Does the growth process discriminate against older workers?

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Abstract

This paper seeks to gain insights on the relationship between growth and employment, particularly when considering heterogeneous agents in terms of age. We introduce in the endogenous job destruction framework *à la* Mortensen and Pissarides (1998) life cycle features. We show that, under the assumption of homogeneous productivity among workers, firms tend to fire more often older workers rather than young ones, when deciding whether to update or not a technology: there is an equilibrium where the creative destruction effect dominates over the capitalization effect for old workers, whereas the capitalization effect dominates for the young workers. This discrimination against older workers can be moderated when we introduce heterogeneity (in terms of productivity) among workers. Indeed, it may be more interesting for the firm to pay the updating cost and train high-productivity old workers, in spite of their advanced age, rather than destroying their job and creating a new one. We also provide empirical support to these theoretical findings using OECD panel data set, and numerical simulations of the model.

Keywords: TFP growth, unemployment by age, old workers' employment rate, capitalization, creative destruction effect, heterogeneous productivity

JEL: J14, J24, J26, O33

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1 Introduction

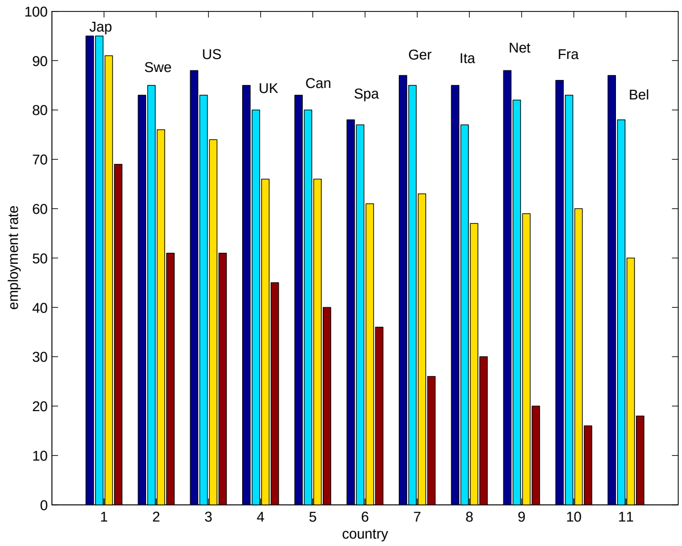
The relationship between growth an unemployment has been largely analyzed by the economic literature. In his seminal work, Pissarides (1990) claims that an acceleration of growth improves the employment rate, because growth increases "freely", as in the Solow model, the expected profits and then provides incentives to open new jobs (the capitalization effect). In contrast, Aghion and Howitt (1994) argue that growth fosters a "creative destruction" process inducing more job destruction and less job creation, yielding higher unemployment rates. What effect (creative destruction vs. capitalization) prevails in case of acceleration in the growth rate? As remarked by Mortensen and Pissarides (1998), if the cost of adopting the new more productive technology in an already existing job is higher than the cost of creating of a new job, growth leads to creative destruction (more unemployment). Conversely, if the updating cost is below the creation cost, growth can lead to capitalization (less unemployment).

Even if from a theoretical point of view the link between growth and employment is ambiguous, the empirical works report more clear results. Using a traditional econometric approach, i.e. a panel of OECD countries, Blanchard and Wolfers (2000) or Pissarides and Vallanti (2007) show that an increase in growth pushes down unemployment. Similarly, using several measures of co-movements in the frequency domain, Tripier (2007) shows that the long-run co-movement between unemployment and productivity is strongly negative, in the US and Europe. Moreover, the simultaneity between the slowdown of the productivity growth and the arise of the unemployment rate in the OECD countries support the view of a positive correlation between these two phenomena. The empirical evidence seems to suggest then that the capitalization effect of growth overcomes the creative destruction effect, leading to a decrease in the long term aggregate unemployment rate.

In this paper, we show that the relative magnitude of the capitalization and the creative destruction effects depends on the age of the worker. When a worker is young, the time horizon during which a firm can recoup the updating cost allowing to renovate an existing technology is longer than for a senior. Then, the optimal policy for the firm is to renovate frequently jobs occupied by young workers whereas separation is the best choice if the job is occupied by an old worker. In this scenario, the capitalization effect can explain the positive relation between the employment rate of young workers and growth, whereas the creative destruction effect explains the low employment rate of older workers. We then argue that a low retirement age (high implicit tax on continued activity) introduces a horizon beyond which firms have less incentives to update jobs occupied by old workers. Because there is heterogeneity among countries concerning the retirement age (implicit tax), we also observe (see figure 1) this heterogeneity concerning the employment rate of old workers 5 year before the retirement age: the cost of renovating these jobs becomes too high relative to the expected associated gains[1] We deduce, from this scenario,

Figure 1: Employment rates from age 30 to 64 for OECD Countries

Legend : 30- 49 (first bar on the left), 50 - 54, 55 - 59 and 60 - 64



Ranking of effective retirement age (from the highest to the lowest)

that the slowdown of the TFP growth rate observed in the OECD countries have a negative impact on the employment rate of the young workers, whereas it reduces the creative destruction process for the older workers. Then, the large decrease of the employment rate of the older worker observed since the beginning of the 80's can be largely due the increase of disincentives to work longer (early retirement schemes, disability program for the older workers, etc...).

We can then test if the acceleration of growth fostered by technological progress has been biased against old workers. Estimates reported in table 1 seem to confirm this intuition. The employment rate of young workers increases with TFP (total factor productivity) growth rate whereas that of old workers decreases. In these regressions, we control for labor market institutions[2],

[1] The impact of short term horizon on the job search behaviors has been analyzed by Hairault, Langot and Sopraseuth (2006), (2007) and Chron, Hairault and Langot (2007).

Table 1: Relationship between the employment rate of the young and old workers and TFP growth

	Employment rate 25-54	Employment rate 55-64
TFP growth	.9758701	-3.28761
	[.2298043 1.721936]	[-4.732897 -1.842322]
Social Security tax		-21.21507
		[-27.86535 -14.56478]
Normal retirement age		1.826419
		[.9542081 2.69863]
Average replacement rate	-.0518935	-.5987909
	[-.0820592 -.0217278]	[-.6778195 -.5197622]
Tax wedge	-.1824758	-.2538403
	[-.2488446 -.116107]	[-.4794317 -.028249]
Employment Protection	-.7203728	10.37075
	[-1.185403 -.2553427]	[8.627823 12.11369]
Union density	-.0230632	-.1237807
	[-.034049 -.0120774]	[-.1869066 -.0606548]
Union coverage	7.576104	7.532248
	[5.825852 9.326355]	[2.1451 12.9194]
Output gap	.4900921	.2511473
	[.3872968 .5928874]	[.0697003 .4325943]
Year	-.1719901	-.5651216
	[-.203243 -.1407373]	[-.6645456 -.4656976]
Country	.0723783	.5306266
	[.0405035 .1042531]	[.4615466 .5997065]
Constant	433.239	1073.156
	[370.8108 495.6673]	[871.2699 1275.042]

Cross-sectional time-series FGLS regression

Countries: BEL, FRA, ITA, GBR, SWE, DNK, USA and CAN

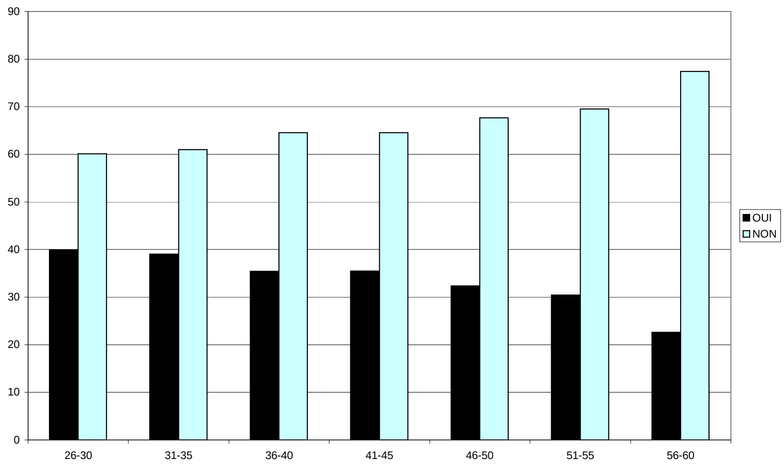
Period: 1982-2003

Data set Bassanini and Duval [2007].

for business cycle, for time specific effects and for country specific heterogeneity. All the labor market institutions are significant and display the expected sign. Concerning the regression for older workers, we also control for the heterogeneity arising from country specific Social Security rules (the implicit tax on continued activity, SS tax, and the normal retirement age).

Furthermore, from a microeconomic point of view, we observe from figure 2, built using data from the French Complementary Survey on Training 2001, that specific human capital investment made by firms in case of technological change[3] declines with age. This also suggest that growth

Figure 2: Training by age.



is biased against old workers. How can can we interpret this finding? Ahituv and Zeira (2000) claim that human capital is technology specific and therefore the introduction and progressive diffusion of novel technologies tends to erode human capital. Moreover, because of their shorter working horizon, seniors have no incentive to follow a training program. These results are supported by Borghans and Weel (2002) who conclude that, by favoring the development of non routine complex task, information and communication technologies (ICT) introduce a bias against older workers, who do not simply have a lower incentive to get trained but also lower

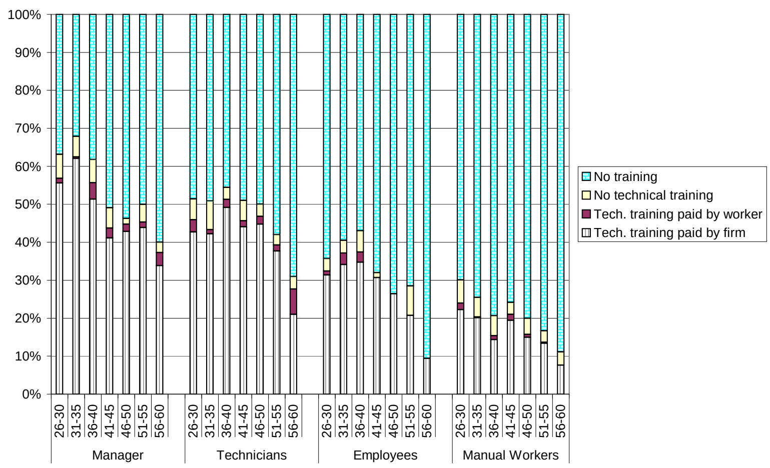
[2] We choose the traditional indicators of the labor market institutions: the average replacement rate, the tax wedge, the employment protection on regular contract, the union density and a indicator of the union coverage.

[3] Workers are asked whether they have been trained or not after the adoption of a novel technology by the firm. They are also asked to specify if the training was aimed at improving their technical skills and who paid for the technical training, the firm or themselves.

capabilities than younger workers[4].

Findings of this literature suggest then that, beyond age heterogeneity, growth has also an heterogenous impact on individuals endowed with divergent characteristics. Actually, on the basis of French data, Behaghel and Greenan (2007) estimate that workers in lower occupations have reduced access to training in firms using new technologies, whereas older managers and technicians/supervisors do get more frequent training. Firms seem hence ready to train some types of old workers in spite of their short working horizon (probably high-productivity workers), while they will not train other types of workers. Data in figure 3, estimated from the French Complementary Survey on Training 2001 confirms this result. We observe from this figure that, when considering workers above 50 years old, specific human capital investment paid by firms is biased in favor of some types of occupations, referred as "high productivity occupations". Around 44% of managers (38% of technicians) between 51 and 55 years old and 34% of those (21% of technicians) between 56 and 60 years old receives training paid by the firm in case of technological change. When considering employees, these percentages are reduced to 21% and 9%, and to 13% and 8% for manual workers. In spite of their shorter working horizon, firms continue to invest in managers and technicians specific human capital (rather than in employees' and manual workers' one), suggesting that productivity gains obtained by updating their skills manage to compensate the training cost.

Figure 3: Training by occupation and age.



In sum, we cannot longer claim that the risk of unemployment induced by an acceleration of growth is equally shared by all workers since, according to our data, certain types of workers

[4]Koning and Glerderblom (2004) confirm this result, on the basis of Dutch data.

support a higher risk. Over the past decade, this topic has been traditionally studied by analyzing the impact of technological progress on the (un)employment rates of skilled and unskilled workers (see for example Berman, Bound, and Griliches (1994), Machin, and Van Reenen (1996) and Moreno-Galbis and Sneessens (2007)). However, to our knowledge, there are few studies analyzing whether a given individual is equally exposed to the consequences of growth along her life-cycle[5]. Our paper will focus on this issue. More precisely we analyze if the acceleration of growth fostered by the diffusion of technological progress is systematically biased against old workers or if we can find the conditions under which growth may be favorable to old workers' employment rate.

The objective of this paper is to develop a theoretical framework explaining under which conditions the capitalization effect induced by technological progress is dominated by the creative destruction effect leading to the observed drop in the employment rate of workers approaching retirement. This result supports the view according to which a low retirement age yields the firm to expect a short time-horizon for seniors and thus disincentives investment on jobs occupied by old workers. This can explain the heterogeneity of the employment rates of the 54-59 year-old workers among countries (see figure 1). The main contribution of this paper, is to reverse the usual causality which claims that workers who have a high-tech job retire latter (see e.g. Friedberg (2003)): in our model, it is the low "normal retirement age" which fosters the reduction of investment in old workers' jobs and favors then an earlier destruction of these jobs. Nevertheless, when dealing with a high productivity old worker it may be more interesting for the firm to update her abilities rather than destroying the job and having to support the cost of opening a new vacancy. By training only high productivity workers, firms induce lower employment rate of low productivity workers who do not benefit from this training.

We use a vintage model inspired from Mortensen and Pissarides (1998). We consider a complete representation of the labor market distinguishing between young and old workers, as well as between high and low productivity workers (last section of the paper). The labor market is then segmented between jobs addressed to old workers and jobs addressed to young workers. A firm hiring an old worker knows that he has a given probability to go on retirement and, similarly, a firm hiring a young worker interiorizes the fact that he will become old at a given probability. Workers are associated to a given vintage technology whose productivity remains

[5]For an empirical study, see Aubert, Caroli, and Roger (2006) who analyze the effects of novel technologies on the labor flows by age

constant along life. In contrast, wages are indexed to the growth rate of the technological frontier, via the unemployment benefit system. A match between the firm and the worker pursues as far as the associated surplus is positive. The equilibrium of the model is not only given by the labor market tension, but also by the scraping time (number of periods of survival) associated to a given vintage.

We find that the acceleration in the rate of technological progress reduces the scrapping time of machines/workers (more job destruction). Besides, firms prefer, in principle, to update technologies associated to younger workers since they can smooth their investment during a larger period of time (old workers have a higher probability to leave on retirement). Nevertheless, this result is modified if we distinguish between old workers having a relatively high productivity and those having a relatively low productivity. In this case, even if the working horizon is short, it is more interesting for the firm to train the high productivity worker rather than destroying the job and creating a new one. Finally the effects of a variation in the impending retirement age are analyzed through a comparative static analysis and we use numerical simulations to analyze the effects of various policy measures.

The paper is organized as follows. Next section presents the main assumptions of the model as well as the agents behavior. In section 3 we describe the main characteristics of the wage posting equilibrium where the firm does not have the possibility to update a technology. We also analyze the effects of an increase in the pace of technological progress. We systematically contrast the results obtained under the hypothesis of rigid wages with those that we would have obtained if wages were flexible. Section 4 introduces the possibility of updating a technology rather than destroying the job in a context where there are no productivity differentials within the group of old and and young workers. Section 5 reviews the conclusions concerning renovation in a context where we allow for productivity heterogeneity within each group. Section 6 presents some numerical results. Finally, section 7 concludes.

2 The model

2.1 Assumptions

We build a matching model based in Mortensen and Pissarides (1998) where the economy is populated by a continuum of firms and workers. Each firm employs only one worker. Workers may be young or old, where the exogenous probability of becoming old is represented by λ_y, so that workers remain young for a period equal to $1/\lambda_y$. Older workers go to retirement with probability λ_o, therefore they are old during $1/\lambda_o$ periods (see figure 4). In this stylized life cycle model, we assume that the risk to become a retiree is supported during a short duration time. We restrict then λ_o and λ_y such that $\lambda_o > \lambda_y$. The evolution of the total population of young and old workers is respectively given by:

$$\dot{P}_y = \lambda_o P_o - \lambda_y P_y \tag{1}$$

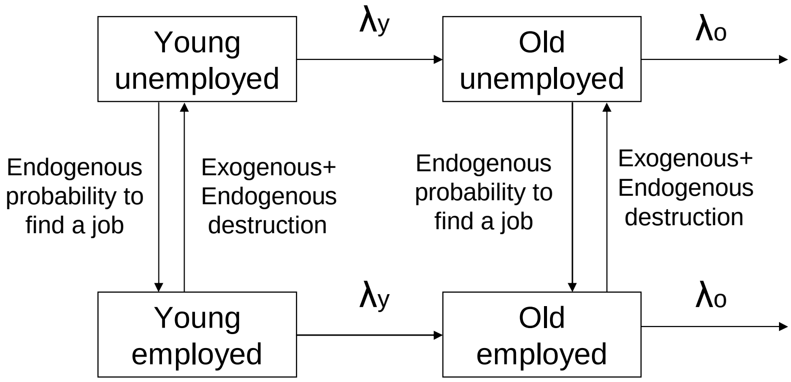
$$\dot{P}_o = \lambda_y P_y - \lambda_o P_o \tag{2}$$

where $\lambda_o P_o$ stands for the fraction of old workers going to retirement and leaving thus the old work force. Because the aggregate work force is assumed to be constant, this proportion of old workers going to retirement must equal the entries into the young work force. Similarly, $\lambda_y P_y$ represents the exits from the young segment and the entries to the old one. Normalizing P_y to one we obtain $P_o = \lambda_y/\lambda_o$.

Firms may direct their open vacancy either to a young or an old worker. Assuming that young and old workers have the same productivity, firms prefer young workers because they can produce for a longer period of time. The free entry condition ensures though that firms post vacancies for both types of workers. It is always in the interest of an old worker, to search in the young labor market segment since there are more job opportunities. Nevertheless, we assume that old workers are not endowed with the required characteristics to become productive in a young worker position. That is, we are assuming that within the firm's organization all characteristics are not perfectly substitutable. For example, for a given productivity level, the organizational process of the firm may require more physical effort if the job is directed to a young worker and more experience if directed to a senior.

Let's denote x productivity level on each job, $(\overline{q}, \underline{p})$ the characteristics of young workers and $(\underline{p}, \overline{q})$ the ones of seniors. A firm searching for a young worker achieves the productivity level x with $(\overline{q}, \underline{p})$, whereas the one searching for a senior requires $(\underline{p}, \overline{q})$ to attain x. If a worker applies to a job not matching its age-characteristic, her production will be nil. Under these assumptions, the decentralized allocation with directed search is a stable equilibrium[6]. When directing the vacancy to a young worker the firm interiorizes though that the worker will become old with a probability λ_y.

[6]See Chron, Hairault and Langot (2007) for a complete discussion on the equilibrium instability with direct search when all the workers have the same characteristics.



New jobs embody the most advanced known technology, however, once created, their productivity remains constant for the rest of their life. Job creation commits the firm to the technology available at the date, unless the firm decides to update later by bearing a fixed renovation cost. Once the job is created several situations may arise. First, the firm can continue to produce with the technology embodied in the job when it was created. Secondly, the firm may decide to pay a fixed renovation cost to update the technology and continue producing with the same worker. Finally, the firm may decide to close down and exit production because the job is not longer profitable. To keep our representation as close as possible to Mortensen and Pissarides (1998) we will also assume that jobs might be destroyed by an exogenous shock with probability δ.

2.2 Notation

We will note X_y all variables referred to young workers and X_o those referred to old. At each moment of time a mass u_i for $i = y, o$ of unemployed workers and a mass v_i for $i = y, o$ of vacant jobs coexist on the labor market. Then jobs and workers meet pairwise at a Poisson rate $M(u_i, v_i)$, where $M(u_y, v_y)$ stands for the matching function in the young segment and $M(u_o, v_o)$ for the matching function for the old segment. This function is assumed to be strictly increasing and concave, exhibiting constant returns to scale. Furthermore it satisfies the Inada conditions and $M(0, v_i) = M(u_i, 0) = 0$. Under these assumptions and knowing that $M(u_i, v_i)$ represents the number of matches per unit of time, we can represent the probability of filling a vacancy as $q(\theta_i) = M(u_i, v_i)/v_i = M(\theta_i, 1)/\theta_i$ for $i = y, o$. Equivalently, the probability of finding a job is given by $p(\theta_i) = M(u_i, v_i)/u_i = M(\theta_i, 1)$ for $i = y, o$.

Independently on whether a vacancy is directed to a young or to an old worker, a firm without a worker advertises a job vacancy at a cost $p(t)c$ per period, where $p(t) = e^{gt}$ is a common growth factor and g is the rate of growth in productivity at the technological frontier. Across newly created jobs, match productivity grows thus at the exogenous rate $g = \dot{p}(t)/p(t)$ (new jobs always embody the most advanced known technology). Once the job is created at date τ its associated productivity, $p(\tau)x$, where x is a constant, does not change, unless the firm implements a new technology at cost $p(t)I$ (updating). The opportunity cost of unemployment is represented by $p(t)b$. Because the outside option of employment increases in response to productivity growth at the technological frontier whereas the job's productivity remains constant, the surplus associated to a match is decreasing through time.

Job creation costs and renovation costs grow with match productivity by assumption to ensure the existence of a steady state with balanced growth.

2.3 The agents' behavior

An open vacancy can remain empty and searching or be filled and start producing. The associated asset value to each of these situations is represented by $V_i(t)$ for $i = y, o$ if the vacancy is empty at current date t. $J_i(\tau, t)$ for $i = y, o$ stands for the value of an existing job at date t which was created at time τ. Similarly, the value of employment in a job at date t which was created at time τ is represented by $W_i(\tau, t)$ for $i = y$, whereas the value of unemployment search at date t is given by $U(t)$. We first consider the case where the renovation cost is so high that the firm waits for obsolescence rather than ever updating the technology.

2.3.1 The firms

When the firm opens a vacancy at date t it supports a cost $p(t)c$, whatever the type of worker, young or old, required to fill the vacancy. There is then a probability $1 - q(\theta_i)$ for $i = y, o$ that the vacancy remains empty next period. On the opposite, there is a probability $q(\theta_i)$ for $i = y, o$ that the vacancy gets filled. The asset value associated to a searching vacancy is then:

$$rV_i(t) = -p(t)c + q(\theta_i)(J_i(t,t) - V_i(t)) + \dot{V}_i(t) \qquad \forall i = y, o \tag{3}$$

In the equilibrium the firm opens vacancies until all rents from new job creation are exhausted, i.e. $V_i(t) = 0$. Market tightness at date t satisfies then:

$$\frac{p(t)c}{q(\theta_i)} = J_i(t,t) \qquad \forall i = y, o \tag{4}$$

Even though we do not assume productivity differentials between young and old workers, the asset value at date t of a job filled at date τ varies depending on whether the job is directed to a young or to an old worker. In the former case, even if at date τ the worker filling the vacancy is young, the firm interiorizes the fact the worker will become old with probability λ_y. In this case, the firm may decide to keep the young worker even if its associated instantaneous profit is negative, if she expects that the worker will become old and will obtain a positive profit compensating the negative one.

Similarly, when the worker is old the asset value of the filled vacancy must take into account for the fact that the worker will go on retirement with probability λ_o. Furthermore, young and old jobs may be destroyed by an exogenous shock with probability δ. The asset values associated to a job filled by a young worker and by an old worker are respectively given by:

$$rJ_y(\tau,t) = \max\left\{\begin{array}{r} p(\tau)x - w_y(\tau,t) - \delta J_y(\tau,t) \\ -\lambda_y(J_y(\tau,t) - J_o(\tau,t)) \\ +\dot{J}_y(\tau,t) \quad ,0 \end{array}\right\} \tag{5}$$

$$rJ_o(\tau,t) = \max\left\{\begin{array}{r} p(\tau)x - w_o(\tau,t) - (\delta+\lambda_o)J_o(\tau,t) \\ +\dot{J}_o(\tau,t) \quad ,0 \end{array}\right\} \tag{6}$$

where $w_i(\tau,t)$ for $i = y, o$ stands for the wage paid at date t in a job created at date τ.

2.3.2 The workers

An unemployed worker receives a flow of earnings $p(t)b$ including unemployment benefits, leisure, domestic productivity, etc. and increasing with the productivity growth at the technology frontier. A young job seeker comes in contact with a vacant slot at rate $p(\theta_y) = \theta_y q(\theta_y)$ and becomes old with probability λ_y. Old job seekers enter in contact with a vacancy at rate $p(\theta_o) = \theta_o q(\theta_o)$ and they quit the labor market with probability λ_o. The asset value of unemployment to both types workers is respectively given by:

$$rU_y(t) = p(t)b + \theta_y q(\theta_y)(W_y(t,t) - U_y(t)) - \lambda_y(U_y(t) - U_o(t)) + \dot{U}_y(t) \tag{7}$$

$$rU_o(t) = p(t)b + \theta_o q(\theta_o)(W_o(t,t) - U_o(t)) - \lambda_o U_o(t) + \dot{U}_o(t) \tag{8}$$

At date t, a job created at τ and filled with a young worker has a productivity $p(\tau)x$ and pays a wage $w_y(\tau,t)$ to the worker. If the job is filled with an old worker, the productivity also equals $p(\tau)x$ and we denote the wage as $w_o(\tau,t)$ (in a wage posting equilibrium $w_y(\tau,t) = w_o(\tau,t)$). Both types of jobs are hit by a shock with probability δ, young workers become old with probability λ_y and old workers quit the labor market with probability λ_o. The present value of a young and an old workers solve:

$$rW_y(\tau,t) = \max \left\{ \begin{array}{r} w_y(\tau,t) - \delta(W_y(\tau,t) - U_y(t)) \\ -\lambda_y(W_y(\tau,t) - W_o(\tau,t)) \\ +\dot{W}_y(\tau,t) \quad ,U_y(t) \end{array} \right\} \tag{9}$$

$$rW_o(\tau,t) = \max \left\{ \begin{array}{r} w_o(\tau,t) - \delta(W_o(\tau,t) - U_o(t)) \\ -\lambda_o W_o(\tau,t) \\ +\dot{W}_o(\tau,t) \quad ,U_o(t) \end{array} \right\} \tag{10}$$

2.3.3 The wage bargaining process

Since search and hiring activities are costly, when a match is formed a joint surplus is generated. At the beginning of every period the firm and the employee negotiate wages through a Nash bargaining process, that splits the joint surplus into fixed proportions at all times. Denoting as $\eta \: \epsilon \: (0,1)$ the bargaining power of workers, we have the following surplus sharing rules, $\forall i = y, o$:

$$S(\tau,t) = J_i(\tau,t) - V_i(t) + W_i(\tau,t) - U_i(t) \tag{11}$$

$$J_i(\tau,t) = (1-\eta)S(\tau,t) \tag{12}$$

$$W_i(\tau,t) - U_i(t) = \eta S(\tau,t) \tag{13}$$

Replacing equations (5), (6), (3), (9), (10), (7) and (8) in expression (11), and applying then one of the sharing rules (either (12) or (13)) leads to the following wage:

$$\begin{aligned} w_i(\tau,t) &= \eta p(\tau)x + (1-\eta)p(t)\left(b + \frac{\eta}{1-\eta}c\theta_i\right) \\ &= \eta p(\tau)x + (1-\eta)p(t)\omega(\theta_i) \qquad \forall i = y, o \end{aligned} \tag{14}$$

The wage equals a weighted average of the worker's productivity on her current job and the flow value of her outside options, since $\omega(\theta_i) = (b + \frac{\eta}{1-\eta}c\theta_i)$ represents the worker's reservation wage (the flow value of the worker's job search option). Even in the absence of an exogenous opportunity cost for employment ($b = 0$), wages in existing jobs (created at some past date τ) grow, in response to wage growth in new jobs induced by technical progress. Because productivity remains constant (at a level $p(\tau)x$), the job may become not profitable at a given moment of time, fostering a job destruction.

2.3.4 The scrapping time for young and old workers

The optimally chosen job destruction age, T_i for $i = y, o$, maximizes the value of a job at all dates after creation $t \geq \tau$. Because old and young workers differ in their probability to go on retirement and in their probability to become old, their scrapping time will also differ among them.

When the firm hires a young worker, we can distinguish among two scenarios (see figure 5):

- When $T_y > T_o$ (panel A figure 5), two possibilities may arise:

  1. Case 1: If the young worker becomes old before T_o, then she will pursue her job until T_o.
  2. Case 2: If the young worker becomes old after T_o, the job is automatically destroyed

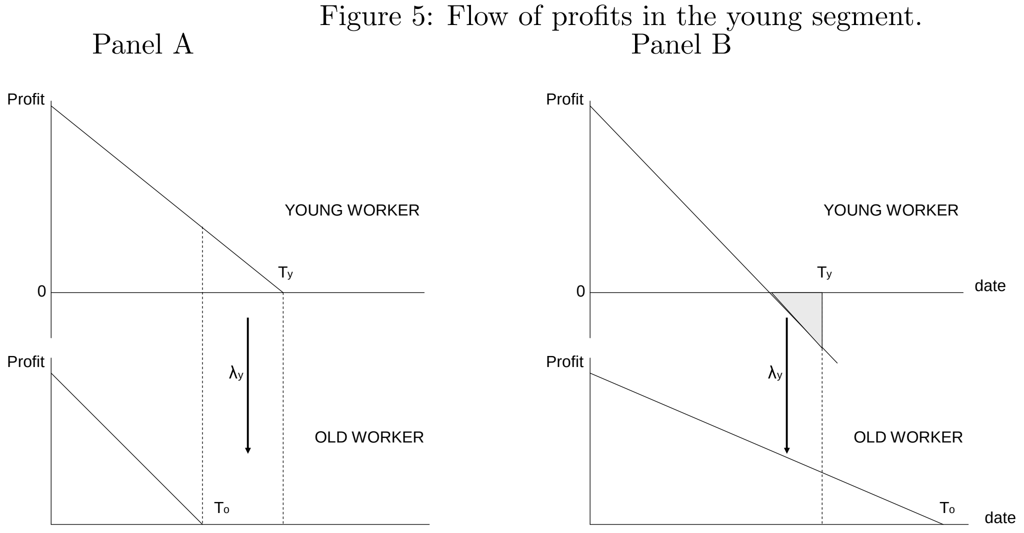
- When the optimal scrapping time of the young worker is below the optimal scrapping time when the worker is old , $T_y < T_o$ (panel B figure 5), T_y represents the maximum period of time the firm will be ready to keep the worker in spite of making a negative profit, since the firm knows that these looses will be compensated as soon as the worker becomes old.

Case 1 of panel A and panel B, can be represented by a single asset value equation in which the firm interiorizes the fact that the worker becomes old with probability λ_o and then the job continues. Case 2 in panel A turns out not to be interesting since the job is either destroyed when the worker becomes old, or if the worker does not become old before T_y, then we have the same analytical procedure as in the old segment of workers.

3 Creative job destruction: why does growth does not discriminate against old workers?

We start taking as reference the case where the employers have all the market power, i.e. the wage posting equilibrium. We believe that the hypothesis of rigid wages corresponds much better to the European situation than the hypothesis of flexible wages. However, to test the robustness of the results, we will systematically compare our predictions to those obtained under the hypothesis of flexible wages (developed in the appendix 8) which reflects more the American situation.

Figure 5: Flow of profits in the young segment.



When the firm hires a young (an old) worker, she knows that the worker will become old (go on retirement) with probability λ_y (λ_o). Therefore, when actualizing the flow of future profits generated by the job, she interiorizes this probability.

$$\begin{aligned} J_y(\tau,t) &= \max_{T_y}\left\{\int_t^{\tau+T_y}[p(\tau)x-w_y(\tau,s)]e^{-(r+\delta+\lambda_y)(s-t)}ds \right.\\ &\left. +\lambda_y\int_t^{\tau+T_y}J_o(\tau,s)e^{-(r+\delta+\lambda_y)(s-t)}ds\right\} \qquad (15)\\ J_o(\tau,t) &= \max_{T_o}\left\{\int_t^{\tau+T_o}[p(\tau)x-w_o(\tau,s)]e^{-(r+\delta+\lambda_o)(s-t)}ds\right\} \qquad (16)\end{aligned}$$

where the wage is equal to $w_i(\tau,s)=p(s)b$ for $i=y,o$.

Given a stationary time path for future labor market tightness, we conjecture that the value of a new job is proportional to productivity on the technology frontier, i.e. $J_i(t,t)=p(t)J$. Replacing in the previous expression when $\tau=t=0$ confirms this conjecture and yields our job destruction rule:

$$\begin{aligned} J_y &= \max_{T_y}\left\{\int_0^{T_y}[x-e^{gs}b]e^{-(r+\delta+\lambda_y)s}ds+\lambda_y\int_0^{T_y}J_o(0,s)e^{-(r+\delta+\lambda_y)s}ds\right\} \qquad (17)\\ J_o &= \max_{T_o}\left\{\int_0^{T_o}[x-e^{gs}b]e^{-(r+\delta+\lambda_o)s}ds\right\} \qquad (18)\end{aligned}$$

Replacing $J_i(t,t)=p(t)J$ in the free entry condition (4) permits to confirm that the labor market tightness is stationary, for $i=y,o$:

$$\frac{c}{q(\theta_i)}=J_i \qquad (19)$$

where we refer to this expression as the job creation rule.

The job destruction curve is an horizontal line in the space (θ_i, J_i) whereas the job creation curve is positively sloped, determining a unique intersection point (θ_i^*, J_i^*) that stands for the equilibrium in the segment $i = y, o$ of workers. Evidently, to be economically meaningful these equilibrium values should be positive, which requires b and c to be sufficiently small.

Property 1a *The equilibrium optimal age of job destruction does not depend on the age of the workers when wages are rigid.*

Proof The equilibrium optimal age of job destruction, is the solution to the problems defined on the right hand side of equations (17) and (18):

$$e^{gT_y^*} = \frac{x}{b}\left[1 + \lambda_y \int_{T_y^*}^{T_o^*} (1 - e^{g(s-T_o^*)})e^{-(r+\delta+\lambda_o)s} ds\right] \tag{20}$$

$$e^{gT_o^*} = \frac{x}{b} \tag{21}$$

Equations (20) and (21) imply that $T_y^* \geq T_o^*$ if $\int_{T_y^*}^{T_o^*}(1 - e^{g(s-T_o^*)})e^{-(r+\delta+\lambda_o)s}ds$ was positive, which cannot be the case if $T_y^* \geq T_o^*$. Hence, there is only one possible solution: $T_y^* = T_o^* \equiv T^*$. This yields the following expression for the scrapping time for $i = y, o$:

$$T^* = \frac{\ln x - \ln b}{g} \tag{22}$$

showing that the scrapping time does not depend on the age of the workers. ■

Replacing (22) in the job destruction curves permits to obtain the relationship between J_i^* and T_i^* for $i = y, o$:

$$J_y^* = x \int_0^{T^*} [1 - e^{g(s-T^*)}]e^{-(r+\delta+\lambda_y)s} ds + \lambda_y \int_0^{T^*} J_o(0, s)e^{-(r+\delta+\lambda_y)s} ds \tag{23}$$

$$J_o^* = x \int_0^{T^*} [1 - e^{g(s-T^*)}]e^{-(r+\delta+\lambda_o)s} ds \tag{24}$$

because $J_o(0, T^*) = 0$ at equilibrium.

Property 1b *The equilibrium value of a young worker job is higher than the equilibrium value of an old worker job. This implies that the hiring rate of the young worker is higher than the senior's hiring rate.*

Proof Equations (23) and (24) clearly show that $J_y^* > J_o^*$ when $\lambda_y = \lambda_o$. The gap between J_y^* and J_o^* increases when $\lambda_y > \lambda_o$, the assumption retained in the paper. Because the job creation curve (equation 19) is an increasing relation between θ_i and J_i, we deduce that $\theta_y > \theta_o$. ■

When wages are flexible, the equilibrium optimal ages of job destruction are given by:

$$
\begin{aligned}
e^{gT_o^*} &= \frac{x}{\omega(\theta_o)} && (25) \\
e^{gT_y^*} &= \frac{x}{\omega(\theta_y)} + \frac{\lambda_y}{(1-\eta)\omega(\theta_y)} J_o(0, T_y^*) && (26)
\end{aligned}
$$

where $J_o(0, T_y^*) = (1-\eta)\int_{T_y^*}^{T_o^*}(x - e^{gw}\omega(\theta_o))e^{-(r+\delta+\lambda_o)w}dw$, and T_o^* is known from the old segment. It is easy to show that T_i^* decreases with θ_i, for $i = y, o$.

> **Property 1c** *As soon as wages are determined by wage bargaining, the equilibrium scrapping time of old and young workers differs. In this case, the only consistent equilibrium is such that:* $T_y^* < T_o^*$, $J_y^* > J_o^*$ *and then* $\theta_y^* > \theta_o^*$.

Proof See appendix 8.

In summary, the equilibrium in the labor market is characterized by a larger creation rate of young positions than of senior positions, independently on the hypothesis made on the wage determination process. This result is confirmed by data on the French economy where we estimate the share of entries of workers aged between 30 and 39 to be around 19% whereas this share is reduced to 4% for those aged above 50. The shorter horizon of old workers is likely to justify this result. Concerning the endogenous destruction rate results differ depending on the way wages are determined. In the presence of rigid, wages, the age of workers does not introduce any heterogeneity since the deterministic wage growth is shared by the two populations. Then, given the particular vintage of a technology introduced at date τ, the waiting time for obsolescence is the same for young and old workers. In contrast, if wages are flexible, the scrapping time differs between both workers categories, young workers having a shorter scrapping horizon. This prediction is not supported by data on the separation rates by age. How can we then justify this result? It seems clear that, at this stage, our theoretical framework is missing something: the possibility of updating a technology (capitalization effect of growth).

Assume now an acceleration in the rate of technical progress represented by a rise in g.

Property 2a *When the growth rate of the technical progress increases ($\uparrow g$)*

(i) The optimal scrapping time T^* decreases.

(ii) This leads to a decrease in both J_y^* and J_o^* and then to less hirings.

(iii) The impact of an increase in the growth rate of technical progress is larger for young workers due to their larger time horizon. Then, when the retirement age is high (λ_o low), the negative impact of the growth is smaller for the old than for the young workers.

(iv) Unambiguously, young and old unemployment rates increase.

Proof Concerning *(i)*, the impact on the scrapping time is negative (see equation (22)): the direct effect of g on T^* tends to reduce the optimal time the firm keeps a technology. Indeed, an acceleration in the pace of technological progress leads the firm to destroy jobs more often in order to adopt the new technology. Then we deduce that $\frac{\partial T^*}{\partial g} < 0$.

Concerning *(ii)*, in none of the segments the job creation curve is affected. The only observed shift in the (θ, J) space (for a constant scrapping time) corresponds to the job destruction curve which moves downwards:

- In the old segment:

$$\frac{\partial J_o}{\partial g}|_{T=cte} = \int_0^T (-se^{gs}b)e^{-(r+\delta+\lambda_o)s}ds < 0 \tag{27}$$

- In the young segment:

$$\begin{aligned}\frac{\partial J_y}{\partial g}|_{T=cte} &= \int_0^T (-se^{gs}b)e^{-(r+\delta+\lambda_y)s}ds \\ &\quad +\lambda_y \int_0^T \frac{\partial J_o(0,s)}{\partial g} e^{-(r+\delta+\lambda_y)s}ds < 0\end{aligned} \tag{28}$$

Because faster growth shifts down the horizontal job destruction curves and does not affect the positive relation between θ_i and J_i defined by the job creation curves, both the equilibrium value of the match surplus and the labor market tightness decline with growth, *i.e.* $\frac{\partial J_i}{\partial g} < 0$ and $\frac{\partial \theta_i}{\partial g} < 0$ for $i = y, o$.

Part *(iii)* can be deduced from (27) and (28). Indeed, at the limit, we have $\lambda_y = \lambda_o$: in this case, $\left|\frac{\partial J_o}{\partial g}|_{T=cte}\right| < \left|\frac{\partial J_y}{\partial g}|_{T=cte}\right|$. Because we assume $\lambda_y < \lambda_o$, we have $\left|\frac{\partial J_o}{\partial g}|_{T=cte}\right| < \left|\frac{\partial J_y}{\partial g}|_{T=cte}\right|$.

Finally, *(iv)* can be easily deduced from equations (**??**) and (**??**). Both unemployment rates increase as the age of obsolescence, T^*, and the market tightness, θ_y^* or θ_o^* decrease.
As proved in appendix 8 these results remain robust when considering flexible wages. ■

In property 1b, we showed that, at the equilibrium, the hiring rate for young workers was larger than that of old workers, $J_y^* > J_o^*$. Property 2a suggests that an increase in the growth rate of technical progress decreases the comparative advantage of young workers. Indeed, a higher growth rate fosters a higher labor cost for a longer period of time if the employee is young than if the employee is old. Can we then deduce that growth discriminates against young workers? This result would be completely counterfactual. We can then conclude again that we are not taking into account an impact mechanism: the creative destruction process is not able by its own to explain the link between growth and unemployment.

4 Renovation: why firms prefer to fire old workers?

We introduce now the possibility for the firm to renovate its technology: the firm pays a fix cost giving access to the best possible technology (capitalization effect). This assumption leads to less caricatural results: with renovation, the creative-destruction process is not the only link between employment and growth. By introducing the possibility of updating, we allow firms to increase their expected profits by incorporating new technologies: if renovation costs are low, one can expect that the "capitalization" effect overstates the "creative-destruction" process. Does this hold for all age categories? More precisely, is the updating decision related to age?

The possibility of updating a technology at date t is possible after incurring in a fixed cost $p(t)I$ that allows the firm to increase the value of a job from $J_i^R(\tau, t)$ to $J_i^R(t, t)$ for $i = y, o$. Under a fixed renovation cost, the firm that chooses to renovate always moves to the current technology frontier. This renovation cost stands for the sum of both the costs of new equipment and the costs of training.

In this new framework, a job can then be destroyed by an exogenous shock, by an endogenous decision of the firm (if the relation becomes not profitable) or the job technology may be updated if the firm pays a fixed renovation cost $p(t)I$ (rather than destroying the job and creating a new one it may be more profitable for the firm to renovate). Evidently, renovation takes place if and only if the renovation horizon T_i^R for $i = y, o$ is smaller than the destruction horizon ($T_i^R < T_i^*$

for $i = y, o$). When renovation occurs, the firm pays an implementation cost $p(\tau + T_i^R)I$ for $i = y, o$ in order to rise the value of the job from $J_i^R(\tau, \tau + T_i^R)$ to $J_i^R(\tau + T_i^R, \tau + T_i^R)$ for $i = y, o$. The value of a filled job when renovation is possible is given by:

$$\begin{aligned} J_o^R(\tau, t) &= \max_{T_o^R} \left\{ \int_t^{\tau + T_o^R} [p(\tau)x - p(t)b] e^{-(r+\delta+\lambda_o)(s-t)} ds \right. \\ &\quad \left. + e^{(r+\delta+\lambda_o)(\tau + T_o^R - t)} [J_o^R(\tau + T_o^R, \tau + T_o^R) - p(\tau + T_o^R)I] \right\} \end{aligned} \tag{29}$$

for a senior, and

$$\begin{aligned} J_y^R(\tau, t) &= \max_{T_y^R} \left\{ \int_t^{\tau + T_y^R} [p(\tau)x - p(t)b] e^{-(r+\delta+\lambda_y)(s-t)} ds \right. \\ &\quad + e^{-(r+\delta+\lambda_y)(\tau + T_y^R - t)} [J_y^R(\tau + T_y^R, \tau + T_y^R) - p(\tau + T_y^R)I] \\ &\quad \left. + \lambda_y \int_t^{\tau + T_y^R} J_o^R(\tau, s) e^{-(r+\delta+\lambda_y)(s-t)} ds \right\} \end{aligned}$$

for a young worker.

As $J_i^R(t, t) = p(t)J_i^R$, when $\tau = t = 0$ we obtain:

$$J_o^R = \max_{T_o^R} \left\{ \int_0^{T_o^R} [x - e^{gs}b] e^{-(r+\delta+\lambda_o)s} ds + e^{(r+\delta+\lambda_o-g)T_o^R} [J_o^R - I] \right\} \tag{30}$$

$$\begin{aligned} J_y^R &= \max_{T_y^R} \left\{ \int_0^{T_y^R} [x - e^{gs}b] e^{-(r+\delta+\lambda_y)s} ds + e^{(r+\delta+\lambda_y-g)T_y^R} [J_y^R - I] \right. \\ &\quad \left. + \lambda_y \int_0^{T_y^R} J_o^R(0, s) e^{-(r+\delta+\lambda_y)s} ds \right\} \end{aligned} \tag{31}$$

These new job destruction curves remain horizontal lines in the (θ_i^R, J_i^R) space for $i = y, o$, whereas we have the same positive sloped job creation curves defined in equation (19), leading to a unique equilibrium labor market equilibrium $(\theta_i^{R*}, J_i^{R*})$ for $i = y, o$.

The optimal choice of the renovation horizon is obtained solving the right hand side problem of equations (30) and (31):

$$b = \frac{x}{e^{gT_o^{R*}}} - (J_o^R - I)(r + \delta + \lambda_o - g) \tag{32}$$

$$b = \frac{x}{e^{gT_y^{R*}}} - (r + \delta + \lambda_y - g)(J_y^R - I) + \frac{\lambda_y}{e^{gT_y^{R*}}} J_o^R(0, T_y^{R*}) \tag{33}$$

Replacing in equation (30) and (31) yields the optimal date at which the firm decides to renovate:

$$I = x \int_0^{T_o^{R*}} [1 - e^{g(s - T_o^{R*})}] e^{-(r+\delta+\lambda_o)s} ds \tag{34}$$

$$\begin{aligned} I &= x \int_0^{T_y^{R*}} [1 - e^{g(s - T_y^{R*})}] e^{-(r+\delta+\lambda_y)s} ds \\ &\quad + \lambda_y \int_0^{T_y^{R*}} [J_o^R(0, s) - e^{g(s - T_y^{R*})} J_o^R(0, T_y^{R*})] e^{-(r+\delta+\lambda_y)s} ds \end{aligned} \tag{35}$$

Property 3a *The optimal date at which the firm decides to renovate a position occupied by an old worker*[7] *(T_o^{R*}) is an increasing function of the fixed renovation cost (I) and the probability that the worker becomes a retiree (λ_0). At the opposite, T_o^{R*} decreases with the growth rate of technical progress (g).*

Proof See equation (34). This result remains robust in the presence of flexible wages (see appendix 8). ■

It is in the interest of the firm to renovate if and only if the renovation cost is lower than the optimal value she obtains without renovation:

$$I \leq J_o^* \quad \Leftrightarrow \quad T_o^{R*} < T^* \quad \text{for a senior}$$
$$I \leq J_y^* \quad \Leftrightarrow \quad T_y^{R*} < T^* \quad \text{for a young worker}$$

Because $J_y^* > J_o^*$ (given the equality in the obsolescence age), the firm may face a situation where the updating cost I is such that $J_y^* > I > J_o^*$. That is, it is in the interest of the firm to update only if the worker is young. For old workers it is not profitable to renovate and the firm prefers to fire the worker and open a new vacancy.

Does technical progress help old workers? If the renovation cost is such that it is optimal to fire old workers ($J_y^* > I > J_o^*$) rather than updating their abilities, then, a rise in g fosters more destruction and less creation on this labor market segment (see property 3) independently on the wage determination process. Destroying the job rather than updating it, remains then the optimal strategy for the firm when dealing with old workers. if it is optimal for firms to renovate all jobs ($J_y^* > J_o^* > I$), the impact of an acceleration in g is ambiguous in both an economy with rigid wages and in an economy with flexible wages. On the one hand, faster growth decreases more rapidly the profits and then accelerates the "creative-destruction process". On the other hand, faster growth yields a higher expected surplus after renovation occurs, reducing the implementation horizon. At the limit, when I is sufficiently small, the capitalization effect induced by growth dominates the creative-destruction effect and then stimulates job creation.

[7] The optimal date at which the firm decides to renovate a position occupied by a young worker (T_y^{R*}) is an increasing function of the fixed renovation cost (I) and of the probability that the worker becomes a retiree. At the opposite, T_y^{R*} decreases with the growth rate of technical progress (g). See equation (35), given that $\partial J_o^R(\tau, t)/\partial \lambda_o < 0$.

Indeed, equation (32) can be rewritten as

$$\begin{aligned}\lim_{I\to 0} J_o^R &= \lim_{(I,T_o^{R*})\to(0,0)} \left\{ I + \frac{x - be^{gT_o^{R*}}}{(r+\delta+\lambda_o-g)e^{gT_o^{R*}}} \right\} \\ &= \frac{x-b}{(r+\delta+\lambda_o-g)}\end{aligned}$$

showing that the "capitalization effect" implies a larger value for jobs and favors hirings[8]. In this case, faster growth can lead to update more frequently all jobs, and in particular the job occupied by old workers.

The predominance of the capitalization effect when I becomes very small, can be interpreted in two ways. First, we can focus on the specific abilities of the workers. If the skills of an old worker are close to the abilities required by new jobs, the training cost is low and then the renovation strategy is optimal. At the opposite, if the gap between the worker's skills and the abilities required by the firm is large, the renovation cost is high: the optimal strategy is to fire the worker because search costs are lower than training costs. This result can give some support to the stylized facts presented in the introduction: high skilled workers are more frequently trained at the end of their life cycle than young ones.

Second, we can focus on the sector specific investment costs. In sectors where the cost of new equipment is low, the probability to renovate a job occupied by an old worker is higher than in sectors where the investment cost is high. This result is clearly observed in the French data for the period 1996-2004, where we estimate that the average share of exits for workers aged above 50 is almost 23% per year in the industry whereas in the service sector it falls to 13%.

What about the impact of the retirement age? Does a low retirement age discriminate against old workers? As the optimal renovation date increases with the probability of retirement, $\partial T_o^{R*}/\partial\lambda_0 > 0$, we are less likely to find $T_o^{R*} < T^*$ when the impending retirement age is low. Indeed, for a high probability of retirement, λ_o, the horizon of the worker is shorter. The probability to recoup the renovation cost is then lower than in an economy characterized by a higher retirement age (low λ_o). Furthermore since $\partial J_o^*/\partial\lambda_o < 0$ the renovation condition $I \leq J_o^*$ becomes also less likely. The increase in λ_o reduces then the probability of updating. This effect is less clear in the presence of flexible wages, since the increase in the optimal renovation date is also associated to the increase in the scrapping time, so at the end we might have $T_o^{R*} \gtrless T_o^*$. Because wages are flexible, we might find that the fall in wages is sufficiently important to induce

[8] Remark that there is a value of I such that the capitalization effect just offset the creative destruction effect (see Mortensen and Pissarides (1998) for further details)

a rise in the scrapping time such that the final updating horizon T_o^{R*} remains lower than T_o^*. In this case it will be in the interest of the firm to update rather than to destroy the job (under the assumption that $I < J_o^*$), in spite of the increased probability of retirement.

Finally, remark that, under the assumption of homogeneous workers, firms either renovate all positions and then there is no endogenous job destruction, or no job is renovated and all jobs are destroyed after T^* periods (case analyzed in section 3). When all jobs are updated, the only source of destruction is the exogenous shock represented by δ, so that the young and old unemployment rates are respectively given by $u_y = \frac{\delta+\lambda_y}{\delta+\lambda_y+p(\theta_y)}$ and $u_o^R = \frac{\delta+\lambda_o u_y}{\delta+\lambda_o+p(\theta_o)}$.

5 Match heterogeneity and aging: why some old workers are retrained?

Results in the previous section do not help us to understand the stylized facts revealed in Behaghel and Greenan (2007) and by figure 3. Why firms should train some old workers if their working horizon is much shorter than the one of young workers? Under which conditions this capitalization effect (renovation of an already existing technology) overcomes the creative destruction effect (destruction of an already existing old job) in the older workers' case? This section aims at providing an answer to these questions.

We now introduce in the model a random idiosyncratic component ϵ in the productivity which becomes $x\epsilon$. ϵ is distributed according to a cumulative function $F(\epsilon)$ and its value is revealed once the worker and the firm meet.

Without renovation opportunity, the destruction age of a job equals

$$\begin{aligned} T^*(\epsilon) &= \frac{\ln(x\epsilon) - \ln b}{g} \qquad \text{if wages are rigid} \\ T^*(\epsilon) &= \frac{\ln(x\epsilon) - \ln \omega(\theta_o)}{g} \qquad \text{if wages are flexible} \end{aligned}$$

for a given match-specific productivity ϵ. Remark that $T^*(\epsilon) \geq 0$ for $\epsilon \geq \frac{b}{x}$ ($\epsilon \geq \frac{\omega(\theta_o)}{x}$ for flexible wages). Finally, this equation shows that $\partial T^*(\epsilon)/\partial\epsilon > 0$ implying that firms keep high productivity workers for a longer period of time. Moreover, property 3 ($\partial T^*(\epsilon)/\partial g < 0$) and property 1a ($\partial T^*(\epsilon)/\partial\lambda_o = 0$) still hold.

The implementation horizon is represented by $T_o^{R*}(\epsilon)$. Implementing a new technology is the best option only if this horizon is lower than the scrapping time, $T_o^{R*}(\epsilon) < T^*(\epsilon)$. If firms have the opportunity to renovate, the updating horizon for old workers[9], denoted $T_o^{R*}(\epsilon)$, solves

$$I = x\epsilon \int_0^{T_o^{R*}(\epsilon)} [1 - e^{g(s-T_o^{R*})}] e^{-(r+\delta+\lambda_o)s} ds$$

We conclude that $\partial T_o^{R*}(\epsilon)/\partial I > 0$, $\partial T_o^{R*}(\epsilon)/\partial \lambda_o > 0$ and $\partial T_o^{R*}(\epsilon)/\partial \epsilon < 0$, implying that the technology is updated more frequently in a good match (furthermore, the equilibrium value J_o^* increases with the productivity level). Indeed a good match is a scarce resource that the firm tries to preserve: renovation insures that the quality of the match is maintained. Conversely, for every new match the firm must draw a new match-specific productivity. This last option, implying lay offs, is preferred by the firm when its actual ϵ is low. These result are robust to the introduction of a bargained wage.

Implementing a new technology is the best option only if $T_o^{R*}(\epsilon) < T^*(\epsilon)$. Therefore, firms renovate only positions having a match-specific productivity greater than a critical value R_i solving[10]:

$$T_o^{R*}(R_o) = T^*(R_o) \Leftrightarrow J_o(R_o) = J_o^R(R_o)$$

Jobs having a productivity level above R_o constitute the set of renovated matches. Because $\partial T_o^{R*}(\epsilon)/\partial I > 0$ we have $\partial R_o/\partial I > 0$, *i.e.* since the implementation horizon decreases with productivity whereas the scrapping horizon $T_o^*(R_o)$ is not affected, a rise in the renovation cost is necessarily associated to an increase in the reservation productivity R_o so as to keep $T_o^{R*}(R_o) = T_o^*(R_o)$). The set of renovated jobs decreases then. This result remains robust when assuming endogenous wages.

Similarly, because $\partial T_o^{R*}(\epsilon)/\partial \lambda_o > 0$, we have that $\partial R_o/\partial \lambda_o > 0$, meaning that the set of renovated jobs decreases with the probability of retirement in a wage posting framework, where the scrapping time remains unaffected by λ_o. In contrast, if wages are flexible, an increase in

[9] The implementation horizon for young workers, $T_y^{R*}(\epsilon)$, is given by:

$$\begin{aligned} I &= x\epsilon \int_0^{T_y^{R*}(\epsilon)} [1 - e^{g(s-T_y^{R*})}] e^{-(r+\delta+\lambda_y)s} ds \\ &\quad + \lambda_y \int_0^{T_y^{R*}(\epsilon)} [J_o^R(0,s) - e^{g(s-T_y^{R*})} J_o^R(0, T_y^{R*})] e^{-(r+\delta+\lambda_y)s} ds \end{aligned}$$

[10] For young workers:

$$T_y^{R*}(R_y) = T^*(R_y) \Leftrightarrow J_y(R_y) = J_y^R(R_y)$$

the probability of retirement yields a rise in both $T_o^{R*}(R_o)$ and $T_o^*(R_o)$. Then, only in the case where the increase in $T_o^{R*}(R_o)$ overcomes the rise in $T_o^*(R_o)$, there will be an adjustment via the increase in the reservation productivity, R_o which will tend to reduce the set of renovated jobs, but much less than in the presence of rigid wages (where only the implementation horizon increased). The flexibility of wages allows thus a larger share of old workers to benefit from training in case of technological change.

In sum, when the retirement age is low (high λ_o), the probability to renovate the job of a senior can be high if the worker has a high match-specific productivity. Even if the working horizon is short, firms take more advantage to preserve this good productivity draw rather than destroying it. This gives some theoretical foundations to the stylized facts presented in the introduction: an old worker with high match-specific productivity (a manager) may have a higher probability to receive training after a technological change than a young worker with low match-specific productivity (an employee or a manual worker). Nevertheless, a low retirement age implies that a smaller fraction of positions occupied by old workers will be updated, particularly in the presence of rigid wages. Then, economies where the official retirement age is low, that is, most European countries, take less advantage of the large right queue of the match-specific productivity distribution.

Regarding the pace of technological progress, an acceleration in g, fosters a reduction in both, the renovation horizon and the scrapping time. Independently on the wage determination process we can distinguish among three possible situations. In the simplest case, R_o remains unchanged by the acceleration in g since the reduction in both the renovation and the scrapping horizon is such that the equality $T_o^{R*}(R_o) = T^*(R_o)$ directly holds. So the set of renovated jobs remains constant. Secondly, if the reduction in the renovation horizon fostered by the rise in g is more important than that of the scrapping time, the reservation productivity R_o increases in order to reestablish the equality $T_o^{R*}(R_o) = T^*(R_o)$. The set of renovated jobs will then fall. Thirdly, if the acceleration in the pace of technological progress yields a more important reduction of the scrapping time, the reservation productivity R_o decreases, fostering a rise in the set of renovated jobs.

Finally, the flow out of unemployment in the old segment is given by $OUT_o = p(\theta_o)u_o(1-F(\frac{b}{x}))+\lambda_o u_o$ and the flow into unemployment by $IN_o = \delta(P_o - u_o) + p(\theta_o)u_o \int_{\frac{b}{x}}^{R_o} e^{-(\delta+\lambda_o)T^*(R_o)}dF(x) +$

$\lambda_y u_y$. Equalizing entries and exits yields the following equilibrium unemployment:

$$u_o = \frac{\lambda_y[u_y + \frac{\delta}{\lambda_o}]}{\delta + \lambda_o + [1 - F(\frac{b}{x}) - \int_{\frac{b}{x}}^{R_o} e^{-(\delta+\lambda_o)T^*(R_o)} dF(x)]p(\theta_o)} \tag{36}$$

Dividing the previous expression by the old work force, P_o, leads to the equilibrium unemployment rate of the old segment:

$$u_o^R = \frac{\delta + \lambda_o u_y}{\delta + \lambda_o + [1 - F(\frac{b}{x}) - \int_{\frac{b}{x}}^{R_o} e^{-(\delta+\lambda_o)T^*(R_o)} dF(x)]p(\theta_o)} \tag{37}$$

Unemployment rates of old workers increase then with the renovation cost (since $T^*(R_o)$ and R_o increase and the labor market tightness decreases with I), whereas the effect of a rise in the retirement probability is more ambiguous (even if the number of employed old workers falls there is also a decrease in the old work force, leading to an ambiguous effect in the unemployment rate).

6 Numerical Simulations

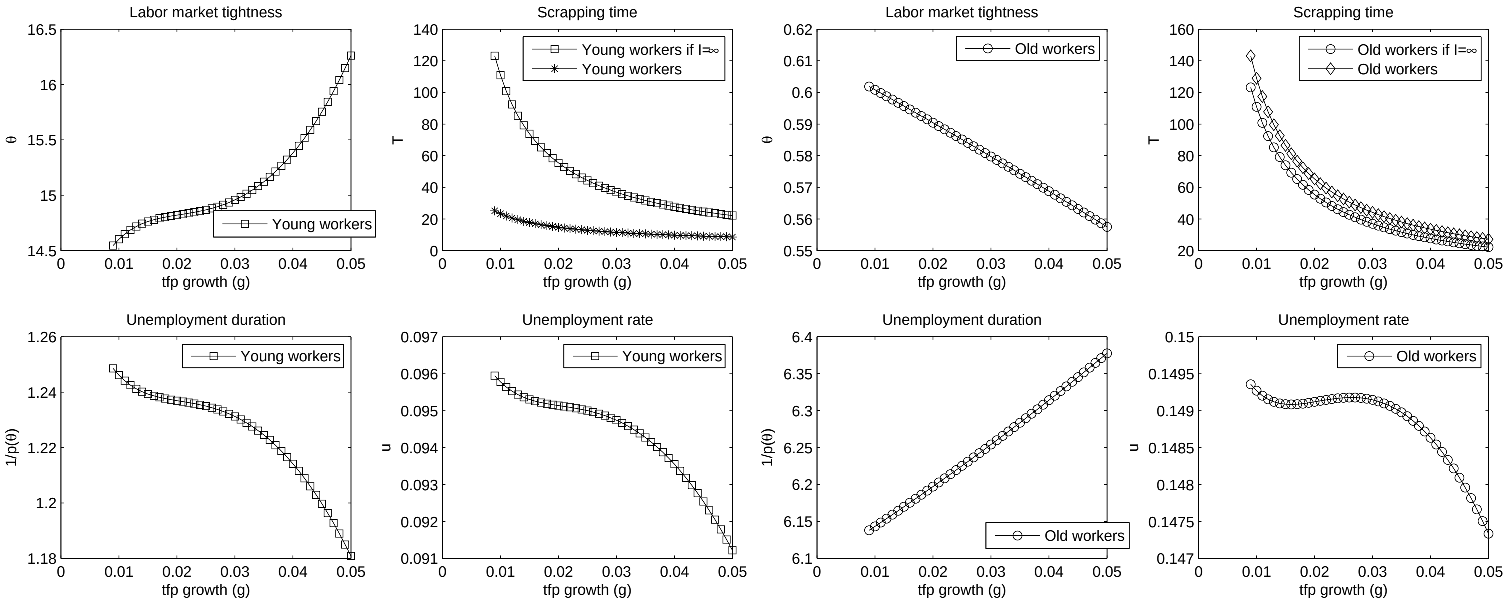
The theoretical analysis shows that the impact of the TFP growth is indeterminate. Which effect dominates, capitalization versus creative-destruction, is a quantitative question. In this section, we report the outcome of computational experiments for the wage posting case. We focus on this wage setting rule because it leads to a rigid real wage. This assumption seems to be more in accordance with the data (see Pissarides and Vallanti (2007)).

Table 2: Baseline Parameters Values: Wage Posting ($\eta = 0$)

Job productivity	x	$=$	1
Interest rate	r	$=$	.04
Matching elasticity	α	$=$	.5
Recruiting cost	$c/q(\theta_y)$	$=$	.3
Exogenous separation rate	δ	$=$	.07
Unemployment benefits	b	$=$	.33
Young age duration	$1/\lambda_y$	$=$	40 years
Old age duration	$1/\lambda_o$	$=$	2 years

A matching function of the Cobb-Douglas form is assumed: $M = \phi v_i^{\alpha} u_i^{1-\alpha}$, for $i = y, o$, where α is the elasticity with respect to the vacancies. The numerical values of the parameters are summarized in the table 6. As in Pissarides and Vallanti (2007), we choose the exogenous destruction rate such that the mean duration of jobs matches its observed value, i.e. 10 years. Then, s solves $10 = (1 - e^{\delta T^*})/\delta$. Given this set of parameters, we choose the implementation

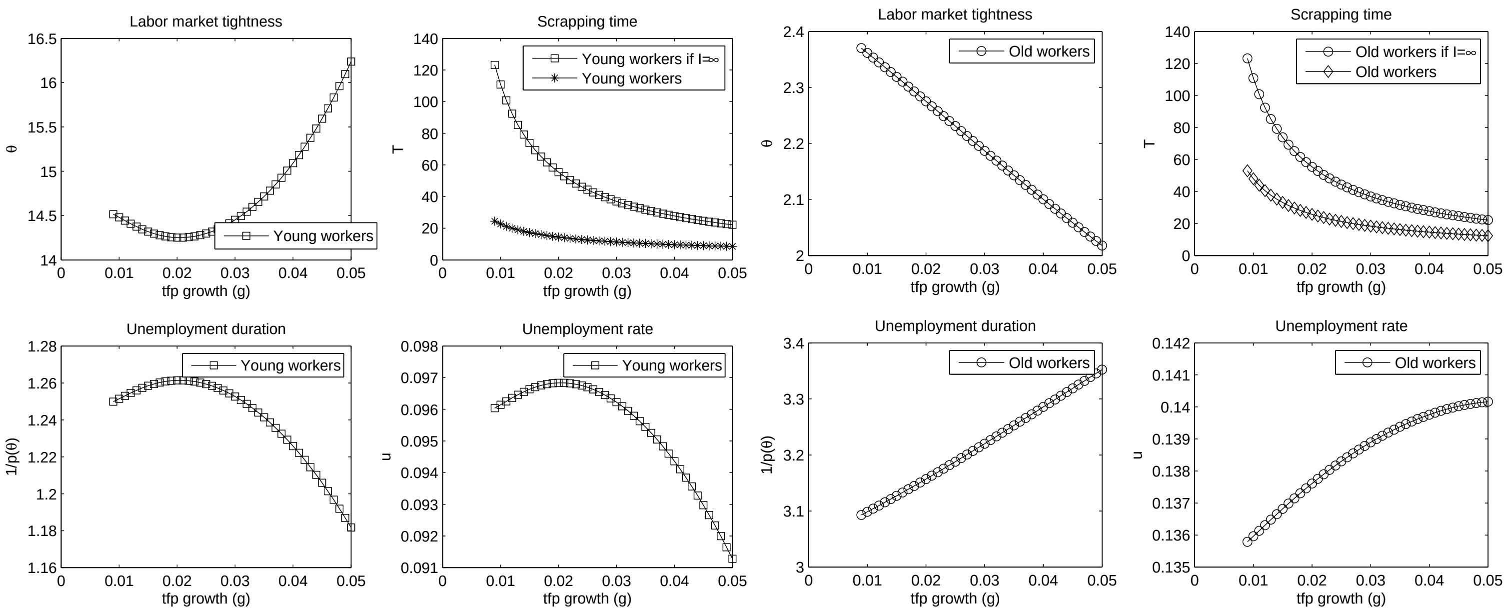
Figure 6: The benchmark calibration



cost I and the scale parameter of the matching function ϕ, such that the capitalization effect dominates the creative destruction effect for the young workers given a realistic unemployment rate: for an annual TFP growth rate equals to 2%, we have, for an European type economy an equilibrium unemployment rate equal to 9.5% for the 16-55 year old workers (the young workers). Then, our estimation of the renovation cost is 1 year and 1 quarter of the the production of a job using the best possible technology. The figure 6 shows that we have $T_y^R < T^*$ for all g, implying that, if the job is occupied by a young worker, the technology is updated before the scraping time. At the opposite, for the old workers, there is no renovation. This calibration can be view as the benchmark case because the model allows to reproduce the positive (negative) impact of the TFP growth rate on the employment rate of the young (old) workers (see the estimations presented in the introduction, table 1). Concerning the magnitude of the capitalization effect, we have the same problem than Pissarides and Vallanti (2007): if the increase in the TFP growth rate is 1 percentage point, the fall in the unemployment rate of the young worker is only 0.1 percentage point. This largely under-estimate the empirical estimations provide in the introduction (see table 1) or by Pissarides and Vallanti (2007). If we reduce the interest rate and the exogenous separation rate, for example if we choose $r = 2\%$ and s such that the mean duration of jobs is 30 years, then, an increase in the TFP growth rate equal to 1 percentage point leads to a fall in the unemployment rate of the young worker equal to 1 percentage point. This sensitivity analysis also confirms the numerical experiments provides in Pissarides and Vallanti (2007).

Finally, for realistic TFP growth rates, *i.e.* for g between 1.5 and 3% per year, figure 6 also shows that unemployment rate of the older worker increases with the growth rate of the TFP. At the opposite, for high TFP growth rates (for a annual growth rate higher than 3%), the model predicts the reverse. Indeed, because the unemployment rate of the young workers largely decrease when the TFP growth rate is high, this reduces the inflow rate in unemployment of the old workers.

Figure 7: The increase of the old worker horizon



We now analyze the impact of an increase in the horizon of the old workers: we now set $1/\lambda_y$=5 years. This numerical experiment can be view as an evaluation of a policy which aims to increase the retirement age[11] The jobs occupied by old workers are now renovated: the separation rate for the jobs occupied by old workers is now reduced to its exogenous component. Nevertheless, the capitalization effect does not dominate: the implementation cost is higher than its threshold

[11]For an other calibration of the matching efficiency parameter ϕ allowing to match a unemployment rate equal to 5%, this simulation can be also view as our model predictions for the US economy.

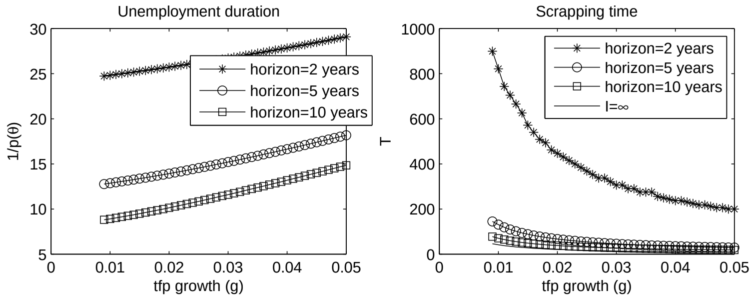
value, under which firms decide to open more vacancies when TFP growth rate increases. This implies that an increase of the TFP growth rate raises the unemployment rate of the old worker (see figure 6). These results suggest that a reform of the social security which simply increases the normal retirement age is not sufficient to reverse the positive correlation between the TFP growth rate and the unemployment rate of the old workers. This result is robust to the calibration of r and s. Indeed, for the older workers, the magnitude of the capitalization effect is low because the horizon of these workers is short. This suggests that the growth process can not act in favor of the employment rate of the older workers.

In the figure 6, we evaluate the impact of the increase of the time horizon before retirement for heterogenous workers: low, median and high skilled workers. We report the results for a calibration of $x \in \{.5; 1; 1.5\}$. For the high skilled workers, the renovation is the better option, for each horizon. At the opposite, the destruction is the better option for the low skilled workers, for each horizon. The median skilled workers are an intermediate case where for a time horizon before retirement longer than 5 years, the best strategy is the renovation. Finally, the figures show that only the high skilled workers having an horizon longer than 5 years, for example a distance form the retirement equal to 10 years, benefit from the capitalization effect. This clearly shows that the growth process discriminates against a large majority of older workers, even if the retirement age is increased. If more growth can lead to renovate the job occupied by high and median skilled workers, this policy does not allow to increase the average employment rate of the older workers.

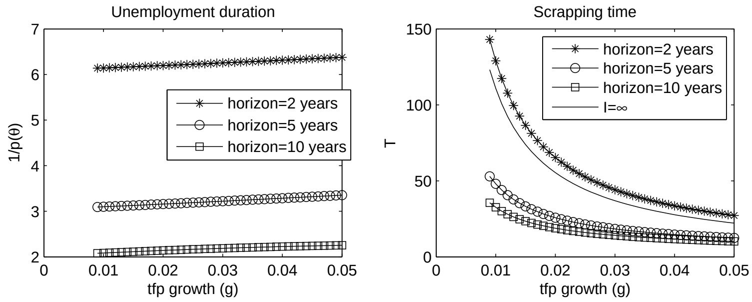
These computational experiments shows that a realistic calibration of our model allows to explain why young workers take advantage for an increase in the growth rate of the technological progress, whereas at the same time old workers are the victims of the creative destruction process. An increase of the time horizon before retirement, though an increase of the normal retirement age, does not allows to increase the employment rate at the end of the life-cycle: the capitalization effect is too small and do not give sufficient incentives to open new vacancies for older workers. Nevertheless, this conclusion is largely due to the lower elasticity of the employment rate to the TFP growth rate implied by the model than the one measured in the data.

Figure 8: The increase of the old worker horizon: low, median and high skilled workers

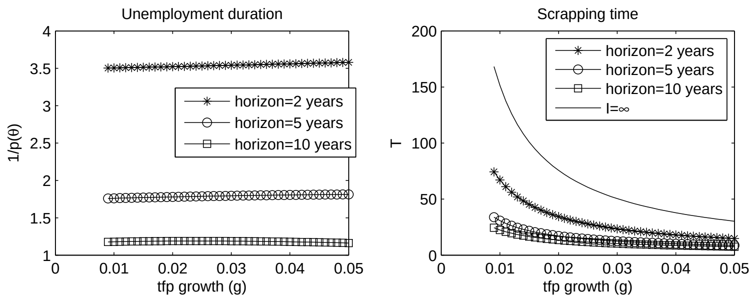
Low skilled workers



Median Skilled workers



High skilled workers



7 Conclusions

Recent studies on the link between growth and unemployment conclude on the existence of a negative relationship between both variables, suggesting that, in case of acceleration in the growth rate, the capitalization effect dominates the creative destruction effect (see Blanchard and Wolfers (2000), Pissarides and Vallanti (2007) or Tripier (2007)). However, on of the main drawbacks of these studies is that, by considering homogeneous agents, they are ignoring the heterogenous effects growth may have on different population segments. This paper analyzes the effects of growth on employment along the life cycle, and more precisely, the effects of growth on the employment rate of old workers.

We develop a vintage model inspired in Mortensen and Pissarides(1998) but distinguishing between young and old workers. We find that when considering the simpler version of the model, where jobs are destroyed as soon as they become not profitable (creative destruction effect) and the possibility of updating a technology (capitalization effect) is not introduced, we are unable to reproduce the observed stylized facts. In order to obtain reasonable results being coherent with data, we need to introduce not only the possibility for the firm to destroy or to update a technology, but also distinguish between high and low productivity workers. To correctly seek the relationship between growth and the employment rate of workers along their life cycle a double heterogeneity, in terms of age and productivity, must then be introduced. Furthermore, firms must have the choice between destroying a job that is not profitable (creative destruction) or updating it (capitalization).

8 Appendix. Robustness : the wage bargaining

8.1 The equilibrium with wage bargaining

Given the wage equation (14) and knowing that the value of a new job is proportional to productivity on the technology frontier, (i.e. $J_i(t,t) = p(t)J$), $\forall i = y, o$, job destruction rules are given by:

$$J_o = \max_{T_o} \left\{ (1-\eta) \int_0^{T_o} [x - e^{gs}\omega(\theta_o(s))] e^{-(r+\delta+\lambda_o)s} ds \right\} \tag{38}$$

$$\begin{aligned} J_y = \max_{T_y} \Big\{ &(1-\eta) \int_0^{T_y} [x - e^{gs}\omega(\theta_y)] e^{-(r+\delta+\lambda_y)s} ds \\ &+ \lambda_y \int_0^{T_y} J_o(0,s) e^{-(r+\delta+\lambda_y)s} ds \Big\} \end{aligned} \tag{39}$$

As in the wage posting case, the job creation rule is given by (19).

The equilibrium optimal ages of job destruction, are the solution to the problems defined on the right hand side of equations (38) and (39):

$$e^{gT_o^*} = \frac{x}{\omega(\theta_o)} \tag{40}$$

$$e^{gT_y^*} = \frac{x}{\omega(\theta_y)} + \frac{\lambda_y}{(1-\eta)\omega(\theta_y)} J_o(0, T_y^*) \tag{41}$$

where $J_o(0, T_y^*) = (1-\eta) \int_{T_y^*}^{T_o^*} (x - e^{gw}\omega(\theta_o)) e^{-(r+\delta+\lambda_o)w} dw$, and T_o^* is known from the old segment. It is easy to show that T_i^* decreases with θ_i, for $i = y, o$.

Replacing (40) and (41) in the job destruction curve permits to obtain the relationship between J_i^* and T_i^*, for $i = y, o$:

$$J_o^* = (1-\eta)x \int_0^{T_o^*} [1 - e^{g(s-T_o^*)}] e^{-(r+\delta+\lambda_o)s} ds \tag{42}$$

$$\begin{aligned} J_y^* &= (1-\eta)x \int_0^{T_y^*} [1 - e^{g(s-T_y^*)}] e^{-(r+\delta+\lambda_y)s} ds \\ &\quad + \lambda_y \int_0^{T_y^*} [J_o(0,s) - e^{g(s-T_y^*)} J_o(0, T_y^*)] e^{-(r+\delta+\lambda_y)s} ds \end{aligned} \tag{43}$$

Proof of property 1c We have two cases:

1. If $T_y^* > T_o^*$, then $J_o(0, T_y^*) = 0$ since the integral cannot be negative. According to $e^{gT_o^*} = \frac{x}{\omega(\theta_o)}$ and $e^{gT_y^*} = \frac{x}{\omega(\theta_y)}$ we should then find that $\theta_y^* < \theta_o^*$, which implies $J_y^* < J_o^*$ given the free entry condition. This is not possible given equations (42) and (43) for $\lambda_y \geq \lambda_o$, because $\frac{\partial J_i^*}{\partial T_i^*} > 0$. Then, this case does not correspond to the equilibrium.

2. At the opposite, if $T_y^* < T_o^*$, one can have $J_o(0, T_y^*) \geq 0$. For the same value of θ ($\theta_o^* = \theta_y^*$), equations (40) and (41) imply that $T_y^* > T_o^*$ if $J_o(0, T_y^*) > 0$. Hence, a consistent equilibrium is such that $\theta_y^* > \theta_o^*$ in order to compensate the impact of $J_o(0, T_y^*) > 0$. If $\theta_y^* > \theta_o^*$, the job creation rule (equation (19)) leads to $J_y^* > J_o^*$. ■

8.2 An increase in the growth rate of technical progress

Property 2b *When the growth rate of technological progress accelerates (g increases):*

(i) There is a decrease in the equilibrium values of J_y^* and J_o^* favoring a reduction in the hiring rates of both segments.

(ii) The fall in the hiring rate of young workers is larger since these workers have a longer horizon.

(iii) Both the scrapping time of the old segment and the scrapping time of the young segment are reduced.

Proof

Part *(i)*: In none of the segments the job creation curve is affected and the only observed shift corresponds to the job destruction curve which moves downwards:

- In the old segment:

$$\frac{\partial J_o}{\partial g}|_{T_o=cte} = (1-\eta)\int_0^{T_o}(-e^{gs}\omega(\theta_o)s)e^{-(r+\delta+\lambda_o)s}ds < 0 \tag{44}$$

- In the young segment:

$$\begin{aligned}\frac{\partial J_y}{\partial g}|_{T_y=cte} &= (1-\eta)\int_0^{T_y}(-e^{gs}\omega(\theta_y)s)e^{-(r+\delta+\lambda_y)s}ds \\ &+ \lambda_y\int_0^{T_y}\frac{\partial J_o(0,s)}{\partial g}e^{-(r+\delta+\lambda_y)s}ds < 0\end{aligned} \tag{45}$$

The new equilibrium in each of the segments is characterized by lower values of J_i^* and θ_i^* for $i = y, o$, implying a fall in the hiring rates.

Part *(ii)* can be deduced from (44) and (45). Indeed, at the limit, we have $\lambda_y = \lambda_o$: in this case, $\left|\frac{\partial J_o}{\partial g}|_{T_o=cte}\right| < \left|\frac{\partial J_y}{\partial g}|_{T_y=cte}\right|$. Because we assume $\lambda_y < \lambda_o$, we have $\left|\frac{\partial J_o}{\partial g}|_{T_o=cte}\right| < \left|\frac{\partial J_y}{\partial g}|_{T_y=cte}\right|$.

We start the proof of part *(iii)* with the analysis of the old segment. The equilibrium asset value of a job occupied by an old worker is given by:

$$J_o^* = (1-\eta)x\int_0^{T_o^*}(1-e^{g(s-T_o^*)})e^{-(r+\delta+\lambda_o)s}ds$$

It is easy to show that the right hand side of the previous equation is increasing in g and T_o^*. At the same time, we have just found that the job destruction curve of the old segment shifts down when g increases, leading to a reduction in the equilibrium value J_o^* (since the job creation curve is not affected by g). Because an increase in the growth rate fosters a reduction in J_o^*, and because we know that the right hand side of (42) is increasing in g and T_o^*, then T_o^* must necessarily decrease in order to induce the reduction in the equilibrium value J_o^*. Therefore $\frac{\partial T_o^*}{\partial g} < 0$.

Concerning the young segment, the equilibrium asset value of a job is given by:

$$J_y^* = (1-\eta)x \int_0^{T_y^*} (1-e^{g(s-T_y^*)})e^{-(r+\delta+\lambda_y)s}ds + \lambda_y \int_0^{T_y^*} [J_o(0,s) - e^{g(s-T_y^*)}J_o(0,T_y^*)]e^{-(r+\delta+\lambda_y)s}ds$$

As previously, we can show that the right hand side of the equation is increasing[12] in g and T_y^*. Because the job destruction curve of the young segment shifts down when g increases, leading to a reduction in the equilibrium value J_y^*, and because we know that the right hand side of (43) is increasing in g and T_y^*, then T_y^* must necessarily decrease in order to induce the reduction in the equilibrium value J_y^*. Therefore $\frac{\partial T_y^*}{\partial g} < 0$.

8.3 An increase in the probability of retirement

Property 2c *When the probability of retirement (λ_o) increases:*

(i) *The decrease in the number of available vacancies (θ_o^* falls), yields firms to keep old workers for a longer period of time (T_o^* increases).*

(ii) *The impact on T_y^* is in principle ambiguous. However, for reasonable values of r, δ, λ_y and g, we can prove that the direct effect (given by a fall in $J_o(0,T_y)$) will dominate over the indirect one (decrease in wages), leading to a reduction in T_y^*.*

(iii) *The hiring rate of older workers decreases more than for younger.*

(iv) *The employment rate of young workers unambiguously decreases whereas the impact is more ambiguous in the old worker's case.*

---

[12] Because $s < T_y^*$ we find:

$$\begin{aligned}
\frac{\partial J_y^*}{\partial T_y^*} &= (1-\eta)x \int_0^{T_y^*} [ge^{g(s-T_y^*)}]e^{-(r+\delta+\lambda_y)s}ds \\
&+ \lambda_y \int_0^{T_y^*} [ge^{g(s-T_y^*)}J_o(0,T_y^*) - e^{g(s-T_y^*)}\frac{\partial J_o(0,T_y^*)}{\partial T_y^*}]e^{-(r+\delta+\lambda_y)s}ds > 0 \\
\frac{\partial J_y^*}{\partial g} &= (1-\eta)x \int_0^{T_y^*} [-(s-T_y^*)e^{g(s-T_y^*)}]e^{-(r+\delta+\lambda_y)s}ds \\
&+ \lambda_y \int_0^{T_y^*} [\frac{\partial J_o(0,s)}{\partial g} - (s-T_y^*)e^{g(s-T_y^*)}J_o(0,T_y^*) \\
&- e^{g(s-T_y^*)}\frac{\partial J_o(0,T_y^*)}{\partial g}]e^{-(r+\delta+\lambda_y)s}ds
\end{aligned}$$

For reasonable parameter values we can prove that $\frac{\partial J_y^*}{\partial g} > 0$

Proof

Part *(i)*: The old segment job creation curve is not affected by the variation in the probability of retirement, whereas the job destruction curve shifts down:

$$\frac{\partial J_o}{\partial \lambda_o}|_{T_o=constant} = (1-\eta)\int_0^{T_o}[x - e^{gs}\omega(\theta)](-s)e^{-(r+\delta+\lambda_o)s}ds < 0$$

The new equilibrium in the old segment will be characterized by a lower value of J_o^* and a lower value of θ_o^*. The reduction in the labor market tightness is associated to a fall in wages that yields an increase in the optimal scrapping time of old jobs ($e^{gT_o^*} = \frac{x}{\omega(\theta_o^*)}$).

Part *(ii)*: Similarly the job creation curve in the young segment is not affected whereas the job destruction curve shifts down $\frac{\partial J_y}{\partial \lambda_o}|_{T_y=constant} = \lambda_y \int_0^{T_y} \frac{\partial J_o(0,s)}{\partial \lambda_o} < 0$. In the new equilibrium the values of J_y^* and θ_y^* are lower. The impact on the optimal scrapping time is in principle ambiguous, since together with the fall in $J_o(0,T_y^*)$ there is also a decrease in wages ($e^{gT_y^*} = \frac{x}{\omega(\theta_y^*)} + \frac{\lambda_y}{(1-\eta)\omega(\theta_y^*)}J_o(0,T_y^*)$). We can though prove, from equation (43), that for reasonable parameter values, T_y^* decreases with the probability of retirement.

$$\begin{aligned} J_y^* &= (1-\eta)x\int_0^{T_y^*}[1-e^{g(s-T_y^*)}]e^{-(r+\delta+\lambda_y)s}ds \\ &+\lambda_y\int_0^{T_y^*}[J_o(0,s) - e^{g(s-T_y^*)}J_o(0,T_y^*)]e^{-(r+\delta+\lambda_y)s}ds \end{aligned}$$

The right hand side of the previous equation (equation (43)) can be proved to be always increasing[13] in T_y^* and, for reasonable parameter values, it is also increasing[14] in λ_o. Then, since we know that a rise in λ_o yields a new equilibrium characterized by a lower value of J_y^*, we must necessarily have $\frac{\partial T_y^*}{\partial \lambda_o} < 0$.

Concerning *(iii)*, we have just shown that $\frac{\partial J_o}{\partial \lambda_o} < 0$ and $\frac{\partial J_y}{\partial \lambda_o} < 0$ (from equations (38) and (39)). As previously done for the case of exogenous wages, we can compute $\frac{\partial J_o(0,s)}{\partial \lambda_o}$ at its minimum by

[13]

$$\frac{\partial J_y^*}{\partial T_y^*} = (1-\eta)x\int_0^{T_y^*}[ge^{g(s-T_y^*)}]e^{-(r+\delta+\lambda_y)s}ds + \lambda_y\int_0^{T_y^*}[ge^{g(s-T_y^*)}J_o(0,T_y^*) - ge^{g(s-T_y^*)}\frac{\partial J_o(0,T_y^*)}{\partial T_y^*}]ds > 0$$

[14]

$$\begin{aligned} &\int_0^{T_y}[\frac{\partial J_o(0,s)}{\partial \lambda_o} - e^{g(s-T_y^*)}\frac{\partial J_o(0,T_y^*)}{\partial \lambda_o}]e^{-(r+\delta+\lambda_y)s}ds = \\ &\int_0^{T_y^*}[\frac{\partial J_o(0,s)}{\partial \lambda_o}e^{-(r+\delta+\lambda_y)s}] - \frac{\partial J_o(0,T_y^*)}{\partial \lambda_o}\int_0^{T_y} e^{-(r+\delta+\lambda_y)s}e^{g(s-T_y^*)} \end{aligned}$$

where $\int_0^{T_y} e^{-(r+\delta+\lambda_y)s}e^{g(s-T_y^*)} = \frac{e^{-gT_y^*}}{r+\delta+\lambda-g}[1 - \frac{1}{e^{(r+\delta+\lambda_y-g)T_y^*}}]$. For reasonable parameter values $\frac{1}{e^{gT_y^*}} - \frac{1}{e^{(r+\delta+\lambda_y)T_y^*}} > r+\delta+\lambda_y - g$ implying that $\frac{\partial J_y^*}{\partial \lambda_o} > 0$.

setting $s = 0$:

$$\overline{\frac{\partial J_o}{\partial \lambda_o}} = \frac{\partial J_o(0,0)}{\partial \lambda_o} = \int_0^{T_o^*} (x - e^{gw}\omega(\theta_o))we^{-(r+\delta+\lambda_o)w}dw \tag{46}$$

which is independent of s. We then have the lower bound of $\frac{\partial J_y}{\partial \lambda_o}$ defined as:

$$\overline{\frac{\partial J_y}{\partial \lambda_o}} = \lambda_y \overline{\frac{\partial J_o}{\partial \lambda_o}} \frac{1 - e^{-(r+\delta+\lambda_y)T_y}}{r + \delta + \lambda_y} \tag{47}$$

where $\frac{\lambda_y}{r+\delta+\lambda_y}$ is always smaller than 1. We can then easily prove that the inequality $1 - \frac{1}{e^{(r+\delta+\lambda_y)T_y}} < 1$ always holds. The term $\overline{\frac{\partial J_o}{\partial \lambda_o}}$ is thus more negative (smaller) than $\overline{\frac{\partial J_y}{\partial \lambda_o}}$. Furthermore, $\overline{\frac{\partial J_y}{\partial \lambda_o}}$ must be more negative (smaller) than $\frac{\partial J_y}{\partial \lambda_o}$. Therefore: $0 > \frac{\partial J_y}{\partial \lambda_o} > \overline{\frac{\partial J_y}{\partial \lambda_o}} > \overline{\frac{\partial J_o}{\partial \lambda_o}}$. We can directly deduce that the hiring rate of the older worker decreases more largely than that of young workers. ■

8.4 The equilibrium situation when the possibility of updating a technology is available

The values of a job filled in the old and the young segments when renovation is possible are respectively given by:

$$J_o = \max_{T_o^R} \left\{ (1-\eta) \int_0^{T_o^R} [x - e^{gs}\omega(\theta_o)]e^{-(r+\delta+\lambda_o)s}ds + e^{-(r+\delta+\lambda_o-g)T_o^R}[J_o - I] \right\} \tag{48}$$

$$\begin{aligned} J_y = \max_{T_y^R} \Bigg\{ & (1-\eta) \int_0^{T_y^R} [x - e^{gs}\omega(\theta_y)]e^{-(r+\delta+\lambda_y)s}ds + e^{(r+\delta+\lambda_y-g)T_y^R}[J_y - I] \\ & +\lambda_y \int_0^{T_y^R} J_o(0,s)e^{-(r+\delta+\lambda_y)s}ds \Bigg\} \end{aligned} \tag{49}$$

with $J_i(t,t) = p(t)J_i$, when $\tau = t = 0$, for $i = y, o$. The optimal choices of the renovation horizon are obtained solving the right hand side problem of equations (48) and (49):

$$\omega(\theta_o) = \frac{x}{e^{gT_o^R}} - \frac{J_o - I}{(1-\eta)}(r + \delta + \lambda_o - g) \tag{50}$$

$$\omega(\theta_y) = \frac{x}{e^{gT_y^R}} - \frac{(r+\delta+\lambda_y - g)}{1-\eta}(J_y - I) + \frac{\lambda_y}{e^{gT_y^R}(1-\eta)} J_o(0, T_y^R) \tag{51}$$

Replacing in equations (48) and (49) yields:

$$I = (1-\eta)x \int_0^{T_o^R} [1 - e^{g(s-T_o^R)}]e^{-(r+\delta+\lambda_o)s}ds \tag{52}$$

$$\begin{aligned} I = {} & (1-\eta)x \int_0^{T_y^R} [1 - e^{g(s-T_y^R)}]e^{-(r+\delta+\lambda_y)s}ds \\ & +\lambda_y \int_0^{T_y^R} [J_o(0,s) - e^{g(s-T_y^R)} J_o(0, T_y^R)]e^{-(r+\delta+\lambda_y)s}ds \end{aligned} \tag{53}$$

for the old workers and for the young workers.

> **Property 3b** *The renovation horizon for old workers (T_o^R) increases with the updating cost and the probability of retirement, whereas it decreases in case of acceleration in the pace of growth*[15].

Proof See equation (52).

A low retirement age discriminates against old workers, since updating will become less probable for two reasons. First, the expected value of a job filled by an old worker (J_o^*) falls, making less likely the probability to update (the inequality $I < J_o^*$ is less likely to hold). Second, in spite of the rise in the scrapping time associated to old positions fostered by the rise in λ_o, the optimal renovation horizon increases too. As a result we may find $T_o^R \lesseqqgtr T_o^*$.

An acceleration in the rate of technological progress reduces the optimal renovation horizon as well as the optimal scrapping time. Simultaneously, the expected value of the job falls, so that we are less likely to find $I < J_o^*$. Renovation becomes therefore less probable.

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[15]For young workers, for reasonable parameter values, the implementation horizon increases with the implementation cost and decreases with the growth rate and the probability of retirement.

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