

Implications of General and Specific Productivity Growth in a Matching Model

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First Version: June 2007

This Version: February 2008

Abstract

In this paper, I explore the implications of incorporating long-run productivity growth into a labor market matching model. I allow productivity growth to be of a general or match-specific nature, and consider an environment in which risk-averse workers are matched with firms and engage in long-term employment contracts, to which the workers cannot commit. The environment gives rise to two new channels through which faster growth may reduce unemployment: intertemporal consumption smoothing and specific productivity growth. A quantitative analysis of the model shows that these two channels are able to generate a negative impact of growth on unemployment comparable to empirical estimates, in contrast to the traditional "capitalization effect". The analysis also suggests that while a greater specificity of productivity growth tends to decrease the unemployment rate and lengthen workers' tenure, it makes these variables more responsive to growth and job destruction rates. These results may potentially explain certain labor market differences among the U.S., Europe and Japan.

*I am deeply indebted to my advisor Andy Atkeson for his continuous guidance and support. I am grateful to Roger Farmer and Christian Hellwig for their invaluable suggestions and encouragement. I have also benefited from the insightful remarks of Ariel Burstein, Matthias Doepke, Hanno Lustig, Lee Ohanian, Pierre-Olivier Weill, Mark Wright, Johnathan Vogel and seminar participants at UCLA and the University of Tokyo. All remaining errors are mine. E-mail: junichif@ucla.edu

1 Introduction

Over the years, economists have proposed various determinants of the "natural", or the "equilibrium steady-state", unemployment rate. A rise in unemployment and a concurrent fall in productivity growth in OECD countries since the latter half of the 1970s led to the inclusion of trend or long-run productivity growth in the potential determinants, and several empirical literatures find its strong negative impact on unemployment¹. A traditional explanation for such a relationship within the Mortensen-Pissarides search and matching framework, which has recently become the standard theory of equilibrium unemployment, is the "capitalization effect" (e.g. Pissarides (2000)) that predicts a negative impact of long-run productivity growth on the steady-state unemployment rate. Intuitively, faster productivity growth increases the present value of output and accordingly the firms' surplus from a match, which encourages firms to post more vacancies, leading to lower unemployment. Recent work by Pissarides and Vallanti (2007), however, finds severe quantitative limitations of this channel, at least under standard assumptions.

In this paper, I examine the implications of incorporating long-run productivity growth into a search and matching model. To overcome the limitations of the capitalization effect, I add new features to a standard model. In particular, I extend the notion of human capital and allow productivity growth to be of a general or match-specific² nature, and consider an environment in which risk-averse workers are matched with firms and engage in long-term employment contracts, to which the workers cannot commit. I then analyze the optimal contract as well as the stationary equilibrium, and claim that such an environment gives rise to two new channels, intertemporal consumption smoothing and specific productivity growth, through which faster growth may reduce unemployment. The intuitions for each of these channels are as follows. First, faster growth increases the benefits of intertemporal consumption smoothing provided by matches, and accordingly the firms' surplus from a match, which increases the creation of vacancies and reduces endogenous separations, both of which serve to lower unemployment. Second, faster growth creates greater match-specific productivity, which provides firms and workers stronger incentives to continue a match by making workers' outside option relatively less attractive, which in turn reduces endogenous separations and hence the unemployment rate.

A quantitative analysis of the model shows that these two channels are able to generate a negative impact of growth on unemployment comparable to empirical estimates. Using

¹See Bruno and Sachs (1985), Blanchard and Wolfers (2000), Fitoussi et al. (2000), Ball and Moffitt (2001), Hatton (2007), and Pissarides and Vallanti (2007).

²Henceforth I use the term "specific" to imply match or firm specific, as opposed to industry or occupation specific.

the data for 20 developed countries for 1960-2000, Blanchard and Wolfers (2000) estimate that a 1% decline in the TFP growth rate leads to an increase in the unemployment rate of 0.25% to 0.7%. Pissarides and Vallanti (2007) use the data for a similar group of countries and time period, and find the effect to be 1.3% to 1.5%, under the levels of unemployment similar to those in the U.S. and in Europe³. Their theoretical model, however, finds the impact from the capitalization effect to be in the order of 0.01% under plausible parameters and the standard assumption of period-by-period Nash bargaining on wages, which is far from the estimates above⁴.

In contrast, a quantitative version of my model performs well in matching the estimates. Under reasonably calibrated parameters, the prediction of my model is in the range of 0.2% to 1.8%. Although it requires some caution to compare results from different empirical and theoretical models, this is a clear improvement over the capitalization effect. As an additional advantage over the capitalization effect, the two new channels in my model turn out to be less dependent on the assumption of an exogenous interest rate. The model also suggests that while a greater specificity of productivity growth, as defined by a larger gap between the productivity growth rate of a worker within and outside the current match, tends to decrease the unemployment rate and lengthen workers' tenure, it makes these variables more responsive to changes in growth and job destruction rates. These features of the model may potentially explain longer tenure and historically lower levels of unemployment rates in Europe and Japan compared to the U.S., as well as their individual labor market experiences, such as the surge of the unemployment rate in Europe, and changes to "lifetime employment" in Japan.

From a theoretical standpoint, the contribution of the paper lies in introducing the concept of specific productivity growth into search and matching models, as well as in examining its implications on long-term employment contracts. Specific productivity growth resembles the well-known notion of specific human capital, but is novel in that the amount of a worker's productivity lost following the termination of a match is dependent on the productivity growth rate. This may be considered as a way of formalizing Mincer

³Blanchard and Wolfers (2000) and Pissarides and Vallanti (2007) divide the TFP growth rate by the labor share to make it labor augmenting. Thus, a "1% drop in the TFP growth rate" here corresponds to a "1% drop in the growth rate of labor augmenting technology" in usual terminology. These coincide in my model as there is no physical capital, allowing a direct comparison of my results with theirs.

⁴The authors state that for the capitalization effect to fully explain their estimated impact of TFP growth on unemployment, (1) wages need to be completely insulated from labor market conditions, and (2) firms need to discount the revenues from a new match over an infinite horizon, despite the finite average duration of a match. Compromising these conditions with the optimizing behaviors of firms and workers appears to be challenging work.

(1993)’s idea that a faster productivity growth may increase investments in human capital which, to the extent that human capital is at least partially firm-specific, may strengthen workers’ attachments to firms⁵.

2 Related Literature

Besides the Mortensen-Pissarides model (Mortensen and Pissarides (1994), Pissarides (2000)), this paper is related to three large strands of literature. The first is the literature on growth and unemployment, some of which I already mentioned. Aghion and Howitt (1994) discuss the two channels through which growth may affect unemployment in a search and matching model with embodied technological change; the aforementioned capitalization effect and the opposing ”creative destruction effect”, which predicts that faster growth increases unemployment by accelerating the obsolescence of technology used in an existing match, and hence shortening the duration of a match. Pissarides and Vallanti (2007) quantitatively evaluate these channels and find that the latter effect dominates under plausible parameters, so that it has to be shut down by making the technology totally disembodied, in order for the model to yield a negative effect of growth on unemployment as they find in data. In a sense, my model pushes their conclusion one step further. Under specific productivity growth, a worker’s productivity in the current match becomes more and more efficient compared to that outside of it, which is the exact opposite of embodied technological change⁶.

The second is the literature on dynamic labor contracts under limited commitment. In particular, part of this paper follows the analysis of Chari et al. (2005), who combine a matching model with dynamic labor contracts with one-sided lack of commitment. However, they do not pursue the implications of productivity growth and, since human capital is general in their model, it does not affect the workers’ commitment problem as in mine.

The third is the literature on general and specific human capital, which dates back to Becker (1964). Wasmer (2006) incorporates the workers’ choice of general and specific human capital in a matching model, and examines under what condition one is preferred

⁵Using the U.S. sectoral data for 1968-87, he finds lower separations and incidences of unemployment in sectors that enjoyed faster long-run productivity growth. This finding complements the negative impact of growth on unemployment at the aggregate level that this paper is concerned with.

⁶Vanhala (2007) shows that a faster growth may, at least locally, lower unemployment if it also reduces the transferability of workers’ skills. His model is a variant of models with embodied technological change, and eventually obsolescence of physical capital destroys all existing matches, unlike in mine.

to the other, but in his model human capital is not related to growth. Ljungqvist and Sargent (1998, 2004, 2005a,b, 2006) and Den Haan et al. (2001) discuss the implications on unemployment of an increased "turbulence", or the probability of skill depreciation when workers become unemployed. The notion of "turbulence" resembles the loss of match-specific productivity in my model, but again the size of "turbulence" is unrelated to growth. Regarding the cross-country differences in the degree of specificity of human capital, Hashimoto and Raisian (1985) find that the returns to firm-specific tenure is higher in Japan than in the U.S., which they claim to be consistent with the larger importance of specific human capital in the former. Wasmer (2006) presumes in his discussions that skills are more job specific in Europe than in the U.S.

3 Model

3.1 General Environment

Time is discrete and goes to infinity, and there is a single perishable good. The economy is populated by a continuum of firms and workers. A worker stays in the labor market for $\bar{T}$ periods. I let t denote the period in the economy (calendar date) and T_A the "age" of an individual worker. A worker is born and begins to be matched with firms at $T_A = -1$, enters the labor force and starts consuming at $T_A = 0$, and retires at $T_A = \bar{T}$. Upon retirement, he receives the utility value V^R .⁷ Each period, new workers with mass equal to $1/\bar{T}$ of the total labor force enter the labor market and replace the retired workers. So the mass of workers in the labor market is a constant which I normalize to one. Firms can borrow or lend at a constant interest rate r , and hence discount their future profits with this rate. In contrast, workers can neither borrow nor lend, and there are no other financial markets.

Workers' period utility is described by a CRRA utility function over consumption, and workers discount future utility with β . Hence their expected lifetime utility is given by

$$E_{-1} \sum_{T_A=0}^{\bar{T}-1} \beta^{T_A} u(C_{T_A}) + \beta^{\bar{T}} V^R \quad (1)$$

where $u(c) = c^{1-\sigma}/(1-\sigma)$ and $\sigma > 0$.

Output is produced by a worker-firm pair; workers and firms are matched each period and the mass of new matches formed is described by a matching function $m(u, s)$, where

⁷The sole purpose of this assumption is to avoid the scaling problem in the numerical analysis, and the value of V^R does not affect the results.

u is the mass of unemployed workers, and s is that of vacancies posted by firms. The matching function is increasing in both arguments, and satisfies $m(0, \cdot) = m(\cdot, 0) = 0$. By posting a vacancy the firm incurs the cost Φ_t , and the vacancy lasts for one period.

Each employed worker is characterized by his age T_A and tenure T_T , or equivalently the number of periods he has been in the current match. By definition, $T_A \in \{0, \dots, \bar{T} - 1\}$ for workers in the labor market, and $T_T \in \{0, \dots, T_A\}$ for each T_A . In period t , a match between a firm and a worker produces

$$A\Psi_t(T_A, T_T) = A(1 + g)^t(1 + g)^{\alpha_T T_T - \alpha_A T_A}, \quad (2)$$

where $A \in [\underline{A}, \bar{A}]$ is the idiosyncratic productivity specific to the match, and $\Psi_t(T_A, T_T)$ is the worker's productivity to be described below. All new matches start with $A = \bar{A}$, and subsequently A remains constant with probability λ , and a new A is drawn from an *i.i.d.* distribution with probability $1 - \lambda$. This distribution has a CDF $G(A)$, which is absolutely continuous with corresponding PDF $g(A)$.

3.2 General and Specific Productivity Growth

A worker's productivity $\Psi_t(T_A, T_T)$ is a deterministic function of his characteristics (T_A, T_T) , and α_A and α_T are parameters common to all matches in the economy that govern the effects of these two variables. Throughout most of the paper I impose a restriction $\alpha_A = \alpha_T \equiv \alpha$, and contrast two cases. The first case is $\alpha = 0$, which I refer to as general productivity growth. Here productivity growth is general in the sense that it is independent of each worker's characteristics (T_A, T_T) , as in a standard neoclassical growth model. The second case is $\alpha > 0$, which I call specific productivity growth. Here $\Psi_t(T_A, T_T)$ is decreasing in age T_A ⁸ and increasing in tenure T_T , so a worker's productivity growth is affected by the accumulation of age and tenure, and is hence partially specific to the current match.

The intuition for specific productivity growth is as follows. Under rapid technology growth, the skills acquired through education are likely to face faster obsolescence, which provides younger workers, controlling for tenure, a larger edge in productivity compared to their seniors. At the same time, faster technological progress tends to require more complicated operations and increases room for on-the-job learning, which enhances the productivity growth accompanying the accumulation of tenure. The assumption $\alpha_A = \alpha_T$

⁸Medoff and Abraham (1980), Medoff and Abraham (1981), and Kotlikoff and Gokhale (1992) find from the U.S. data that age negatively affects productivity.

implies that these effects exactly offset each other, so that the workers with the same $T_A - T_T$, for example those staying in the same match since $T_A = 0$, are equally productive⁹.

Below are some important relationships that follow from the above formulation.

$$\frac{\Psi_{t+1}(T_A, T_T)}{\Psi_t(T_A, T_T)} = 1 + g \quad (3)$$

$$\frac{\Psi_{t+1}(T_A + 1, T_T + 1)}{\Psi_t(T_A, T_T)} = 1 + g \quad (4)$$

$$\frac{\Psi_{t+1}(T_A + 1, 0)}{\Psi_t(T_A, 0)} = (1 + g)^{1-\alpha} \leq 1 + g \quad (5)$$

(3) implies that the productivity of a worker with given (T_A, T_T) grows at $1 + g$. Thus, the economy grows at $1 + g$ in a stationary equilibrium, in which the distribution of workers over (A, T_A, T_T) is constant. (4) implies that a worker's productivity grows at $1 + g$ within the current match regardless of the value of α , while (5) indicates that it grows only at $(1 + g)^{1-\alpha}$ outside the current match. The gap between these two growth rates is increasing in α , and accordingly, I call α the specificity of productivity growth.

3.3 Labor Contract

I now describe the labor contract between a firm and a worker. What I refer to below as the "firm's value" is its expected present value of profits, and the "worker's value" is his expected present value of utility. When a firm and a worker with age T_A are matched at date t , they write a contract that specifies the stream of wages¹⁰ $\{C_s(\{A_z\}_{z=t+1}^s, T_A)\}_{s=t+1}^{\bar{T}-1-T_A+t}$, contingent on the history of idiosyncratic productivity A , and the value of T_A at date t . Notice that the value of T_A at date t is the sufficient statistics for the sequence of worker's productivity Ψ_t , since Ψ_t is a deterministic function of (T_A, T_T) , whose values increase by 1 each period, with $T_T = 0$ at the beginning of the match. As the good is non-storable and workers have no access to financial markets, the stream of wages is also the stream of consumption of the worker.

Firms can commit to the contract. However, each period a match is destroyed exogenously with probability γ , which I refer to as an exogenous separation. Also, a bad draw of A may lead to an endogenous separation, or equivalently dismissal of the worker, in which case the firm incurs firing costs F_t . Let $\Pi_t(A, V, T_A, T_T)$ denote the firm's value of continuing the match at date t , when it is matched with a worker with characteristics

⁹I relax this assumption in Section 6.1.

¹⁰The stream of wages starts from $t + 1$, when the match starts producing, and continues until the worker's last period in the labor force $\bar{T} - 1$, which corresponds to $\bar{T} - 1 - T_A + t$ in the calendar date.

(T_A, T_T) , the idiosyncratic productivity of the match is A , and it has the promise to provide the worker the value V . I assume that an endogenous separation occurs if and only if (i) A has changed, and (ii) the following condition holds.

$$\Pi_t(A, V_t^{un}(T_A), T_A, T_T) < -F_t \quad (6)$$

Here, $V_t^{un}(T_A)$ is the worker's value of unemployment, which equals his outside option. Making use of this condition, the firm's value of the match is given by $-F_t$ if (i) and (ii) above are satisfied, and $\Pi_t(A, V, T_A, T_T)$ otherwise.

This rule for endogenous separation is similar to the one adopted by Chari et al. (2005)¹¹, and its precise nature deserves more explanation. Clearly the firm never wishes to separate under values of A that do not satisfy (6), because for such A the firm can earn more than $-F_t$ by sustaining the match and providing the worker $V_t^{un}(T_A)$. But without assuming the separation rule above, the firm's separation decisions next period will depend on the current promised value, and when this value is large, the firm may wish to sustain the match even if (6) holds. Since such consideration largely complicates the analysis without altering the main mechanisms of the paper, I impose the separation rule above that makes the separation outcomes depend only on (A, T_A, T_T) ¹². The interpretation is that when A changes, the match ends if it is unable to provide the firm its outside option value $-F_t$, if the firm must provide the worker at least his outside option value $V_t^{un}(T_A)$. One may consider this as a restriction on the kind of commitment the firm can make¹³.

In contrast to firms, workers cannot commit to the contract. They can walk away from the current contract at any date, become unemployed, and search for new jobs. When unemployed, workers receive unemployment benefits $B_t^{un}(T_A)$ ¹⁴.

The optimal contract maximizes the firm's value of a new match, subject to the worker's participation constraints and the requirement of providing the worker the initial promised value $V_t^{new}(T_A)$, which depends on the initial value of T_A ¹⁵. $V_t^{new}(T_A)$ is deter-

¹¹Chari et al. (2005) do not include (i), but unlike in my model, in theirs a separation is never optimal when A does not change. The economic justification for (i) is that it can be difficult, due to legal restrictions or resistance of unions, to fire a worker when there is no change in his performance.

¹²I am currently working on an extension in which the firm may optimally choose whether to separate when A changes, under both one-sided and two-sided lack of commitment. Preliminary results imply that the negative impact of growth on unemployment presented in this paper is robust to such changes.

¹³If firms cannot commit at all, $\Pi_t(A, V, T_A, T_T) \geq -F_t$ must hold for any V promised in the contract. This is much stricter than (6), which only requires $\Pi_t(A, V_t^{un}(T_A), T_A, T_T) \geq -F_t$ in a sustained match.

¹⁴I do not model how unemployment benefits are financed. One may consider that the government has foreign assets with proceeds large enough to finance them, or interpret them as home production.

¹⁵If a firm and a worker of age T_A are matched at date t , the worker's initial value is $V_{t+1}^{new}(T_A + 1)$, as the employment starts at $t + 1$.

mined by the Nash bargaining, so that it maximizes the Nash product of the firm and the worker's surplus from the match, subject to their participation constraints¹⁶. Formally,

$$\begin{aligned} V_t^{new}(T_A) &= \arg \max_V \{ \Pi_t(\bar{A}, V, T_A, 0)^\theta (V - V_t^{un}(T_A))^{1-\theta} \} \\ s.t. \quad &\Pi_t(\bar{A}, V, T_A, 0) \geq 0, \quad V \geq V_t^{un}(T_A) \end{aligned} \quad (7)$$

where θ is the firm's bargaining power.

In (7), note that $A = \bar{A}$ and $T_T = 0$ for a new match. As the firm and the worker's outside options are respectively 0 and $V_t^{un}(T_A)$, $\Pi_t(\bar{A}, V, T_A, 0)$ and $V - V_t^{un}(T_A)$ are their surpluses from the match. I assume $\Pi_t(\bar{A}, V_t^{un}(T_A), T_A, 0) \geq 0$ for all t and T_A , so there always exists a V that satisfies (7), and all new matches lead to employment relationships.

3.4 Recursive Optimal Contract

I assume that the cost of posting a vacancy Φ_t , unemployment benefits B_t^{un} , and firing costs F_t all grow deterministically at rate $1 + g$, and that the value of retirement V_t^R grows at rate $(1 + g)^{1-\sigma}$. I can then find a stationary representation of the firm's problem by detrending profit, output and wage by $(1 + g)^t$, and the variables related to the workers' utility by $(1 + g)^{(1-\sigma)t}$. This allows me to formulate a recursive optimal contract in detrended variables, which I denote by lower case letters.

In this recursive formulation, the state variables for a matched firm are idiosyncratic productivity A , promised value v , and the worker's characteristics (T_A, T_T) . The worker's value of a new match is given by¹⁷

$$\begin{aligned} v^{new}(T_A) &= \arg \max_v \{ \pi(\bar{A}, v, T_A, 0)^\theta (v - v^{un}(T_A))^{1-\theta} \} \\ s.t. \quad &\pi(\bar{A}, v, T_A, 0) \geq 0, \quad v \geq v^{un}(T_A) \end{aligned} \quad (8)$$

and the separation rule (6) becomes

$$\pi(A, v^{un}(T_A), T_A, T_T) < -f. \quad (9)$$

Now, when the match continues, the firm chooses the current wage $c \geq 0$, and the worker's state-contingent promised value next period $v'(A')$, to solve:

¹⁶As the firm can commit to the contract, its participation constraint needs to be satisfied only at the beginning of the match.

¹⁷Under the setup of the paper, this condition turns out to be equivalent to (7).

$$\pi(A, v, T_A, T_T) = \max_{c, v'(A')} \left\{ A\psi(T_A, T_T) - c \right. \\ \left. + [(1 - \gamma)(1 + g)/(1 + r)] E[\hat{\pi}(A, v'(A'), T_A + 1, T_T + 1)] \right\} \quad (10)$$

$$s.t. \quad u(c) + \beta(1 + g)^{1-\sigma} E[\hat{u}(A, v'(A'), T_A + 1, T_T + 1)] = v \quad (11)$$

$$v'(A') \geq v^{un}(T_A + 1), \quad \forall A' \in [\underline{A}, \bar{A}] \quad (12)$$

The promise-keeping constraint (11) requires the firm's choice of c and $v'(A')$ to provide the worker the promised value v . Let p denote the probability that an unemployed worker finds a job. The participation constraints (12) require the promised value next period to be at least as large as the worker's value of unemployment, given by

$$v^{un}(T_A) = u(b^{un}(T_A)) + \beta(1 + g)^{1-\sigma} [pv^{new}(T_A + 1) + (1 - p)v^{un}(T_A + 1)] \quad (13) \\ , T_A \in \{0, \dots, \bar{T} - 2\}$$

$$v^{un}(\bar{T} - 1) = u(b^{un}(T_A)) + \beta(1 + g)^{1-\sigma} v^R,$$

where the latter follows as workers retire at $T_A = \bar{T}$ and receive the value v^R .

In (10) and (11), $E[\hat{\pi}]$ and $E[\hat{u}]$ are respectively the firm and the worker's (detrended) continuation value of the match, as I describe below. First, let us define a function that indicates whether (9), a necessary condition for an endogenous separation, is satisfied;

$$g^e(A, T_A, T_T) = 1 \text{ if } \pi(A, v^{un}(T_A), T_A, T_T) < -f \quad (14) \\ = 0 \text{ otherwise}$$

Then, for $T_A \in [0, \bar{T} - 2]$, the firm's continuation value of the match is obtained from its value of the match next period under three cases; A doesn't change, A changes but the match is sustained, and A changes and leads to an endogenous separation. Therefore,

$$E[\hat{\pi}(A, v'(A'), T_A + 1, T_T + 1)] \equiv \lambda \pi(A, v'(A), T_A + 1, T_T + 1) \\ + (1 - \lambda) \left(\int_{\underline{A}}^{\bar{A}} [(1 - g^e(A', T_A + 1, T_T + 1)) \pi(A', v'(A'), T_A + 1, T_T + 1) \right. \\ \left. + g^e(A', T_A + 1, T_T + 1))(-f)] dG(A') \right) \quad (15)$$

Similarly, the worker's continuation value of the match is computed from the value he receives next period, which equals the state-contingent value provided by the firm if the match continues, and the value of unemployment if it ends due to an exogenous or

endogenous separation. Thus,

$$\begin{aligned}
& E[\hat{u}(A, v'(A'), T_A + 1, T_T + 1)] \\
& \equiv (1 - \gamma) \left[\lambda v'(A) + (1 - \lambda) \int_{\underline{A}}^{\bar{A}} (1 - g^e(A', T_A + 1, T_T + 1)) v'(A') dG(A') \right] \\
& + \left[(1 - \gamma)(1 - \lambda) \int_{\underline{A}}^{\bar{A}} g^e(A', T_A + 1, T_T + 1) dG(A') + \gamma \right] v^{un}(T_A + 1)
\end{aligned} \tag{16}$$

For $T_A = \bar{T} - 1$, $E[\hat{\pi}] = 0$ and $E[\hat{u}] = v^R$ because the worker retires next period, which provides zero value to the firm and v^R to the worker.

3.5 Stationary Recursive Equilibrium

To close the model, I impose a zero profit condition for posting a vacancy. Let q denote the probability of filling a vacancy. Both p and q must be consistent with the matching function, so that

$$p = \frac{m(u, s)}{u}, \quad q = \frac{m(u, s)}{s} \tag{17}$$

The zero profit condition is then given by

$$\phi = q \frac{1 + g}{1 + r} \frac{\mu^{nb} \pi^{new}(0) + \sum_{T_A=0}^{\bar{T}-2} \mu^{un}(T_A) \pi^{new}(T_A + 1)}{\mu^{nb} + \sum_{T_A=0}^{\bar{T}-2} \mu^{un}(T_A)} \tag{18}$$

where $\pi^{new}(T_A)$ is the firm's value of a new match with a worker with age T_A , defined as

$$\pi^{new}(T_A) \equiv \pi(\bar{A}, v^{new}(T_A), T_A, 0), \quad T_A \in \{0, \dots, \bar{T} - 1\},$$

and $\mu^{un}(T_A)$ and $\mu^{nb} = 1/\bar{T}$ are the distributions of unemployed and new born workers.

I now define a stationary recursive equilibrium of the model.

Definition 1 *A stationary recursive equilibrium is*

- *A list of functions*

$$\pi(A, v, T_A, T_T), \quad g^e(A, T_A, T_T), \quad g^c(A, v, T_A, T_T), \quad g^{v'(A')}(A, v, T_A, T_T)$$

- $\bar{T}$ *vectors* v^{un} *and* v^{new}
- *Probabilities* p *and* q

- *Distributions of workers $\mu^{un}(T_A)$ and $\mu^{em}(A, v, T_A, T_T)$ for $T_A \in \{0, \dots, \bar{T} - 1\}$, $T_T \in \{0, \dots, T_A\}$ such that:*
 1. The value function $\pi(A, v, T_A, T_T)$ solves the Bellman equation (10), $g^e(A, T_A, T_T)$ is as defined in (14), and $g^c(A, v, T_A, T_T)$ and $g^{v'(A')}(A, v, T_A, T_T)$ are the optimal policy rules for the current wage and the promised value next period.
 2. The values of unemployed workers v^{un} are given by (13).
 3. The values of new workers v^{new} are determined by the Nash bargaining problem (8).
 4. A zero profit condition for posting a vacancy (18) holds.
 5. The probabilities of finding a job, p , and of filling a vacancy, q , satisfy (17).
 6. Distributions of unemployed workers $\mu^{un}(T_A)$ and of employed workers $\mu^{em}(A, v, T_A, T_T)$ are stationary¹⁸.

4 Analytical Results from the Model

In this section and the next, I present the implications of the model. One drawback of moving away from the typical assumption in the search and matching framework of risk-neutral workers and period-by-period Nash bargaining on wages is that, analytical results become hard to obtain. Accordingly, most of my results are based on a numerical analysis, but there are several implications of the model I am able to obtain analytically, which I present below. These turn out to be important in understanding the mechanism underlying the quantitative results.

4.1 Wage Rule

The first set of results concerns the path of wages in a match, which are important in relation to my later discussion of the intertemporal consumption smoothing channel.

Proposition 2 *(1) The path of the wage c is described as*

$$\begin{aligned} c' &= c[\beta(1+r)]^{1/\sigma}/(1+g) \quad \text{if the participation constraint doesn't bind} \\ &\geq c[\beta(1+r)]^{1/\sigma}/(1+g) \quad \text{if the participation constraint binds} \end{aligned}$$

(2) Conditional on the continuation of the match, the wage and the promised value next period are independent of the value of A next period.

¹⁸Appendix B describes the precise laws of motion that these distributions must satisfy.

Proof. See Appendix A. ■

Proposition 2(1) implies that when the participation constraint doesn't bind, the non-detrended wage C drifts upwards or downwards with factor $[\beta(1+r)]^{1/\sigma}$; as a special case, C remains constant if $\beta(1+r) = 1$. Given a discount factor β and an interest rate r , such path of wages is least expensive for the firm to provide the worker a given promised value. But when the participation constraint binds, the firm must increase the wage and provide the worker the value of his outside option $v^{un}(T_A)$.

4.2 Threshold Productivity

The separation rule (9) leads to a threshold property for the idiosyncratic productivity.

Proposition 3 (1) *For all (T_A, T_T) , there exists a threshold for the idiosyncratic productivity A_{T_A, T_T}^* , such that an endogenous separation occurs if and only if the newly drawn A satisfies $A < A_{T_A, T_T}^*$.*

(2) *The threshold productivity A_{T_A, T_T}^* is decreasing in f if $A_{T_A, T_T}^* \in (\underline{A}, \bar{A})$.*

Proof. See Appendix. ■

Proposition 3(1) implies that the threshold property of the idiosyncratic productivity for an endogenous separation, which is a standard result in the literature, holds also in my model. Proposition 3(2) tells us that the role of firing costs in the model is to lower this threshold productivity and hence, all else equal, reduce endogenous separations.

Proposition 4 *A_{T_A, T_T}^* is non-increasing in tenure T_T .*

Proof. See Appendix. ■

Proposition 4 implies that for a given T_A , the hazard rate of separation falls in tenure under specific productivity growth. This negative relationship between the separation hazard and tenure is consistent with the empirical findings in the literature, for example Mincer and Jovanovic (1982) and Pries (2004).

5 Numerical Analysis

5.1 Overview

I now move on to a numerical analysis. After calibrating the model and solving it numerically¹⁹, I first examine how the unemployment rate u and the workers' tenure,

¹⁹The computational procedure is described in Appendix C.

under different values of α , vary with the growth rate g in the stationary equilibrium. I then perform a similar experiment by varying the exogenous separation rate γ .

Recall that α , the specificity of productivity growth, tells the gap between the growth rates of a worker's productivity within and outside the current match. The aforementioned literature on the cross-country differences on the specificity of human capital suggests that one should consider α to be higher in Europe and Japan than in the U.S.

5.2 Calibration

In order to conduct the numerical analysis, I need to pick certain functional forms and parameter values. I follow common practice in the literature and set the firm's bargaining power θ to 0.5. I assume that when the value of A changes, a new A is drawn from a uniform distribution with support $[\underline{A}, \bar{A}]$. I then normalize $\underline{A}$ to 1 and set $\bar{A}$ to 3^{20} .

I let one model period correspond to one quarter. I follow Chari et al. (2005) and assume that the matching function is given by a Cobb-Douglas function $m(u, s) = Bu^\varkappa s^{1-\varkappa}$, with $\varkappa = 0.4$ and $B = 0.776$. I set the coefficient of risk aversion σ to 2, and the discount factor β to 0.99. I then choose r such that $\beta(1+r) = 1$, which is the standard choice in a small open economy model. I let $\bar{T} = 160$, so that each worker stays in the labor force for 40 years. In all examples below, I set firing costs f to 0.

I calibrate the rest of the parameters by solving the model. I pick them to match the targets from the U.S. economy below, in the baseline case of $g = 0.005$, $\alpha = 0$ and $\gamma = 0.025$. These values of g and α correspond to 2% annual and general productivity growth, and the value of γ is based on the job destruction rate in the U.S. manufacturing sector, reported in Davis et al. (1996)²¹. The calibration target 1. follows Chari et al. (2005), and 3. is close to the figure used in Shimer (2005). Under general productivity growth, I assume $b(T_A) = b_0$ for all T_A , and set b_0 so that the ratio of unemployment benefits to the average wage, averaged over T_A , approximately equals 40%.

1. u/s ratio of 1.3153, computed from the average duration of vacancy and unemployment
2. Unemployment rate of 5.5%, the average figure for the 1991-2005 period, as documented in the OECD Labor Market Statistics

²⁰These assumptions on $\underline{A}$ and $\bar{A}$ imply a standard deviation for A of $2/\sqrt{12}$, which is within the range of values used in the literature. For example, this figure equals $1/\sqrt{12}$ in Ljungqvist and Sargent (2004), $1/\sqrt{12} \sim 7.07/\sqrt{12}$ in Den Haan et al. (2005), and $15/\sqrt{12}$ in Chari et al. (2005).

²¹I compute γ from their annual job destruction rate, as they state that "the annual job flow measures provide a better indication of permanent job reallocation activity" (p18).

3. Value of unemployment benefits and home production of 40%

The procedure above leads to the following set of parameters.

Parameters Selected with no Empirical Counterpart		
θ	0.5	Firm's bargaining power
$\underline{A}$	1	Lowest level of match productivity
$\bar{A}$	3	Highest level of match productivity

Parameters Calibrated without Solving the Model		
$\varkappa$	0.4	Unemployment elasticity in matching function
B	0.7776	Scale in matching function
σ	2	Coefficient of risk aversion
β	0.99	Discount factor
r	$1/\beta - 1$	Interest rate
$\bar{T}$	160	Number of periods in labor force
f	0	Firing Costs

Parameters Selected by Solving the Model		
λ	0.835	Persistence in match productivity
ϕ	4.4437	Cost of posting a vacancy
b_0	0.82	Unemployment benefits for general productivity growth

Finally, a potential complication for the comparisons of results arises from the choice of $b(T_A)$ under specific productivity growth, as the workers' productivity varies with T_A and T_T . Making $b(T_A)$ proportional to the productivity of an unemployed worker (i.e. $T_T = 0$) makes the average replacement rate too low compared to the general productivity growth case, while setting it proportional to the productivity of a career worker (i.e. $T_T = T_A$) leads to an opposite implication. I choose to set $b(T_A) = b_0(1 + g)^{-0.8\alpha T_A}$; under this rule, the average replacement rate turns out to be slightly higher for specific productivity growth in almost all cases, which should lead, if anything, to conservative estimates of the effect of growth on unemployment under specific productivity growth.

5.3 Results on Growth and Unemployment

Two New Channels and the Capitalization Effect

The key result from the numerical analysis is that a faster growth rate leads to a substantial fall in the unemployment rate, due to the two new channels in the model.

The first is intertemporal consumption smoothing. The canonical Mortensen-Pissarides model assumes that both firms and workers have linear utility, and that the wages are determined according to period-by-period Nash bargaining. In contrast, in my model, workers have CRRA utility, and wages are determined by long-term contracts. This setup provides a match with an additional margin of creating surplus, which is to smooth worker's consumption over time. Under reasonable parameters, faster growth enlarges this margin. While the full mechanism is complicated, the intuition works as follows. Ignore the participation constraint as well as the changes in idiosyncratic productivity, and consider the general productivity growth case. As we observed earlier, the optimal wage rule is to let the non-detrended wage grow by $[\beta(1+r)]^{1/\sigma}$ each period. On the other hand, the output produced in the match as well as the worker's consumption when unemployed (i.e. unemployment benefits) grow at rate $1+g$. The larger the gap between these two growth rates, the more valuable it becomes to smooth the worker's consumption through a match. This gap expands as g rises for any $g > 0$ if $\beta(1+r) \leq 1$, and the same applies for $\beta(1+r) > 1$ when $g > [\beta(1+r)]^{1/\sigma} - 1$. In such regions of g , faster growth increases the firm's surplus of the match, which reduces unemployment by increasing the creation of vacancies and reducing endogenous separations.

The second channel is specific productivity growth, which limits the rise in the value of a worker's outside option by making his productivity outside the current match grow slower than that within it. This directly reduces endogenous separations under the separation rule (9), and furthermore, makes the participation constraint less binding. This relaxation of the worker's commitment problem enables more intertemporal consumption smoothing and serves to increase the joint, and hence the firm's, surplus of the match, which reduces unemployment through the aforementioned mechanism.

These channels improve the two shortcomings of the capitalization effect, the channel through which growth reduces unemployment in a standard matching model. First, as I present below, they generate effects that are large enough to be compared with empirical estimates. This contrasts with the result mentioned earlier that the capitalization effect is very small, when wages are determined according to the standard assumption of period-by-period Nash bargaining. Second, as I show in section 6.2, they are less dependent on the assumption of an exogenous interest rate than the capitalization effect.

Results on Growth and Unemployment

To assure that the results below are not due to the capitalization effect, I first describe what will happen in my model if I shut down the two new channels. To do this, I set the *CRRA* parameter σ to 0 to eliminate the benefits from consumption smoothing, and

set $\alpha = 0$ to make productivity growth general²². As I increase g from 0 to 0.01 under these parameters, the unemployment rate increases from 4.96% to 5.93%. This result, the opposite of what the capitalization effect predicts in a standard model, is explained as follows. Faster growth, all else equal, increases firms' profits and makes posting vacancies more attractive, which raises the job-finding rate and hence the flow out of unemployment. But a higher job-finding rate also improves the workers' outside option, which increases endogenous separations and hence the flow into unemployment. In the example above, the latter effect dominates so that a rise in g raises the unemployment rate. But since this latter effect is absent in a model with no endogenous separations, I consider below the impact of growth on unemployment through the capitalization effect to be zero rather than positive, based on the very small figures found in Pissarides and Vallanti (2007) mentioned earlier. I then follow the same practice for the impact of growth on tenure²³.

Figure 1 plots the unemployment rate, computed from the model, against the (annualized) growth rate. The three series correspond to (i) $\alpha = 0$, the baseline case of general productivity growth, and two cases with specific productivity growth, namely (ii) $\alpha = 0.1$ and (iii) $\alpha = 0.3$. For all three cases, the exogenous separation rate γ is set to 0.025. Recall from (4) and (5) that the growth rate of a worker's productivity within the current match equals g for all three cases, while that outside the current match is roughly $(1 - \alpha)g$, using the approximation $\log(1 + g) \approx g$.

In Figure 1, the unemployment rate takes the same value for all series when $g = 0$, because the production function (2) is independent of α in such a case. We observe that a higher growth rate reduces the unemployment rate in all three cases. As I indicate in the figure, the changes in the unemployment rate under general productivity growth (case (i)) are due to the intertemporal consumption smoothing channel, while the additional changes for the specific productivity growth cases ((ii) and (iii)) are generated by the specific productivity growth channel²⁴. When $\alpha = 0.1$, a fall in growth rate from 4% to 3% increases the unemployment rate by 0.19%, and that from 1% to 0% by 1.17%. When $\alpha = 0.3$, the corresponding effects are 0.15% and 1.75%. Although a direct comparison requires caution due to differences in the basic environment of the model, these results are of similar magnitudes to the empirical estimates in Blanchard and Wolfers (2000) and

²²I change b to 0.8095, λ to 0.4495, and ϕ to 1.4965 from the benchmark case, in order to satisfy the calibration targets mentioned earlier.

²³Under specific productivity growth, faster growth lowers unemployment even when $\sigma = 0$. When $\alpha = 0.1$, a rise in g from 0 to 0.01 reduces u from 4.96% to 3.98%, indicating that the specific productivity growth channel, to some extent, works independently of the intertemporal consumption smoothing channel.

²⁴Similar decomposition applies to Figures 2-6.

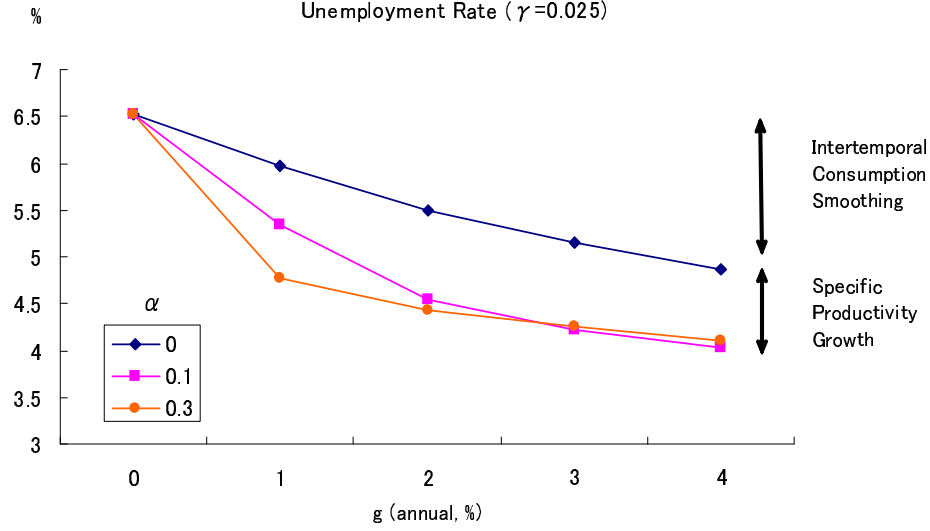


Figure 1: Growth and Unemployment

Pissarides and Vallanti (2007) reported earlier.

Notice that specific productivity growth is an “inferior” technology compared to general productivity growth, in the sense that the productivity ψ of a worker with any (T_A, T_T) is weakly smaller in the former. Figure 1 shows that such an inferior technology (cases (ii) and (iii)) may nonetheless yield lower unemployment rates through the mechanism I described earlier. Therefore, in contrast to the typical literature on general and specific human capital that assumes a trade-off between them by considering that the latter is more efficient in a given match, specific productivity growth in my model may have a positive role, even when it is only as productive as general productivity growth within a given match. It is also worth emphasizing that the results are similar in cases (ii) and (iii), when g is reasonably large. Thus, we need not be too concerned with the choice of parameter α for specific productivity growth.

Effects for Different Age Groups

As the sharp rise of youth unemployment during the European unemployment experience suggests, changes in the unemployment rate may affect different age groups of workers in different magnitudes. Thus, I now decompose the population into three different age groups, namely youth (age less than 5 years), prime-age (age equal to or larger than 5 years and less than 35 years), and old (age equal to or larger than 35 years), and

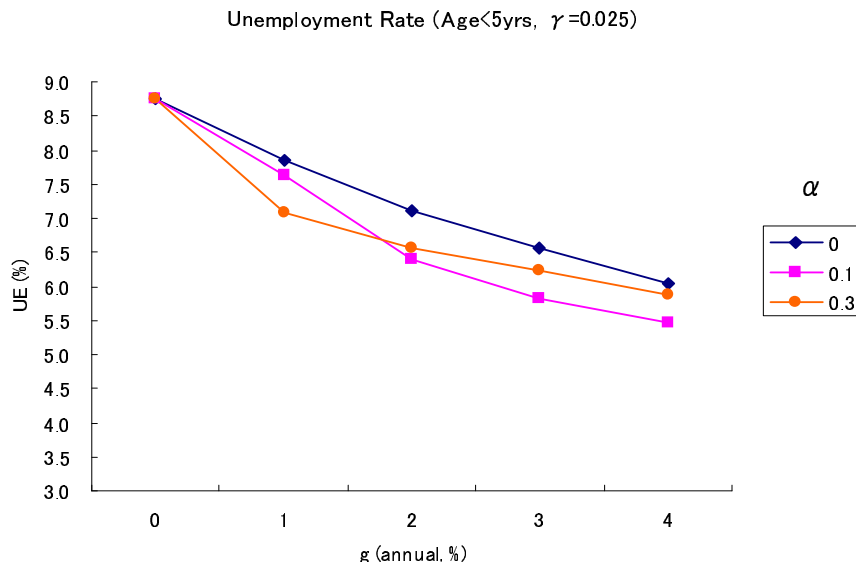


Figure 2: Growth and Unemployment (Youth, $\gamma = 0.025$)

examine the effect of growth on unemployment for the respective groups. Recall that what I call "age" is, more precisely, the time since entering the labor force, and that workers stay in the labor force for 40 years. The parameters used, including the values of α , are the same as in Figure 1.

Not surprisingly, the unemployment rates for the prime-age workers (Figure 3), who have a 75% share in the population, reveal similar responses to the changes in growth rates as the entire workforce do (Figure 1). Comparison of Figure 2-4 illustrates that the unemployment of old workers is the least affected by the changes in growth rates, especially under general productivity growth. Intuitively, the approaching retirement age largely limits the length of the match for these workers, which weakens the intertemporal consumption smoothing channel and accordingly the negative impact of growth on unemployment. In contrast, young workers not only face higher unemployment rates than older age groups, but also experience larger movements in unemployment when the growth rate varies. That the youth unemployment rates are higher and respond more in absolute terms to the variations in economic conditions than those of adult workers is in fact true in a wide range of countries, not just Europe²⁵. In the model, this result is mostly driven by the responses of the job-finding rate to the changes in growth rates; a slower growth leads to a lower job-finding rate, which mostly hurts the youth, who start

²⁵See e.g. Blanchard (2006) and O'Higgins (1997).



Figure 3: Growth and Unemployment (Prime-age, $\gamma = 0.025$)

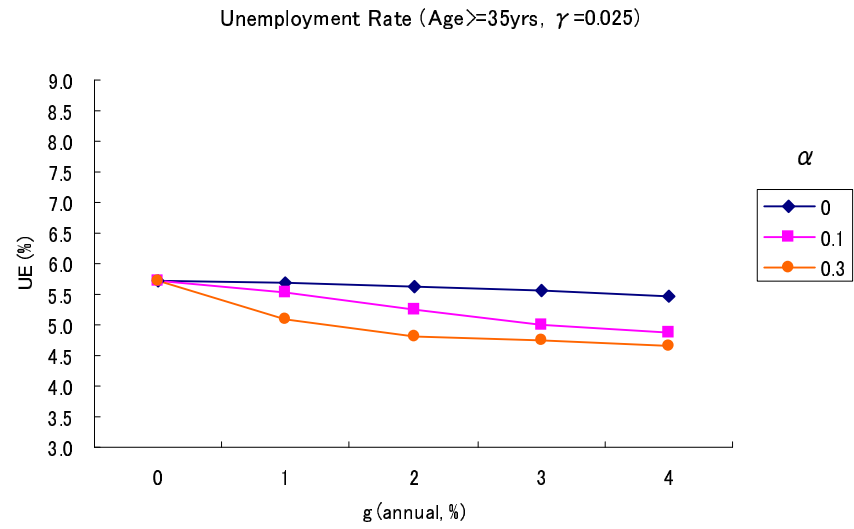


Figure 4: Growth and Unemployment (Old, $\gamma = 0.025$)

their working lives without jobs.

5.4 Results on Growth and Tenure

Next I present the model's implications on workers' tenure²⁶. The main finding is that a higher growth rate tends to lengthen workers' tenure, especially under specific productivity growth.

Figure 5 and 6 respectively plot, for $\gamma = 0.03$ and $\gamma = 0.015$, the median tenure of workers against the growth rate. These figures of γ are within a plausible range of job destruction rates for OECD countries (OECD (1996)). In each figure, as before, there are three series corresponding to $\alpha = \{0, 0.1, 0.3\}$.

The findings from these two figures are as follows. First, for all series, the median tenure rises as the growth rate rises. Second, the rise in tenure is larger for larger values of α , that is, when productivity growth is more match-specific. Third, the median tenure is much more responsive to the growth rates when γ , the exogenous separation rate, is small. The results are quite similar when I plot instead the average tenure.

The intuition for these results on tenure parallels that for unemployment. A higher growth rate increases the firm's value of a match through the intertemporal consumption smoothing channel. As we observe from (9), a larger value of the firm reduces threshold productivity A_{T_A, T_T}^* and hence endogenous separations, which translates into a longer tenure. A greater specificity of productivity growth further reduces endogenous separations by lowering the workers' value of an outside option; it directly lowers the threshold productivity A_{T_A, T_T}^* , and also tends to increase the firm's value of the match by relaxing the commitment problem of the worker, which again leads to less separations. Such effects are larger under faster growth, when the ability of specific productivity growth to relax the commitment problem, as well as the benefits from doing so, are both larger.

The model's prediction of a negative impact of growth slowdown on tenure, or more generally on job security, is interesting in view of the widespread concerns of decreased job security in the OECD countries. While there is little evidence of major declines in job security in these countries at the aggregate level (OECD (1997)), disaggregated analyses tend to find indications of reduced job security for at least some subgroups of workers over the past 20 or 30 years, during which these countries experienced slowdowns

²⁶In the baseline case of the model, the average and median tenure of employed workers are respectively 5.5 and 3.7 years. The corresponding figures for the U.S. in 1995 are 7.4 and 4.2 years (OECD (1997)), so the model underestimates workers' tenure. Such underestimation is also true in Chari et al. (2005), and it appears to be a common feature of the models that attempt to match both the unemployment rate and the job-finding rate in data.

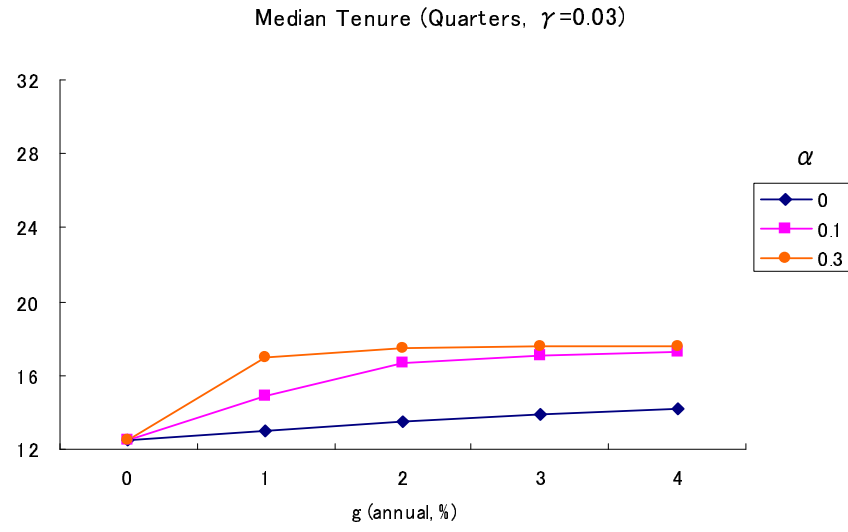


Figure 5: Growth and Median Tenure ($\gamma = 0.03$)

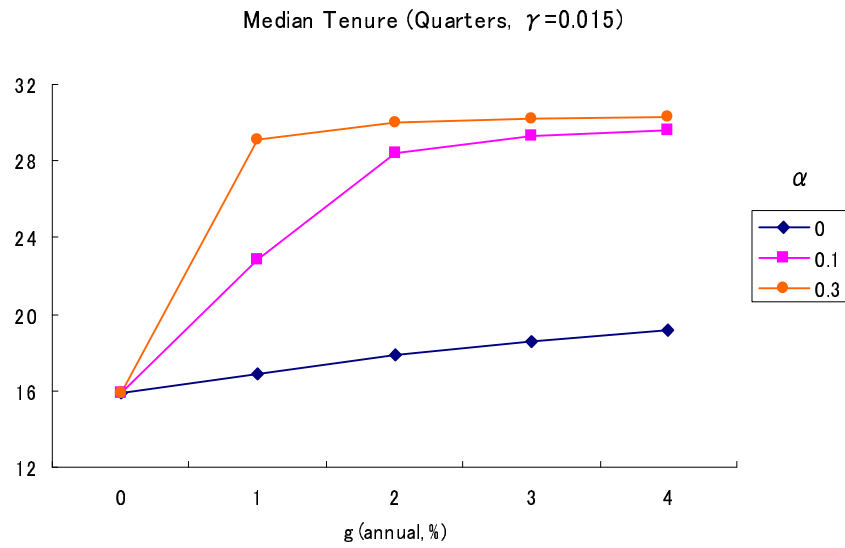


Figure 6: Growth and Median Tenure ($\gamma = 0.015$)

in productivity growth²⁷. More work is warranted to reach a definitive conclusion, but such findings appear to provide some support for the implication of the model.

5.5 Responsiveness to Growth and Exogenous Separation Rates

The results above indicate that an economy with a large value of α , corresponding to a large specificity of productivity growth, tends to enjoy lower unemployment and longer tenure of workers. It is worth emphasizing, however, that unemployment and tenure are also more responsive to changes in the growth and exogenous separation rates in such an economy. Such properties in response to the changes in g can be observed from Figure 1-6. As the values of unemployment rate and median tenure are common to all α for $g = 0$, larger variations of these variables under more match-specific productivity growth imply their larger sensitivity to g under such cases.

I show below that similar results hold with respect to the changes in exogenous separation rate γ . Figure 7 plots the unemployment rate against γ under $g = 0.005$, or 2% annual growth, again for $\alpha = \{0, 0.1, 0.3\}$. The figure again illustrates that unemployment rates are lower under specific productivity growth, and that they rise along with the exogenous separation rate. More significantly, the curves have steeper slopes for larger values of α ; as the exogenous separation rate rises, the unemployment rate rises more quickly when productivity growth is more match-specific.

Figure 8 illustrates the impact of γ on the median tenure for the same series as in Figure 7. It shows that the greater the specificity of productivity growth (i.e. the larger the value of α), the more sensitive the median tenure is to the changes in γ .

Compared to general productivity growth, specific productivity growth lowers the value of workers' outside option, which serves to decrease the unemployment rate and lengthen tenure through the mechanism I described earlier. However, under a large γ , most separations turn out to be exogenous rather than endogenous, which leaves little room for such a mechanism to work.

We can think of specific productivity growth as a costly commitment device, whose costs, which take the form of partial destruction of workers' productivity, materialize when matches terminate. A larger exogenous separation rate increases this contingency through forced termination of matches, and raises the social cost of specific productivity growth. Thus the level of the exogenous separation rate, which I identify in this paper with the

²⁷See Valletta (1999) for the analysis of changes in job security in the U.S., Gregg and Wadsworth (2002) in the U.K., Givord and Maurin (2004) in France, Bergemann and Mertens (2004) in West Germany, and Kato (2001) in Japan.

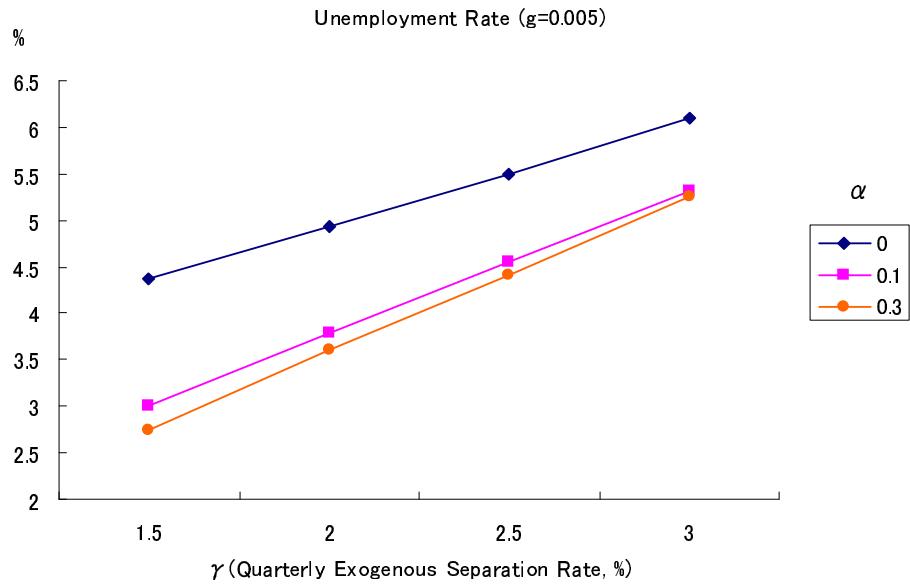


Figure 7: Exogenous Separation Rate and Unemployment

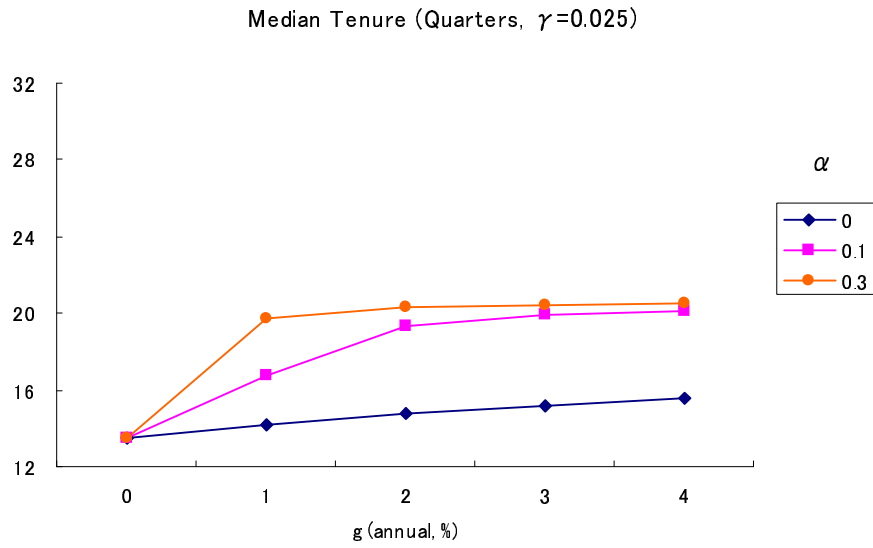


Figure 8: Exogenous Separation Rate and Tenure

empirical job destruction rate, may have a crucial impact on the relative desirability of general and specific productivity growth for the economy.

6 Extensions

6.1 Relaxing the Parameter Restriction

I have focused throughout on the case $\alpha_T = \alpha_A = \alpha$, which implies that the effects of age and tenure on productivity exactly offset each other. While this parameter configuration facilitated the comparison between general and specific productivity growth, the literature that investigates the effects of age and tenure on productivity suggests that the case $\alpha_T > \alpha_A$, which implies that older workers are more productive conditional on $T_A - T_T$, may be a more realistic description of specific productivity growth. I show what happens in such case, by allowing α_A and α_T to differ from each other, and replacing the series for $\alpha = 0.3$ in Figure 1 with the one for $(\alpha_A, \alpha_T) = (0.1, 0.3)$. These values of α_A and α_T imply that under 2% annual growth, a worker who has spent his entire career in the same match (i.e. $T_A = T_T = 159$) is approximately 27% more productive than a worker with the same age and zero tenure, and 17% more so than a worker who has just entered the labor force. This appears to be within a plausible range of the effects of age and tenure on productivity found in the literature²⁸. I also set $b(T_A) = b_0(1 + g)^{(0.2\alpha_T - \alpha_A)T_A}$, which is consistent with my assumption under $\alpha_T = \alpha_A = \alpha$.

Figure 9 shows that increasing the tenure effect on productivity amplifies the effect of growth on unemployment, as indicated by the vertical distance between the series for $(\alpha_A, \alpha_T) = (0.1, 0.1)$ and $(0.1, 0.3)$. When $(\alpha_A, \alpha_T) = (0.1, 0.3)$, a fall in growth rate from 4% to 3% increases the unemployment rate by 0.24%, and that from 1% to 0% by 1.8%.

²⁸Using the U.S. manufacturing data, Lichtenberg (1981) estimates that compared to workers with less than 6 months of tenure, those with 25 or more months of tenure are 4.2 times more productive in the durable sector, and 18.5 times so in the non-durable sector. Recent literature typically finds much more modest, yet at least initially positive, effects of tenure on productivity. The estimates from the Japanese manufacturing firms reported in Fukao et al. (2006) imply that a worker with 40 years of tenure is roughly 34% more productive than a worker with the same age and zero tenure. Hellerstein and Neumark (1995) estimate from the Israeli manufacturing data that the unskilled workers aged 35 – 54 and 55+ are respectively 20% and 36% more productive than those under 35. Since they do not control for tenure, their results should be considered as the combined effects of tenure and age. Estimates from the U.S. data using a similar method, reported in Hellerstein et al. (1999) and Hellerstein and Neumark (2004), show a much smaller productivity gain for older workers. In the latter, while workers aged 35-54 are 11% more productive than those under 35, those over 55 are 13.5% less so.

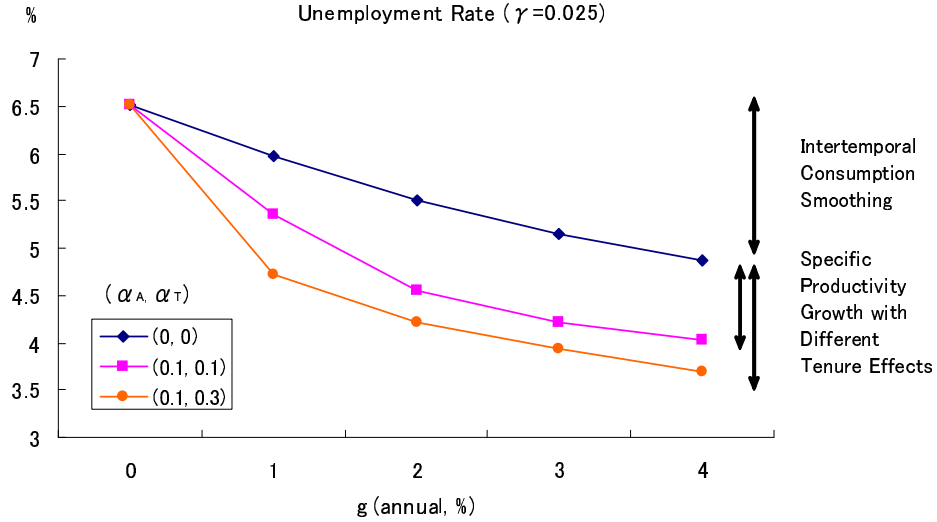


Figure 9: Growth and Unemployment with a Larger Tenure Effect

6.2 What if the Interest Rate is not Exogenous ?

As I mentioned earlier, the traditional capitalization effect depends crucially on the assumption of an exogenous interest rate. If, for example, the interest rate is endogenously determined in a closed economy populated by infinitely-lived agents with CRRA preference, $r - g$ rises when g rises, if the intertemporal elasticity of substitution is less than one. As analyzed in Pissarides (2000), in such a case, faster growth will increase the effective discount rate and reduce the present value of surplus from a match, so that the capitalization effect will be reversed. The negative capitalization effect of growth on unemployment will survive if the intertemporal elasticity of substitution is greater than one, but it will be quantitatively smaller compared to the constant interest rate case.

Some readers may hence be tempted to ask whether this assumption is also crucial in my model. While a full-blown general equilibrium analysis is beyond the scope of this paper, I argue below that the effects in this paper are likely to survive under a general equilibrium setting, although they will be reduced quantitatively.

It is important to notice that the basic structure of my model, with no accumulation of physical capital, is similar to an OLG endowment model in which the endowment exhibits the same life-cycle profile for all generations, but grows at a constant rate g across generations. As shown in Appendix D, in such an economy, the interest rate r

in the golden-rule steady state equals g^{29} . Thus, if faster growth reduces unemployment in my model under the alternative assumption $r = g$, the same is likely to be true in a general equilibrium version of the model.

I hence focus on the growth effect on unemployment, and redo my quantitative analysis for the baseline case, this time imposing $r = g$. In order to facilitate the comparison with the earlier result, I pick new values for some of the parameters so that the calibration targets are again matched for $g = 0.005^{30}$. Figure 10 compares the response of the unemployment rate under such an assumption with the earlier result obtained under a fixed interest assumption, under general productivity growth. We observe that a rise in g still reduces the unemployment rate under $r = g$, although in magnitude it is roughly half of that under the fixed interest rate assumption. This result is remarkable when we consider that the capitalization effect is exactly zero when $r = g$, as the changes in r and g exactly offset each other, and keep the present value of output unchanged.

The intuition for this result is as follows. Again ignoring the participation constraint as well as the changes in idiosyncratic productivity, the optimal path of non-detrended wages is given by

$$C_{t+1}/C_t = [\beta(1+r)]^{1/\sigma} = [\beta(1+g)]^{1/\sigma}$$

while the output and unemployment benefits grow at $1+g$. The gap between these two growth rates, $[\beta^{-1}(1+g)^{\sigma-1}]^{1/\sigma}$, is increasing in g for $\sigma > 1$. Hence if $\sigma > 1$, faster growth increases the potential benefits of intertemporal consumption smoothing through a match, thereby serving to reduce the unemployment rate.

6.3 What if Workers are Allowed to Save ?

I have assumed throughout that workers have no access to financial markets. It is natural to ask the implications of relaxing this assumption, for example, by allowing workers to save, since that will give workers an additional means for consumption smoothing and will potentially reduce the role of consumption smoothing through matches. An analysis of such an environment requires including the worker's stock of savings as an additional state variable in the firm's problem; the optimal contract then prescribes, as a function of all state variables, the amount of savings the worker should carry into the next period.

²⁹The problem can be handled similarly as the one with growth in population, rather than in endowment. There may exist other steady-state interest rates as analyzed in Gale (1973), but I will not explore this issue.

³⁰This results in changing λ to 0.8455 and ϕ to 4.8342, while using the same values as before for all other parameters.

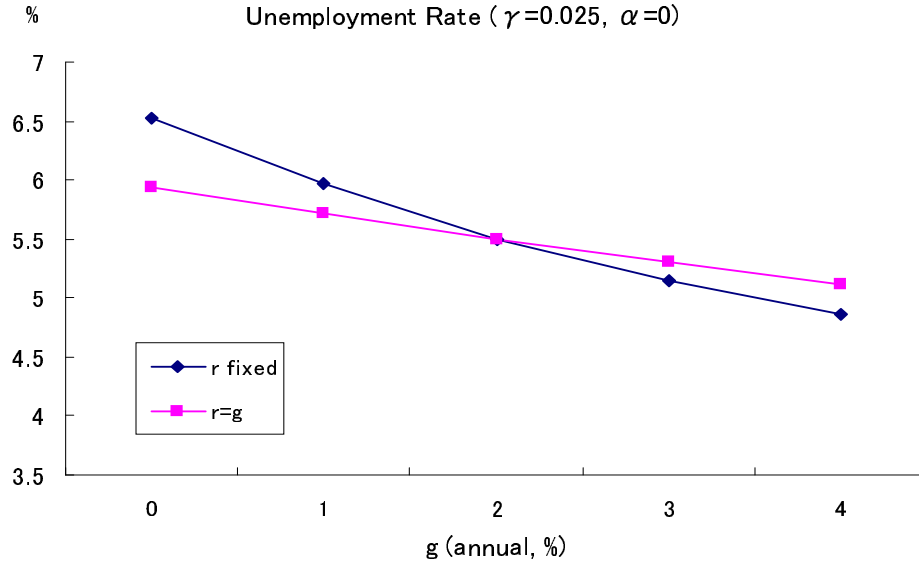


Figure 10: Growth and Unemployment (r constant vs $r = g$)

It is easy to observe that the impact of allowing for savings depends crucially on the exogenous separation rate. Suppose there are neither exogenous nor endogenous separations, so that a worker stays in the same match until retirement. In such a case, allowing workers to save is redundant when firms can borrow and lend, and hence does not affect the consumption profile in the optimal contract. This means that the workers' savings are useful only as buffer stocks against future separations. Now, in determining the amount of savings, the optimal contract weighs the opposing effects on two constraints. While increasing savings reduces the current consumption, it improves the worker's outside option next period by mitigating the drop in consumption in the contingency of unemployment. When the chances of exogenous separations are high and the gap between the worker's consumption within and outside the match is large, the latter effect dominates and relaxes the promise-keeping constraint. On the other hand, an increase in the outside option value tightens the participation constraint. Finding out the relative importance of these two effects requires a numerical analysis, which is left for future research. We can infer from the case with no possibility of separation, however, that when the exogenous separation rate is small, the costs from the latter is likely to outweigh the benefits from the former, which limits the role of savings and makes the resulting allocations, as well as the implications of growth, close to those in

this paper³¹.

7 Conclusion

In this paper, I examined the implications of incorporating long-run productivity growth into a matching model. I proposed two new channels, namely intertemporal consumption smoothing and specific productivity growth, through which faster growth may reduce unemployment. A quantitative analysis of the model indicated that under plausible parameters, these two new channels are able to generate a negative impact of growth on unemployment comparable to empirical estimates. Moreover, the analysis showed that specific productivity growth tends to (i) reduce the unemployment rate, (ii) lengthen worker's tenure, and (iii) make these variables more responsive to changes in growth and exogenous separation rates.

I believe that the framework of this model may prove useful in explaining various labor market differences and episodes across countries. For example, the model predictions are consistent with the historically lower unemployment rates and longer tenure in Europe and Japan, where it seems reasonable to assume that productivity growth was relatively more specific compared to the U.S. Moreover, the larger labor market consequences that these two regions faced concurrent with the productivity slowdown, in the form of a surge of the unemployment rate in Europe since the 1970s, and changes to the "lifetime employment" in Japan since the 1990s³², also appear to be consistent with the implications of the model. The model structure is also rich enough to allow cross-country differences in labor market institutions, such as the levels of unemployment benefits and firing costs, which may be an interesting route for future research.

One limitation of the present model is its wage implications. While the model predicts wage growth over time, it does not generate a cross-sectional wage profile that is increasing in age and tenure, as is typically found in the labor economics literature. Also, since the model does not clearly distinguish quits and layoffs, it is not well suited to address

³¹On the other hand, allowing workers to borrow is likely to have a much larger impact on the optimal contract as well as on the equilibrium allocations, because that will provide workers a means of consumption smoothing during unemployment, over which firms do not have direct control.

³²Much of the literature that investigates the effect of the economic slowdown since the 1990s on the Japanese labor market finds that, while the job security for the core workforce who are covered by the "lifetime employment" system has not declined, the fraction of young workers who manage to become part of the core workforce fell. This reflects the increase in temporary and part-time, as well as discouraged, workers among youths. For details, see Kato (2001), Hashimoto and Higuchi (2005), Ono (2006), and Shimizutani and Yokoyama (2006).

the issue of the cost of displaced workers. I conjecture that these may be improved by incorporating into the model stochastic skill appreciation while a worker is in a match, and skill depreciation when leaving it; I leave such extensions for future studies.

Appendix

Appendix A: Proof of Proposition 2

Recall the firm's problem given by equations (10)-(12) (I ignore the non-negativity constraint on c , which does not bind as $u'(0) = \infty$), and let η_1 and $\eta_2(A')$ respectively denote the Lagrange multipliers on the promise keeping and participation constraints. By setting up the Lagrangian adequately, many terms cancel out in the FOCs and the envelope conditions, leading to

$$1 = \eta_1 u'(c) \quad (19)$$

$$\frac{1+g}{1+r} \frac{\partial \pi(A', v'(A'), T_A+1, T_T+1)}{\partial v} + \beta(1+g)^{1-\sigma} \eta_1 + \eta_2(A') = 0 \quad (20)$$

$$\frac{\partial \pi(A, v, T_A, T_T)}{\partial v} = -\eta_1 \quad (21)$$

, $\forall A' \in [A_{T_A+1, T_T+1}^*, \bar{A}]$. These equations can be combined to yield

$$\eta_2(A') = \frac{1+g}{1+r} \frac{1}{u'(c')} - \beta(1+g)^{1-\sigma} \frac{1}{u'(c)} \quad (22)$$

$$= \frac{1+g}{1+r} \left(-\frac{\partial \pi(A', v'(A'), T_A+1, T_T+1)}{\partial v} \right) - \beta(1+g)^{1-\sigma} \left(-\frac{\partial \pi(A, v, T_A, T_T)}{\partial v} \right) \quad (23)$$

Therefore, whenever the participation constraint doesn't bind (i.e. $\eta_2(A') = 0$),

$$\frac{1}{1+r} \frac{1}{u'(c')} = \beta(1+g)^{-\sigma} \frac{1}{u'(c)}$$

But since $u'(c) = c^{-\sigma}$, it follows that

$$(c')^\sigma = \beta(1+r)(1+g)^{-\sigma} c^\sigma$$

or equivalently $c' = c[\beta(1+r)]^{\frac{1}{\sigma}}/(1+g)$. When the participation constraint binds, $\eta_2(A') \geq 0$ which yields $c' \geq c[\beta(1+r)]^{\frac{1}{\sigma}}/(1+g)$. This proves Proposition 2(1).

For the proof of Proposition 2(2), first note that in the firm's problem given by (10)-(12), A affects the current and future output, but not the cost of providing a given promised value v , so that the optimal choice of c must be independent of A . Since (19) and (21) imply

$$\frac{\partial \pi(A, v, T_A, T_T)}{\partial v} = -\frac{1}{u'(c)},$$

then, $\frac{\partial \pi(A, v, T_A, T_T)}{\partial v}$ must be independent of A . Thus, c' and $\frac{\partial \pi(A', v'(A'), T_A+1, T_T+1)}{\partial v}$ are independent of A' .

Now, consider the case in which the participation constraint doesn't bind next period. In this case, $v'(A')$ is the solution to (23), where $\eta_2(A') = 0$. Noting that $\frac{\partial \pi(A', v'(A'), T_A+1, T_T+1)}{\partial v}$ and $\frac{\partial \pi(A, v, T_A, T_T)}{\partial v}$ are independent of A' , it then follows that $v'(A')$ is also independent of A' .

Next, note that the RHS of (12) does not depend on A' , because the match-specific productivity does not carry beyond the current match. This observation, combined with the result above that the optimal $v'(A')$ does not depend on A' when the participation constraint does not bind, indicates that whether or not the participation constraint binds is independent of A' . Thus, when the participation constraint binds next period, it binds for all $A' \in [A_{T_A+1, T_T+1}^*, \bar{A}]$, and the wage and the promised value next period c' , $v'(A')$ are given by

$$\begin{aligned} v'(A') &= v^{un}(T_A + 1) \\ c' &= \left(-\frac{\partial \pi(A', v^{un}(T_A + 1), T_A + 1, T_T + 1)}{\partial v} \right)^{\frac{1}{\sigma}}, \end{aligned}$$

which are both independent of A' from the argument above. Therefore, in either case c' and $v'(A')$ are independent of A' , which proves Proposition 2(2). ■

Appendix B: Proof of Proposition 3

I first claim that $\pi(A, v, T_A, T_T)$ is strictly increasing in A . To see this, first note that for $T_A = \bar{T} - 1$,

$$\frac{\partial \pi(A, v, T_A, T_T)}{\partial A} = \psi(T_A, T_T) > 0$$

For $T_A = 1, 2, \dots, \bar{T} - 2$, apply the envelope theorem on (10)-(12) to obtain

$$\frac{\partial \pi(A, v, T_A, T_T)}{\partial A} = \psi(T_A, T_T) + [(1 - \gamma)(1 + g)/(1 + r)] \lambda \frac{\partial \pi(A, v'(A), T_A + 1, T_T + 1)}{\partial A}$$

, where $v'(A)$ is the optimally chosen promised value tomorrow when tomorrow's state is A . Since the first term in this equation is positive, and we can recursively confirm for $T_A = \bar{T} - 2, \bar{T} - 3, \dots$ that the second term is also positive, it follows that $\frac{\partial \pi(A, v, T_A, T_T)}{\partial A} > 0$ for all $T_A = 1, 2, \dots, \bar{T} - 1$, and hence $\pi(A, v, T_A, T_T)$ is strictly increasing in A .

Therefore the threshold A_{T_A, T_T}^* is the value that satisfies

$$\pi(A_{T_A, T_T}^*, v^{un}(T_A), T_A, T_T) = -f, \quad (24)$$

where $A_{T_A, T_T}^* = \underline{A}$ if $\pi(\underline{A}, v^{un}(T_A), T_A, T_T) > -f$ and $A_{T_A, T_T}^* = \bar{A}$ if $\pi(\bar{A}, v^{un}(T_A), T_A, T_T) < -f$, which proves Proposition 3(1). Moreover, 3(2) is immediate from the monotonicity of $\pi(A, v^{un}(T_A), T_A, T_T)$ in A . ■

Appendix C: Proof of Proposition 4

If we can show that $\pi(A, v, T_A, T_T)$ is weakly increasing in T_T , the proposition follows from the definition of A_{T_A, T_T}^* , as $\pi(A, v, T_A, T_T)$ is increasing in A . We show this by backward induction.

(i) First, let $T_A = \bar{T} - 1$. Since the worker retires next period and receives v^R , the promise keeping constraint implies $g^c(A, v, \bar{T} - 1, T_T) = u^{-1}(v - \beta(1 + g)^{1-\sigma}v^R)$. Thus,

$$\pi(A, v, \bar{T} - 1, T_T) = A\psi(\bar{T} - 1, T_T) - u^{-1}(v - \beta(1 + g)^{1-\sigma}v^R),$$

which implies that $\pi(A, v, \bar{T} - 1, T_T)$ is weakly increasing in T_T , as $\psi(\bar{T} - 1, T_T)$ is so. Moreover, $A_{\bar{T}-1, T_T}^*$ is non-increasing in T_T . Hence for $T_1 < T_2$, $\pi(A, v, \bar{T} - 1, T_1) \leq \pi(A, v, \bar{T} - 1, T_2)$ for any A and v , and $A_{\bar{T}-1, T_1}^* \geq A_{\bar{T}-1, T_2}^*$.

(ii) Next, let $T_A = \bar{T} - 2$. Recall that for the state variables $(A, v, \bar{T} - 2, T_1 - 1)$, the associated policy functions are $g^c(A, v, \bar{T} - 2, T_1 - 1)$ and $g^{v'(A')}(A, v, \bar{T} - 2, T_1 - 1)$, and the threshold productivity next period is $A_{\bar{T}-1, T_1}^*$.

Consider now the problem of the firm, when the state variables are $(A, v, \bar{T} - 2, T_2 - 1)$. In particular, first suppose that the threshold productivity next period is $A_{\bar{T}-1, T_1}^*$, instead of the true threshold $A_{\bar{T}-1, T_2}^*$. Note that the optimal policy functions under $(A, v, \bar{T} - 2, T_1 - 1)$, namely $c = g^c(A, v, \bar{T} - 2, T_1 - 1)$ and $v'(A') = g^{v'(A')}(A, v, \bar{T} - 2, T_1 - 1)$, consist a suboptimal but feasible solution for such a problem, since both the promise-keeping and the participation constraint are unchanged from the firm's problem under $(A, v, \bar{T} - 2, T_1 - 1)$. Letting $\tilde{\pi}(A, v, \bar{T} - 2, T_2 - 1)$ denote the corresponding value of continuing the match for the firm,

$$\begin{aligned} & \tilde{\pi}(A, v, \bar{T} - 2, T_2 - 1) - \pi(A, v, \bar{T} - 2, T_1 - 1) \\ &= A\{\psi(\bar{T} - 2, T_2) - \psi(\bar{T} - 2, T_1)\} \\ &+ [(1 - \gamma)(1 + g)/(1 + r)] \left\{ \lambda [\pi(A, v'(A), \bar{T} - 1, T_2) - \pi(A, v'(A), \bar{T} - 1, T_1)] \right. \\ &+ (1 - \lambda) \int_{A_{\bar{T}-1, T_1}^*}^{\bar{A}} [\pi(A', v'(A'), \bar{T} - 1, T_2) - \pi(A', v'(A'), \bar{T} - 1, T_1)] dG(A') \Big\} \\ &\geq 0 \end{aligned}$$

where the inequality follows since $\psi(\bar{T} - 2, T_2) \geq \psi(\bar{T} - 2, T_1)$, and $\pi(A, v, \bar{T} - 1, T_2) \geq \pi(A, v, \bar{T} - 1, T_1)$ as shown in (i).

Next, suppose $A_{\bar{T}-1, T_1}^*$ is replaced by the true threshold $A_{\bar{T}-1, T_2}^*$, and as before, consider the policy functions $c = g^c(A, v, \bar{T} - 2, T_1 - 1)$ and $v'(A') = g^{v'(A')}(A, v, \bar{T} - 2, T_1 - 1)$. The difference from the previous case is that this time, the match is sustained

for $A \in [A_{\bar{T}-1, T_2}^*, A_{\bar{T}-1, T_1}^*)$, while $v'(A') = v^{un}(\bar{T} - 1)$ for such values of A as before. Such a modification should weakly increase the firm's value of continuing the match, as $\pi(A, v^{un}(\bar{T} - 1), \bar{T} - 1, T_2) > -f$ for $A \in (A_{\bar{T}-1, T_2}^*, A_{\bar{T}-1, T_1}^*]$ by the definition of $A_{\bar{T}-1, T_2}^*$ and the monotonicity of $\pi(A, v, T_A, T_T)$ in A . More precisely, letting $\hat{\pi}(A, v, \bar{T} - 2, T_2 - 1)$ the resulting value of continuing the match for the firm,

$$\begin{aligned} & \hat{\pi}(A, v, \bar{T} - 2, T_2 - 1) - \tilde{\pi}(A, v, \bar{T} - 2, T_2 - 1) \\ &= [(1 - \lambda)(1 - \gamma)(1 + g)/(1 + r)] \int_{A_{\bar{T}-1, T_2}^*}^{A_{\bar{T}-1, T_1}^*} [\pi(A', v^{un}(\bar{T} - 1), \bar{T} - 1, T_2) + f] dG(A') \end{aligned}$$

so that

$$\hat{\pi}(A, v, \bar{T} - 2, T_2 - 1) \geq \tilde{\pi}(A, v, \bar{T} - 2, T_2 - 1)$$

Finally, the firm's value of continuing the match obtained using the optimal policy functions, $\pi(A, v, \bar{T} - 2, T_2 - 1)$, can be no smaller than $\hat{\pi}(A, v, \bar{T} - 2, T_2 - 1)$. Thus,

$$\pi(A, v, \bar{T} - 2, T_2 - 1) \geq \hat{\pi}(A, v, \bar{T} - 2, T_2 - 1)$$

Combining these inequalities, we obtain

$$\pi(A, v, \bar{T} - 2, T_2 - 1) \geq \pi(A, v, \bar{T} - 2, T_1 - 1)$$

so that $\pi(A, v, \bar{T} - 2, T_T)$ is non-decreasing in T_T , and $A_{\bar{T}-2, T_T}^*$ is non-increasing in T_T .

(iii) Following the same argument for $T_A = \bar{T} - 3, \bar{T} - 4, \dots, \bar{T} - 1 - T_1$ (so that $T_T = \{0, T_2 - T_1\}$), for all possible combinations of (T_1, T_2) , it follows that $\pi(A, v, T_A, T_T)$ is weakly increasing in T_T for all (T_A, T_T) , as we wished to show. This completes the proof. ■

Appendix D: Law of Motion for the Distribution of Workers

I describe below, for different values of T_A , the law of motion that the distribution of workers must satisfy in the stationary recursive equilibrium.

(i) $T_A = 0$

The unemployed workers are those who were born last period and did not find a job, so that

$$\mu^{un}(T_A) = (1 - p)\mu^{nb},$$

where $\mu^{nb} = 1/\bar{T}$.

The employed workers are those who were born last period and found a job. These workers receive $v^{new}(T_A)$ and new matches start with $A = \bar{A}$, so that

$$\begin{aligned}\mu^{em}(A, v, T_A, T_T) &= p\mu^{nb} \text{ for } (A, v) = (\bar{A}, v^{new}(T_A)) \\ &= 0 \text{ otherwise}\end{aligned}$$

(ii) $T_A \in \{1, \dots, \bar{T} - 1\}$

The unemployed workers consist of those who were unemployed last period and did not find a job, as well as those who were employed last period and experienced exogenous or endogenous separations. Therefore,

$$\begin{aligned}\mu^{un}(T_A) &= (1 - p)\mu^{un}(T_A - 1) + \sum_{T_T=1}^{T_A} \left\{ \int_{\underline{A}}^{\bar{A}} \int_V [\gamma + (1 - \gamma)(1 - \lambda) \cdot \right. \\ &\quad \left. \int_{\underline{A}}^{\bar{A}} g^e(A', T_A, T_T) dG(A')] \mu^{em}(A, v, T_A - 1, T_T - 1) dv dA \right\}\end{aligned}$$

The employed workers with $T_T = 0$ are those who were unemployed last period and found a job. Thus,

$$\begin{aligned}\mu^{em}(A, v, T_A, T_T) &= p\mu^{un}(T_A - 1) \text{ for } (A, v) = (\bar{A}, v^{new}(T_A)) \\ &= 0 \text{ otherwise}\end{aligned}$$

On the other hand, the employed workers with $T_T \in \{1, \dots, T_A\}$ are those who were employed last period and remained in the match. Therefore,

$$\begin{aligned}\mu^{em}(A, v, T_A, T_T) &= (1 - \gamma) \left\{ \lambda \int_{\{\tilde{v} | g^{v'}(A)(A, \tilde{v}, T_A - 1, T_T - 1) = v\}} \mu^{em}(A, \tilde{v}, T_A - 1, T_T - 1) d\tilde{v} + (1 - \lambda) g(A) \right. \\ &\quad \cdot (1 - g^e(A, T_A, T_T)) \int_{\underline{A}}^{\bar{A}} \int_{\{\tilde{v} | g^{v'}(A)(\tilde{A}, \tilde{v}, T_A - 1, T_T - 1) = v\}} \mu^{em}(\tilde{A}, \tilde{v}, T_A - 1, T_T - 1) d\tilde{v} d\tilde{A} \left. \right\}\end{aligned}$$

Appendix E: Computational Procedure

I solve the model following the procedure below.

1. Choose an initial guess for u/s , and compute the job-finding rate p and vacancy-filling rate q from the matching function $m(s, u) = Bu^\alpha s^{1-\alpha}$.
2. For $T_A = \bar{T} - 1$, solve the firm's problem and obtain the functions $\pi(A, v, T_A, T_T)$, $g^e(A, T_A, T_T)$, $g^c(A, v, T_A, T_T)$, $g^{v'(A')}(A, v, T_A, T_T)$, as well as the threshold productivity A_{T_A, T_T}^* , for $T_T \in \{1, \dots, T_A\}$. This is easy because the worker retires next period and receives v^R for sure. In other words, the firm just chooses the wage c to satisfy the promise keeping constraint, taking as given this value of retirement the worker receives next period. Also, use the Nash bargaining condition (8) to compute $\pi^{new}(T_A)$ and $v^{new}(T_A)$.
3. For $T_A = \bar{T} - 2$, compute $v^{un}(T_A)$ from (13) and then solve the firm's problem for $T_T \in \{1, \dots, T_A\}$, using the results obtained in 2. Also, use the Nash bargaining condition (8) to compute $\pi^{new}(T_A)$ and $v^{new}(T_A)$. Iterate this process until you obtain all relevant objects for $T_A \in \{0, \dots, \bar{T} - 1\}$ and $T_T \in \{0, \dots, T_A\}$.
4. Using the values of A_{T_A, T_T}^* and p , recursively compute the stationary distributions of employed and unemployed workers, $\mu^{em}(A, v, T_A, T_T)$ and $\mu^{un}(T_A)$, for $T_A \in \{0, \dots, \bar{T} - 1\}$ and $T_T \in \{0, \dots, T_A\}$.
5. Using the values of $\pi^{new}(T_A)$ and $\mu^{un}(T_A)$ for $T_A \in \{0, \dots, \bar{T} - 1\}$ obtained above, check the zero-profit condition (18). If not satisfied, update the guess for u/s and go back to 1. Iterate 1.~5. until convergence.

Appendix F: Equilibrium Interest Rate in an OLG Economy with Endowment Growth

Consider an OLG endowment economy in which each agent lives for $N + 1$ periods. Each generation has the same population, normalized to one. In each period there is a single perishable good, and all agents have an identical preference over the consumption of this good during the $N + 1$ periods of their life, and their period utility is expressed by a CRRA utility function. Let $t = 0$ be the initial date of the model, and call the generation born at date t as generation t .

Denote by e_j^t the endowment of generation t in the j th period of its life. I assume $e_j^{t+1} = (1 + g)e_j^t$ for all t and $j \in \{0, 1, 2, \dots, N\}$, so that the endowment exhibits the same life-cycle profile, but grows at a constant rate g across generations.

Now, suppose that agents are able to borrow from or lend to an infinitely lived financial intermediaries at an interest rate r_t , which is to be determined in equilibrium. I focus on the steady state equilibrium, so that $r_t = r$ for all t . As in Magill and Quinzii (2003), I apply a "symmetry" condition to "old" generations at date 0, so that they start off with borrowings or savings in a way similar to the later generations in the corresponding stage of life. Let us denote by $f_j^t(r)$ the excess demand function of generation t in the j th period of its life, which is unique under the assumptions above. Since agents have an identical homothetic preference and since the endowment grows at g across generations,

$$f_j^{t+1}(r) = (1 + g)f_j^t(r) \quad (25)$$

The lifetime budget constraint for generation 0 is given by

$$f_0^0(r) + \left(\frac{1}{1+r}\right)f_1^0(r) + \dots + \left(\frac{1}{1+r}\right)^N f_N^0(r) = 0 \quad (26)$$

As I only consider the steady state and as (25) holds, it suffices to consider the market clearing condition in a single date. The condition at $t = N$ is

$$f_N^0(r) + f_{N-1}^1(r) + \dots + f_0^N(r) = 0 \quad (27)$$

By substituting (25) into (27) and dividing by $(1 + g)^N$,

$$f_0^0(r) + \left(\frac{1}{1+g}\right)f_1^0(r) + \dots + \left(\frac{1}{1+g}\right)^N f_N^0(r) = 0 \quad (28)$$

Therefore, if $r = g$, the excess demand functions that satisfy (26) will satisfy (28), so that $r = g$ is an equilibrium interest rate. Moreover, this is a "golden rule" interest rate because the corresponding consumption allocation coincides with the steady state allocation that maximizes the lifetime utility of a typical generation under (25) and (28) evaluated at $r = g$, which are the aggregate feasibility constraints in the steady state.

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