF(x) + \mu \int_{\bar{x}_e^T}^{\infty} p^T(x) \frac{-\rho}{1-\rho} dF(x) \right)^{\frac{-(1-\rho)}{\rho}} \\
&= \left(\frac{\gamma(1-\rho) - \rho}{\rho^{\frac{\rho}{1-\rho}} (1-\rho) \gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} \right)^{\frac{1-\rho}{\rho}}
\end{aligned} \tag{142}$$

The demand in a country for a good produced by a firm with productivity $x \geq \bar{x}_d^T$ is

$$\begin{aligned}
c(x) &= p^T(x) \frac{-1}{1-\rho} (P^T)^{\frac{\rho}{1-\rho}} (\bar{\ell} + \pi^T) \\
&= \frac{\rho(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^T) x^{\frac{1}{1-\rho}}}{(1-\rho) \gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} .
\end{aligned} \tag{143}$$

The cutoff for operating, $\bar{x}_d^T$, must satisfy

$$p^T(\bar{x}_d^T) c^T(\bar{x}_d^T) - \frac{c^T(\bar{x}_d^T)}{\bar{x}_d^T} - f_d = 0, \tag{144}$$

so

$$\frac{(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^T)(\bar{x}_d^T)^{\frac{\rho}{1-\rho}}}{\gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} - f_d = 0. \tag{145}$$

Similarly, the cutoff for exporting, $\bar{x}_e^T$, must satisfy

$$p^T(\bar{x}_e^T) c(\bar{x}_e^T) - \frac{c(\bar{x}_e^T)}{\bar{x}_e^T} - f_e = 0, \tag{146}$$

Or, equivalently,

$$\frac{(\gamma(1-\rho) - \rho)(\bar{\ell} + \pi^T)(\bar{x}_e^T)^{\frac{\rho}{1-\rho}}}{\gamma \mu \left((\bar{x}_d^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + (\bar{x}_e^T)^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right)} - f_e = 0. \tag{147}$$

Therefore:

$$y^T(x) = \begin{cases} c^T(x) & \bar{x}_d^T \leq x < \bar{x}_e^T \\ 2c^T(x) & x \geq \bar{x}_e^T \end{cases}. \tag{148}$$

Industry profits are:

$$\begin{aligned}
\pi^T &= \mu \int_{\bar{x}_d^T}^{\infty} \left(p^T(x) c^T(x) - \frac{c^T(x)}{x} - f_d \right) dF(x) \\
&\quad + \mu \int_{\bar{x}_e^T}^{\infty} \left(p^T(x) c^T(x) - \frac{c^T(x)}{x} - f_e \right) dF(x) \quad . \\
&= (1-\rho)(\bar{\ell} + \pi^T) - \mu \left((\bar{x}_d^T)^{-\gamma} f_d + (\bar{x}_e^T)^{-\gamma} f_e \right)
\end{aligned} \tag{149}$$

Notice that (145), (147), and (149) give us a system of 3 equations and 3 unknowns to be solved for $\bar{x}_d^T$, $\bar{x}_e^T$, and π^T . Also notice that

$$\frac{\bar{x}_e^T}{\bar{x}_d^T} = \left(\frac{f_e}{f_d} \right)^{\frac{1-\rho}{\rho}} . \tag{150}$$

The solution to the system is:

$$\bar{x}_d^T = \left(\frac{\mu(\gamma-\rho) f_d \left(1 + \left(f_e / f_d \right)^{\frac{\rho-\gamma(1-\rho)}{\rho}} \right)}{(\gamma(1-\rho)-\rho) \bar{\ell}} \right)^{1/\gamma} \tag{151}$$

$$\bar{x}_e^T = \left(\frac{f_e}{f_d} \right)^{\frac{1-\rho}{\rho}} \left(\frac{\mu(\gamma-\rho) f_d \left(1 + \left(f_e / f_d \right)^{\frac{\rho-\gamma(1-\rho)}{\rho}} \right)}{(\gamma(1-\rho)-\rho) \bar{\ell}} \right)^{1/\gamma} \tag{152}$$

$$\pi^T = \frac{\rho \bar{\ell}}{\gamma - \rho} . \tag{153}$$

4. Proof of proposition 7.

We need to show that $P^T < P^A$, where P^A is given by (135) and P^T is given by (142). Using these expressions, together with the values for $\bar{x}_d^A$, $\bar{x}_d^T$, and $\bar{x}_e^T$, the inequality reduces to showing that

$$\left(\bar{x}_d^T / \bar{x}_d^A \right)^m > \left(1 + \left(\bar{x}_e^T / \bar{x}_d^T \right)^m \right)^{-1} \tag{154}$$

for $m = (\rho - \gamma(1-\rho)) / (1-\rho)$. A simple substitution shows that the inequality holds, given our assumptions on γ . ■

5. Complete solution to the dynamic Heckscher-Ohlin model with diversification

5.1. Autarky

The analytical solution is

$$r_{it}^A = A_1 D (k_{it}^A)^{A_1-1} \quad (155)$$

$$w_{it}^A = A_2 D (k_{it}^A)^{A_1} \quad (156)$$

$$p_{ijt}^A = \frac{a_j D}{D_j} (k_{it}^A)^{A_1-\alpha_j} \quad (157)$$

$$y_{ijt}^A = D_j (k_{it}^A)^{\alpha_j} \quad (158)$$

$$k_{ijt}^A = \frac{a_j \alpha_j}{A_1} k_{it}^A \quad (159)$$

$$\ell_{ijt}^A = \frac{a_j (1-\alpha_j)}{A_2} \quad (160)$$

$$c_{ijt}^A = (1-\beta A_1) D_j (k_{it}^A)^{\alpha_j} \quad (161)$$

$$x_{ijt}^A = \beta A_1 D_j (k_{it}^A)^{\alpha_j} \quad (162)$$

where

$$k_{it}^A = \beta A_1 D (k_{i,t-1}^A)^{A_1} = (\beta A_1 D)^{\frac{1-A_1^i}{1-A_1}} \bar{k}_{i0}^{A_1^i} . \quad (163)$$

5.2. Free trade

As in the text, let

$$\gamma_i = \frac{\bar{k}_{i0}}{\bar{k}_0} \quad (164)$$

and

$$\bar{k}_0 = \frac{\sum_{i=1}^n L_i \bar{k}_{i0}}{\sum_{i=1}^n L_i} . \quad (165)$$

We analyze the case where $\kappa_2 \leq \gamma_i \leq \kappa_1$, $i = 1, \dots, n$.

The solution is

$$r_t^T = A_1 D (k_t^T)^{A_1-1} \quad (166)$$

$$w_t^T = A_2 D (k_t^T)^{A_1} \quad (167)$$

$$p_{jt}^T = \frac{a_j D}{D_j} (k_t^T)^{A_1-\alpha_j} \quad (168)$$

$$c_{ijt}^T = (1 - \beta A_1)(\gamma_i A_1 + A_2) D_j (k_t^T)^{\alpha_j} \quad (169)$$

$$x_{ijt}^T = \beta A_1 (\gamma_i A_1 + A_2) D_j (k_t^T)^{\alpha_j} \quad (170)$$

$$y_{ijt}^T = \mu_{ij} D_j (k_t^T)^{\alpha_j} \quad (171)$$

$$k_{ijt}^T = \mu_{ij} \frac{a_j \alpha_j}{A_1} k_t^T \quad (172)$$

$$\ell_{ijt}^T = \mu_{ij} \frac{a_j (1 - \alpha_j)}{A_2}, \quad (173)$$

where

$$k_t^T = \beta A_1 D (k_{t-1}^T)^{A_1} = (\beta A_1 D)^{\frac{1-A_1^t}{1-A_1}} \bar{k}_0^{A_1^t}. \quad (174)$$

FIGURES

Figure 1

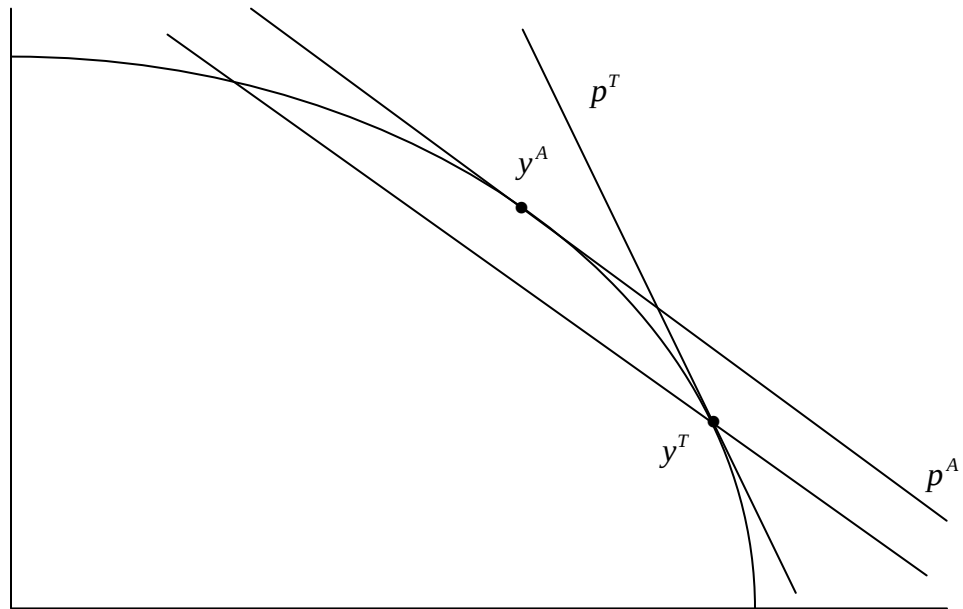


Figure 2

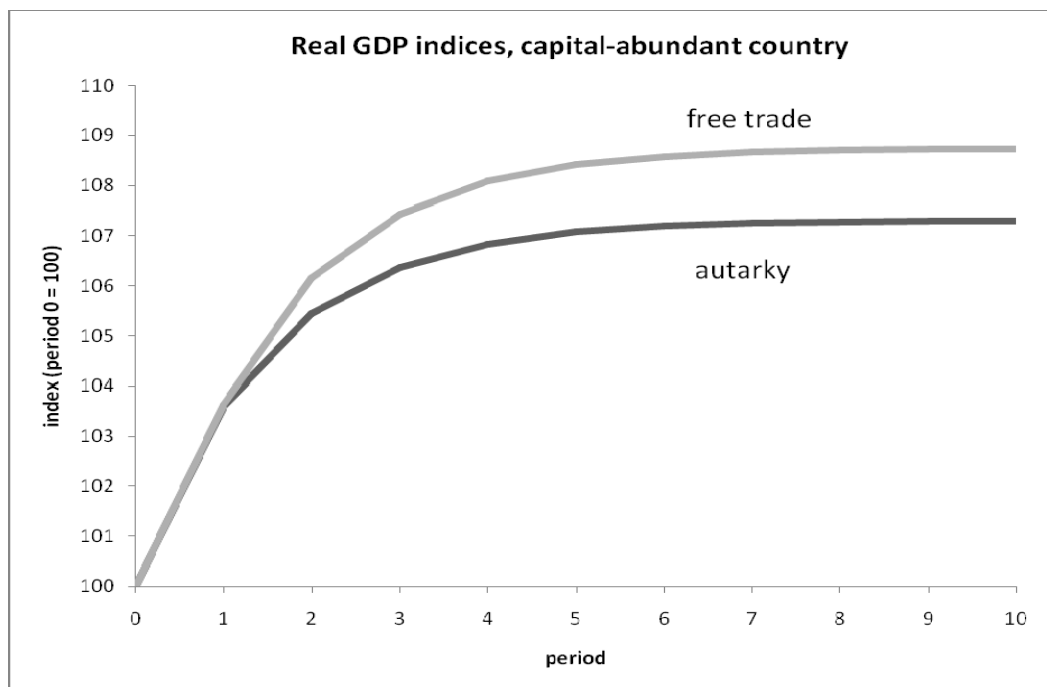


Figure 3

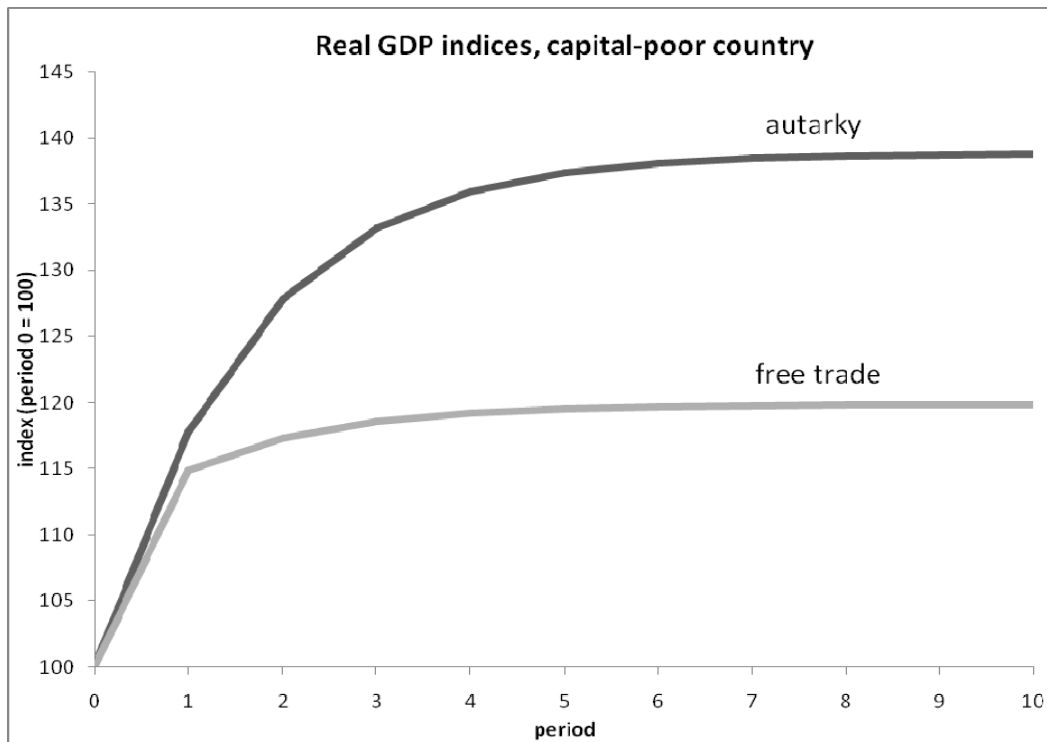


Figure 4

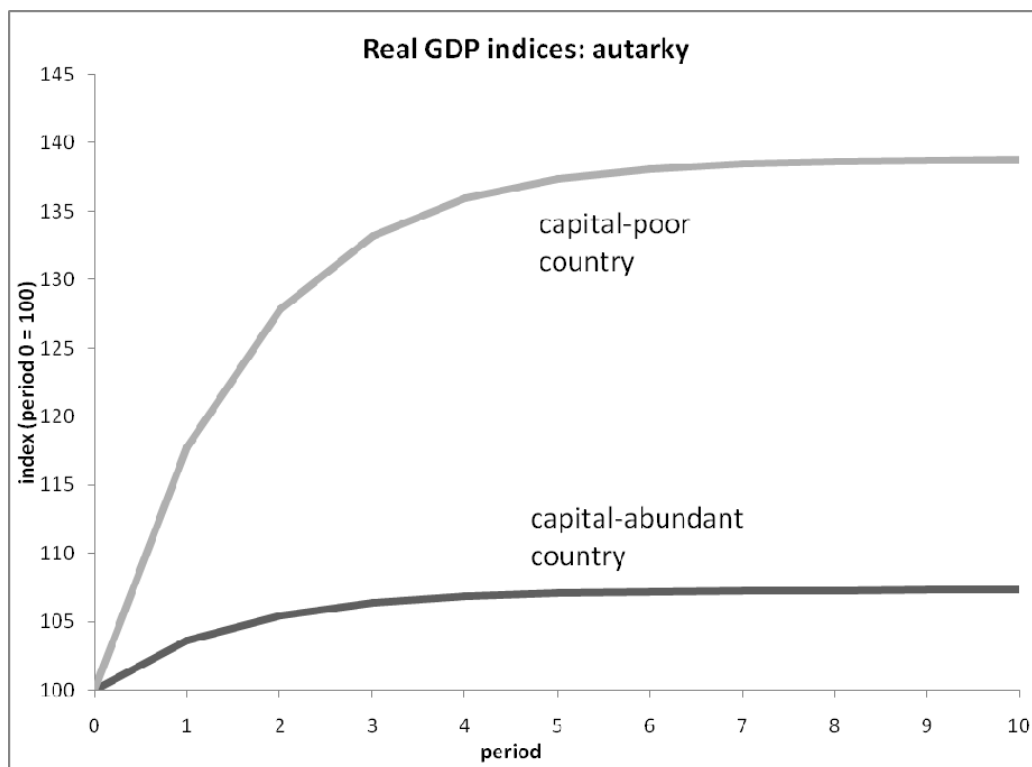


Figure 5

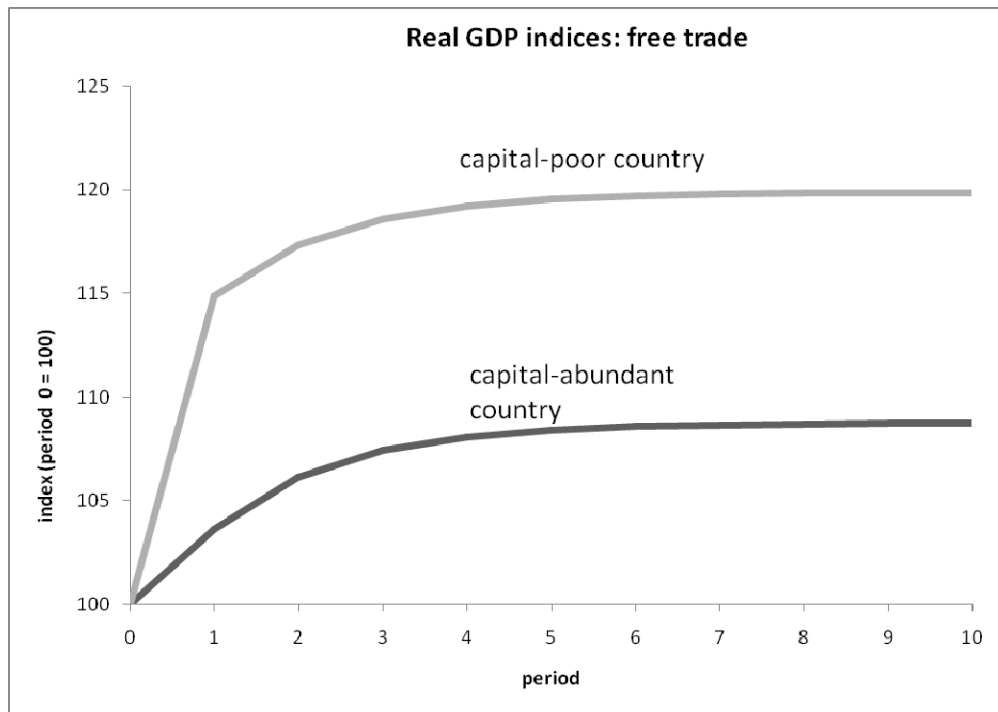


Figure 6

