

Uncertainty and entry into export markets ^{*}

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January 2008

PRELIMINARY AND INCOMPLETE.

Abstract

We face uncertainty in most economic decisions we take. This is particularly true in the case of a firm entering a foreign market where there is uncertainty about the size of the market, the distribution channels, the adequacy of the firm's product to local tastes, etc. Despite its obvious importance, this ingredient appears to have been largely overlooked by the literature explaining the direction and volume of international trade flows. We incorporate this informational uncertainty into a model with heterogeneous firms similar to the one proposed by Melitz (2003). The model exhibits informational externalities that arise via informational complementarities: in markets with less uncertainty, the most productive firms always find it optimal to enter. Once a firm enters that foreign market, her success/failure reveals information to other domestic firms who, given the new information, optimally decide whether to enter. We characterize the conditions under which, given initial entry, informational externalities are strong enough to reach an equilibrium with full information. The model delivers an explanation for the recent dynamic evolution of trade flows, at the intensive margin at the country level and the extensive margin at the firm/product level. The model also provides insights on the persistence of bilateral trade flows, zero trade flows, and why we observe empirically less entrance by small firms than the Melitz model predicts.

^{*}We would like to thank Chrysostomos Tabakis for comments, discussions, and suggestions on earlier drafts of this paper. We also would like to thank Alex Cuñat, Aitor Erce, Luis A. Puch, Juan Ruiz and seminar participants at the Bank of Spain macro lunch, the Bank of Spain, CEMFI and CREI seminars, the ETSG 2007 and ELSNIT 2007 conferences and the 2007 Moneda y Credito Symposium and the 2007 Simposio de Análisis Económico for helpful comments.

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1 Introduction and Literature Review

The recent availability of more disaggregated data on international trade transactions has allowed trade economists to gain a deeper insight into the microeconomics of trade. A large body of literature (see, for instance, Roberts and Tybout, 1996, Bernard, Jensen, Redding, and Schott, 2007 and Bernard, Redding, and Schott 2007) have documented that exporting firms are very different from strictly domestic firms: they are larger (both in terms of sales and employment) and they are more productive (both in terms of valued added per worker and of total factor productivity). Trade has also been found to be quite persistent both in terms of the export status of firms and on their trade volumes, as documented, for instance, in Roberts and Tybout (1997) or Opromolla and Irarrazabal (2006).

More over, the large increase in trade volumes we have observed in the recent decades has taken place mostly on two margins. First, at the intensive margin at the country level: in the direction of trade statistics data set, over 92% of the increase in trade volume between 1970 and 2005 has taken place among pairs of countries that were already trading in 1970. Second, at the extensive margin in terms of firms or products, as reported by Bernard, Redding, and Schott, 2006, among others: more firms and products are responsible for the increase at the intensive margin at the country level. Helpman, Melitz, and Rubinstein (2007) provides a comprehensive analysis of these two margins.

Parallel to this empirical literature, an emerging theoretical literature (see Melitz, 2003 and Chaney, 2007) has also risen aiming to capture - in its different versions - several of these stylized facts. Despite this effort, there are still some questions lacking a convincing explanation, in particular the persistence of trade flows and the dynamic behavior of the extensive and intensive margin both at the country and firm level. This is mainly because these are models built for another purpose and static in nature.¹ For instance, Helpman, Melitz, and Rubinstein (2007) provide an explanation for the relative importance of the intensive versus the extensive margin at the country level but, being a static model is silent about the dynamics of those margins. The same is true for Chaney (2007) when explaining the importance of those margins at the firm level. Finally, although the Melitz-Chaney framework does a good job at explaining why exporters are different than importers, it implies too much entry by smaller firms and a ranking of destinations that is at odds with the data, as pointed out by Eaton, Kortum and Kramarz (2005).

In an attempt to capture some of this unexplained facts we build on these models

¹The original Melitz model is a dynamic model, but only the steady state is characterized.

by introducing an element that has been largely overlooked and which can account for the stylized facts above: *uncertainty*. In principle, there are several ways in which uncertainty could affect a firm's decision to start exports to a given foreign market. From the demand side, a firm might be uncertain about the exact size of demand for its good in the foreign market or about the adequacy of the product to local tastes. Another source of uncertainty could come from the supply side as the same firm might also not be fully aware of the legal requirements for selling its good in that particular market or the cost and adequacy of supply chains in the given country. In both cases, the firm can make an investment (for instance, performing a market study) which would reduce the degree of uncertainty and allow for improved decision-making.

Nevertheless, in this paper we emphasize that an alternative source of information about foreign markets could originate in the observation by a firm of the actions and successes (or failures) of its rival domestic firms who also attempt breaking into that market. In other words, we allow for partial (or full, in some cases) attenuation of the uncertainty through informational externalities from those who have full information (those that have already entered a given market) to those that do not (outsiders).

To this end, we build a model that combines a framework of monopolistic competitive firms which are heterogeneous in their productivity levels as in Melitz (2003) and we model the decision-making process of firms under uncertainty in a fashion close to Rob (1991). In our setup, firms breaking into export markets need to pay a known one-time sunk cost as well as an unknown per-period cost of presence in the foreign markets. After paying the sunk cost, the firm becomes informed about the true value of the per-period costs and can decide whether to continue operations in the foreign export market (which will do if the per-period revenues cover this per-period fixed cost). Prior knowledge of the per-period cost would allow firms to avoid paying the sunk cost whenever it is not profitable for them. As noted before, we allow firms to get information about this cost from the observation of the actions of the rest of firms in that particular market. The result is a learning game with endogenous timing where, if there is entry into a given market, more productive firms are the first to attempt entry into the export market while less productive firms optimally post-pone their entry waiting for more information about the true value of the cost to be revealed.

Our framework is simple enough as to allow us to characterize the equilibrium path of entry only using the general properties of the Melitz model, without any parametrization. Our model delivers several implications. A first one is that a

large degree of uncertainty could preclude the existence of trade flows for a given country pair.² This will be the case in those instances when not even the most productive firm finds it profitable, in expected terms, to break into the foreign export market. On the other hand, in markets with little uncertainty, the most productive firms always find it optimal to enter. Once a firm enters that foreign market, her success/failure reveals information to other domestic firms who, given the new information, optimally decide whether to enter. However, it is possible that entry stops before all the information about a market is revealed. This is the case whenever the marginal entrant does not find it profitable to attempt entry given that informational externalities stop being strong enough to compensate the risk of entering a non-profitable market. We characterize the conditions under which, given initial entry, informational externalities are strong enough to reach an equilibrium with full information. Reaching the full information scenario depends, among other variables, on the elasticity of substitution between products in a given sector.

Information externalities also help explain the existence and persistence of international trade flows, as well as the relative importance of the intensive margin at the country level (since breaking into new markets is subject to uncertainty and could lead to a waiting game) and of the extensive margin at the product level (since most of the increase in aggregate trade flows would come from new firms starting the export on new varieties as they learn from other firms through their actions in foreign markets). Finally, the model can shed light on why we observe empirically less entrance by small firms than the Melitz-Chaney model predicts.

Despite its obvious importance, uncertainty appears to have been largely overlooked by the literature explaining the direction and volume of international trade flows. An exception is the literature on trade and networks, pioneered by Rauch (1996, 1999), which emphasizes the role of trade networks across countries in the presence of uncertainty.³ We see our framework as complementing that literature. Even without the formation of formal networks, uncertainty, and the observation of other firms actions in foreign markets can deliver similar implications for trade flows. There are also recent contributions on the role of uncertainty on causing hysteresis and persistence in export markets. Oromolla and Irarrazabal (2006) and Das, Roberts, and Tybout (2007) are an example of this, but none of them analyze the role of informational externalities on determining the direction and dynamics of trade flows. Finally, there are well known contributions that address the issue

²Segura-Cayuela and Vilarrubia (2007) provide evidence that uncertainty may be an important ingredient in the formation of trade links.

³See Krauthaim (2007) for a recent contribution with endogenous networks within a country, in the context of the Melitz framework.

of entry into a given market under some kind of uncertainty and information externalities, such as Caplin and Leahy (1993) and (1998), or Rob (1991), but to our knowledge nobody has studied these ingredients in an international markets setup or with heterogeneous agents.

The rest of the paper proceeds as follows. Section 2 describes the general set up, analyzes the static problem and solves for the dynamics of entry. Section 3 describes the properties of the equilibrium. Finally, Section 4 concludes and discuss future extensions.

2 General Setup

We build on the framework by Melitz (2003) and Chaney (2007) with firms interacting over several periods. Time is discrete, infinite, and we index it by t . At the beginning of each period, firms make production and pricing decisions and, at the end of the period, firms outside (who have not entered) the export market make the decision whether to remain out or enter that market given the information available to them (to be defined later).

We consider a world consisting of J countries, indexed by $j = 0, 1, \dots, J$ where $j = 0$ represents what we indistinctly refer to as the domestic or home country while we refer to $j > 0$ as the foreign countries or export markets.

2.1 Consumers

Each country, j , is populated by a continuum of hand-to-mouth representative agents who have per-period preferences defined over the available set of goods Ω_t^j , which consist on an agricultural good A and a bundle of manufacturing goods M , distributed among S sectors. Each sector s contains an endogenously determined set of varieties. That is, per-period preferences are given by:⁴

$$u_t^j = c_{A,t}^j + \alpha \ln c_{M,t}^j, \quad \alpha > 0, \quad \text{where} \quad (1)$$

$$c_{M,t}^j = \prod_{s=1}^S \left[\int_{\omega \in \Omega_t^j} x_{st}^j(\omega)^{\frac{\sigma_s-1}{\sigma_s}} d\omega \right]^{\frac{\sigma_s}{\sigma_s-1} \cdot \mu_s}, \quad \text{with} \quad \sum_{s=1}^S \mu_s = 1, \quad \sigma_s > 1, \quad (2)$$

and consumers are subject to the budget constraint (normalizing the price of A to 1) $P_t^j \cdot c_{M,t}^j + c_{A,t}^j = Y_t^j$, where $x_{st}(\omega)$ represents the quantity consumed of variety ω from sector s at time t , σ_s is the elasticity of substitution between varieties from

⁴Hand-to-mouth consumers are introduced to simplify the analysis and to center it on the supply side of the economy, abstracting from the consumers intertemporal problem.

sector s and it is assumed to be the same for all varieties within a sector regardless of whether they are domestically produced or imported varieties, c_A is the tradable, homogeneous good produced using a constant returns to scale technology which requires one unit of labor per unit produced, P_t^j is the ideal price index of the manufacturing bundle and Y_t^j is income in country j at period t .

Throughout the paper we will assume that demand for the homogeneous good A is large enough as for each country to produce that good, which allows us to normalize the wage rate in each country to 1. We also assume this homogeneous good sector is large enough to accommodate any kind of fluctuations in labor demand coming from heterogeneous goods producers, which simplifies the dynamic analysis of the model (that is, we assume that α is small enough). To see this notice that for a given total income of Y_t^j in country j , demands for goods A and M are given by

$$c_{M,t} = \alpha P_t^{j-1}, \quad c_{A,t}^j = Y_t^j - \alpha.$$

Thus, the above assumptions allows us to isolate the manufacturing sector from income effects arising from entry and exit in that sector that would complicate the dynamics of the model. In other words, sector A will accommodate any income shocks to the domestic economy (particularly those coming from creation and destruction of firms).⁵

Consumers are endowed with one unit of labor, which they supply inelastically, and a single share of a diversified portfolio of all domestic firms. Whatever profits earned by firms (both domestically and in foreign markets) are completely redistributed to the consumers in units of the homogeneous good. Given country j 's income, Y_t^j , demand for a given heterogeneous variety of sector s in country j is given by:

$$x_{st}^j(\omega) = \frac{\hat{p}_{st}^j(\omega)^{-\sigma_s} \cdot \mu_s \cdot \alpha}{P_{st}^{j1-\sigma_s}} = \lambda_{st}^j \hat{p}_{st}^j(\omega)^{-\sigma_s}, \quad (3)$$

where $\hat{p}_{st}^j(\omega)$ is the price actually paid by country j 's consumers for variety ω of sector s , and P_{st}^j is the ideal price index of sector s in country j which is defined as:

$$P_{st}^j = \left[\int_{\omega \in \Omega_{st}^j} \hat{p}_{st}^j(\omega)^{1-\sigma_s} d\omega \right]^{1/(1-\sigma_s)}. \quad (4)$$

⁵See Pfluger (2004) for a detailed discussion on this demand structure.

2.2 Producers, Static Problem

We focus our analysis on the export market and, thus, we do not explicitly model entry and exit in the domestic market by domestic firms. Instead, we assume that the domestic side is determined as in the standard Melitz framework and that the needed regularity conditions are satisfied. We assume that each sector in each country has a continuous, but countable, set N_s^j of firms that produce a differentiated good and interact in monopolistic competition fashion.

Each individual firm has measure zero and firms are heterogeneous with respect to their productivity level φ which is drawn from the cumulative distribution function $H_s(\varphi)$ with support $[\underline{\varphi}_s, \overline{\varphi}_s]$, where $\overline{\varphi}_s > \underline{\varphi}_s > 0$. Given our structure, no firm has an incentive to produce the same variety as another firm which allows us to simply index firms according to their productivity level (φ). For simplicity, we assume that the distribution and support of productivities are the same in all sectors and countries. Furthermore, we assume that each domestic firm knows the productivity level of every other domestic firm (possibly from repeated interaction in the domestic market, possibly because the price a firm charges in the domestic market reveals information about its productivity). Firms discount future profits at a per-period rate of β .

Firms set their price to maximize their own profits and engage in monopolistic competition, i.e. when setting their own prices, they do not take into account their own effect neither on the wage or on the ideal price index of their sector. The production of one unit of output for a firm with productivity parameter φ has a variable cost of $w^j/\varphi = 1/\varphi$. Sales in the domestic market are only subject to this cost. However, when selling abroad, there are three additional costs that firms need to incur:

Transportation cost We assume transportation costs to be of the iceberg kind and denote them as τ_s^{ij} which represents the number of units that need to be shipped from country j in order for one unit of good to reach country i in sector s . Our assumptions regarding the domestic market imply that $\tau_s^{ii} = 1$, $\tau_s^{ij} > 1$ for $i \neq j$ and we further assume, without loss of generality, that these costs are symmetric i.e. $\tau_s^{ij} = \tau_s^{ji}$.

Sunk cost There is a known one-time sunk cost that a firm needs to pay in order gain access the foreign markets. We denote it by c_s^{ij} which represents the sunk cost that a firm in sector s of country j would need to sink in order to be able to access country i .

Fixed per-period cost We further assume that there is an *unknown* per-period fixed cost to the continued presence of a firm of country i in the foreign export market s in country j which we denote by f_s^{ij} with an analogous interpretation to the sunk cost. The value of this cost is distributed *i.i.d.* across markets and sectors according to the cdf $G_s(f_s^{ij})$ over the range $\left[\underline{f}_s^{ij}, \bar{f}_s^{ij}\right]$.

The presence of fixed and sunk export costs implies that not all domestic firms are going to necessarily be active in the export market (even if there was perfect information about its value).

These assumptions mean to capture the uncertainty that a firm faces when making the decision on whether to access a foreign market. We assume that the value of f_s^{ij} is revealed to firms that actually access the foreign market (after having paid c_s^{ij}). This introduces an asymmetry in the information set of entrants into the export market with respect to that of non-entrants. Furthermore, we assume that, after having paid c_s^{ij} and observing f_s^{ij} , the firm can costlessly switch off its exporting operations to country i . One possible simple, yet realistic, interpretation of this cost structure is that c_s^{ij} represents the (sunk) cost of adapting the firm's production structure to the possibility of entering export markets while f_s^{ij} represents the cost of sustaining a presence export market i for a firm in sector s .

The market structure implies that firms' pricing decisions are such that a firm in sector s with productivity φ charges a domestic price of:

$$p_s^j(\varphi) = \frac{\sigma_s}{\sigma_s - 1} \cdot \frac{1}{\varphi} \quad (5)$$

that is, firms in a sector charge a common and constant markup (that only depends on the elasticity of substitution with respect to other varieties within the same sector) over their own marginal cost, given by their productivity.

Notice that the domestic price paid by country j 's consumers differs from the one paid by consumers in country i because of the existence of iceberg transportation costs that, in our setup, introduce a wedge between the domestic and foreign price of any given variety. Thus, the price that a firm in country i charges to consumers in country j is

$$\widehat{p}_s^{ij}(\varphi) = \tau_s^{ij} \cdot p_s^j(\varphi) = \tau_s^{ij} \cdot \frac{\sigma_s}{\sigma_s - 1} \cdot \frac{1}{\varphi}. \quad (6)$$

Given this pricing decision, we can compute the per-period export revenues (gross of fixed and sunk costs) of a firm with productivity φ from sector s in country j

that sells its good in country i as:

$$r_{st}^{ij}(\varphi) = \frac{1}{\sigma_s} \lambda_{st}^j \cdot \left(\frac{\sigma_s}{\sigma_s - 1} \cdot \frac{\tau_s^{ij}}{\varphi} \right)^{1-\sigma}, \quad (7)$$

where $r_{st}^{ij}(\varphi)$ only depends on time (t) exclusively via entry (and exit) into export markets (i.e. through the general equilibrium effect that entry of other firms has on the ideal price index).

In the next subsection we characterize the dynamic path of entry in export markets.

2.3 Dynamic Equilibrium

In what follows we analyze the dynamics of entry for a single sector s in a given market j , that is, the decision of entering a given export market.

We assume that outsiders only learn from the strict past, that is, decisions about entry at time t can only be based on observation of actions and market outcomes at times prior to t . Outsiders can not observe the realized profits (net of fixed costs) of domestic firms active in the export market but they can observe their actions, i.e. whether or not they remain active in the export market following entry. These assumptions amount to assuming that it is hard for a firm to disentangle the exact net profits of exporting for any given firm but can easily observe its presence and whether they are active in a given export market.

When solving for the equilibrium path of entry into export markets, we focus on the role of informational externalities which implies ignoring the general equilibrium effects on price indices (as in Melitz, 2003).⁶ That is, we do not take into account strategic complementarities/substitutabilities through payoffs. The reason for that is twofold. First, we want to focus our analysis on the role that informational asymmetries and externalities have in this setup. Secondly, introducing strategic complementarities and substitutabilities, although certainly interesting, would complicate our analysis beyond the scope of this paper.⁷ We also abstract from other strategical considerations such as firms trying to conceal information from outsiders by attempting to conceal their actions.

Our information structure implies that the only relevant information to a firm when making the entry decision is the identity of the least productive firm that

⁶Remember we assume that each firm is a price-taker of zero measure which allows us to ignore any kind of strategic complementary/substitutability.

⁷For instance, if we had strategic complementarities, the solution would be a dynamic global game, with heterogeneous agents and endogenous timing.

exported in any period prior to t since this determines the upper bound for the per-period fixed export cost: if a firm observes another firm with productivity φ' at time t being active in the export market, this implies that the fixed per-period cost can be at most $r_{st}(\varphi')$ since this firm has learnt the true value of f and it would only choose to remain active if doing so was profitable (that is, it can cover at least the per-period fix cost, but not necessarily the sunk cost).

In other words, if a firm is active in the export market, given that c_s^{ij} is sunk, it needs to be the case that per-period revenue is no less than the per-period fixed cost of being in the export market, $f_{st}^{ij} \leq r_{st}^{ij}(\varphi_{st}^{ij})$, where φ_{st}^{ij} is the productivity (and identity) of the least productive firm from country j active in the export market i in sector s at time t . Thus, a firm making the entry decision at time t can infer that the per-period fixed cost is distributed according to $G\left(f_s^{ij} | f_{st}^{ij} \leq f_{st}^{ij}\right)$ where:

$$G(f_s^{ij} | f_{st}^{ij} \leq f_{st}^{ij}) = \frac{G(f_s^{ij})}{G(f_{st}^{ij})}$$

We characterize a stationary (Markovian) equilibrium path of entry in the fashion of Rob (1991). Entry flow at any point in time depends only on the state, which in this setup is defined by the existing set of firms which are in the export market and the available information about the fixed cost.

In a given period t , we define χ_{st}^{ij} as the set of domestic firms active in the export market, and denote as δ_{st+1}^{ij} the flow of entrants between t and $t+1$. The equilibrium is going to be sequential in productivity (more productive firms enter first), which means that if a firm with a given productivity is active in the export market, all firms more productive than her should be too. This means that it is sufficient to know the identity of the least productive active exporter in the previous period, φ_t , to characterize χ_{st}^{ij} . For the same reason, given φ_{st}^{ij} , it is enough to know φ_{st+1}^{ij} to characterize δ_{st+1}^{ij} .⁸

Notice that, given the observation of other firm's action in previous periods, an outsider firm can infer more information regarding the true value of f_s^{ij} . More specifically, if she observes that the identity of the last entrant, φ_{st+1}^{ij} , differs from that of the last active firm, $\widetilde{\varphi_{st+1}^{ij}}$ (with associated per period cost of $\widetilde{f_{st+1}^{ij}}$), this means that either no other firm tried to enter in the previous period, or some did but stopped producing (had negative per-period profits). Thus, at each period t the

⁸We chose to proceed in this way for simplicity. Alternatively we could solve for the equilibrium without sequentiality and then show that it has to be the case. But it is trivial to show that entry has to be sequential. Because of the properties of W , if there is initial entry in period 0 then we have entry of a continuous set of the most productive firms. If this is the case, in period 1 the same is going to happen. And so on. See the Appendix for details.

prior about the distribution of f_s^{ij} is given by

$$G_t(f_s^{ij}) = \begin{cases} G(f_s^{ij}) \text{ over } [\underline{f}_s^{ij}, \bar{f}_s^{ij}] & \text{if } t = 0 \\ G(f_s^{ij} | f_s^{ij} \leq f_{st}^{ij}) & \text{if } f_{st+1}^{ij} = \widetilde{f_{st+1}^{ij}} < f_{st-t'}^{ij} \quad \forall t' > 0 \\ f_{st}^{ij} & \text{otherwise,} \end{cases} \quad (8)$$

where we use the inferred fixed cost from the break-even condition of producers to define the Bayesian updating.

We now compute what the best response for an outsider firm is (i.e. that was not active in the export market) in period t given the behavior followed by the rest of firms. Given $G_t(f_s^{ij})$ and the strategies of the rest of outsiders, it is easy to characterize what the expected release of new information is going to be in subsequent periods. The release of information in future periods can occur in two ways. If all entrants in the current period remain in the export market, an outsider can further truncate its prior about the distribution of f_s^{ij} . On the other hand, if some firms enter the export market and subsequently leave it, the outsider can infer the exact value of f_s^{ij} by equating it to the revenue of the last remaining active firm. The firm can, therefore, optimally decide whether to enter today or wait for the arrival of this new information given the strategies and action of other firms.

From the discussion we know that at time t all outsiders have productivities $\varphi \leq \varphi_{st}^{ij}$, where φ_{st}^{ij} is the productivity of the least productive firm from sector s in country j which is active in export market i at time t . For an outsider to make the decision to enter, she has to find it profitable to enter today instead of waiting for the (potential) release of new information. Thus, to characterize the flow of entrants today we are searching for the firm that is indifferent between entering this period and postponing entry for one additional period.⁹ We assume that, when indifferent between the two options, a firm decides to enter.

For later use, we find it convenient to define $V^e(\varphi)$ (with corresponding per-period revenue $r_s^{ij}(\varphi)$) as the value of entry (the present discounted of future revenues) that a firm with productivity φ would have under perfect information given

⁹The value of waiting for n periods before entering trivially falls at a rate of β^n for firms that weakly prefer to enter than waiting for one period, and since we know that $\beta < 1$, waiting for any number of periods $n > 1$ is strictly dominated by waiting for only one period. The reason is trivial. If waiting for more information until tomorrow does not convince a firm to wait, waiting for two periods is even less preferred, as less information will be released then. Thus, in characterizing the optimal equilibrium path we only need to consider those firms which are deciding between entering today and waiting for one additional period. Of course some firms may be indifferent between entering today and waiting for two periods, but we do not need to worry about this case since these firms do not define the equilibrium path of entry.

that everybody with non-negative profits also decides to enter that market:

$$V^e(\varphi) = \sum_{t=0}^{\infty} \beta^t (r(\varphi) - f_s^{ij}) - c = \frac{1}{1-\beta} (r(\varphi) - f_s^{ij}) - c_s^{ij} \quad (9)$$

We find it convenient to define $f_s^{ij}(\varphi) = r(\varphi) - (1-\beta) \cdot c_s^{ij}$ which corresponds to the maximum fixed cost a firm with productivity φ can incur under no uncertainty. Conversely, we use $\varphi_s^{ij} = \varphi \left(f_s^{ij} \right)$ to denote the productivity (and identity) of the last firm that would have non-negative profits for a given true value of f_s^{ij} , that is

$$\varphi \text{ s.t. } \varphi = r^{-1}(f_s^{ij} + (1-\beta) \cdot c) \quad (10)$$

Define gross revenue for a firm with productivity φ (who is outside the export market) given the state variable (the cutoff productivity to enter the export market) and this period's entry flow as:

$$r_{st}^{ij}(\varphi) \equiv r(\varphi; \varphi_{st}^{ij}, \varphi_{st+1}^{ij}). \quad (11)$$

Clearly the function $r(\varphi; \varphi_{st}^{ij}, \varphi_{st+1}^{ij})$ is increasing in the first argument and decreasing in the other two.¹⁰ This implies that, if there is inefficient entry (by firms with productivity below $\varphi \left(f_s^{ij} \right)$), gross revenue for those firms are below those under perfect information. In terms of our model, this means that $r(\varphi; \varphi_{st}^{ij}, \varphi_{st+1}^{ij}) > r(\varphi)$ $\forall \varphi$ as long as $\varphi_{t+1} \geq \varphi(f)$.

In what follows we focus our analysis in firms in a given sector of the domestic country breaking into a single foreign market and, therefore, drop the sub and super indexes. The dynamics of the rest of sectors in the rest of foreign markets are analogous.¹¹

We use $V^e(\varphi; \varphi_t, \varphi_{t+1})$ to denote the value of entering the foreign markets at time t and $V^w(\varphi; \varphi_t, \varphi_{t+1})$ the value of waiting i.e. of postponing the entry decision for one extra period. We characterize the equilibrium path of entry when information has not been fully revealed. The entry decision when there is full information is trivial, entry stops after this point. We incorporate this possibility later. Notice that we abuse notation by using φ_s and f_s interchangeably.

¹⁰Notice the slight abuse of notation. First, the effect of φ_{st}^{ij} and φ_{st+1}^{ij} on revenues goes through λ_{st}^j , and second, it is enough to know φ_{st+1}^{ij} (total entry up until today) to determine revenues.

¹¹Notice that because of the Cobb-Douglas aggregation of sectors we can analyze each sector separately without loss of generality.

From the previous discussion, the value of entering in period t is given by:

$$V_t^e(\varphi; \varphi_t, \varphi_{t+1}, c) = r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) - c + E_t \left[\sum_{j=1}^{\infty} \beta^j \max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} | f \leq f_t \right]. \quad (12)$$

That is, if a firm enters the export market today, she gets this period's gross revenues $-r(\varphi; \varphi_t, \varphi_{t+1})$, covers the sunk cost c and the fixed cost which, in expected terms, is $E_t(f|f \leq f_t)$ for a given f_t . From the next period onwards, the firm knows the exact value of the per-period fixed cost and earns an infinitely-lived stream of profits if the fixed cost is below her per-period revenues, or exits the export market and gets nothing if the fixed cost is above per-period revenue (but the firm can not recoup the sunk cost c).

The value of waiting and entering the following period if there is still uncertainty is given by:

$$V_t^w(\varphi; \varphi_t, \varphi_{t+1}, c) = \beta \left(p_I \cdot E_t \left[\max\{0, \frac{1}{1-\beta} [r(\varphi) - f] - c\} | f_{t+1} \leq f < f(\varphi) \} \right] \right) + \beta \times \left((1 - p_I) \cdot E_t \left[\sum_{j=1}^{\infty} \beta^j \max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} - c | f \leq f_t \right] \right). \quad (13)$$

In words, if a firm decides to postpone entry, then she gets (and pays) nothing at time t . The payoff to the firm in future periods has two components which depend on the amount of information that is released between t and $t+1$. First, with some probability (p_I), the firm might become perfectly informed about the true value of f in the next period. This will occur whenever there are unsuccessful entrants and the firm can infer the true value of f from the per-period revenues of the remaining least active firm in the export market. In this case, the firm optimally decides whether to enter the export market at time $t+1$ if its revenues are sufficiently large to cover the per-period fixed cost as well as the annuity value of the sunk cost. We refer to p_I as the probability of becoming perfectly informed i.e. of the information set degenerating to a singleton.

The second component of the waiting option corresponds to the payoff that the firm gets in the complementary case, i.e. when perfect information about the true value of f is not released but, nevertheless, some information is still revealed from the fact that the support of f gets further truncated thanks to the mass of (successful) entrants of time t . Since firms are rational, they anticipate this possibility at time t .

Given the flow of entry today, the probability of becoming perfectly informed is given by

$$p_I = \Pr(f_{t+1} < f < f_t | f \leq f_t) = \frac{G(f_t) - G(f_{t+1})}{G(f_t)}.$$

Using this expression, we can rewrite the value of waiting for one additional period as:

$$\begin{aligned} V_t^w(\varphi; \varphi_t, \varphi_{t+1}, c) &= \frac{\beta}{1-\beta} \frac{G(f(\varphi)) - G(f_{t+1})}{G(f_t)} \cdot \\ &\quad \max\{0, (r(\varphi) - (1-\beta) \cdot c - E_t(f | f_{t+1} < f < f(\varphi)))\} + \\ &\quad \frac{G(f_{t+1})}{G(f_t)} \cdot E_t \left[\sum_{j=1}^{\infty} \beta^j (\max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} - c) | f \leq f_{t+1} \right]. \end{aligned}$$

That is, if a firm decides to wait and all information is released, it will only enter whenever it is profitable to do so with certainty, $f < f(\varphi)$. If information does not get fully revealed, the firm will be in a similar position as that of today but with $f \leq f_{t+1}$.

We denote by $W_t(\varphi; f_t, f_{t+1}, c)$ the difference between the value of entering or waiting one period for a firm of productivity φ given the value of f_t and the expected value of f_{t+1} ,

$$W_t(\varphi; \varphi_t, \varphi_{t+1}, c) = V_t^e(\varphi; \varphi_t, \varphi_{t+1}) - V_t^w(\varphi; \varphi_t, \varphi_{t+1}, c) \quad (14)$$

We can rewrite this as (see Appendix for details):

$$\begin{aligned} W_t(\varphi; \varphi_t, \varphi_{t+1}, c) &= r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f | f \leq f_t) - c \left(1 - \beta \frac{G(\varphi_{t+1})}{G(\varphi_t)} \right) \\ &\quad + \frac{\beta}{1-\beta} \frac{G(f_t) - G(f_{t+1})}{G(f_t)} E_t[\max\{0, r(\varphi) - f\} - \\ &\quad \max\{0, (r(\varphi) - (1-\beta) \cdot c - f) | f_{t+1} < f < f_t\}], \end{aligned}$$

which is very convenient as the last term is always non-negative. To characterize the equilibrium we make the assumption that $f_t - E_t(f | f \leq f_t)$ is increasing, which is satisfied by most of the common distribution functions. We proceed by stating some properties of this function

Lemma 1 $\frac{\partial W(\cdot)}{\partial \varphi} > 0$, that is, earlier entry is more attractive the more productive a firm is.

Proof. See Appendix ■

Lemma 1 just states that entry is always more attractive for more productive firms. This just follows from the fact that more productive firms can afford larger fixed costs because they have larger revenues, which implies that the incentives to wait for the release of additional information is less valuable for them.

Lemma 2 $\frac{\partial W(\cdot)}{\partial f_{t+1}} \geq 0$, that is, the lower the entry flow in the current period, the larger the incentive of firms for earlier entry into export markets.

Proof. See Appendix ■

Lemma 2 just states that the less information is going to be released in a given period, the smaller the incentives to wait. That is, if not many firms are going to enter in a given period, the benefits derived from waiting are small when compared to entry.

Also, the appendix shows that $(\partial W(\cdot)/\partial f_{t+1}) \propto \max\{g(f_{t+1}) \times (f(\varphi) - f_{t+1}), 0\}$. Whenever $f(\varphi) < f_{t+1}$, by waiting the firm may ensure a positive net present value. Thus, reducing f_{t+1} decreases the probability of that happening, which in turn makes entry more attractive. If $f(\varphi) > f_{t+1}$ then changing f_{t+1} does not have any effect on that probability, as the firm is almost certain to have zero net present value from entry, which in turn implies that the relative value of entry does not change.

Lemma 3 If $f_{t+1} < f_t$, and $W(\varphi_{t+1}; \varphi_t, \varphi_{t+1}, c) = 0$, then it must be the case that $W(\varphi_{t+1}; \varphi_{t+1}, \varphi_{t+1}, c) > 0$.

Proof. See Appendix ■

$W(\varphi_{t+1}; \varphi_t, \varphi_{t+1}, c) = 0$ is the condition required for entry today up to f_{t+1} given entry yesterday up to f_t (see Proposition 1 for a discussion of this). Lemma 3 just states that if in period t there was entry up to the firm characterized by f_{t+1} then, in period $t + 1$, a firm with a slightly lower productivity (the most productive firm that did not enter in period t) strictly prefers to enter if she is the only entrant.

Proposition 1 (zero trade flows)

$$W_t(\overline{\varphi}; \overline{\varphi}, \overline{\varphi}, c) = r(\overline{\varphi}) - E(f) - c(1 - \beta G(\overline{\varphi})) > 0 \quad (\text{Condition 1})$$

is a necessary and sufficient condition for positive trade flows to exist.

Proof. See Appendix ■

Proposition 1 just states that in order to have entry into a given market, it has to be profitable to do so at least for the most productive firm if she is the only one to enter. Notice the difference with what we would have if we did not consider the option to wait. In that case the most productive firm would enter if $r(\bar{\varphi}) > E(f) - c(1 - \beta)$. That is, given the option to wait, firms are more reluctant to enter. The reason is that information is valuable because it may allow them to save the sunk cost and thus, they only enter if it is profitable enough to compensate for the risk of doing a mistake, $G(\bar{\varphi})$.

Proposition 2 (*sequential entry*) *Given $r(\bar{\varphi}) - E(f) - c(1 - \beta \cdot G(\bar{\varphi})) > 0$, the last firm to attempt entry at time t will be the one with productivity $\tilde{\varphi}$ such that $W(\tilde{\varphi}; \varphi_t, \varphi_{t+1}, c) = 0$ unless the distribution of priors about f has previously collapsed because of the existence of unsuccessful entrants to the export market. In that case entry stops. Entry is characterized by the fix point of that condition, $W(\varphi_{t+1}; \varphi_t, \varphi_{t+1}, c) = 0$*

Proof. See Appendix ■

Thus the equilibrium sequence of entry is characterized by the equation $W(\varphi_{t+1}; \varphi_t, \varphi_{t+1}, c) = 0$ unless somebody enters the export market but does not remain active the following period, that is, someone does not cover the per-period sunk cost. In other words, there is entry until the true cost is revealed. Analyzing the law of motion defined by $W(\varphi_{t+1}, \varphi_t, \varphi_{t+1}, c) = 0$ more closely,

$$r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) = c \cdot \left(1 - \beta \frac{G(\varphi_{t+1})}{G(\varphi_t)}\right). \quad (15)$$

What this equation shows is that in order to be indifferent between entering today and waiting for an additional period, the cost of waiting (potentially lost revenue today, $r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t)$) needs to be equal to the expected gain thanks to the reduced uncertainty next period. The expected gain from this reduced uncertainty is just given by the fact that with some probability information might be released, allowing the firm to save the sunk cost in those cases where it would not have been optimal to enter.

The last part of the equilibrium characterization consists of establishing whether it is the case that the equilibrium path of entry always leads to full information. To see this, notice that the law of motion implies convergence to a productivity level

which we denote by φ_∞ and that is defined by:¹²

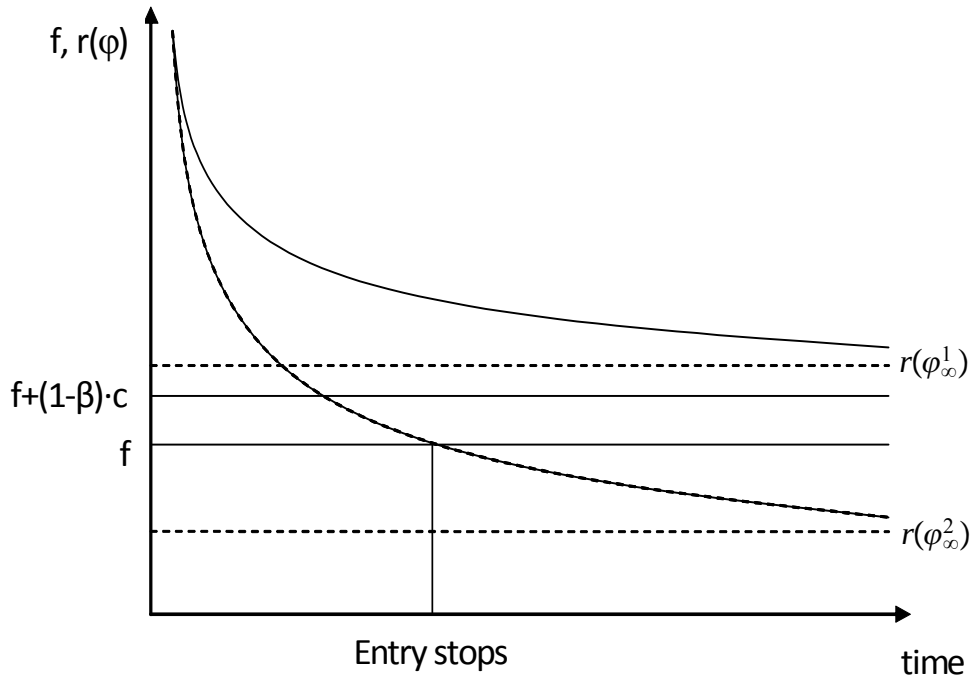
$$r(\varphi_\infty; \varphi_\infty, \varphi_\infty) - E_t(f|f \leq f_\infty) = c(1 - \beta). \quad (16)$$

It is obvious that if $E_t(f|f \leq f_\infty) < f$, entry is going to stop before full information is revealed. The next proposition summarizes the long run properties of the equilibrium:

Proposition 3 *Given $r(\bar{\varphi}) - E(f) - c(1 - \beta \cdot G(\bar{\varphi})) > 0$, there are two types of equilibria. In the first one (if $f < E_t(f|f \leq f_\infty)$) the last entrant converges (possibly not in finite time) to φ_∞ defined in (16). In the second case (if $f \geq E_t(f|f \leq f_\infty)$), entry stops when one or more firms do not even cover the per period fixed cost.*

Proof. In text ■

Figure 1 summarizes these 2 cases.



The first case, where the productivity of the last entrant converges to φ_∞^1 , reflects those instances where entry does not reveal enough information (or at sufficiently fast pace) as to convince outsiders to attempt entry and sink the cost even when, under perfect information, these outsiders would have positive profits from entry into

¹²Convergence follows from the properties of W derived in the *Lemmas* above.

export markets. The other possibility is the one depicted by the second case (φ_∞^2) where entry is faster and only stops due to the existence of unsuccessful entrants and the true value of f being revealed. The following corollary summarizes which conditions make it more likely to have full information being revealed:

Corollary 4 *Full information is more likely to be revealed when the demand of the sector is less elastic, the actual fixed cost is large, the tails of the distribution of the fixed cost are thick, and firms are more impatient (that is the smaller β is).*

The corollary is just a straightforward interpretation of the condition $f \geq E_t(f|f \leq f_\infty)$. The more elastic the demand of a sector is, the smaller the number of firms that the market can support (everything else being equal) and thus the less likely the two curves are going to cross. The thicker the tail of the distribution of f , the more weight firms put on low values of f . Expecting f to be low with high probability is going to lead to more entry, making it more likely for the two curves to cross. Finally, the more impatient firms are, the less they value the option of waiting for more information, which leads to more entry and makes it more likely that full information will be revealed.

The next section discusses the properties of the equilibrium in more details along with some additional implications of the model.

3 Properties of the equilibrium

3.1 Hysteresis and the lack of entrance of small firms

Notice that, when information is fully revealed, by construction some firms have made a mistake and stay in the market with negative net present value (inclusive of sunk costs), that is, we have long-run hysteresis. Although empirically relevant, we see this as a flaw of our model, consequence of having discrete time and not having an exogenous probability of death. We conjecture that with continuous time we would have still two types of equilibria, one with full information being revealed, but where there are no mistakes and with convergence to the true cost, as in Caplin and Leahy (1993).

One interesting result is the fact that we can generate an explanation for what Eaton, Kortum and Kramarz (2005) denote as the failure of the Zip's Law. These authors note that in the data we observe less entrance by small firms than that predicted by Melitz model. In our model, this would correspond to the case when information stops being revealed at some point. In other words, one could argue that a reason why we observe less entry of smaller than the Melitz model would

predict is due to the existence of uncertainty, and that informational externalities are, in some cases, not strong enough as to fully lift this uncertainty.

3.2 Asymmetric trade flows and "contagion" effects

From our framework, and similar to Helpman, Melitz, and Rubinstein (2007), it is obvious that we can account for the existence of asymmetric trade flows (or, more concretely, for the asymmetric existence of positive trade flows). As long as there are different degrees associated with different markets and/or informational flows are not symmetric between two countries, we would expect to see these asymmetric flows, even if the true fixed costs are the same. This implication is something we investigate carefully in the empirical section.

Also, the model delivers what we have termed as "*contagion*" effects. Notice that in the standard Melitz model, the more firms were present in a given market yesterday the less likely a given firm is going to enter today because of general equilibrium effects. This is because the increased size of exporters puts pressure on endowments, namely labor, which drives up their cost and reduces the incentives to export of other firms. In our model, we have an additional force, that works on the opposite direction. The more firms are present in a given market, the more information is released to outsiders, which could potentially overcome the general equilibrium forces (unfortunately we are silent about this second force as our structure kills those general equilibrium effects). Whether this is empirically relevant is something we plan to evaluate in the future.

3.3 A view of the recent history of world trade from our framework

Imagine we are back in the aftermath of WWII, traditional trade routes and relations have been largely disrupted and assume that no country is trading with each other. Which trade flows would our model predict?

Countries would start trading with other countries and in sectors for which they have better information. Once these links have been formed, no more trading partners and/or sectors would be added to those flows unless new information comes about or uncertainty is somehow reduced. However, those countries and in those sectors on which they started trading, more information is going to be revealed thanks to informational externalities from those firms that attempt entry to those who stay out. As this information is revealed more firms are going to enter those given sectors in those particular countries, which in turn is going to reveal even more information in a dynamic that feeds off itself.

Thus, our model would predict dynamics consistent with what we observe in the data, this is that most of the recent increases in trade volumes have occurred at the intensive margin at the country level (between countries that were already trading) and at the extensive margin at the firm level (new firms/products enter those markets where other firms are also trading). Of course we do not claim that this is the whole story, but we claim that uncertainty and informational externalities through the action of exporters might contribute to explain these dynamics.

4 Conclusions and extensions

In this paper we have emphasized an element that has been largely overlooked in the international trade literature, uncertainty. We build a setup based on the current workhorse model of international trade – the Melitz model – where firms are faced with uncertainty and informational externalities when breaking into foreign export markets. We have shown that such a simple modification helps explain some of the not so well understood facts present in the data. Uncertainty and learning from the actions of others (informational externalities) may be an important ingredient in understanding why some small firms do not enter, the persistence of trade volumes (or the lack of them) and, more importantly, the recent dynamic evolution of the relative importance of the extensive and intensive margins, both at the country and the firm/product level.

It is important to point out that in this paper we have abstracted from several interesting issues. This was deliberate, as another intention of this paper was to build a simple framework that would allow us to easily build upon it. The model is simple enough that can be solved using the general properties of the Melitz model without any parametrization. But, among other things, we have abstracted from correlated costs across sectors and/or countries. In the future, we plan to study the dynamics of entry once informational externalities cross the country-sector dimension, that is, when information about a sector in a given country may help learning about the same sector in other countries or other sectors in the same country. This framework might also help us gain new insights in the export vs. FDI decision, especially if we view it as a potentially sequential process (instead of as a dichotomy) where learning through exporting may be the previous step to doing FDI.

Another application might lie in studying the importance that private information plays in multiproduct firms and whether this is an explanation for the fact that these firms enter more markets per product than single product firms. This setup also seems well-suited to understand FDI flows since the decision to setup a foreign

subsidiary might be subject to the same uncertainties that we have emphasized in this model. Finally, most of the uncertainty about new markets may have a lot to do with institutional ingredients. In this matter this framework may help shed some light on the recent debate on the impact of institutions on international trade and FDI.

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5 Theory Appendix

5.1 Derivation of $W_t(\varphi, \varphi_t, \varphi_{t+1}, c)$

Notice that we can write $V_t^e(\varphi, \varphi_t, \varphi_{t+1}, c)$ as

$$\begin{aligned} V_t^e(\varphi; \varphi_t, \varphi_{t+1}, c) &= r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) - c + \\ &\frac{G(f_t) - G(f_{t+1})}{G(f_t)} E_t \left[\sum_{j=1}^{\infty} \beta^j \max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} | f_{t+1} \leq f \leq f_t \right] + \\ &\frac{G(f_{t+1})}{G(f_t)} E_t \left[\sum_{j=1}^{\infty} \beta^j \max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} | f \leq f_{t+1} \right], \quad (17) \end{aligned}$$

that is, we can decompose it in two terms, one containing what happens after today if information is fully revealed today and another if not. Notice that if information is fully revealed then from tomorrow and on $r(\varphi, \varphi_{t+j}, \varphi_{t+j+1}) = r(\varphi)$ and we can rewrite $V_t^e(\varphi, \varphi_t, \varphi_{t+1}, c)$ (after some algebra) as:

$$\begin{aligned} V_t^e(\varphi; \varphi_t, \varphi_{t+1}, c) &= r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) - c + \\ &\frac{\beta}{1 - \beta} \frac{G(f_t) - G(f_{t+1})}{G(f_t)} E_t [\max\{0, r(\varphi) - f\} | f_{t+1} \leq f \leq f_t] + \\ &\frac{G(f_{t+1})}{G(f_t)} E_t \left[\sum_{j=1}^{\infty} \beta^j \max\{0, r(\varphi; \varphi_{t+j}, \varphi_{t+j+1}) - f\} | f \leq f_{t+1} \right] \end{aligned}$$

Now it is easy to see that

$$\begin{aligned} W_t(\varphi; \varphi_t, \varphi_{t+1}, c) &= r(\varphi; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) - c \left(1 - \beta \frac{G(\varphi_{t+1})}{G(\varphi_t)} \right) \\ &+ \frac{\beta}{1 - \beta} \frac{G(f_t) - G(f_{t+1})}{G(f_t)} E_t [\max\{0, r(\varphi) - f\} - \\ &\quad \max\{0, (r(\varphi) - (1 - \beta) \cdot c - f) | f_{t+1} < f < f_t\}], \end{aligned}$$

which is the expression on the main text.

5.2 Proof that the Equilibrium is Sequential in Productivities

In a given period t define as χ_t as the set of domestic firms active in the export market, and denote as δ_{t+1} as the flow of entrants between t and $t + 1$, and lets not restrict the identity of those already in the market and those that will enter today

to be continuous in productivity. In this case

$$W_t(\varphi; \chi_t, \delta_{t+1}, c) = V_t^e(\varphi; \chi_t, \delta_{t+1}, c) - V_t^w(\varphi; \chi_t, \delta_{t+1}, c),$$

where this expressions can be found by replacing φ_t and φ_{t+1} by χ_t and δ_{t+1} respectively on the main text. It is straightforward to check that it is still the case that $\partial W_t(\varphi; \chi_t, \delta_{t+1}, c)/\partial \varphi > 0$. This means that no matter who is already in the market and who enters this period, entry is more attractive for more productive firms. This immediately implies that if a firm φ' enters in t , all firms outside the market with $\varphi > \varphi'$ will too. Finally, when there is nobody in the market, this immediately implies that if the last firm that enters in the first period has productivity φ'' , then all firms with $\bar{\varphi} > \varphi' > \varphi''$ will enter too. *Q.E.D.*

This result allows us to simplify the analysis on the text and look only for sequential entry, simplifying the analysis and notation.

5.3 Proof of Lemma 1

We take the derivative of the $W(\cdot)$ with respect to $r(\varphi; \varphi_t, \varphi_{t+1})$ and use the fact that $\partial r(\varphi; \varphi_t, \varphi_{t+1})/\partial \varphi > 0$, which implies that $\text{sign}(\partial W(\cdot)/\partial r(\varphi; \varphi_t, \varphi_{t+1})) = \text{sign}(\partial W(\cdot)/\partial \varphi)$. This yields, after rearranging terms,

$$\begin{aligned} \frac{\partial W(\cdot)}{\partial r(\varphi, \cdot, \cdot)} &= 1 + \frac{\beta}{1 - \beta} \frac{G(f_t) - G(f_{t+1})}{G(f_t)} + \\ &\frac{\beta}{1 - \beta} \cdot \frac{\partial}{\partial r(\varphi, \cdot, \cdot)} E_t[\max\{0, r(\varphi) - f\} - \max\{0, (r(\varphi) - (1 - \beta) \cdot c - f)\} | f_{t+1} < f < f_t]. \end{aligned}$$

Notice that $\max\{0, \partial r(\varphi, \cdot, \cdot) - f\} \geq \max\{0, (\partial r(\varphi, \cdot, \cdot) - (1 - \beta) \cdot c - f)\}$ which immediately implies that $\partial W(\cdot)/\partial r(\varphi, \cdot, \cdot) > 0$. *Q.E.D.*

5.4 Proof of Lemma 2

Taking the derivative of $W(\cdot)$ with respect to f_{t+1} and after rearranging terms it is straightforward to check that

$$\begin{aligned} \left(\frac{\partial W(\cdot)}{\partial f_{t+1}} \right) &= \frac{\partial r(\varphi, \cdot, \cdot)}{\partial f_{t+1}} + \beta \frac{g(f_{t+1})}{G(f_t)} c + \\ &\frac{\beta}{1 - \beta} E_t[\max\{0, r(\varphi) - f_{t+1}\} - \max\{0, (r(\varphi) - (1 - \beta) \cdot c - f_{t+1})\}] \end{aligned}$$

Notice that $\partial r(\varphi, \cdot, \cdot) / \partial f_{t+1} > 0$ (less entry increases profits today). Also, the third term is trivially positive as it is the expectation of a positive number (the first max is always larger than the second). This completes the proof.

Q.E.D.

5.5 Proof of Lemma 3

Notice that the last term of $W(\varphi; \varphi_{t+1}, \varphi_{t+1}, c)$ cancels out for $\varphi = \varphi_{t+1}$,

$$W(\varphi_{t+1}; \varphi_{t+1}, \varphi_{t+1}, c) = r(\varphi_{t+1}; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_{t+1}) - c \left(1 - \beta \frac{G(\varphi_{t+1})}{G(\varphi_t)} \right).$$

Adding and subtracting $E_t(f|f \leq f_t)$ we get

$$\begin{aligned} W(\varphi_{t+1}; \varphi_{t+1}, \varphi_{t+1}, c) &= r(\varphi_{t+1}; \varphi_t, \varphi_{t+1}) - E_t(f|f \leq f_t) - c \left(1 - \beta \frac{G(\varphi_{t+1})}{G(\varphi_t)} \right) - \\ &\quad E_t(f|f \leq f_{t+1}) + E_t(f|f \leq f_t) - E_t(f|f \leq f_{t+1}), \end{aligned}$$

which because of Condition (15) simplifies to

$$W(\varphi_{t+1}; \varphi_{t+1}, \varphi_{t+1}, c) = E_t(f|f \leq f_t) - E_t(f|f \leq f_{t+1}) > 0.$$

Q.E.D.

5.6 Proof of Proposition 1

Straightforward. To have entry in period 0 we need that the value of entry relative to waiting for the most productive firm, if he was the only firm to enter, is positive, that is $W_t(\bar{\varphi}; \bar{\varphi}, \bar{\varphi}, c) = r(\bar{\varphi}, \bar{\varphi}, \bar{\varphi}) - E(f) - c(1 - \beta c(1 - \beta G(\bar{\varphi}))) > 0$, which replacing by its expression translates into Condition 1 in the main text.. Thus, at least firm $\bar{\varphi}$ enters the export market if the condition is satisfied. If Condition 1 is not satisfied, Lemma 1 ensures that nobody enters.

5.7 Proof of Proposition 2

We know from Proposition 1 that at least the top firm enters in period 1 if *Condition 1* is satisfied. By continuity of the function $W(\varphi; \varphi_t, \varphi_{t+1}, c)$ on both φ and φ_{t+1} and Lemma 1 and 2, there exist a firm $\tilde{\varphi}_1$ such that $W(\tilde{\varphi}_1; \varphi_0, \tilde{\varphi}_1, c) = 0$ and $W(\varphi; \varphi_0, \tilde{\varphi}_1, c) > 0$ for all $\varphi > \tilde{\varphi}_1$ (and increasing in φ) which implies that all these firms enter the market. Now in period 2 two things can happen. Uncertainty can totally reveal and we reach an equilibrium in which the last entrant is given by

$W(\varphi_1; \varphi_0, \varphi_1, c) = 0$, where the last firm that will produce will have productivity equal to φ_1 if there was no mistake, or it will be the φ such that $r(\varphi) > f$ if there were mistakes. On the other hand, if uncertainty did not reveal and we had entry up to a level φ_1 given by the condition above, by Lemma 3 we know that $W(\varphi_1; \varphi_1, \varphi_1, c) > 0$, which implies that the marginal firm that did not enter yesterday wants to enter today. This condition in period 2 is equivalent to *Condition 1* for the first period, which implies that the reasoning done for the first period goes through now. This keeps going until entry hits the real value of the fixed cost. *Q.E.D.*